IR CONSTRAINTS ON IRREPS OF QUANTUM GRAVITY

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Dark World to Swampland 2024 Nov 13 Daejeong IBS

Bootstrapping quantum gravity

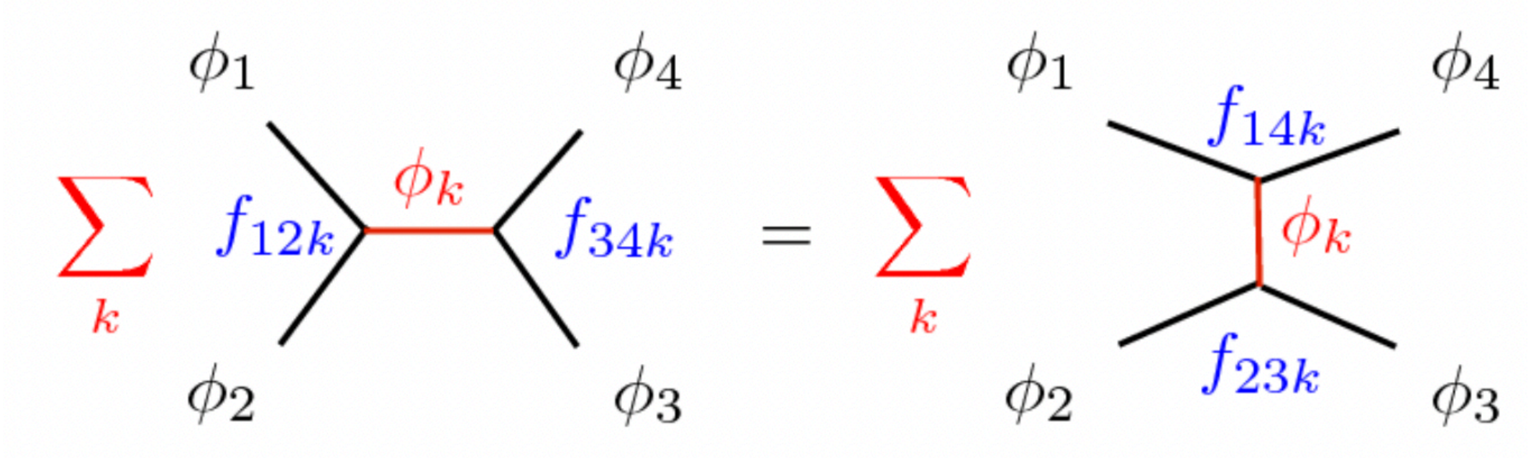
CFT bootstrap

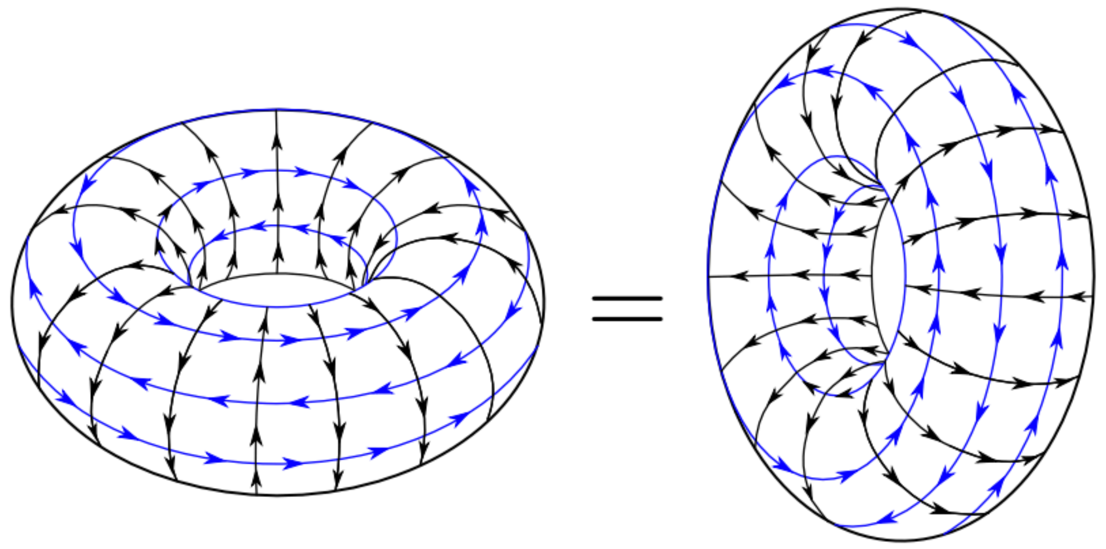
S-matrix Bootstrap

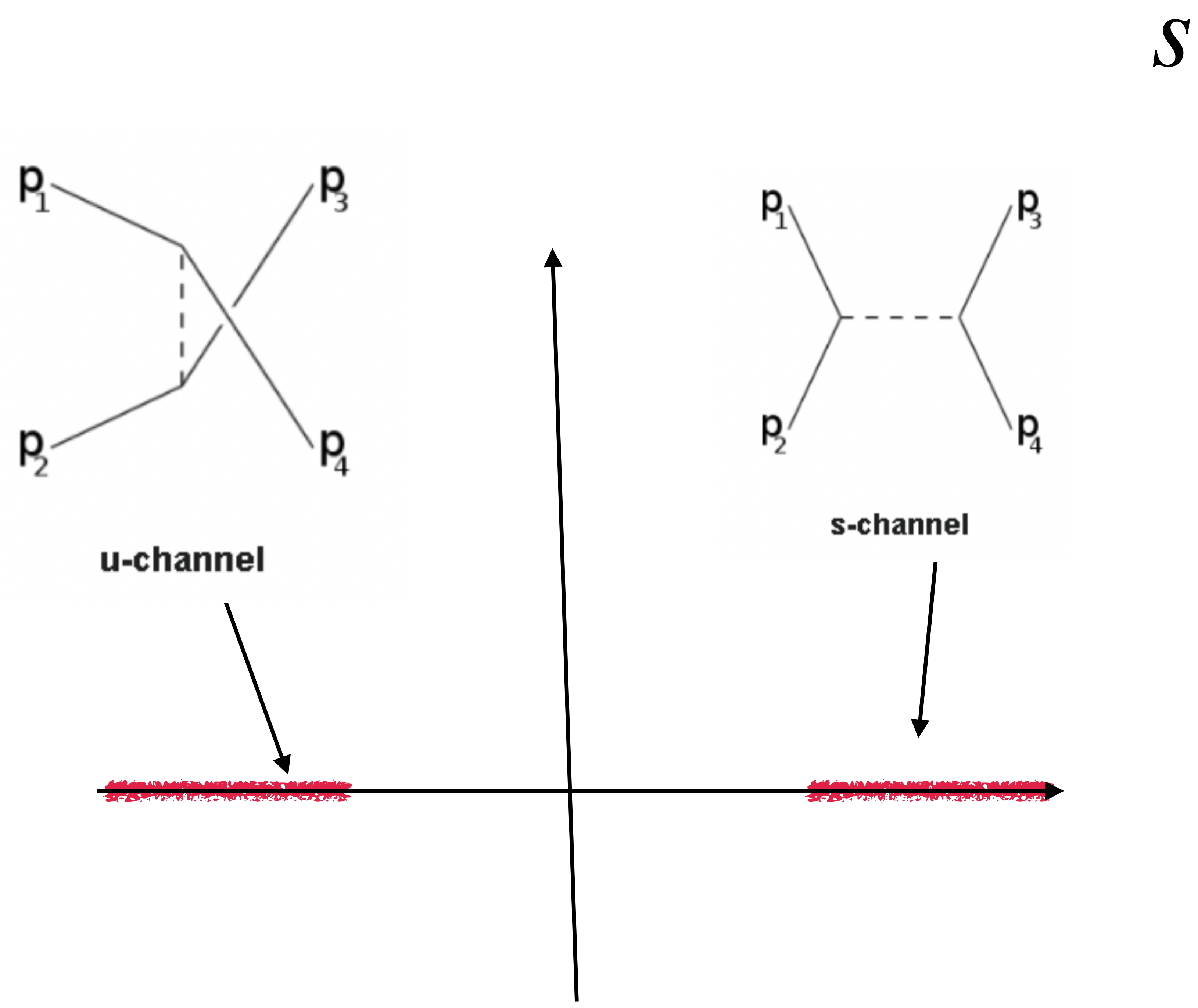
$$f(z,\bar{z}) = \langle \phi(0)\phi(1)\phi(z,\bar{z})\phi(\infty)\rangle$$

$$Z\left[\tau,\bar{\tau}\right] = Z\left[-\frac{1}{\tau},-\frac{1}{\bar{\tau}}\right]$$

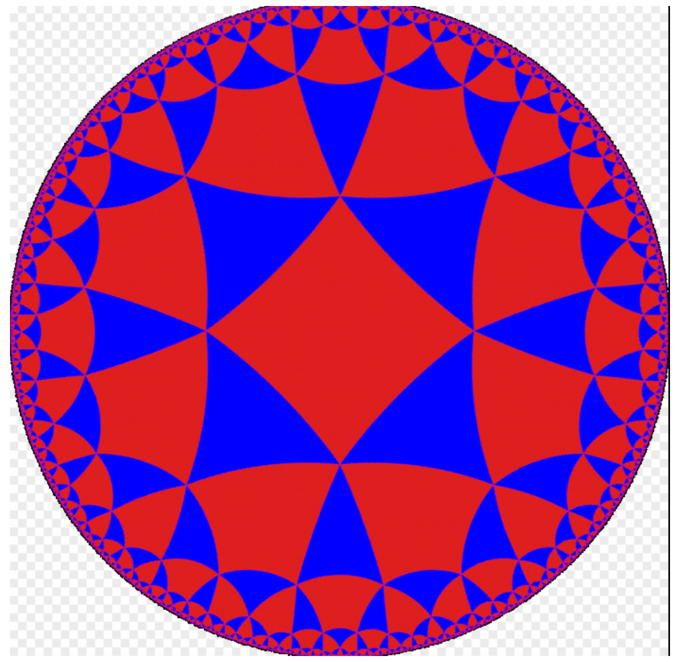
$$M(s,t)$$





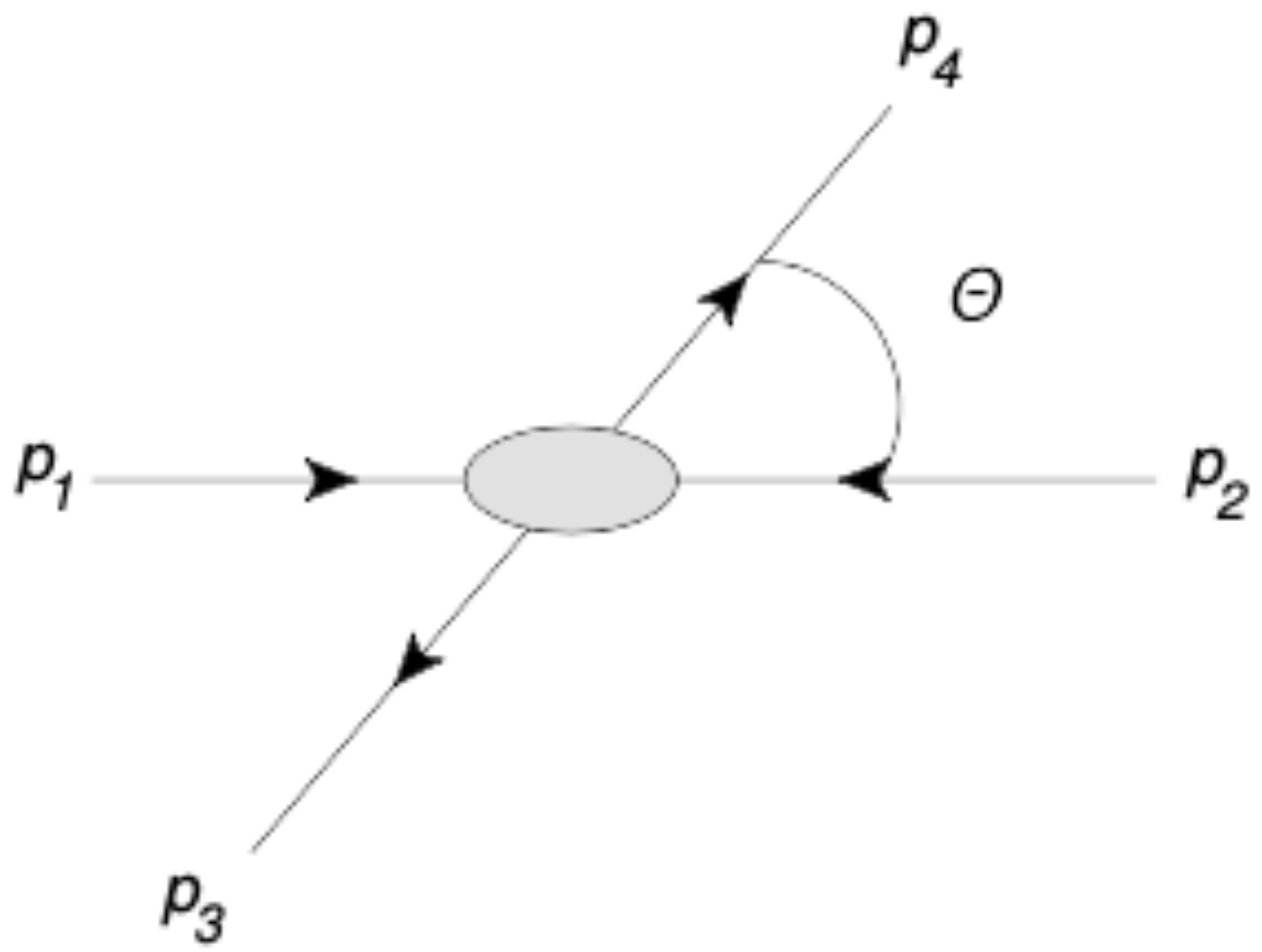


AdS/CFT



Gravity is the oldest force → It is long range, and universal → Difficult to UV complete

Let's consider the four-point amplitude



$$t = (p_1 - p_4)^2 = 2E^2(1 - \cos\theta)$$

$$s = (p_1 + p_2)^2 = 4E^2$$

$$A(s,t) \to A(s,\theta)$$

Pure contact EFTs are easy to UV complete

$$\mathscr{L} = \partial\phi\partial\phi + G_f(\partial\phi\partial\phi)^2$$

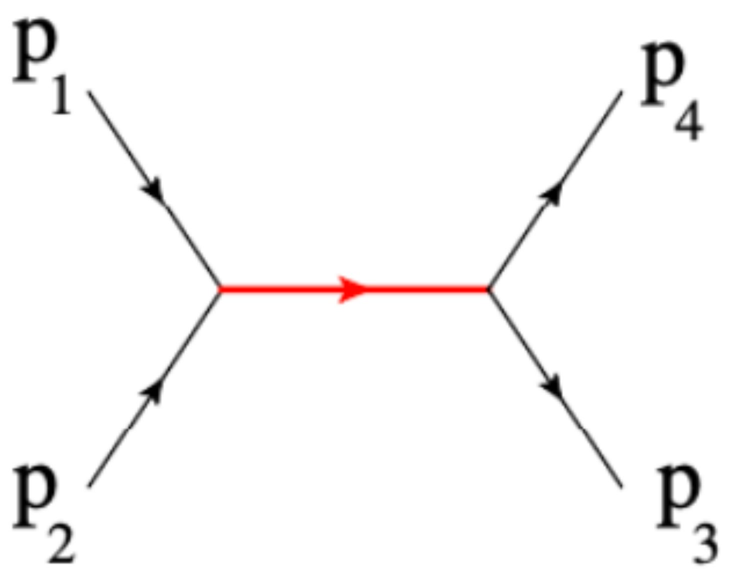
The amplitude is purely polynomial, can be completed by adding new thresholds

$$M(1234) = -G_F s \rightarrow G_f m^2 \frac{s}{s - m^2}$$

>0

Partial wave unitarity demands positive residue

$$\frac{G_f m^2}{8\pi} \leq \frac{1}{2} \rightarrow m < 1TeV$$



EFTs with massless poles are difficult to UV complete

$$\mathscr{L} = \sqrt{-g}(R + \partial\phi\partial\phi)$$

The

$$M(1234) = G_N \frac{(s^2 + t^2 + u^2)^2}{stu}$$

Play the same game!

$$\frac{1}{s} \to \frac{M^2}{s(M^2 - s)}$$

$$s = 0 \quad \to \quad \frac{1}{s}$$

$$s = M^2 \to -\frac{1}{(s - M^2)}$$

Unitarity of the massless and massive pole is in tension!

