# How Does a Quantum State Self-Gravitate?

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# The Presenter

- Who am I?

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- Who am I?
- This guy!

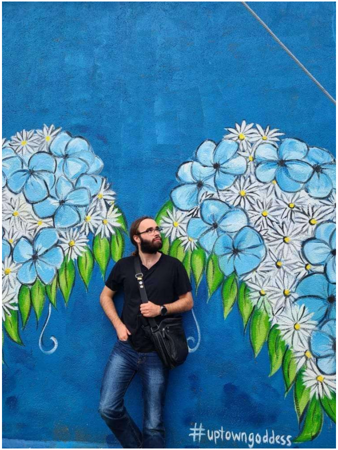


Figure: Gaia Noseworthy, your presenter!

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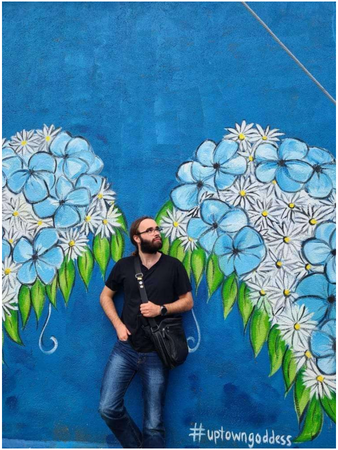


Figure: Gaia Noseworthy, your presenter!

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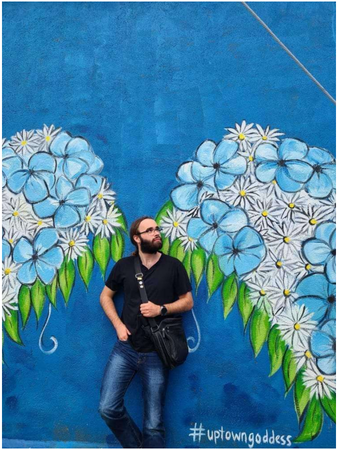


Figure: Gaia Noseworthy, your presenter!

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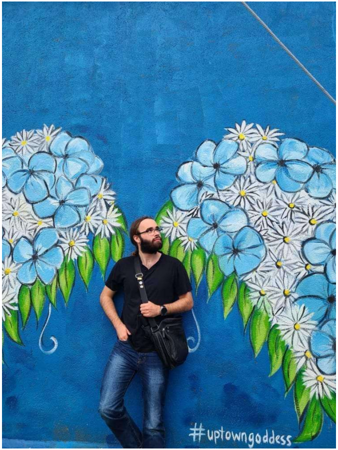


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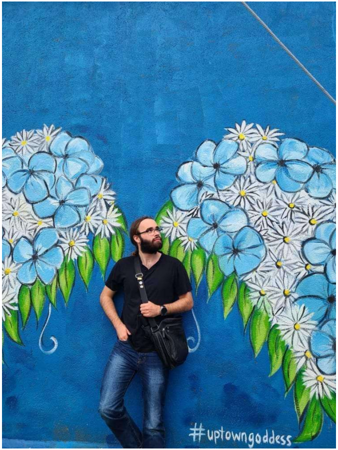


Figure: Gaia Noseworthy, your presenter!

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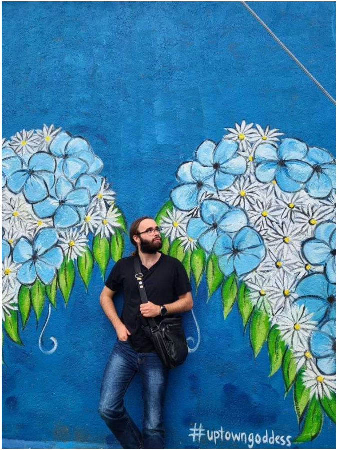


Figure: Gaia Noseworthy, your presenter!

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- Birthday: Jan 21
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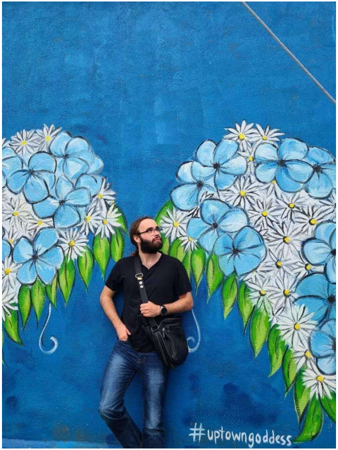


Figure: Gaia Noseworthy, your presenter!

