Recent highlights in precision calculations and related challenges

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The role of precision theory

After the discovery of the Higgs boson in 2012 no evidence of new phenomena has been reported yet

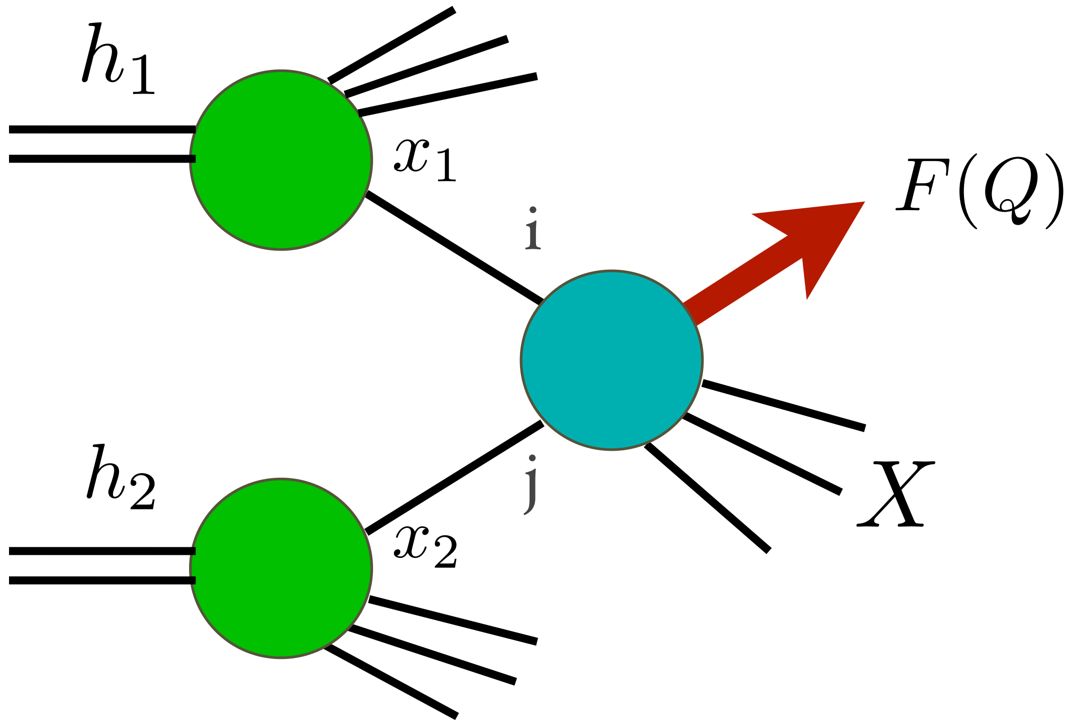
The LHC has accumulated only about 5-10% of the expected data, and surprises are still possible but it is difficult to expect a striking signal in the coming years

The most likely scenario is the one in which one or more consistent (small) deviations with respect the SM appear

→ **The more accurate theory predictions are, the sooner can we be sensitive to these small deviations**

Precision theory increases the discovery reach of the LHC and anticipates possible discoveries

Our starting point



High-p_T interactions are characterised by the presence of a hard scale Q (invariant mass of a lepton pair, high-p_T jet, heavy-quark mass…)

→ **Can be controlled through the factorisation theorem**

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2)\, \hat{\sigma}_{ij}(x_1 P_1, x_2 P_2, Q^2, \alpha_S(\mu_R); \mu_F^2, \mu_R^2) + \mathcal{O}\left(\left(\frac{\Lambda_{\rm QCD}}{Q}\right)^p\right)$$

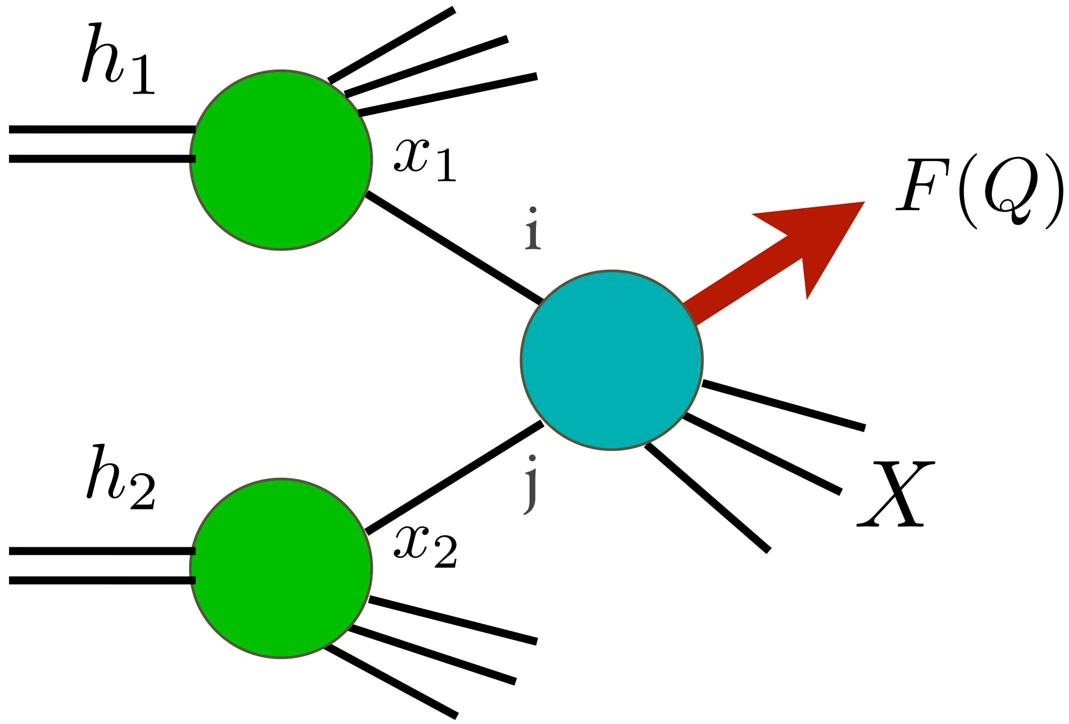
Parton distributions: universal but not perturbatively computable

