CHAOS THEORY AND ARRHYTHMIA: A DYNAMICAL APPROACH TO CARDIAC CARE

ICT AWARDS: DIPLOMA OF THE YEAR 2024

LUM BOROVCI

Science Week @FShN, September 2024


21
EPENESA
57
STOP HATE
HAMLIN
3

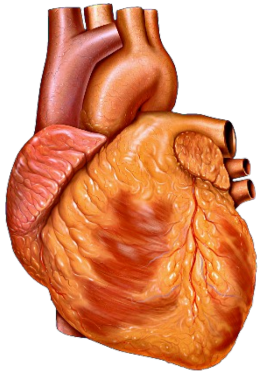
What went wrong?

THE RACE TO THE HEART

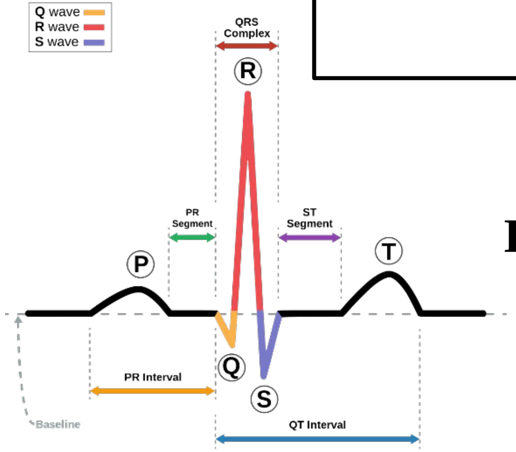
BEGINS

What went Wrong?

A Primer in Cardiac Dynamics

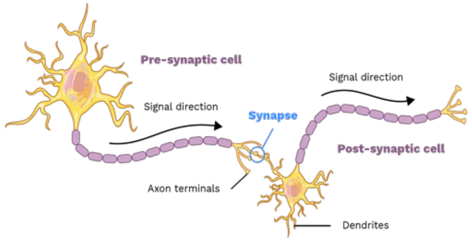


HEART



ELECTROCARDIOGRAPHIC SIMULATIONS

CENTRAL NERVOUS SYSTEM



Execution Process:

Cardio-Respiratory Breathing

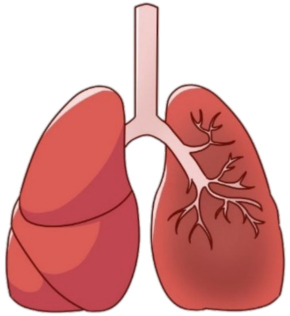
Hydro-mechanical System

Commanding Process:

Stochastic Modeling of

Action Potentials of Neurons

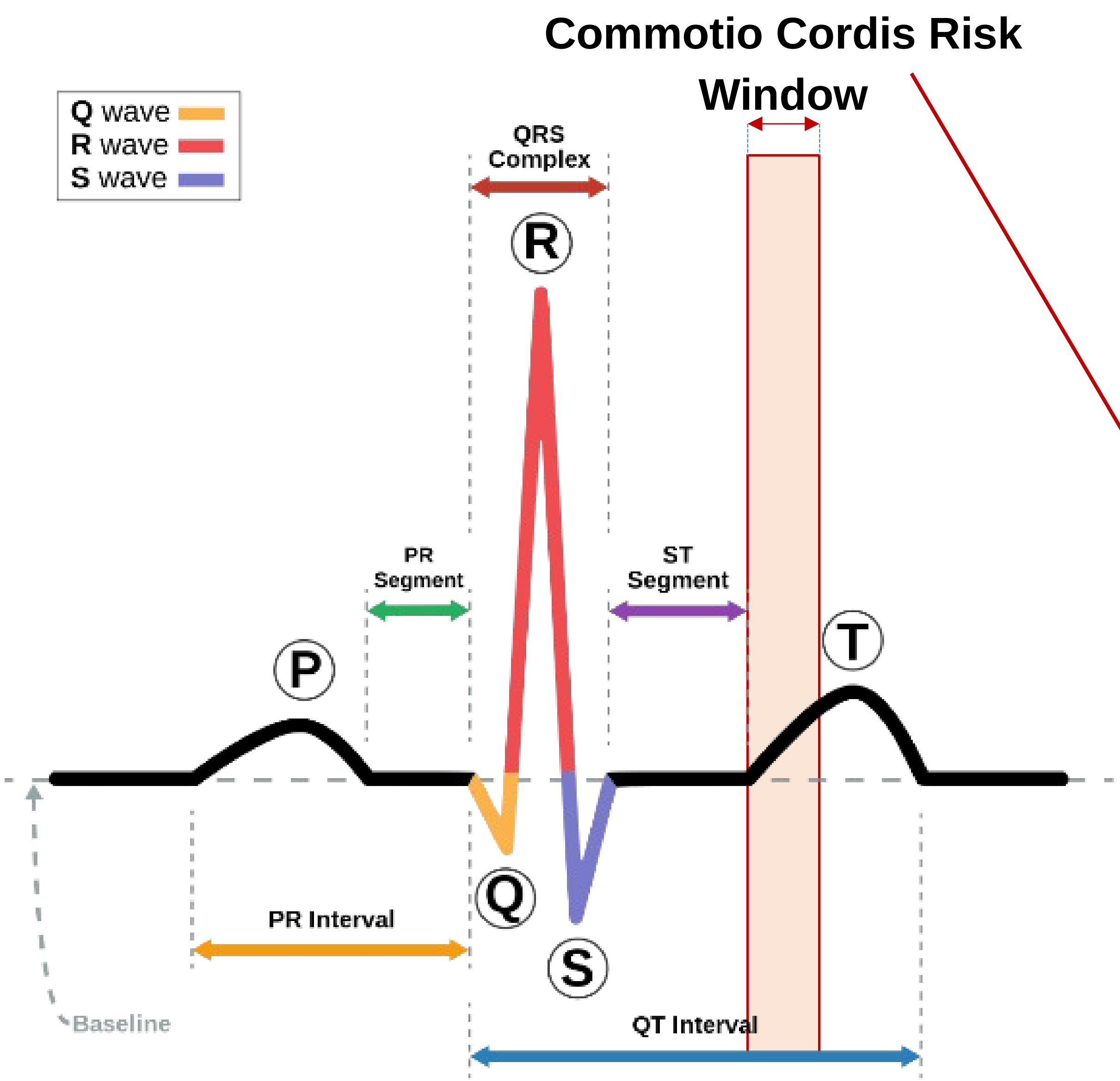
Electrical System



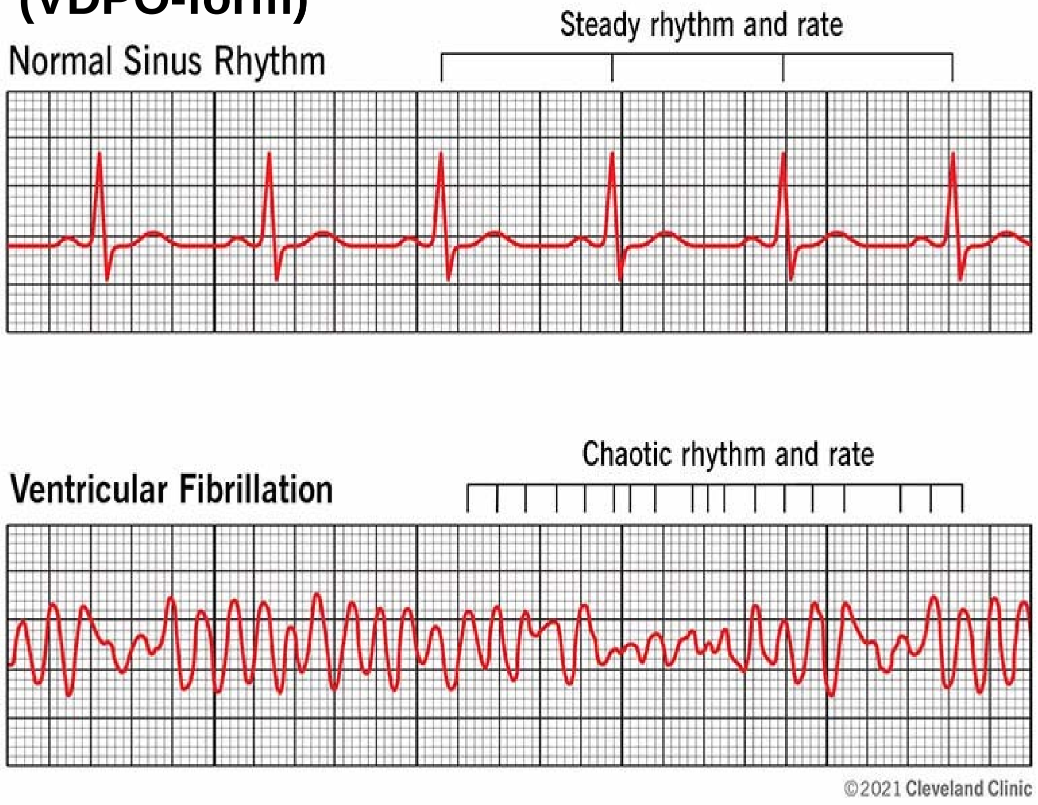
LUNGS

What went Wrong?

