Infinite distances in symmetric moduli spaces

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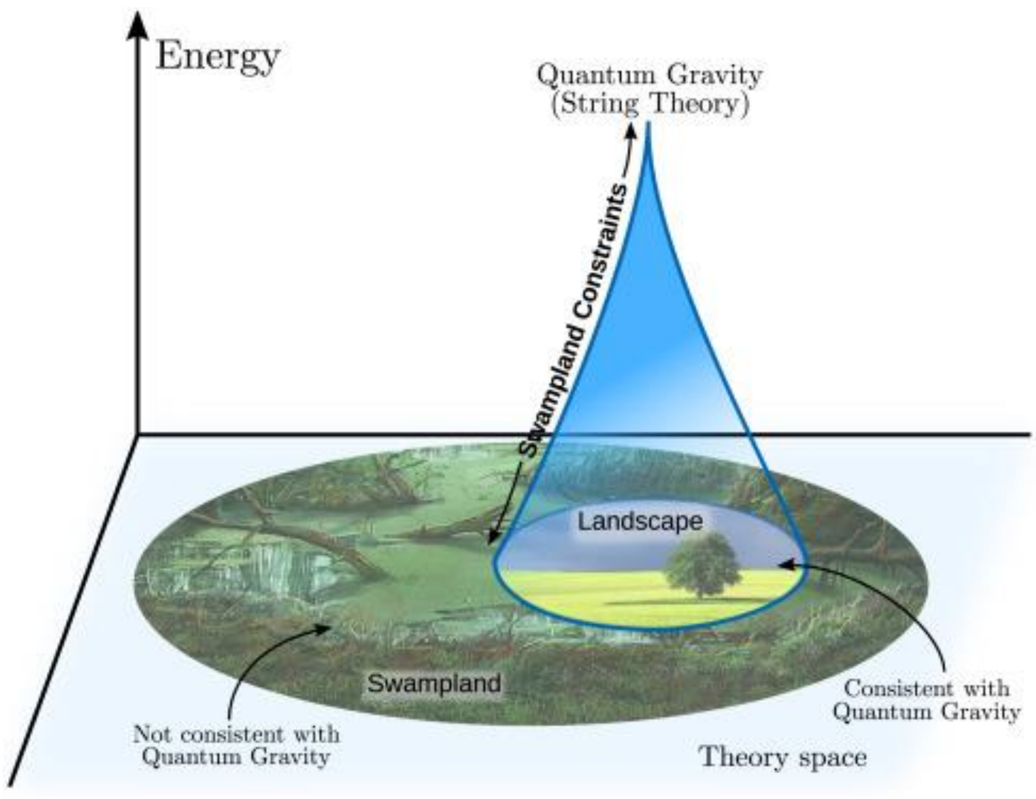
The Swampland Program

[Ooguri, Vafa, ’06]

Which are the EFTs that can be consistently coupled to Quantum Gravity?



[van Beest, Calderon-Infante, Mirfedereski, Valenzuela, ’21]

The Distance Conjecture

$$S = \frac{M_{\text{Pl}}^{d-2}}{2} \int d^d x \sqrt{-g_d} (R_d - G_{ij} \partial_\mu \phi^i \partial^\mu \phi^j)$$

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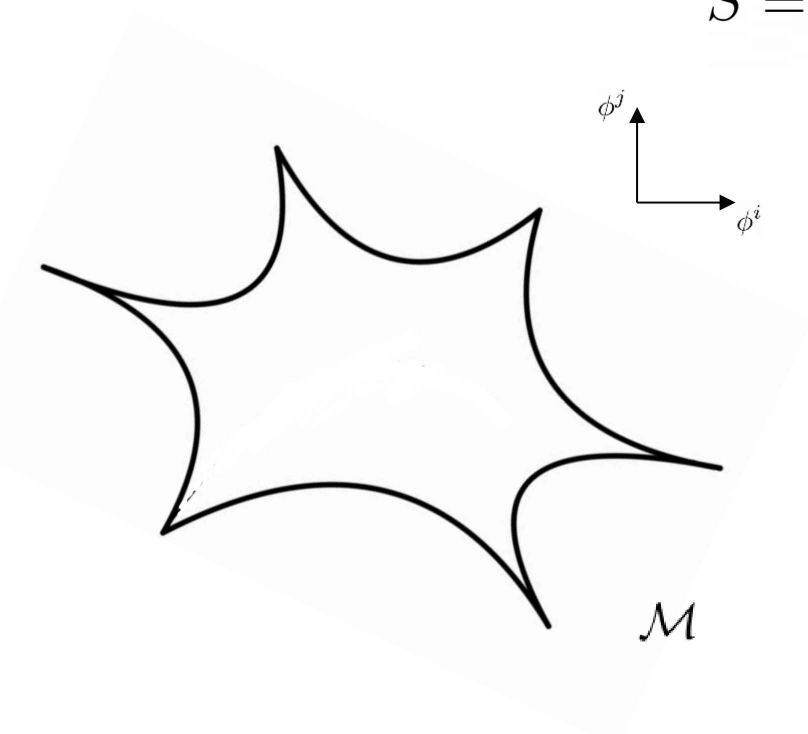

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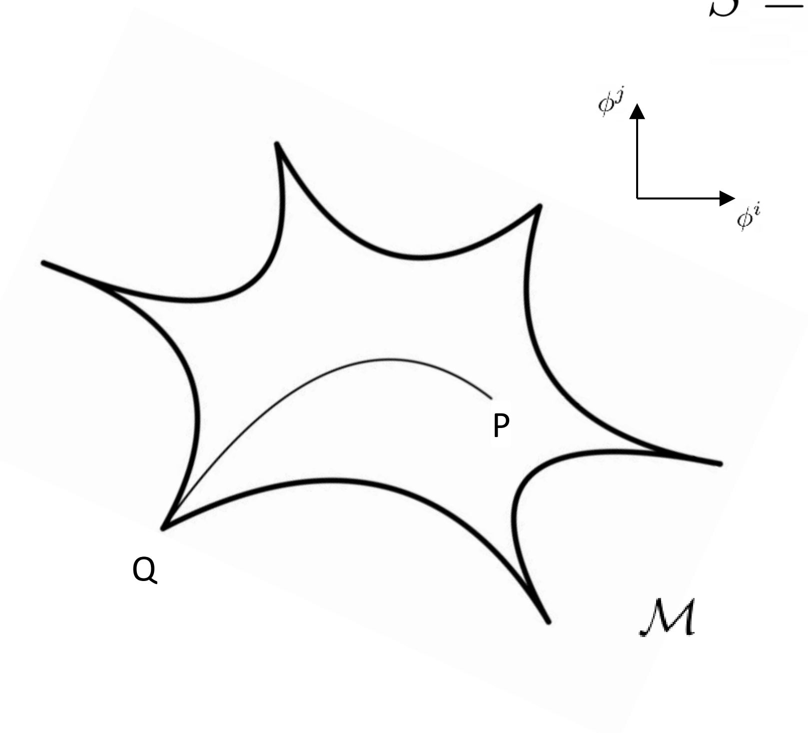

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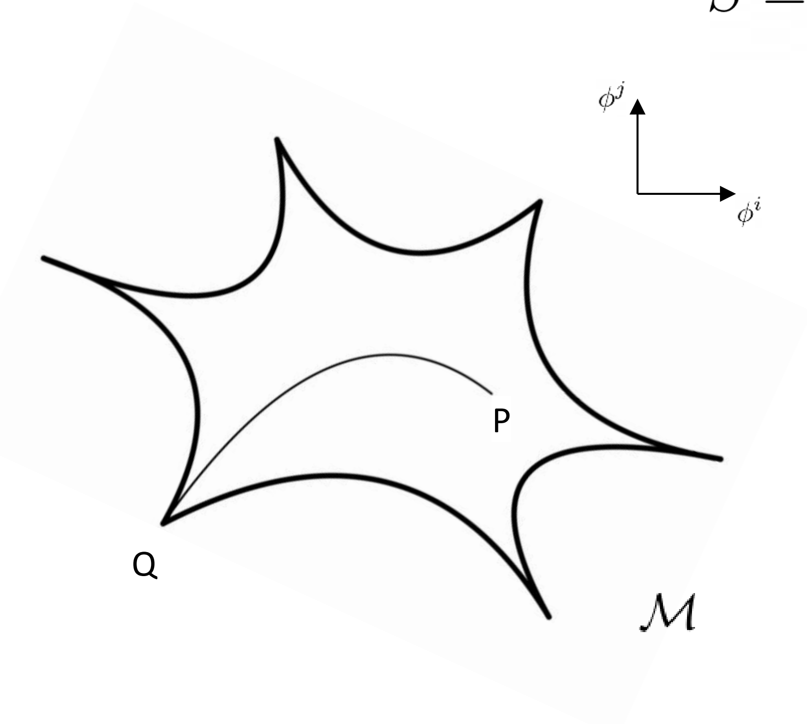

Moving in moduli space from a point P towards a point Q an infinite geodesic distance away, an infinite tower of states becomes exponentially light (in Planck units) as

$$M(Q) = M(P) e^{-\alpha d(P;Q)}$$

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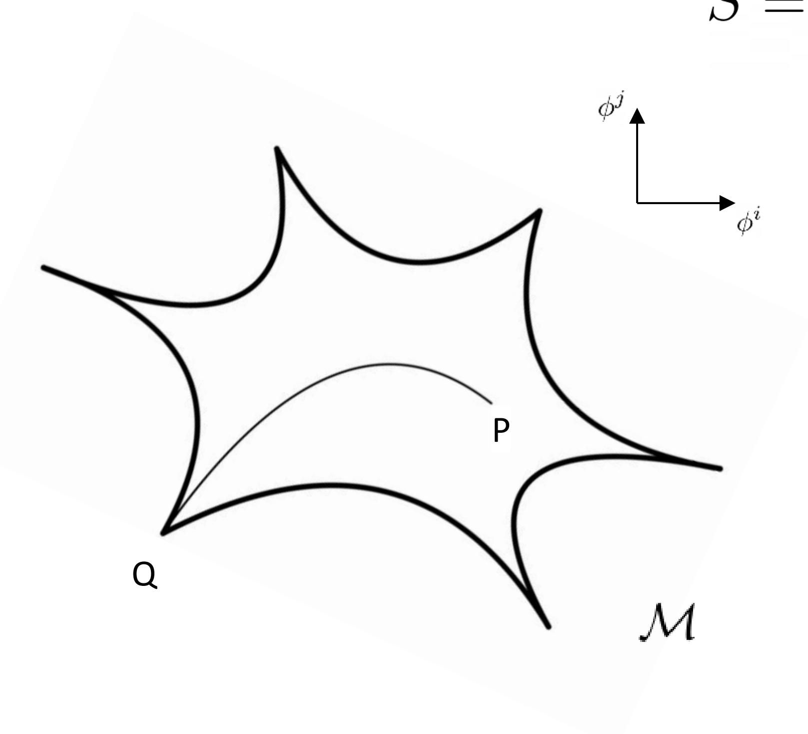

