"Strong lensing of quasars"

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Gravitational Lensing

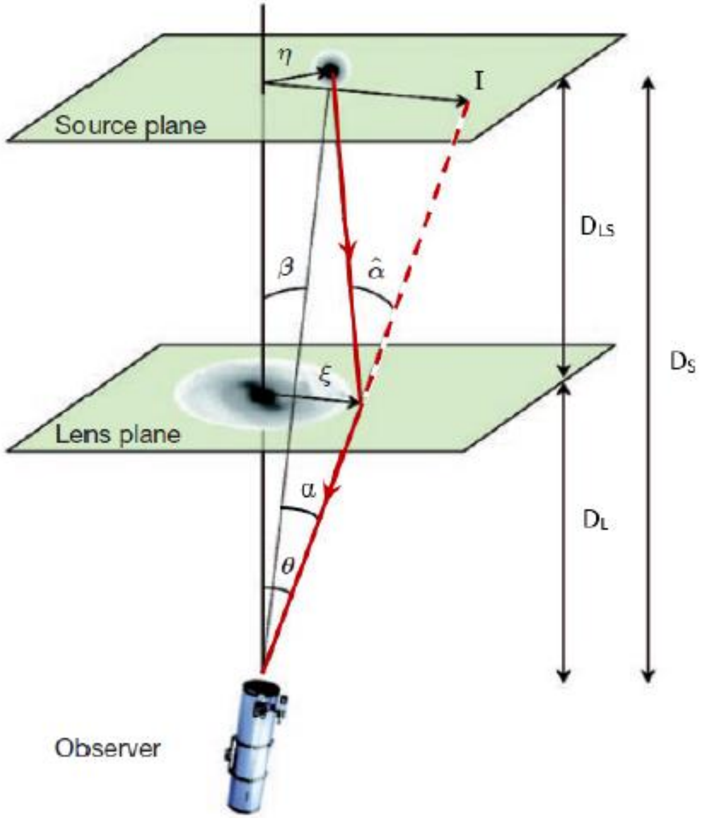
Gravitational lensing is deflection of light from a distant source by the gravity of a massive object

- Lens equation (Einstein, 1936)

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}\left(\vec{\theta}\right),$$

$$\vec{\beta} = 0 \quad \Rightarrow \quad \theta_E = \sqrt{\frac{4GM\left(\theta\right)}{c^2}\frac{D_{LS}}{D_s D_L}}$$

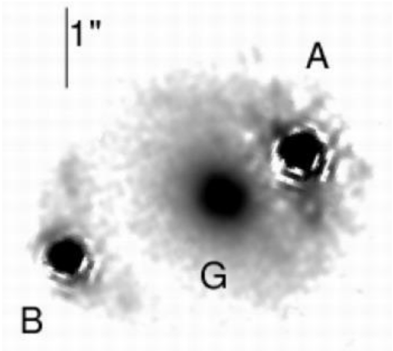
- *Dl*, *Ds* and *Dls* are the observer-lens, observer-source and lens-source angular diameter distances



Strong Lensing

In the case of strong lensing, the lens is a galaxy between the Earth and the source (quasar, supernova, galaxy)

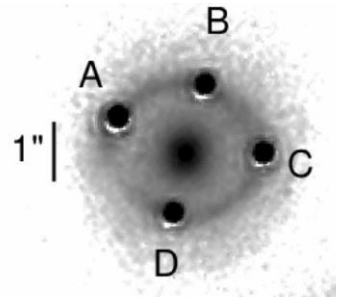
- When a quasar is lensed by galaxies, can be formed multiple images (double, triple or quadruple).



The two-image lens HE1104–1805. G is the lens galaxy and A and B are the quasar images,ZQ=2.32

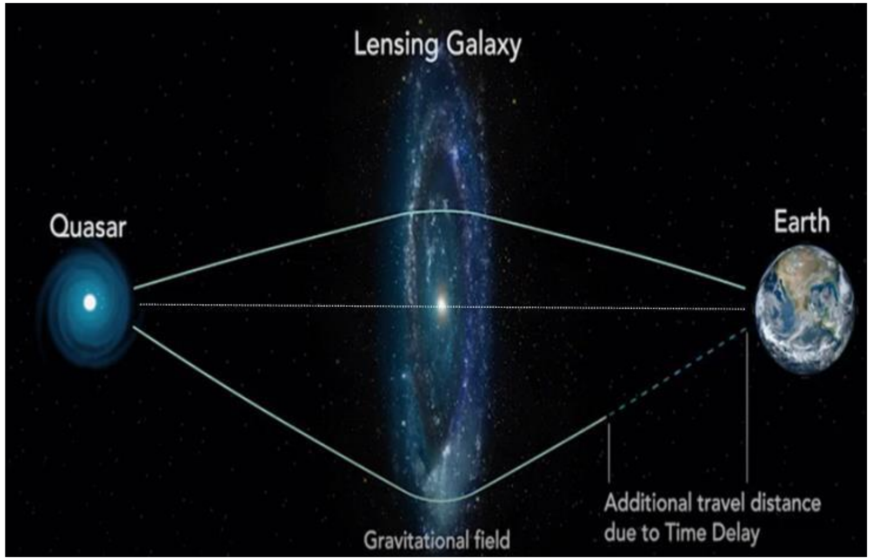
- By strong lensing of quasars can be learn:

- the mass distribution of the lensing galaxy
- cosmological parameters,
- understand the physics of quasars



The four-image lens HE0435–1223. ZQ=1.639 and ZG=0.454

Time delay…



- The geometric time delay:

$$t_{geom} = \frac{1+z_L}{c}\frac{D_L D_S}{D_{LS}}\left[\frac{1}{2}\left(\vec{\theta}-\vec{\beta}\right)^2\right].$$

- The gravitational time delay, time dilation effect (Shapiro delay):

$$t_{grav} = -\frac{1+z_L}{c}\frac{D_L D_S}{D_{LS}}\Psi\left(\vec{\theta}\right)+const.$$