Commotio Cordis and Ventricular Fibrillation

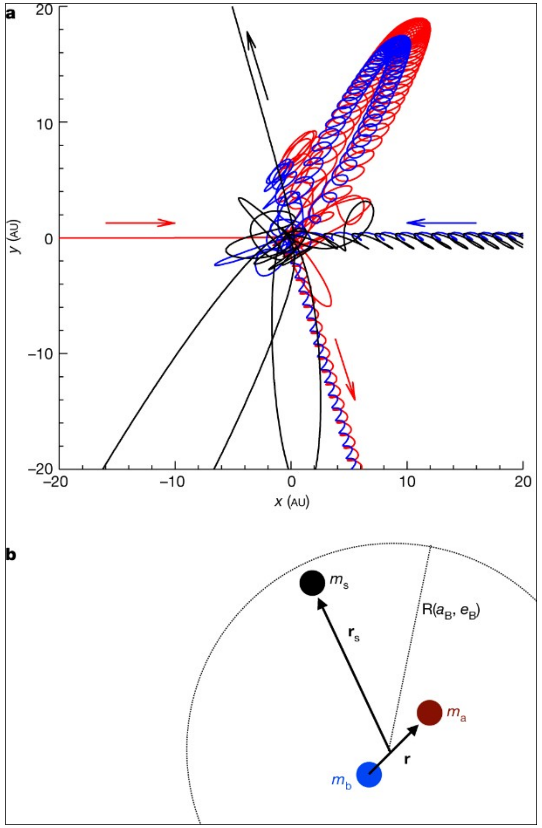


1. Relaxation Oscillations (VDPO-form)



2. Chaotic Oscillations (Lorenz-form)

Introducing Nonlinear Dynamics
Their relevance, interpreting and graphing

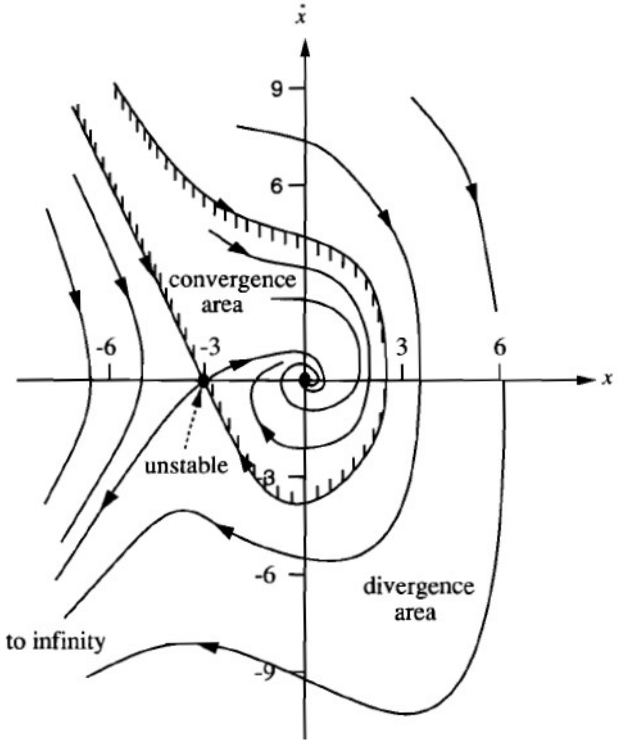


The n-body problem and (not so) elliptical planetary orbits
(1687 - Principia)

DIFFERENTIAL EQUATIONS

1. Analytical Methods (very few)
2. Numerical Methods (less computational ability back then)

3. Phase Diagrams (Poincare's Idea)



Paradigmatic NLD Systems

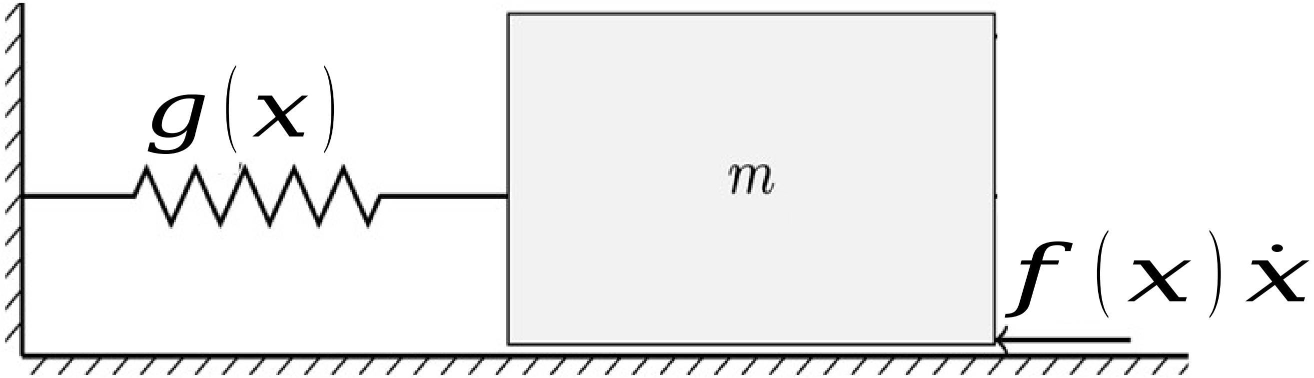
From Lienard's Equation to the Van Der Pol System

Second-Order, Ordinary, Nonlinear, **Autonomous**

$$\ddot{x} + f(x)\dot{x} + g(x) = 0$$

$$\ddot{x} = -f(x)\dot{x} - g(x)$$

- a nonlinear damping force
- a nonlinear restoring force



Our analytical ability reaches its limits very quickly!

Lienard's Transformation

$$\ddot{x} + f(x)\dot{x} + g(x) = 0 \rightarrow \begin{cases} ¿\,\dot{x} = y \\ ¿\,\dot{y} = -f(x)\,y - g(x) \end{cases}$$

Lienard's Theorem: For a set of Conditions, shows the existence of a stable limit cycle

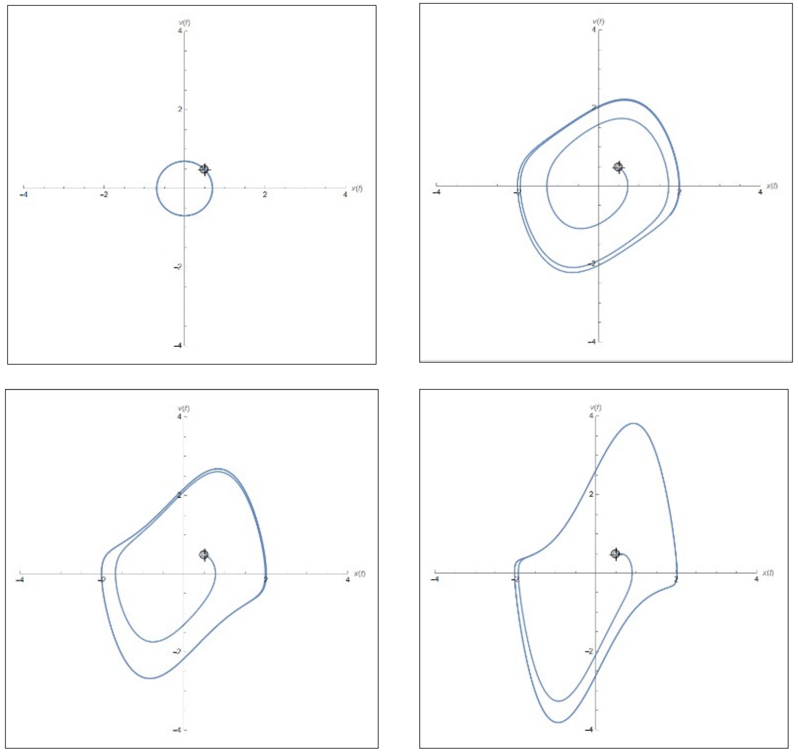
For:

