On the Fate of Quantum Black Holes

Edward Wilson-Ewing

University of New Brunswick

with Lorenzo Cipriani, Francesco Fazzini, Viqar Husain, Jarod Kelly & Robert Santacruz

Kelly, Santacruz, WE, PRD **102** (2020) 106024, arXiv:2006.09302
Kelly, Santacruz, WE, CQG **38** (2021) 04LT01, arXiv:2006.09325
Husain, Kelly, Santacruz, WE, PRL **128** (2022) 121301, arXiv:2109.08667
Husain, Kelly, Santacruz, WE, PRD **106** (2022) 024014, arXiv:2203.04238
Fazzini, Husain, WE, PRD **109** (2024) 084052, arXiv:2312.02032
Cipriani, Fazzini, WE, arXiv:2404.04192

Atlantic General Relativity Meeting 2024
University of New Brunswick

Reminder: Motivation

Black holes are one of the few places quantum gravity effects are expected to be important.

There are two main problems in black hole physics that any successful theory of quantum gravity should be able to solve:

- **Singularity**
  Black hole space-times in general relativity are singular. Can quantum gravity resolve the singularity?
- **Information loss problem**
  Hawking radiation is thermal. If a black hole fully evaporates, an initial pure state seems to evolve to a thermal state. Can quantum gravity somehow restore unitarity?

Black Hole Collapse

Goal: study quantum gravity effects in black holes, according to loop quantum gravity, starting from the initial collapse.

There are (at least) two good reasons to study black hole collapse:

1. **How is the singularity avoided?**
   This is presumably a dynamical process, so we should study space-times where (classically) the singularity forms dynamically.
2. **The role of matter**
   Classically, vacuum is often thought sufficient since matter from the collapse will eventually hit the singularity and 'disappear'. But what if there is no singularity?
   - → During collapse an inner horizon forms; this is missed in vacuum.

For these reasons, we studied the Lemaître-Tolman-Bondi (LTB) space-time: spherically symmetric, with a dust field, building on a lot of earlier work studying black holes in LQG.

Main Steps

1. Start with classical GR, using the Hamiltonian framework with Ashtekar-Barbero variables and impose spherical symmetry;

Main Steps

1. Start with classical GR, using the Hamiltonian framework with Ashtekar-Barbero variables and impose spherical symmetry;

2. Gauge-fix the scalar constraint by using the dust field as a clock, and gauge-fix the diffeo constraint by using the areal gauge:

$$\mathrm{d}s^2 = -1\,\mathrm{d}t^2 + 2N^x\,\mathrm{d}t\mathrm{d}x + f(x,t)\,\mathrm{d}x^2 + x^2\mathrm{d}\Omega^2;$$

Main Steps

1. Start with classical GR, using the Hamiltonian framework with Ashtekar-Barbero variables and impose spherical symmetry;
2. Gauge-fix the scalar constraint by using the dust field as a clock, and gauge-fix the diffeo constraint by using the areal gauge:

$$\mathrm{d}s^2 = -1\,\mathrm{d}t^2 + 2N^x\,\mathrm{d}t\mathrm{d}x + f(x,t)\,\mathrm{d}x^2 + x^2\mathrm{d}\Omega^2;$$

3. Discretize along the radial coordinate;

Main Steps

1. Start with classical GR, using the Hamiltonian framework with Ashtekar-Barbero variables and impose spherical symmetry;
2. Gauge-fix the scalar constraint by using the dust field as a clock, and gauge-fix the diffeo constraint by using the areal gauge:

$$\mathrm{d}s^2 = -1\,\mathrm{d}t^2 + 2N^x\,\mathrm{d}t\mathrm{d}x + f(x,t)\,\mathrm{d}x^2 + x^2\mathrm{d}\Omega^2;$$

3. Discretize along the radial coordinate;
4. Do a loop quantization at each point along the radial lattice;

Main Steps

1. Start with classical GR, using the Hamiltonian framework with Ashtekar-Barbero variables and impose spherical symmetry;

2. Gauge-fix the scalar constraint by using the dust field as a clock, and gauge-fix the diffeo constraint by using the areal gauge:

$$\mathrm{d}s^2 = -1\,\mathrm{d}t^2 + 2N^x\,\mathrm{d}t\mathrm{d}x + f(x,t)\,\mathrm{d}x^2 + x^2\mathrm{d}\Omega^2;$$

3. Discretize along the radial coordinate;

4. Do a loop quantization at each point along the radial lattice;

5. Extract effective dynamics from the quantum theory, and take the continuum limit.

Loop Quantization: Holonomies

A key step in the loop quantization is to calculate holonomies of the Ashtekar-Barbero connection.