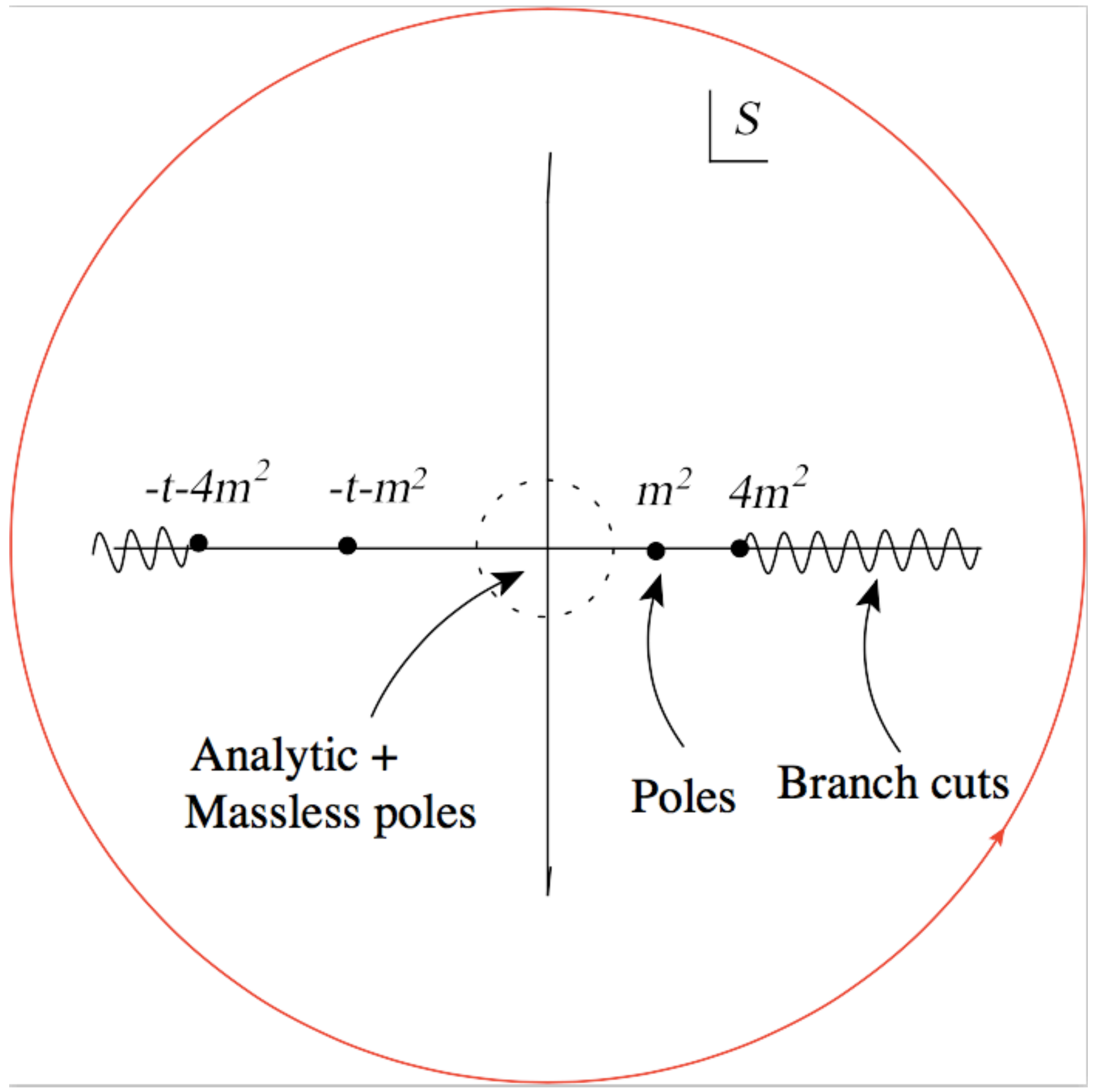
The fact that gravity is long range force (the massless pole) makes it hard to unitarize !

What does the fact that gravity must be unitized say about our world ??

Causality implies twice subtraction

Haring, Zhiboedov 2202.08280

$$\oint_{\infty} \frac{ds'}{2\pi i(s'-s)} \frac{M^{abcd}(s',t)}{s'(s'+t)} = 0\,,$$



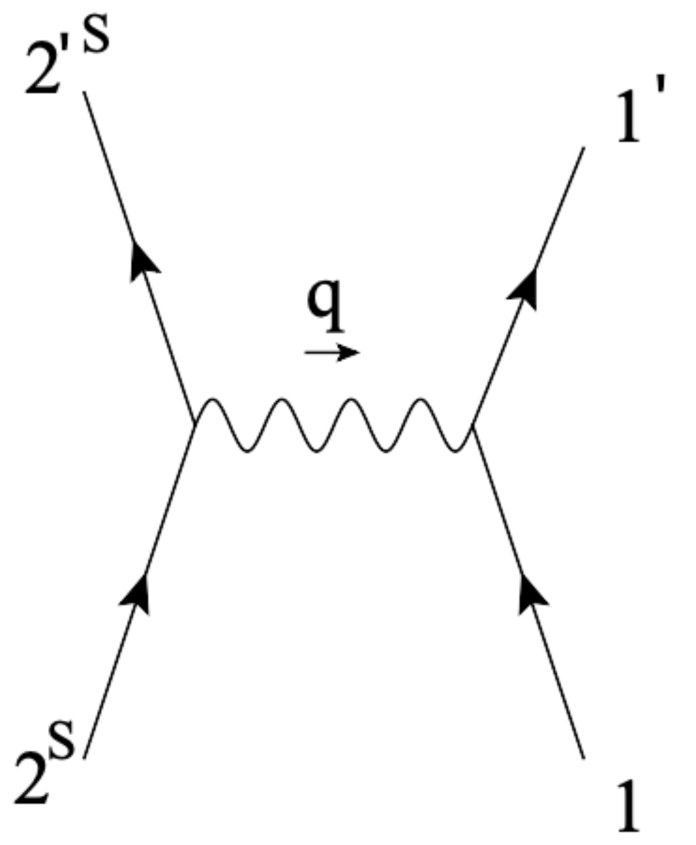
Relates the low energy parameters with unitarity of UV

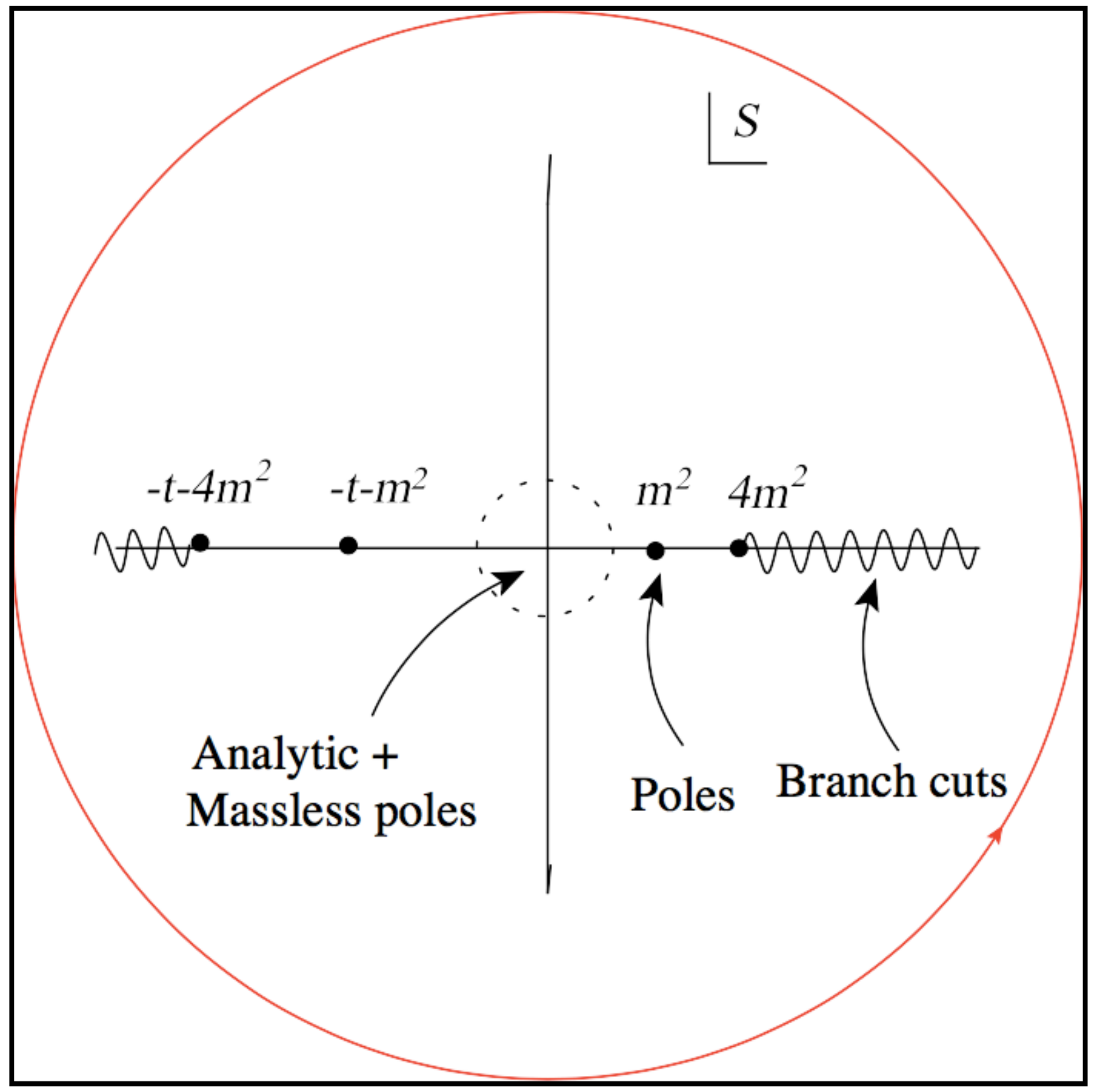
$$(\mathrm{Res}_{s'=0} + \mathrm{Res}_{s'=-t} + \mathrm{Res}_{s'=s}) \frac{M^{abcd}(s',t)}{(s'-s)s'(s'+t)} =$$

$$\int_{M^2}^{\infty} \frac{ds'}{\pi s'(s'+t)} \left(\frac{\mathrm{Im} M^{abcd}(s',t)}{(s'-s)} + \frac{\mathrm{Im} M^{abcd}(-s'-t,t)}{(-s'-t-s)} \right)$$

In fact the low energy theory for long range force is constrained by the UV (anti-Wilsonian!)

$$V(r) = -G\frac{m_1 m_2}{r} \rightarrow \quad V(q) = -\frac{4\pi G m_1 m_2}{|\vec{q}|^2} \quad \rightarrow \langle 1'2'|\mathcal{S}|1,2\rangle_{IR} \sim -\frac{s^2}{t}$$



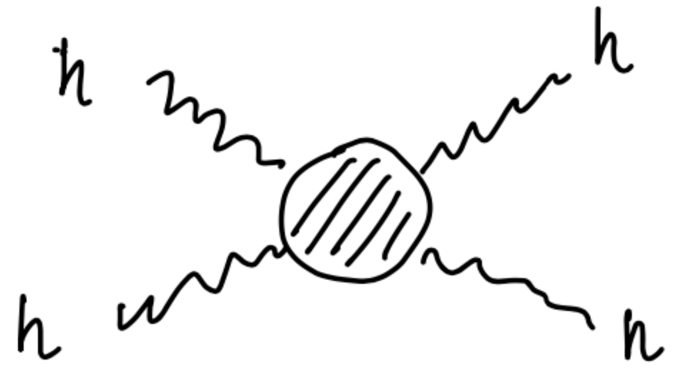


$$0 = \oint \frac{ds}{s^3}\langle 1'2'|\mathcal{S}|1,2\rangle(s,t) = a\frac{1}{t} + \int_{s_0}^{\infty}\frac{ds}{s^3} Im\left[\langle 1'2'|\mathcal{S}|1,2\rangle(s,0)\right]$$

from optical theorem $$a\frac{1}{t} = -\int_{s_0}^{\infty}\frac{ds}{s}\sigma < 0$$

Gravity must be attractive !

The Gravitation S-matrix

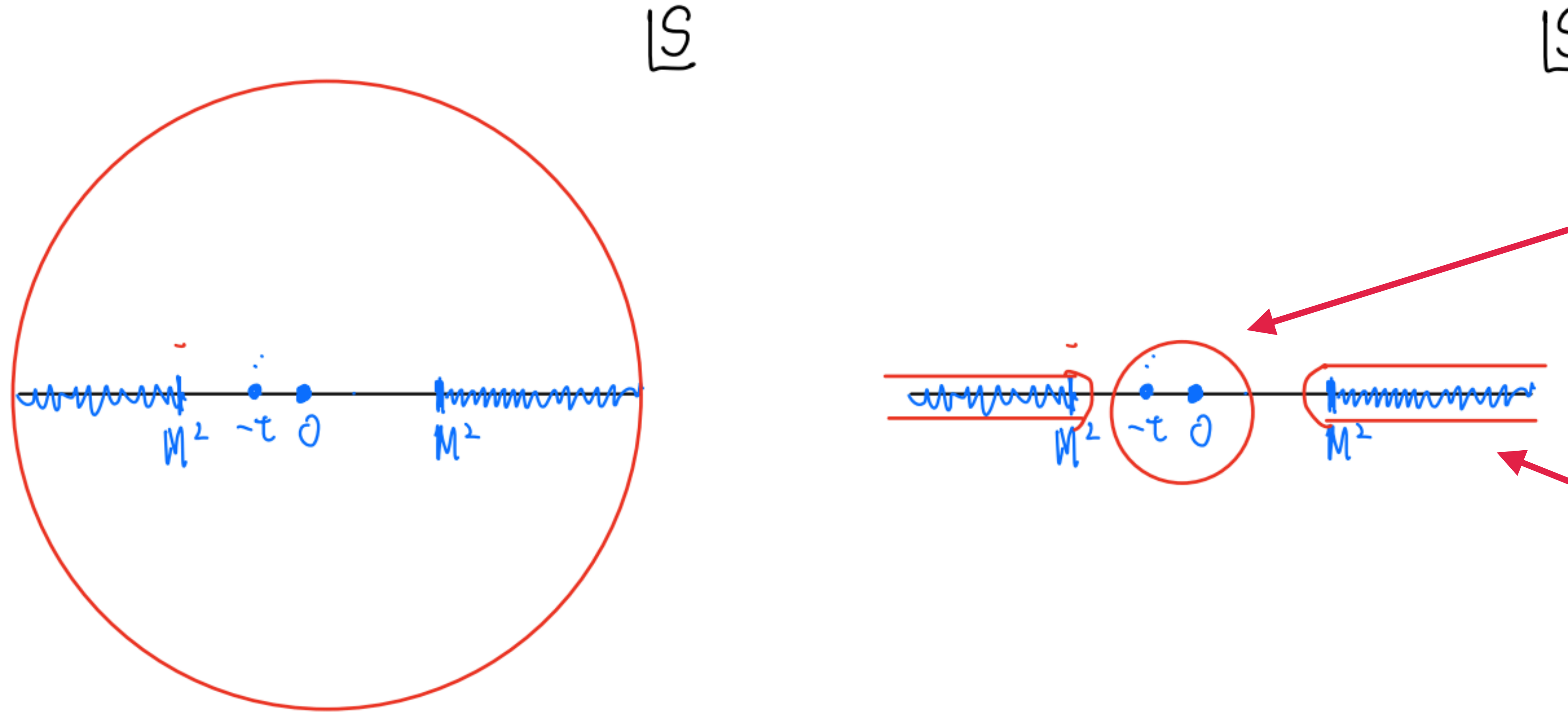


EFT information is embedded in the low-energy limit of $M(s,t)$

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2} \left(R + \hat{\alpha}_1 \frac{R^2}{M^2} + \hat{\alpha}_2 \frac{R^3}{M^4} + \hat{\alpha}_4 \frac{R^4}{M^6} \cdots \right)$$

For perturbative completion we can keep large, loops are suppressed

$$M^{\rm IR}(s,t) = R^4 \left(\frac{1}{stu} + \{\text{massless poles from } R^2, R^3\} + \sum_{k,q} b_{k,q} s^{k-q} t^q \right)$$



$$b_{k,q} = \frac{1}{2\pi i} \frac{\partial^q}{\partial t^q} \oint \frac{ds}{s^{k-q+1}} M(s,t)$$

$$\frac{1}{\pi} \frac{\partial^q}{\partial t^q} \int_{m^2}^{\infty} \frac{ds}{s^{k-q+1}} \mathrm{Im}[M(s,t)]$$

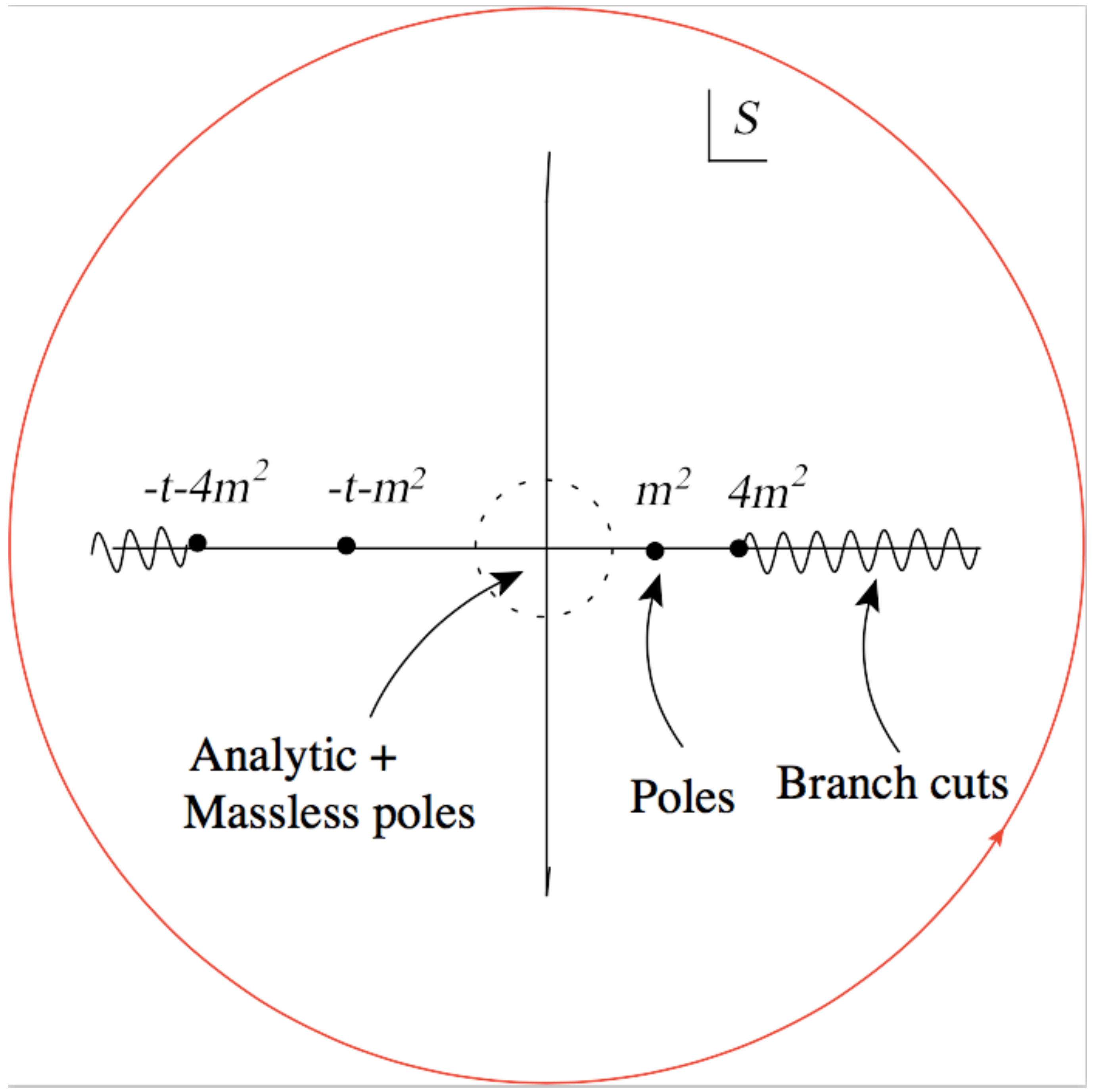
Dispersion relations for S-matrix

Arising from perturbative completion

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2} \left(R + \hat{\alpha}_1 \frac{R^2}{M^2} + \hat{\alpha}_2 \frac{R^3}{M^4} + \hat{\alpha}_4 \frac{R^4}{M^6} \cdots \right)$$

