EFFECTIVE GROUP FIELD THEORY METRIC FOR THE UNIVERSE

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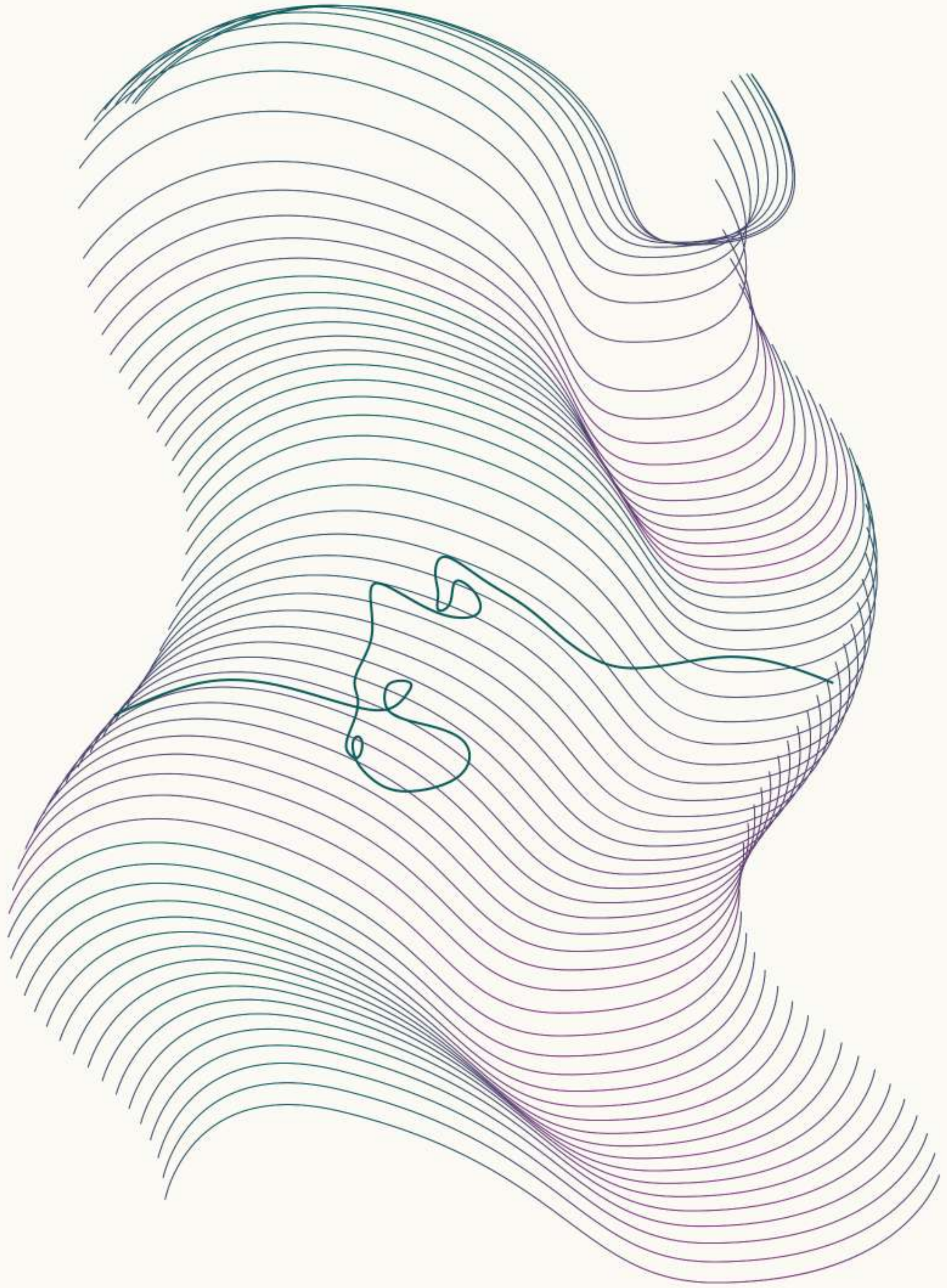

