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Quantum model of field emission from dielectric coatings

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Relevance

- Dielectric coating is considered as one of the options to increase resistance to breakdowns
- Under high vacuum, native oxides or foreign adsorbates can easily form on the cathode surface.
- ☐ There is a theory that takes into account the presence of a dielectric layer on the metal surface within the quasi-classical approximation, but does not take into account quantum effects.

Probability of electron tunnelling from a dielectric-coated surface

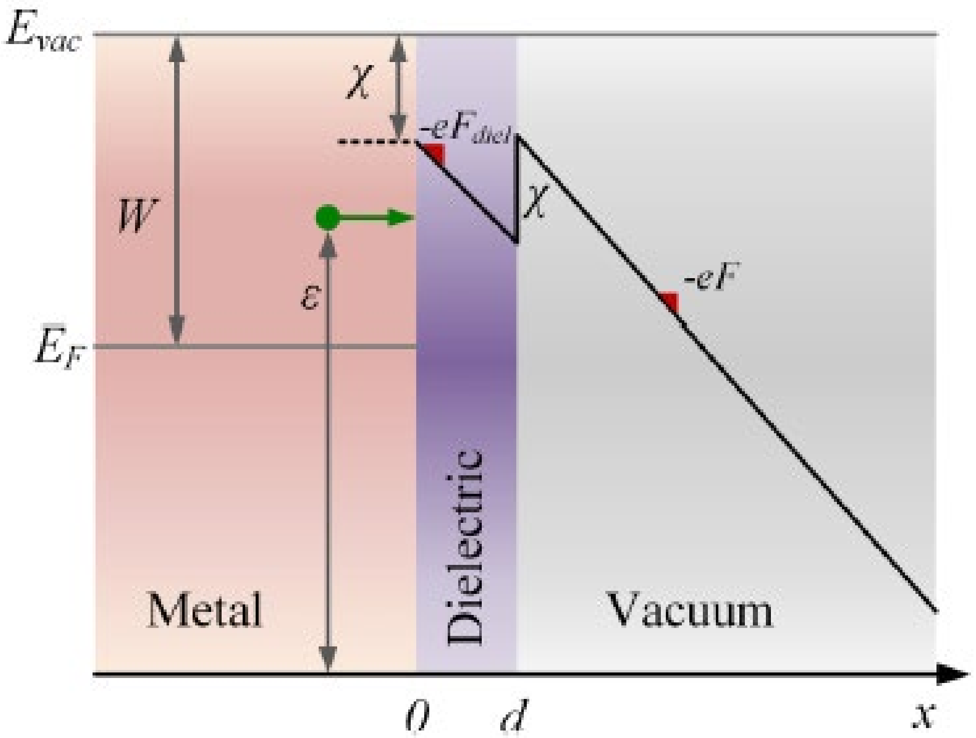

Figure 1 Schematic representation of a potential barrier for a metal-dielectric-vacuum system

Exact solution[1]:

$$\boxed{D(\varepsilon) = \frac{4\alpha\xi}{\pi^3}\frac{1}{\Gamma^2 + \Delta^2}}$$

$$\Gamma = A_3 U + \alpha B_3 Y - \xi\,(A_3' V + \alpha B_3' Z), \text{ i}$$

$$\Delta = B_3 U - \alpha A_3 Y + \xi\,(\alpha A_3' Z - B_3' V),$$

$$\alpha = |\zeta| = \frac{1}{k_0}\left(\frac{2meE_{diel}}{\hbar^2}\right)^{\frac{1}{3}}, \quad U = A_1 B_2' - B_1 A_2',$$

$$V = A_1 B_2 - B_1 A_2,\ Y = A_1' B_2' - B_1' A_2',\ Z = A_1' B_2 - B_1' A_2,$$

A_1, B_1, A_1', B_1', A_2, B_2, A_2', B_2', A_3, B_3, A_3', B_3' - different variations of Airy functions and their derivatives

[1] Y. Zhou, P. Zhang, «Theory of field emission from dielectric coated surfaces» Physical review research, vol. 2, pp. 043439-1-043439-10, 2020.