Due to spherical symmetry, holonomies along an edge on a great circle of a sphere at fixed radius $x = x_0$ do not depend on the angular coordinates. The only input required is the coordinate (angle) length $\bar{\mu}$ of the holonomy,

$$h_\theta(\bar{\mu}) = \mathcal{P} \exp \int_0^{\bar{\mu}} \mathrm{d}\theta \; b(x_0)\, \tau_2 = \cos\left(\frac{\bar{\mu} b}{2}\right) \mathbb{I} + 2 \sin\left(\frac{\bar{\mu} b}{2}\right) \tau_2 .$$

Loop Quantization: Holonomies

A key step in the loop quantization is to calculate holonomies of the Ashtekar-Barbero connection.

Due to spherical symmetry, holonomies along an edge on a great circle of a sphere at fixed radius $x = x_0$ do not depend on the angular coordinates. The only input required is the coordinate (angle) length $\bar{\mu}$ of the holonomy,

$$h_\theta(\bar{\mu}) = \mathcal{P} \exp \int_0^{\bar{\mu}} \mathrm{d}\theta \; b(x_0)\, \tau_2 = \cos\left(\frac{\bar{\mu} b}{2}\right) \mathbb{I} + 2 \sin\left(\frac{\bar{\mu} b}{2}\right) \tau_2 .$$

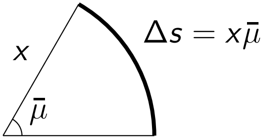
In LQC, the fundamental discreteness of LQG geometric operators motivates an operator for (components of) the field strength to be based on holonomies with a small but finite length of order $\sim \ell_{\rm Pl}$ (not an infinitesimal length).

Loop Quantization: Fixing $\bar{\mu}$

$\bar{\mu}$ is the coordinate length of the edge in the θ direction, and we want the physical length to be $\ell_{\rm Pl}$ [Ashtekar, Pawłowski, Singh, 2005].
To fix $\bar{\mu}$, we use the metric.

Loop Quantization: Fixing $\bar{\mu}$

$\bar{\mu}$ is the coordinate length of the edge in the θ direction, and we want the physical length to be $\ell_{\rm Pl}$ [Ashtekar, Pawłowski, Singh, 2005].
To fix $\bar{\mu}$, we use the metric.



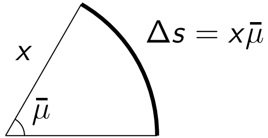
In this case, the path for the holonomy is an arc with angle $\bar{\mu}$ of a great circle of radius x. To get the physical length $\Delta s = \ell_{\rm Pl}$ requires

$$\bar{\mu} = \frac{\ell_{\rm Pl}}{x}.$$

For LQC improved dynamics in spherical symmetry, see also [Boehmer, Vandersloot, 2007; Chiou, Ni, Tang, 2012; Gambini, Olmedo, Pullin, 2020].

Loop Quantization: Fixing $\bar{\mu}$

$\bar{\mu}$ is the coordinate length of the edge in the θ direction, and we want the physical length to be $\ell_{\rm Pl}$ [Ashtekar, Pawłowski, Singh, 2005].
To fix $\bar{\mu}$, we use the metric.



In this case, the path for the holonomy is an arc with angle $\bar{\mu}$ of a great circle of radius x. To get the physical length $\Delta s = \ell_{\rm Pl}$ requires

$$\bar{\mu} = \frac{\ell_{\rm Pl}}{x}.$$

For LQC improved dynamics in spherical symmetry, see also [Boehmer, Vandersloot, 2007; Chiou, Ni, Tang, 2012; Gambini, Olmedo, Pullin, 2020].

This procedure fails for coordinates that become null, but generalized Painlevé-Gullstrand coordinates are never null.