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[Baume, Blumenhagen, Buratti, Calderon-Infante, Castellano, Cecotti, Corvilain, Cribiori, Etheredge, Font, Gendler, Grimm, Heidenreich, Herraez, Ibañez, Joshi, Kaya, Klaewer, Lee, Lerche, Li, Lockhart, Lust, McNamara, Marchesano, Martucci, Montella, Montero, Ooguri, Palti, Perlmutter, Qiu, Rastelli, Rudelius, Ruiz, Stout, Uranga, Vafa, Valenzuela, van de Heisteeg, Weigand, Wiesner, Wolf, …]

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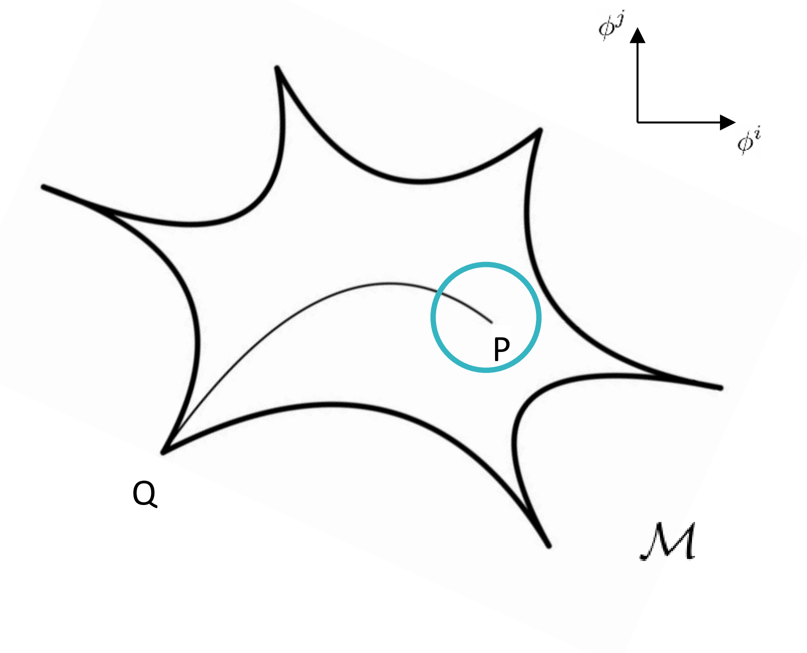

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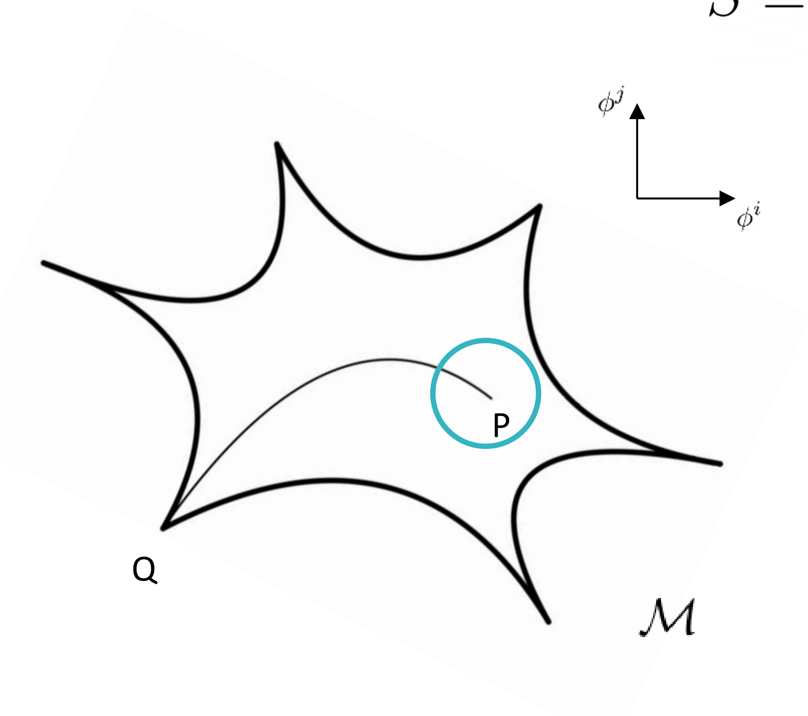

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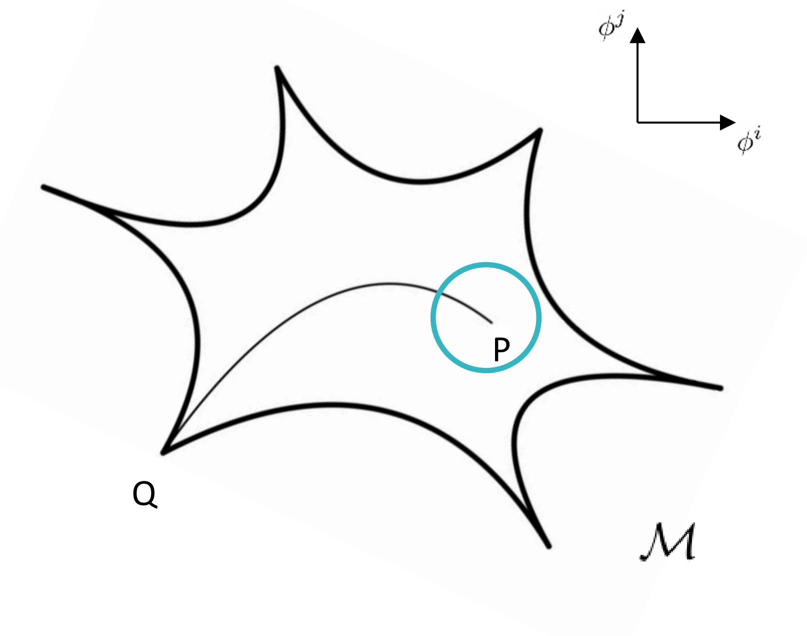

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Emergent String Conjecture [Lee, Lerche, Weigand, '19]: the tower is (dual to)

- Kaluza-Klein states
- Oscillators of a critical, weakly coupled and tensionless string

Symmetric Moduli Spaces

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- Because we like circles and symmetries [Bruno, private communication]

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- Complete and explicit characterization of boundaries/infinite distance limits

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Moduli spaces of theories with 32 or 16 supercharges

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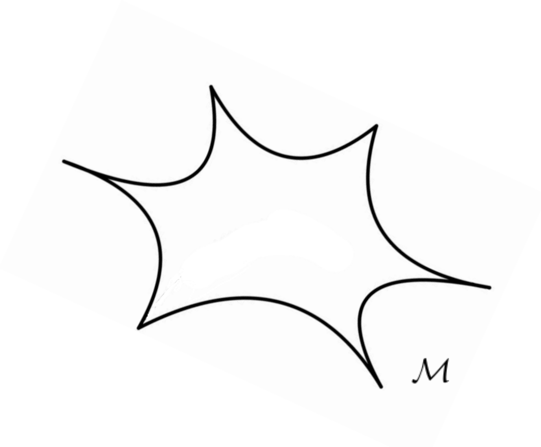

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G: (Connected) group of isometries of $\mathcal{M}$

K: Subgroup of isometries fixing one point, o

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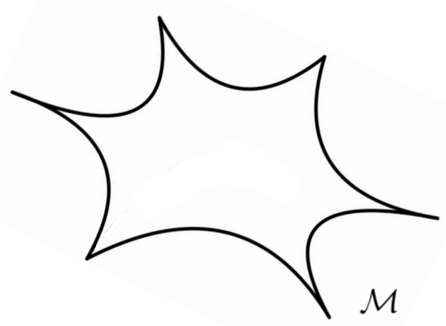

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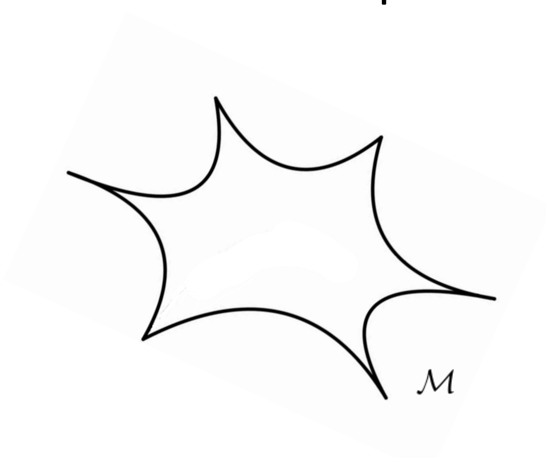

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From string theory: action of dualities $\quad G(\mathbb{Z}) \Big\backslash \frac{G(\mathbb{R})}{K}$

⟶ non-compact but finite volume [Ooguri, Vafa, '06]