- The total time delay: $t\left(\vec{\theta}\right)=\frac{1+z_L}{c}\frac{D_L D_S}{D_{LS}}\left[\frac{1}{2}\left(\vec{\theta}-\vec{\beta}\right)^2-\Psi\left(\vec{\theta}\right)\right]+const$

- The relative time delays between two images (1) and (2) is observable:

$$\Delta t_{1,2} = \frac{1+z_L}{c}\frac{D_L D_S}{D_{LS}}\left[\frac{1}{2}\left(\vec{\theta_1}-\vec{\beta}\right)^2-\frac{1}{2}\left(\vec{\theta_2}-\vec{\beta}\right)^2-\Psi\left(\vec{\theta_1}\right)+\Psi\left(\vec{\theta_2}\right)\right].$$

Time delay

- Time delays can be written as:

$$\Delta t_{1,2} = \frac{D_{\Delta t}}{c}\Delta\tau_{1,2}$$

- where $D_{\Delta t} = \left(1+z_l\right)\frac{D_L D_S}{D_{LS}}$ and $\tau\left(\vec{\theta}\right) = \frac{1}{2}\left(\vec{\theta}-\vec{\beta}\right)^2 - \Psi\left(\vec{\theta}\right)$

$D_{\Delta t}$ → Time delay distance

$\tau\left(\vec{\theta}\right)$ → Fermat's potential

- If Fermat's potential difference is estimated by lens data, can be determined "time-delay distance" $D_{\Delta t}$, which is inversely proportional to H_0

$$D_L = \frac{c}{H_0\left(1+z_L\right)}\int_0^{z_L}\frac{dz}{E\left(z\right)};$$

$$D_S = \frac{c}{H_0\left(1+z_S\right)}\int_0^{z_S}\frac{dz}{E\left(z\right)};$$

$$D_{LS} = D_S - \frac{1+z_L}{1+z_S}D_L;$$

$$E\left(z\right) = \sqrt{\Omega_m\left(1+z\right)^3 + \Omega_k\left(1+z\right)^2 + \Omega_\lambda}$$

Magnification

- Fermat's potential $\tau\left(\vec{\theta}\right)=\frac{1}{2}\left(\vec{\theta}-\vec{\beta}\right)^2-\Psi\left(\vec{\theta}\right)$

$$\nabla\tau\left(\vec{\theta}\right)=0 \quad \Leftrightarrow \quad \vec{\beta}=\vec{\theta}-\vec{\nabla}\Psi\left(\vec{\theta}\right) \quad \Leftrightarrow \quad \vec{\alpha}\left(\vec{\theta}\right)=\vec{\nabla}\Psi\left(\vec{\theta}\right)$$

- The second order derivative of Fermat's potential:

$$A=\frac{\partial\vec{\beta}}{\partial\vec{\theta}}=\left(\delta_{ij}-\frac{\partial^2\Psi\left(\vec{\theta}\right)}{\partial\theta_i\partial\theta_j}\right)=\begin{pmatrix}1-k-\gamma_1 & -\gamma_2\\ -\gamma_2 & 1-k+\gamma_1\end{pmatrix}$$

where

$$k(\theta)=\frac{1}{2}\left(\frac{\partial^2\psi}{\partial\theta_1^2}+\frac{\partial^2\psi}{\partial\theta_2^2}\right)$$

$$\gamma_1(\theta)=\frac{1}{2}\left(\frac{\partial^2\psi}{\partial\theta_1^2}-\frac{\partial^2\psi}{\partial\theta_2^2}\right)$$

$$\gamma_2(\theta)=\frac{\partial^2\psi}{\partial\theta_1\partial\theta_2}$$

$$\gamma^2=\gamma_1^2+\gamma_2^2$$

The magnification of a image is the inverse of the determinant of the matrix A

$$\mu(\theta)=\left(\det A\right)^{-1}=\left[\left(1-k\right)^2-\gamma^2\right]^{-1}$$

Singular Isothermal Sphere (SIS)

- The mass density distribution is described by:

$$\rho = \frac{\sigma^2}{2\pi G r^2}$$

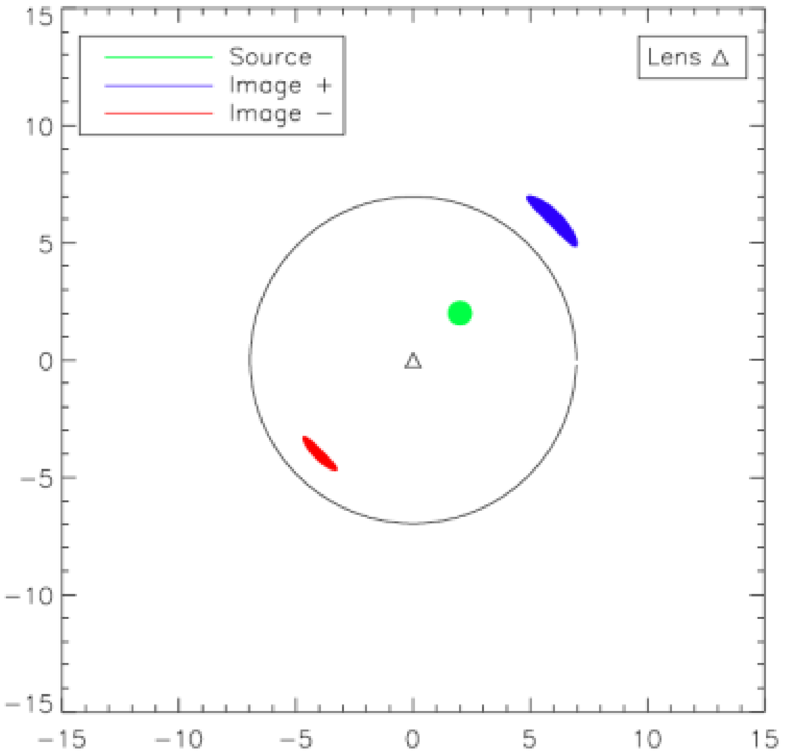
- The lens equation and the solutions:

$$\theta = \beta + \theta_0 \frac{|\theta|}{\theta} \qquad \theta_1 = \beta + \theta_0 \qquad \theta_2 = \beta - \theta_0$$