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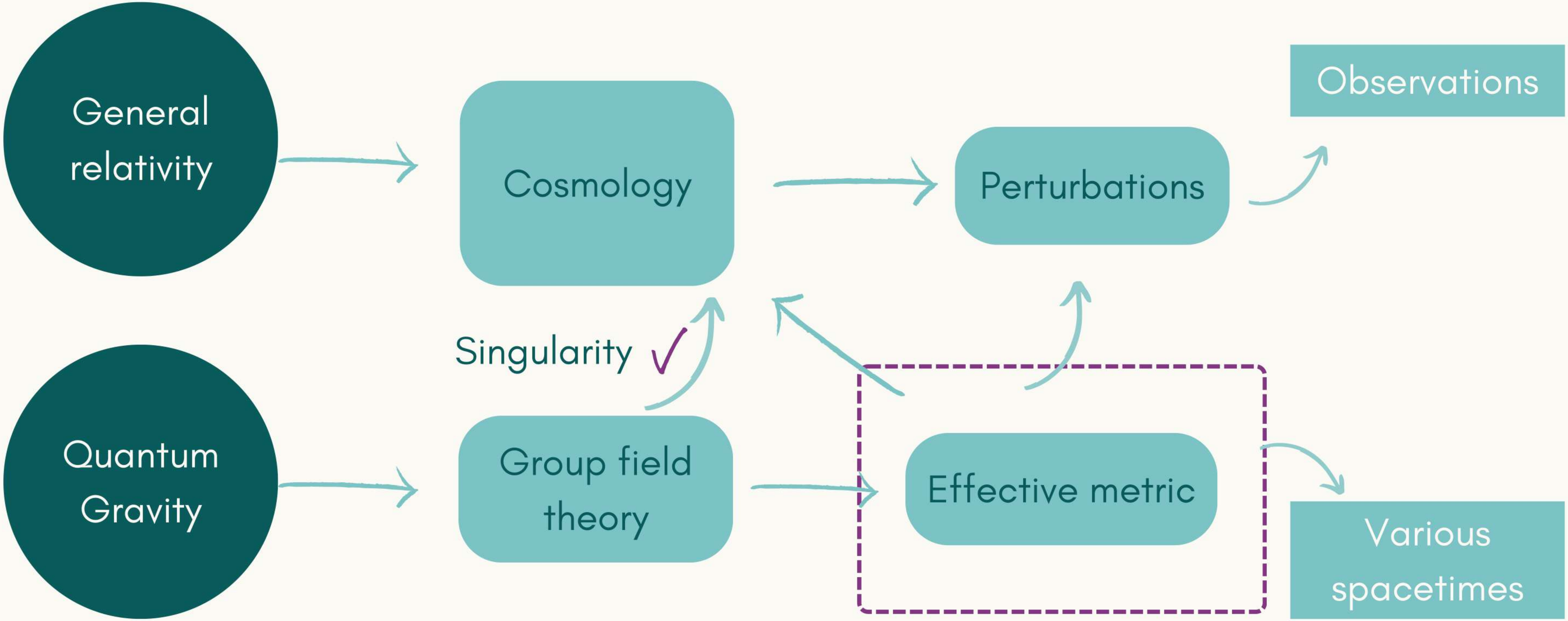
OVERVIEW



General relativity

Quantum Gravity

Cosmology

Group field theory

Singularity ✓

Perturbations

Effective metric

Observations

Various spacetimes

CONTENT

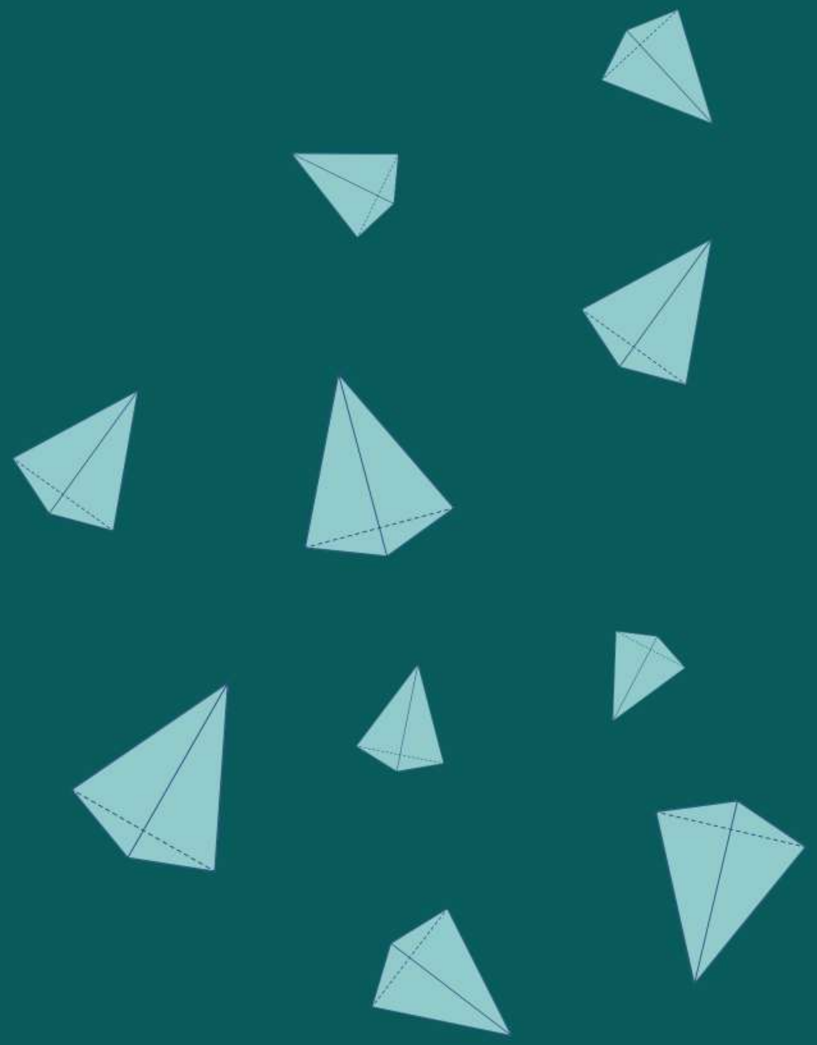
- Group field theory, relational coordinate system

