Evolution of the momentum compaction factor

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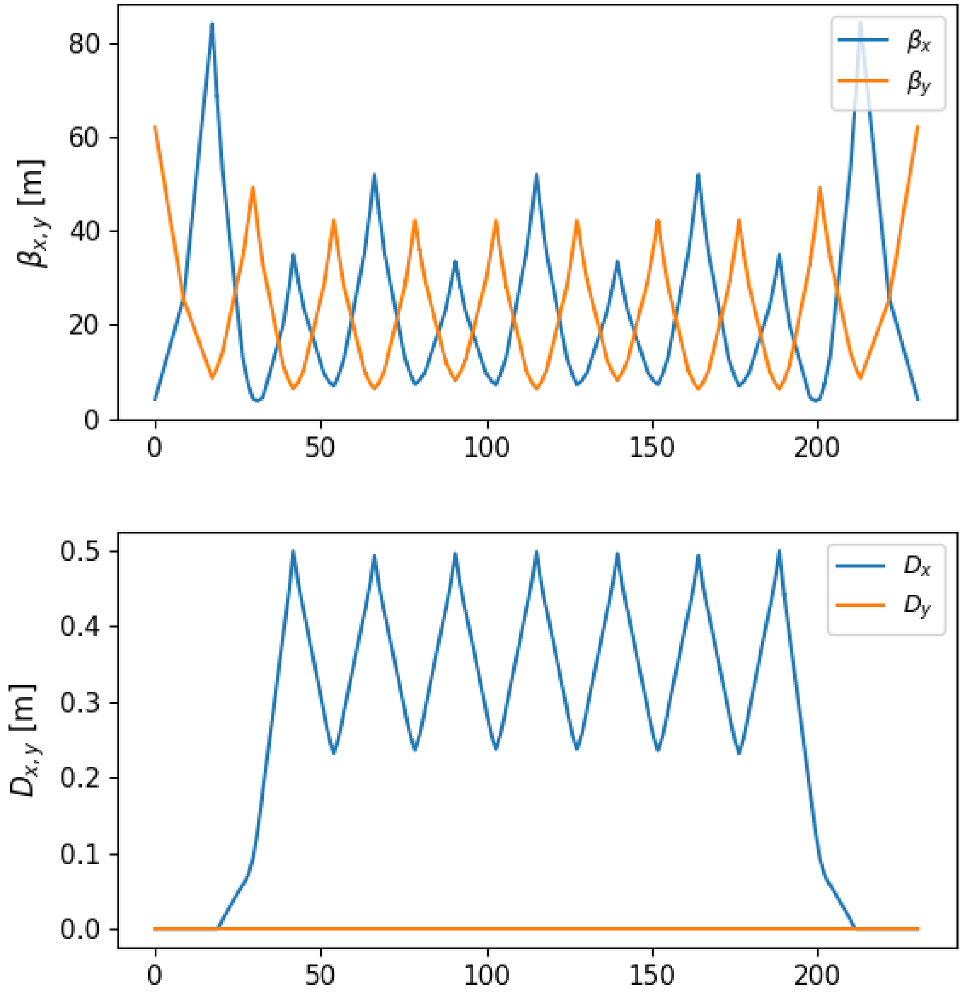
α_c with current FODO+DS lattice

- Momentum compaction factor:

$$\alpha_c = \frac{dL/L}{dp/p} = \frac{1}{L}\oint \frac{D_x(s)}{\rho(s)}ds$$

- RCS 2
- FODO structure (μ=90°) + Dispersion suppressor
- Geometry:
  - n_a = 26 arcs
  - n_c= 8 cells/arc
  - Pattern : [SC,NC,SC]
- Momentum compaction factor: **α_c = 0.00031**
- Synchrotron tune: Q_s = 0.181 (V_{tot}=11.22GV)



Analytical approach

- Classical FODO formula for 1 dipole family, depends on number of cells/ length of a cell

$$\alpha_{c,\,1f} = \frac{\theta_c \langle D \rangle}{L_c} = \left(\frac{\pi}{n_c}\right)^2 \left(\frac{1}{sin^2\left(\frac{\mu}{2}\right)} - \frac{1}{12}\right) \times FF_{arc}$$

(to take into account for RF insertion and DS)

- FODO formula for 2 dipole families:

$$\alpha = \left(\frac{\pi}{n_c}\right)^2 \left\{ \frac{1}{\sin^2\frac{\mu}{2}} - \frac{1}{4} + \frac{L_T}{6L_{\text{arc}}} + \frac{ab + n_d}{6L_{\text{arc}} n_d (1 + n_d)} \left[a(1-b)L_T + 4n_c(n_d(2+n_d) - ab(1+2n_d))L_{dd} \right] \right\} \times FF_{arc}$$

with $a = \frac{L_{T,2}}{L_T}$ $b = \frac{h_2}{h^*}$

- **scales as $1/n_c^2$**

FODO + DS	0.000313
FODO $\alpha_{c,\,1f}$	0.000371
FODO $\alpha_{c,\,2f}$	0.000368 (injection) 0.000370 (extraction)

Very similar results

Evolution of α_c

- With current optics: quads are too long (≈ 7m for a 24m long cell)

Different options:

- Increase L_c and decrease n_c → if $L_c \times 2$: $\alpha_c \times 4$
- Other optics
- Combined magnets

➢ Changes that will impact α_c

Synchrotron tune, energy acceptance and bucket area

- Synchrotron tune: $Q_s = \sqrt{-\frac{h\boldsymbol{\eta}}{2\pi\beta_s^2 E_s} eV\cos(\phi_s)}$
- Energy acceptance : $\Delta E \propto \sqrt{\frac{E_s e\, V\, \beta_s^2}{\pi h\, \boldsymbol{\eta}}}$
- Bucket area: $A_0 = \frac{16\tau\beta_s}{2\,\pi h}\sqrt{\frac{E_s\, e\, V}{2\pi\, h\, \boldsymbol{\eta}}}$

For Φ_s=0 or Φ_s=π

→ Smaller α_c, smaller Q_s : impact on need of distributed RF

- $\eta = \alpha_c - \frac{1}{\gamma^2}$ slip factor
- E_s particle energy
- eV max. energy gain per transit
- h harmonic number
- τ transit time

Collective effects, RF: what are the requirements (min, max) for α_c ?