Three years of thermodynamics of scalar-tensor gravity

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work with J. Côté, A. Giusti, S. Giardino, S. Zentarra, L. Heisenberg, A. Mentrelli, S. Jose, M. Miranda, P.-A. Graham, A. Leblanc, R. Vanderwee, J. Houle, T. Françonnet, N. Karolinski (BU, ETH Zürich, U. Heidelberg, U. Bologna, U. Sherbrooke, U. Naples) spread over 14 articles in PRD, JCAP, EPJC, GRG

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Outline

1. Effective fluid description of scalar-tensor/Horndeski gravity
2. 1st order thermodynamics of this *dissipative* fluid: "temperature of gravity", states of equilibrium, "hot" singularities
3. "Standard" scalar fields in GR $+$ Einstein frame ST gravity: trade temperature with chemical potential
4. A more refined picture when $\Box\phi \neq 0$
5. Outlooks

Motivation

There are many motivations to modify Einstein's GR:

- Quantum corrections introduce deviations from GR as extra degrees of freedom, higher order field equations, ... (ex: Starobinski inflation, low-energy limit of bosonic string is $\omega = -1$ Brans-Dicke gravity, ...).
- Explaining the present acceleration of the cosmic expansion without the *ad hoc* dark energy. The ΛCDM model fits the data but is incomplete and completely unsatisfactory from the theoretical point of view.
- GR is not well-tested on many scales or in all regimes. Even Newtonian gravity is doubted (MOND).

Scalar-tensor gravity is the prototypical alternative to GR; $f(R)$ gravity, a subclass, is extremely popular to explain the current acceleration of the universe without an *ad hoc* dark energy.

Introduces one extra scalar (massive) d.o.f. ϕ in addition to the two massless spin 2 polarizations of GR

ST gravity has evolved into Horndeski gravity, believed to be the most general ST theory with 2nd order equations of motion (until DHOST came) $\rightarrow$ avoid Ostrogradsky instability.

$$S\left[g_{ab}, \phi\right] = \int d^4x \sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right) + S^{(m)}$$

where $X \equiv -\dfrac{1}{2}\nabla_c\phi\nabla^c\phi$ and

$$
\begin{aligned}
\mathcal{L}_2 &= G_2(\phi, X)\,, \\
\mathcal{L}_3 &= -G_3(\phi, X)\,\Box\phi\,, \\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_a\nabla_b\phi)^2\right]\,, \\
\mathcal{L}_5 &= G_5(\phi, X)\,G_{ab}\nabla^a\nabla^b\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_a\nabla_b\phi)^3\right. \\
&\quad \left.+2\,(\nabla_a\nabla_b\phi)^3\right]\,,
\end{aligned}
$$

the $G_i(\phi, X)$ are regular functions ($i = 2, 3, 4, 5$), $G_{i\phi} \equiv \partial G_i/\partial\phi$, *etc.*

The multi-messenger event GW1708017/GRB1708017A, a neutron star binary merger, restricts Horndeski (at least in the late universe) to the "viable" class in which gravitational waves propagate at speed c:

$$\boxed{G_5 = G_{4X} = 0}$$

(also avoids instabilities and allows an Einstein frame formulation). Contains "1st generation" scalar-tensor (*e.g.*, Brans-Dicke) and $f(R)$ gravity.

Field equations of viable Horndeski:

$$G_4 G_{ab} - \nabla_a \nabla_b G_4 + \left[\Box G_4 + \frac{G_2}{2} - X G_3 \right] g_{ab}$$
$$+ \frac{1}{2} \left(G_{3X} \Box \phi - G_{2X} \right) \nabla_a \phi \nabla_b \phi + \nabla_{(a} \phi \nabla_{b)} G_3 = T_{ab}^{(m)}$$

rewrite as effective Einstein equations

$$G_{ab} = T_{ab}[\phi] + \frac{T_{ab}^{(m)}}{G_4} \equiv T_{ab}^{(2)} + T_{ab}^{(3)} + T_{ab}^{(4)} + \frac{T_{ab}^{(m)}}{G_4}$$