The coefficients can be derived from a contour integral of $M(s,t)$

$$b_{n+q,q} = \frac{\partial^q}{\partial t^q} \oint \frac{ds}{s^{n+1}} M(s,t)$$



- Analyticity: M(s,t) is analytic away from the real s-axes for fixed t

$$M(s,t^*) = M^{\rm sub} + \int_{M^2}^{\infty} \frac{{\rm Im}[M(s,t^*)]}{s-m^2} + \int_{M^2}^{\infty} \frac{{\rm Im}[M(s,t^*)]}{u-m^2}$$

$$M(s,t^*) = M^{\rm sub} + \int_{M^2}^{\infty} \frac{{\rm Im}[\rho_j(s)] P_j^s \left(1+\frac{2t^*}{m^2}\right)}{s-m^2} + \int_{M^2}^{\infty} \frac{{\rm Im}[\rho_j(s)] P_j^u \left(1+\frac{2t^*}{m^2}\right)}{u-m^2}$$

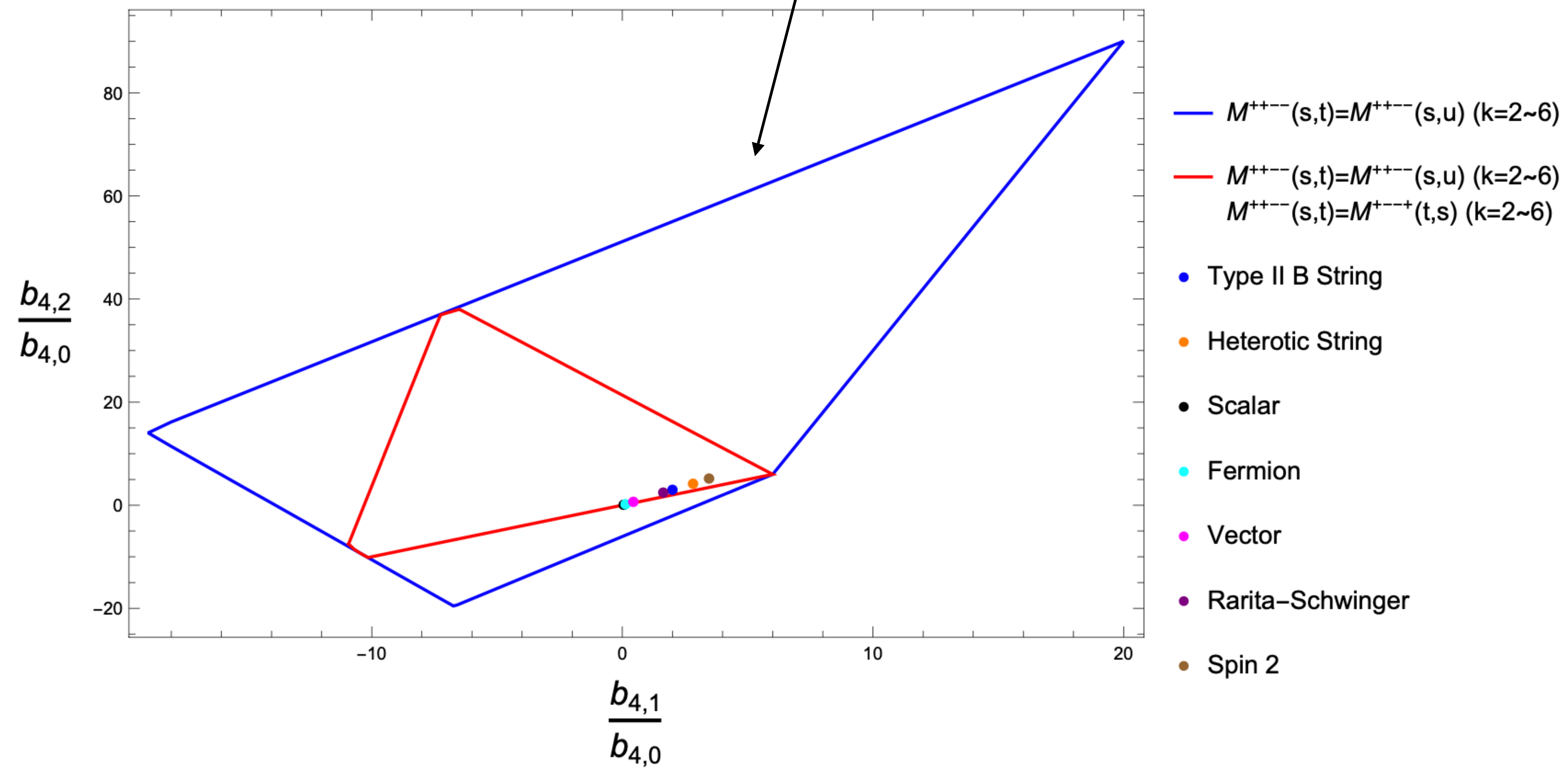
- Unitarity: $0 \le {\rm Im}[\rho_j(s)]$

positivity
optical theorem

For fixed derivative couplings, with sdpb Li-Yuan Chiang, Wei Li, He-Chen Wen, Laurentiu Rodina, Y-T H 2201.07177

$D^8 R^4$

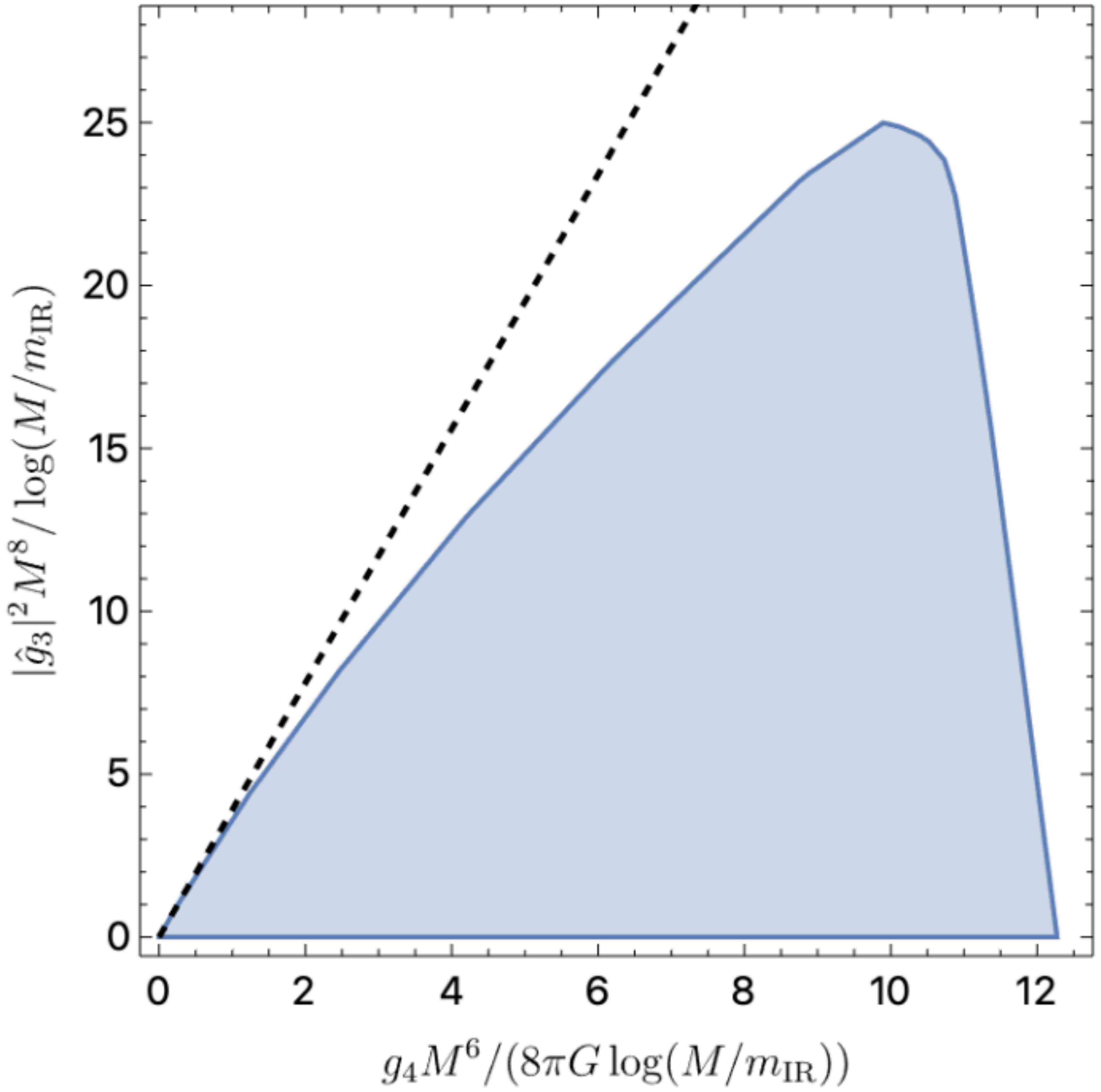
Z. Bern D. Kosmopoulos, A Zhiboedov 2103.12729



Bounds with respect to G_N

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big[R - \frac{1}{3!}\left(\alpha_3 R^{(3)} + \tilde{\alpha}_3\tilde{R}^{(3)}\right) + \frac{1}{4}\left(\alpha_4 (R^{(2)})^2 + \alpha_4'(\tilde{R}^{(2)})^2 + 2\tilde{\alpha}_4 R^{(2)}\tilde{R}^{(2)}\right) + \ldots\Big] .$$

$$\widehat{g}_3 = \alpha_3 + i\tilde{\alpha}_3, \qquad g_4 = 8\pi G(\alpha_4 + \alpha_4'), \qquad \widehat{g}_4 = 8\pi G(\alpha_4 - \alpha_4' + i\tilde{\alpha}_4)$$



Caron-Huot, Li, Parra-Martinez, Simmons-Duffin 2201.06602

We have seen that gravity is tamed in the UV imposes important bounds on the EFT we see in the IR

Do what we see in the IR constraint that in the UV ??

Let's assume that we see a symmetry in the IR, say SO(n)

At low energies we see

$$\mathcal{A}^{i_1 i_2 i_3 i_4} = \mathcal{A}^{i_1 i_2 i_3 i_4}_{\text{Grav}} + B^{i_1 i_2 i_3 i_4}_{\text{poly}}$$

$$\mathcal{A}^{i_1 i_2 i_3 i_4}_{\text{Grav}} = 8\pi G_N \left(\frac{(t-u)^2}{s} \delta^{i_1 i_2} \delta^{i_3 i_4} + \text{Perm} \right) .$$

$$\begin{aligned} \text{fundamental :} \quad & B^{i_1 i_2 i_3 i_4}_{\text{poly}}(s,t) = B(s,t)\delta^{i_1 i_2}\delta^{i_3 i_4} + B(u,s)\delta^{i_1 i_3}\delta^{i_2 i_4} + B(t,u)\delta^{i_1 i_4}\delta^{i_2 i_3} \\ \text{adjoint :} \quad & B^{i_1 i_2 i_3 i_4}_{\text{poly}}(s,t) = \sum_{\sigma \in S_3} \text{Tr}[i_1 \sigma(i_2)\sigma(i_3)\sigma(i_4)] B_1(1,\sigma(2),\sigma(3),\sigma(4)) \\ & + B_2(s,t)\text{Tr}[i_1 i_2]\text{Tr}[i_3 i_4] + B_2(t,u)\text{Tr}[i_1 i_4]\text{Tr}[i_2 i_3] + B_2(u,t)\text{Tr}[i_1 i_3]\text{Tr}[i_2 i_4] . \end{aligned}$$

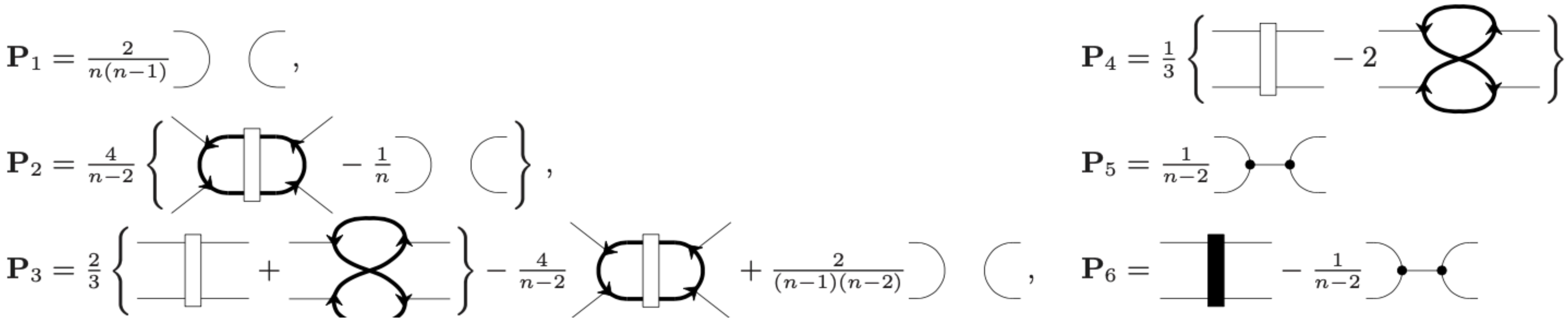
The partial wave expansion for colored amplitudes take the form

$$\mathcal{A}^{i_1 i_2 i_3 i_4} = \sum_{J,R} n^{(D)}_{J,R} \, f_{J,R}(s) \mathbb{P}^{i_1 i_2 ; i_3 i_4}_{R} \mathbb{G}^{(D)}_{J} \left(1 + \frac{2t}{s} \right)$$

For adjoint matter fields

$$\mathcal{A}^{i_1 i_2 i_3 i_4} = \sum_{J,R} n_{J,R}^{(D)} f_{J,R}(s) \mathbb{P}_R^{i_1 i_2; i_3 i_4} \mathbb{G}_J^{(D)} \left(1 + \frac{2t}{s}\right)$$

Projection operators



The dispersion relation now takes the form

$$\oint_\infty \frac{ds'}{2\pi i(s'-s)} \frac{\mathcal{A}^{i_1 i_2 i_3 i_4}(s',t)}{s'(s'+t)} = 0\,,$$

$$\frac{B_{\text{poly}}^{i_1 i_2 i_3 i_4}(s,t)}{s(s+t)} - \frac{B_{\text{poly}}^{i_1 i_2 i_3 i_4}(0,t)}{st} + \frac{B_{\text{poly}}^{i_1 i_2 i_3 i_4}(-t,t)}{(s+t)t} - \frac{8\pi G}{t}\mathbb{P}_0^{i_2 i_3; i_4 i_1} =$$

$$n_{J,R}^{(D)} \sum_{JR} \mathbb{G}_J^{(D)}(cos(\theta)) \int_{M^2}^{\infty} \frac{ds'}{\pi s'(s'+t)} \text{Im}[f_{J,R}(s')] \left(\frac{\mathbb{P}_R^{i_1 i_2; i_3 i_4}}{(s'-s)} + \frac{\mathbb{P}_R^{i_1 i_3; i_2 i_4}}{(s'+t+s)}\right)$$

Using the fact that projectors are orthogonal we can put everything in the t-channel basis

$$\frac{B_{\text{poly}}^{i_1i_2i_3i_4}(s,t)}{s(s+t)} - \frac{B_{\text{poly}}^{i_1i_2i_3i_4}(0,t)}{st} + \frac{B_{\text{poly}}^{i_1i_2i_3i_4}(-t,t)}{(s+t)t} - \frac{8\pi G}{t}\mathbb{P}_0^{i_2i_3;i_4i_1} =$$

$$n_{J,R}^{(D)}\sum_{JR}\mathbb{G}_J^{(D)}(cos(\theta))\int_{M^2}^{\infty}\frac{ds'}{\pi s'(s'+t)}\text{Im}[f_{J,R}(s')]\left(\frac{\mathbb{P}_R^{i_1i_2;i_3i_4}}{(s'-s)}+\frac{\mathbb{P}_R^{i_1i_3;i_2i_4}}{(s'+t+s)}\right)$$

$$\mathbb{P}^s = M_{st}\mathbb{P}^t, \quad \mathbb{P}^u = M_{ut}\mathbb{P}^t\,.$$

