Distorted static black holes with a bubble

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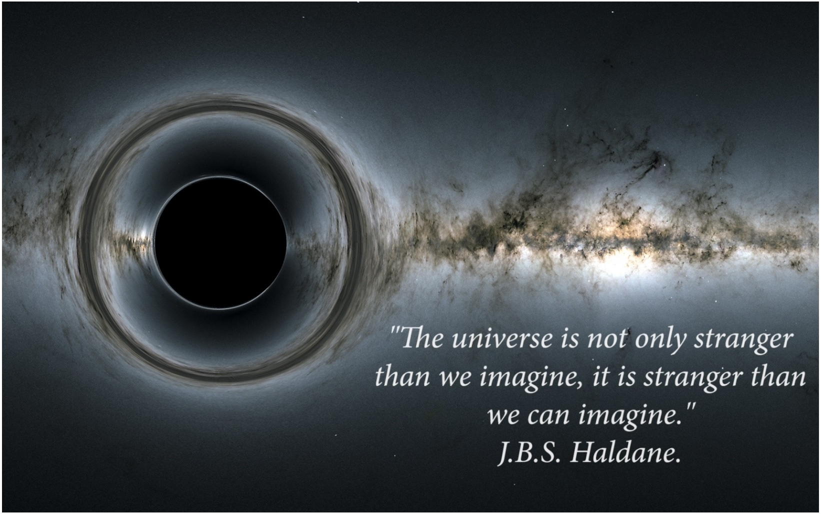
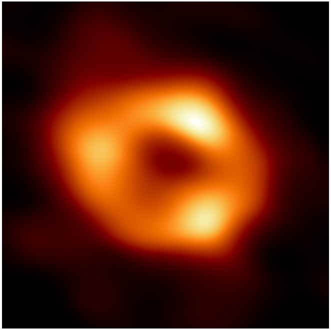
Abstract

We construct a family of local static, vacuum five-dimensional solutions with two commuting spatial isometries describing a black hole with a S^3 horizon and a 2-cycle 'bubble' in the domain of outer communications. The solutions are obtained by adding distortions to an asymptotically flat seed solution. We show that the conical singularities in the undistorted geometry can be removed by an appropriate choice of the distortion.



The simulation of a supermassive black hole. Credit: NASA's Goddard Space Flight Center; background, ESA/Gaia/DPAC.

Black holes in four dimensions

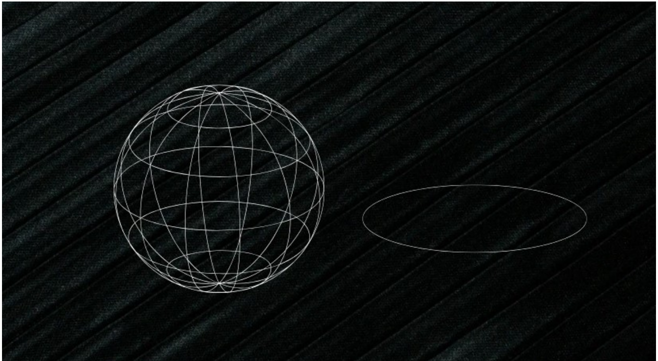


The first image of Sagittarius A*, the supermassive black hole at the centre of our galaxy.Credit: EHT Collaboration.

Well known black hole solutions have a simple exterior region. This means there are no interesting topological features outside the horizon. this is a consequence of the topological censorship theorem of Friedman, Scheich, and Witt (gr-qc/9305017 [gr-qc])

Black holes in higher dimensions

- In five and higher dimensions, this theorem allows for more complicated topology in the exterior region.
- For example, we can have 'bubbles' (i.e. two-dimensional 'holes' that do not collapse to a point) outside the event horizon. (see for example the paper of Horowitz, Kunduri and Lucietti (e-Print: 1704.04071 [hep-th]).)
- It suffers from conical singularities.

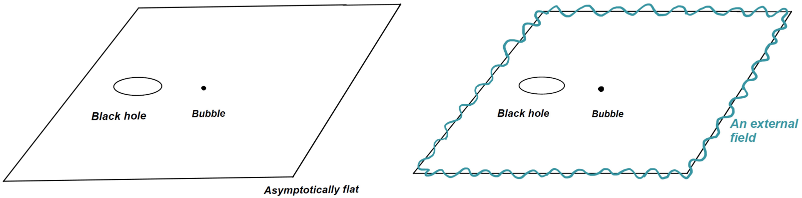


Why distorted black holes?