- GFT operators for an effective metric

- Application to cosmology
    - background
    - perturbations

INTRODUCTION

Group field theory



GROUP FIELD THEORY

- Field theory on a group manifold
  - ○ bosonic group field $\varphi(g_i, \chi^A)$ e.g. $\varphi : \mathrm{SU}(2)^4 \times \mathbb{R}^4 \to \mathbb{R}$

    real valued functions

    group elements
- Field excitations: building blocks of spacetime
  - ○ tetrahedron or spin network vertex

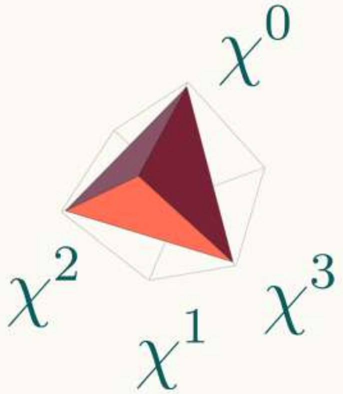
GROUP FIELD THEORY

- Field theory on a group manifold
  - bosonic group field $\varphi(g_i, \chi^A)$ real valued functions group elements e.g. $\varphi : \mathrm{SU}(2)^4 \times \mathbb{R}^4 \to \mathbb{R}$
- Field excitations: building blocks of spacetime
  - tetrahedron or spin network vertex

- Mode decomposition $\varphi(g_i, \chi^A) = \sum_J \varphi_J(\chi^A) D_J(g_i)$ $J = (\vec{j}, \vec{m}, \iota)$
- Action $S[\varphi] = \mathcal{K}[\varphi] + V[\varphi]$
  - here: second order kinetic term $\mathcal{L} = \sum_J \left(\frac{1}{2} \mathcal{K}_J^{(0)} \varphi_J - \frac{1}{2} \mathcal{K}_J^{(2)} (\partial_A \varphi_J)^2 \right)$ $\partial_A = \frac{\partial}{\partial \chi^A}$

RELATIONAL COORDINATE SYSTEM

[Gielen ('18)]
[Kotecha, Oriti ('18)]
[Marchetti, Oriti ('21), ('22)]



- Couple four massless scalar fields $\varphi : \mathrm{SU}(2)^4 \times \mathbb{R}^4 \to \mathbb{R}$
    - relate to coordinate system in which $\partial_\mu \chi^A = \delta^A_\mu$
    - one clock field χ^0, three 'spatial' fields $\chi^1\ \chi^2\ \chi^3$

[Wilson-Ewing ('18)], [Gielen, Polaczek ('20), ('21)]

- Deparametrised approach
    - single out clock field χ^0
    - introduce canonical momentum $\pi_J = -\mathcal{K}^{(2)} \partial_0 \varphi_J$
    - Fourier decomposition w.r.t. spatial fields $\varphi_J(\chi^A) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{\mathrm{i}\vec{k}\vec{\chi}} \varphi_{J,k}(\chi^0)$

QUANTIZATION

[Gielen, Polaczek (`21)]

- Equal time commutation relations

$$[\varphi_{J,k}(\chi^0), \pi_{J',k'}(\chi^0)] = \mathrm{i}\,\delta_{JJ'}(2\pi)^3\delta(\vec{k}+\vec{k'})$$

- Hamiltonian

$$\boxed{H = \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\sum_J \frac{\mathcal{K}_J^{(2)}}{2}\left(-\frac{1}{|\mathcal{K}_J^{(2)}|^2}\pi_{J,-k}(\chi^0)\pi_{J,k}(\chi^0) + \omega_{J,k}^2\,\varphi_{J,-k}(\chi^0)\varphi_{J,k}(\chi^0)\right)}$$

$$\omega_{J,k}^2 = m_J^2 + \vec{k}^2$$

$$m_J^2 = -\frac{\mathcal{K}^{(0)}}{\mathcal{K}^{(2)}}$$

- Two types of Hamiltonian: `squeezing' $\omega_{J,k}^2 > 0$ and `oscillating' $\omega_{J,k}^2 < 0$
- Restrict to single J mode