where

$$
\begin{aligned}
T^{(2)}_{ab} &= \frac{1}{2G_4}\left(G_{2X}\nabla_a\phi\nabla_b\phi + G_2\, g_{ab}\right) , \\
T^{(3)}_{ab} &= \frac{1}{2G_4}\left(G_{3X}\nabla_c X\nabla^c\phi - 2XG_{3\phi}\right) g_{ab} , \\
&\quad -\frac{1}{2G_4}\left(2G_{3\phi} + G_{3X}\Box\phi\right)\nabla_a\phi\nabla_b\phi \\
&\quad -\frac{G_{3X}}{G_4}\nabla_{(a}X\nabla_{b)}\phi , \\
T^{(4)}_{ab} &= \frac{G_{4\phi}}{G_4}\left(\nabla_a\nabla_b\phi - g_{ab}\Box\phi\right) + \frac{G_{4\phi\phi}}{G_4}\left(\nabla_a\phi\nabla_b\phi + 2X\right) .
\end{aligned}
$$

Most current works on Horndeski gravity are formal (not a criticism): disformal transformation as a solution-generating technique, disformal classes, *etc.* The physics is certainly hidden.

Effective fluid description of Horndeski gravity

Already known for special theories, or geometries, or "1st gen" scalar-tensor gravity. Possible if $\nabla^c\phi$ is timelike and future-oriented.

Kinematic quantities. Define effective **4-velocity**

$$u^c \equiv \frac{\nabla^c\phi}{\sqrt{2X}}$$

Corresponding spatial **3-metric**

$$h_{ab} \equiv g_{ab} + u_a u_b \,,$$

4-acceleration

$$\dot{u}^a \equiv u^c \nabla_c u^a = -\frac{1}{2X}\left(\nabla^a X - \frac{\nabla_c X \nabla^c X}{2X}\nabla^a\phi\right)$$

4-velocity gradient

$$\nabla_b u_a = \sigma_{ab} + \frac{\Theta}{3} h_{ab} + \cancel{\omega_{ab}} + \dot{u}_a u_b \,.$$

Decompose projected velocity gradient in the usual way:

$$V_{ab} \equiv h_a{}^c\, h_b{}^d \nabla_d u_c = \sigma_{ab} + \frac{\Theta}{3}\, h_{ab}\,,$$

where **expansion** and **shear** are

$$\begin{aligned}
\Theta &= \nabla_c u^c = \frac{1}{\sqrt{2X}}\left(\Box\phi + \frac{\nabla^a\phi\nabla^b\phi\nabla_a\nabla_b\phi}{2X}\right) \\
\sigma_{ab} &= \frac{1}{\sqrt{2X}}\left[\nabla_a\nabla_b\phi - \frac{\nabla_{(a}X\nabla_{b)}\phi}{X} - \frac{\nabla_c X\nabla^c\phi}{4X^2}\,\nabla_a\phi\nabla_b\phi \right. \\
&\quad \left. -\frac{h_{ab}}{3}\left(\Box\phi - \frac{\nabla_c X\nabla^c\phi}{2X}\right)\right]
\end{aligned}$$

Restrict, for now, to viable Horndeski, rewrite field equations as effective Einstein eqs.

$$G_{ab} = T_{ab}[\phi] + \frac{T_{ab}^{(m)}}{G_4} \equiv T_{ab}^{(2)} + T_{ab}^{(3)} + T_{ab}^{(4)} + \frac{T_{ab}^{(m)}}{G_4}$$

then $T_{ab}[\phi]$ has the form of a *dissipative* fluid tensor

$$\boxed{T_{ab} = \rho u_a u_b + P h_{ab} + \pi_{ab} + q_a u_b + q_b u_a}$$

with energy density

$$\begin{aligned}
\rho \quad = \quad & T_{ab}u^a u^b = \rho^{(2)} + \rho^{(3)} + \rho^{(4)} = \frac{1}{2G_4}\left(2XG_{2X} - G_2\right) \\
& -\frac{1}{2G_4}\left(-G_{3X}\nabla_c X \nabla^c \phi + 2XG_{3\phi} + 2XG_{3X}\Box\phi\right) \\
& +\frac{G_{4\phi}}{G_4}\left(\Box\phi - \frac{\nabla_c X \nabla^c \phi}{2X}\right),
\end{aligned}$$