Effect of dielectric thickness **d** on the probability of electron tunnelling

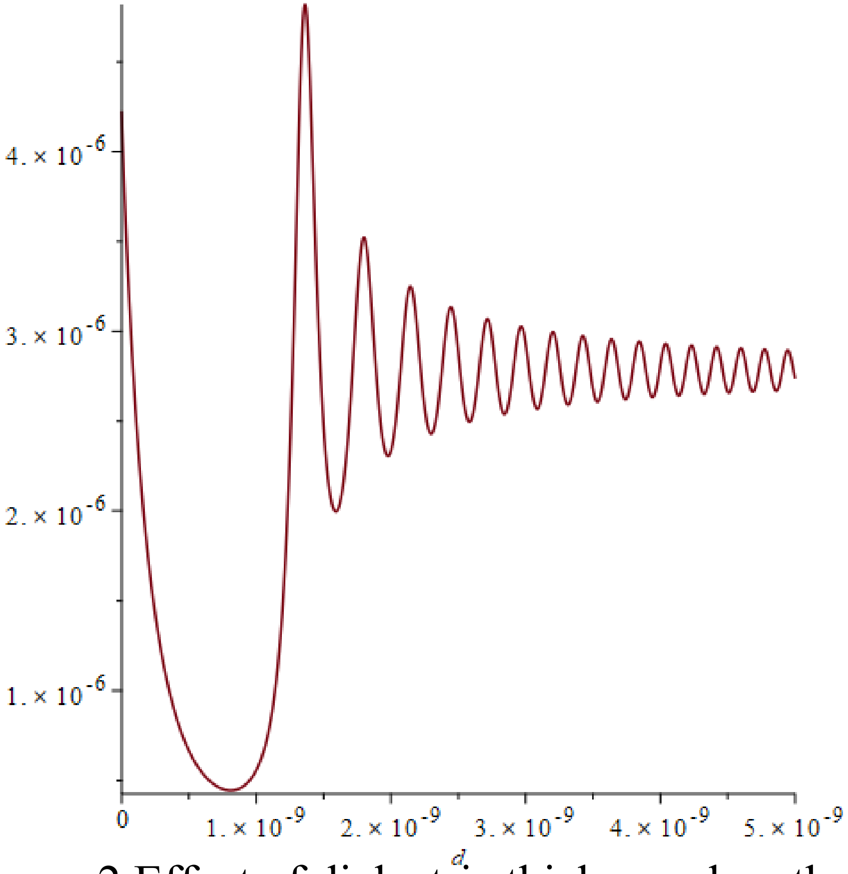

Figure 2 Effect of dielectric thickness d on the probability of electron tunnelling from the surface of a dielectric-coated metal

Practical use for calculations is complicated by cumbersome writing and the use of a large number of special functions. At the same time, this expression can be simplified for cases where the arguments of the Airy functions fulfil one of the conditions:

$$0 < d < d_0 \text{ or } d > d_0, \text{ where } d_0 \sim \varepsilon_{diel} \frac{W}{eE}.$$

It is also worth noting that the thickness of the dielectric d is limited by the free path length of the electrons. For example, for the dielectric coating SiO_2, which is often used to obtain field emission from coated cathodes, the average free path length of the electron is about 3 nm.