- Potential: $\Psi(\theta) = \frac{4\pi\sigma^2}{c^2}\frac{D_{LS}}{D_S}|\theta| = \theta_0|\theta|$

- Relative time delays between images: $\Delta t_{1,2} = \frac{1+z_L}{2c}\frac{D_L D_S}{D_{LS}}\left(\theta_1^2 - \theta_2^2\right)$
- Magnification of images:

$$\mu\left(\vec{\theta}\right) = \frac{|\theta|}{|\theta| - \theta_0}$$



Non Singular Isothermal Sphere (NIS)…

- The mass density distribution of the galaxy:

$$\rho(r) = \frac{\sigma^2}{2\pi G}\left(\frac{1}{r^2 + r_c^2}\right)$$

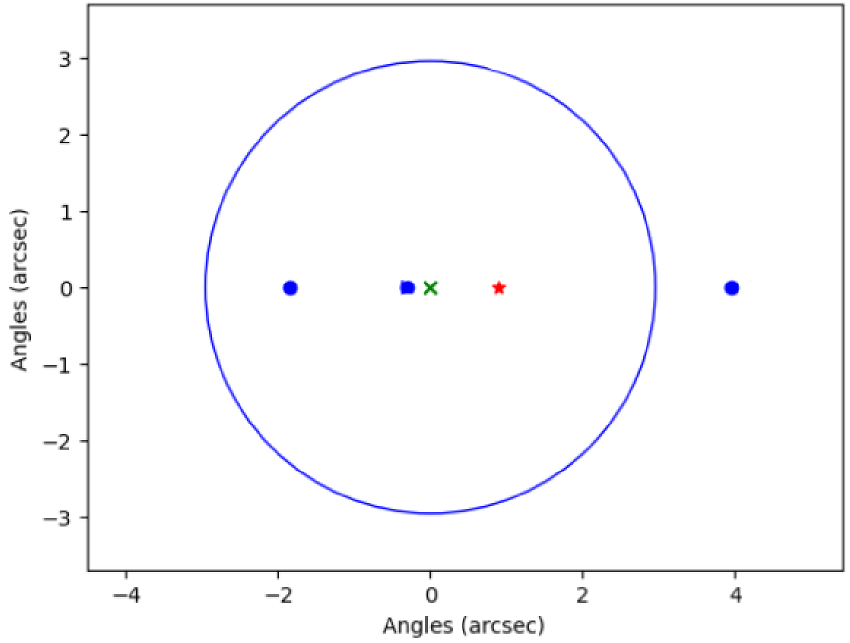
- The Einstein angle and the lens equation:

$$\theta_{ENIS} = \sqrt{\theta_0^2 - 2\theta_0\theta_c} \qquad \vec{\beta} = \vec{\theta}\left(1 - \frac{\theta_0}{\theta^2}\left(\sqrt{\theta^2 + \theta_c^2} - \theta_c\right)\right)$$

- In this case three images are formed, the positions of which are determined by solving the equation.

$$\theta^3 - 2\beta\theta^2 + \left(\beta^2 - \theta_0\left(\theta_0 - 2\theta_c\right)\right)\theta - 2\beta\theta_0\theta_c = 0$$

$$\theta_c < 1/2\theta_0$$



Non Singular Isothermal Sphere (NIS)

- Potential: $\Psi\left(\theta\right)=\theta_0\left[\sqrt{\theta^2+\theta_c^2}-\theta_c\ln\left(\sqrt{\theta^2+\theta_c^2}+\theta_c\right)\right]$

- Relative time delays between two external images:

$$\Delta t_{1,2}=\frac{1+z_L}{c}\frac{D_L D_S}{D_{LS}}\left[\frac{1}{2}\left(\vec{\theta}_1-\vec{\beta}\right)^2-\frac{1}{2}\left(\vec{\theta}_2-\vec{\beta}\right)^2-\Psi\left(\vec{\theta}_1\right)+\Psi\left(\vec{\theta}_2\right)\right]$$

- Magnification:

$$\mu\left(\vec{\theta}\right)=\frac{1}{\left(1-\frac{\theta_0}{2\sqrt{\theta^2+\theta_c^2}}\right)^2-\left[\frac{\theta_0\left(\sqrt{\theta^2+\theta_c^2}-\theta_c\right)}{\theta^2}-\frac{\theta_0}{2\sqrt{\theta^2+\theta_c^2}}\right]^2}.$$