- and
- ~ constant parameter

We obtain the Van Der Pol System (1927):

$$\begin{cases} ¿\,\dot{x} = \mu - \dfrac{x^3}{3} - y \\ ¿\,\dot{y} = \dfrac{1}{\mu}\,x \end{cases}$$

=0 =0.5

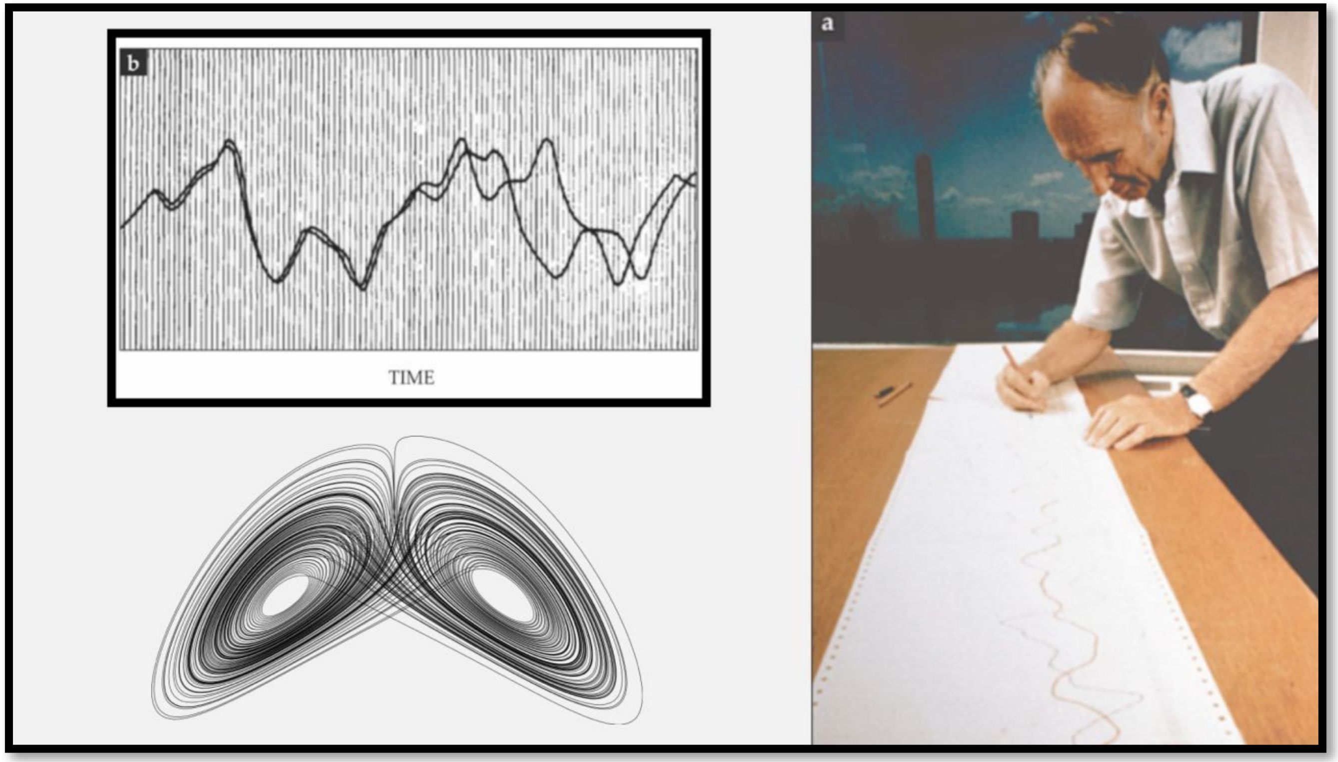


=1 =2

Deterministic Chaos

$$\begin{cases} \geq 3\, state\ variables \\ \geq 1\, nonlinear\ term \end{cases}$$

Lorenz System (1963):

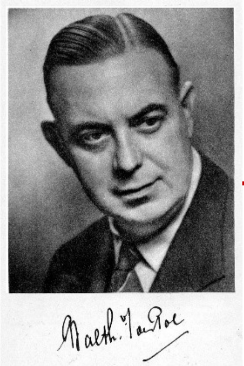


What went Wrong?

Nonlinear Dynamics: *a framework for treating the heart*

1. **Relaxation Oscillations (VDPO-form)**

$$\begin{cases} ¿\ \dot{x} = \mu - \frac{x^3}{3} - y \\ ¿\ \dot{y} = \frac{1}{\mu} x \end{cases}$$

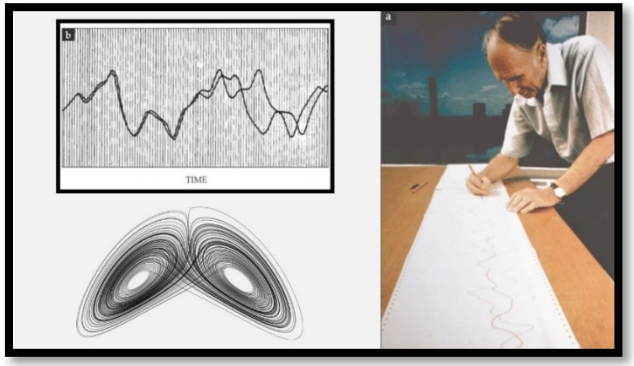


**as a normally beating heart*

$$\begin{cases} \geq 3\, state\, variables \\ \geq 1\, nonlinear\, term \end{cases}$$

2. Chaotic Oscillations (Lorenz-form)

$$\begin{cases} \dot{x} = -\sigma(x - y) \\ \dot{y} = rx - y - \mathbf{xz} \\ \dot{z} = \mathbf{xy} - bz \end{cases}$$

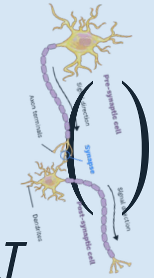


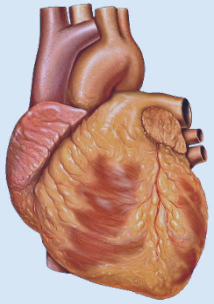
**as an arrhythmic heart*

How exactly do we do this?

- One VDPO Neuron as an **Ergodic Sum:** <u>a sample is a good representative of the entirety of the set!</u>

$$\sum_{i=FHN}^{\Omega \ni i} (\;) \Rightarrow$$





Prospective Framework

Chua’s Circuit

Exhibiting NLD in Analog Conditions

The next question naturally was whether such p
could be produced using continuous, real-time
whether they existed purely in the digital domai