EFFECTIVE METRIC

Construction

CONSERVATION LAWS FROM SYMMETRY

- Shift symmetry $\chi^A \mapsto \chi^A + \epsilon^A$

- Classically conserved current from free scalar field action

$$(j^\mu)^A = -\sqrt{-g}g^{\mu\nu}\partial_\nu\chi^A = -\sqrt{-g}g^{\mu A}$$

- Klein Gordon equation $\partial_\mu (j^\mu)^A = 0$

- GFT stress energy tensor

$$T^{AB} := -\frac{\partial \mathcal{L}}{\partial(\partial_A\varphi)}\partial_B\varphi + \delta^{AB}\,\mathcal{L}$$

- Conservation law $\partial_A T^{AB} = 0$

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- Conservation law $\partial_A T^{AB} = 0$

- Define

$$j^{AB} := (j^A)^B = \sqrt{-g}\, g^{AB}$$

- Construct operators

$$T^{AB} \to \mathcal{T}^{AB}$$

- Identify in relational coordinate system

$$j^{AB} = \langle \mathcal{T}^{AB} \rangle$$

QUANTIZE GFT ENERGY MOMENTUM TENSOR

$$\begin{tikzcd} T^{AB} \arrow[d] \\ T_k^{AB} \arrow[d] \\ \mathcal{T}_k^{AB} \arrow[d] \\ :\mathcal{T}_k^{AB}: \end{tikzcd}$$

- Fourier decomposition
- Replace $\varphi_k,\ \pi_k$ with operators
- Impose normal ordering for $a_k,\ a_k^\dagger$

- E.g. $$T_k^{00} = \frac{1}{2}\int \frac{\mathrm{d}^3\gamma}{(2\pi)^3}\left[\frac{\pi_\gamma \pi_{k-\gamma}}{\mathcal{K}^{(2)}} - \mathcal{K}^{(2)}\left(m^2 - \vec{\gamma}\cdot(\vec{k}-\vec{\gamma})\right)\varphi_\gamma\varphi_{k-\gamma}\right]$$
- Conservation law $\partial_0 : \mathcal{T}_k^{0B} : + \mathrm{i}\sum_a k_a : \mathcal{T}_k^{aB} : = 0$ holds at operator level

EFFECTIVE METRIC

Application to cosmology

GROUP FIELD THEORY COSMOLOGY

- Single massless scalar field χ^0 as a clock
- Condensate phase: state determined by single particle wavefunction

- Effective scale factor from volume operator $\langle \hat{V} \rangle = a^3$
- Single J mode dominates at late times

[Gielen ('16)], ...]

- Effective Friedmann equation
  - bounce
  - recover GR at late times
- Phenomenology
  - interactions, multiple modes, anisotropy, perturbations

[Oriti, Pang, ('21)]
[Gielen, Calcinari, ('23)]
[de Cesare, Pithis, Sakellariadou, ('16)]
[Marchetti, Oriti ('21) ('22)], [Jercher, Marchetti, Pithis ('23)]

EFFECTIVE FLRW METRIC FROM GFT I

- Recall: $j^{AB} = \langle \mathcal{T}^{AB} \rangle \ = \sqrt{-g}\, g^{AB}$
- Fock coherent state is sufficiently semiclassical

[Gielen, Polaczek (`20)]

$$a_k|\sigma\rangle = \sigma(\vec{k})|\sigma\rangle$$

- State determines physical scenario
  - flat FLRW metric

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}x^i\mathrm{d}x^j$$

  - Gaussian peaked around homogeneous background mode $\vec{k}_0 = 0$

$$\sigma(\vec{k}) = \frac{\mathcal{A} + \mathrm{i}\mathcal{B}}{c_\sigma}\, e^{-\frac{(\vec{k}-\vec{k}_0)^2}{2s^2}}$$

Contrast to standard cosmology: No clear split into background and perturbations

EFFECTIVE FLRW METRIC FROM GFT II

- Classical currents

$$j^{AB} = \begin{pmatrix} |\pi_0| & 0 \\ 0 & -\frac{a^4}{|\pi_0|}\delta^{ab} \end{pmatrix}$$

clock field conjugate momentum

- Identification with operator expectation values

$$|\pi_0| = \langle \mathcal{T}_0^{00} \rangle = |m|(\mathcal{B}^2 - \mathcal{A}^2)\,, \qquad \langle \mathcal{T}_0^{0b} \rangle = 0, \qquad \langle \mathcal{T}_0^{ab} \rangle = 0$$

$$a^4 = -|\pi_0| \langle \mathcal{T}_0^{aa} \rangle = m^2(\mathcal{B}^2 - \mathcal{A}^2)\left((\mathcal{A}^2 + \mathcal{B}^2)\cosh(2|m|\chi^0) - 2\mathcal{A}\mathcal{B}\sinh(2|m|\chi^0)\right)$$