The algorithm of the quasar lensing

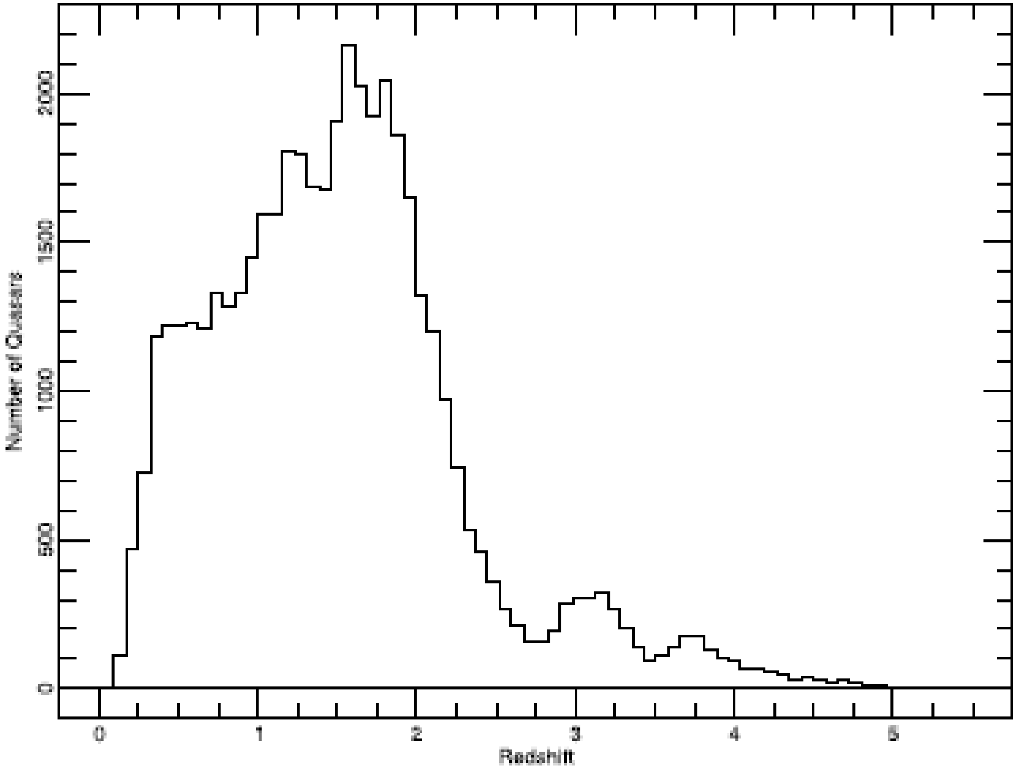
- Generate *the redshift of the quasar*, following the corresponding distribution (Schneider et al., 2005) ;
- Generate *the redshift of the galaxy*, following the corresponding distributions (Appenzeller et al., 2004) Constrain that the galaxy redshift is lower than quasar redshift. The number of galaxies that fulfilling this condition for each quasar is found by the cumulative number of galaxies
- For each generated galaxy, we find *its mass* (Davidzon et. al., 2017), and its *velocity dispersion* (Zahid et al., 2016);
- Based on the model (SIS) for the galaxies, we define the *Einstein angle* for the couple quasar/galaxy;
- Extract a uniformly distributed number in the interval (0,1) and, considering 200 billions of galaxies in the Universe, keep the couple when $n < 10^8\, \theta_E^2 / 14$ (aligned couple), otherwise reject it (non aligned couple);
- For each aligned couple calculate *Δt and magnification* of images. Comparing its Einstein angle with the angular resolution of the instrument, estimate the number of lensed quasars expected to be observed by it.

The cosmological parameters were used for the above calculations:

$$\Omega_m = 0.30 \quad \Omega_k = 0 \quad \Omega_\lambda = 0.70 \quad H_0 = 70 \ \text{km s}^{-1}\,\text{Mpc}^{-1}$$

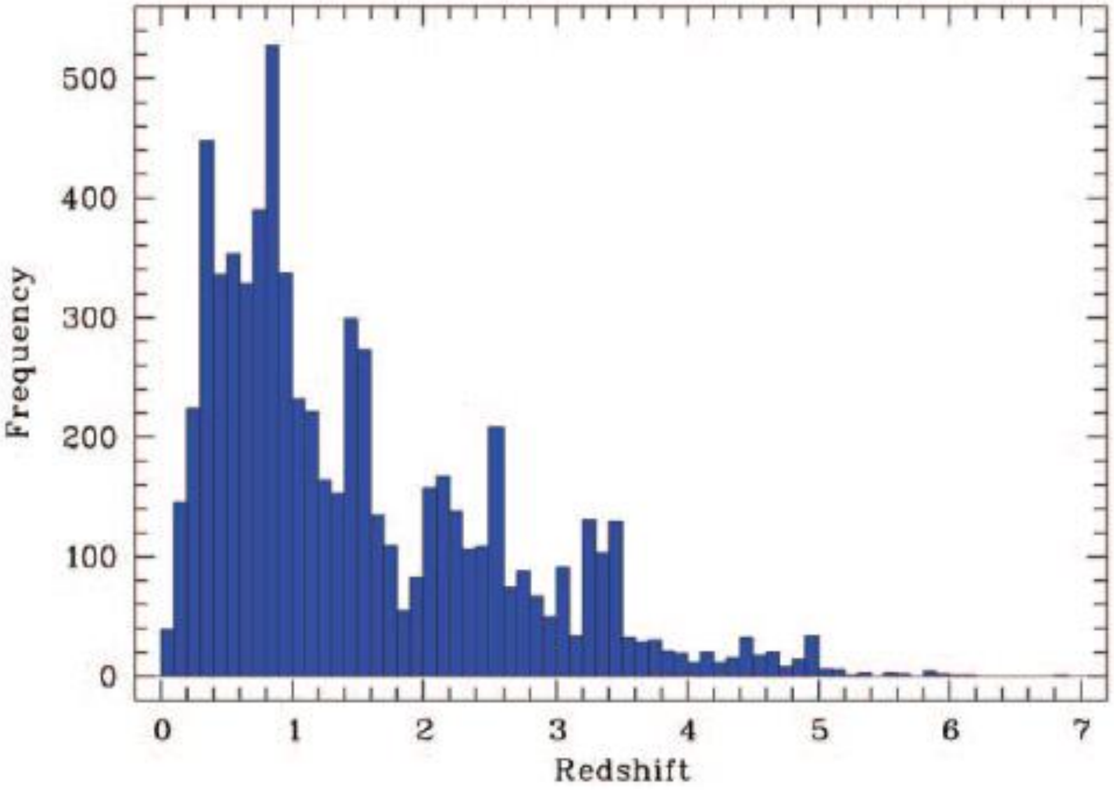
Quasar redshift distribution (Histogram)

The redshift histogram of 46,420 quasars. The redshift bins have a width of 0.076. The dips at redshifts of 2.7 and 3.5 are caused by the lower efficiency of the selection algorithm at these redshifts, **Schneider et al. 2005**.



Galaxy redshift distribution (Histogram)

The frequency is the number of galaxies within redshift intervals of 0.1. Conspicuous maxima at distinct redshifts (e.g. at z=0.3, 0.8, 2.4, and 3.4) reflect the local sponge-like large scale structure of the matter distribution in the universe, **Appenzeller et al. 2004**.