Why do we study distorted black holes?

- The study of black holes such as Schwarzschild black holes is analogous to the study of a single charge in an empty space where no interaction is present. It is highly idealized.
- Real black holes interact with external matters and fields.

In our case, we consider a black hole-bubble surrounded in the exterior region by other gravitational sources. We Construct regular vacuum solutions by distorting black hole-bubble configuration by an external field.



The Weyl solution in four dimensions

- The Weyl form is used to describe any axisymmetric, static, and vacuum spacetime.
- g_{ab} is a diagonal matrix.

The Weyl metric in 4-D

$$ds^2 = -e^{2U}dt^2 + e^{-2U}\left(e^{2\gamma}(dr^2 + dz^2) + r^2 d\phi^2\right)$$

- It is described by two orthogonal Killing vector fields that commute with each other.
- U(r, z) is an arbitrary axisymmetric solution of Laplace's equation in a three-dimensional flat space in cylindrical coordinates (r, z, ϕ).

The Laplace equation for an axisymmetric function

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial z^2} = 0$$

- While $\gamma(r,z)$ satisfies the auxiliary equations which are integrable because U(r,z) is harmonic.

$$\frac{\partial\gamma}{\partial r} = r\left(\left(\frac{\partial U}{\partial r}\right)^2 - \left(\frac{\partial U}{\partial z}\right)^2\right)$$
$$\frac{\partial\gamma}{\partial z} = 2r\left(\frac{\partial U}{\partial r}\right)\left(\frac{\partial U}{\partial z}\right).$$

- Since U is harmonic, it can be regarded as a Newtonian potential produced by certain (axisymmetric) sources.

The Schwarzschild solution

- The exterior Schwartzschild metric in 4-dimensions is

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad .$$

  where $t \in \mathbb{R}, r > 2m$, and (θ, ϕ) are standard coordinates on S^2.
- If we write it in the Weyl form we will have

$$U = -\frac{1}{2} \log \left[\frac{M - z + \sqrt{(M-z)^2 + r^2}}{-M - z + \sqrt{(M+z)^2 + r^2}} \right] \quad .$$

Based on the constraint, the function U_2 must be the potential produced by semi-infinite rods for $z \geq M$ and $z \leq -M$.

Sources for a four dimensional Schwarzschild solution.

The Weyl solution in five dimensions

- A five dimensional Weyl metric can be locally expressed in the form

$$ds^2 = -e^{2U_0}dt^2 + e^{2\nu}(dr^2 + dz^2) + e^{2U_1}d\psi^2 + e^{2U_2}d\phi^2$$

- It is described by three orthogonal Killing vector fields, $\partial_t, \partial_\psi, \partial_\phi$ that commute with each other.
- The metric functions $U_i = U_i(r, z)$, $i = 0, 1, 2$ are each axisymmetric solutions of the Laplace equation in a 3D flat space in cylindrical coordinates (r, z, ϕ).

The Laplace equation for an axisymmetric function

$$\frac{\partial^2 U_i}{\partial r^2} + \frac{1}{r}\frac{\partial U_i}{\partial r} + \frac{\partial^2 U_i}{\partial z^2} = 0\ .$$

- U_i functions are not all independent, but satisfy the constraint

$$\sum_i U_i = \log r + c \, .$$

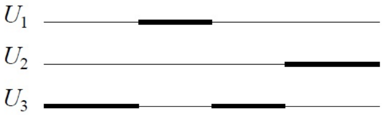
- $\log r$ represents a solution to Laplace's equation, which is the Newtonian potential sourced by a one-dimensional rod with an infinite length along the $z-$axis, having a uniform mass density of $1/2$.
- $\nu = \nu(r, z)$ satisfies the first order equations

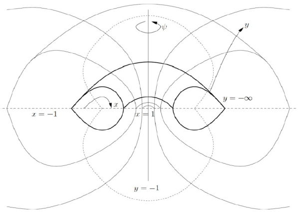
$$\partial_r \nu = -\frac{1}{2r} + \frac{r}{2} \sum_{i=0}^{2} \left((\partial_r U_i)^2 - (\partial_z U_i)^2 \right)$$

$$\partial_z \nu = r \sum_{i=0}^{2} \left((\partial_r U_i)(\partial_z U_i) \right) \, .$$

The Black ring

The static black ring is a vacuum solution that describes an asymptotically flat black hole with an event horizon that is not topologically a three-sphere S^3. Instead the horizon is topologically a ring $S^1 \times S^2$. The metric will have three orthogonal commuting Killing vector fields, making it a Weyl solution.