Tunnelling electrons through a thin dielectric layer

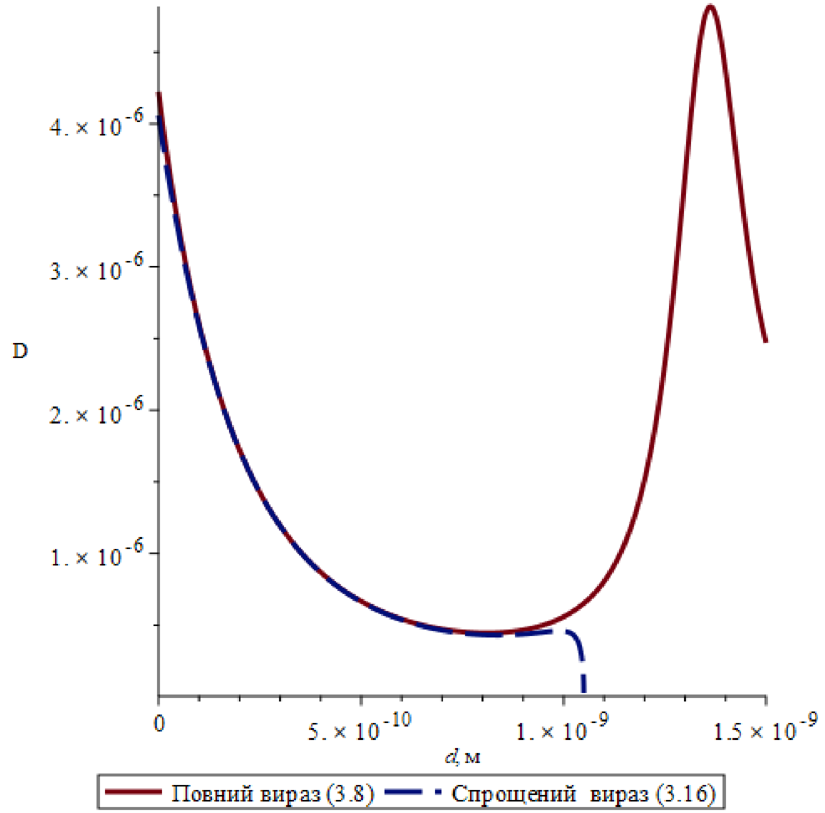

Figure 3 Comparison of the transmission coefficients of the potential barrier

Thickness of the dielectric layer $0 < d < d_0$

Transmission coefficient:

$$D = \frac{16e^{-\frac{4\left(C_1^{\frac{3}{2}} - C_2^{\frac{3}{2}} + \frac{C_3^{\frac{3}{2}}}{\varepsilon_{diel}}\right)k\varepsilon_{diel}}{3eE}}\sqrt{C_1}\sqrt{C_2}\sqrt{C_3}\sqrt{\varepsilon}}{\left(\sqrt{C_2}+\sqrt{C_3}\right)^2(V_0-\chi)}$$

where the following designations are introduced:

$C_1 = V_0 - \chi - \varepsilon$, $C_2 = C_1 - deE_{diel}$, $C_3 = C_2 + \chi$.

Tunnelling electrons through a thick dielectric layer

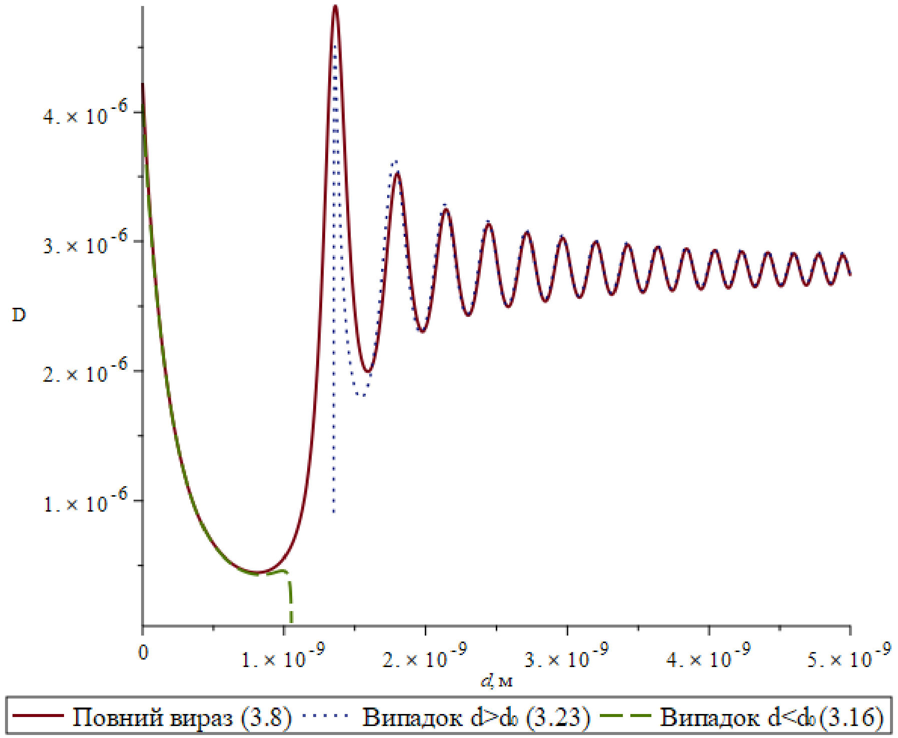

Figure 4 Comparison of the transmission coefficients of the potential barrier

Thickness of the dielectric layer $d > d_0$

Transmission coefficient:

$$D = \frac{4\sqrt{eE_{diel}}\sqrt{eE_{diel}(d-d_{ef})+\chi}\sqrt{d-d_{ef}}\sqrt{ed_{ef}E_{diel}-\chi}\sqrt{\varepsilon}}{e^{\frac{4k(V_0-\varepsilon-\chi)^{\frac{3}{2}}}{3eE_{diel}}}(V_0-\chi)\left(\chi\cos^2(z)+eE_{diel}\left(d-d_{ef}\right)\right)}$$

where the following designations are introduced:

$$d_{ef} = \frac{V_0-\varepsilon}{eE_{diel}},\ z = \frac{2(gy_2)^{\frac{3}{2}}}{3} + \frac{\pi}{4}.$$

Field emission from the metal coated surface

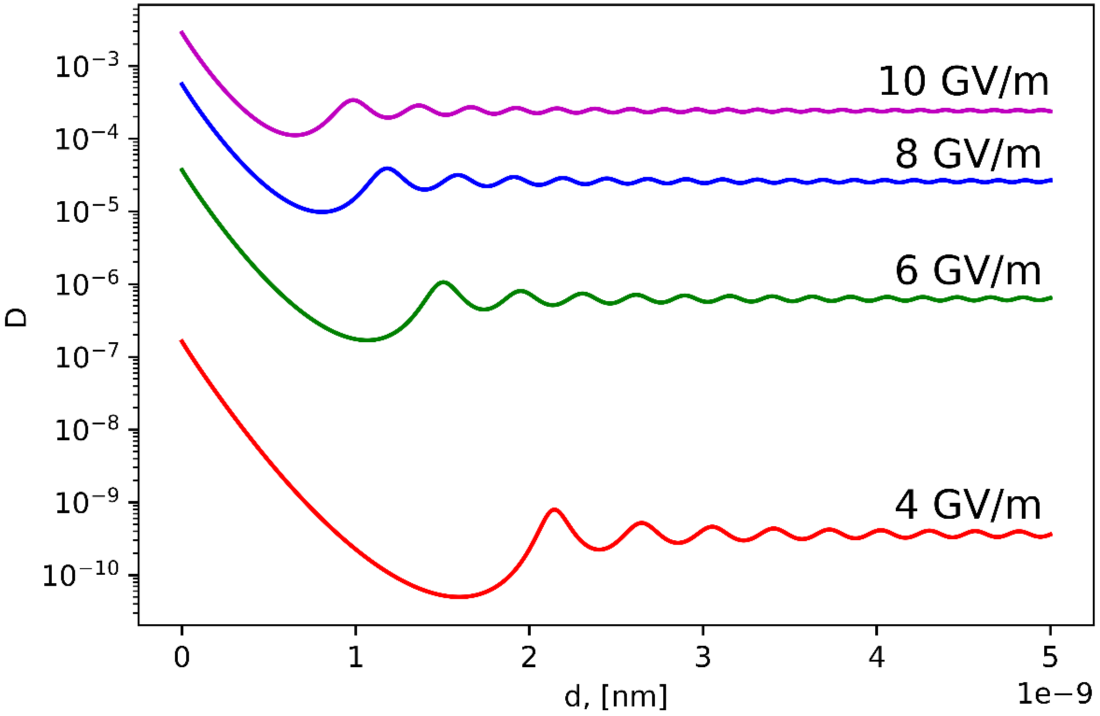

Figure 5 Effect of dielectric thickness d and electric field E on the probability of electron tunnelling

