Einstein Static Universe in String Theory

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work in progress with J. Heckman and N. Macpherson

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Introduction

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- used in GR singularity theorems [Hawking, Penrose '70...]
- most matter fields satisfy it

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$$\Lambda g^4_{00}$$

total derivative

$$\geq 0$$

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- In string theory, various possible loopholes:
  - Romans mass: but it doesn't help [Maldacena, Nuñez '00]
  - O-planes
  - Stringy corrections to EH action
  - U-duality transition functions?

Expand our focus: FLRW

$$-\mathrm{d}t^2 + a^2(t)\underbrace{\mathrm{d}s^2_{\mathrm{MS}_3}}_{\text{maximally symmetric}}$$

- Taking $\mathrm{d}s_4^2 = \mathrm{FLRW} \Rightarrow \; a'' \leq 0$

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- But: acceleration becomes possible if internal metric depends on time

[Russo, Townsend '18; Marconnet, Tsimpis '22; Andriot, Tsimpis, Wrase '24...]

- Even laxer: different warping factors for time and space...

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- Recent idea: start from Einstein Static Universe (ESU)

[Heckman, Lawrie, Lin, (Sakstein,) Zoccarato '18,'19]

    - It can be supersymmetric
    - Its perturbations can lead to accelerated expansion
    - Realized from NS5 backreaction?

This talk: a **classification** of supersymmetric ESU

Plan:

- Why ESU?
- How to impose supersymmetry
- (Preliminary) results

ESU

- Recall original ESU:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\frac{k + \dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho$$

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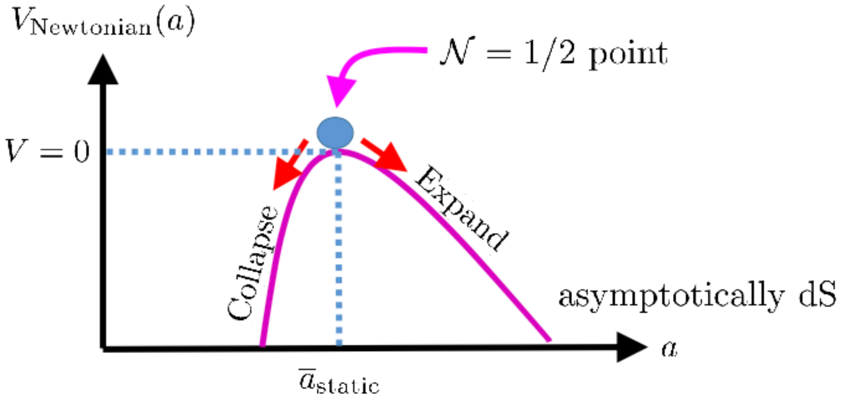
- Unstable: if we do realize it, a perturbation might lead to a more realistic cosmology



[Heckman, Lawrie, Lin, (Sakstein,) Zoccarato '18,'19]

- How to realize it in string theory?

near-horizon limit of an NS5:
linear-dilaton background

$$\mathrm{d}s^2 = \mathrm{d}s^2_{\text{Mink}_4} + \mathrm{d}x^2 + \mathrm{d}y^2 + N(\mathrm{d}\rho^2 + \mathrm{d}s^2_{S^3})$$
$$\mathrm{e}^{\phi} = \sqrt{N}\mathrm{e}^{-\rho}\,, \qquad H = 2N\mathrm{vol}_{S^3}$$

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- so perhaps let's wrap NS5's on a cycle inside a CY...

[Heckman, Lawrie, Lin, Zoccarato '18,'19]

$$\mathbb{R}_{\mathrm{time}} \times M_5 \subset \mathbb{R}^4 \times \text{Kähler}$$

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back-reaction should produce

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{unlike with D-branes, no AdS emerges}

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- overall expectation: $\mathbb{R}_{\text{time}} \times S^3$ with linear dilaton / F-theory mix

- $S^3 \to S^3/\mathbb{Z}_K$, large K ⇨ duality to M-theory on $\mathbb{R}_{\text{time}} \times S^2$

[Heckman, Lawrie, Lin, Zoccarato '18,'19]

- maybe then only "$\mathcal{N} = 1/2$"? namely, susy in exotic $(2, 8)$ signature

perhaps also beneficial for perturbing to realistic cosmology.

- However, in this work we investigated supersymmetry in Lorentzian signature.