Hard partonic cross section: process dependent but computable in perturbation theory

Power-suppressed contributions

The factorisation picture is systematically improvable (until the power-suppressed contributions become quantitative relevant…)

Our starting point



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Fixed order predictions

Fixed order computations constitute the backbone of theory predictions at high-energy colliders

- Conceptually clean: systematic expansion in QCD and EW couplings (but technically more and more challenging as order increases)
- Compared to resummed computations, necessary when multiple scales are present, less prone to ambiguities
- Completely solved at NLO (both QCD and EW)

Openloops, Gosam, Madloop, NLOX, Recola….

- Still, difficult to assess theory uncertainties

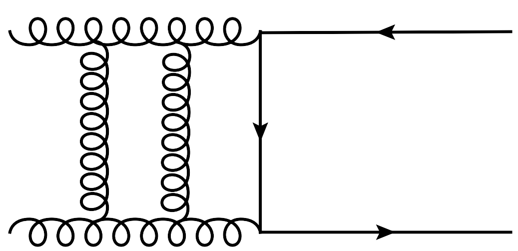
see e.g. recent public discussion at https://indico.cern.ch/event/1368033

- Since $\alpha_S \gg \alpha$ the QCD effects are often (but not always !) the most important

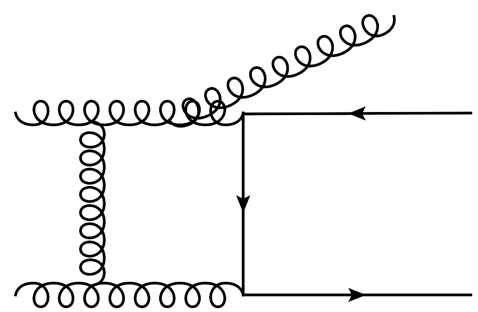
How do we do these calculations ?

In short: we integrate matrix elements over phase space but…
….at each order we have more loops and more legs and

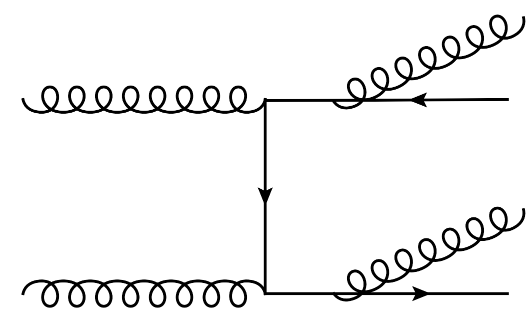
- amplitudes develop infrared (IR) singularities
- we need to be fully differential to adapt to realistic experimental cuts



virtual



real-virtual



real

Amplitudes:

At tree-level and one-loop they can be computed automatically

From two-loop on no general solution exists and complexity grows in #loops and #scales