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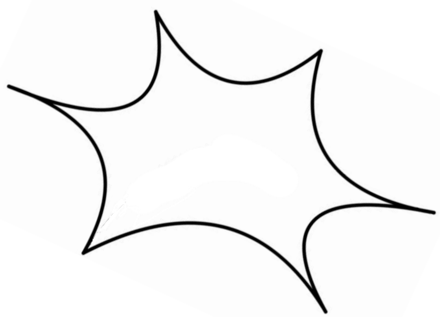
e.g. $d = 9$

$r = 17$



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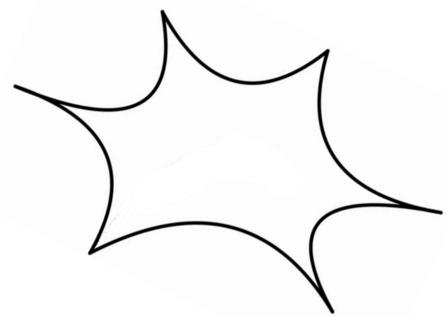

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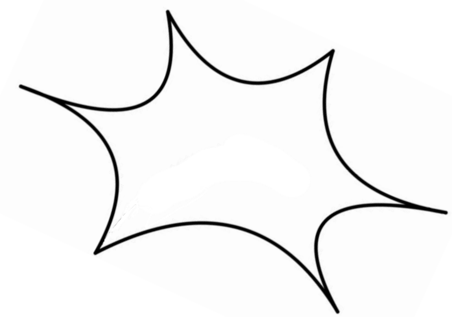

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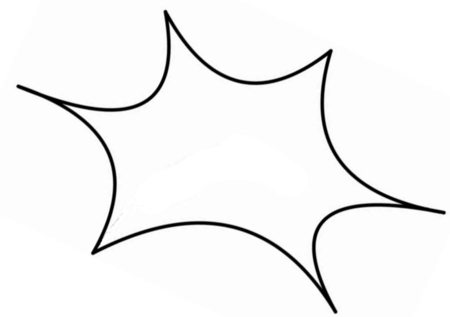

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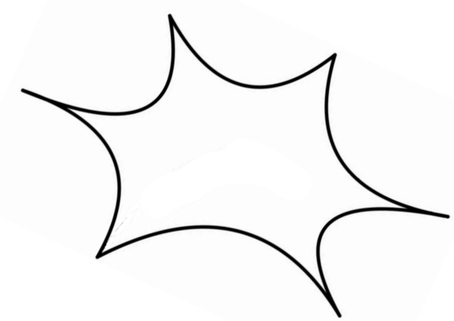

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→ characterize the symmetries of infinite distance limits

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Geometry of moduli spaces ⟷ **Spectrum** of the theory

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- Structure of the boundary

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we know how the moduli are embedded here

Non-compact moduli ⟶ Cartan generators of $\mathfrak{g}$

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⟶ follow the behavior along a geodesic

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[Borel, Ji '06]

Use the geodesic flow to study the boundary of these spaces.

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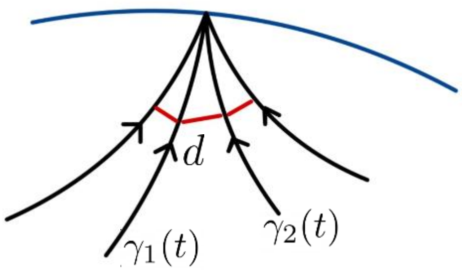

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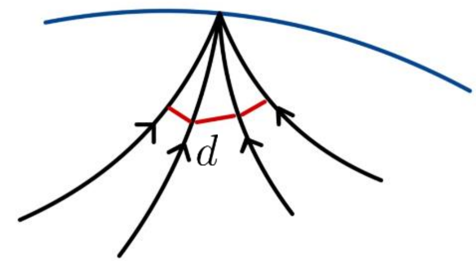

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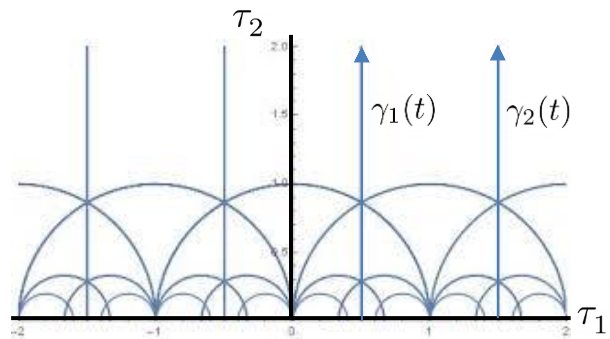

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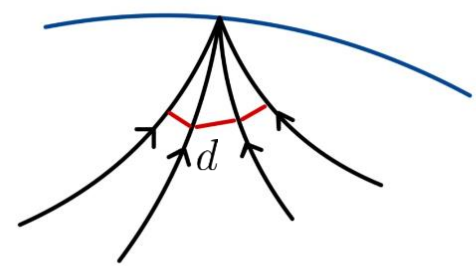

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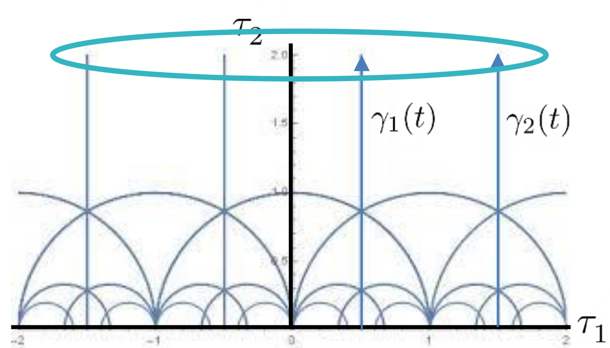

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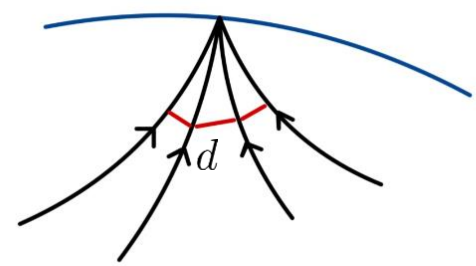

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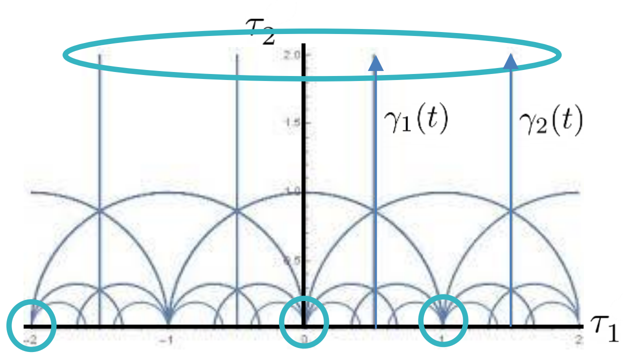

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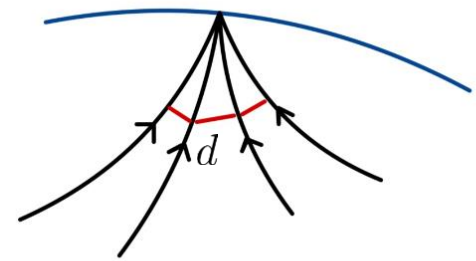

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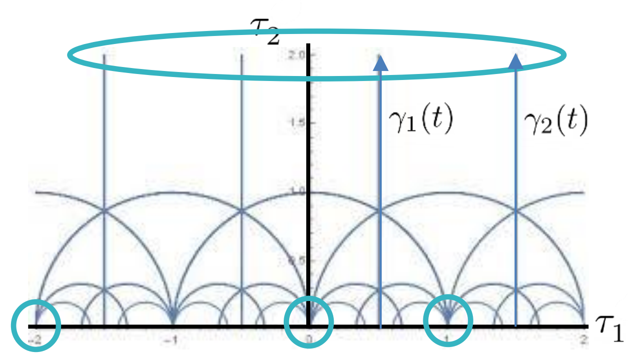

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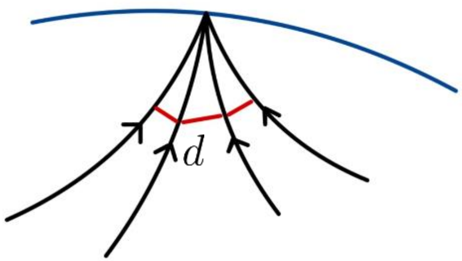

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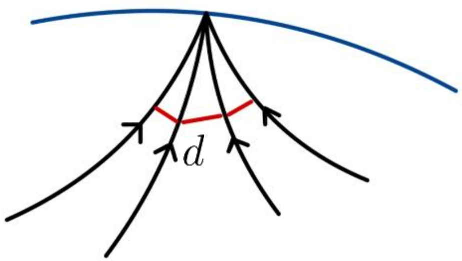

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e.g. $\frac{SL(2,\mathbb{R})}{SO(2)}$ $\qquad \tau = i\infty$ stabilised by $P = \begin{pmatrix} a & b \\ 0 & a^{-1} \end{pmatrix}$

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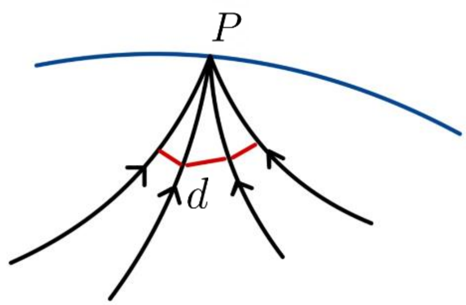
Use the geodesic flow to study the boundary of these spaces.

