New Static Spherically Symmetric Spacetime Teleparallel $F(T)$ solutions

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Table of Featured Topics

1. Summary of Teleparallel Gravity and Geometry

Teleparallel Gravity: A torsion-based theory of gravity: $F(T)$-type.

Fundamental Principles and Quantities:

A. Zero Curvature $R^a{}_b = 0$ and Zero Non-Metricity $Q_{ab\mu} = 0$.

B. Co-frame $h^a = h^a{}_\mu \, dx^\mu$: gravitational effects (Non-proper frame: we add some inertial effects).

C. Spin-Connection $\omega^a{}_{b\mu}$: inertial effects only.

D. Gauge Metric g_{ab}: gauge determination where the physical processes are occuring.

Some precisions:

A. For frame changing: we use Lorentz Transformation $\Lambda^a{}_b$.

B. Physical quantities like T, $T^a{}_{\mu\nu}$, $S_{ab}{}^\mu$, $g_{\mu\nu}$ depend on the previous fundamental quantities ($g_{\mu\nu}$ is not a fundamental quantity).

Main Features of Teleparallel Gravity:

A. We only need to satisfy the Strong equivalence and Covariance principle (Inertial and Gravitational masses could not be equivalent and the theory still hold!).

B. Action Integral in Teleparallel Gravity ($F(T)$-type):

$$S_{F(T)} = \int d^4 x \left[\frac{h}{2\kappa} F(T) + \mathcal{L}_{Matter} \right] \tag{1}$$

C. From the Least-Action Principle, the Field Equations (FEs) are:

$$\kappa\, \Theta_{(ab)} = F'(T)\, \overset{\circ}{G}_{ab} + F''(T)\, S_{(ab)}^{\ \ \ \mu}\, \partial_\mu T + \frac{g_{ab}}{2} \left[F(T) - T\, F'(T) \right], \tag{2a}$$

$$0 = F''(T)\, S_{[ab]}^{\ \ \ \mu}\, \partial_\mu T, \tag{2b}$$

where $\overset{\circ}{G}_{ab}$ is the Einstein tensor, $\Theta_{(ab)}$ the Energy-Momentum, T the Torsion scalar, g_{ab} the gauge metric, $S_{ab}^{\ \ \mu}$ the Superpotential (Torsion dependent) and κ the coupling constant.

Additional Precisions:

A. Eqns (2a) and (2b): symmetric and antisymmetric parts.

B. $\Theta_{(ab)}$ is obtained from $\mathcal{L}_{Matter}$ and satisfying $\nabla_\nu \Theta^{\mu\nu} = 0$ (Null hypermomentum $\mathfrak{T}^{\mu\nu} = 0$ case).

C. Teleparallel Equivalent of General Relativity (TEGR) ($F(T) = T$): $\kappa \Theta_{(ab)} = \overset{\circ}{G}_{ab}$ as Einstein Eqns in General Relativity (GR).

D. Non-null Hypermomentum cases (i.e. $\mathfrak{T}^{\mu\nu} \neq 0$) lead to more complex conservation laws.

2. Spherically Symmetric Spacetimes

We will study the Teleparallel Spherical Symmetric spacetime Geometries. Examples are: Schwarzschild spacetime, TRW, TdS and others.

A. We like working on a Proper Frame: we look for gravitational effects.

B. Non-proper frames: inertial effects would mingle with the gravitational effects and make difficult the analysis.

C. Spherically symmetric spacetimes: we work with orthonormal frame (the simplest frame), but non-proper. No extra DOF !

D. We use the following coframe in the orthonormal gauge (i.e. $g_{ab} = \eta_{ab} = Diag\,[-1, 1, 1, 1]$):

$$h^a{}_\mu = Diag\,[A_1(t,r), A_2(t,r), A_3(t,r), A_3(t,r)\sin\theta]\,. \tag{3}$$

E. Spin-connection is $\omega_{abc} = f_j(X_i(\psi,\,\chi),\theta) \neq 0$ on this orthonormal frame and find two antisymmetric FEs.

There are two cases arising from the antisymmetric FEs:

- 1st case: $\sin\chi = 0$: $\chi = n\,\pi$ where $n \in \mathbb{Z}$,
  $\cos\chi = \cos(n\,\pi) = \pm 1 = \delta$, and

$$(\partial_t\, T) = \left[\frac{\delta\, A_1\, \sinh\,\psi + \partial_t\, A_3}{\delta\, A_2\, \cosh\,\psi + \partial_r\, A_3}\right](\partial_r\, T)\,. \tag{4}$$

- 2nd case: $A_1\, \cosh\,\psi\,\partial_r\, T = A_2\, \sinh\,\psi\,\partial_t\, T$.