Effective Dynamics

In homogeneous cosmology, the quantum dynamics of sharply-peaked states is very well approximated by a set of LQC effective dynamics defined on the classical phase space that include $\hbar$ corrections [Ashtekar, Pawłowski, Singh, 2005; Taveras, 2008; Rovelli, WE, 2013; Bojowald, Brahma, 2015].

The key input to derive the effective equations is to neglect quantum fluctuations, but retain modifications due to finite-length holonomy corrections.

Effective Dynamics

In homogeneous cosmology, the quantum dynamics of sharply-peaked states is very well approximated by a set of LQC effective dynamics defined on the classical phase space that include $\hbar$ corrections [Ashtekar, Pawłowski, Singh, 2005; Taveras, 2008; Rovelli, WE, 2013; Bojowald, Brahma, 2015].

The key input to derive the effective equations is to neglect quantum fluctuations, but retain modifications due to finite-length holonomy corrections.

Effective dynamics can also be derived for LTB space-times, it is generally expected that they will again be a good approximation for sharply-peaked states so long as we don't probe Planck-length scales [Zhang, 2021].

Planck curvature scales are ok: the Kretschmann invariant $K \sim M^2/x^6$, so Planck curvature arises at $x \sim (\ell_{\rm Pl}^2 M)^{1/3} \gg \ell_{\rm Pl}$.

Effective Equations

The effective dynamics capture leading-order loop quantum gravity effects, and are generated by a Hamiltonian (density)

$$\mathcal{H}^{\rm eff} = -\frac{1}{2G}\left[\frac{E^b}{\ell_{\rm Pl}^2 x}\partial_x\left(x^3\sin^2\frac{\ell_{\rm Pl}b}{x}\right) + \frac{x}{E^b} + \frac{E^b}{x}\right].$$

I will focus on the 'marginally bound' class of solutions

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + 2N^x\,\mathrm{d}t\mathrm{d}x + {\color{red}1}\,\mathrm{d}x^2 + x^2\mathrm{d}\Omega^2.$$

Effective Equations

The effective dynamics capture leading-order loop quantum gravity effects, and are generated by a Hamiltonian (density)

$$\mathcal{H}^{\rm eff} = -\frac{1}{2G}\left[\frac{E^b}{\ell_{\rm Pl}^2 x}\partial_x\left(x^3 \sin^2\frac{\ell_{\rm Pl} b}{x}\right) + \frac{x}{E^b} + \frac{E^b}{x}\right].$$

I will focus on the 'marginally bound' class of solutions

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + 2N^x\,\mathrm{d}t\mathrm{d}x + 1\,\mathrm{d}x^2 + x^2\mathrm{d}\Omega^2.$$

There remains one degree of freedom b (the connection component in angular directions) that satisfies the non-linear equation of motion

$$\dot{b} + \frac{1}{2\ell_{\rm Pl}^2 x}\partial_x\left(x^3\sin^2\frac{\ell_{\rm Pl} b}{x}\right) = 0.$$

To find solutions to non-linear wave equations, it is typically necessary to allow weak solutions.

Weak Solutions

Weak solutions are not differentiable, so they cannot solve a differential equation—but they can solve an integral form of the equation of motion. For the conservation equation

$$\dot{u} + \partial_x[f(u)] = 0,$$

weak solutions $u(x, t)$ satisfy

$$\int_{x_1}^{x_2} \mathrm{d}x \; u \Big|_{t=t_1}^{t=t_2} \; + \; \int_{t_1}^{t_2} \mathrm{d}t \; f(u) \Big|_{x=x_1}^{x=x_2} \; = \; 0,$$

for all x_1, x_2, t_1, t_2.

When the weak solution is discontinuous, the discontinuity is called a shock wave.

Weak Solutions in General Relativity

Examples of weak solutions in general relativity are thin shell solutions obtained using Israel's junction conditions [Israel, 1966], and the Dray-'t Hooft shock wave [Dray, 't Hooft, 1985].

It has also been argued that weak solutions should be considered for the LTB space-time in general relativity [Nolan, 2003; Lasky, Lun, Burston, 2006].

We will allow for weak solutions in the LQC effective dynamics for LTB space-times.

- Analytical methods are useful for simple configurations.
- Otherwise, numerics are typically necessary—we use the standard Godunov algorithm.