- Spatially flat metric, Lorentzian signature dependent on initial conditions $\mathcal{B}^2 > \mathcal{A}^2$

EFFECTIVE FRIEDMANN EQUATION

- Resulting Friedmann equation $$H^2 = \frac{1}{4}m^2\left(1 - \frac{|\pi_0|^4}{a^8}\right) \xrightarrow[\text{late times}]{} \frac{1}{4}m^2$$

- Classical Friedmann equation with a single scalar field $$H^2 = \frac{\kappa}{6}$$ $\kappa = 8\pi G$

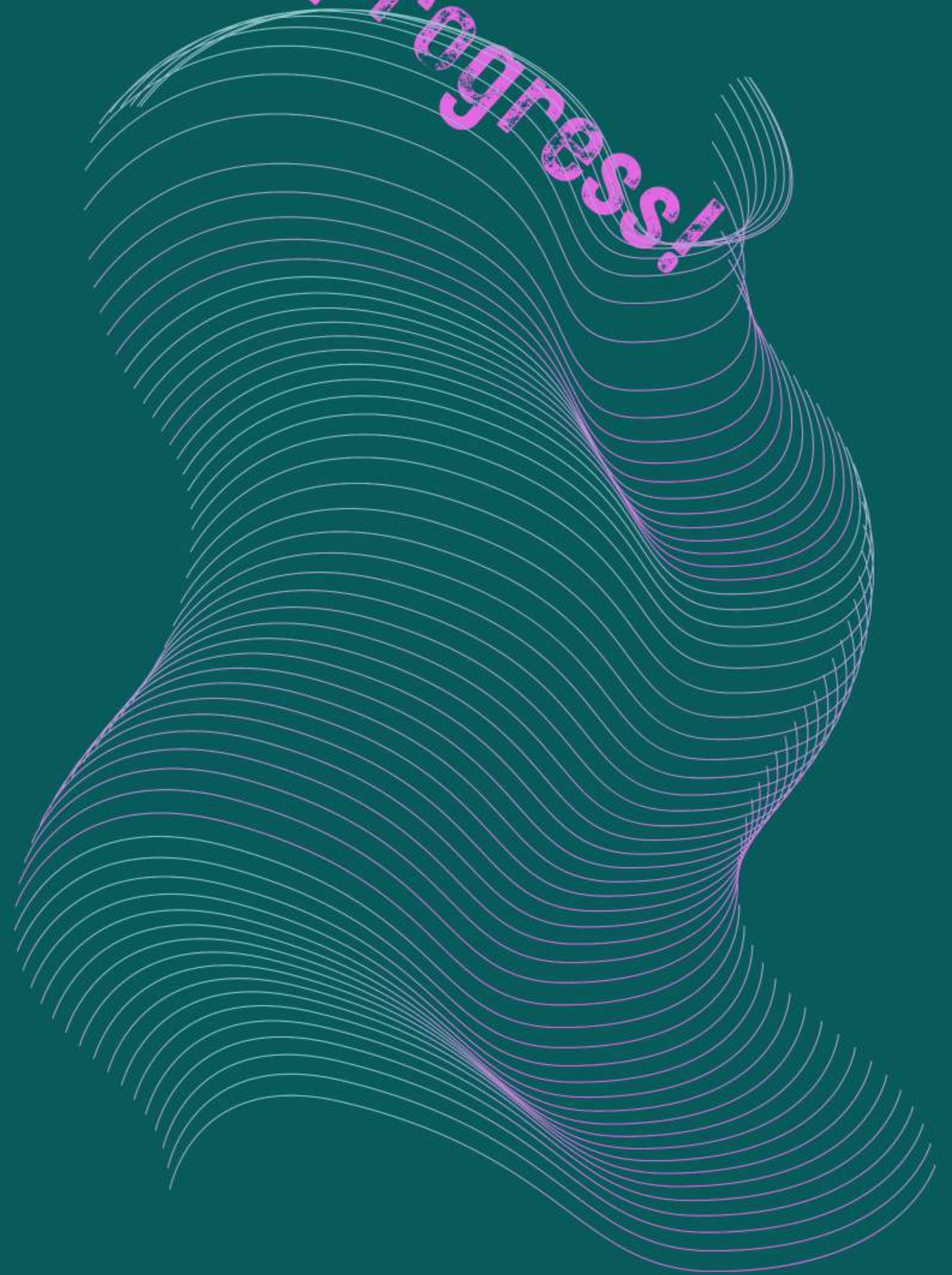
- GFT with a clock field $$H^2 = \frac{\kappa}{6}\left(1 + \frac{v_0}{a^3} + \frac{\mathcal{Y}}{a^6}\right)$$ negative constant (initial condition)