The matrix M_{st} for adjoint representation given as:

$$\begin{pmatrix}
\frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(N-1)N} \\
\frac{N+2}{N} & \frac{N^2-8}{2(N-2)N} & \frac{N-4}{(N-2)N} & \frac{2(N+2)}{(2-N)N} & \frac{(N-4)(N+2)}{2(N-2)N} & \frac{4}{(2-N)N} \\
\frac{(N-3)(N+1)(N+2)}{6(N-1)} & \frac{(N-4)(N-3)(N+1)}{6(N-2)(N-1)} & \frac{N^2-6N+11}{3(N-2)(N-1)} & \frac{(N+1)(N+2)}{3(N-2)(N-1)} & \frac{(N-3)(N+1)(N+2)}{6(2-N)(N-1)} & \frac{(N-4)(N+1)}{3(2-N)(N-1)} \\
\frac{(N-3)(N-2)}{12} & \frac{3-N}{6} & \frac{1}{6} & \frac{1}{6} & \frac{N-3}{6} & -\frac{1}{6} \\
1 & \frac{N-4}{2(N-2)} & \frac{1}{2-N} & \frac{2}{N-2} & \frac{1}{2} & 0 \\
\frac{(N-3)(N+2)}{4} & \frac{N-3}{2-N} & \frac{N-4}{2(2-N)} & \frac{N+2}{2(2-N)} & 0 & \frac{1}{2}
\end{pmatrix}$$

and similarly for M_{ut}:

$$\begin{pmatrix}
\frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(1-N)N} & \frac{2}{(1-N)N} \\
\frac{N+2}{N} & \frac{N^2-8}{2(N-2)N} & \frac{N-4}{(N-2)N} & \frac{2(N+2)}{(2-N)N} & \frac{(4-N)(N+2)}{2(N-2)N} & \frac{4}{(N-2)N} \\
\frac{N^3-7N-6}{6(N-1)} & \frac{(N-4)(N-3)(N+1)}{6(N-2)(N-1)} & \frac{N^2-6N+11}{3(N-2)(N-1)} & \frac{(N+1)(N+2)}{3(N-2)(N-1)} & \frac{N^3-7N-6}{6(N-2)(N-1)} & \frac{(N-4)(N+1)}{3(N-2)(N-1)} \\
\frac{(N-3)(N-2)}{12} & \frac{3-N}{6} & \frac{1}{6} & \frac{1}{6} & \frac{3-N}{6} & \frac{1}{6} \\
1 & \frac{N-4}{2(N-2)} & \frac{1}{2-N} & \frac{2}{N-2} & -\frac{1}{2} & 0 \\
\frac{(N-3)(N+2)}{4} & \frac{N-3}{2-N} & \frac{N-4}{2(2-N)} & \frac{N+2}{2(2-N)} & 0 & -\frac{1}{2}
\end{pmatrix}.$$

Using the fact that projectors are orthogonal we can put everything in the t-channel basis

$$\frac{B_{\text{poly}}^{i_1 i_2 i_3 i_4}(s,t)}{s(s+t)} - \frac{B_{\text{poly}}^{i_1 i_2 i_3 i_4}(0,t)}{st} + \frac{B_{\text{poly}}^{i_1 i_2 i_3 i_4}(-t,t)}{(s+t)t} - \frac{8\pi G}{t}\mathbb{P}_0^{i_2 i_3; i_4 i_1} =$$

$$n_{J,R}^{(D)} \sum_{JR} \mathbb{G}_J^{(D)}(cos(\theta)) \int_{M^2}^{\infty} \frac{ds'}{\pi s'(s'+t)} \text{Im}[f_{J,R}(s')] \left(\frac{\mathbb{P}_R^{i_1 i_2; i_3 i_4}}{(s'-s)} + \frac{\mathbb{P}_R^{i_1 i_3; i_2 i_4}}{(s'+t+s)} \right)$$

↑

$$\sum_J \int_{M^2}^{\infty} ds' \text{Im}[\mathbf{f}_J(s')] \left(M_{st} h_{s,J}(s',t) + M_{ut} h_{u,J}(s',t) \right)$$

Now if we can find a vector $v = (1, \cdots)$ such that

$$\boxed{[M_{st}v]_i = [M_{ut}v]_i = 0}$$

Then the RHS takes the form

$$\frac{8\pi G_N}{-t} + Poly(t) = \sum_{j \neq i} \sum_J \int_{M^2}^{\infty} ds' \text{Im}[f_{J,j}(s')]$$

$$([M_{st}v]_j h_{s,J}(s',t) + [M_{ut}v]_j h_{u,J}(s',t))$$

Using the fact that projectors are orthogonal we can put everything in the t-channel basis

$$\frac{8\pi G_N}{-t} + Poly(t) = \sum_{j\neq i}\sum_{J}\int_{M^2}^{\infty} ds' \mathrm{Im}[f_{J,j}(s')] \left([M_{st}v]_j h_{s,J}(s',t) + [M_{ut}v]_j h_{u,J}(s',t)\right)$$

On the LHS i is absent

if we set $f_{J,j} = 0$ for all $j \neq i$,

$$\frac{8\pi G_N}{-t} + \cdots = 0$$

Impossible !!!

The spectrum with just i is ruled out

Exp: Let's start with SO(n) fundamentals

$$\mathcal{A}^{i_1 i_2 i_3 i_4} = \mathcal{A}^{i_1 i_2 i_3 i_4}_{\text{Grav}} + B^{i_1 i_2 i_3 i_4}_{\text{poly}}$$

$$\mathcal{A}^{i_1 i_2 i_3 i_4}_{\text{Grav}} = 8\pi G_N \left(\frac{(t-u)^2}{s} \delta^{i_1 i_2} \delta^{i_3 i_4} + \text{Perm} \right) . \qquad B^{i_1 i_2 i_3 i_4}_{\text{poly}}(s,t) = B(s,t)\delta^{i_1 i_2}\delta^{i_3 i_4} + B(u,s)\delta^{i_1 i_3}\delta^{i_2 i_4} + B(t,u)\delta^{i_1 i_4}\delta^{i_2 i_3}$$

We have 3 irreps entering the exchange

$$\mathcal{A}^{i_1 i_2 i_3 i_4} = \sum_{J,R} n^{(D)}_{J,R}\, f_{J,R}(s) \mathbb{P}^{i_1 i_2; i_3 i_4}_{R} \mathbb{G}^{(D)}_{J} \left(1 + \frac{2t}{s}\right)$$

S-channel:

$$\begin{aligned}
\mathbb{P}^{i_1 i_2; i_3 i_4}_{1} &= \frac{\delta^{i_1 i_2}\delta^{i_3 i_4}}{n}, \\
\mathbb{P}^{i_1 i_2; i_3 i_4}_{2} &= \frac{1}{2}\left(\delta^{i_1 i_4}\delta^{i_2 i_3} + \delta^{i_1 i_3}\delta^{i_2 i_4} - \frac{2}{n}\delta^{i_1 i_2}\delta^{i_3 i_4}\right) \\
\mathbb{P}^{i_1 i_2; i_3 i_4}_{3} &= \frac{1}{2}\left(\delta^{i_1 i_4}\delta^{i_2 i_3} - \delta^{i_1 i_3}\delta^{i_2 i_4}\right) .
\end{aligned}$$

The dispersion relation contains 3 equalities

$$\frac{B^{i_1 i_2 i_3 i_4}_{\text{poly}}(s,t)}{s(s+t)} - \frac{B^{i_1 i_2 i_3 i_4}_{\text{poly}}(0,t)}{st} + \frac{B^{i_1 i_2 i_3 i_4}_{\text{poly}}(-t,t)}{(s+t)t} - \frac{8\pi G}{t}\mathbb{P}^{i_2 i_3; i_4 i_1}_{0} =$$

$$n^{(D)}_{J,R} \sum_{JR} \mathbb{G}^{(D)}_{J}(cos(\theta)) \int_{M^2}^{\infty} \frac{ds'}{\pi s'(s'+t)} \text{Im}[f_{J,R}(s')] \left(\frac{\mathbb{P}^{i_1 i_2; i_3 i_4}_{R}}{(s'-s)} + \frac{\mathbb{P}^{i_1 i_3; i_2 i_4}_{R}}{(s'+t+s)} \right)$$

$$\mathbb{P}^{s} = M_{st}\mathbb{P}^{t}, \quad \mathbb{P}^{u} = M_{ut}\mathbb{P}^{t}$$

$$M_{st} = \begin{pmatrix} \frac{1}{2} - \frac{1}{n} & \frac{(n-1)(2+n)}{2n} & -\frac{2+n}{2n} \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ -\frac{1}{2} & \frac{1}{2}(n-1) & \frac{1}{2} \end{pmatrix}$$

The dispersion relation contains 3 equalities

$$\frac{B^{i_1 i_2 i_3 i_4}_{\text{poly}}(s,t)}{s(s+t)} - \frac{B^{i_1 i_2 i_3 i_4}_{\text{poly}}(0,t)}{st} + \frac{B^{i_1 i_2 i_3 i_4}_{\text{poly}}(-t,t)}{(s+t)t} - \frac{8\pi G}{t}\mathbb{P}_0^{i_2 i_3; i_4 i_1} =$$

$$n^{(D)}_{J,R} \sum_{JR} \mathbb{G}^{(D)}_J(cos(\theta)) \int_{M^2}^{\infty} \frac{ds'}{\pi s'(s'+t)} \text{Im}[f_{J,R}(s')] \left(\frac{\mathbb{P}_R^{i_1 i_2; i_3 i_4}}{(s'-s)} + \frac{\mathbb{P}_R^{i_1 i_3; i_2 i_4}}{(s'+t+s)} \right)$$

$$\mathbb{P}^s = M_{st}\mathbb{P}^t, \quad \mathbb{P}^u = M_{ut}\mathbb{P}^t$$

$$M_{st} = \begin{pmatrix} \frac{1}{2} - \frac{1}{n} & \frac{(n-1)(2+n)}{2n} & -\frac{2+n}{2n} \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ -\frac{1}{2} & \frac{1}{2}(n-1) & \frac{1}{2} \end{pmatrix}$$

$$\boxed{[M_{st}v]_i = [M_{ut}v]_i = 0}$$

Irreps	$\mathbf{v}$
$\{1\}$	$\left(-\frac{(n-1)(2+n)}{n-2}, 1, 0\right)$
$\{2\}$	$(-1, 1, 0)$
$\{3\}$	$(n-1, 1, 0)$

The UV completion of a gravitationally coupled fundamental matter demands at least 2 irreps

(Sing, Adj) (Anti, Adj) (Sing, Anti)

We now recycle the argument and consider (Sing, Adj) and let's scatter SO(n) Adj

$$\mathcal{A}^{i_1 i_2 i_3 i_4} = \mathcal{A}_{\text{Grav}}^{i_1 i_2 i_3 i_4} + B_{\text{poly}}^{i_1 i_2 i_3 i_4}$$

$$\mathcal{A}_{\text{Grav}}^{i_1 i_2 i_3 i_4} = 8\pi G_N \left(\frac{(t-u)^2}{s} \delta^{i_1 i_2} \delta^{i_3 i_4} + \text{Perm} \right) .$$

$$\text{adjoint}: \quad B_{\text{poly}}^{i_1 i_2 i_3 i_4}(s,t) = \sum_{\sigma \in S_3} \text{Tr}[i_1 \sigma(i_2) \sigma(i_3) \sigma(i_4)] B_1(1, \sigma(2), \sigma(3), \sigma(4))$$
$$+ B_2(s,t) \text{Tr}[i_1 i_2] \text{Tr}[i_3 i_4] + B_2(t,u) \text{Tr}[i_1 i_4] \text{Tr}[i_2 i_3] + B_2(u,t) \text{Tr}[i_1 i_3] \text{Tr}[i_2 i_4] .$$

We have 6 irreps that can be exchanged

$$\mathbb{P}_1^{i_1 i_2; i_3 i_4} = \frac{2}{n(n-1)} \text{Tr}[i_1, i_2] \text{Tr}[i_3, i_4] ,$$
$$\mathbb{P}_2^{i_1 i_2; i_3 i_4} = \frac{4}{(n-2)} \left(\text{Tr}[i_1, i_2, (i_3, i_4)] - \frac{1}{n} \text{Tr}[i_1, i_2] \text{Tr}[i_3, i_4] \right) ,$$
$$\mathbb{P}_3^{i_1 i_2; i_3 i_4} = \frac{2}{3} \left(\text{Tr}[i_1, (i_3] \text{Tr}[i_4), i_2] + \text{Tr}[i_1, i_4, i_2, i_3] \right) - \frac{4}{n-2} \text{Tr}[i_1, i_2, (i_3, i_4)] + \frac{2}{(n-1)(n-2)} \text{Tr}[i_1, i_2] \text{Tr}[i_3, i_4] ,$$
$$\mathbb{P}_4^{i_1 i_2; i_3 i_4} = \frac{1}{3} \left(\text{Tr}[i_1, (i_3] \text{Tr}[i_4), i_2] - 2 \text{Tr}[i_1, i_4, i_2, i_3] \right) ,$$
$$\mathbb{P}_5^{i_1 i_2; i_3 i_4} = \frac{2}{(n-2)} \text{Tr}[[i_1, i_2], [i_3, i_4]] ,$$
$$\mathbb{P}_6^{i_1 i_2; i_3 i_4} = \text{Tr}[i_1, [i_3] \text{Tr}[i_4], i_2], [i_3, i_4]] - \frac{2}{(n-2)} \text{Tr}[[i_1, i_2], [i_3, i_4]] .$$

(.

Using the fact that projectors are orthogonal we can put everything in the t-channel basis

$$\frac{B^{i_1 i_2 i_3 i_4}_{\text{poly}}(s,t)}{s(s+t)} - \frac{B^{i_1 i_2 i_3 i_4}_{\text{poly}}(0,t)}{st} + \frac{B^{i_1 i_2 i_3 i_4}_{\text{poly}}(-t,t)}{(s+t)t} - \frac{8\pi G}{t}\mathbb{P}_0^{i_2 i_3; i_4 i_1} =$$
$$n^{(D)}_{J,R} \sum_{JR} \mathbb{G}^{(D)}_J(cos(\theta)) \int_{M^2}^{\infty} \frac{ds'}{\pi s'(s'+t)} \text{Im}[f_{J,R}(s')] \left(\frac{\mathbb{P}_R^{i_1 i_2; i_3 i_4}}{(s'-s)} + \frac{\mathbb{P}_R^{i_1 i_3; i_2 i_4}}{(s'+t+s)} \right)$$

The matrix M_{st} for adjoint representation given as:

$$\begin{pmatrix} \frac{2}{(n-1)n} & \frac{2}{(n-1)n} & \frac{2}{(n-1)n} & \frac{2}{(n-1)n} & \frac{2}{(n-1)n} & \frac{2}{(n-1)n} \\ \frac{n+2}{n} & \frac{n^2-8}{2(n-2)n} & \frac{n-4}{(n-2)n} & \frac{2(n+2)}{(2-n)n} & \frac{(n-4)(n+2)}{2(n-2)n} & \frac{4}{(2-n)n} \\ \frac{(n-3)(n+1)(n+2)}{6(n-1)} & \frac{(n-4)(n-3)(n+1)}{6(n-2)(n-1)} & \frac{n^2-6n+11}{3(n-2)(n-1)} & \frac{(n+1)(n+2)}{3(n-2)(n-1)} & \frac{(n-3)(n+1)(n+2)}{6(2-n)(n-1)} & \frac{(n-4)(n+1)}{3(2-n)(n-1)} \\ \frac{(n-3)(n-2)}{12} & \frac{3-n}{6} & \frac{1}{6} & \frac{1}{6} & \frac{n-3}{6} & -\frac{1}{6} \\ 1 & \frac{n-4}{2(n-2)} & \frac{1}{2-n} & \frac{2}{n-2} & \frac{1}{2} & 0 \\ \frac{(n-3)(n+2)}{4} & \frac{n-3}{2-n} & \frac{n-4}{2(2-n)} & \frac{n+2}{2(2-n)} & 0 & \frac{1}{2} \end{pmatrix},$$

and similarly for M_{ut}:

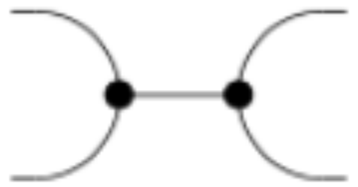
$$\begin{pmatrix} \frac{2}{(n-1)n} & \frac{2}{(n-1)n} & \frac{2}{(n-1)n} & \frac{2}{(n-1)n} & \frac{2}{(1-n)n} & \frac{2}{(1-n)n} \\ \frac{n+2}{n} & \frac{n^2-8}{2(n-2)n} & \frac{n-4}{(n-2)n} & \frac{2(n+2)}{(2-n)n} & \frac{(4-n)(n+2)}{2(n-2)n} & \frac{4}{(n-2)n} \\ \frac{n^3-7n-6}{6(n-1)} & \frac{(n-4)(n-3)(n+1)}{6(n-2)(n-1)} & \frac{n^2-6n+11}{3(n-2)(n-1)} & \frac{(n+1)(n+2)}{3(n-2)(n-1)} & \frac{n^3-7n-6}{6(n-2)(n-1)} & \frac{(n-4)(n+1)}{3(n-2)(n-1)} \\ \frac{(n-3)(n-2)}{12} & \frac{3-n}{6} & \frac{1}{6} & \frac{1}{6} & \frac{3-n}{6} & \frac{1}{6} \\ 1 & \frac{n-4}{2(n-2)} & \frac{1}{2-n} & \frac{2}{n-2} & -\frac{1}{2} & 0 \\ \frac{(n-3)(n+2)}{4} & \frac{n-3}{2-n} & \frac{n-4}{2(2-n)} & \frac{n+2}{2(2-n)} & 0 & -\frac{1}{2} \end{pmatrix}.$$

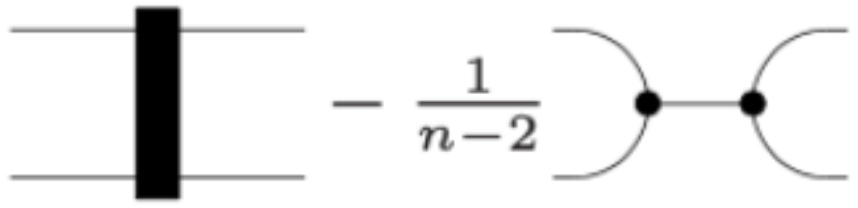
$$\boxed{[M_{st}v]_i = [M_{ut}v]_i = 0}$$