Geodesics (distance induced from the Killing form on $\mathfrak{g}$)

$$\gamma(t) = ge^{tX} \cdot o, \quad g \in G, \quad t \in \mathbb{R}, \quad X \in \mathfrak{g} - \mathfrak{k}$$

⟶ **points at infinity :** equivalence classes of asymptotic geodesics, with equivalence relation

$$\lim_{t\to+\infty} d(\gamma_1(t), \gamma_2(t)) < \infty$$



We can explicitely and systematically parametrize the boundary only with group theory (**parabolic subgroups** as stabilizers of a point at infinty).

e.g. $\frac{SL(2,\mathbb{R})}{SO(2)}$ $\tau = i\infty$ stabilised by $P = \begin{pmatrix} a & b \\ 0 & a^{-1} \end{pmatrix}$

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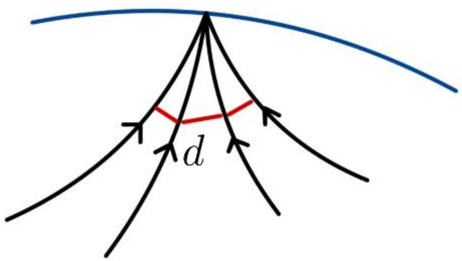

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$$[\gamma(t)] = \mathrm{diag}(e^{\lambda_1 t}, \ldots, e^{\lambda_n t})$$

→ only the non-compact moduli flow

Discrete quotient: SL(2,R)

Moduli space of Type IIB in 10 dimensions, or of T^2 at fixed volume.

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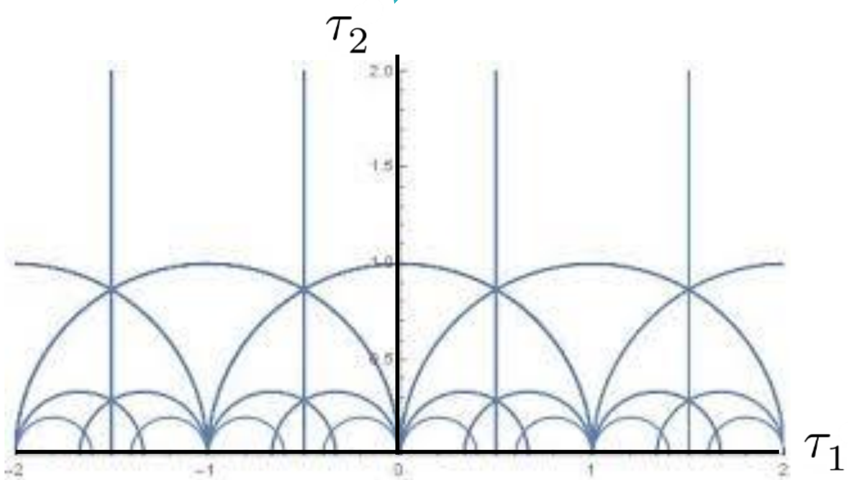
Upper half plane $\mathbb{H}^2$

$$ds^2 = \frac{d\tau_1^2 + d\tau_2^2}{\tau_2^2}$$



Boundary: $\mathbb{R} \cup \{i\infty\} \sim S^1$

Geodesics end at the boundary

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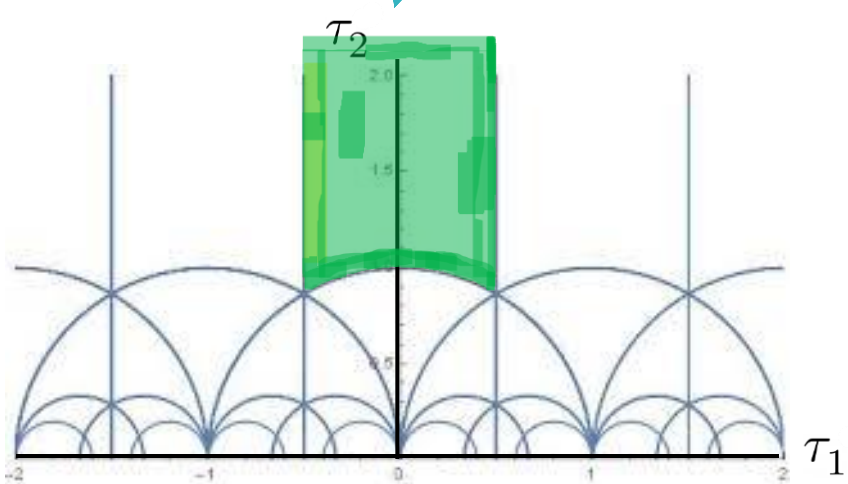

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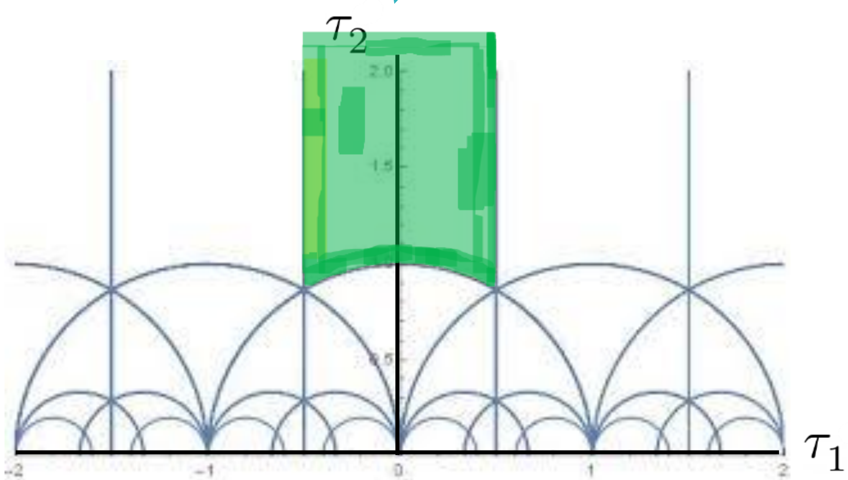

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[Keurentjes, '06]

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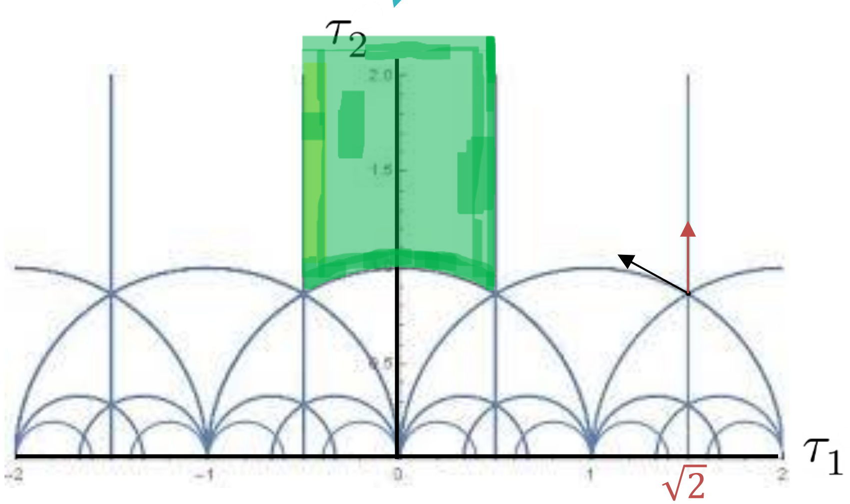

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  Still of the form

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$$\downarrow$$

$$\mathcal{H}(t)$$

Towards the Distance Conjecture

Assumptions: (motivated by string compactifications)

- Existence of a lattice of states $\Sigma \hookrightarrow V$ on which G acts

$$d_V(v,w) = v^T g^T g w, \quad v,w \in V,\ g \in \frac{G}{K}$$

- Completeness of the spectrum

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Always at least one $\lambda_i < 0$

16 supercharges & gauge symmetries

$\frac{O(d,d+16)}{O(d)\times O(d+16)}$ and $\frac{O(d,d+8)}{O(d)\times O(d+8)}$ have a heterotic description: perturbative gauge sector.

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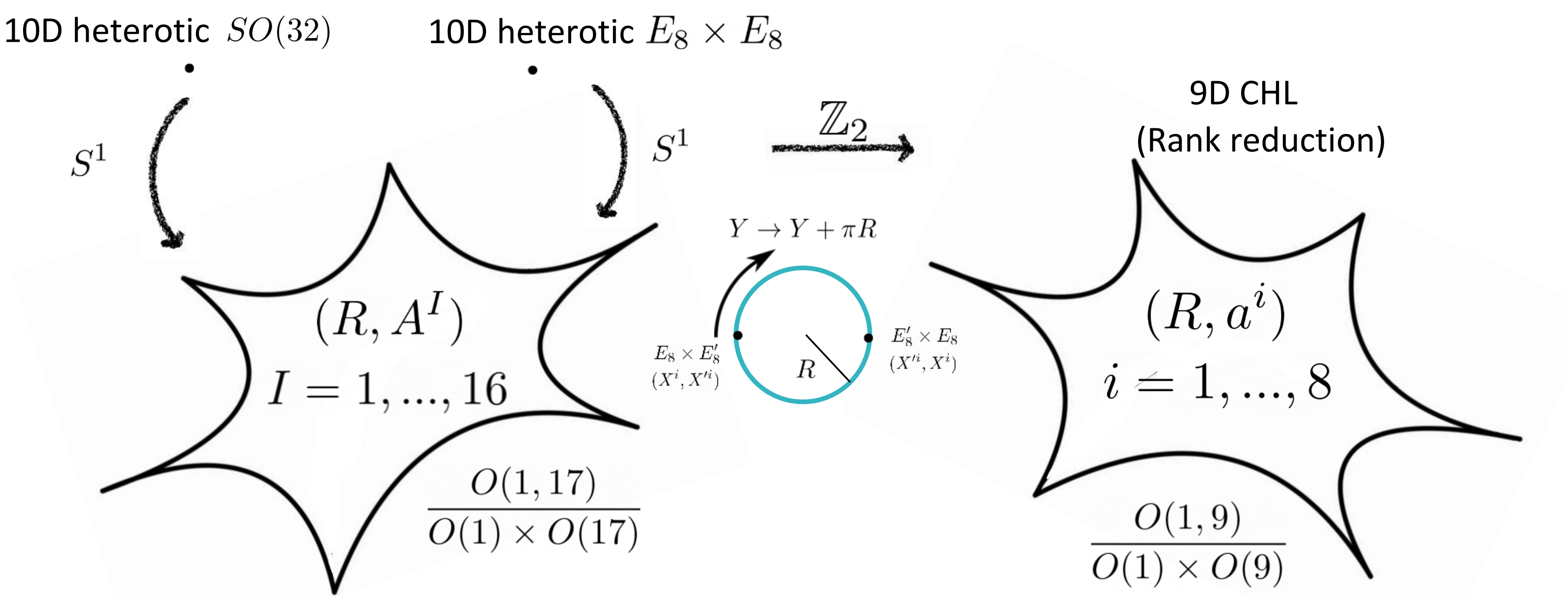