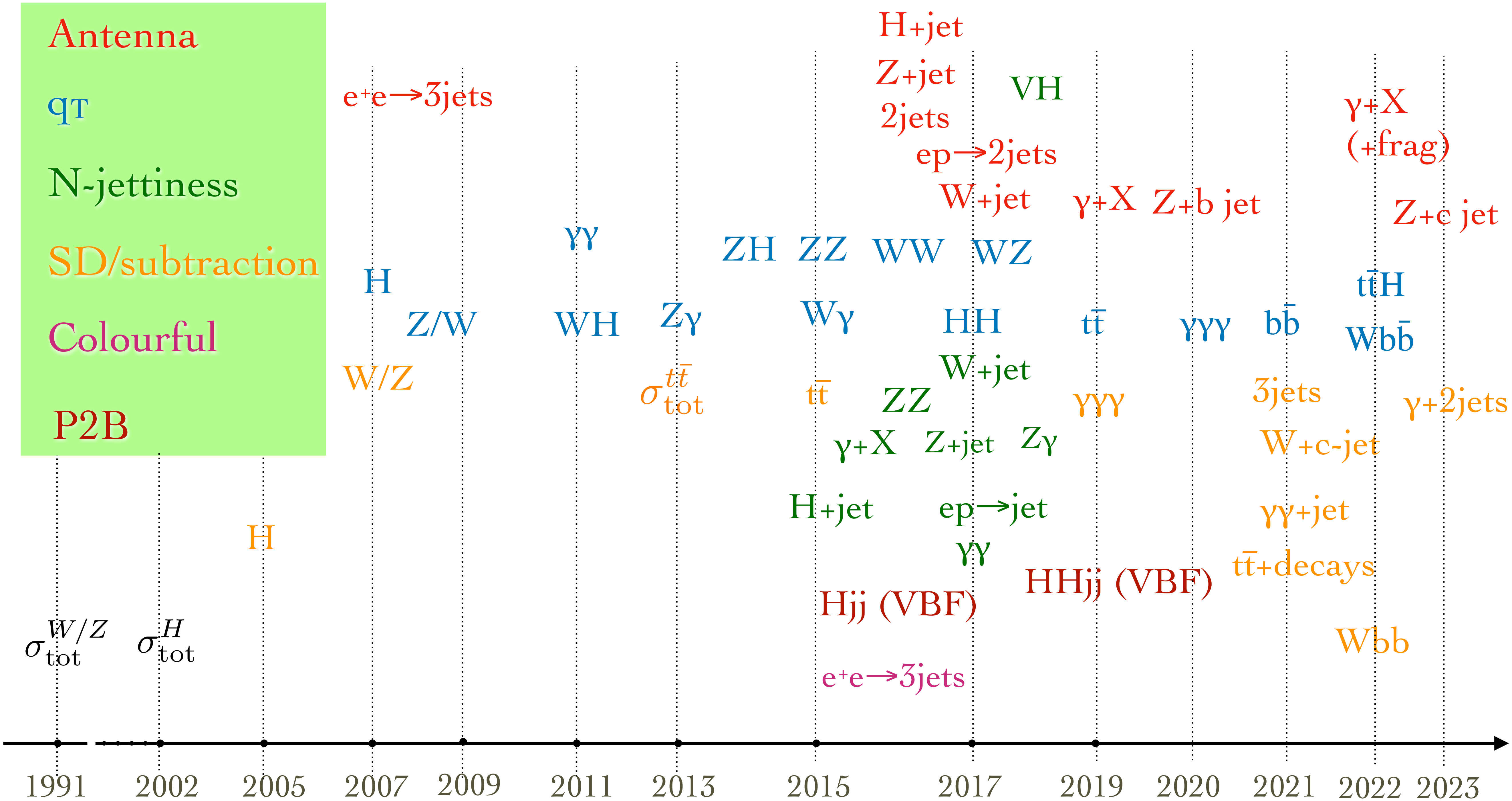
Subtraction/slicing schemes:

Organise and cancel IR singularities

Efficiency becomes crucial as multiplicity increases

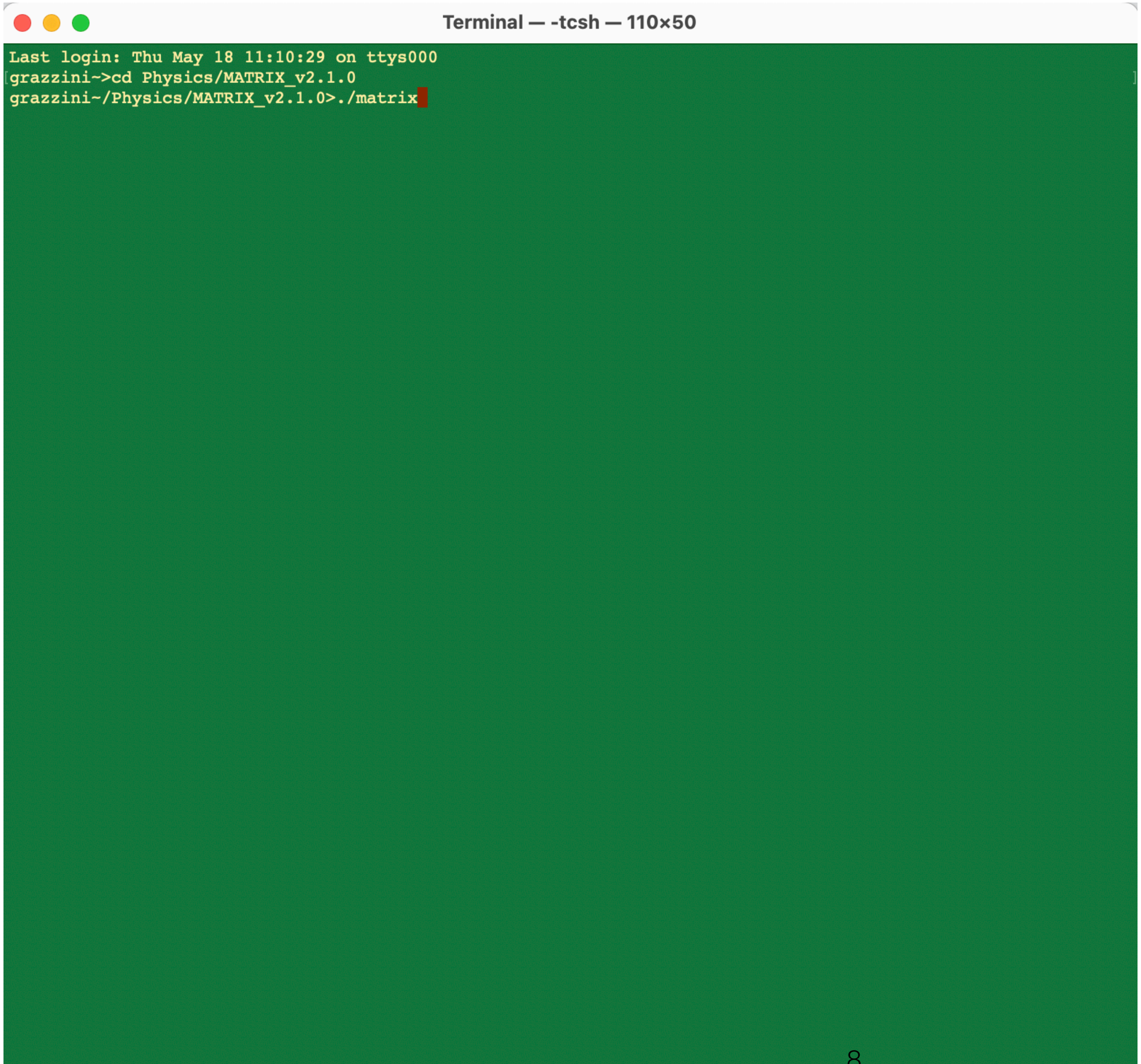
Cross validation between independent calculations essential