All subsets of 3 irreps and lower are ruled out
Except

{1,5,6}

$\mathbf{P}_1 = \frac{2}{n(n-1)}$  ,

$\mathbf{P}_5 = \frac{1}{n-2}$ 

$\mathbf{P}_6 =$ 

One must have a new irrep in the spectrum

reps	v
$\{1,2,3\}$	$\left(1, \frac{2-n-n^2}{-2+n}, \frac{2n+3n^2+n^3}{6(-2+n)}, \frac{1}{6}(n-n^2), 0, 0\right)$
$\{1,2,4\}$	$\left(1, \frac{4-5n+n^2}{2(-2+n)}, \frac{16n-3n^2-n^3}{12(-2+n)}, \frac{-n+n^2}{12}, 0, 0\right)$
$\{1,2,5\}$	$\left(1, -\frac{2(-1+n)}{-2+n}, \frac{n}{-2+n}, 0, 0, 0\right)$
$\{1,2,6\}$	$\left(1, \frac{8-6n-3n^2+n^3}{4(-2+n)}, \frac{16n+10n^2-n^3-n^4}{24(-2+n)}, \frac{2n-3n^2+n^3}{24}, 0, 0\right)$
$\{1,3,4\}$	$\left(1, \frac{4-5n+n^2}{2(-2+n)}, \frac{12n+5n^2-6n^3+n^4}{12(-2+n)}, \frac{-3n+4n^2-n^3}{12}, 0, 0\right)$
$\{1,3,5\}$	$\left(1, \frac{-16+16n+n^2-n^3}{(-8+n)(-2+n)}, \frac{4n+3n^2-n^3}{6(-2+n)}, \frac{2n-3n^2+n^3}{6(-8+n)}, 0, 0\right)$
$\{1,3,6\}$	$\left(1, \frac{-32+26n+11n^2-5n^3}{2(-8+n)(-2+n)}, \frac{6n+7n^2-n^4}{12(-2+n)}, \frac{-6n+11n^2-6n^3+n^4}{12(-8+n)}, 0, 0\right)$
$\{1,4,5\}$	$\left(1, \frac{4-5n+n^2}{2(-2+n)}, \frac{20n-9n^2+n^3}{12(-2+n)}, \frac{n-n^2}{12}, 0, 0\right)$
$\{1,4,6\}$	$\left(1, \frac{4-5n+n^2}{2(-2+n)}, \frac{24n-17n^2+6n^3-n^4}{12(-2+n)}, \frac{3n-4n^2+n^3}{12}, 0, 0\right)$
$\{2,3,4\}$	$\left(1, -\frac{2(-2+n+n^2)}{-4-n+n^2}, \frac{6n+7n^2-n^4}{3(-4-n+n^2)}, \frac{6n-11n^2+6n^3-n^4}{6(-4-n+n^2)}, 0, 0\right)$
$\{2,3,5\}$	$\left(1, -\frac{4(-2+n+n^2)}{-8+n+n^2}, \frac{8n+10n^2+n^3-n^4}{3(-8+n+n^2)}, \frac{2n-5n^2+4n^3-n^4}{6(-8+n+n^2)}, 0, 0\right)$
$\{2,3,6\}$	$\left(1, -\frac{2(8-6n-3n^2+n^3)}{16-9n+n^2}, \frac{2(-6n-7n^2+n^4)}{3(16-9n+n^2)}, \frac{6n-11n^2+6n^3-n^4}{6(16-9n+n^2)}, 0, 0\right)$
$\{2,4,5\}$	$\left(1, -\frac{4(-1+n)}{-4+n}, \frac{10n-n^3}{3(-4+n)}, \frac{-2n+3n^2-n^3}{6(-4+n)}, 0, 0\right)$
$\{2,4,6\}$	$\left(1, \frac{(-1+n)(-8-2n+n^2)}{-8+n^2}, \frac{48n+2n^2-9n^3+n^4}{6(-8+n^2)}, \frac{-6n+11n^2-6n^3+n^4}{3(-8+n^2)}, 0, 0\right)$
$\{2,5,6\}$	$\left(1, 0, \frac{-4+n^2}{3}, \frac{2-3n+n^2}{6}, 0, 0\right)$
$\{3,4,5\}$	$\left(1, -\frac{2(-4+n)(-1+n)}{8-5n+n^2}, \frac{-12n-5n^2+6n^3-n^4}{3(8-5n+n^2)}, -\frac{(-3n+n^2)(2-3n+n^2)}{6(8-5n+n^2)}, 0, 0\right)$
$\{3,4,6\}$	$\left(1, \frac{-2+n+n^2}{2}, \frac{-6n-7n^2+n^4}{12}, \frac{6n-11n^2+6n^3-n^4}{12}, 0, 0\right)$
$\{3,5,6\}$	$\left(1, -\frac{2(-4+n^2)}{-8+n}, 0, \frac{6n-5n^2+n^3}{2(-8+n)}, 0, 0\right)$
$\{4,5,6\}$	$\left(1, -1+n, \frac{-3n+n^2}{2}, 0, 0, 0\right)$

Comment1: The ``completeness hypothesis" in bootstrap

Find a vector $v = (1, \cdots)$ such that

$$[M_{st}v]_i = [M_{ut}v]_i = 0$$

We consider the image of the s-and u-channel projectors in the full color space. If the complement of a certain set of projectors contains the t-channel singlet, then a spectrum that only includes the said set is inconsistent.

For the conjecture to hold, one should then prove that if any of the irreps is removed from the spectrum, one can always identify a particular scattering where said irreps is allowed, and its absence will lead to the graviton singlet living in the complement of remaining irreps.

Comment2: the story is completely different for vector poles

What if the massless pole is in the adjoint representation ?

For spin-1 we will need one subtraction to pick up $\frac{s}{t}$ let's assume it is valid

Representations	$\mathbf{v}$
$\{1\}$	$(0,0,0,0,1,-1)$
$\{2\}$	$\left(0,0,0,0,1,\frac{-8-2n+n^2}{8}\right)$
$\{3\}$	$\left(0,0,0,0,1,\frac{6+n-n^2}{2(-4+n)}\right)$
$\{4\}$	$(0,0,0,0,1,-3+n)$
$\{6\}$	$(0,0,0,0,1,0)$

All single irrep completions are ruled out except adjoint, it is closed

Comments:

- Assuming twice subtraction, and crossing, the fact that gravity is long range impose constraint on the spectrum of UV Completion

- The fact that graviton is spin-2 is also important in that it actually enters the dispersion relation.

- Iterating the procedure we see that the necessary spectrum is not closed

Current/Next stage:

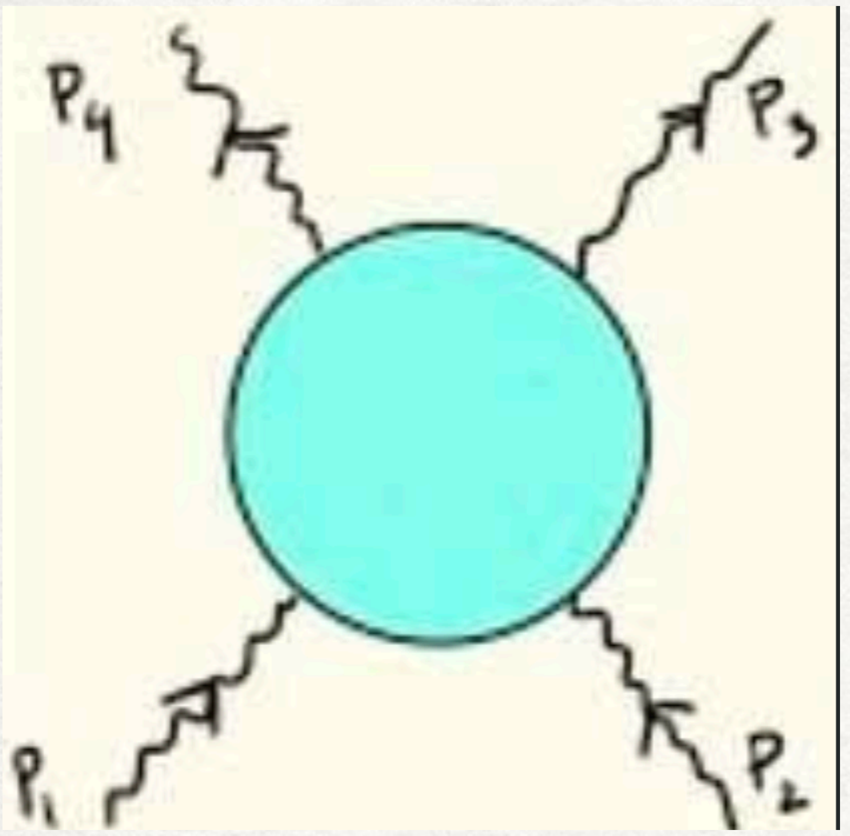
- (Global vs Gauge): We are doing twice subtraction, which does not capture the massless gauge pole. There is no-distinction between local and global symmetries.

- Consider one subtraction for certain smeared amplitudes (Haring, Zhiboedov 2202.08280)

- Consider four-dimensional helicity states

If the symmetry is gauge, in known models of quantum gravity (string theory), gauge groups are constrained $SO(32)$ or $E_8 \times E_8$

This constraint is directly visible in the S-matrix



$$\mathcal{A}(1^{-a}, 2^{-b}, 3^{+c}, 4^{+d}) = \langle 12 \rangle^2 [34]^2 \left[\frac{1}{M_P^2} \left(\frac{\mathbb{P}_1^s}{s} + \frac{\mathbb{P}_1^t}{t} + \frac{\mathbb{P}_1^u}{u} \right) + \frac{g_{\mathrm{YM}}^2}{3} \left(\frac{\mathbb{P}_{\mathrm{Adj}}^s - \mathbb{P}_{\mathrm{Adj}}^t}{st} + \frac{\mathbb{P}_{\mathrm{Adj}}^t - \mathbb{P}_{\mathrm{Adj}}^u}{tu} + \frac{\mathbb{P}_{\mathrm{Adj}}^u - \mathbb{P}_{\mathrm{Adj}}^s}{su} \right) \right]$$

$$\begin{cases} \mathbb{P}_1^s & \delta^{ab}\delta^{cd} \\ \mathbb{P}_{\mathrm{Adj}}^s & f^{abe}f^{edc} \end{cases}$$

UV complete

$$\mathcal{A}^{\mathrm{UV}}(1^{-a}, 2^{-b}, 3^{+c}, 4^{+d}) = \Gamma^{\mathrm{str}} \mathcal{A}(1^{-a}, 2^{-b}, 3^{+c}, 4^{+d})$$

$$\Gamma^{\mathrm{str}} = -\frac{\Gamma(-\alpha' s)\,\Gamma(-\alpha' t)\,\Gamma(-\alpha' u)}{\Gamma(\alpha' s)\,\Gamma(\alpha' t)\,\Gamma(\alpha' u)}$$

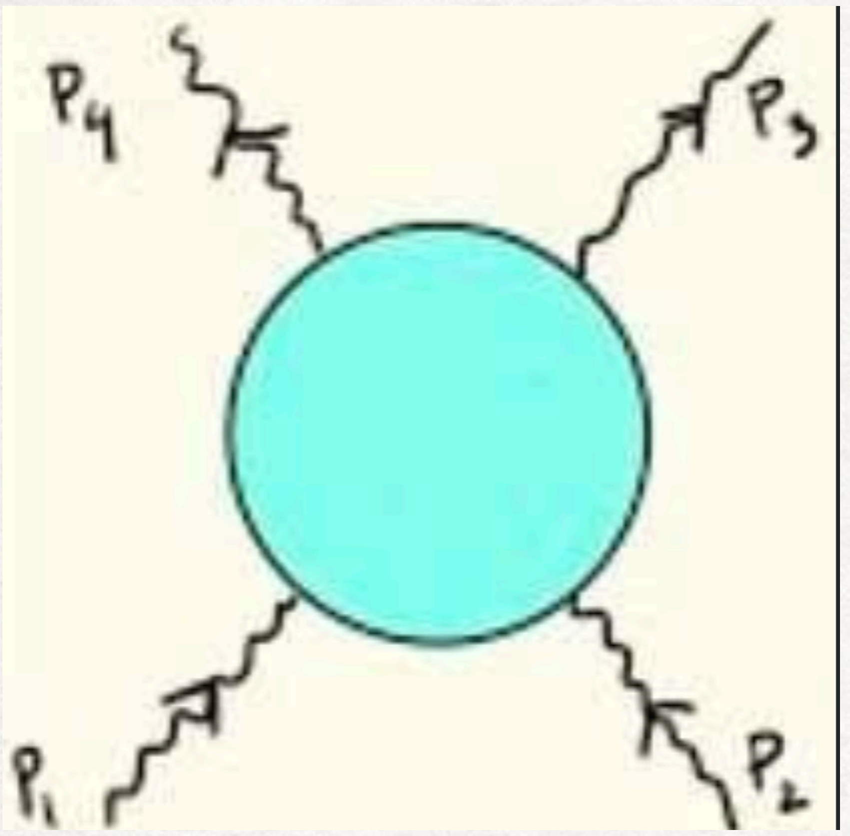
Require the residues on factorization poles to be consistent with unitarity

$$\lim_{s \to m^2} \mathcal{A}(1^a 2^b 3^c 4^d) \sim \frac{1}{s - m^2} \sum_J \rho_{J,\alpha} \mathbb{P}_\alpha^{abcd} \mathbb{G}_j(\cos\theta)$$

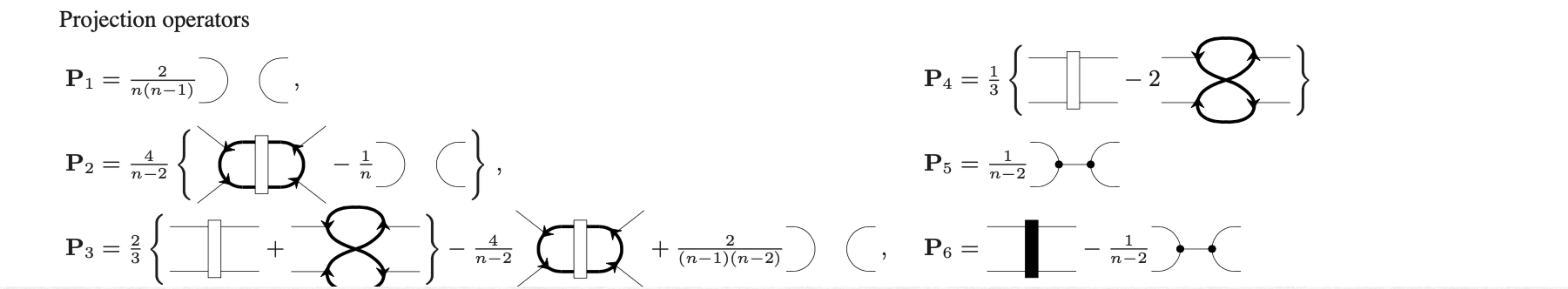
$$\rho_{J,\alpha} > 0$$

If the symmetry is gauge, in known models of quantum gravity (string theory), gauge groups are constrained $SO(32)$ or $E_8 \times E_8$

This constraint is directly visible in the S-matrix



$$\lim_{s\to m^2} M(1^a 2^b 3^c 4^d) \sim \frac{1}{s-m^2}\sum_J \rho_{J,\alpha}\mathbb{P}^{abcd}_{\alpha}\,\mathbb{G}_j(\cos\theta)$$



Level 1

$$\left\{\frac{-1+8\,\mathrm{gym}^2\,\mathrm{Mp}^2+\frac{32}{(-1+n)\,n}+x^2}{8\,\mathrm{Mp}^2},\ \frac{\mathrm{gym}^2\,(-4+n)}{2\,(-2+n)}+\frac{4}{\mathrm{Mp}^2\,(-1+n)\,n},\ -\frac{\mathrm{gym}^2}{-2+n}+\frac{4}{\mathrm{Mp}^2\,(-1+n)\,n},\ \frac{2\,\mathrm{gym}^2}{-2+n}+\frac{4}{\mathrm{Mp}^2\,(-1+n)\,n},\ \frac{\mathrm{gym}^2\,x}{2},\ 0\right\}$$

→

$$\left\{\mathrm{gym}^2\,\mathrm{Mp}^2-\frac{1}{9}+\frac{4}{(-1+n)\,n}>0,\ \frac{4\,(-2+n)}{(-1+n)\,n}>\mathrm{gym}^2\,\mathrm{Mp}^2\right\}$$