isotropic pressure

$$
\begin{aligned}
P &= \frac{h^{ab}T_{ab}}{3} = \bar{P} + P_{viscous} = P^{(2)} + P^{(3)} + P^{(4)} \\
&= \left[\frac{G_2}{2G_4}\right] + \left[\frac{1}{2G_4}\left(G_{3X}\nabla_c X\nabla^c\phi - 2XG_{3\phi}\right)\right] \\
&\quad + \left[-\frac{G_3}{3G_4}\left(2\Box\phi + \frac{\nabla_c X\nabla^c\phi}{2X}\right) + \frac{2XG_{4\phi\phi}}{G_4}\right],
\end{aligned}
$$

trace-free anisotropic stress tensor

$$
\begin{aligned}
\pi_{ab} &= T_{cd}h_a{}^c h_b^d - Ph_{ab} = \pi_{ab}^{(2)} + \pi_{ab}^{(3)} + \pi_{ab}^{(4)} \\
&= 0 + 0 + h_{ab}\left[\frac{G_{4\phi}}{3G_4}\left(2\Box\phi + \frac{\nabla_c X\nabla^c\phi}{2X}\right) - \frac{2XG_{4\phi\phi}}{G_4}\right],
\end{aligned}
$$

and heat flux density

$$
\begin{aligned}
q_a &= -T_{cd}u^c h_a{}^d = q_a^{(2)} + q_a^{(3)} + q_a^{(4)} \\
&= 0 - \frac{G_{3X}\sqrt{2X}}{2G_4}\left(\nabla_a X + \frac{\nabla_c X \nabla^c \phi}{2X}\nabla_A \phi\right) \\
&\quad + \frac{G_{4\phi}}{G_4\sqrt{2X}}\left(\nabla_a X + \frac{\nabla_c X \nabla^c \phi}{2X}\nabla_a \phi\right) .
\end{aligned}
$$

h_{ab}, π^{ab}, q^a are purely spatial.

1st order thermodynamics of viable Horndeski

With the *caveat* that we have an *effective* dissipative fluid $T_{ab}[\phi]$, take the consequences seriously. What do we know about dissipative fluids? Eckart's 1st order thermodynamics, based on 3 constitutive relations:

$$
\begin{aligned}
P_{viscous} &= -\zeta\,\Theta\,, \\
\pi_{ab} &= -2\eta\,\sigma_{ab}\,, \\
q_a &= -\mathcal{K} h_{ab}\left(\nabla^b \mathcal{T} + \mathcal{T}\dot{u}^b\right)\,,
\end{aligned}
$$

$\mathcal{T}$ = temperature, $\mathcal{K}$ = thermal conductivity. For the ϕ-fluid,

$$
q_a = -\frac{\sqrt{2X}\left(G_{4\phi} - XG_{3X}\right)}{G_4}\,\dot{u}_a \rightarrow \mathcal{K}\mathcal{T}
$$

$$\mathcal{KT} = \frac{\sqrt{2X}\,(G_{4\phi} - XG_{3X})}{G_4}$$

$\phi = \text{const.} \leftrightarrow \mathcal{KT} = 0 \leftrightarrow$ GR "state of equilibrium".

$$\eta = -\frac{\sqrt{X}\,G_{4\phi}}{\sqrt{2}\,G_4}\,, \qquad \zeta = -\frac{(G_{4\phi} - 3XG_{3X})}{3G_4}\sqrt{2X}$$

Example

For example, for "1st gen" ST gravity with action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega(\phi)}{\phi} \nabla^c\phi \nabla_c\phi - V(\phi) \right] + S^{(m)}$$

we have $G_{eff} \simeq 1/\phi$ and (if $\nabla^c\phi$ is timelike and future-oriented)

$$\begin{aligned} \mathcal{KT} &= \frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi}, \\ \eta &= -\frac{\mathcal{KT}}{2}, \\ \zeta &= -\frac{\mathcal{KT}}{3} \end{aligned}$$

$$\phi = \text{const.} \implies \mathcal{KT} = 0 \quad \text{(GR)}$$

Approach to GR equilibrium

One can derive a "heat equation" describing the approach to the GR equilibrium, or departures from it

$$\frac{d(\mathcal{K}\mathcal{T})}{d\tau} = \left(\frac{\Box\phi}{\sqrt{2X}} - \Theta\right)\left[\mathcal{K}\mathcal{T} - \frac{(2X)^{3/2}}{G_4}\left(G_{3X} + XG_{3XX}\right)\right]$$
$$-\frac{2X}{G_4^2}\left[G_4 G_{4\phi\phi} - XG_4 G_{3X\phi} - G_{4\phi}\left(G_{4\phi} - XG_{3X}\right)\right]$$