Sources for a black ring

Unbalanced black hole-bubble solution

U_i defined by

$$e^{2U_0} = \frac{\mu_0}{\mu_a}, \quad e^{2U_1} = \frac{r^2\mu_b}{\mu_0\mu_c}, \quad e^{2U_2} = \frac{\mu_a\mu_c}{\mu_b}.$$

$0 < a < b < c$ and

$$\mu_k = \sqrt{r^2 + (z-k)^2} - (z-k)$$

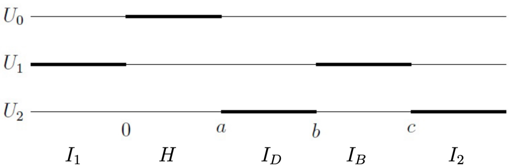
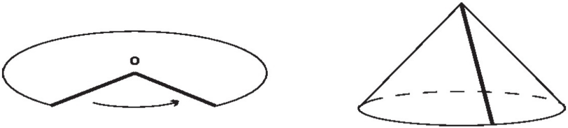


Figure: The rod structure for a black hole with a bubble [A. Rose "Constructing static black hole-soliton spacetimes" MSc. thesis, Memorial University of Newfoundland (2019).]

Conical singularity



Conical singularity, M. Kenmoku, S. Uchida, T. Matsuyama, Int.J.Mod.Phys. D12 (2003) 677-687

Regularity condition:$\phi \sim \phi + \Delta\phi$, $\Delta\phi = 2\pi \lim_{r\to 0} \sqrt{\frac{r^2 e^{2\nu}}{|K|^2}}$. Along the disk rod, regularity requires $b + c = a$. It is not possible.

$$\sqrt{\lim_{r\to 0} \frac{r^2 e^{2\nu}}{e^{2U_2}}} = \sqrt{\frac{b(b-a)}{c(c-a)}}, \quad a < z < b < c$$

Along the bubble rod I_B, $\Delta\phi = 2\pi\sqrt{\frac{(c-b)^2}{c(c-a)}} < 2\pi$

Distorted black hole with a bubble

- It describes a black hole-bubble system that exists within the gravitational field of external sources.
- This is in analogy with the interior or exterior multiple expansions in the electromagnetism.

$\widehat{X}$ is understood to represent the deformation. The deformation fields $\widehat{U}$ and $\widehat{W}$ satisfy the Laplace equation. In the cylindrical coordinates the solution of Laplace equation is

$$\widehat{X}(r,z) = \sum_{n\geq 0} \left[A_n R^n + B_n R^{-(n+1)} \right] P_n(\cos\vartheta)$$

$$R = \frac{\sqrt{r^2+z^2}}{m}, \quad \cos\vartheta = \frac{z}{R}$$

$P_n(\cos\vartheta)$ represent the Legendre polynomials of the first kind. and $\widehat{X}$ refers to either $\widehat{U}$ or $\widehat{W}$.

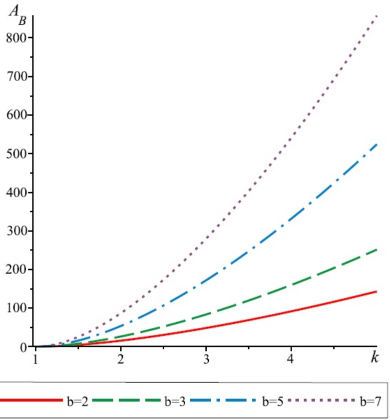
Conical regularity condition

The regularity of the metric is achieved provided we choose the distortion parameters (A_1, B_1) to satisfy

$$e^{B_1} = \left(\frac{c(c-a)}{b(b-a)} \right)^{\frac{1}{2(c-b)}} e^{A_1}$$

which leads to

$$e^{A_1} = \left[\frac{c(c-a)}{(c-b)^2} \left(\frac{b(b-a)}{c(c-a)} \right)^{\frac{b+a}{c-b}} \right]^{\frac{1}{6a}}$$



Area of the bubbleA_B for dipole distortions of the DBHB for several values of b. In this plot a = 1, c = kb.