NNLO QCD progress



NNLO results lead to much better description of the data

MATRIXv2.1

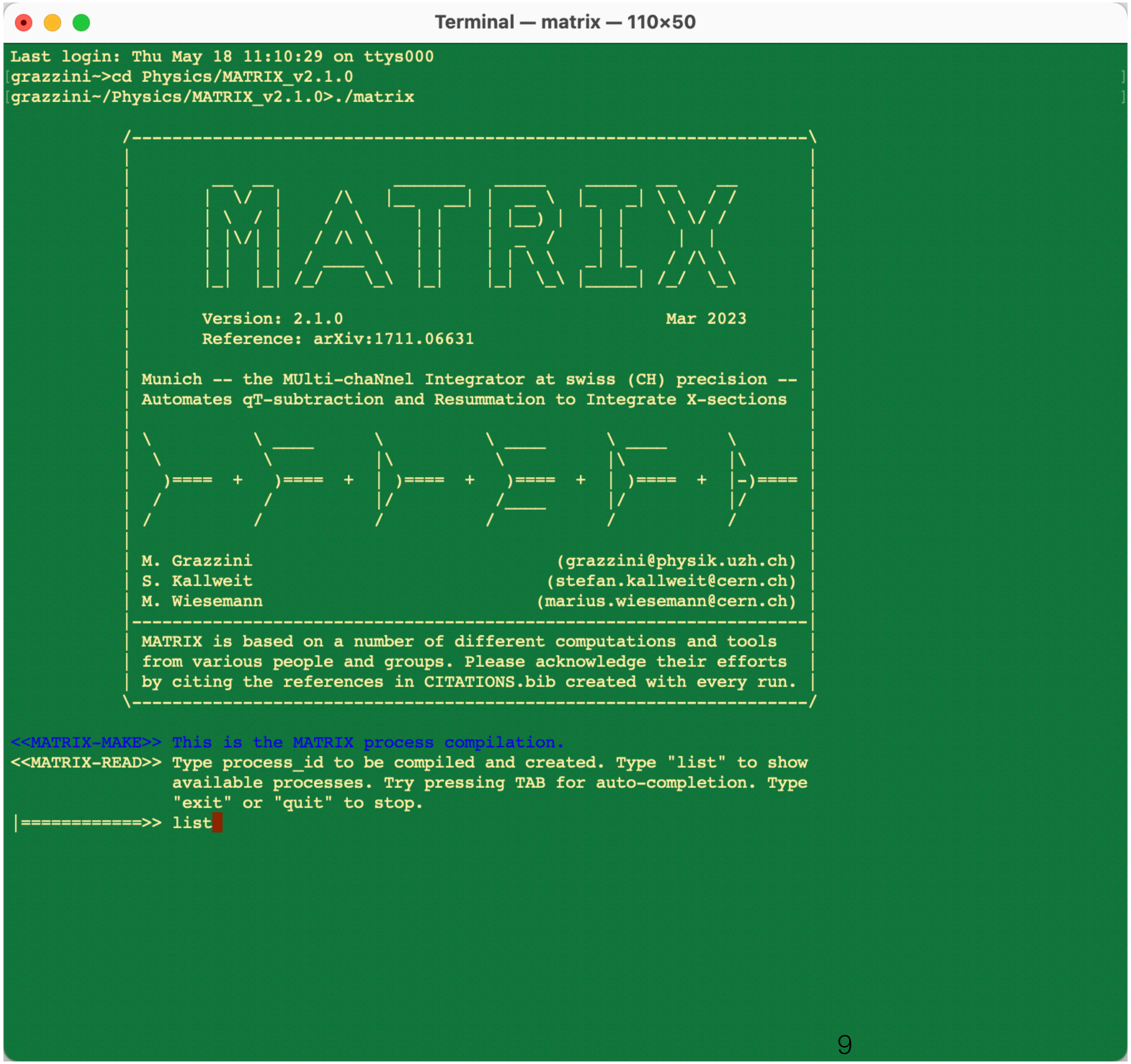


MATRIX allows the user to evaluate fully differential cross sections for a wide class of processes at hadron colliders in NNLO QCD, NLO EW and NLO QCD for the loop-induced contribution

Publicly available here

http://matrix.hepforge.org

MATRIXv2.1



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Terminal — matrix — 110×50

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 |                                                                  |
 | M. Grazzini                              (grazzini@physik.uzh.ch) |
 | S. Kallweit                             (stefan.kallweit@cern.ch) |
 | M. Wiesemann                           (marius.wiesemann@cern.ch) |
 |------------------------------------------------------------------|
 | MATRIX is based on a number of different computations and tools  |
 | from various people and groups. Please acknowledge their efforts |
 | by citing the references in CITATIONS.bib created with every run. |
 \------------------------------------------------------------------/