Shell-Crossing Singularities

Theorem: [Fazzini, Husain, WE, 2024]

In the marginally bound case, for all initial conditions for the dust field such that the energy density $\rho > 0$ is continuous, of compact support, and such that $\int_0^a \mathrm{d}x\, x^2 \rho > 0$ for some a, a shell-crossing singularity will form.

- This happens at the latest $\frac{2}{3} t_{\mathrm{Pl}}$ after the bounce,
- The occurrence of a shell-crossing singularity signals the formation of a shock,
- A shell-crossing singularity is a weak singularity: tidal forces do not diverge and it is possible to evolve past it, in this case in terms of a weak solution.

Formation of a Shock Wave

Typically, soon after the bounce a shock wave forms. This is because the interior bounces, but the vacuum exterior is stationary. There is a discontinuity in the gravitational field b, and in the metric.

To calculate the speed of the outgoing shock, we use the standard Rankine-Hugoniot condition which gives

$$\dot{L} = \frac{L^2}{2\ell_{\rm Pl}^2} \cdot \frac{[\sin^2 \frac{\ell_{\rm Pl} b}{x}]}{[b]}, \qquad [f] = \lim_{x \to L^+} f(x) - \lim_{x \to L^-} f(x).$$

Formation of a Shock Wave

Typically, soon after the bounce a shock wave forms. This is because the interior bounces, but the vacuum exterior is stationary. There is a discontinuity in the gravitational field b, and in the metric.

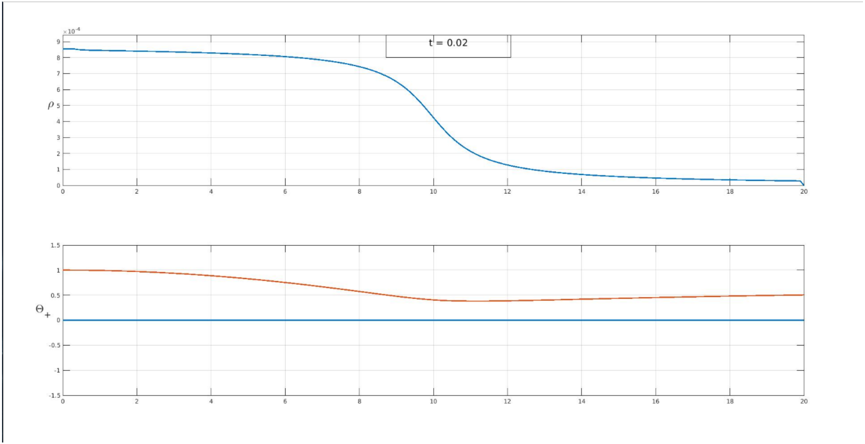
To calculate the speed of the outgoing shock, we use the standard Rankine-Hugoniot condition which gives

$$\dot{L} = \frac{L^2}{2\ell_{\rm Pl}^2} \cdot \frac{[\sin^2 \frac{\ell_{\rm Pl} b}{x}]}{[b]}, \qquad [f] = \lim_{x \to L^+} f(x) - \lim_{x \to L^-} f(x).$$

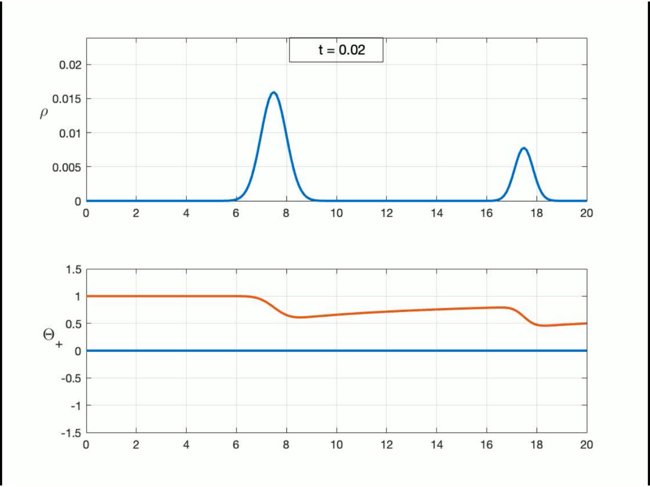
The lifetime of the black hole (neglecting the short collapse time) is

$$T = \int_{L_{\rm bounce}}^{2GM} {\rm d}L \, \dot{L}^{-1} = \frac{8\pi}{3} \cdot \frac{M^2}{m_{\rm Pl}} + \mathcal{O}(M).$$