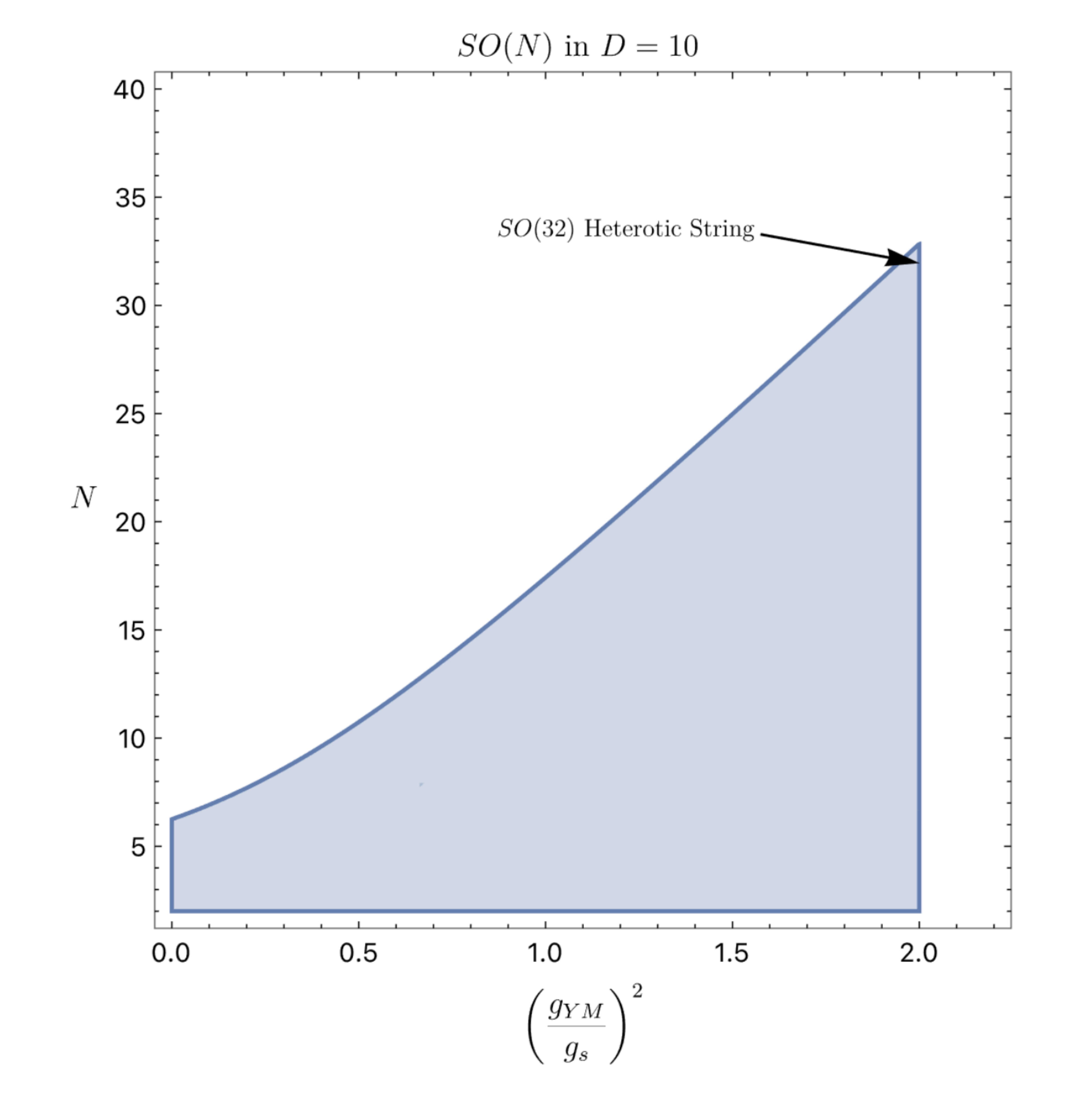
No solutions for n> 35

Require the residues on factorization poles to be consistent with unitarity

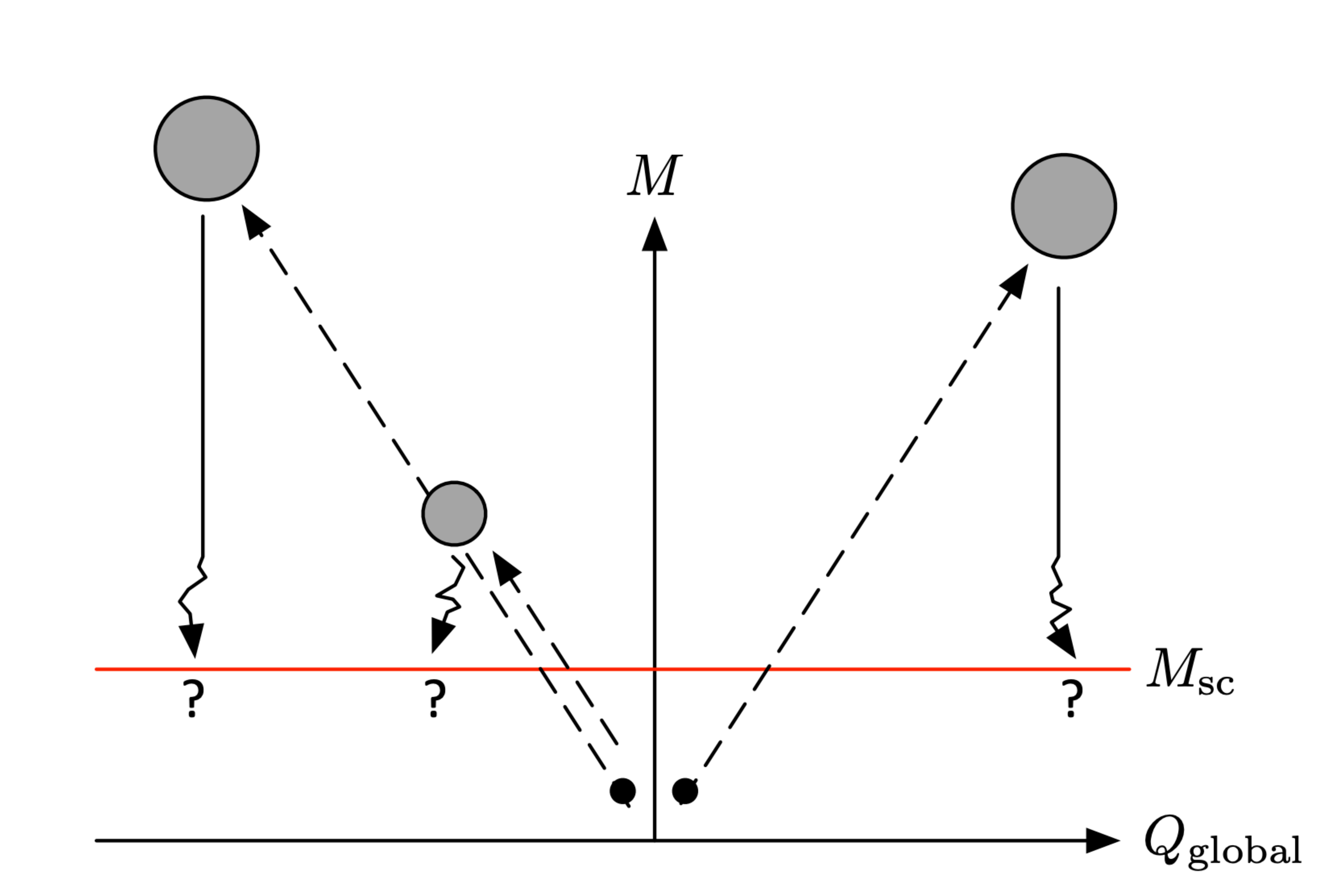
$$\lim_{s\to m^2} \mathcal{A}(1^a 2^b 3^c 4^d) \sim \frac{1}{s-m^2} \sum_J \rho_{J,\alpha} \mathbb{P}_\alpha^{abcd}\, \mathbb{G}_j(\cos\theta)$$

$$\rho_{J,\alpha} > 0$$

Brad Bachu, Aaron Hillman, 2212.03871



It is generally believed that all global symmetries are broken, or become gauged, in the full theory of quantum gravity

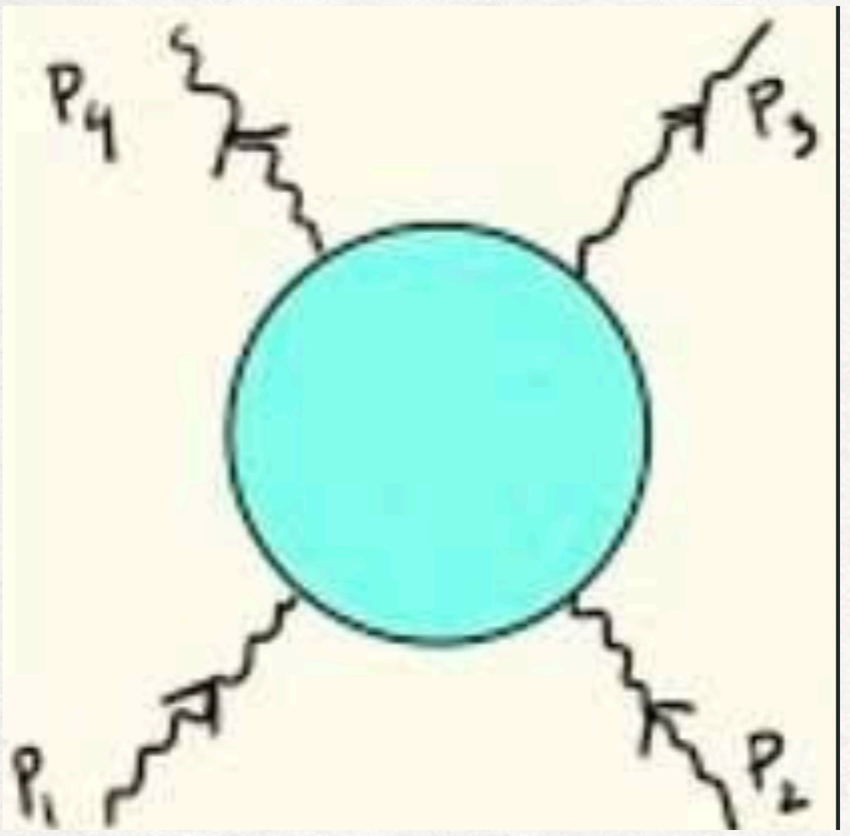


T. Banks and N. Seiberg, "Symmetries and Strings in Field Theory and Gravity," *Phys. Rev.* **D83** (2011) 084019, arXiv:1011.5120 [hep-th].

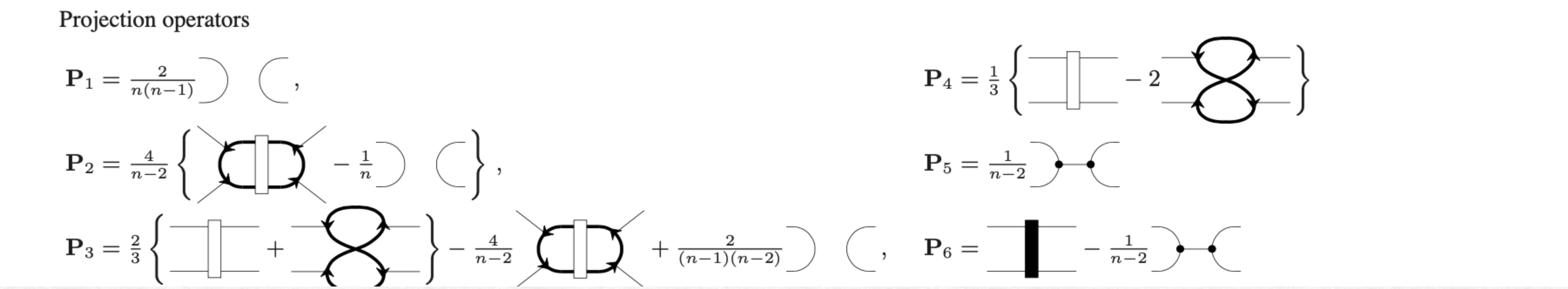
The gravitational collapse of global-charged objects creates black holes of arbitrarily large global charge. After Hawking radiation, this leads to an infinite number of microstates violating he Bekenstein-Hawking entropy formula

If the symmetry is gauge, in known models of quantum gravity (string theory), gauge groups are constrained SO(32) or $E_8 \times E_8$

This constraint is directly visible in the S-matrix



$$\lim_{s\to m^2} M(1^a 2^b 3^c 4^d) \sim \frac{1}{s-m^2}\sum_J \rho_{J,\alpha}\mathbb{P}^{abcd}_\alpha\,\mathbb{G}_j(\cos\theta)$$



Level 1

$$\left\{\frac{-1+8\,\mathrm{gym}^2\,\mathrm{Mp}^2+\frac{32}{(-1+n)\,n}+x^2}{8\,\mathrm{Mp}^2},\ \frac{\mathrm{gym}^2\,(-4+n)}{2\,(-2+n)}+\frac{4}{\mathrm{Mp}^2\,(-1+n)\,n},\ -\frac{\mathrm{gym}^2}{-2+n}+\frac{4}{\mathrm{Mp}^2\,(-1+n)\,n},\ \frac{2\,\mathrm{gym}^2}{-2+n}+\frac{4}{\mathrm{Mp}^2\,(-1+n)\,n},\ \frac{\mathrm{gym}^2\,x}{2},\ 0\right\}$$

→

$$\left\{\mathrm{gym}^2\,\mathrm{Mp}^2-\frac{1}{9}+\frac{4}{(-1+n)\,n}>0,\ \frac{4\,(-2+n)}{(-1+n)\,n}>\mathrm{gym}^2\,\mathrm{Mp}^2\right\}$$

No solutions for n> 35

A proof of species bound

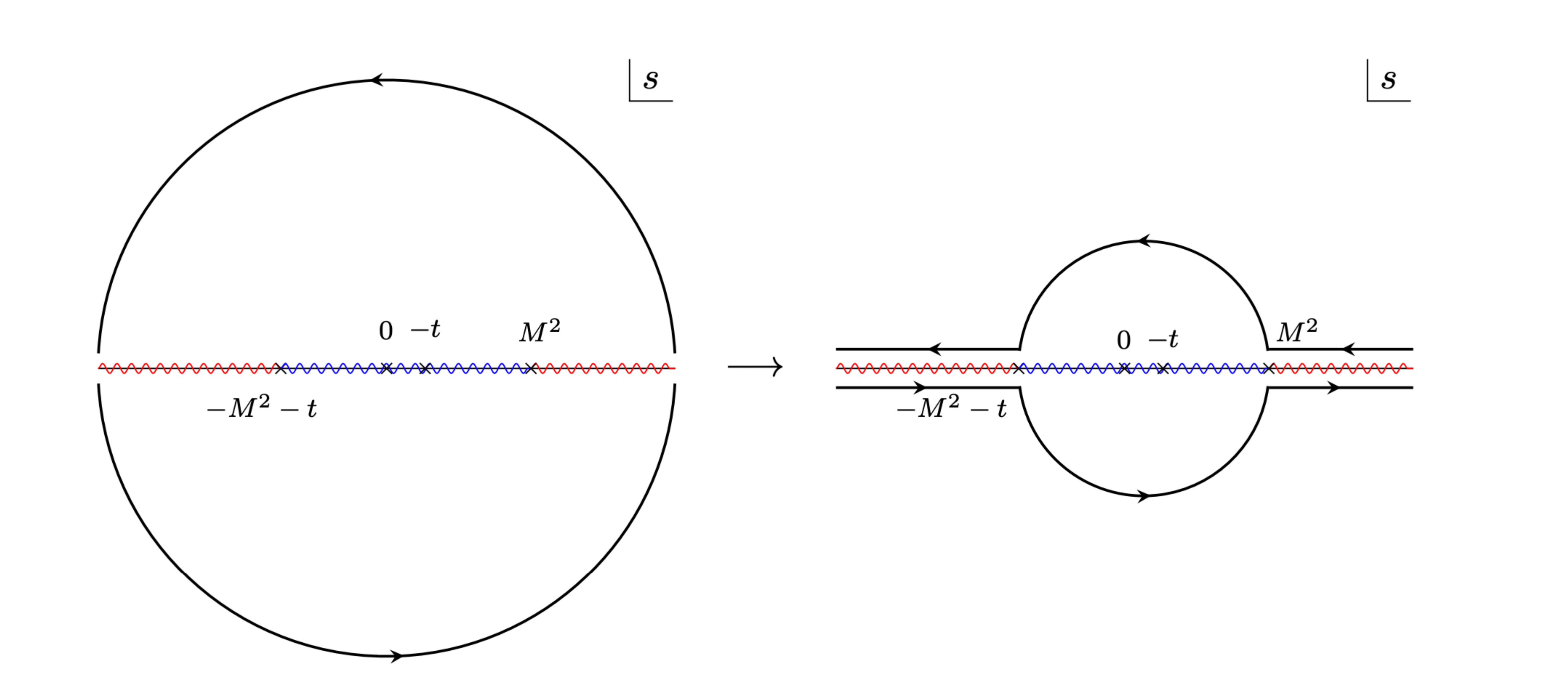
Simon Caron-Huot, Yue-Zhou Li arXiv:2408.06440

A longstanding observation is that in the presence of a large number N of matter fields below the scale Λ, the "quantum gravity cutoff" Λ should be parametrically lower than the Planck mass

$$\Lambda^{d-2} N(\Lambda) \lesssim \mathcal{O}(1) M_{\rm pl}^{d-2}$$

Consider the four-graviton amplitude $\mathcal{M}(1^+2^-3^-4^+) = \langle 23\rangle^4 [14]^4 f(s,u)$

Which satisfies Kramers-Kronig-type sum rules



$$B_2(p) \equiv \oint_{\mathcal{C}_+\cup\mathcal{C}_-} \frac{ds}{2\pi i}(s-t) f(s, u=-p^2) = 0$$

$$\longrightarrow \quad \frac{8\pi G}{p_\perp^2} = \int \frac{ds}{\pi}(2s-p^2)\,\mathrm{Im}\, f(s,-p_\perp^2)$$

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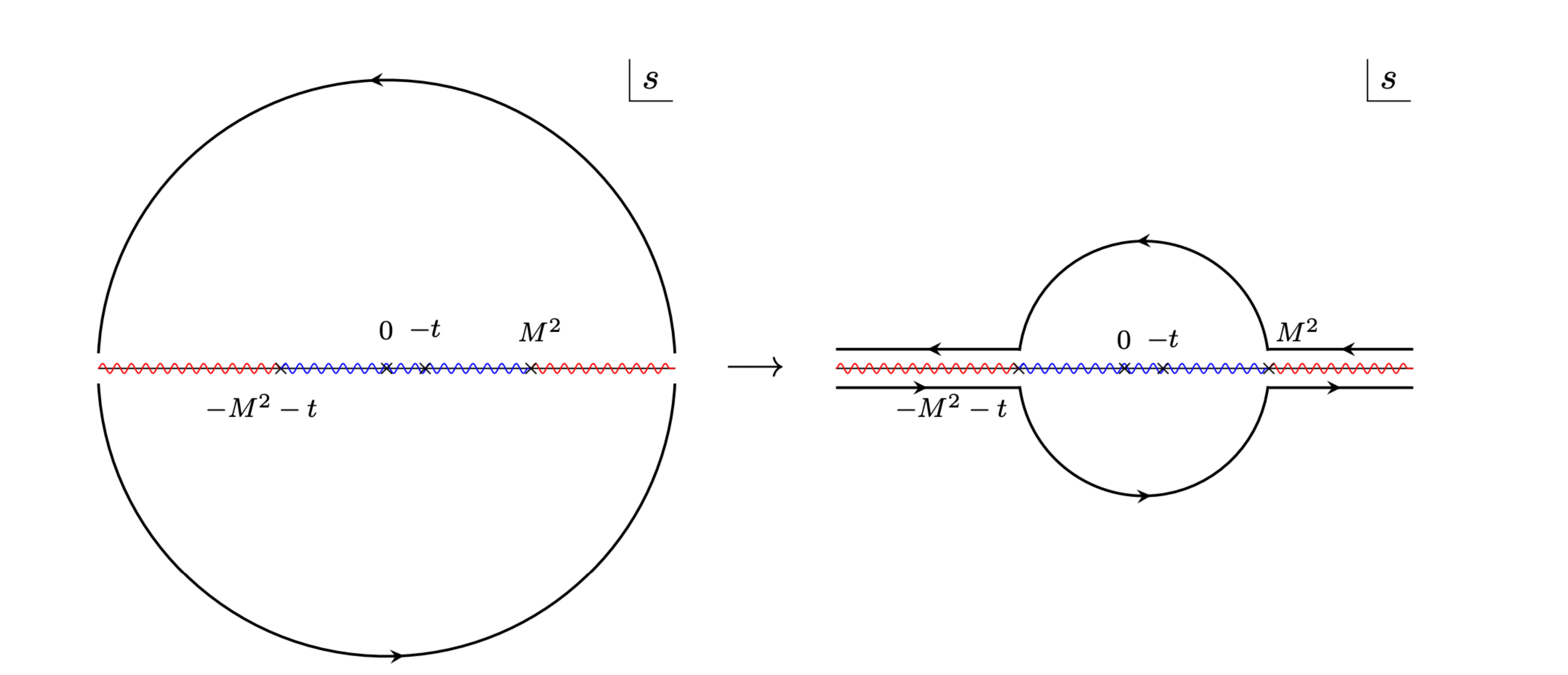
Consider the four-graviton amplitude

$$\mathcal{M}(1^+2^-3^-4^+) = \langle 23\rangle^4[14]^4 f(s,u)$$

$$-\sum_{k=2,3}\int_0^M pdp\,\psi_k(p)B_k(p)\Big|_{\rm low} = \sum_{k=2,3}\int_0^M pdp\,\psi_k(p)B_k(p)\Big|_{\rm high}$$

$$-B_2(p)\Big|_{\rm low} = \sum_{\pm}\int_{M^2}^{p^2-M^2}\frac{ds}{2\pi i}(p^2-2s)f(s,-p^2) = \frac{8\pi G}{p^2} + \text{loops},$$

$$B_2(p)\Big|_{\rm high} = 16\int_{M^2}^{\infty}\frac{ds}{s^4}\,(2s-p^2)\left[\sum_{J\geq 0,\,\text{even}}|\bar{c}_J^{++}(s)|^2 P_J\left(1-\tfrac{2p^2}{s}\right) + \sum_{J\geq 4}|\bar{c}_J^{+-}(s)|^2\tilde{d}^J_{4,4}\left(1-\tfrac{2p^2}{s}\right)\right]$$