[Chaudhuri, Hockney, Lykken '95]
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[Font, Fraiman, Graña, Nuñez, Parra de Freitas ‘18-'21]

<u>Space-time</u>	<u>Worldsheet</u>
• States of the theory	• Operators on the 2d worldsheet
• Mass of the state M	• Conformal dimension $(h,\bar{h})$
• Mediator of a gauge interaction $\{A_\mu^a\}$ with $M=0$	• Current algebra $\{J^a\}$ with $(h,\bar{h})=(1,0)$

Symmetry enhancements from the worldsheet

Space-time gauge bosons (left moving sector, $M=0$)	⟷	Worldsheet currents (holomorphic, $(h,\bar{h})=(1,0)$)

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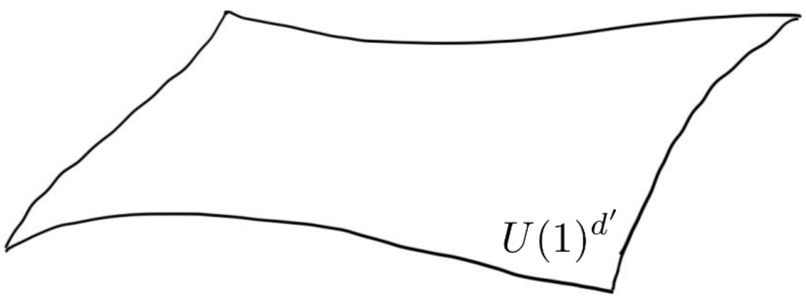

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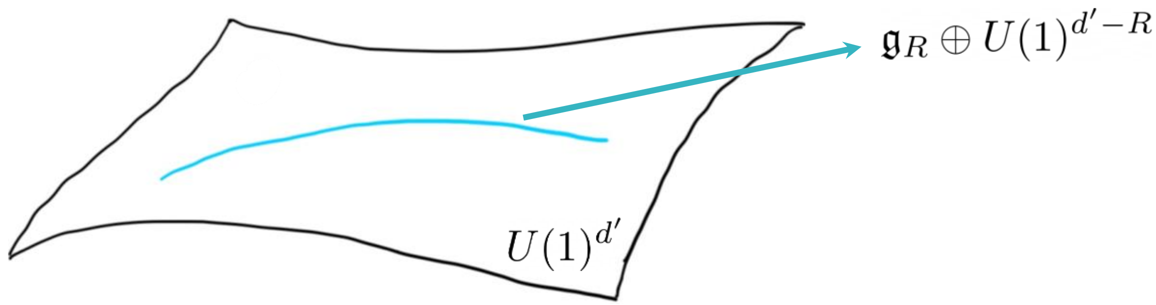

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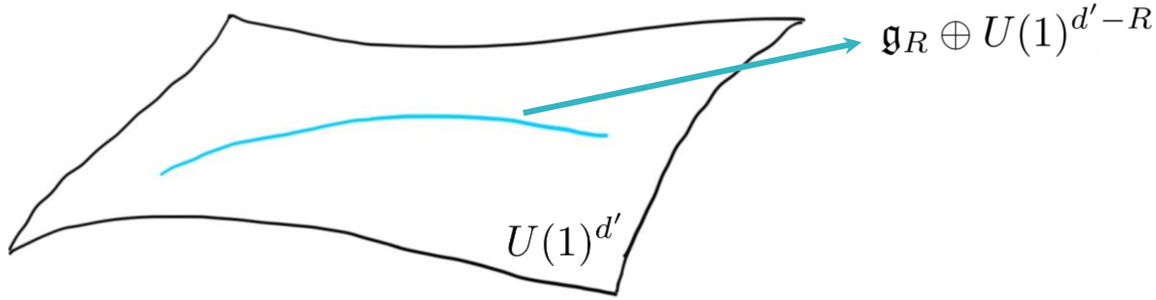

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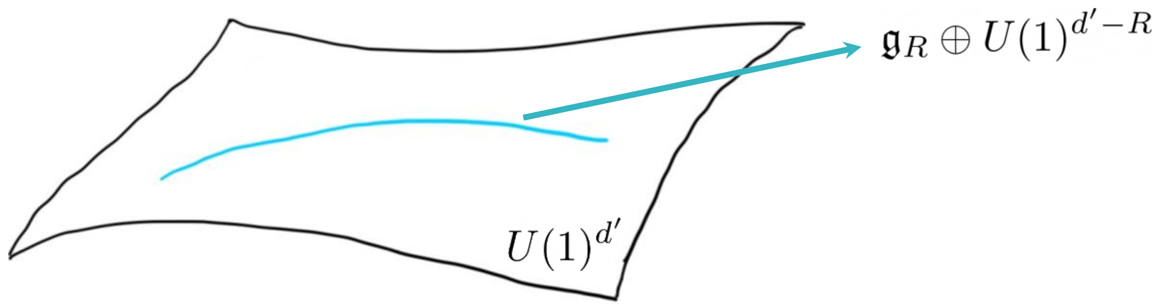

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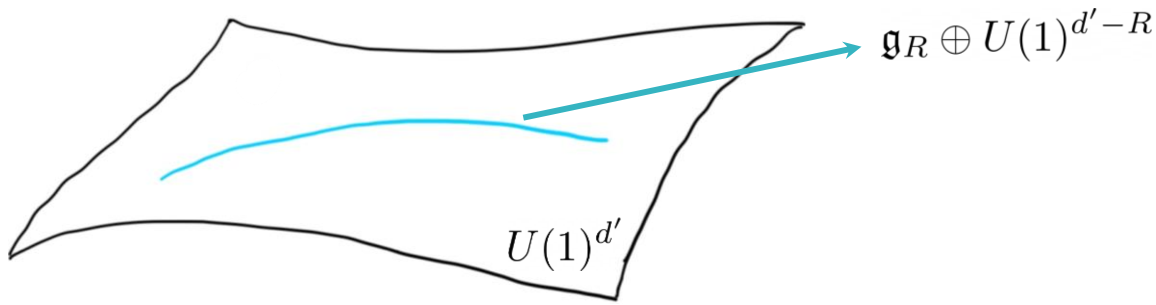

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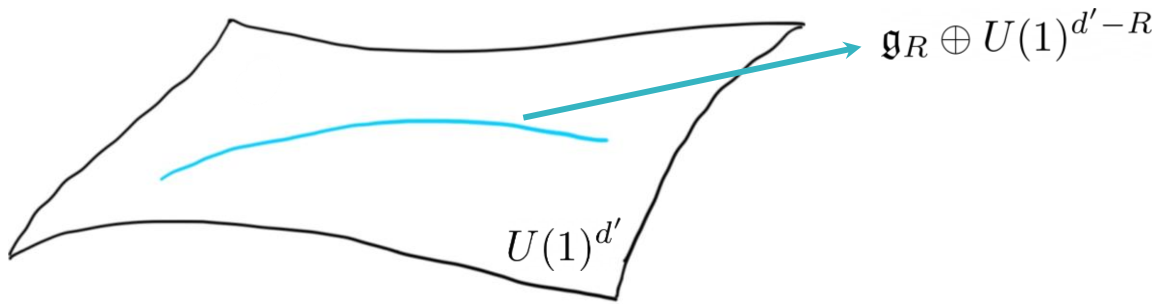

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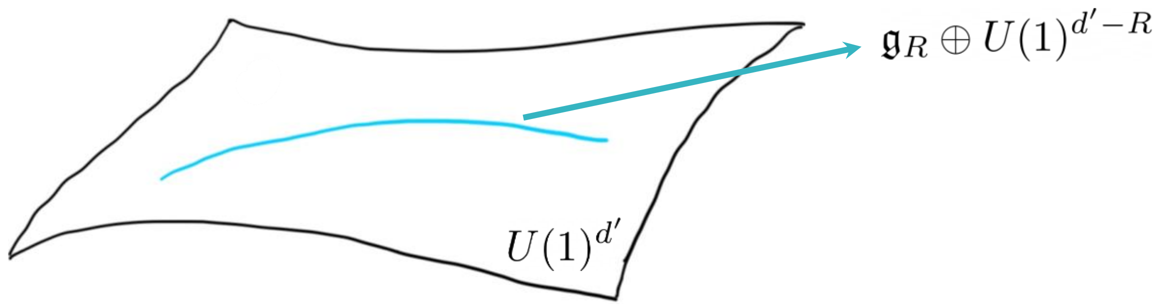

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$$f^{ab}{}_c = \mathcal{F}^{abd} k_{dc}$$

Question

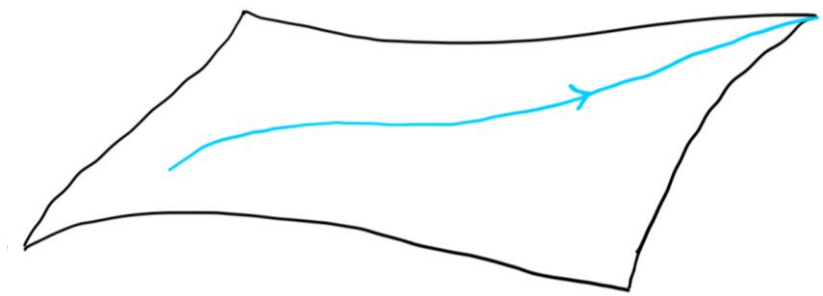
Symmetry enhancements ⟷ massive vectors becoming massless

Infinite distance points ⟷ infinte towers of massless states

What about symmetry enhancements at infinite distance in moduli space?