Galaxy mass distribution

SMF (stellar mass function) is described by **a Schechter function** (Schechter 1976):

$$\Phi(M)dM = \left[\Phi_1^*\left(\frac{M}{M_*}\right)^{\alpha_1} + \Phi_2^*\left(\frac{M}{M_*}\right)^{\alpha_2}\right]\exp\left(-\frac{M}{M_*}\right)\frac{dM}{M_*}$$

The parameters are estimated by **Davidzon et al. 2017**, best fitting the data of COSMOS2015 catalogue

Redshift	$\log \mathcal{M}_\star$ $[h_{70}^{-2}\,\mathcal{M}_\odot]$	α_1	$\Phi_{\star 1}$ $[10^{-3}\,h_{70}^3\,\mathrm{Mpc}^{-3}]$	α_2	$\Phi_{\star 2}$ $[10^{-3}\,h_{70}^3\,\mathrm{Mpc}^{-3}]$
			Total sample		
$0.2 < z \leqslant 0.5$	$10.78^{+0.13}_{-0.14}$	$-1.38^{+0.08}_{-0.25}$	$1.187^{+0.633}_{-0.969}$	$-0.43^{+0.62}_{-0.60}$	$1.92^{+0.73}_{-0.78}$
$0.5 < z \leqslant 0.8$	$10.77^{+0.09}_{-0.08}$	$-1.36^{+0.05}_{-0.06}$	$1.070^{+0.287}_{-0.315}$	$0.03^{+0.43}_{-0.43}$	$1.68^{+0.33}_{-0.33}$
$0.8 < z \leqslant 1.1$	$10.56^{+0.05}_{-0.05}$	$-1.31^{+0.05}_{-0.06}$	$1.428^{+0.306}_{-0.308}$	$0.51^{+0.35}_{-0.34}$	$2.19^{+0.40}_{-0.41}$
$1.1 < z \leqslant 1.5$	$10.62^{+0.08}_{-0.07}$	$-1.28^{+0.05}_{-0.05}$	$1.069^{+0.222}_{-0.240}$	$0.29^{+0.40}_{-0.42}$	$1.21^{+0.23}_{-0.22}$
$1.5 < z \leqslant 2.0$	$10.51^{+0.08}_{-0.07}$	$-1.28^{+0.06}_{-0.06}$	$0.969^{+0.202}_{-0.208}$	$0.82^{+0.48}_{-0.52}$	$0.64^{+0.18}_{-0.17}$
$2.0 < z \leqslant 2.5$	$10.60^{+0.15}_{-0.12}$	$-1.57^{+0.12}_{-0.21}$	$0.295^{+0.173}_{-0.177}$	$0.07^{+0.70}_{-0.74}$	$0.45^{+0.12}_{-0.12}$
$2.5 < z \leqslant 3.0$	$10.59^{+0.36}_{-0.36}$	$-1.67^{+0.26}_{-0.26}$	$0.228^{+0.300}_{-0.300}$	$-0.08^{+1.73}_{-1.73}$	$0.21^{+0.14}_{-0.38}$
$3.0 < z \leqslant 3.5$	$10.83^{+0.15}_{-0.15}$	$-1.76^{+0.13}_{-0.11}$	$0.090^{+0.064}_{-0.039}$		
$3.5 < z \leqslant 4.5$	$11.10^{+0.21}_{-0.21}$	$-1.98^{+0.14}_{-0.13}$	$0.016^{+0.020}_{-0.009}$		
$4.5 < z \leqslant 5.5$	$11.30^{+1.22}_{-1.22}$	$-2.11^{+0.34}_{-0.22}$	$0.003^{+0.002}_{-0.002}$		

Galactic velocity dispersion

Analyzing the SDSS and SHELS data, **Zahid et al. 2016**, proposed a relation between the stellar velocity dispersion (*σ)* and the stellar mass (*M)* of the galaxies, which is a power law with a break point,

$$\begin{cases} \sigma(M) = \sigma_b \left(\dfrac{M}{M_b} \right)^{\alpha_1} \; for M \leq M_b \\ \sigma(M) = \sigma_b \left(\dfrac{M}{M_b} \right)^{\alpha_2} \; for M \geq M_b . \end{cases}$$

$$\log\left(M_b / M_{\odot}\right) = 10.26,$$
$$\log\left(\sigma_b\right) = 2.073,$$
$$\alpha_1 = 0.403,$$
$$\alpha_2 = 0.293$$

Results and conclusions…

- 100 000 strong lensing events.