  [Gielen, Polaczek (`20)]

- Effective Friedmann equation from volume operator $$a^3 = \langle\sigma|V|\sigma\rangle \propto \langle\sigma|N|\sigma\rangle$$
- Here: $$a^4 \propto \langle\sigma|N|\sigma\rangle$$

THE NEXT STEP...

Cosmological perturbations



Work in progress!

PERTURBATIONS FROM OPERATORS

- General relativity: add small inhomogeneous perturbations on homogeneous background

$$\mathrm{d}s^2 = -N^2(1+2\tilde{\Phi})\mathrm{d}t^2 + 2Na\,\partial_i B\,\mathrm{d}t\mathrm{d}x^i \; + a^2\left((1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\right)\mathrm{d}x^i\mathrm{d}x^j$$

- Perturbation of effective quantity $f = \langle\hat{f}\rangle$ given by $\delta f_k = \langle\hat{f}_k\rangle - \delta_{0,k}\langle\hat{f}_0\rangle$
- Effective metric identification gives ($k \neq 0$)

$$T_k^{00} = |\pi_0|(\tilde{\Phi} + 3\psi + k^2 E) \qquad T_k^{a\neq b} = 2\frac{a^4}{|\pi_0|}k_a k_b E$$

$$T_k^{aa} = \frac{a^4}{|\pi_0|}(\tilde{\Phi} - \psi - k^2 E + 2k_a^2 E) \qquad T_k^{0b} = \mathrm{i}a^2 k_b B$$

Access to all perturbative quantities

EXAMPLE DYNAMICS OF PERTURBATIONS

- Focus on

$$E = \frac{|\pi_0|}{2a^4}\frac{T^{a\neq b}}{k_a k_b} = -\mathrm{sgn}(\mathcal{K}^{(2)})\frac{|\pi_0|}{16\omega_{k/2}a^4}(C_k + 2N_k)$$

- GFT equation of motion
  - late times

$$E'' - k^2 E = 0$$

  - agreement with GR for $k \to 0$
  - Euclidean signature

- Classical equation of motion
  - single field

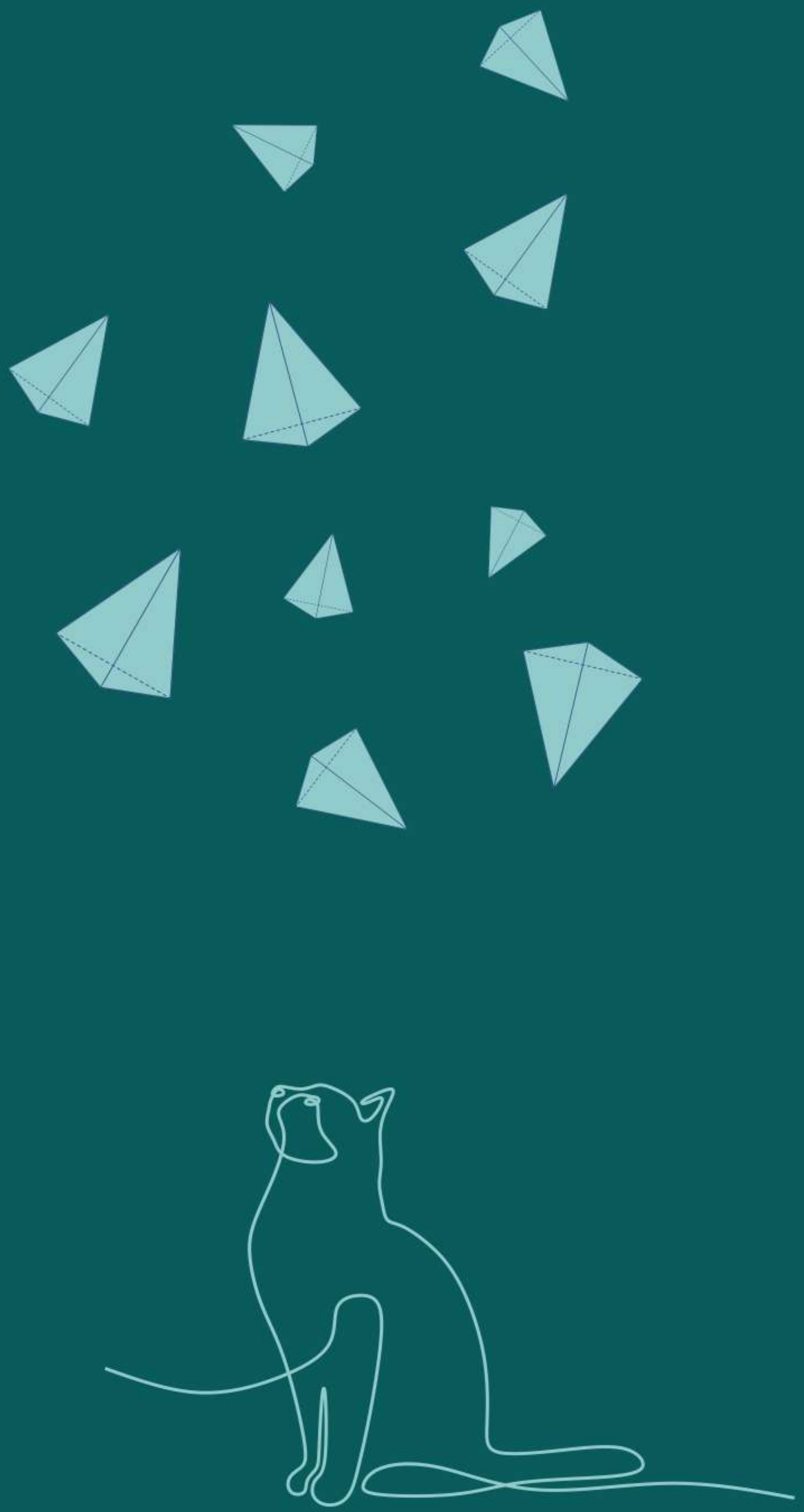
$$\frac{(\pi^0)^2}{a^4}E'' + k^2 E = 0$$