<<MATRIX-MAKE>> This is the MATRIX process compilation.
<<MATRIX-READ>> Type process_id to be compiled and created. Type "list" to show
                available processes. Try pressing TAB for auto-completion. Type
                "exit" or "quit" to stop.
|============>> list
--------------------------------------------------------------------------------------------------------
process_id    ||   process                            ||   description
--------------------------------------------------------------------------------------------------------
pph21         >>   p p --> H                          >>   on-shell Higgs production (NNLO)
ppz01         >>   p p --> Z                          >>   on-shell Z production (NNLO,NLO EW)
ppw01         >>   p p --> W^-                        >>   on-shell W- production with CKM (NNLO)
ppwx01        >>   p p --> W^+                        >>   on-shell W+ production with CKM (NNLO)
ppeex02       >>   p p --> e^- e^+                    >>   Z production with decay (NNLO,NLO EW)
ppnenex02     >>   p p --> v_e^- v_e^+                >>   Z production with decay (NNLO,NLO EW)
ppenex02      >>   p p --> e^- v_e^+                  >>   W- production with decay and CKM (NNLO,NLO EW)
ppexne02      >>   p p --> e^+ v_e^-                  >>   W+ production with decay and CKM (NNLO,NLO EW)
ppaa02        >>   p p --> gamma gamma                >>   gamma gamma production (NNLO)
ppeexa03      >>   p p --> e^- e^+ gamma              >>   Z gamma production with decay (NNLO)
ppnenexa03    >>   p p --> v_e^- v_e^+ gamma          >>   Z gamma production with decay (NNLO)
ppenexa03     >>   p p --> e^- v_e^+ gamma            >>   W- gamma production with decay (NNLO)
ppexnea03     >>   p p --> e^+ v_e^- gamma            >>   W+ gamma production with decay (NNLO)
ppzz02        >>   p p --> Z Z                        >>   on-shell ZZ production (NNLO)
ppwxw02       >>   p p --> W^+ W^-                    >>   on-shell WW production (NNLO)
ppemexmx04    >>   p p --> e^- mu^- e^+ mu^+          >>   ZZ production with decay (NNLO,NLO gg,NLO EW)
ppeeexex04    >>   p p --> e^- e^- e^+ e^+            >>   ZZ production with decay (NNLO,NLO gg,NLO EW)