➢ **Roman Space Telescope (NASA infrared Space Telescope)**

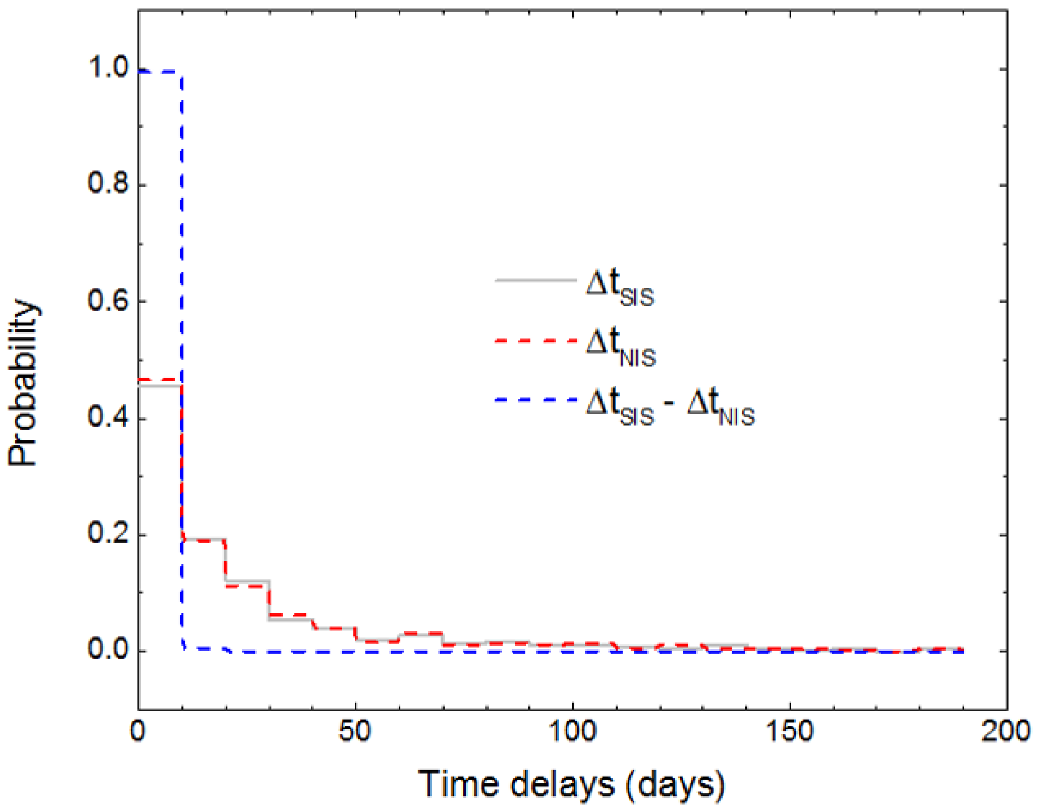
- Angular resolution 0.1 arcsec
- launch in May 2027

- 85% of lensed quasars are caused by a single galaxy.

- The time delays range from 3.5 hours to 2000 days.

- The difference between them $\Delta t_{SIS} - \Delta t_{NIS}$ range from 0.1 hours to 73 days.

- The width of the bin is 10 days.

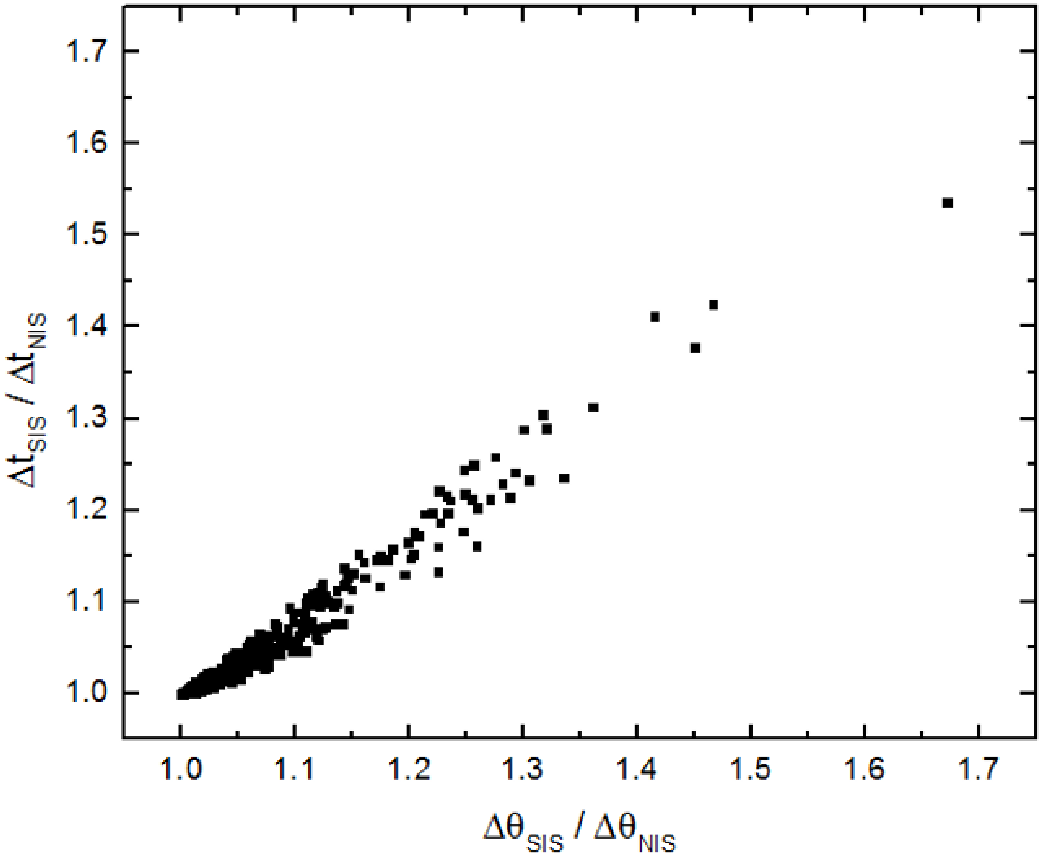


Results and conclusions...

There is a linear relation between $\Delta t_{SIS} / \Delta t_{NIS}$ and $\Delta\theta_{SIS} / \Delta\theta_{NIS}$