τ = proper time along fluid lines
Similar in spirit to Jacobson's thermodynamics of spacetime, but very different:

- minimal assumptions
- less fundamental (no QFT)
- explicit $\mathcal{T}$ and equation describing approach to equilibrium.

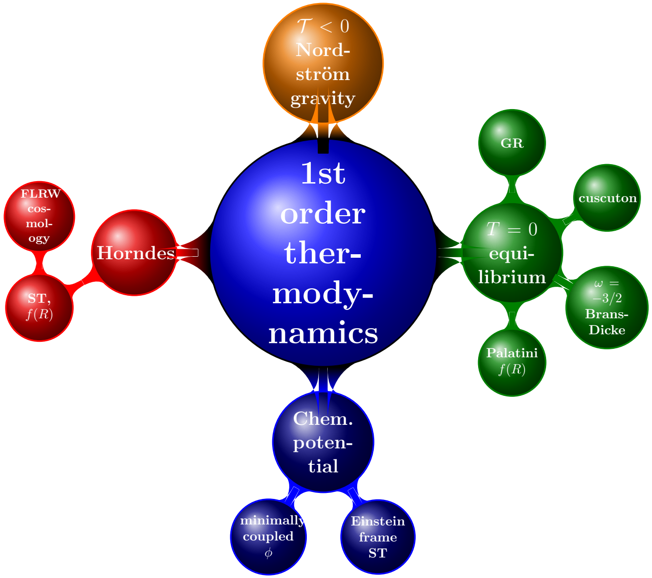
Easier to interpret in "1st gen" ST gravity and in simplified scenarios:

$$\frac{d\,(K\mathcal{T})}{d\tau} = 8\pi\,(K\mathcal{T})^2 - \Theta\, K\mathcal{T} + \frac{\Box\phi}{\sqrt{-\nabla^e\phi\nabla_e\phi}}$$

- Electrovacuum, $\omega =$ const., $V(\phi) = 0 \rightarrow \Box\phi = 0$:

  $$\Theta < 0 \rightarrow \frac{d(K\mathcal{T})}{d\tau} > 8\pi(K\mathcal{T})^2$$

  or, $K\mathcal{T}$ diverges away from the GR equilibrium.
  Deviations of ST gravity from GR will be extreme near spacetime singularities (singularities are "hot").
- Electrovacuum, $\Theta > 0$: then $-\Theta\, K\mathcal{T}$ can dominate $(K\mathcal{T})^2$, the solution $K\mathcal{T}$ can approach 0: diffusion to GR equilibrium (expansion cools gravity).
  But, if $K\mathcal{T}$ is large, the positive term dominates r.h.s. and drives solution away from GR:
  approach to GR equilibrium state not always expected.



1st order thermodynamics
$\mathcal{T} < 0$ Nordström gravity
$T = 0$ equilibrium
GR
cuscuton
$\omega = -3/2$ Brans-Dicke
Palatini $f(R)$
Chem. potential
minimally coupled ϕ
Einstein frame ST
Horndes
FLRW cosmology
ST, $f(R)$

Other states of equilibrium?

Are states of equilibrium other than GR possible?
Well, why is ST gravity an excited state w.r.t. GR? We have introduced an extra (scalar) degree of freedom ϕ in addition to the two massless spin 2 modes of GR contained in the metric g_{ab}. When this d.o.f. is dynamical and propagates we have "more dynamics" than GR and an excited states.
Some "pathological" theories of gravity are known in which ϕ is non-dynamical:

- $\omega = -3/2$ Brans-Dicke theory (in matter);
- Palatini $f(R)$gravity (in matter);
- cuscuton gravity;
- ....