# A History of Physical Theories

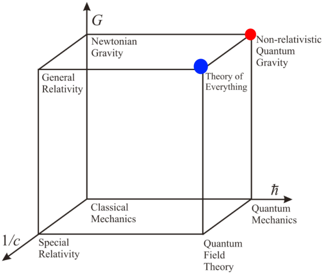


Figure: The cube of physical theories, with the Theory of Everything in blue and Non-Relativistic Quantum Gravity in red.

- It has been a long-term goal to couple gravity with quantum mechanics.
- Modern physical theories aim to unify three concepts: Relativity ($c$), Quantum Theory ($\hbar$), and Gravity ($G$).
- With no Theory of Everything found, we can look towards the other unknown: Non-Relativistic Quantum Gravity.

# The Newton-Schrödinger Equation

1. Originally defined by Diósi, then Penrose. The Newton-Schrödinger System gives us an approach to Non-Relativistic Quantum Gravity.
2. Topic of 1 master's thesis, 2 PhD theses, and numerous papers in multiple sub-fields.
3. Defined by 2 equations: The Time-Dependent Schrödinger equation with gravitational potential,

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + m\Phi\Psi$$

and the Poisson equation

$$\nabla^2\Phi = 4\pi Gm|\Psi|^2$$

# Our Goal

Previous studies focused on simple wavefunctions in spherical symmetry. So, we will seek to answer 3 new questions:

1. How does the NS System behave in other domains, like a circle?
2. How does the NS System behave with multi-peaked initial conditions?
3. How do test particles behave in the NS System?

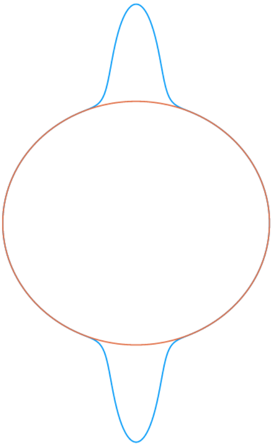

Figure: A two-peaked Gaussian initial wavefunction on a circle.

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# The NS Equations in Dimensionless Coordinates

1. Firstly, let us rescale the system using three fundamental constants:

$$l_{NS} = \frac{\hbar^2}{m^3 G}, \quad m_{NS} = m, \quad t_{NS} = \frac{\hbar^3}{m^5 G^2}$$

2. Final NS System in dimensionless coordinates:

$$i\frac{\partial \Phi}{\partial t} = -\frac{1}{2}\nabla^2 \Psi + \Phi\Psi, \quad \nabla^2 \Phi = |\Psi|^2$$

3. Our initial value problem: Constants $A, B$, momentum $p_i$, initial angle $\theta_i$, and initial width $\sigma_i$

$$\Psi(0,\theta) = e^{-A\left(\frac{\theta-\theta_0}{\sigma_0}\right)^2 + ip_0\theta - B\left(\frac{\theta-\theta_1}{\sigma_1}\right)^2 + ip_1\theta}$$

# The System on a Circle

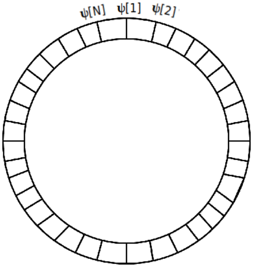


Figure: Positioning of edge points on the circle.

1. Circular boundary conditions:

$$\Psi(t,0) = \Psi(t,2\pi)$$
$$\Phi(t,0) = \Phi(t,2\pi)$$

2. NS System through $\nabla$ becomes one-dimensional:

$$i\frac{\partial \Phi}{\partial t} = -\frac{1}{2}\frac{d^2}{d\theta^2}\Psi + \Phi\Psi$$
$$|\Psi|^2 = \frac{d^2}{d\theta^2}\Phi$$

3. Four initial conditions: Single peak, double peak with equal spacing, double peak with unequal spacing, double peak with different sizes.

# Solving for Gravitational Potential

Multiple approaches exist:

1. In general: Use iterative methods like the Conjugate Gradient or UMFPACK matrix equation solvers.
   (i) Discretize the derivative:
   $$\frac{d^2\Phi}{d\theta^2} = \frac{\Phi_{i+1} - 2\Phi_i + \Phi_{i-1}}{(\Delta\theta)^2}$$
   (ii) Rewrite our equation:
   $$\frac{\Phi_{i+1} - 2\Phi_i + \Phi_{i-1}}{(\Delta\theta)^2} = |\Psi|^2$$
   (iii) Represent this as an $M \times M$ tri-diagonal matrix $A$.
   (iv) Implement Boundary conditions: The corners of $A$ must also be $\Phi/(\Delta\theta)^2$ for the first and last points to interact.
   (v) Solve the linear equation $A\Phi = |\Psi|^2$.
2. Alternatively, use a spectral method (The Fourier Transform).