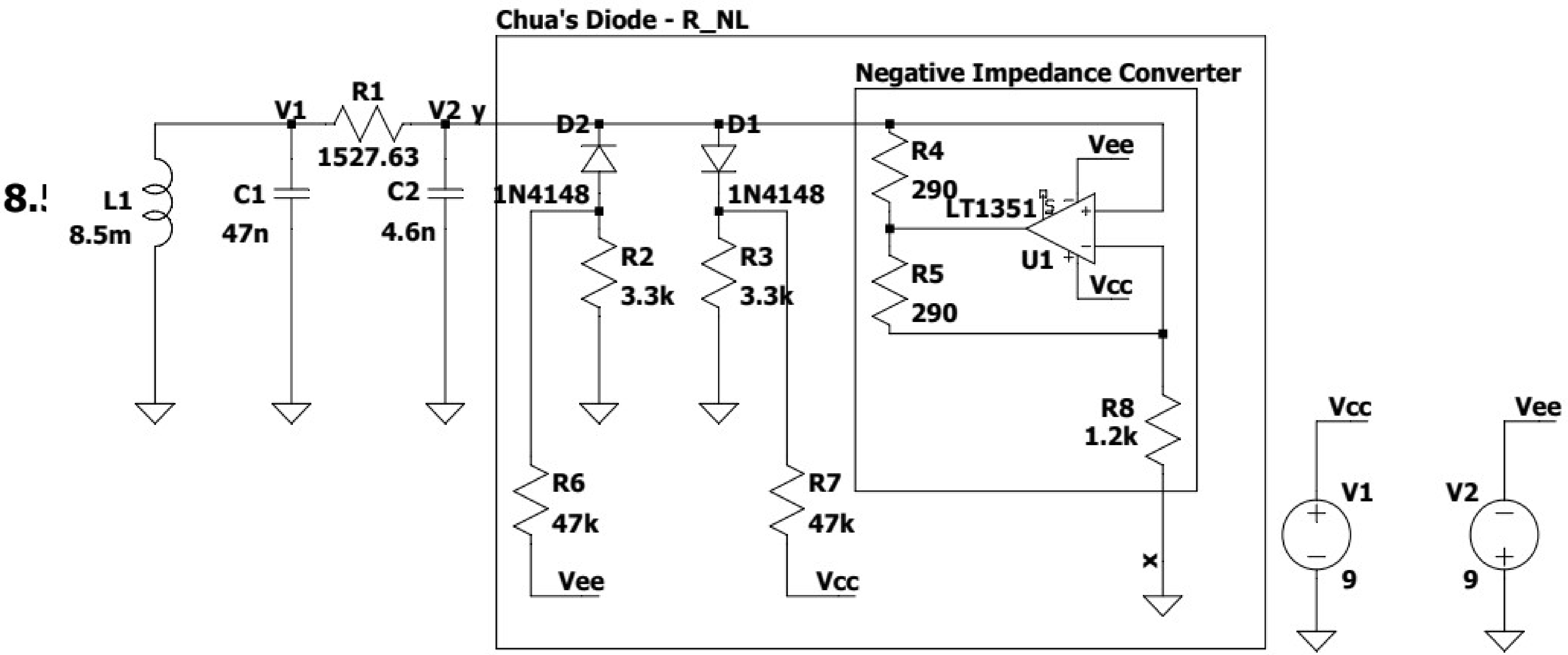
Provisional Circuit Topology to achieve c
chaos:

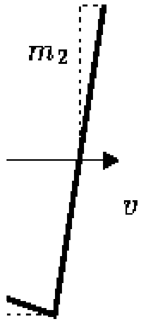
- 3 or more Energy Storage Elements (i.e. Cap
  Inductors)
  that act as state variables to the Circuit Equ
- 1 or more Nonlinear Element coupling two
  of the Energy Storage Elements Mathemati
- 1 or more locally active Resistor

The Chua Circuit that mirrors the Heart

Active Electronic Implementation of NLD Systems

$$\frac{V_{C_1} - V_{C_2}}{R}$$

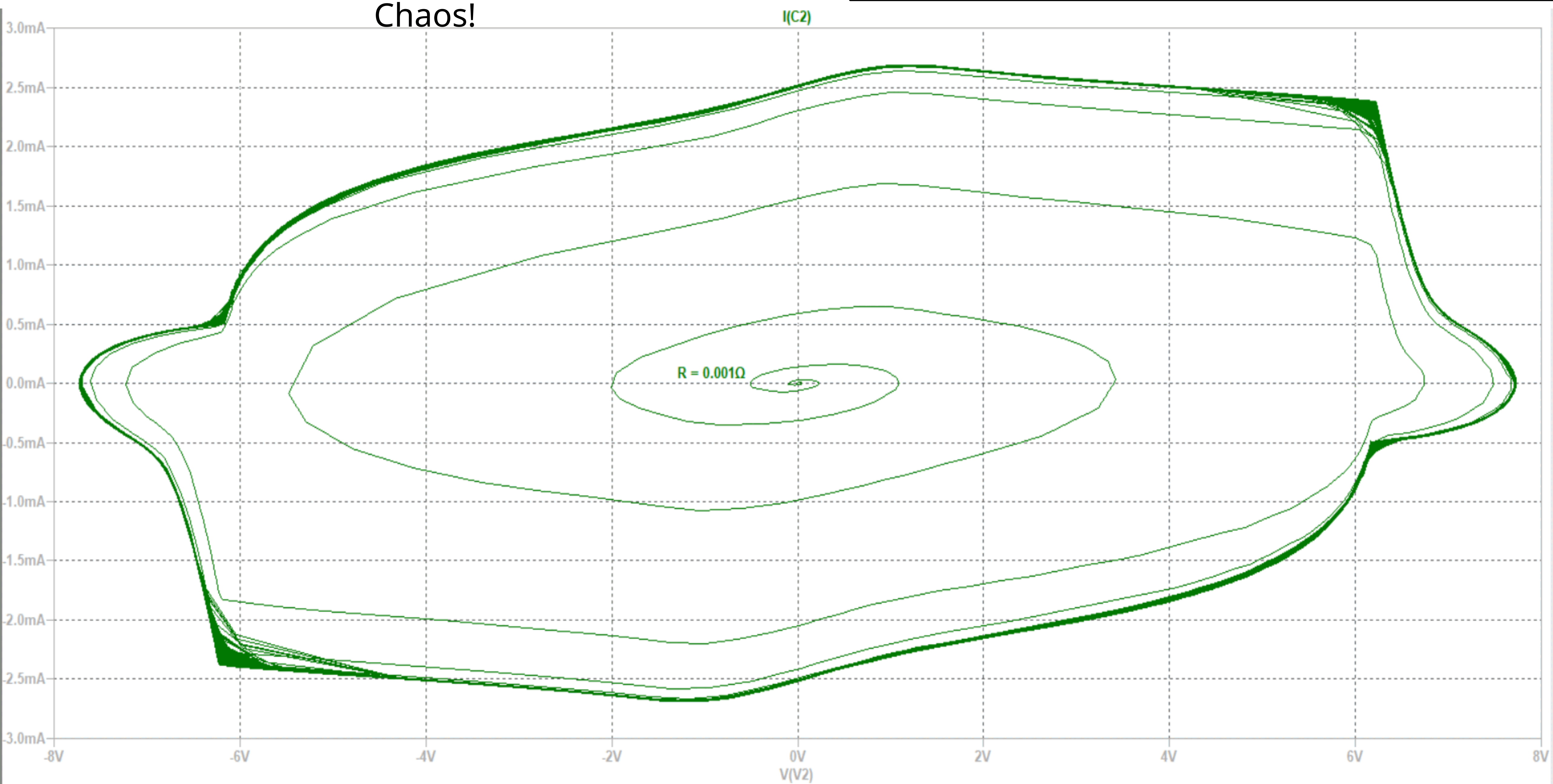




I) NLD System Analysis

I) NLD Analysis
~Attractor Topologies

:
from relaxed, VDPO Oscillations to Lorenz-form Chaos!



Chua's Diode Load Line Analysis (LLA) demonstrated with three Load Lines:

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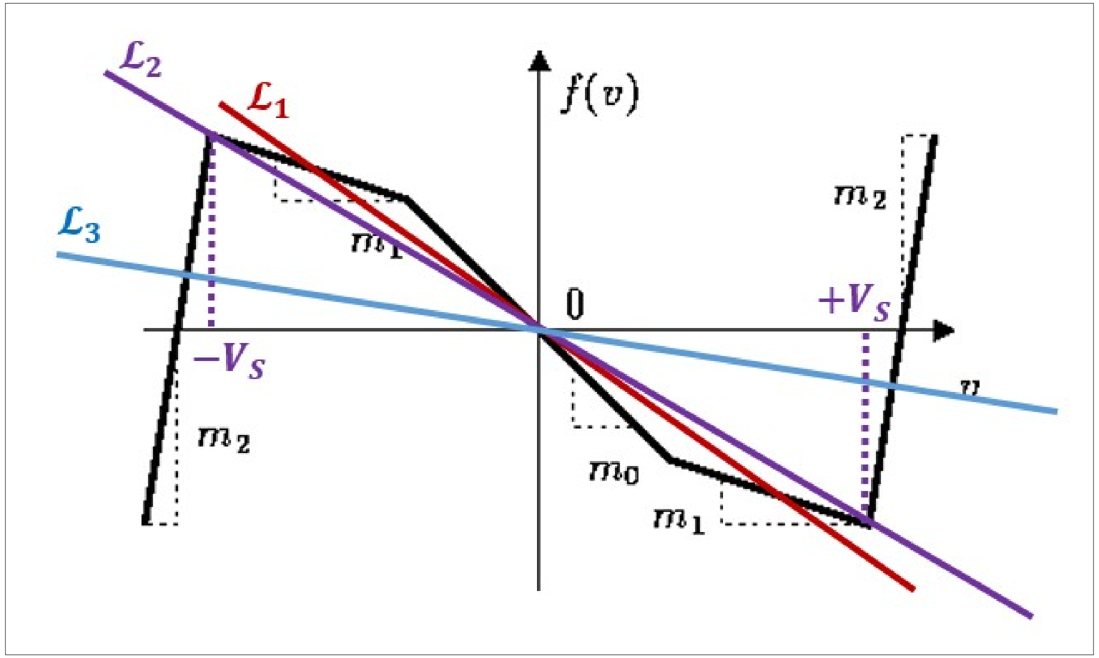
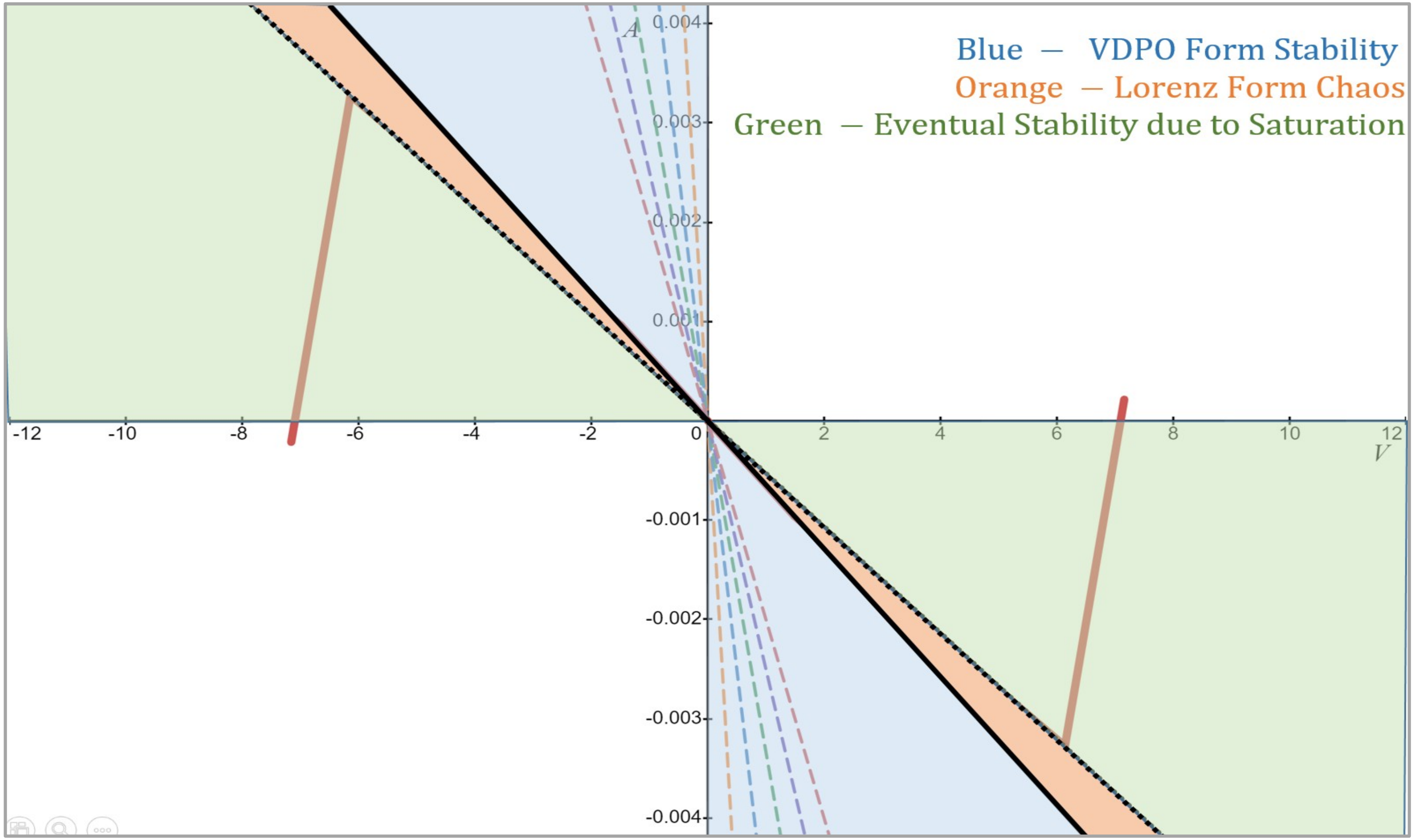


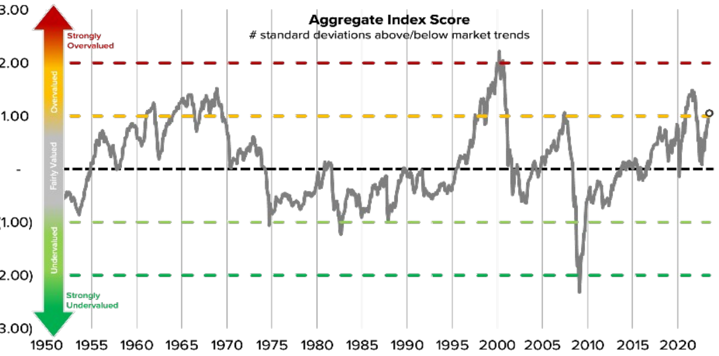
Chaotic Circuit regime is the set of all possible Load Line intersections within the closed interval .

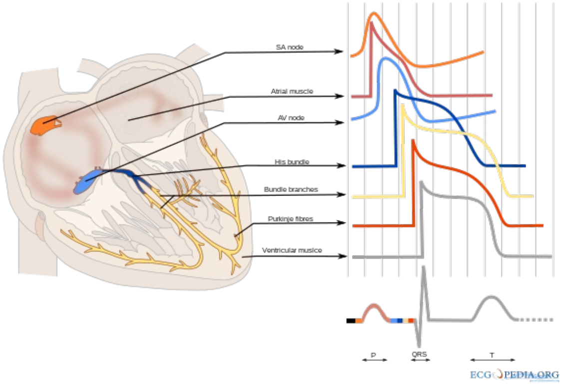
II) Load Line Analysis

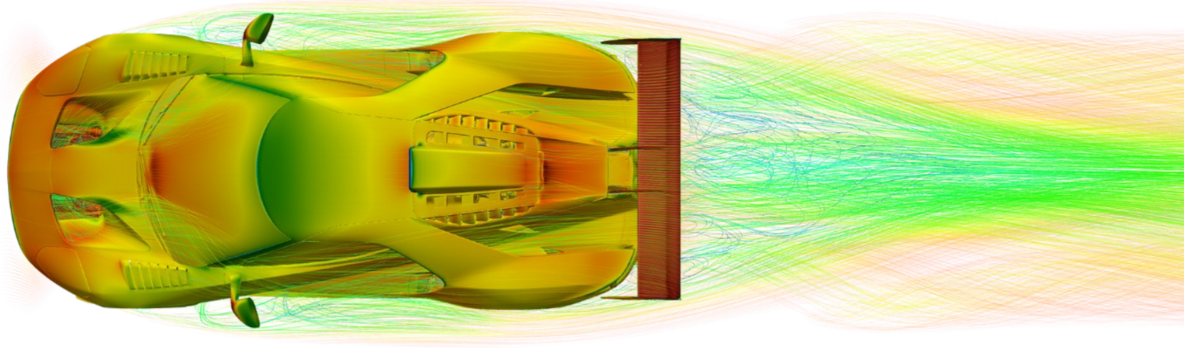
~Mapping of the Circuit Regimes



Strenuous Integrating and analytically impenetrable DEs? Let’s try the ML Approach!