Numerical Results: Example of Star-like Collapse



Numerical Results: No Mass Inflation



Numerical Results: Summary

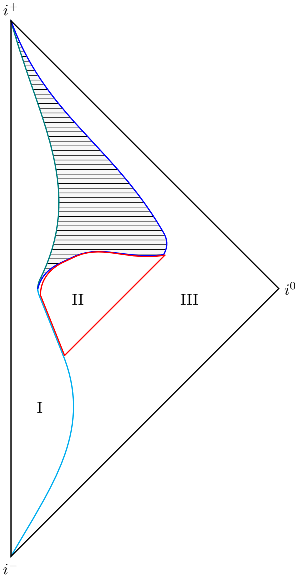
- There is no singularity, it is replaced by a bounce.
- A shock wave forms, at the latest $\sim t_{\rm Pl}$ after the bounce.
- The bounce is stable: there is no mass inflation [Poisson, Israel, 1989] and no instability to infalling matter like there is for white holes in general relativity [Eardley, 1974].
- The lifetime seems robust: we find $T = \dfrac{8\pi M^2}{3m_{\rm Pl}}$ in various analytic and numeric solutions.

Conformal Diagram

This is a sketch of the conformal diagram inferred from the numerical solution for the collapse of a Gaussian distribution of dust.

The hatched area is excised with the boundaries identified: this is the location of the shock wave.

For the vacuum case, see [Münch, 2021].



Implications for Information Loss

There is no singularity, and no event horizon: it should be possible to recover information once the apparent horizons are gone.

1. LQG corrections at the horizon are negligible:
   $\Rightarrow$ Hawking radiation will occur as usual.

2. The predicted black hole lifetime $T \sim M^2/m_{\rm Pl}$ is much less than the Page time $\sim M^3/m_{\rm Pl}^2$.
   $\Rightarrow$ The entropy of Hawking radiation $S_{HR} \ll A_{BH}/4\ell_{\rm Pl}^2$ always.

3. The Hawking radiation will presumably be entangled with matter/gravitational fields in the shock wave. These degrees of freedom will be accessible to outside observers once the apparent horizon vanishes when the shock passes beyond $x = 2GM$.

More work is required, but it seems likely that there is no information loss problem here.

Summary

- The black hole singularity is resolved and replaced by a bounce,
- There are apparent horizons but no event horizon,
- A shock wave forms, and slowly moves outwards,
- The lifetime of the black hole is $\sim M^2/m_{\rm Pl}$,
- Information loss seems to be avoided.

$\Rightarrow$ Non-marginally bound solutions: similar results [Cipriani, Fazzini, WE, 2024].

Summary

- The black hole singularity is resolved and replaced by a bounce,
- There are apparent horizons but no event horizon,
- A shock wave forms, and slowly moves outwards,
- The lifetime of the black hole is $\sim M^2/m_{\rm Pl}$,
- Information loss seems to be avoided.

⇒ Non-marginally bound solutions: similar results [Cipriani, Fazzini, WE, 2024].

Next steps:

- Effective models: Cosmological sector, other matter fields, avoid fixing gauges [Alonso-Bardaji, Brizuela, 2021; Giesel, Liu, Rullit, Singh, Weigl, 2023].
- Beyond effective models: Hawking radiation, quantum fluctuations, full quantum description.

Summary

- The black hole singularity is resolved and replaced by a bounce,
- There are apparent horizons but no event horizon,
- A shock wave forms, and slowly moves outwards,
- The lifetime of the black hole is $\sim M^2/m_{\rm Pl}$,
- Information loss seems to be avoided.

$\Rightarrow$ Non-marginally bound solutions: similar results [Cipriani, Fazzini, WE, 2024].

Next steps:

- Effective models: Cosmological sector, other matter fields, avoid fixing gauges [Alonso-Bardaji, Brizuela, 2021; Giesel, Liu, Rullit, Singh, Weigl, 2023].
- Beyond effective models: Hawking radiation, quantum fluctuations, full quantum description.

Thank you for your attention!