# The Crank-Nicolson Method Numerically

- For time-evoltion we use the Crank-Nicolson Method: A normalization-preserving multi-step model.
- The time derivative is simple to modify:

$$\frac{\partial \Psi}{\partial t} = \frac{\Psi_i^{n+1} - \Psi_i^n}{\Delta t}$$

- For space derivatives, each piece of the equation:

$$\frac{\partial^2 \Psi}{\partial \theta^2} = \frac{1}{2(\Delta\theta)^2}\Big(\left[\Psi_{i+1}^{n+1} - 2\Psi_i^{n+1} + \Psi_{i-1}^{n+1}\right] + \left[\Psi_{i+1}^{n} - 2\Psi_i^{n} \quad + \Psi_{i+1}^{n}\right]\Big)$$

$$\frac{\partial \Psi}{\partial \theta} = \frac{1}{\Delta\theta}\left(\left[\Psi_{i+1}^{n+1} - \Psi_i^{n+1}\right] + \left[\Psi_{i+1}^{n} - \Psi_i^{n}\right]\right)$$

$$\Phi\Psi = \frac{\Phi_i^n}{2}\left(\Psi_i^{n+1} + \Psi_i^n\right)$$

# Test Particles

- Finally, how do test particles move within the system, assuming they don't impact the system itself?
- Define the position of a test particle as $X$.
- Write a formula for its acceleration:

$$\frac{d^2X}{dt^2} = -\nabla\Phi \to \frac{X^{i+1} - 2X^i + X^{i-1}}{(\Delta t)^2} = -\nabla\Phi$$

- Rewrite to solve iteratively:

$$X^{n+1} = 2X^n - X^{n-1} - \frac{(\Delta t)^2}{m_t}\nabla\Phi^n$$

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# Example Single Peak Initial Conditions

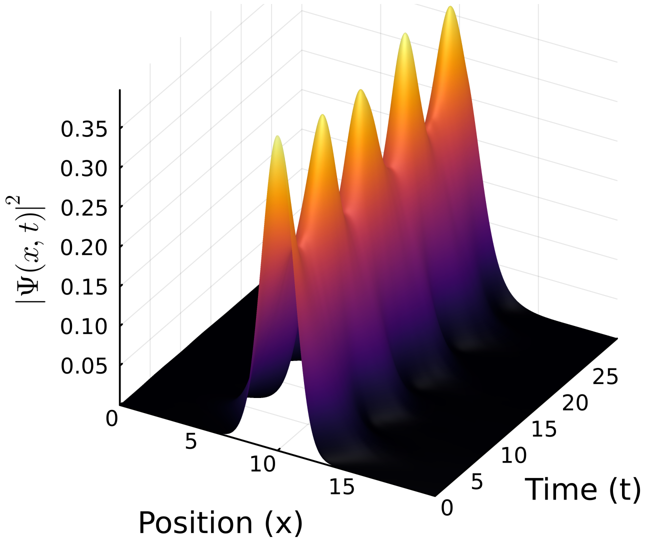


Figure: Amplitude of the evolution of a single peak Gaussian wavefunction on the circle.

# Equal Spacing $\theta_0 = \frac{\pi}{2}, \theta_1 = \frac{3\pi}{2}$

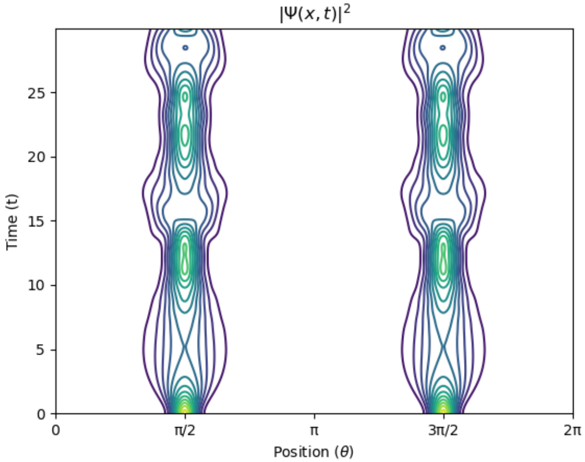


Figure: Amplitude of the evolution of an equally spaced double Gaussian wavefunction on the circle.