A proof of species bound

Simon Caron-Huot, Yue-Zhou Li arXiv:2408.06440

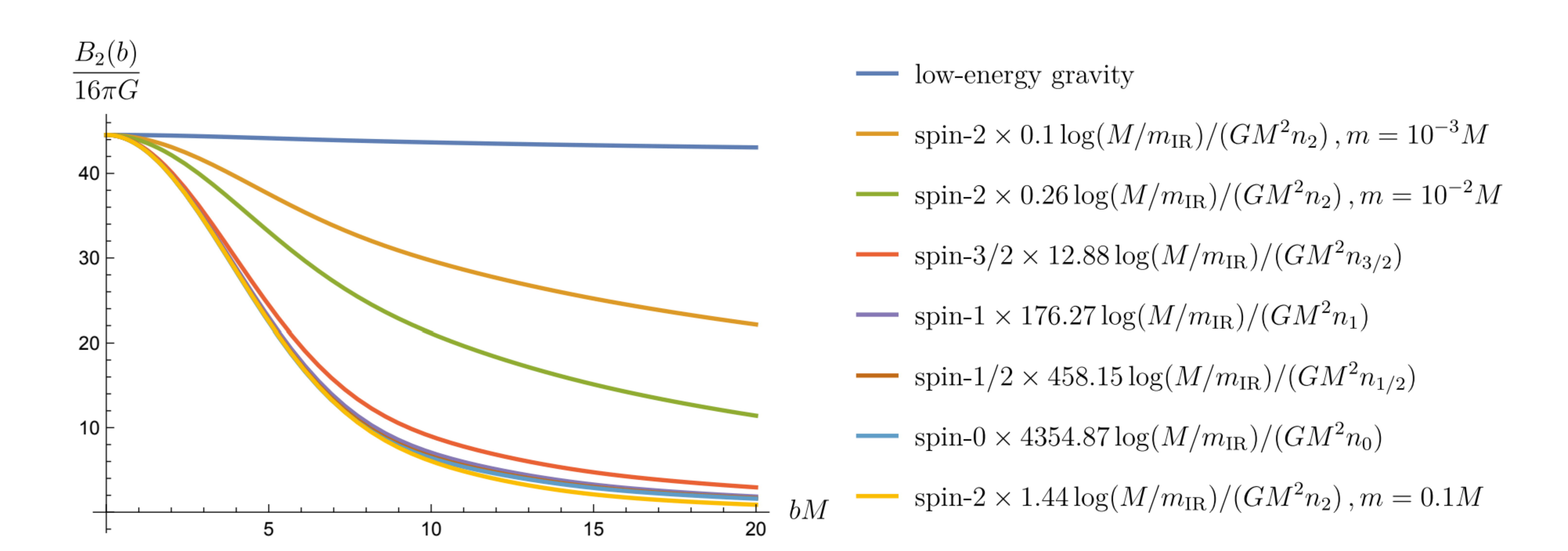
A longstanding observation is that in the presence of a large number N of matter fields below the scale Λ, the "quantum gravity cutoff" Λ should be parametrically lower than the Planck mass

$$\Lambda^{d-2}N(\Lambda) \lesssim \mathcal{O}(1)M_{\rm pl}^{d-2}$$

$$B_2(p) \equiv \oint_{\mathcal{C}_+\cup\mathcal{C}_-} \frac{ds}{2\pi i}(s-t)f(s,u=-p^2),$$

smeared bounds →

$$B_2(b) = \int_0^M dp\,(1-p)^2 p J_0(bp) B_2(p)$$



Exp

$$n_0 + 5.6n_{1/2} + 9.2n_1 + 157n_{3/2} + 491.7n_2 \log\frac{M}{m_2} < \left(1429.6\log\frac{M}{m_{\rm IR}} - 1735.6\right)\frac{1}{GM^2}$$

Plan:

Consider a general colored EFT (Adjoint or Fundamental), derive optimal bounds on the Wilson coefficients

$$M^{abcd}(s,t) = g^2\left(P^s_{adj}\frac{t-u}{s} + P^t_{adj}\frac{u-s}{t} + P^u_{adj}\frac{s-t}{u}\right) + 8\pi G\left(P^s_I\frac{tu}{s} + P^t_I\frac{us}{t} + P^u_I\frac{st}{u}\right) + B^{abcd}(s,t),$$

$$B^{abcd}(s,t) = \sum_{\sigma\in S_3}\mathrm{Tr}(T^aT^{\sigma(b)}T^{\sigma(c)}T^{\sigma(d)})B_1(1,\sigma(2),\sigma(3),\sigma(4)) + B_2(s,t)\mathrm{Tr}(T^aT^b)\mathrm{Tr}(T^cT^d) + B_2(t,u)\mathrm{Tr}(T^aT^c)\mathrm{Tr}(T^bT^d) + B_2(u,s)\mathrm{Tr}(T^aT^d)\mathrm{Tr}(T^cT^b).$$

$$B_1(1234) = \sum_{k,q\leq k,q\in\text{even}} g_{kq}t^{k-q}(s-u)^q,$$

$$B_2(s,t) = \sum_{k,q\leq k,q\in\text{even}} G_{kq}s^{k-q}(t-u)^q,$$

We can also consider supersymmetry. Consider maximal SUSY (16 super charges)

$$M^{abcd} = \delta^8(Q) f(s,t) \qquad \delta^8(Q) \sim s^2$$

This leads to zero subtraction dispersion relations for the couplings since

$$\lim_{s\to\infty} f(s,t) < s^0$$

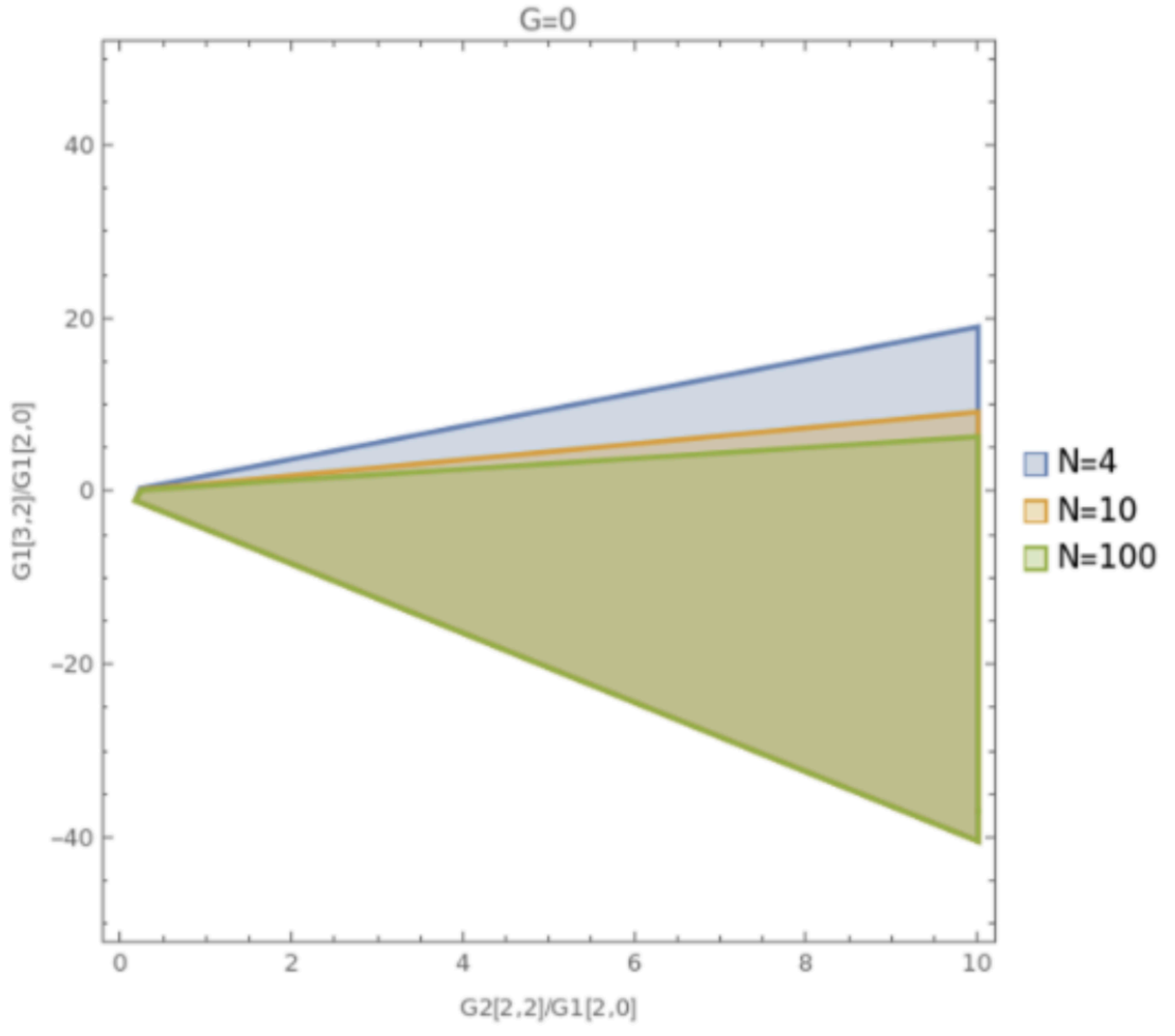


Figure 5. Adjoint representation, maximal SUSY, $G = 0$

We see N dependence even without Gravity !

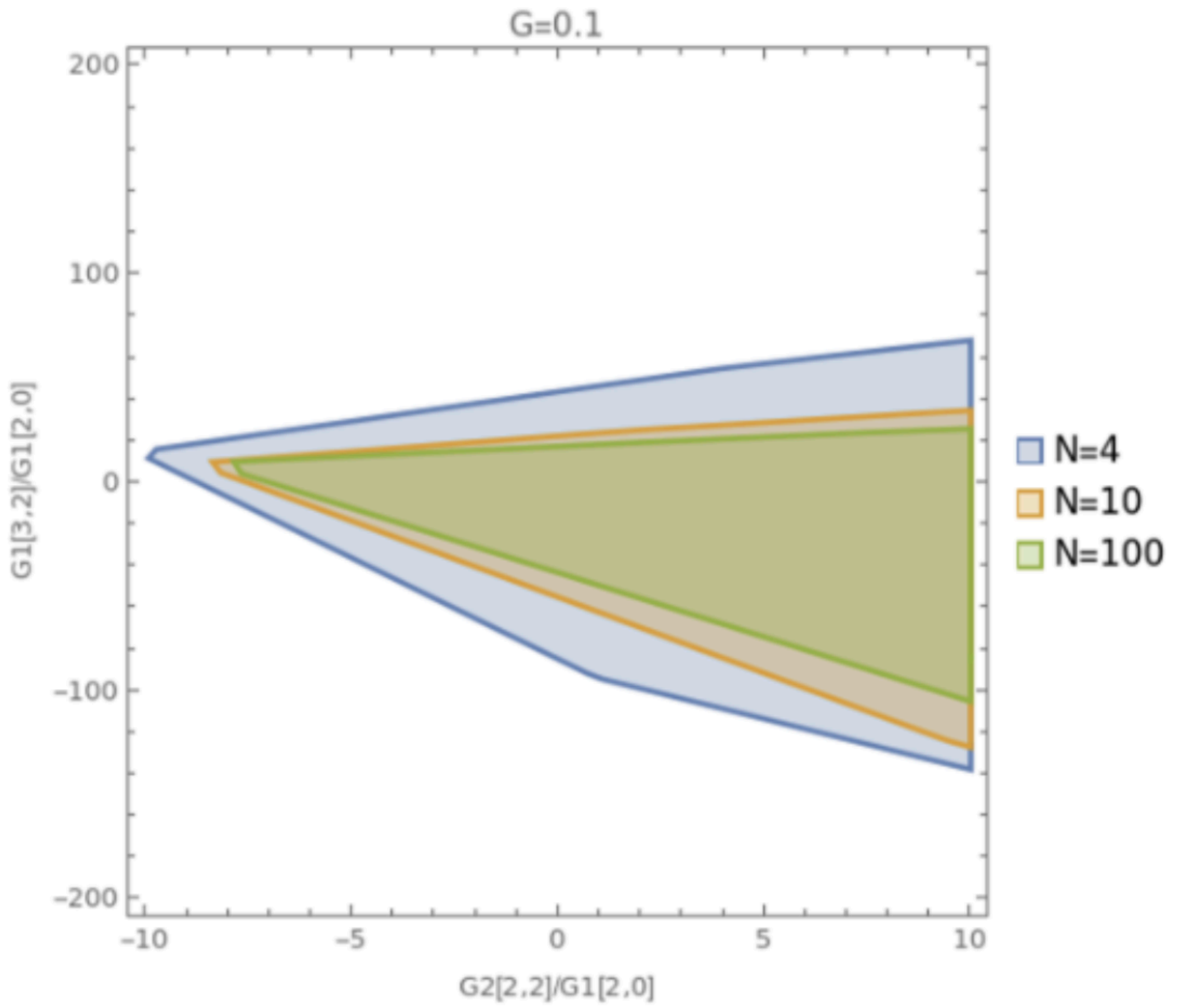
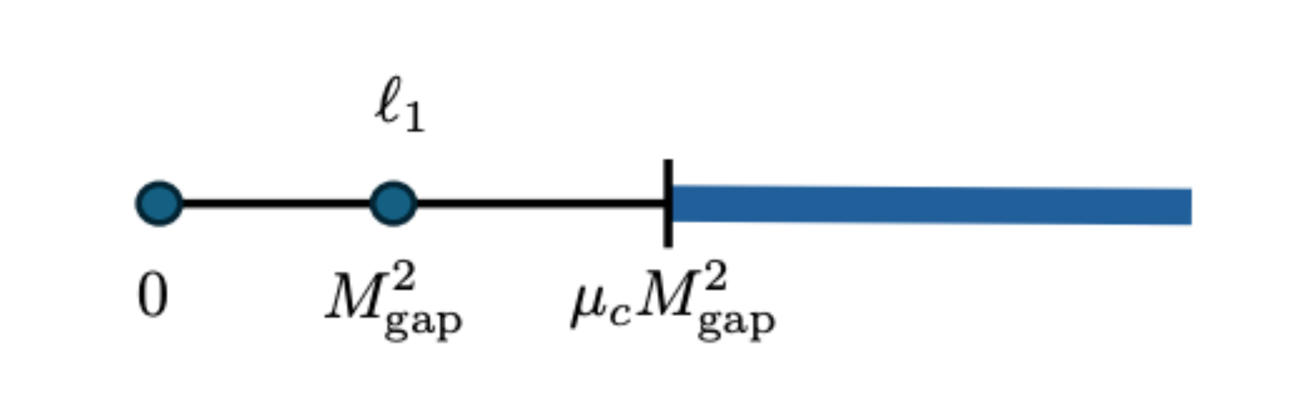
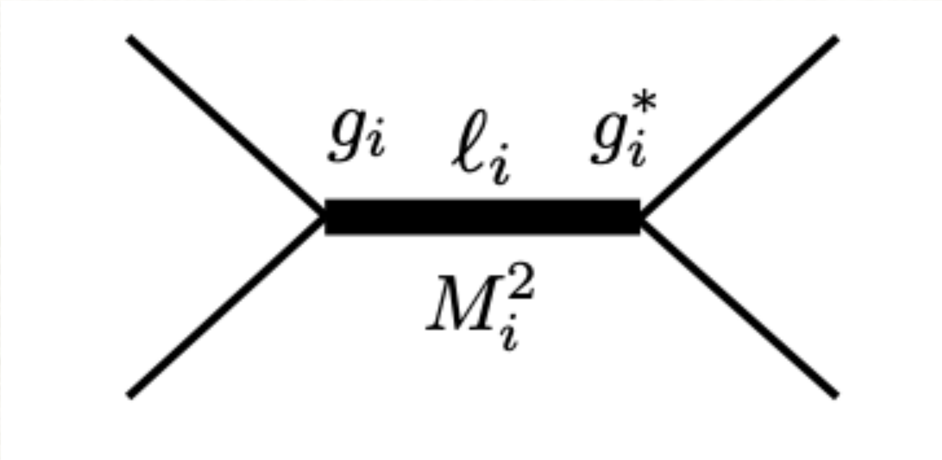