Symmetry enhancements from the worldsheet

T^{d+k}



$\mathcal{M}_{d+k}$

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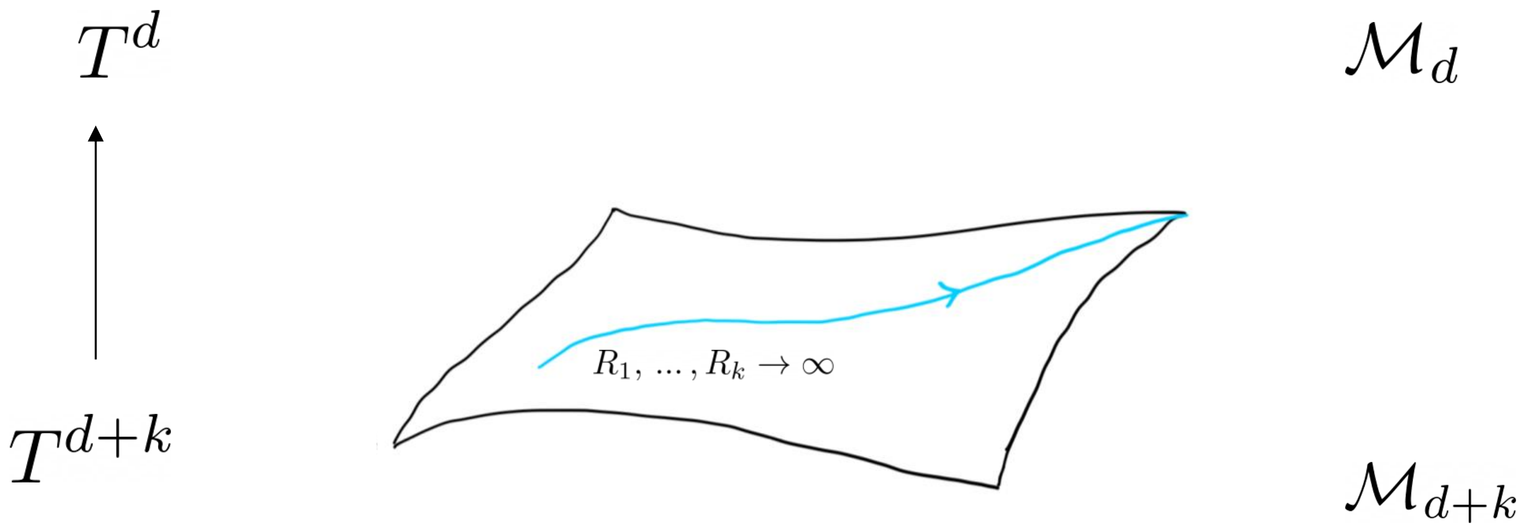

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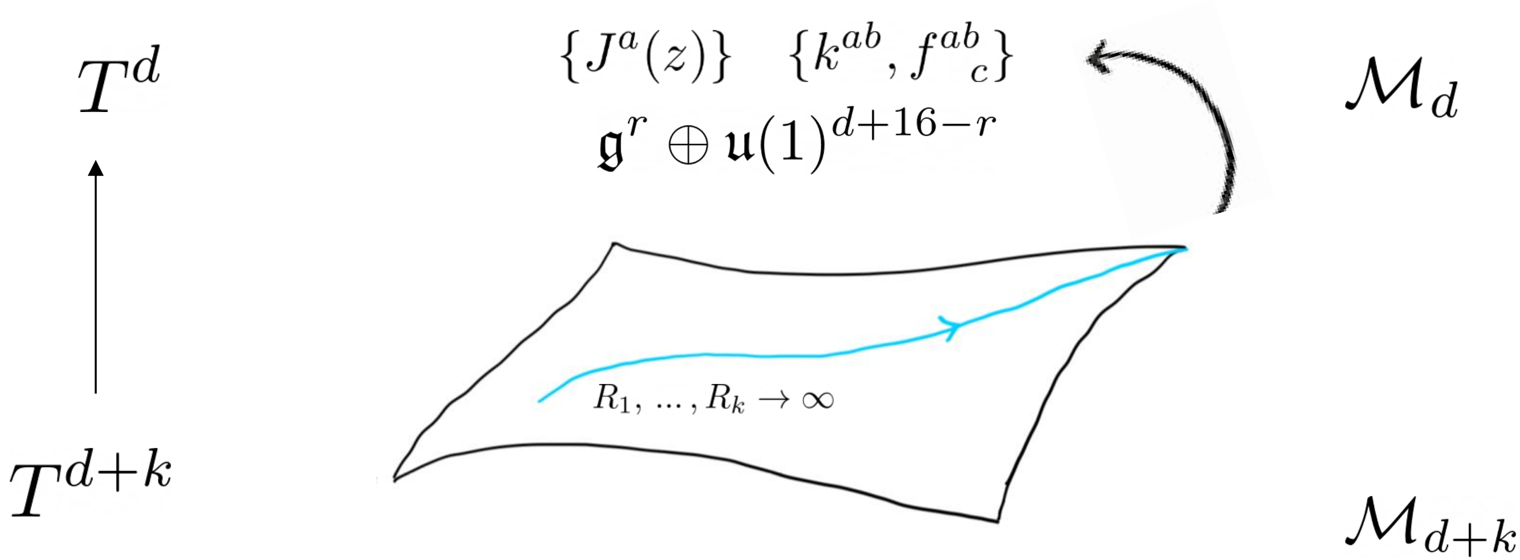
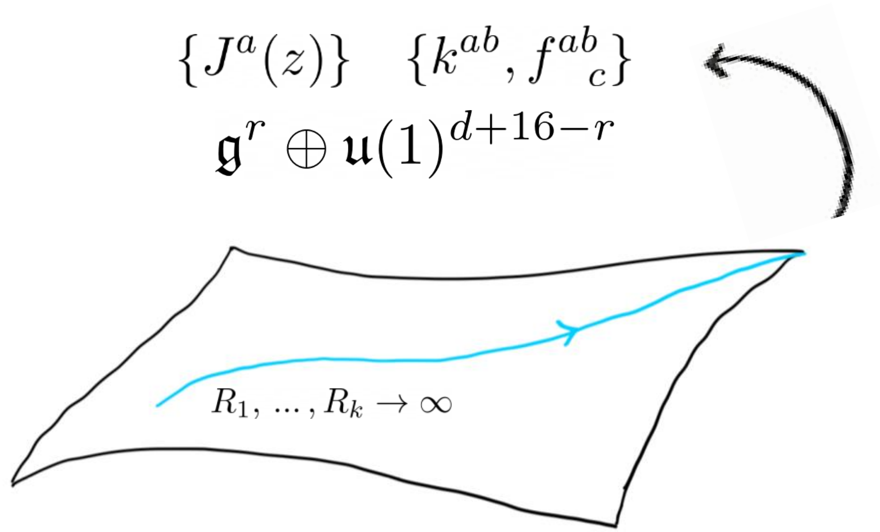


Symmetry enhancements from the worldsheet

T^d $\{J^a(z)\}$ $\{k^{ab}, f^{ab}{}_c\}$ $\mathcal{M}_d$

$\mathfrak{g}^r \oplus \mathfrak{u}(1)^{d+16-r}$

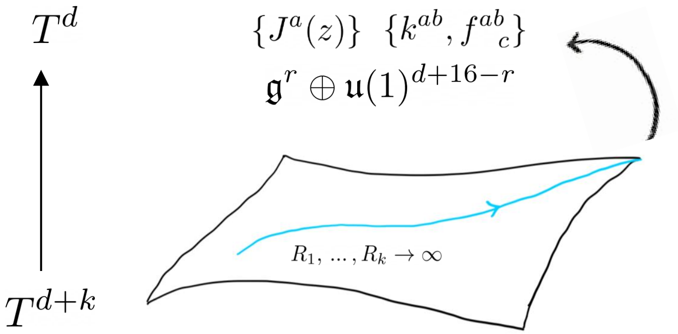


T^{d+k} $\mathcal{M}_{d+k}$

Question: what algebra arises in the limit from the lower dimensional point of view?

Space-time gauge bosons (left moving sector, $M \to 0$) ⟷ Worldsheet currents (holomorphic, $(h, \bar{h}) \to (1, 0)$)

Heterotic on T^{d+k}: decompactifying k directions



On $\mathcal{M}_{d+k}$ there are infinitely many asymptotic currents $(h, \bar{h}) \to (1, 0)$

$$\mathcal{J}^a_{\{n_j\}}(z,\bar{z}) = J^a(z) e^{i n_j X^j(z,\bar{z})}, \; j = 1, \ldots, k$$

$$\{n_j\} \in \mathbb{Z}$$

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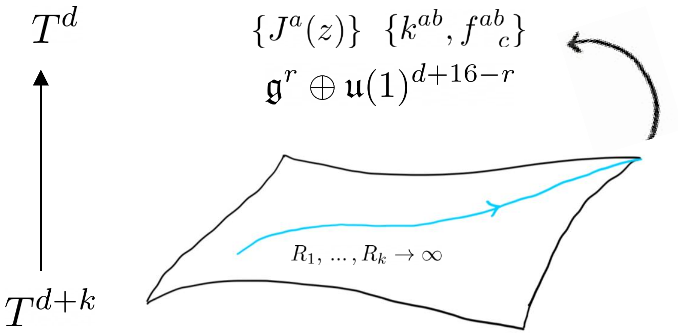

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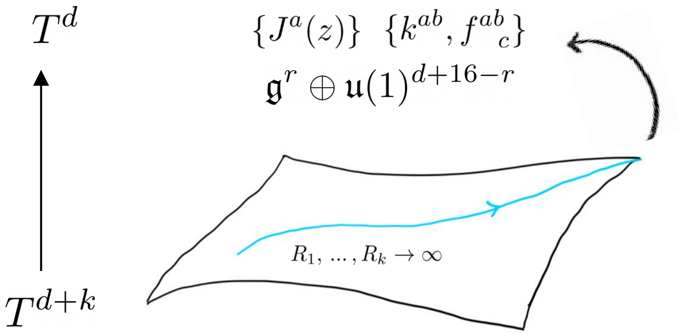