They should lead to $\mathcal{KT} = 0$. Analyzed in VF, A. Giusti, S. Jose, S. Giardino PRD 106, 024029 (2022).

$\omega = -3/2$ **Brans-Dicke theory**

Equation of motion for ϕ in Brans-Dicke theory

$$(2\omega + 3)\,\Box\phi = 8\pi T^{(m)} + \phi\,\frac{dV}{d\phi} - 2V$$

is lost when $\omega = -3/2$ (reduces to an algebraic constraint), ϕ is non-dynamical. In vacuo the theory is known to be GR+Λ; in matter

$$\mathcal{KT} = \frac{\sqrt{\left|\nabla^c T^{(m)} \nabla_c T^{(m)}\right|}}{\phi\left|V' - \phi V''\right|}$$

becomes almost completely arbitrary.

Cuscuton gravity

A special Horndeski theory introduced to obtain dark energy. It is known that ϕ is non-dynamical

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi} \pm \mu^2 \sqrt{2X} - V(\phi) \right] + S^{(m)}$$

When you write down the field eqs. as effective Einstein eqs., you find

$$G_{ab} = 8\pi T_{ab}^{(\phi)} , \quad T_{ab} = (P + \rho)\, u_a u_b + P g_{ab}$$

perfect fluid, no dissipation! So $q^a = 0$, $\mathcal{K}\mathcal{T} = 0$

The curious case of Nordström gravity (not a ST theory)

Nordström purely scalar theory of gravity predates GR; considered interesting by Einstein, was very short-lived; sometimes used as toy model by theorists.

All solutions are conformally flat, $\tilde{g}_{ab} = \Omega^2 \eta_{ab}$. The only degree of freedom is the scalar Ω, which satisfies

$$\Box \Omega = 0$$

Compute the Einstein tensor for such metrics, obtain

$$G_{ab} = 8\pi \, \tilde{T}^{(\Omega)}_{ab} = -\frac{2\tilde{\nabla}_a \tilde{\nabla}_b \Omega}{\Omega} + \tilde{g}_{ab} \frac{\tilde{\nabla}^c \Omega \, \tilde{\nabla}_c \Omega}{\Omega^2} \, .$$

Assume

$$\tilde{u}_a \equiv \frac{\tilde{\nabla}_a \Omega}{\sqrt{-\tilde{g}^{cd} \, \tilde{\nabla}_c \Omega \, \tilde{\nabla}_d \Omega}}$$

timelike$+$ future-oriented, then

$$\tilde{q}^{(\Omega)}_a = -\tilde{T}^{(\Omega)}_{cd} \, \tilde{u}^c \, \tilde{h}_a^{\ d} = \frac{\sqrt{2\tilde{X}}}{4\pi \, \Omega} \, \dot{\tilde{u}}_a$$

and

$$\boxed{\mathcal{K}\mathcal{T} = -\frac{\sqrt{2\tilde{X}}}{4\pi\,\Omega} = -\frac{\sqrt{-\tilde{g}^{ef}\,\tilde{\nabla}_e\Omega\,\tilde{\nabla}_f\Omega}}{4\pi\,\Omega} < 0}$$

Nordström gravity has *less* d.o.f. than GR so is a de-excited state (first case in which the formalism is extended beyond ST/Horndeski gravity).

Einstein frame formulation of ST gravity

There exists another representation of ST gravity using different variables (a different "conformal frame"). Instead of

$$S_{ST} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}\nabla^c\phi\nabla_c\phi - V(\phi)\right] + S^{(m)}$$

use the "Einstein frame" variables $\left(\tilde{g}_{ab}, \tilde{\phi}\right)$ defined by

$$\tilde{g}_{ab} \equiv \phi\, g_{ab}\,, \qquad d\tilde{\phi} = \sqrt{\frac{|2\omega+3|}{16\pi}}\,\frac{d\phi}{\phi}$$

the action becomes

$$S_{EF} = \int d^4x\sqrt{-g}\left[\frac{\tilde{R}}{16\pi} - \frac{1}{2}\tilde{g}^{ab}\nabla_a\tilde{\phi}\nabla_b\tilde{\phi} - U(\tilde{\phi}) + \frac{\mathcal{L}^{(m)}}{\phi^2(\tilde{\phi})}\right]$$

with

$$U(\tilde{\phi}) = \frac{V(\phi)}{16\pi\phi^2}\Big|_{\phi=\phi(\tilde{\phi})}$$

Now the scalar field $\tilde{\phi}$ does not couple explicitly to gravity (to R) so $\tilde{T}_{ab}^{(\tilde{\phi}}$ has a *perfect* fluid structure. No dissipation, no heat flux, $\mathcal{T} = 0$ (but $\tilde{\phi}$ couples explicitly to matter).