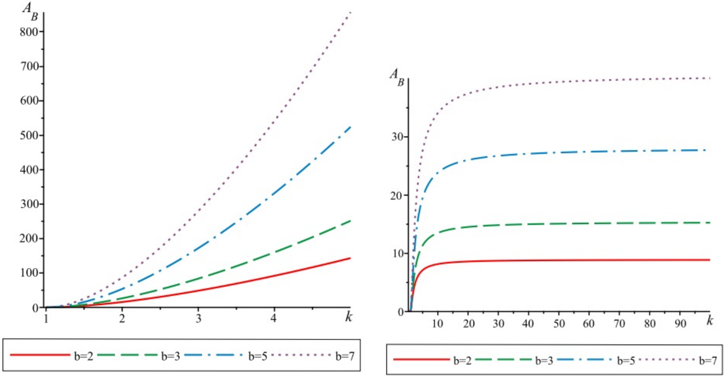
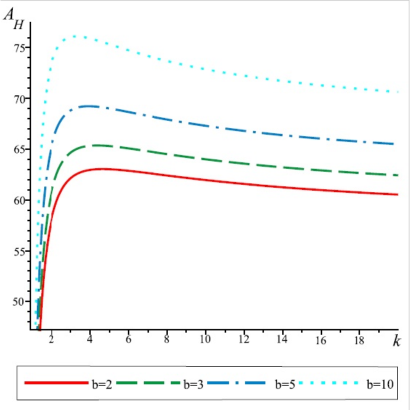
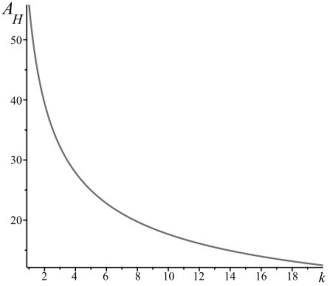


Figure 3: Area of the bubble A_B for dipole distortions of the BHB-AF solution (left) and the BHB-NF solution (right) for several values of b. For both cases $a = 1$, $c = kb$.



Area of the horizon A_H for dipole distortions of the DBHB for different values of b. In this plot, a = 1 and c = kb.



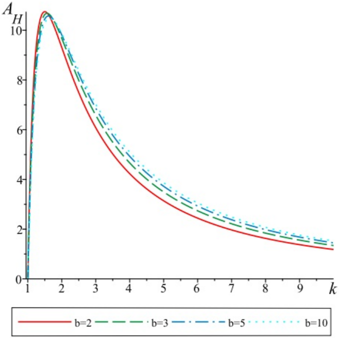
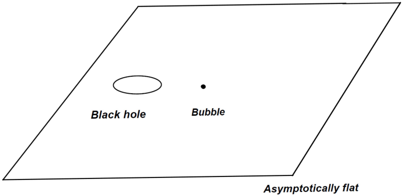
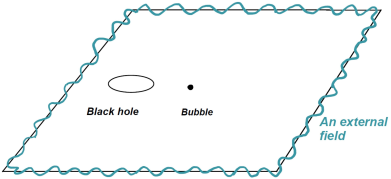


Figure 5: Left side: area of the horizon, A_H, for dipole distortions of AF black hole-bubble solution for different values of b. In this plot $a = 1$ and $c = kb$. Right side: Area of the horizon, A_H, for dipole distortions of the BHB-nf b. In this plot $a = 1$ and $c = kb$.

Summary

- We have constructed completely regular vacuum solutions in 5-D by relaxing the asymptotic flatness condition.
- We studied a five dimensional local distorted black hole-bubble solution.
- The solution contains a non-collapsing S^2 bubble outside smooth event horizon with spatial cross-section topology S^3
- The distortion are produced by distant sources in a asymptotic region far from the event horizon.
- This distorted solution is a local solution valid in the region interior to the sources.





References

- F. Tomlinson, *Five-dimensional electrostatic black holes in a background field*, Class. Quantum Grav, 39 135008 (2022)
- Emparan, R., and Reall, H.S., *Generalized Weyl solutions*, Phys. Rev. D, 65, 084025, (2002)
- H. Weyl, Ann. Phys. (Leipzig) 54 (1917) 117.
- R. Emparan and H. S. Reall, Black Holes in Higher Dimensions , Living Rev. Rel. 11 (2008) 6.
- S. Abdolrahimi, A. A. Shoom and D. N. Page, *Distorted 5-dimensional vacuum black hole*, Phys. Rev. D 82, 124039 (2010).

THANK YOU!