Data-Driven Dynamical Systems

A Machine Learning Approach to System Dynamics – the Lorenz Attractor

Lum Borovci – PISU – Artificial Intelligence and its Applications

July 2023

Old School vs. New School

First-Principles vs. Data Driven Approach

First Principles Approach → Data-Driven Approach

$$\nabla \cdot \mathbf{D} = \rho \qquad \vec{F} = m\vec{a}$$

$$\nabla \cdot \mathbf{B} = 0$$

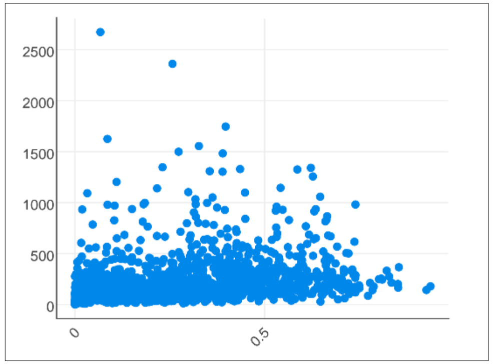
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\frac{\partial \psi}{\partial t} = -i\left(\frac{1}{\hbar}\hat{H}\psi\right)$$

The direction of motion of the amplitude of the wave function = Rotation of -90° (Complex energy at a particular point divided by Planck's constant)

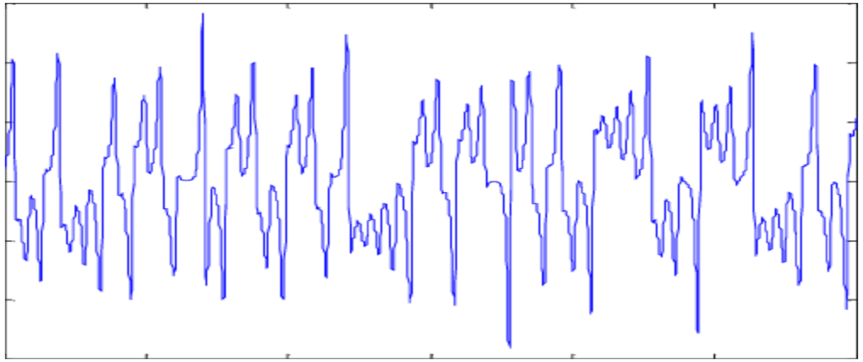
Use general laws as system constraints (conservation of energy, momentum, mass, etc.)



(Galileo's work on planetary orbits

1. Data Collection

2. ML Model proposes system dynamics according to data



3. Result assessment via accuracy measurement

DIFFERENTIAL EQUATIONS (almost always intractable analytically)

(Newton's work on planetary orbits)

SOLUTION

What makes this clock tick?

An inside view of Data-Driven Neural Networks

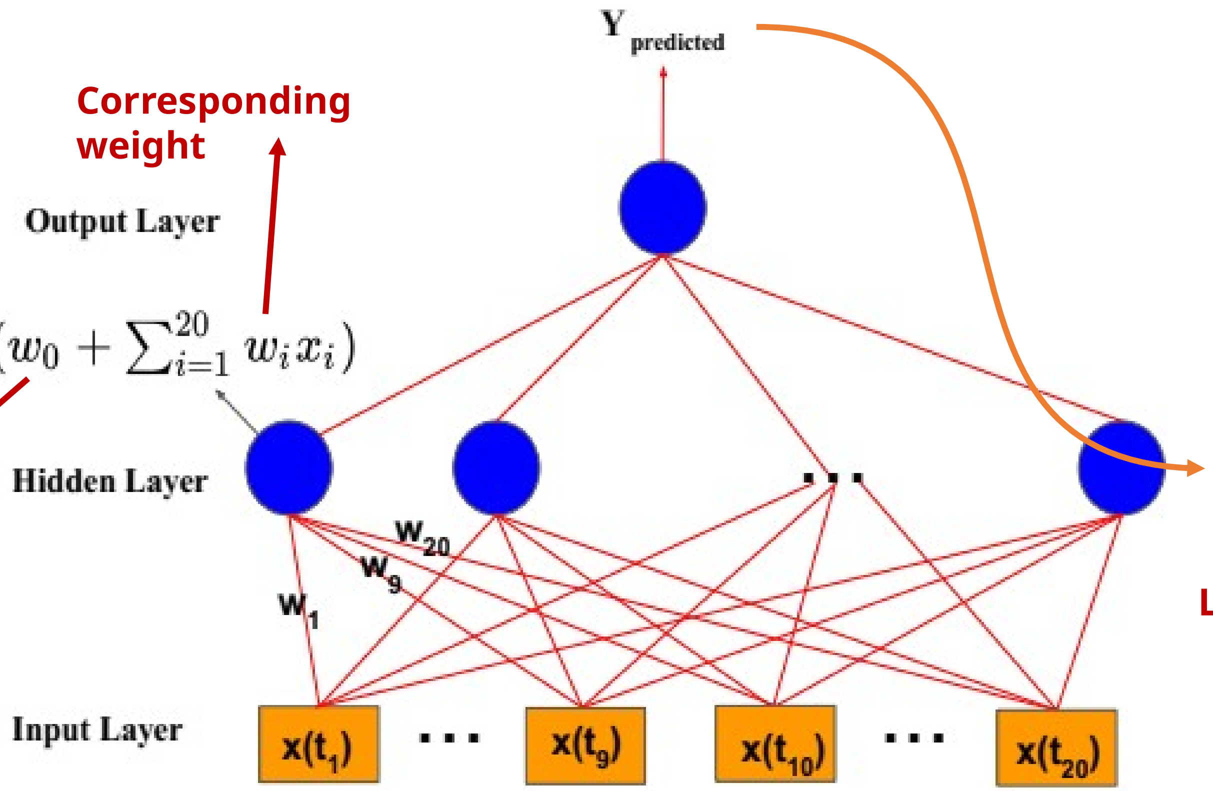
Activation function (Sigmoid)

Corresponding weight

$\widehat{y}_i = g(w_0 + \sum_{i=1}^{20} w_i x_i)$

Potential output

bias term



Schematic for a set of data points where

Backpropagation (Reinforcement NN)
To update the weights

$w := w + \eta \frac{\partial L(w)}{\partial t}$

Loss Function

Learning rate

$L(w) = \frac{1}{N} \sum_{i=1}^{N} \left(\widehat{y}_i - f(x_i, w)\right)^2$

Obtained output

Desired Output

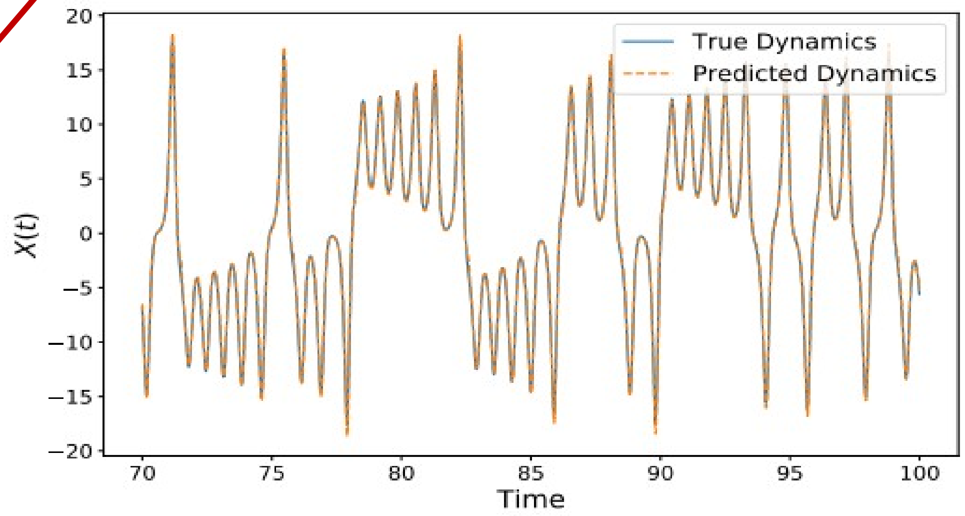
& Regularization (t
avoid overfitting)

Roy, S., Rana, D. Machine Learning in Nonlinear Dynamical Systems. *Reson* **26**, 953–970 (2021). https://doi.org/10.1007/s12045-021-1194-0

How much air is in our system?
The Root Mean Square Error Measurement

Chosen metric for error measurement:

: no noise

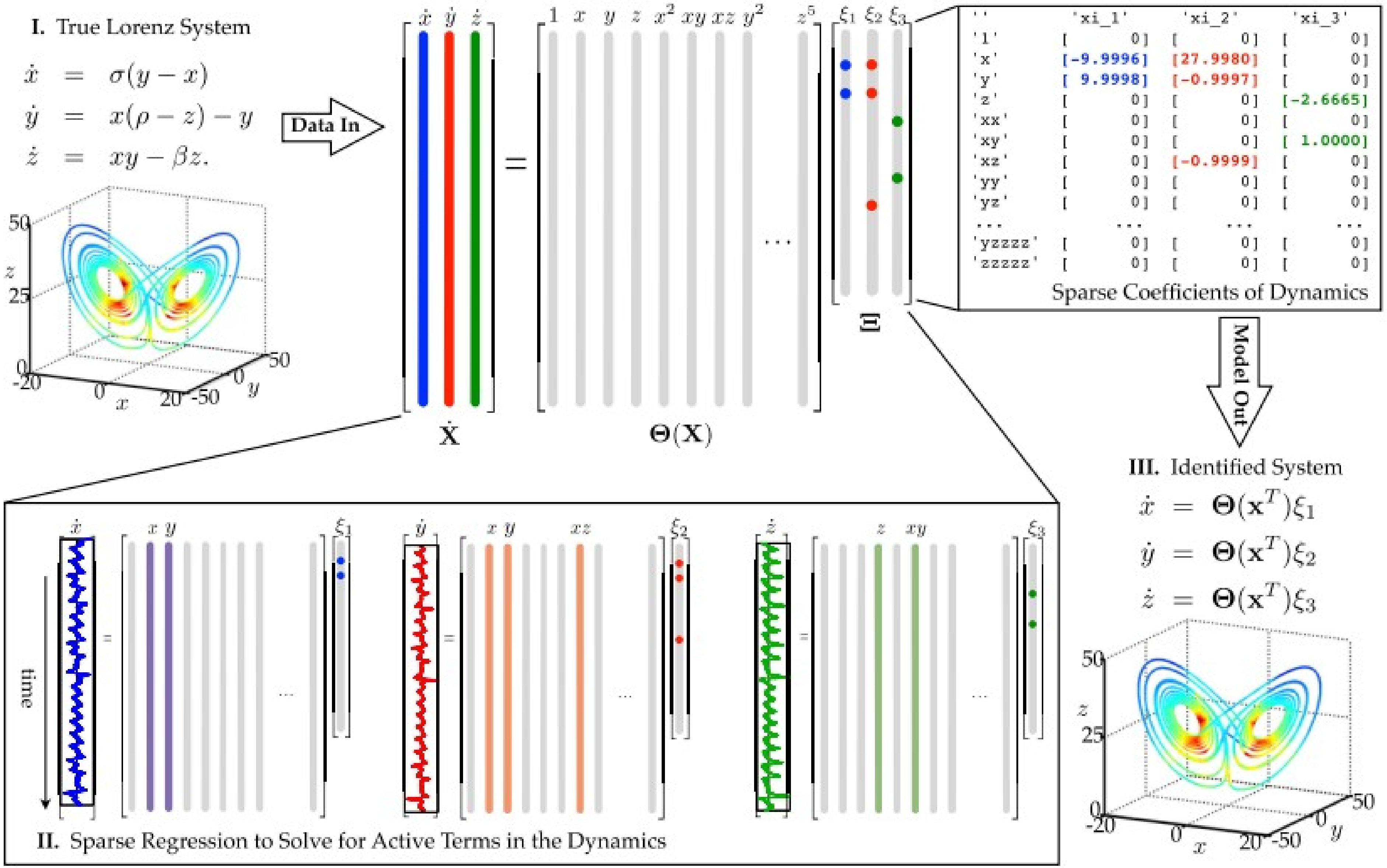


: Gaussian Noise

<u>Only assumption used by the method:</u> Knowing that physical systems have only a few terms (i.e. aren't of dozen polynomial terms, but of 5 or 6 at the most special cases), this allows to build a Matrix of sparse entries, consisting of a few terms.

SINDy Overview

Sparse Identification of Nonlinear Dynamics



Brunton, Steven L., Joshua L. Proctor, and J. Nathan Kutz. "Discovering governing equations from data by sparse identification of nonlinear dynamical systems." *Proceedings of the national academy of sciences* 113.15 (2016): 3932-3937.

Back to the Surgery Room!

Hidden Attractors and Improving Cardiac Reanimation Practices

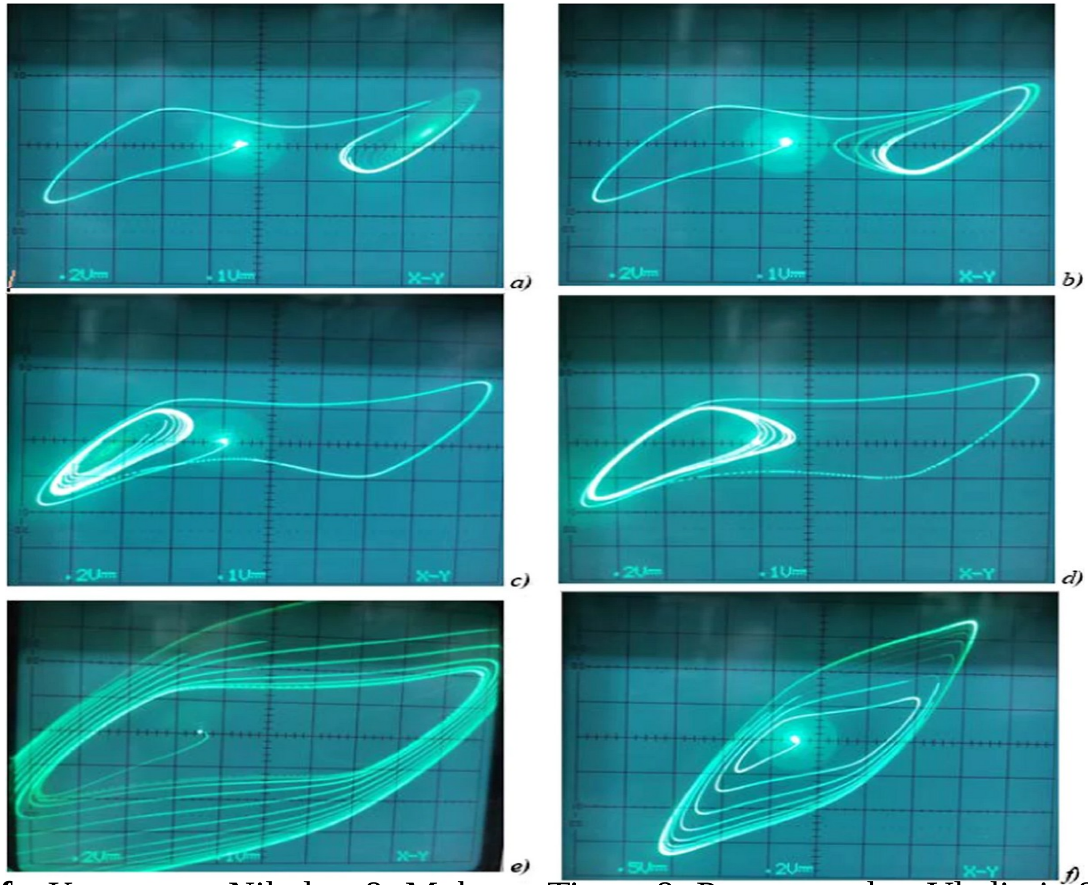
The Frontiers of NLD in Cardiac Dynamics

Hidden Attractors: More Data on CR Breathing!