Where did the thermal description go?

We need to understand better standard nonminimally coupled scalars in GR. The standard picture is: a minimally coupled ϕ is equivalent to a *perfect* fluid with the dictionary (Piattella 2013)

$$
\begin{aligned}
u^a &\equiv \nabla^a \phi / \sqrt{2X} \quad \text{(4-velocity)} \\
\rho &= 2X\mathcal{L}_X - \mathcal{L} \quad \text{(energy density)} \\
P &= \mathcal{L} \quad \text{(pressure)} \\
n &= \sqrt{2X}\,\mathcal{L}_X \quad \text{(particle number density)} \\
\frac{s}{n} &= \phi \quad \text{(entropy density)} \\
T &= \frac{-\mathcal{L}_\phi}{\sqrt{2X}\,\mathcal{L}_X} \quad \text{(temperature)}
\end{aligned}
$$

$$
\begin{aligned}
\mu &= \frac{2X\mathcal{L}_X + \phi\mathcal{L}_\phi}{\sqrt{2X}\,\mathcal{L}_X} = \sqrt{2X} - \phi T \quad \text{(chemical potential)} \\
T_{ab}^{(\phi)} &= (P+\rho)\,u_a u_b + P g_{ab}
\end{aligned}
$$

where $\mathcal{L}_\phi \equiv \partial\mathcal{L}/\partial\phi$ and $\mathcal{L}_X \equiv \partial\mathcal{L}/\partial X > 0$.

Two problems:

- In general the fluid is accelerated, $\dot{u}^a \neq 0$. Then, according to Eckart's generalization of the Fourier law

$$
q_a = -\mathcal{K} h_{ab} \left(\nabla^b \mathcal{T} + \mathcal{T} \dot{u}^b \right) \neq 0 \,,
$$

  there must be a heat flux, which contradicts the perfect fluid structure.
- Both $\mathcal{T}$ and μ can be negative

Fixed in VF, S. Giardino, A. Giusti, R, Vanderwee EPJC 83, 24 (2023).
Revised dictionary:

$$\begin{aligned}\mathcal{T} &= 0\\ \mu &= \sqrt{2X}\end{aligned}$$

The argument: 1st law is

$$d\left(\frac{\rho}{n}\right) + P\, d\left(\frac{1}{n}\right) = T\, d\left(\frac{s}{n}\right)$$

Take s and n as independent variables →

$$T(s,n) = \frac{1}{n}\frac{\partial \rho}{\partial (s/n)}\Big|_n = \frac{\partial \rho}{\partial s}\Big|_n$$

perfect fluid = no dissipative effects → $T = 0$, agrees with ST thermodynamics.
Assuming $\phi = \phi(s,n)$ and $X = X(s,n)$ leads to

$$0 = \frac{\partial \rho}{\partial s}\Big|_n = -\mathcal{L}_\phi \, \frac{\partial \phi}{\partial s}\Big|_n$$

satisfied if $\mathcal{L}_\phi = 0$ or $\frac{\partial \phi}{\partial s}\Big|_n = 0$.

Since, in general, $\mathcal{L}$ contains a potential, consistency requires

$$\left.\frac{\partial\phi}{\partial s}\right|_n = 0 \rightarrow \mathcal{T} = 0$$

then $P = \mathcal{L}$ and

$$\mu = \frac{P+\rho}{n} = \sqrt{2X}\,,$$

Einstein frame ST gravity has $\mathcal{T} = 0$ but $\mu \neq 0$ and is still a non-equilibrium state. GR with $\phi = \text{const.}$ and $\mu = 0$ is still the equilibrium state.

Bring back $\Box\phi$: what happens?