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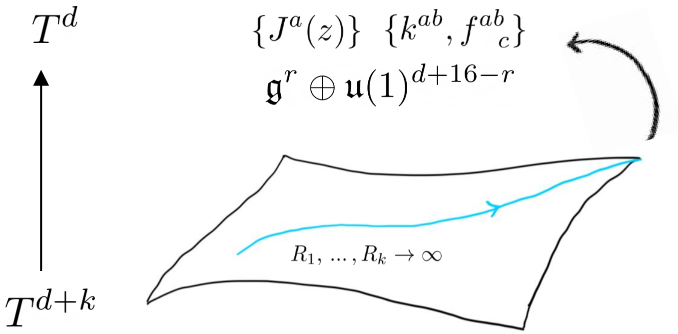

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$$k^{am;bn} = k^{ab}\, \delta^{m+n,0}$$

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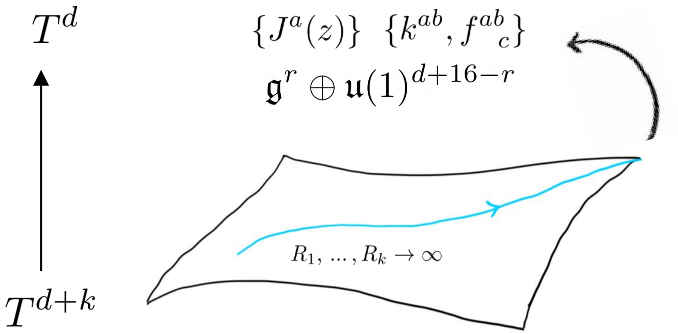

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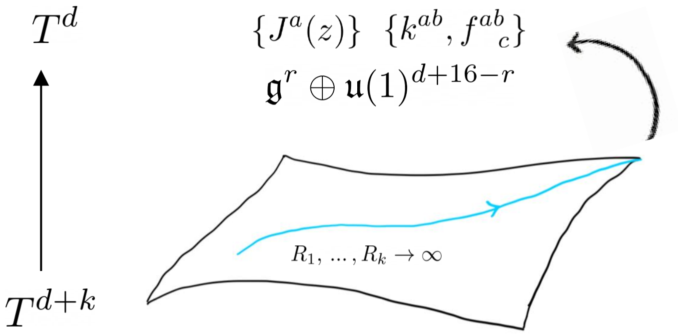

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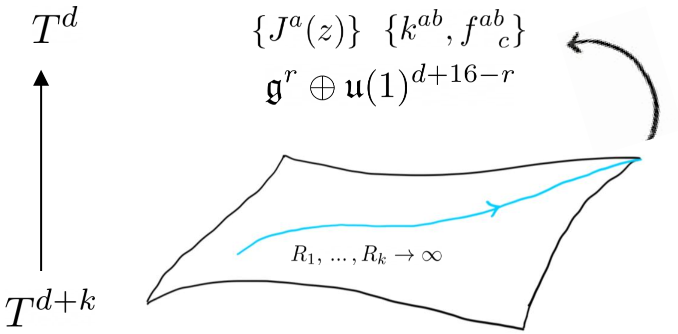

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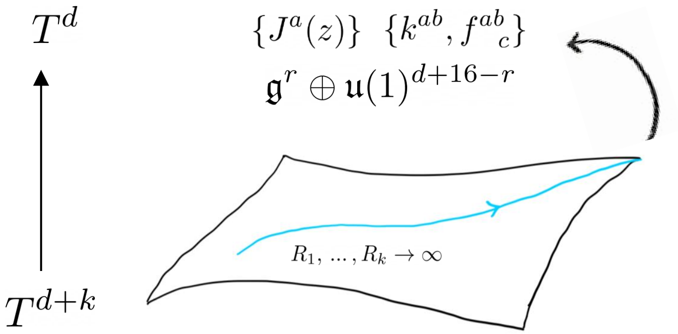
$k^{\mathrm{am};b\mathrm{n}} = k^{ab}\,\delta^{\mathrm{m+n},0}$ → C^j orthogonal to all the other generators and itself

$f^{\mathrm{am};b\mathrm{n}}{}_{c\mathrm{p}} = f^{ab}{}_c\,\delta^{\mathrm{m+n+p},0}$ → Loop algebra

$f^{\mathrm{am};b\mathrm{n}}{}_j = -i\mathrm{m}k^{ab}\delta^{\mathrm{m+n},0}$ → C^j central extension

$\mathrm{m},\,\mathrm{n},\,\mathrm{p} \in \mathbb{Z}$

Heterotic on T^{d+k}: decompactifying k directions



On $\mathcal{M}_{d+k}$ there are infinitely many asymptotic currents $(h, \bar{h}) \to (1, 0)$

decompactified bosons

$$\mathcal{J}^a_{\{\mathrm{n}_j\}}(z, \bar{z}) = J^a(z) e^{i\mathrm{n}_j X^j(z,\bar{z})}, \; j = 1, \ldots, k$$

$$\{n_j\} \in \mathbb{Z}$$

$$C^j(z) = i\partial X^j(z)$$

They form an algebra characterized by the following data:

$$k^{am;bn} = k^{ab}\, \delta^{\mathrm{m+n},0}$$

$$f^{am;bn}{}_{cp} = f^{ab}{}_c\, \delta^{\mathrm{m+n+p},0}$$

$$f^{am;bn}{}_j = -i\mathrm{m} k^{ab} \delta^{\mathrm{m+n},0}$$

$$\mathrm{m},\ \mathrm{n},\ \mathrm{p} \in \mathbb{Z}$$

$$(\widehat{\mathfrak{g}^r} \oplus \widehat{\mathfrak{u}(1)}^{\,d+16-r})/\sim$$

Affine version of the higher dimensional algebra

Heterotic string on S^1: decompactification limits

- $E_8 \times E_8$ theory in 9d
  - $R \to \infty$ and finite A : $E_9 \oplus E_9/\sim$
  - $R \to 0$ and $A = (0_7, 1, 0_7, 1)$: $\widehat{D}_{16}$

- $\mathrm{Spin}(32)/\mathbb{Z}_2$ theory in 9d
  - $R \to \infty$ and finite A: $\widehat{D}_{16}$
  - $R \to 0$ and $A = \left(\left(\frac{1}{2}\right)_8, 0_8\right)$: $E_9 \oplus E_9/\sim$

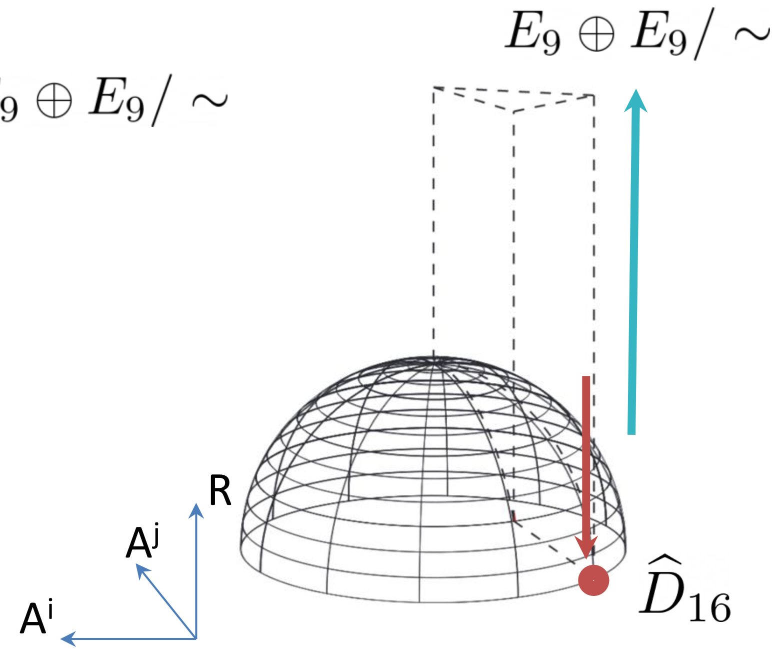
$$J_n^a(z) \equiv J^a(z) e^{inY(z)}$$

$$[J_n^{\hat{I}}, J_m^{\hat{J}}] = in\delta^{\hat{I}\hat{J}}\delta_{n+m,0}\partial y\,,$$

$$[J_n^{\hat{I}}, J_m^{\alpha}] = \pi_\alpha^{\hat{I}} J_{n+m}^{\alpha}\,,$$

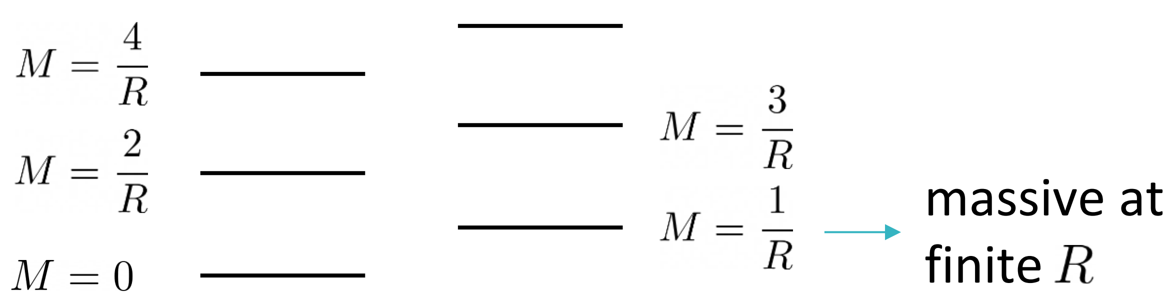
$$[J_n^{\alpha}, J_m^{\beta}] = \begin{cases} \epsilon(\alpha,\beta) J_{n+m}^{\alpha+\beta} & \alpha+\beta\,\mathrm{root}, \\ \pi_\alpha^{\hat{I}} J_{n+m}^{\hat{I}} + in\delta_{n+m,0}\partial y & \alpha = -\beta\,, \\ 0 & \mathrm{otherwise} \end{cases}$$