- Previous studies of GFT perturbations: perturbed volume

$$k^2 E - 3\psi = \frac{\delta V_k}{V}$$

[Gielen ('18)],[Marchetti, Oriti ('21) ('22)]
[Jercher, Marchetti, Pithis ('23)] ⟶ Recover GR

CONCLUSION



CONCLUSION

- We introduce new GFT operators that allow to reconstruct an effective metric in a relational coordinate system for suitable states

- Background cosmology
    - recover Friedmann equation for single scalar field in GR
    - different relation between scale factor and number operator $a^4 \propto \langle\sigma|N|\sigma\rangle$
- Cosmological perturbations
    - can be reconstructed explicitly
    - mismatch with GR dynamics

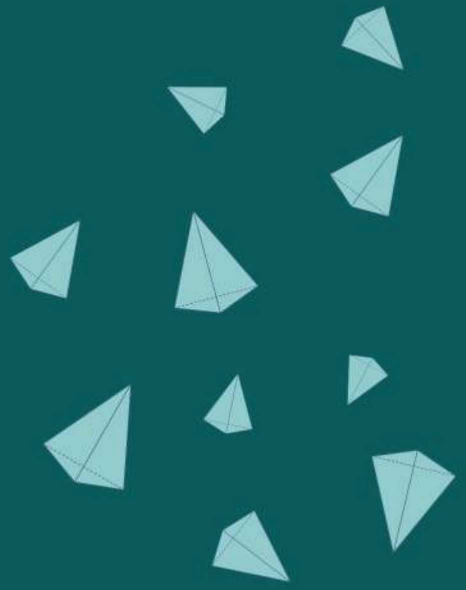
OUTLOOK

- Metric properties
- Application to other spacetimes
  - state choice

- Cosmological phenomenology
  - correct GR limit
  - connection to observables: study gauge invariant variables

THANK YOU!

Questions...?



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- Squeezing Hamiltonian:

$$\omega_{J,k}^2 > 0 \qquad H_{J,k} = \mathrm{sgn}(\mathcal{K}_J^{(2)})\frac{|\omega_{J,k}|}{2}\left(a_{J,k}a_{J,-k} + a_{J,k}^\dagger a_{J,-k}^\dagger\right)$$

- Restrict to singe J mode

$$\omega_{J,k}^2 = m_J^2 + \vec{k}^2$$

$$m_J^2 = -\frac{\mathcal{K}^{(0)}}{\mathcal{K}^{(2)}}$$

$$\varphi_{J,k}(\chi^0) = \frac{1}{2\alpha_{J,k}}(A_{J,k} + A_{J,-k}^\dagger)$$

$$\pi_{J,k}(\chi^0) = -\mathrm{i}\alpha_{J,k}(A_{J,k} - A_{J,-k}^\dagger)$$

$$\alpha_{J,k} = \sqrt{\frac{|\omega_{J,k}||\mathcal{K}^{(2)}|}{2}}$$

QUANTIZE GFT ENERGY MOMENTUM TENSOR

$$T^{AB} \to T_k^{AB} \to \mathcal{T}_k^{AB} \to : \mathcal{T}_k^{AB} :$$

- Fourier decomposition
- Replace $\varphi_k,\ \pi_k$ with operators
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- E.g.