ppeexnmnmx04  >>   p p --> e^- e^+ v_mu^- v_mu^+      >>   ZZ production with decay (NNLO,NLO gg,NLO EW)
ppemxnmnex04  >>   p p --> e^- mu^+ v_mu^- v_e^+      >>   WW production with decay (NNLO,NLO gg,NLO EW)
ppeexnenex04  >>   p p --> e^- e^+ v_e^- v_e^+        >>   ZZ/WW production with decay (NNLO,NLO gg,NLO EW)
ppemexnmx04   >>   p p --> e^- mu^- e^+ v_mu^+        >>   W-Z production with decay (NNLO,NLO EW)
ppeeexnex04   >>   p p --> e^- e^- e^+ v_e^+          >>   W-Z production with decay (NNLO,NLO EW)
ppeexmxnm04   >>   p p --> e^- e^+ mu^+ v_mu^-        >>   W+Z production with decay (NNLO,NLO EW)
ppeexexne04   >>   p p --> e^- e^+ e^+ v_e^-          >>   W+Z production with decay (NNLO,NLO EW)
ppttx20       >>   p p --> top anti-top               >>   on-shell top-pair production (NNLO)
ppaaa03       >>   p p --> gamma gamma gamma          >>   gamma gamma gamma production (NNLO)
|============>>
```

MATRIX allows the user to evaluate fully differential cross sections for a wide class of processes at hadron colliders in NNLO QCD, NLO EW and NLO QCD for the loop-induced contribution

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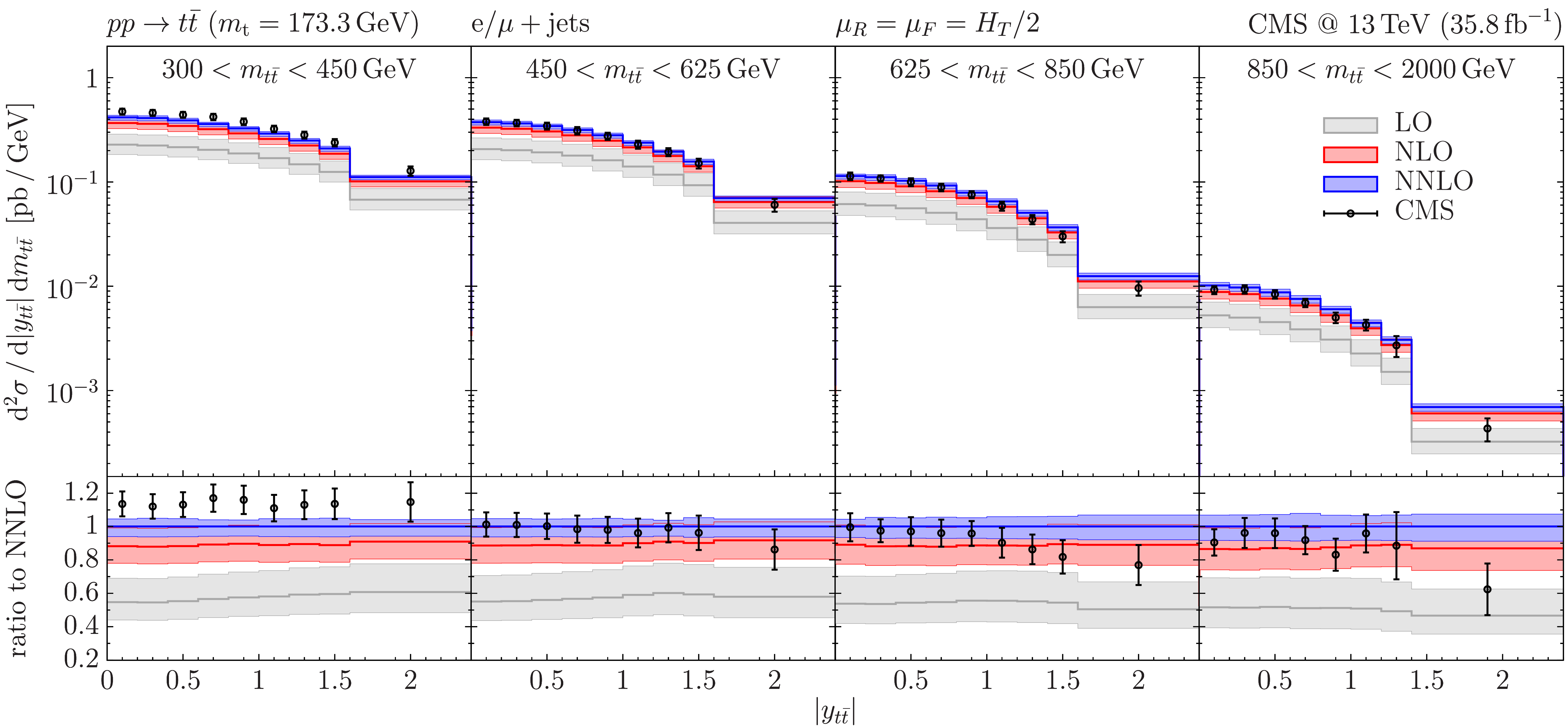
Version 2.1 includes top-pair production

MATRIXv2.1

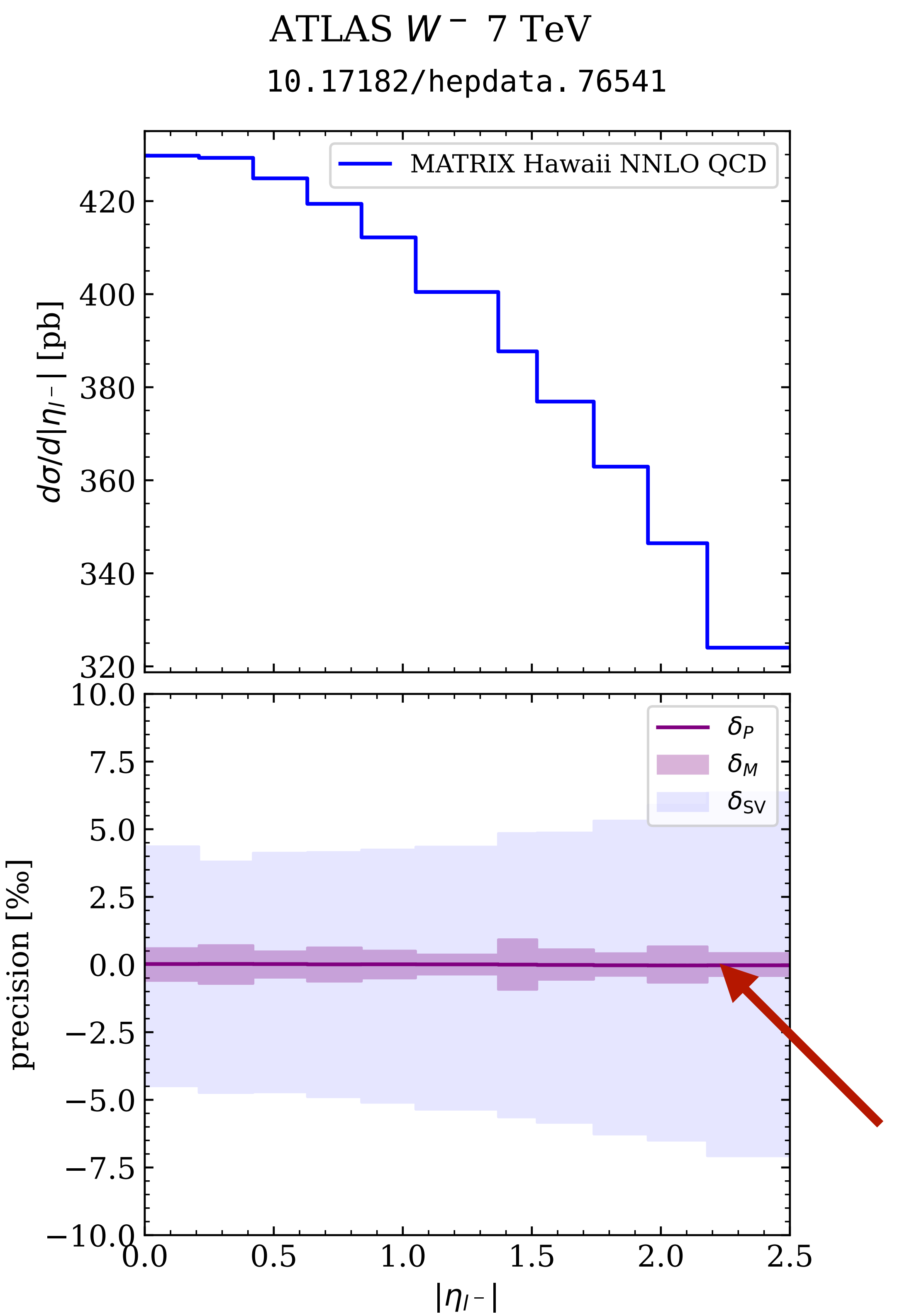
Multidifferential distributions



NNLO corrections significantly improve the agreement with the data

NEW: PDF uncertainties: MATRIX+PineAPPL interface

Devoto, Jezo, Kallweit, Schwan (in preparation)



Will allow PDF fits without relying on K-factors, but directly using NNLO predictions for all the processes in MATRIX

Plots: courtesy S. Devoto

$2 \to 2$: maturity

Benchmark $2 \to 2$ processes VV, $Q\bar{Q}$ ($Q = t, b$), V+jet available since quite some time

More recently:

- Flavoured jets: Z+b, Z+c, W+c

  Gauld et al (2020,2023)
  Czakon et al. (2023)
  Gehrmann et al (2023)

- Production and decay $pp \to WH(H \to b\bar{b})$

  Behring et al (2020)

- Mass effects in H+jet and $gg \to ZZ, ZH$ at NLO

  Kerner, Jones, Luisoni (2018)
  Del Duca et al (2023)
  Degrassi et al. (2021-24), Kerner et al (2022-24)….