A. There are 4 independent components for each set of general symmetric Spherically Symmetric FEs (two sets of FEs).

B. By default: 3 affine symmetry operators (3 KVs).

C. There are special cases with further symmetries:

   - Static spherically symmetric spacetimes (r-dependent only).
   - Kantowski-Sachs spacetimes (t-dependent only).
   - X_4 similarity with λ-parameter.
   - Teleparallel de Sitter (TdS) ($\lambda = 0$) and Robertson-Walker (TRW) spacetimes.

3. Static Field Equations and Solutions

3.1 The FEs and their caracterictics

A. Coframe and spin-connection: $(A_1,\, A_2,\, A_3) = (A_1(r),\, A_2(r),\, A_3(r))$ and $\omega_{abc} = f_j(X_i(\psi(r),\, \chi(r)), \theta)$.

B. Static antisymmetric FEs are:

$$
\begin{aligned}
0 &= \frac{F''\,(T)\;\partial_r\,T}{\kappa\,A_2\,A_3}\;[\cos\,\chi\;\sinh\;\psi] \\
0 &= \frac{F''\,(T)\;\partial_r\,T}{\kappa\,A_2\,A_3}\;[\sin\,\chi\;\cosh\;\psi]\,.
\end{aligned} \tag{5}
$$

C. The only solution is $\sin\,\chi = 0$ and $\sinh\psi = 0$: $\chi = n\,\pi$ and $\psi = 0$ where $n \in \mathbb{Z}$, $\cos\,\chi = \cos\,(n\,\pi) = \pm 1 = \delta$ and $\cosh\psi = 1$ leading to the non-zero spin-connection components:

$$
\omega_{233} = \omega_{244} = \frac{\delta}{A_3} \qquad \omega_{344} = -\frac{\cot\,\theta}{A_3}. \tag{6}
$$

D. 4 KV, but the 4th symmetry leads to static solutions ($X_4 = \partial_t$).

E. This solution leads to the 3 non-trivial symmetric FEs set of the form:

$$\begin{aligned} 0 &= -F''(T)\,(\partial_r T)\,k_1 + F'(T)\,g_1, && \text{(7a)} \\ \kappa\,[\rho + P] &= -2\,F''(T)\,(\partial_r T)\,k_2 + 2\,F'(T)\,g_2, && \text{(7b)} \\ \left[\kappa\,\rho + \frac{F(T)}{2}\right] &= -2\,F''(T)\,(\partial_r T)\,k_3 + 2\,F'(T)\,g_3. && \text{(7c)} \end{aligned}$$

$g_i = g_i(A_1(r), A_2(r), A_3(r))$ and $k_i = k_i(A_1(r), A_2(r), A_3(r))$ where $i = 1, 2, 3$.

F. For vacuum ($P = \rho = 0$), eqns (7a)–(7c) become:

$$\begin{aligned} \frac{g_2}{k_2} &= \frac{g_1}{k_1}, && \text{(8a)} \\ \frac{g_2}{k_2} &= \partial_r\,[\ln\,F'(T(r))]\,, && \text{(8b)} \\ \frac{F(T)}{4} &= F'(T)\,(g_3 - g_2)\,. && \text{(8c)} \end{aligned}$$

3.2 Vacuum FEs for $A_3 =$ constant solution

A. For $A_3 = c_0$ and the coordinate choice $A_2 = b_0 = 1$: static FEs become:

$$A_1'' + \frac{A_1}{c_0^2} = 0, \tag{9a}$$

$$\partial_r \left[\ln F'(T)\right] = 0, \tag{9b}$$

$$\partial_r \left[\ln F(T)\right] = -\frac{\delta\, c_0\, A_1\, T'(r)}{4\, A_1'}, \tag{9c}$$

$$T(r) = -\frac{2}{c_0^2} - \frac{4\delta\, A_1'}{c_0\, A_1}. \tag{9d}$$

B. The solution is $F(T) = F_1 \left[T + \frac{2}{c_0^2}\right]$, leading to GR solutions: not really teleparallel.

C. Eqn. (9a) leads to an oscillating $A_1(r)$ of frequency $\omega_0 = \frac{1}{c_0}$: not physically relevant.

3.3 Vacuum FEs with $A_3 = r$ coordinate choice

The static FEs with $\mathbf{A_3} = \mathbf{r}$ and torsion scalar are:

$$0 = (\delta\, A_2 + 1) \left(\frac{\partial_r^2 A_1}{A_1}\right) + \left(\frac{\partial_r A_1}{A_1}\right)^2 - \delta \left(\frac{\partial_r A_1}{A_1}\right) (\partial_r A_2) + \frac{1}{r^2} (\delta\, A_2 + 1) \left(A_2^2 - 1\right), \tag{10a}$$

$$0 = (\delta\, A_2 + 1)\, \partial_r \left[\ln\, F'\left(T(r)\right)\right] - \partial_r \left[\ln\left(A_1 A_2\right)\right], \tag{10b}$$

$$\frac{F(T)}{4} = \frac{F'(T)}{r^2 A_2^2} \left[- (\delta\, A_2 + 2) \left(\frac{r\, \partial_r A_1}{A_1}\right) - (\delta\, A_2 + 1) \right]. \tag{10c}$$

$$T(r) = -\, 2 \left(\frac{\delta}{r} + \frac{1}{A_2\, r}\right) \left(\frac{\delta}{r} + \frac{1}{A_2\, r} + 2 \frac{\partial_r A_1}{A_2\, A_1}\right). \tag{10d}$$

3.4 Power-Law solutions

A. For $A_1(r) = a_0\, r^a$, $A_2(r) = b_0\, r^b$ and $A_3 = r$: eqns. (10a) – (10d) become

$$0 = \frac{1}{b_0^2\, r^{2b}} \left[\delta\, \left(a^2 - a(1+b) - 1 \right) + \frac{\left(2a^2 - a - 1 \right)}{b_0\, r^b} \right] + \left[\frac{1}{b_0\, r^b} + \delta \right], \tag{11a}$$

$$F'(T(r)) = F_1 \exp\left[(a+b) \int \frac{dr}{r\, (\delta\, b_0\, r^b + 1)} \right], \tag{11b}$$

$$F'(T(r)) = -\frac{b_0\, r^{b+2}\, F(T(r))}{4 \left[\left(\delta + \frac{2}{b_0\, r^b} \right)\, a + \left(\delta + \frac{1}{b_0\, r^b} \right) \right]}, \tag{11c}$$

$$T(r) = -\frac{2}{r^2} \left[\left(\delta + \frac{1}{b_0\, r^b} \right)^2 + \frac{2\, a}{b_0\, r^b}\, \left(\delta + \frac{1}{b_0\, r^b} \right) \right]. \tag{11d}$$

B. From eqn. (11a): $\mathbf{b = 0}$ is the only possible solution.

3.5 Exact solutions

A. For $\mathbf{A_2} = \mathbf{b_0} =$ constant ($b = 0$ cases), we obtain as eqns. (10a) – (10c):

$$0 = \left[\frac{A_1''}{A_1} + \frac{1}{r^2}\,\left(b_0^2 - 1\right)\right]\left(\delta\, b_0 + 1\right) + \left(\frac{A_1'}{A_1}\right)^2 = 0, \tag{12a}$$

$$F'(T(r)) = F_1\, A_1^{\frac{1}{(\delta\, b_0+1)}}, \tag{12b}$$

$$F'(T(r)) = -\frac{b_0^2\, r^2\, F(T(r))}{4\left[\left(\delta\, b_0 + 2\right)\,\left(\frac{r\,A_1'}{A_1}\right) + \left(\delta\, b_0 + 1\right)\right]}, \tag{12c}$$

where F_1 is an integration constant and $\delta b_0 \neq -1$.

B. The solution for eqn (12a) is:

$$\mathbf{A_1(r) = a_0\, r^{\frac{(\delta\, b_0+1)}{2\,(\delta\, b_0+2)}(1+S)}\,\left(1 + y_1\, r^{-S}\right)^{\frac{(\delta\, b_0+1)}{(\delta\, b_0+2)}}}, \tag{13}$$

where $\delta\, b_0 \neq -2$ and $\delta\, b_0 \neq \pm 1$ and

$$\mathbf{S = \pm\sqrt{1 - 4\,(\delta\, b_0 - 1)\,(\delta\, b_0 + 2)}}. \tag{14}$$

C. Eqns. (12b) and (12c) become:

$$F'(T(r)) = F_2\, r^{\frac{(1+S)}{2\,(\delta\, b_0+2)}} \left(1 + y_1\, r^{-S}\right)^{\frac{1}{(\delta\, b_0+2)}}, \tag{15a}$$

$$F'(T(r)) = -\frac{b_0^2\, r^2\, F(T)}{2\,(\delta\, b_0+1)\,(3-S)\left[1 + \frac{2S}{(3-S)[1+y_1\, r^{-S}]}\right]}, \tag{15b}$$

where $S \neq 3$ and $F_2 = F_1\, a_0^{\frac{1}{(\delta\, b_0+1)}}$.