Supersymmetry

- $(\mathrm{Mink}_d, \mathrm{AdS}_d) \times M_{10-d}$: various geometrical methods
    - G-structures / spinorial geometry
    - generalized (complex) geometry / pure spinors
    - exceptional geometry

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- For example, $d = 4$, one internal spinor η_+:

$$J_{mn} = \mathrm{i}\eta_+^\dagger \gamma_{mn} \eta_+$$
$$\Omega_{mnp} = \overline{\eta_+} \gamma_{mnp} \eta_+$$

$$\eta_+ \otimes \eta_+^\dagger = \mathrm{e}^{-\mathrm{i}J}$$
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SU(3) structure
$\cap$
SO(6)

⇘ forms (J, Ω) determine metric g_{mn}

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- More generally, *pure* forms $\Phi_\pm \equiv \eta_+^1 \otimes \eta_\pm^{2\,\dagger} \quad \longleftrightarrow \quad (g_{mn}, \eta_+^1, \eta_+^2)$

[Hitchin '02, Gualtieri '04]

supersymmetry $\Longleftrightarrow$ *pure spinor equations* for $\Phi_\pm$

[Graña, Minasian, Petrini, AT '05]

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$$K \equiv \frac{1}{2}(V^1 + V^2)$$
$$\tilde{K} \equiv \frac{1}{2}(V^1 - V^2)$$

pick new forms | $\hat{V}^a \cdot V^a = 1;\quad (\Psi, \hat{V}_a) \mapsto$ metric g_{mn}

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$$\text{supersymmetry} \Longleftrightarrow \left\{ L_K g = 0\,, \quad \iota_K H = \mathrm{d}\tilde{K} \quad | \right.$$

[AT '11]

[Hackett-Jones, Smith '04;
Koerber, Martucci '07;
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supersymmetry $\Longleftrightarrow$ [AT '11]

$$\left\{ \begin{array}{l} L_K g = 0\,, \quad \iota_K H = d\tilde{K} \\ \boxed{d_H \Psi = (\iota_K + \tilde{K}\wedge)F} \end{array} \right.$$

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exterior equation

$$\left(\psi_- \otimes \overline{\epsilon^2} \cdot \hat{V}_2, \pm \mathrm{d}_H(\mathrm{e}^{-\phi}\Psi \cdot \hat{V}_2) + \sigma_1 \Psi - F\right) = 0$$
$$\left(\hat{V}_1 \cdot \epsilon^1 \otimes \overline{\psi_\pm}, \mathrm{d}_H(\mathrm{e}^{-\phi}\hat{V}_1 \cdot \Psi) - \sigma_2 \Psi - F\right) = 0$$
$$\forall \psi_\pm$$

pairing equations

$$\sigma_a \equiv \frac{1}{2}\mathrm{e}^{\phi}\mathrm{d}^\dagger(\mathrm{e}^{-2\phi}\hat{V}_a)$$
$$(\alpha, \beta) \equiv (\alpha \wedge \lambda(\beta))_{\text{top}}$$

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- Pairing equations are *unsatisfactory...*
  - If K timelike, they can be replaced by

[Legramandi, Martucci, AT '19]

$$\mathrm{e}^{2\phi}\mathrm{d}(\mathrm{e}^{-2\phi}\tilde{\Omega}) = *(\tilde{K}\wedge H) \mp 16\mathrm{e}^{\phi}(\iota_M\Psi, \iota^M F)_6$$

$$\mathrm{d}*\tilde{K} = -4\mathrm{e}^{\phi}(\Psi, (10-\mathrm{deg})F)$$

with a calibration interpretation [sort of]

  - For $\mathrm{Mink}_4 \times M_6$ they are redundant

$$\Psi = \mp 2\mathrm{e}^{A} v\wedge \mathrm{Re}\Phi_{\mp} + 2\mathrm{e}^{2A}\mathrm{Re}(\omega\wedge\Phi_{\pm}) \pm 2\mathrm{e}^{3A} * v\wedge \mathrm{Im}\Phi_{\mp}$$

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- ... but hey, *better than nothing*
  - useful for AdS in several dimensions

    [Apruzzi, Fazzi, Rosa, AT '13;
    Apruzzi, Fazzi, Passias, Rosa, AT '14...]
  - collapse to single equation for AdS3

    [Dibitetto, Lo Monaco, Passias, Petri, AT '18;
    Macpherson, AT '22; Legramandi, Macpherson, Passias '23...]
  - AdS2

    [Legramandi, Macpherson, Passias '23]
  - in our case, they provide several constraints on RR flux.

Results

- Minimal amount of susy ⇨

$$\epsilon_1 = \chi_1 \otimes \theta_+ \otimes \xi + \text{conj.},$$
$$\epsilon_2 = \chi_2 \otimes \theta_\mp \otimes \xi + \text{conj.},$$

7d

aux.