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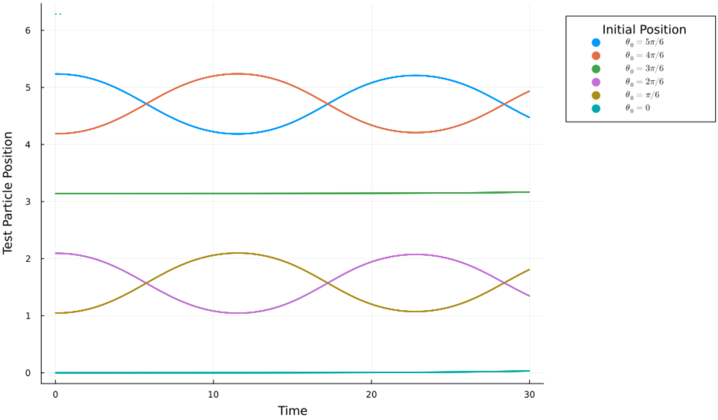


Figure: Movement of 2 test particles on the equally spaced double Gaussian located at $[2\pi/3, \pi/2]$ with mass ratio 1.

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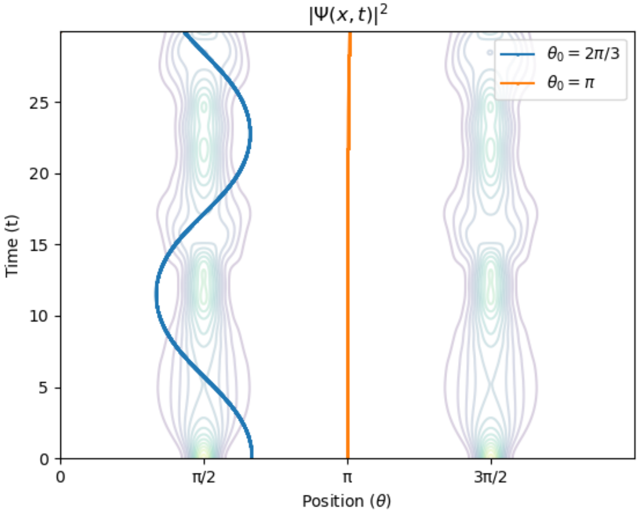


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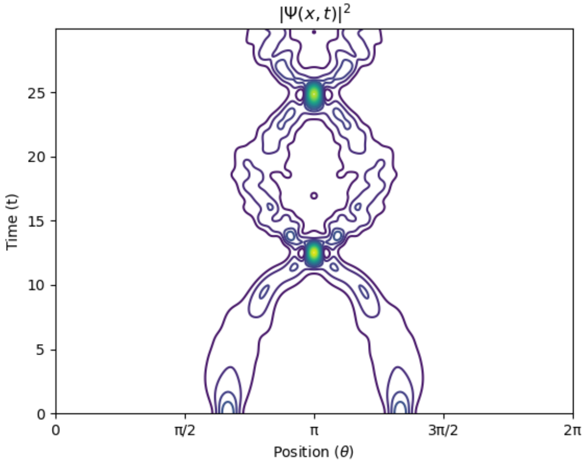


Figure: Amplitude of the evolution of unequally spaced spaced double Gaussian wavefunction on the circle.

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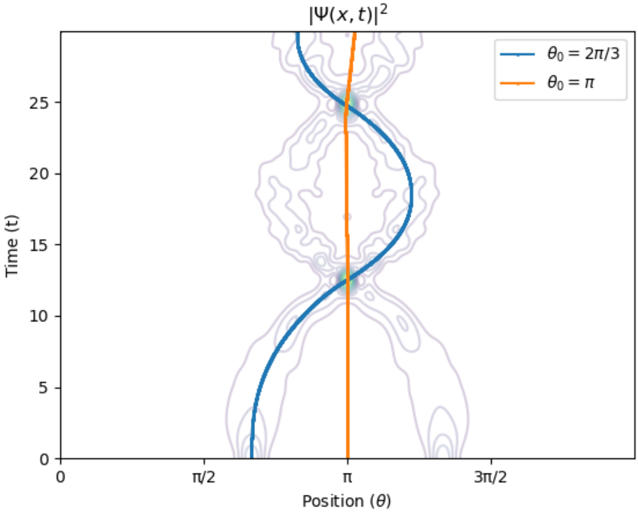


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- Thanks for listening!