Decompactification to 10d $E_8 \times E_8$



Decompactification to 10d $SO(32)$

CHL string: decompactification limit

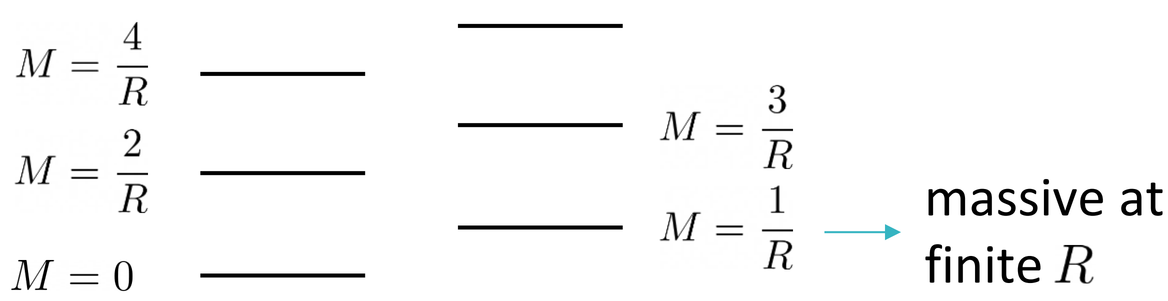


CHL string: decompactification limit

The conserved currents in the limit $R \to \infty$ are:

- The even-momentum towers of the $(\mathfrak{e}_8)_2$ currents

$$\mathcal{J}^a_{+;\mathrm{n}}(z,\bar{z}) = J^a_+(z)e^{i2\mathrm{n}Y(z,\bar{z})},\ \mathrm{n}\in\mathbb{Z}$$



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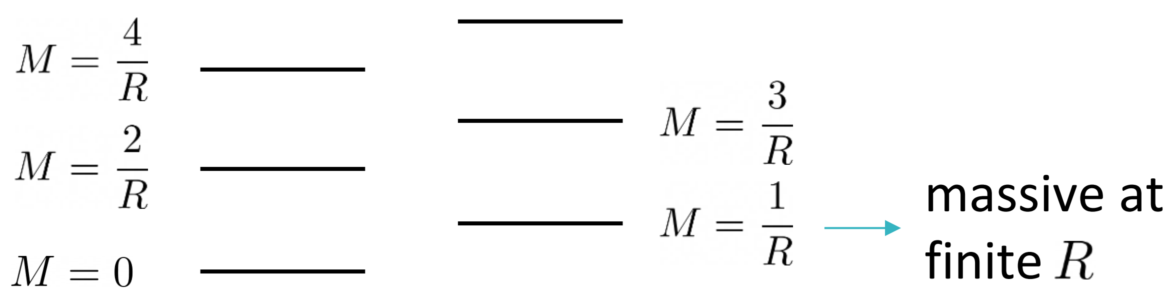
$$\mathcal{J}^{\dot{a}}_{-;\mathrm{r}}(z,\bar{z}) = J^{\dot{a}}_-(z)e^{i2\mathrm{r}Y(z,\bar{z})},\ \mathrm{r} \in \mathbb{Z} + \frac{1}{2}$$



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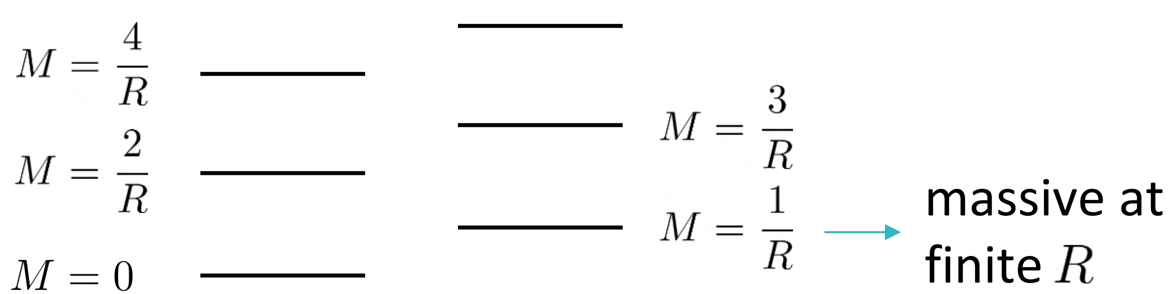
- The circle boson

$$C(z) = i\partial Y(z)$$



CHL string: decompactification limit

$$\mathcal{J}^{a}_{+;\mathrm{n}}(z,\bar{z}) \qquad \mathcal{J}^{\dot{a}}_{-;\mathrm{r}}(z,\bar{z}) \qquad C(z)$$

Their algebra is characterized by:

$$\begin{aligned}
\tilde{k}^{a\mathrm{m};b\mathrm{n}} &= 2\delta^{a,b}\delta^{\mathrm{m}+\mathrm{n},0} \\
\tilde{f}^{a\mathrm{m};b\mathrm{n}}{}_{c\mathrm{p}} &= f^{ab}{}_{c}\delta^{\mathrm{m}+\mathrm{n}+\mathrm{p},0} \\
\tilde{f}^{a\mathrm{m};b\mathrm{n}}{}_{Y} &= -2i\mathrm{m}\sqrt{2}\delta^{a,b}\delta^{\mathrm{m}+\mathrm{n},0}
\end{aligned}$$

$$\mathrm{m},\ \mathrm{n},\ \mathrm{p} \in \mathbb{Z}$$

$$\begin{aligned}
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where $f^{ab}{}_{c}$ are the structure constants of E_8 .

CHL string: decompactification limit

[VC, Melnikov, '24]

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where $f^{ab}{}_{c}$ are the structure constants of E_8 .

This is a twisted version of the algebra $(E_9 \oplus E_9)/\sim$.

CHL string: decompactification limit

[VC, Melnikov, '24]

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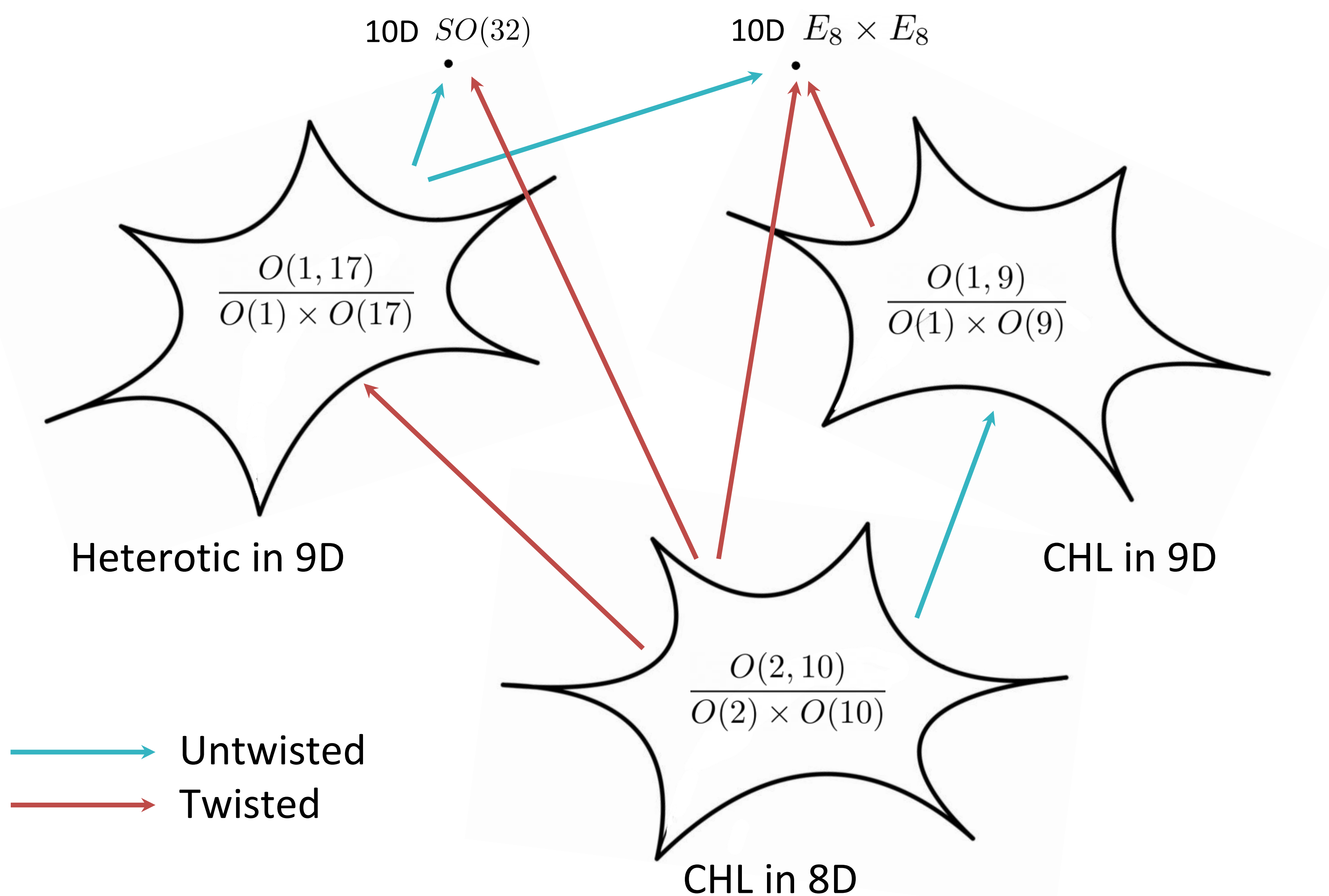
It is the only affine algebra –no limit with $\widehat{D}_{16}$.

CHL string: decompactification limit



Conclusions

Characterization of infinite distance limits in symmetric moduli spaces.

- Complete structure of the boundary and argument supporting the SDC.
- For 16 supercharges, presence of (twisted) affine algebras in decompactification limits.

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Thank you!