Figure 6. Adjoint representation, maximal SUSY, $G = 0.1$

Additional N dependence when gravity is turned on

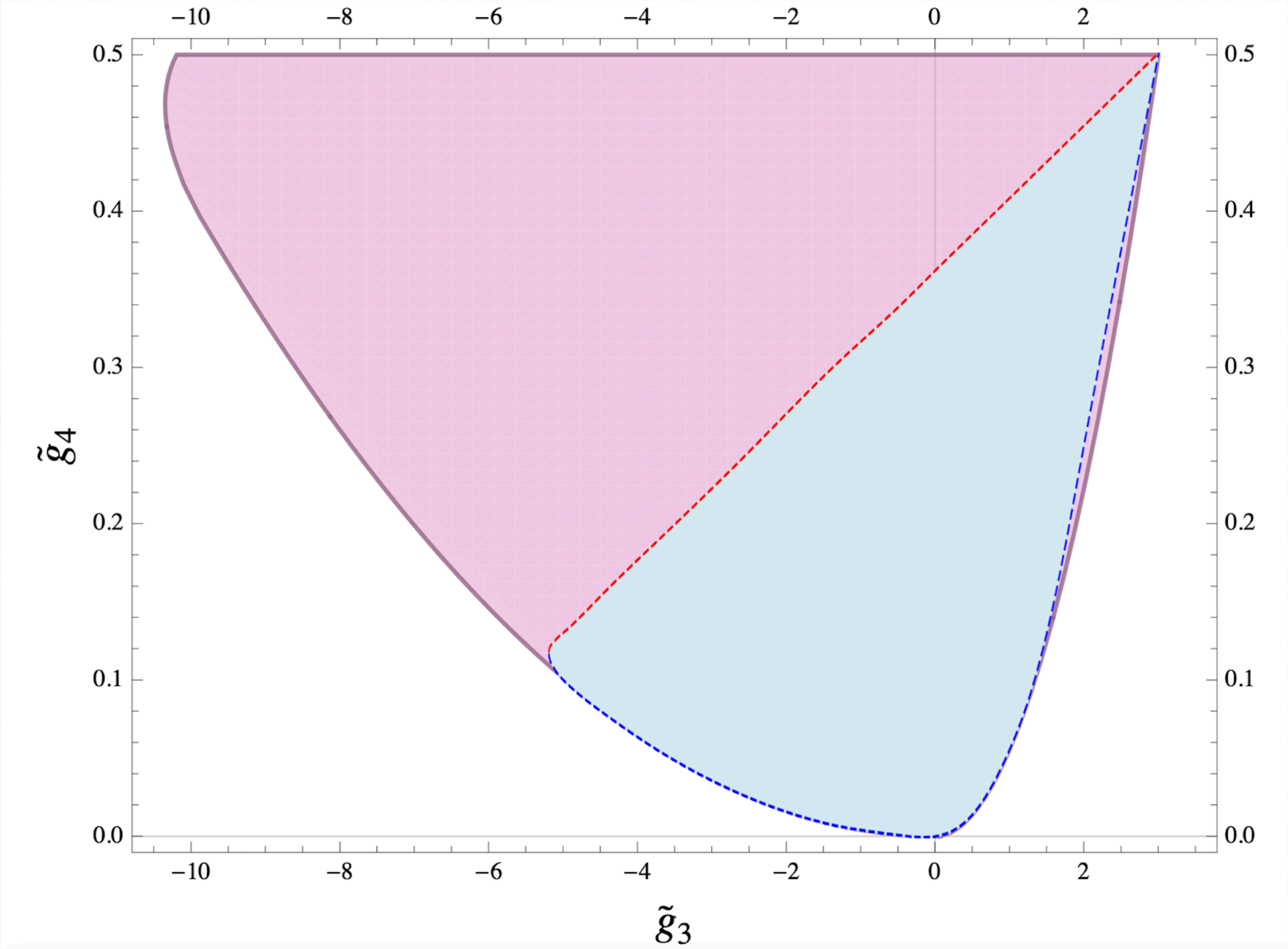
Let us assume that at the gap, there is an isolated state



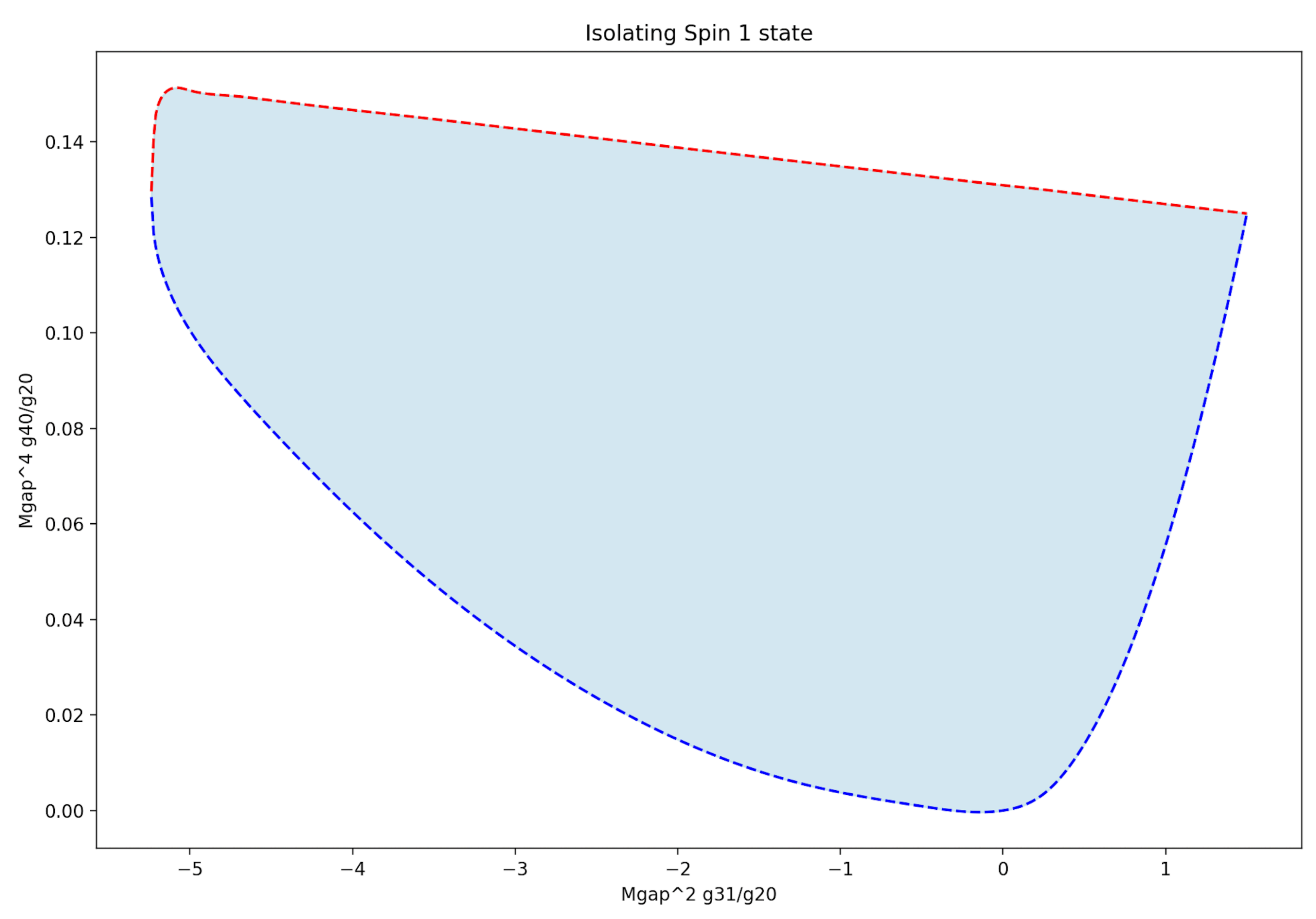


Assuming just a single scalar state, vs higher spin

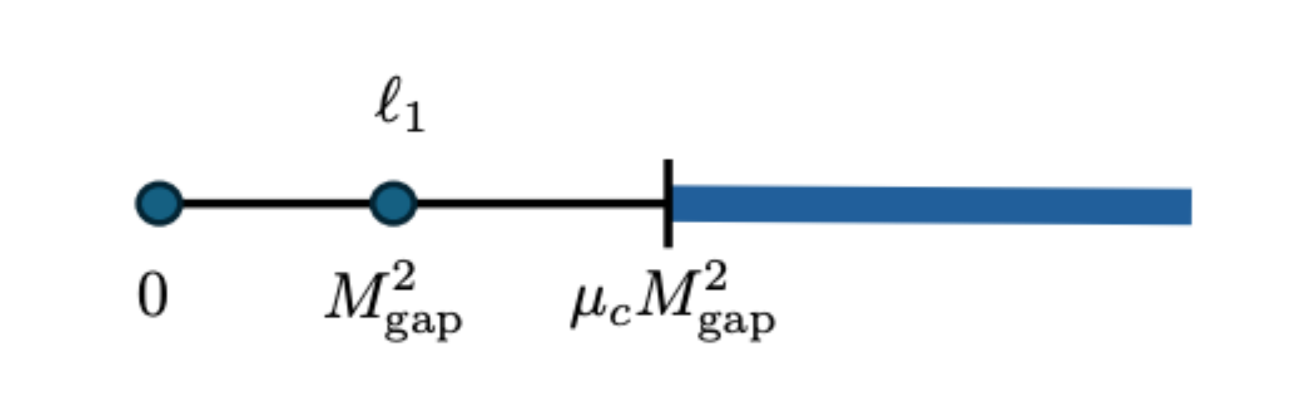
S=0

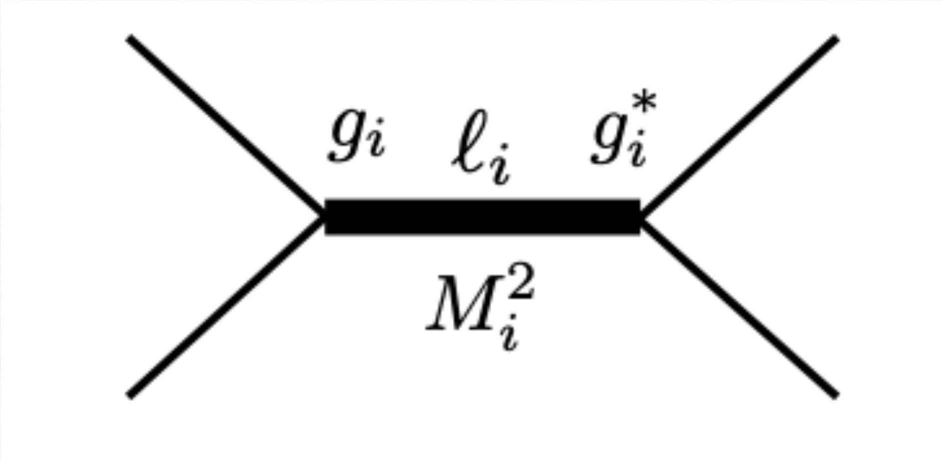


S=1

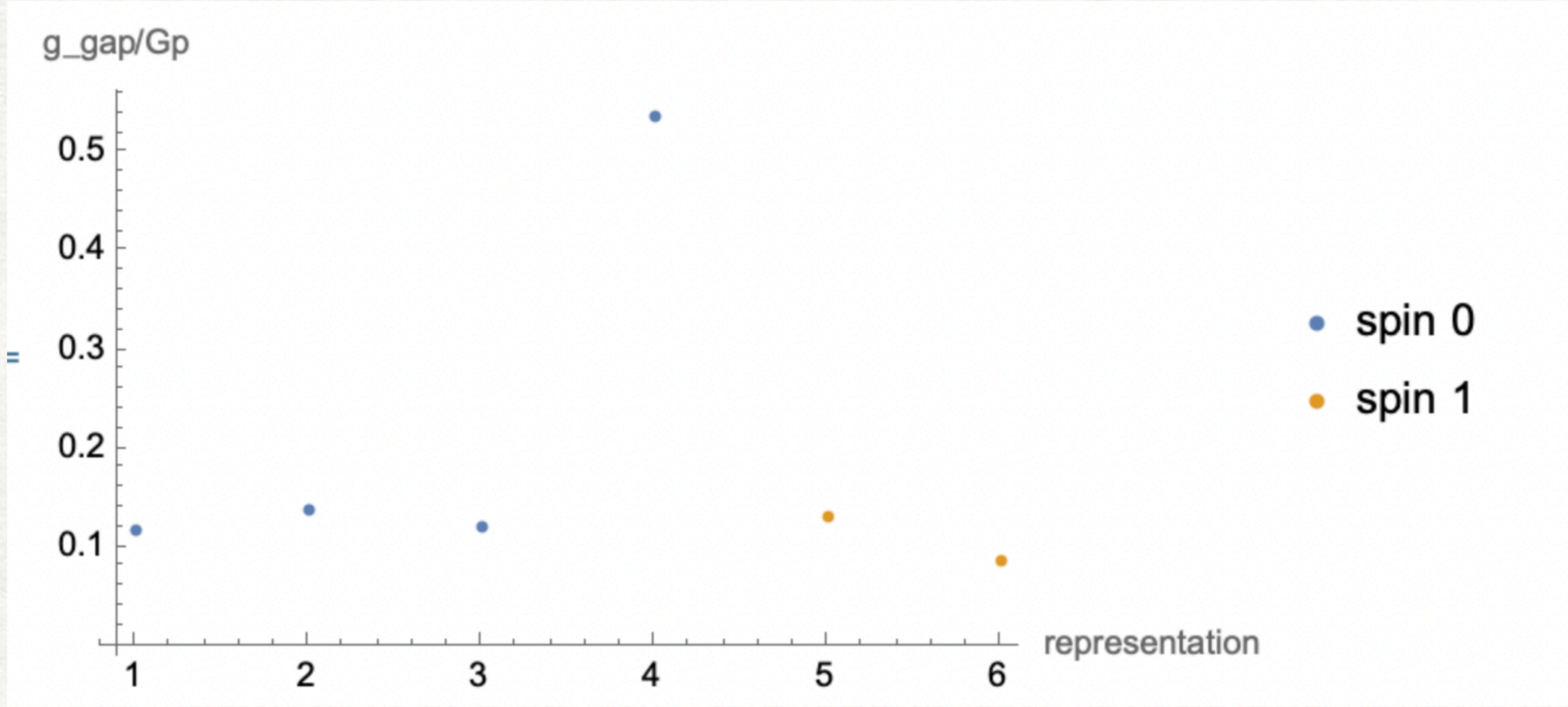


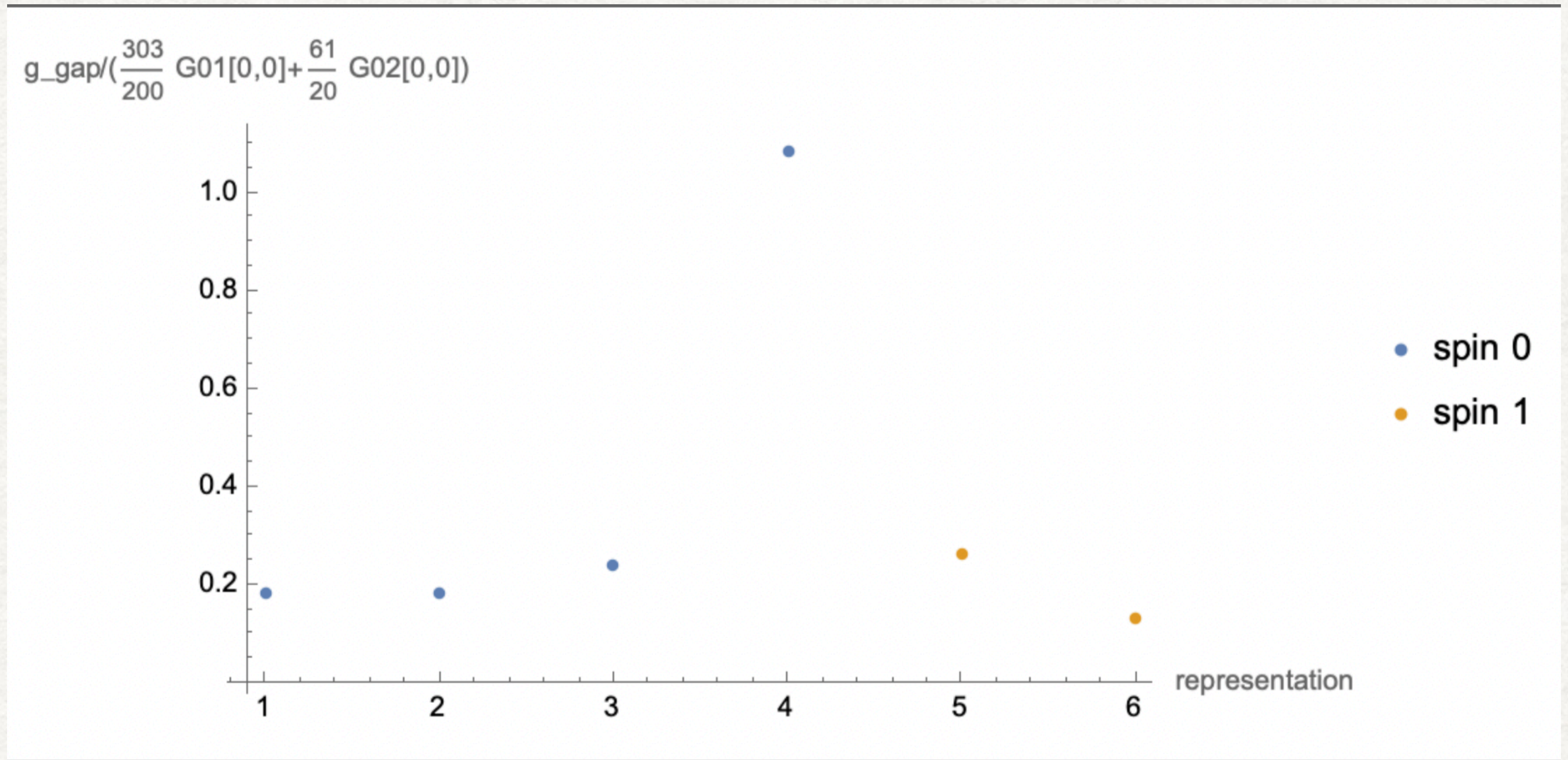
Let us assume that at the gap, there is an isolated state





We can probe the maximal value of the coupling vs EFT Wilson coefficients





Dominated by the symmetric representation !