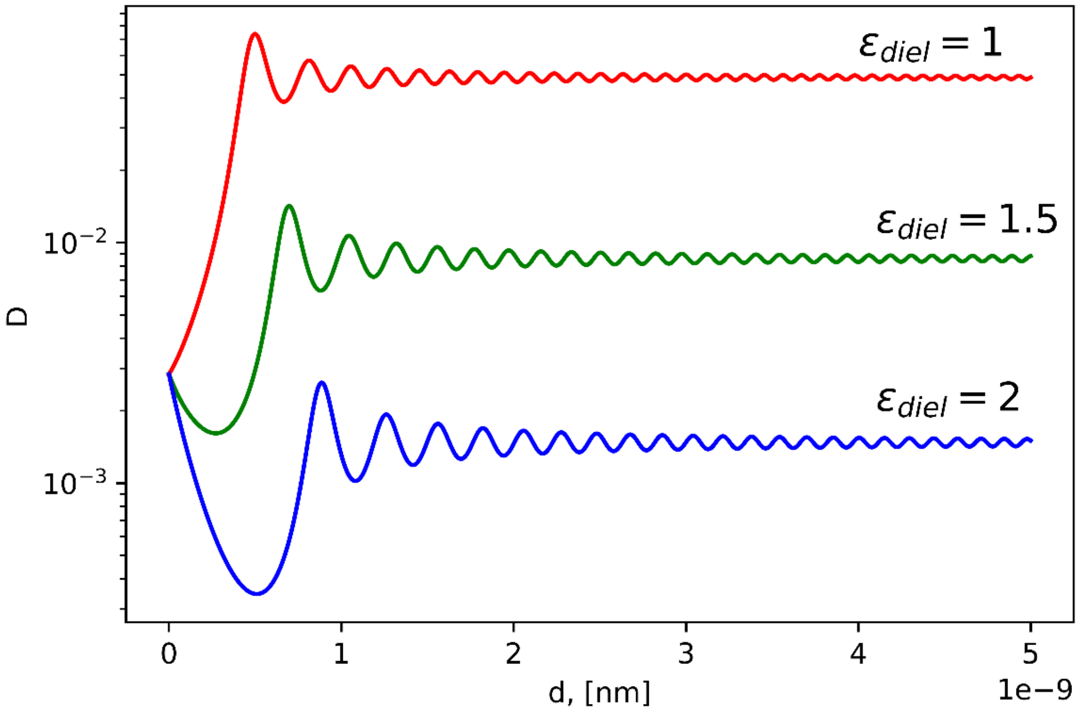

Figure 6 Influence of dielectric thickness d and dielectric constant ε_{diel}on the probability of electron tunnelling

Field emission from the metal coated surface

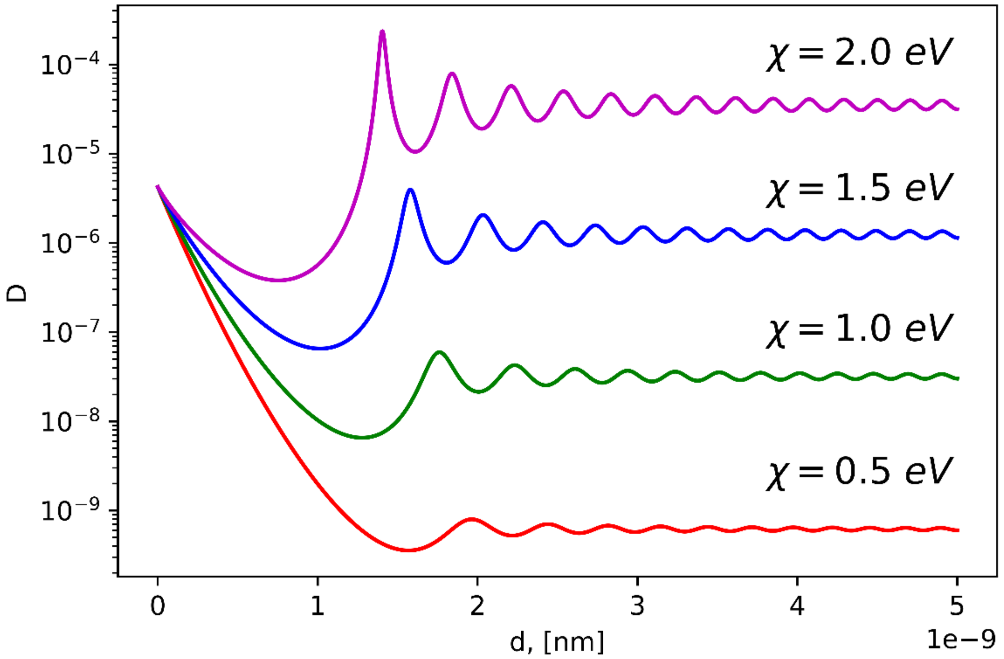

Figure 7 Effect of dielectric thickness d and affinity energy χ on the probability of electron tunnelling