$$T_k^{00} = \frac{1}{2}\int \frac{\mathrm{d}^3\gamma}{(2\pi)^3}\left[\frac{\pi_\gamma \pi_{k-\gamma}}{\mathcal{K}^{(2)}} - \mathcal{K}^{(2)}\left(m^2 - \vec{\gamma}\cdot(\vec{k}-\vec{\gamma})\right)\varphi_\gamma \varphi_{k-\gamma}\right]$$

$$: \mathcal{T}_k^{00} : = \int \frac{\mathrm{d}^3\gamma}{(2\pi)^3}\frac{\mathrm{sgn}(\mathcal{K}^{(2)})}{4\sqrt{|\omega_\gamma||\omega_{k-\gamma}|}}\left[2\beta^+_{k,\gamma} : A^\dagger_{-\gamma}A_{k-\gamma} : + \beta^-_{k,\gamma}\left(: A^\dagger_{-\gamma}A^\dagger_{\gamma-k} : + : A_\gamma A_{k-\gamma} :\right)\right]$$

- Conservation law $\partial_0 : \mathcal{T}_k^{0B} : + \mathrm{i}\sum_a k_a : \mathcal{T}_k^{aB} : = 0$ holds at operator level

GR IN RELATIONAL FRAMEWORK

- GFT Friedmann equation: $$H^2 = \frac{1}{4}m^2\left(1 - \frac{|\pi_0|^4}{a^8}\right) \xrightarrow[\text{late times}]{} \frac{1}{4}m^2$$

- Classically, the gradients of spatial fields change background Friedmann equation

$$H^2 = \left(\frac{a'}{a}\right)^2 = \frac{\kappa}{6}\left(1 + 3\frac{a^4}{\pi_0^2}\right), \qquad \frac{a''}{a} = \frac{\kappa}{6}\left(1 + 9\frac{a^4}{\pi_0^2}\right)$$

- GFT bounce at $a^4 = \pi_0^2$
  - Can only reproduce GR with single massless scalar field at late times
- Gradient terms absent also in other GFT approaches
  - Consistency with GFT literature

→ Conflict between homogeneity on GFT side and relational coordinate system

CLASSICAL PERTURBATIONS

- Perturbed stress energy tensor

$$^{(x)}\delta T^0_0 = \frac{\pi_0^2}{a^6}\tilde{\Phi} + \boxed{\frac{1}{a^2}\left(-3\psi + \nabla^2 E\right)}$$

$$^{(x)}\delta T^i_i = -\frac{\pi_0^2}{a^6}\tilde{\Phi} + \boxed{\frac{1}{a^2}\left(-\psi + (\nabla^2 - 2\partial_i^2)E\right)}$$

Needs to vanish for adiabatic perturbations: $\frac{\delta P}{\delta \rho} = \frac{P'}{\rho'}$

- Non-adiabatic perturbations in GR with relational coordinate system
  - Additional terms not negligible, as bounce happens at $a^4 = \pi_0^2$

EFFECTIVE FLRW METRIC FROM GFT II

- Classical currents

$$j^{AB} = \begin{pmatrix} |\pi_0| & 0 \\ 0 & -\frac{a^4}{|\pi_0|}\delta^{ab} \end{pmatrix}$$

clock field conjugate momentum

- Background quantities given by k=0 mode, use saddle point approximation

$$\int dx\, e^{-\frac{(x-\mu)^2}{2s^2}} f(x) \approx f(\mu) \int dx e^{-\frac{(x-\mu)^2}{2s^2}}$$

- Identification with operator expectation values

$$|\pi_0| = \langle \mathcal{T}_0^{00} \rangle = |m|(\mathcal{B}^2 - \mathcal{A}^2), \qquad \langle \mathcal{T}_0^{0b} \rangle = 0, \qquad \langle \mathcal{T}_0^{ab} \rangle = 0$$

$$a^4 = -|\pi_0| \langle \mathcal{T}_0^{aa} \rangle = m^2(\mathcal{B}^2 - \mathcal{A}^2)\left((\mathcal{A}^2 + \mathcal{B}^2)\cosh(2|m|\chi^0) - 2\mathcal{A}\mathcal{B}\sinh(2|m|\chi^0)\right)$$

- Spatially flat metric , Lorentzian signature dependent on initial conditions $\mathcal{B}^2 > \mathcal{A}^2$