Killing spinor on S^3:

$$\nabla_i \xi = -\tfrac{i}{2}\sigma_i \xi$$

⇩

$$L_K \epsilon_a = 0$$

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- Two classes:
  - K timelike: $\Rightarrow V^1, V^2$ identify two directions t, z in 7d

    ⇨ internal $M_6 \sim \{z, 5\text{d}\}$

further split:

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  - K null: $V^1 \propto V^2$

most AdS vacua are here.

we still *assume* two
such directions exist, but now:

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- K timelike: ext. equation ⇨ $\eta_2 = \underset{\nwarrow \text{complex one-form}}{W} \cdot \eta_1$

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With some simplifying Ansatz: a T-dual to the D8–D0–F1 class [Imamura '01]

which we can now greatly generalize.

$$\Delta_{\underline{8}} S + \frac{1}{2}\partial_z^2 S^2 = 0$$

here $S^3 \times M_5$.

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[K3]: $\eta_2 = \eta_1$ SU(2)-structure in M_5 $\mathrm{d}J_a + 2\mathcal{B} \wedge J_a = 0$

contains:

- AdS$_3$ $\times$ S^3 $\times$ K3 with D5, KK5 [Lozano, Macpherson, Nuñez, Ramirez '20]
- linear dilaton solution with D5s
- more!

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[S] Another class with a locally determined metric [as in timelike case]

- an extra S^2 emerges

Conclusions

- ESU gives a possible new starting point for cosmology in string theory
    - We launched a systematic investigation of such solutions
- Promising classes with a linear dilaton or holomorphic axio-dilaton

Backup

- generalized D8–D0–F1 class [Imamura '01]

$$\Delta_8 S + \tfrac{1}{2}\partial_z^2 S^2 = 0$$

$$F_0 K = \partial_z S$$

$$S = S(x_i, z)$$

$\mathbb{R}^8$

$$\mathrm{d}s^2 = -S^{-1}K^{-1/2}\mathrm{d}t^2 + SK^{1/2}\mathrm{d}z^2 + K^{1/2}\mathrm{d}x^2_{\mathbb{R}^8}$$

- our (further generalization of) its dual

$$P = P(z,\rho) \quad f = f(z, x_i, \rho) \quad h(z, x_i, \rho)$$

$$\mathrm{d}s^2 = \tfrac{1}{h}\left(-\tfrac{1}{\sqrt{fP}}\mathrm{d}t^2 + \sqrt{fP}\mathrm{d}z^2\right)+\sqrt{\tfrac{P}{f}}d\tau^2+\sqrt{\tfrac{f}{P}}\left(\mathrm{d}s^2_{\mathbb{R}^3} + P(\mathrm{d}\rho^2 + \rho^2\mathrm{d}s^2(\mathrm{S}^3)\right)\right)$$

PDEs look complicated...

- linear dilaton with D5s

$$\mathrm{d}s^2 = \frac{\sqrt{\rho}}{\sqrt{h}}\left(-\mathrm{d}t^2 + \mathrm{d}z^2 + ds^2_{S^3}\right) + \frac{\sqrt{h}}{\rho}\mathrm{d}s^2_{\mathrm{K3}} + \frac{1}{4\rho^{\frac{3}{2}}\sqrt{h}}\mathrm{d}\rho^2, \qquad e^{\phi} = \frac{1}{\sqrt{h}\sqrt{\rho}}$$

$$F_1 = H = F_5 = 0, \qquad F_3 = 2u\mathrm{vol}_{S^3} - \star_4 \mathrm{d}h$$

h harmonic on K3

- general class [F]

$$\mathrm{d}s^2 = \frac{q\mathrm{e}^{-2C}}{\sin^2\alpha}(Dt^2 + D\phi^2) + \mathrm{e}^{2C}\mathrm{d}s^2_{S^3} + \mathrm{d}s^2_{M_4} + \mathrm{e}^{2C}\left(\frac{q'}{2q}\right)^2\mathrm{d}\rho^2$$

$$\partial_\rho\left(\frac{\cos\alpha}{e^{2A}\sin^2\alpha}\right) + \frac{q'}{2e^{2A+2C}\sin^2\alpha q}h_0 = 0 \qquad \mathrm{d}_4\left(\frac{a_1}{e^{2A}\sin^2\alpha a_2}\right) = \mathrm{d}_4^c\left(\frac{\cos\alpha}{e^{2A}\sin^2\alpha}\right)$$

+Bianchi identity for fluxes