The Appearance of Hidden Attractors ***(b)*** and ***(d)*** of the inner limit cycles ***(a)*** and ***(c)*** in the Chua Circuit via:

- **Specific Initial Conditions**
- **Very Delicate Measurement Methods**

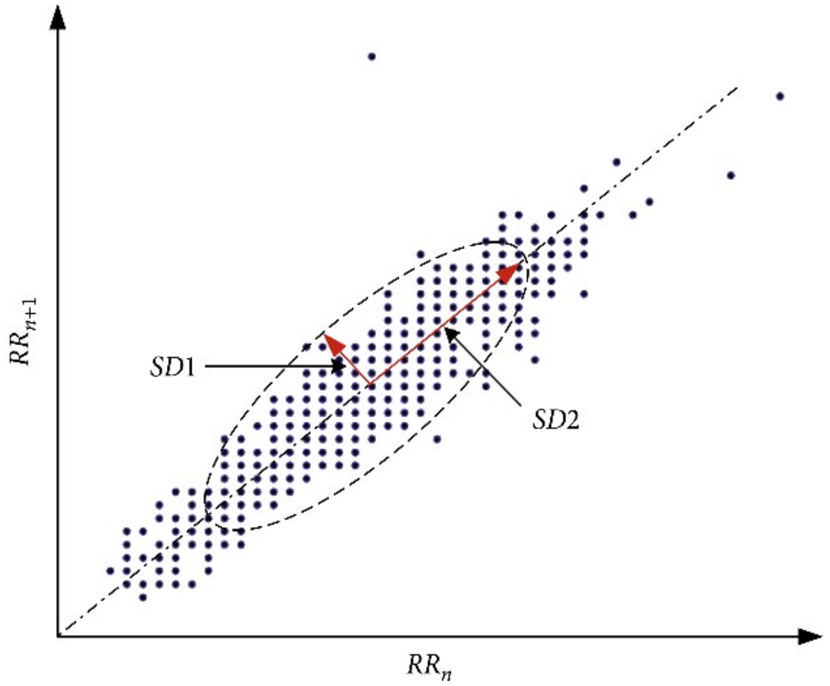
How do we read Hidden Attractors?



Ref.: Kuznetsov, Nikolay & Mokaev, Timur & Ponomarenko, Vladimir & Seleznev, Evgeniy & Stankevich, Nataliya & Chua, Leon. (2022). Hidden attractors in Chua circuit: mathematical theory meets physical experiments. Nonlinear Dynamics.

The Frontiers of NLD in Cardiac Dynamics

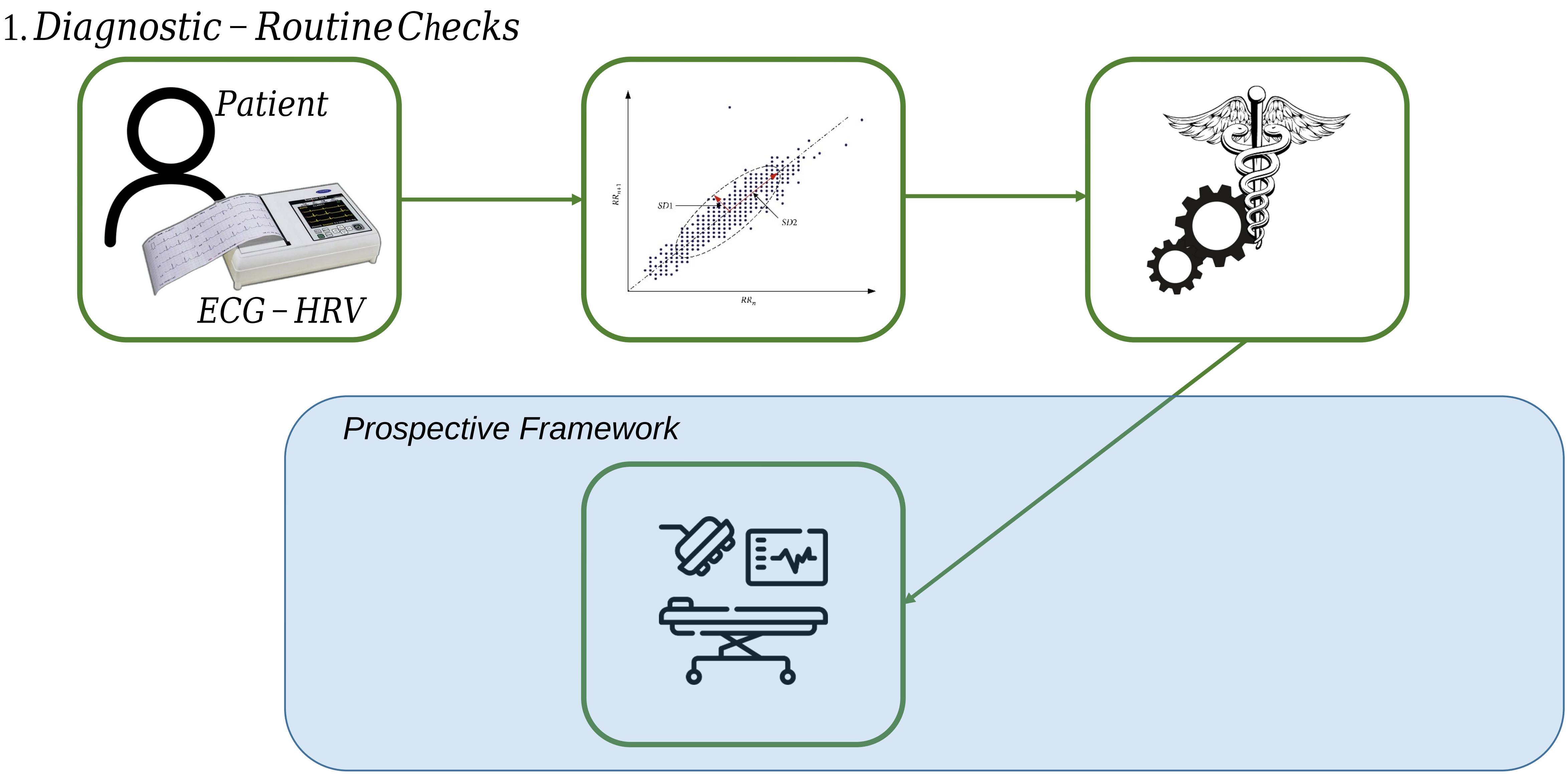
HRV Plots – Hidden Attractor Identifiers!



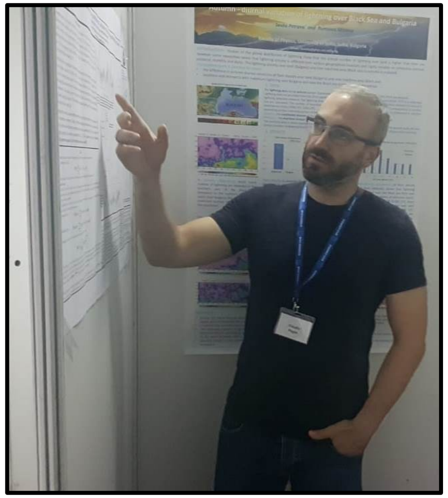
Poincare Plot of a sample HRV Analysis. The trend in the Plot can be defined by an ellipse of major axis and minor axis .

AN IMPROVED CARDIAC RESPONSE SCHEMATIC

USING AND EMPLOYING *HIDDEN-ATTRACTOR RICH* HRV DATA


1. Diagnostic – Routine Checks
Patient
ECG – HRV
SD1
SD2
Prospective Framework

Physical Engineering Framework
Development and Scaling Models into Application



"The physical interpretation of this approach provides extensive data on how the system works on a given set of conditions and how it reacts to external effects that may perturb this behavior. Lum's paper adapts this entire framework to examine cardiac dynamics and shows promising results, paving the way for making data-driven decisions to tweak equipment and practices for treating arrhythmias."

- Dr. Klaudio Peqini, Thesis Co-Supervisor

The development and validation of the mathematical models and methodologies employed in the paper were made possible with the aid of distinguished professors from the Faculty of Natural Sciences:

-Dr. Arban Uka (Supervisor)
-Dr. Klaudio Peqini (Co-Supervisor)
-Dr. Ervin Kafexhiu

FShN, Tirana, May-July 2023

The Response Schematic

Assistance from Medical Experts and Practitioners: Testing, Trialing and Validation

"The driving mechanism behind cardio-respiratory breathing is the network of neurons that command both the heart and the lungs autonomously. This system can be represented using a macroscopic model of stochastic distribution of the FitzHugh-Nagumo model for Action Potentials. Lum presents and works with variations of the Van Der Pol Oscillator, the framework which allows us to build this model, and consequently probe its behavior. In the grand scheme of things, and potentially in future pieces of work, such simulations, if deemed representative to their physical counterparts after testing in medical labs, may give crucial information to medical practitioners treating cardiac arrhythmia."

- *Prof. Ass. Dr. Fatos Sada*

Cross-validation of the claims made on the Response Schematic with regard to CR Breathing has been made with distinguished medicine students with research interests in Pulmonology

UNIZKM, Tirana, June-July 2023

Thank you!

Lumi ☺