D. Eqn. (10d) for the torsion scalar becomes:

$$T(r) = \frac{T_0}{r^2}\left[1 + \frac{T_1}{[1 + y_1\, r^{-S}]}\right], \tag{16}$$

where T_0 and T_1 are constants depending on b_0 and S.

E. From eqns (15a) and (15b) we obtain $F(T)$:

$$\mathbf{F(T(r))} = -\frac{\mathbf{2\,(\delta\, b_0+1)\,(3-S)}}{\mathbf{b_0^2}}\, \mathbf{F_2}\left[\mathbf{1 + F_3\, \left(1 + y_1\, r^{-S}\right)^{-1}}\right]$$
$$\times \left(\mathbf{1 + y_1\, r^{-S}}\right)^{\frac{1}{(\delta\, \mathbf{b_0}+2)}}\, \mathbf{r}^{\frac{(1+S)}{2\,(\delta\, \mathbf{b_0}+2)}-2}, \tag{17}$$

where $F_3 = \frac{2S}{(3-S)}$ (for $S \neq 3$).

F. $\mathbf{y_1 = 0}$ (Power-law solutions for $F(T)$): From eqns. (16) and (17) for all S values, we find:

$$T(r) = \frac{T_0\,(1+T_1)}{r^2}, \tag{18a}$$

$$\mathbf{F(T)} = -\frac{\mathbf{2\,(\delta\,b_0+1)\,(3-S)}}{\mathbf{b_0^2}}\,\mathbf{F_2}\,\left(\mathbf{1+F_3}\right)\left(\frac{\mathbf{T}}{\mathbf{T_0\,(1+T_1)}}\right)^{\mathbf{1-\frac{(1+S)}{4\,(\delta\,b_0+2)}}}. \tag{18b}$$

We have the power-law solution of Golovnev-Guzman (ArXiv:2103.16970).

G. $\mathbf{y_1 \neq 0}$: Eqn. (16) becomes:

$$\left(\frac{T}{T_0}\right)\left(r^2 + y_1\,r^{2-S}\right) - y_1\,r^{-S} - (1+T_1) = 0. \tag{19}$$

To find $F(T)$ explicitly, we need to find r as a function of T in eqn. (19).

H. There are some solvable cases for specific values of S:

1. $\mathbf{S=0}$: With $\delta\, b_0 = \frac{\pm\sqrt{10}-1}{2}$, $T_0 = -\frac{2}{27}\left(25 \pm 34\sqrt{10}\right)$, $T_1 = 0$, $F_2 = F_1\, a_0^{\frac{2}{9}\left(-1\pm\sqrt{10}\right)}$ and $F_3 = 0$, Eqn. (19) leads to $r(T) = \left(\frac{T}{T_0}\right)^{-\frac{1}{2}}$ and eqn. (17) will be:

$$\mathbf{F(T) = -\frac{4}{27}\left(31 \pm 13\sqrt{10}\right) F_2\, (1+y_1)^{\frac{2}{(3\pm\sqrt{10})}} \left(\frac{T}{T_0}\right)^{1-\frac{1}{2\left(3\pm\sqrt{10}\right)}}}, \tag{20}$$

We obtain a power-law solution for $F(T)$.