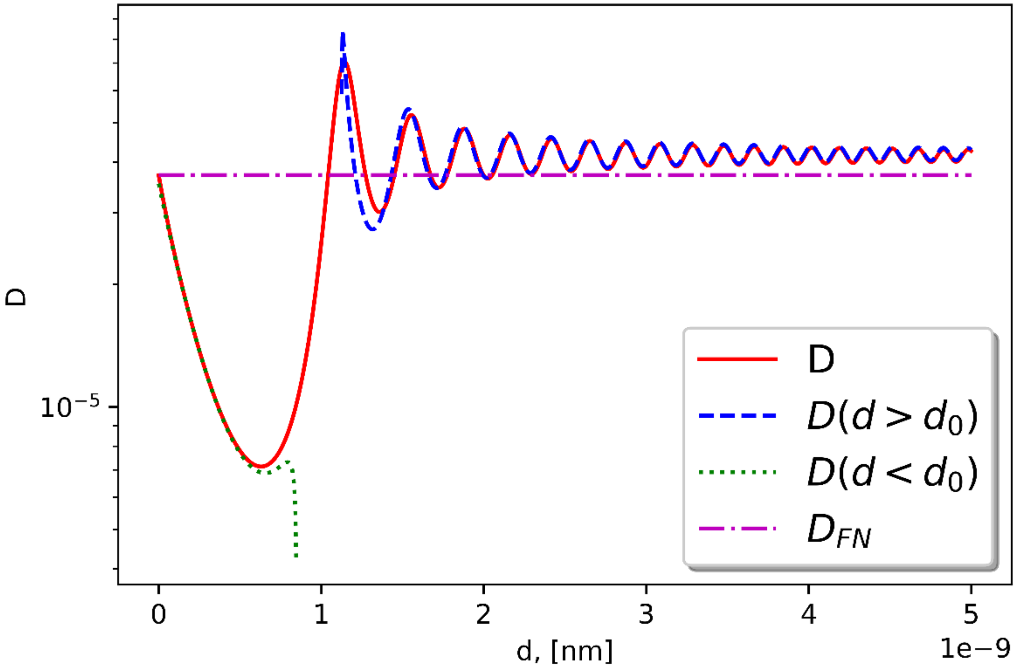

Figure 8 Transmission coefficient of the potential barrier at the metal-dielectric coating interface

Comparison with the quasi-classical model

$$J = \frac{e^3E^2}{16\pi^2\hbar\varphi B^2}\exp\left[-\frac{4\sqrt{2m}}{3e\hbar E}\varphi^{\frac{3}{2}}C\right]$$

$$B = \varepsilon_{diel}\left[\sqrt{\frac{\varphi_{\text{eff}}}{\varphi}} - H(\varphi_{\text{eff}} - eE_{diel}d)\sqrt{\frac{\varphi_{\text{eff}} - eE_{diel}d}{\varphi}}\right] + H(\varphi - eE_{diel}d)\sqrt{\frac{\varphi - eE_{diel}d}{\varphi}}$$

$$C = \varepsilon_{diel}\left[\left(\frac{\varphi_{\text{eff}}}{\varphi}\right)^{\frac{3}{2}} - H(\varphi_{\text{eff}} - eE_{diel}d)\left(\frac{\varphi_{\text{eff}} - eE_{diel}d}{\varphi}\right)^{\frac{3}{2}}\right] + H(\varphi - eE_{diel}d)\left(\frac{\varphi - e\varphi_{diel}d}{\varphi}\right)^{\frac{3}{2}}$$

$H\ (x)$ is the Heaviside function

In the absence of a dielectric layer, $\varphi_{eff} = \varphi$, $d = 0, \varepsilon_{diel} = 1$, B and C become equal to 1, and the equation becomes the Fowler-Nordheim equation