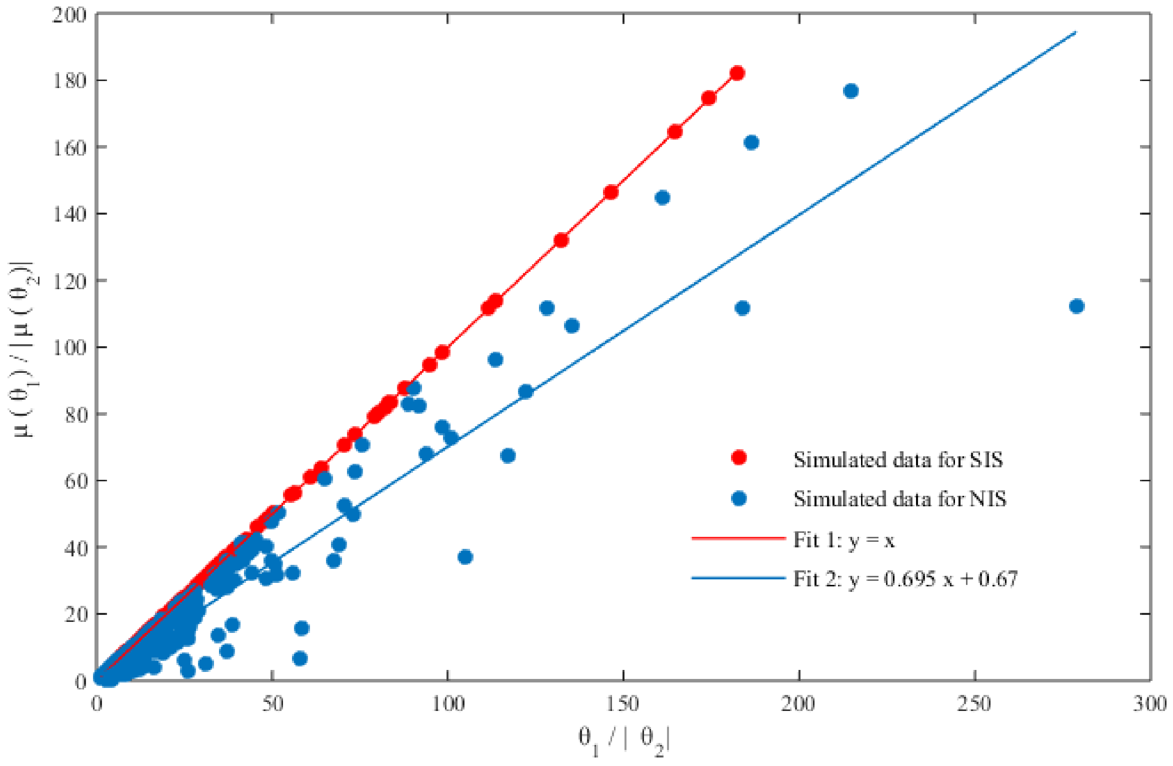
$$\Delta\theta_{SIS} = \theta_{1SIS} - \theta_{2SIS}$$

$$\Delta\theta_{NIS} = \theta_{1NIS} - \theta_{2NIS}$$



Results and conclusions…

- For the SIS model (in the red color), it is evident that the quantities increase linearly
- The slope of the line for SIS is 1.
- For the NIS model (in the blue color), a linear relationship is suggested by a Pearson correlation coefficient of 0.95 and a Spearman’s correlation of 0.94, indicating a monotonic increase.
- The slope of the line for the NIS model is 0.7

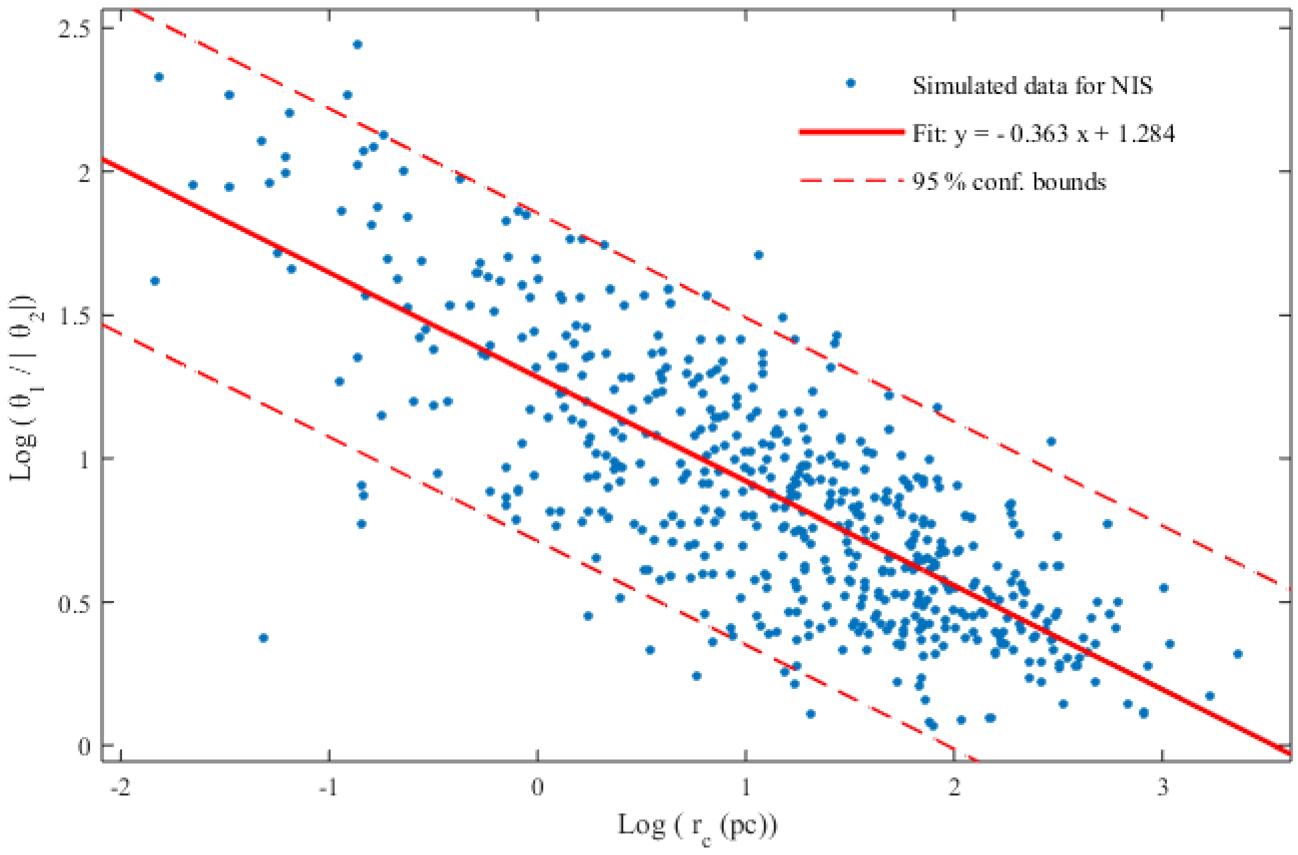


Results and conclusions...