The approach to/departure from the (GR) equilibrium is now ruled by

$$\frac{d\left(K\mathcal{T}\right)}{d\tau} = 8\pi\left(K\mathcal{T}\right)^2 - \theta\,K\mathcal{T} + \frac{\Box\phi}{8\pi\phi}$$

$\Box\phi$ does not have definite sign *a priori*, so it can contribute in different ways. Look at examples → richer range of thermal behaviours of modified gravity than previously thought:

- Sultana-Wyman solution of conformally coupled scalar field theory (N. Karolinski + VF 2024, PRD 109, 084042) ;
- Bianchi I universes in Brans-Dicke gravity (J. Houle + VF arXiv:2404.19470)

Sultana-Wyman solution

Wyman had a minimally coupled scalar field solution; Sultana extended it to GR+Λ; then mapped it from the Einstein to the Jordan frame of conformally coupled scalar field theory:

$$ds^2 = -\kappa r^2 d\tau^2 + \left(1 + \alpha^2 \tau^2\right)\left(\frac{2dr^2}{1 - 2\Lambda r^2/3} + r^2 d\Omega^2_{(2)}\right)$$

$$\phi(\tau) = \frac{\Lambda}{\kappa\left(1 + \alpha^2\tau^2\right)^2}, \qquad \alpha = \sqrt{\frac{6}{\kappa}}$$

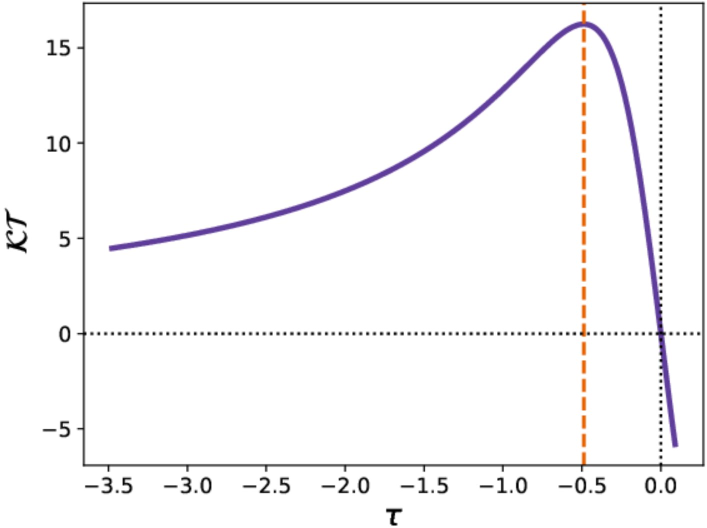
$$0 < r < \sqrt{\frac{3}{2\Lambda}}, \qquad -\infty < \tau < +\infty$$

Inhomogeneous but $\phi = \phi(t)$ only, spherical, time-dependent; naked singularity at $r = 0$;

$$\text{3-volume } V^{(3)} \simeq \left(1 + \alpha^2\tau^2\right)^{3/2} \to +\infty \qquad \text{as } \tau \to -\infty$$

$$\mathcal{K}\mathcal{T} = \frac{-2\alpha\tau}{\sqrt{6}\,\alpha r\left(1 + \alpha^2\tau^2\right)} \qquad (\tau < 0)$$

$\mathcal{KT} \to +\infty$ as $r \to 0^+$ (“hot” singularity);
$\Box\phi$ affects the evolution of $\mathcal{KT}$. Non-monotonic $\mathcal{KT}$ unlike all previous solutions studied:



CONCLUSIONS

- There is an effective fluid equivalent of ST/Horndeski gravity; it is a *dissipative* fluid.
- The constitutive relation of Eckart's 1st order thermodynamics give a "temperature of gravity" $\mathcal{T}$ and an equation describing approach to the GR equilibrium. Same spirit as emergent gravity and Jacobson's thermodynamics of spacetime, but very different.
- Theories with non-dynamical ϕ are also states of equilibrium.
- Einstein frame ST: trade $\mathcal{T}$ with μ, thermal with chemical equilibrium.
- Many open problems under study; extend validity to spacelike $\nabla^a\phi$; understand Horndeski better; more complicated constitutive relations $\leftrightarrow$ other theories of gravity? Eckart's 1st order thermo is not causal; ...

THANK YOU

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