2. $\mathbf{S=2}$: With $\delta\, b_0 = \frac{\pm\sqrt{6}-1}{2}$, $T_0 = -\frac{2}{75}\left(117 \pm 62\sqrt{6}\right)$, $T_1 = -\frac{8}{5}\left(1 \mp \sqrt{6}\right)$, $F_2 = F_1\, a_0^{\frac{2}{5}\left(-1\pm\sqrt{6}\right)}$ and $F_3 = 4$, eqn. (19) leads to:

$$r^2(T) = \frac{1}{2}\left(\frac{T_0(1+T_1)}{T} - y_1\right)\left[1 \pm \sqrt{1 + \frac{4\, y_1\, \frac{T_0}{T}}{\left(\frac{T_0(1+T_1)}{T} - y_1\right)^2}}\right].$$

$$\mathbf{F(T) = -\frac{4}{25}\left(19 \pm 9\sqrt{6}\right) F_2\, \frac{\left(5\, r^2(T) + y_1\right)}{\left(r^2(T) + y_1\right)^{1-\frac{2}{3\pm\sqrt{6}}} \left(r(T)\right)^{2+\frac{1}{3\pm\sqrt{6}}}}}. \tag{21}$$

3. $\mathbf{S=-2}$: With $\delta\, b_0 = \frac{\pm\sqrt{6}-1}{2}$, $T_0 = -2\left(7 \pm 2\sqrt{6}\right)$, $T_1 = -\frac{8}{75}\left(9 \mp \sqrt{6}\right)$, $F_2 = F_1\, a_0^{\frac{2}{5}\left(-1\pm\sqrt{6}\right)}$ and $F_3 = -\frac{4}{5}$, eqn. (19) leads to:

$$r^2(T) = \frac{1}{2}\left[\left(\frac{T}{T_0}\right)^{-1} - \frac{1}{y_1}\right]\left[1 \pm \sqrt{1 + \frac{4(1+T_1)\left(\frac{T}{T_0}\right)}{y_1\left[1 - \frac{1}{y_1}\left(\frac{T}{T_0}\right)\right]^2}}\right].$$

$$\mathbf{F(T)} = -\frac{\mathbf{4}}{\mathbf{5}}\left(\mathbf{19 \pm 9\sqrt{6}}\right)\mathbf{F_2}\,\frac{\left(\mathbf{1 + 5y_1\, r^2(T)}\right)}{\left(\mathbf{1 + y_1\, r^2(T)}\right)^{\mathbf{1-\frac{2}{3\pm\sqrt{6}}}}\left(\mathbf{r(T)}\right)^{\mathbf{2+\frac{1}{3\pm\sqrt{6}}}}}, \tag{22}$$

4. $\mathbf{S} = \pm\mathbf{1}$ and $\delta\,\mathbf{b_0} = -\mathbf{2}$: Eqn. (12a) becomes ($y = \ln A_1$):

$$0 = y'' + \frac{3}{r^2}. \qquad (23)$$

The solution is $A_1(r) = a_0\, r^3\, e^{y_1\, r}$, $T(r) = \frac{5}{2}\, r^{-2} + y_1\, r^{-1}$ and then

$$r^{-1}(T) = \frac{1}{5}\left[-y_1 \pm \sqrt{y_1^2 + 10\,T}\right], \qquad (24)$$

$$\mathbf{F(T)} = \frac{\mathbf{F_2}}{\mathbf{3125}}\left[-\mathbf{y_1} \pm \sqrt{\mathbf{y_1^2} + \mathbf{10\,T}}\right]^{\mathbf{5}} \exp\left[\frac{\mathbf{5y_1}}{\left[\mathbf{y_1} \mp \sqrt{\mathbf{y_1^2} + \mathbf{10\,T}}\right]}\right], \qquad (25)$$

where $F_2 = \frac{F_1}{a_0}$ and $y_1 \neq 0$.

5. $y_1 = 0$: $A_1(r) = a_0\, r^3$ and $F(T(r)) = \frac{F_1}{a_0}\, r^{-5}$. Then eqn. (24) becomes $r^{-1}(T) = \sqrt{\frac{2}{5}}\, T^{\frac{1}{2}}$ and eqn (25) is:

$$\mathbf{F(T)} = \left(\frac{\mathbf{2}}{\mathbf{5}}\right)^{\frac{5}{2}} \mathbf{F_2\,T^{\frac{5}{2}}}. \qquad (26)$$

Here we have a new and different power-law solution.