- Inclusion of fragmentation

  - identified hadrons — Czakon et al (2021,2022)

  - photons — Gehrmann et al (2022)

- Mixed QCD-EW corrections (more later)

  Bonciani, Buonocore, Kallweit Rana,Tramontano, Vicini, MG (2021)
  Buccioni et al (2022)

$2 \to 3$: the frontier

5 massless partons

- $pp \to \gamma\gamma\gamma$ — Czakon et al (2019); Kallweit, Sotnikov, Wiesmann (2020)
- $pp \to \gamma\gamma j$ — Czakon et al (2021)
- $pp \to jjj$ — Czakon et al (2021)
- $pp \to \gamma jj$ — Badger et al (2023)

multiscale processes

- $pp \to Wb\bar{b}$ (first massless then small mass b) — Poncelet et al. (2022); Buonocore et al (2023)
- $pp \to t\bar{t}H$ — Catani, Devoto, Kallweit, Mazzitelli, Savoini, MG (2022)
- $pp \to t\bar{t}W$ — Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini, MG (2022)
- $pp \to b\bar{b}Z$ (through NNLOPS) — Sotnikov et al (2024)
- $pp \to W\gamma\gamma$ (2-loop amplitude) — Badger et al (2024)

$2 \to 3$: the frontier

Calculations of the two-loop virtual corrections with one or more masses typically performed in approximated form

Often in the leading-colour (LC) approximation ($N_c \gg 1$)

Other approximations exploit particular kinematical limits (e.g. soft or collinear approximations, small mass limits…)

One maybe obvious technical comment: whatever approximation is used, the singular terms have to be included exactly, in order to achieve a IR finite result

Quality of approximations may depend on the definition of the finite remainder

Differences between LC and full color can be relatively large

Sotnikov et al (2023)

First exact $2 \to 3$ appeared for $pp \to \gamma jj$ (here subleading colour terms small)

Badger et al (2023)

➡ In general quality of approximations need to be checked case by case

ttW

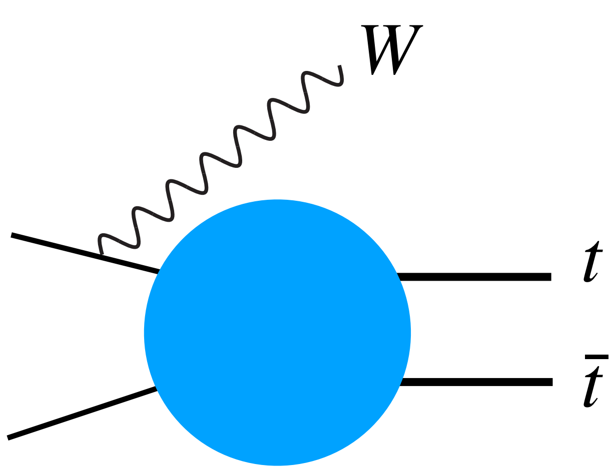
Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini, MG (2023)

Among the ttV signatures, ttW is special because it involves both EW and top sectors

It is at the same time a signal and a background to ttH and tttt and new physics searches

Since the top quark quickly decays into a W and a b jet, the signature is characterised by 3 W bosons