- As the core radius rc increases, the ratio θ1/|θ2| decreases.

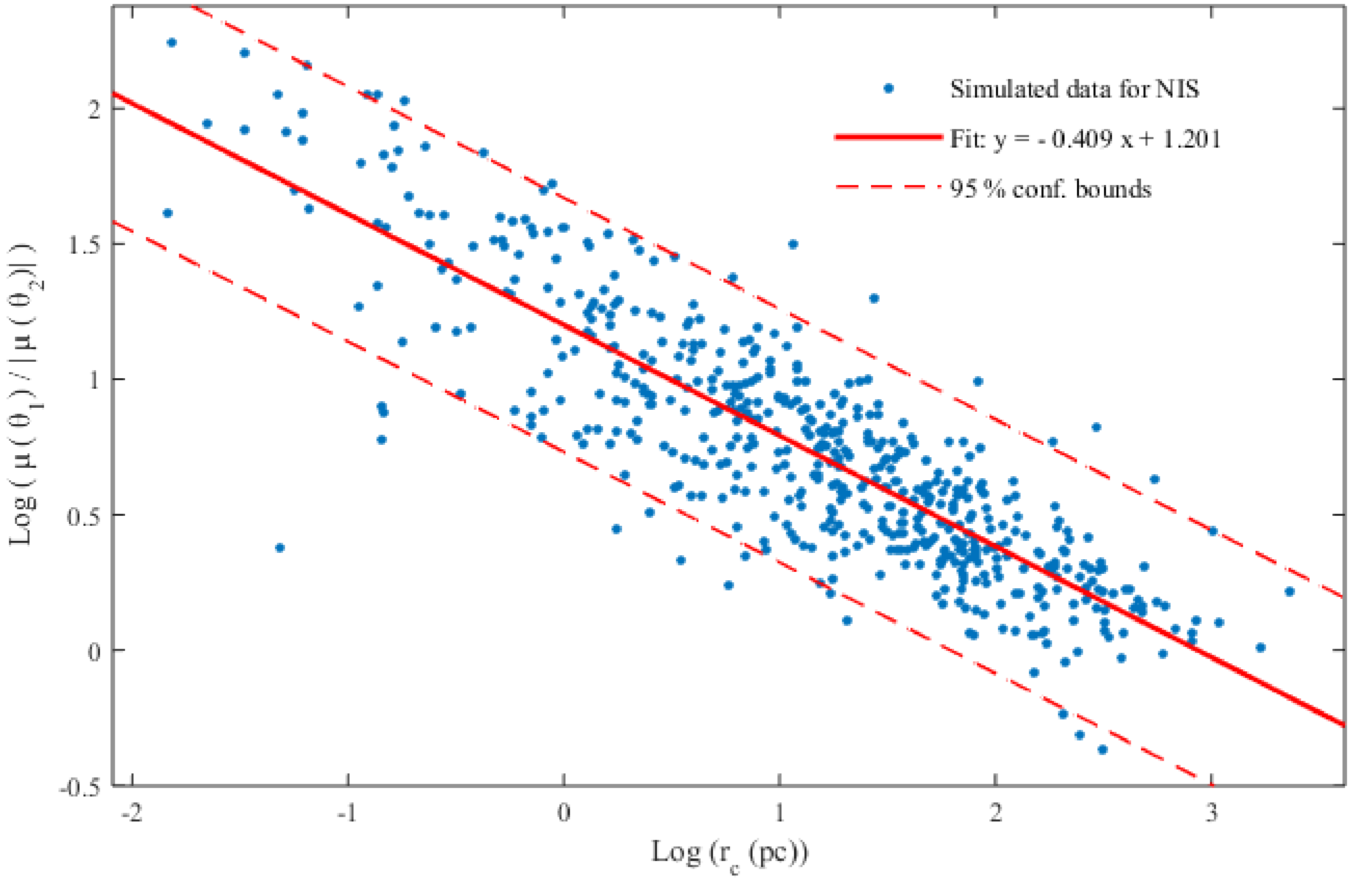
- Pearson's correlation coefficient is −0.77 and a Spearman's correlation coefficient is −0.74.

- The line representing the curve of the best fit has a slope of −0.363.



Results and conclusions

- As the core radius rc increases, the ratio μ (θ1) /|μ (θ2) | decreases.
- A Pearson's correlation coefficient is −0.86 and a Spearman's correlation coefficient of −0.85.
- The line representing the curve of the best fit has a slope of −0.409, further confirming this inverse relation.



Isothermal elliptical lens model (SIE)

- Surface mass density:

$$\Sigma(\vec{\xi}) = \frac{\sigma_v^2}{2G}\frac{\sqrt{f}}{\sqrt{\xi_1^2 + f^2\xi_2^2}},$$

- Potential:

$$\Psi(x,\varphi) = x\frac{\sqrt{f}}{f'}\left[\sin\varphi \arcsin\left(f'\sin\varphi\right) + \cos\varphi \operatorname{arcsinh}(f'/f\cos\varphi)\right].$$

- Lens equation:

$$\boldsymbol{y} = \boldsymbol{x} - \frac{\sqrt{f}}{f'}\left[\operatorname{arsinh}\left(\frac{f'}{f}\cos\varphi\right)\boldsymbol{e}_1 + \arcsin(f'\sin\varphi)\,\boldsymbol{e}_2\right]$$

- Time delays:

$$c\,\delta t = 2\pi^2(1+z_\mathrm{d})\frac{v^4}{c^4}\frac{D_\mathrm{d}D_\mathrm{ds}}{D_\mathrm{s}}\frac{f'^2|\sin(2\varphi)|}{\sqrt{f}\Delta}(\delta x)^3$$

- Magnification:

$$|\mu| = \frac{4\sqrt{f}\Delta}{f'^2|\sin(2\varphi)|}\frac{1}{\delta x}$$

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Thank you!