6. $\mathbf{S} = \pm\mathbf{1}$ and $\delta\,\mathbf{b_0} = \mathbf{1}$: Eqn. (12a) reduces to ($y = \ln A_1$):

$$0 = y'' + \frac{3}{2}\,y'^2. \qquad (27)$$

The solution is $A_1(r) = a_0\,(r+y_1)^{\frac{2}{3}}$, $T(r) = -\frac{8}{r^2}\left(1 + \frac{2r}{3(r+y_1)}\right)$, $r(T)$ and $F(T)$ become:

$$r(T) = -\frac{y_1}{3} + \frac{1}{3T}\left[-(y_1\,T)^3 + 2\sqrt{2}\,T\,\sqrt{27\,y_1^4\,T^3 - 312\,y_1^2\,T^2 + 8000\,T} - 48\,y_1\,T^2\right]^{\frac{1}{3}}$$
$$-\frac{\left(40 - y_1^2\,T\right)}{3}\left[-(y_1\,T)^3 + 2\sqrt{2}\,T\,\sqrt{27\,y_1^4\,T^3 - 312\,y_1^2\,T^2 + 8000\,T} - 48\,y_1\,T^2\right]^{-\frac{1}{3}}. \qquad (28)$$

$$\mathbf{F(T) = -8\,F_2\,\frac{(2r(T) + y_1)}{(r(T)+y_1)^{\frac{2}{3}}\;r^2(T)}}, \qquad (29)$$

where $F_2 = F_1\sqrt{a_0}$.

7. $y_1 = 0$: $A_1(r) = a_0\,r^{\frac{2}{3}}$, $r(T) = \sqrt{\frac{40}{3}}\,(-T)^{-\frac{1}{2}}$ and eqn. (29) is:

$$\mathbf{F(T) = -16\left(\frac{3}{40}\right)^{\frac{5}{6}}\,F_2\,(-T)^{\frac{5}{6}}}, \qquad (30)$$

where $T \leq 0$, as for Golovnev-Guzman solution.

8. **Limits on S**: From eqn. (14), real values of δb_0 are possible when $-\sqrt{10} \leq S \leq +\sqrt{10}$.
9. Extremum test on eqn. (14) for S: $\delta b_0 = -\frac{1}{2}$, but non explicit $r(T)$ expression from eqn. (16) in this case.

4. Features on Perfect Fluids Field Equations Solutions

A. There are dozens of $F(T)$ solutions for linear perfect fluid (EoS: $\mathbf{P} = \alpha\,\rho$), perfect dust fluid $\alpha = 0$ and non-linear perfect fluids.

B. The perfect fluid energy-momentum tensor is:

$$\Theta_{(ab)} = T_{ab} = (\mathbf{P} + \rho)\, u_a\, u_b + g_{ab}\, \mathbf{P}. \tag{31}$$

C. We have in addition to satisfy fluid conservation laws $\nabla_\nu\, \Theta^{\mu\nu} = 0$ for $\mathfrak{T}^{\mu\nu} = 0$ case. For vacuum solution: it is trivially satisfied!

D. Power-law solutions ansatz leads not only to $b = 0$, but to several values of b with specific $F(T)$ solutions, not only pure power of T.

E. Non-linear perfect fluids: using a power-term correction ρ^w leading to the EoS $\mathbf{P} = \alpha\,\rho + \beta\,\rho^{\mathbf{w}}$ where $w > 1$ and $\beta \ll \alpha$ in several situations (small correction).

F. We can use several types of ansatz, not only power-law !

G. $A_3 =$ constant set of solutions: there are new non-trivial teleparallel $F(T)$ solutions, not only restricted to GR solutions.

5. Conclusion and Perspectives

A. We found and solved the static spherically symmetric FEs in Teleparallel Gravity (r-dependent spacetimes).

B. We found several static new vacuum spherically symmetric $F(T)$ solutions, not only power-law solutions.

C. Static new perfect fluid solutions: dozens of new $F(T)$ solutions.

D. Time-dependent (Kantowski-Sachs) vacuum and perfect fluid solutions: several new $F(T)$ solutions. Coming soon...

E. Extra Degree of Freedom and the Strong Coupling problems by using perturbation limits. Coming soon...

F. Studying specific solution of NGR theory. Coming soon...

G. Studying much more cosmological models by applying perturbations to Teleparallel Spherical Symmetric spacetimes.

6. For Further Details

My recent papers on the subject:

1. A.A. Coley, A. Landry, R.J. van den Hoogen and D.D. McNutt, **Spherically symmetric teleparallel geometries**, European Physical Journal C **84**, 334 (2024) **ArXiv:2402.07238**.
2. A. Landry, **Static spherically symmetric perfect fluid solutions in teleparallel $F(T)$ gravity**, Axioms **13** (5), 333 (2024) **ArXiv:2405.09257**.
3. A. Landry, **Kantowski-Sachs spherically symmetric general and perfect fluids solutions in teleparallel $F(T)$ gravity**, In final preparation.

Thank You Very Much !