Comparison with the quasi-classical model

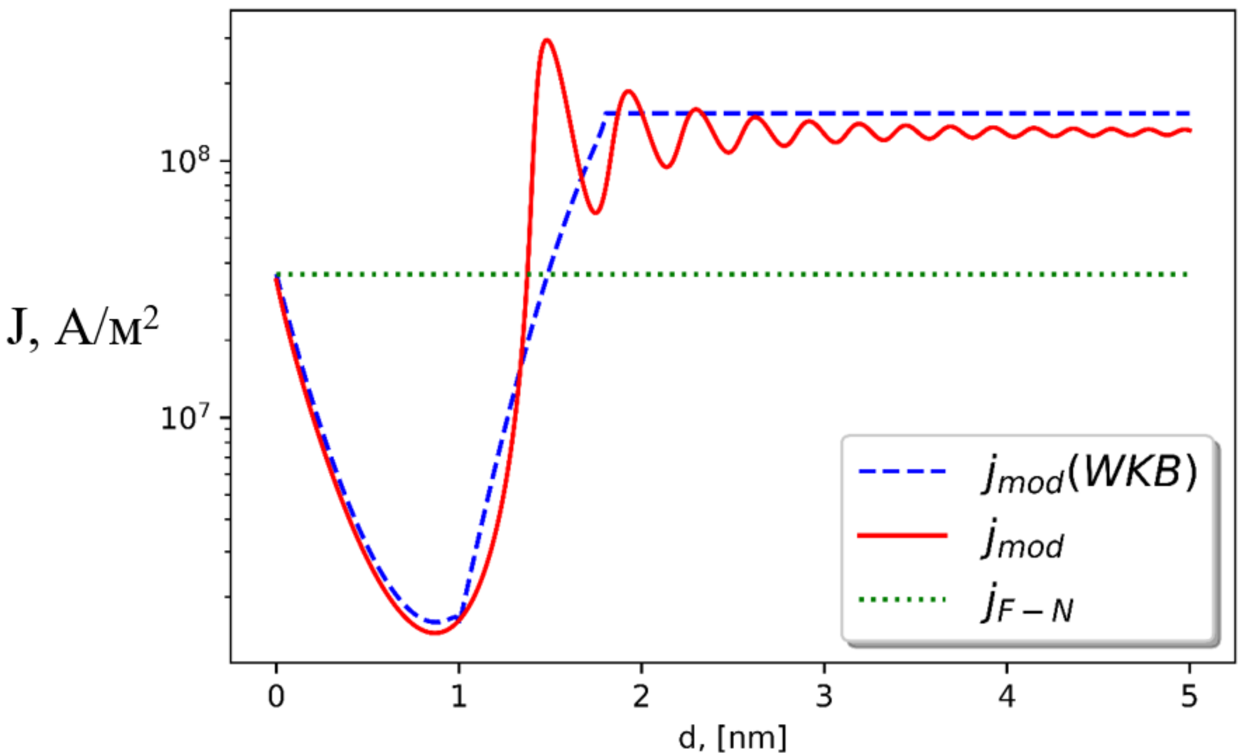

Figure 9. Emission current density J from a metal with a dielectric coating compared to the current density from an uncoated surface The dielectric has $\varepsilon_{diel} = 2$ and χ=2 eV

- The emission current density J from a dielectric-coated metal (solid lines) and the modified Fowler-Nordheim equation (dashed lines) compared to the current density from an unmodified surface. The dielectric has $\varepsilon_{diel} = 2$ and χ=2 eV.
- Both models are in good agreement when scaled up, with the quantum model giving a higher emission current density overall. The quantum model shows a resonant behavior in the dependence of $J\ vs\ d$ that cannot be detected by the quasi-classical modified Fowler-Nordheim equation.

Conclusions

- The quantum model reveals resonant features in the current and provides more accurate predictions of current density compared to the modified Fowler–Nordheim equation, especially when a dielectric layer is present.
- A combination of low dielectric permittivity $\varepsilon_{\mathrm{diel}}$ and high electron affinity of the dielectric χ increases the emission current density, sometimes even exceeding the emission from an uncoated surface.
- High permittivity $\varepsilon_{\mathrm{diel}}$ and low electron affinity χ reduce the emission current.
- It was found that thin coatings $0 < d < d_0$ suppress emission with a minimum at a characteristic thickness.
- With further increase in thickness $d > d_0$ field emission current grows, and its density oscillates near an asymptotic value, with the amplitude of oscillations decreasing with thickness.

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Thanks for your attention!