→ It provides an irreducible source of same-sign dilepton pairs relevant for many BSM searches



It is special compared to other $ttF\,(F = H, Z, \gamma)$ signatures because the W can only be emitted by the initial-state light quarks (no gg channel at LO)

ttW rate consistently higher than SM predictions

Here we use two different approximations of the missing two-loop amplitude

1) Use soft approximation for W emission with momentum k and polarisation $\varepsilon(k)$ to express ttW amplitude in terms of the $q\bar{q} \to t\bar{t}$ amplitude

$$\mathcal{M}(\{p_i\}, k, \mu_R; \epsilon) \simeq \frac{g}{\sqrt{2}} \left(\frac{p_2 \cdot \varepsilon(k)}{p_2 \cdot k} - \frac{p_1 \cdot \varepsilon(k)}{p_1 \cdot k} \right) \mathcal{M}_L(\{p_i\}, \mu_R; \epsilon)$$

$q_L\bar{q}_R \to t\bar{t}$ virtual amplitude

Bärnreuther et al. (2013)
Mastrolia et al (2022)

2) Start from massless W+4 parton amplitudes

Abreu et al. (2021)

Use a "massification" procedure to obtain the leading terms in a $m_Q/Q \ll 1$ expansion

Penin (2006)
Moch, Mitov (2007)
Becher, Melnikov (2007)

$$\mathcal{M}(\{p_i\}, k; \mu_R; \epsilon) \simeq Z_{[q]}^{(m_Q|0)}(\alpha_S(\mu), m_Q/\mu, \epsilon) \mathcal{M}^{(m_Q=0)}(\{p_i\}, k; \mu_R; \epsilon)$$

Universal perturbatively computable factor

Successfully applied to the NNLO computation of Wbb

Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini (2023)

The computation

The starting point is the q_T subtraction formula

$$d\sigma = \mathcal{H} \otimes d\sigma_{\rm LO} + \left[d\sigma_{\rm R} - d\sigma_{\rm CT}\right]$$

All the ingredients in this formula for $t\bar{t}H$ are now available and implemented in MATRIX except the two-loop virtual amplitudes entering $\mathcal{H}$

We define

$$\mathcal{H} = H\delta(1-z_1)\delta(1-z_2) + \delta\mathcal{H} \qquad H^{(n)} = \frac{2\mathrm{Re}\left(\mathcal{M}^{(n)}_{\rm fin}\mathcal{M}^{(0)*}\right)}{|\mathcal{M}^{(0)}|^2}$$

with

$$H = 1 + \frac{\alpha_S(\mu_R)}{2\pi}H^{(1)} + \left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2 H^{(2)} + \ldots. \qquad |\mathcal{M}_{\rm fin}(\mu_{IR})\rangle = \mathbf{Z}^{-1}(\mu_{IR})|\mathcal{M}\rangle$$

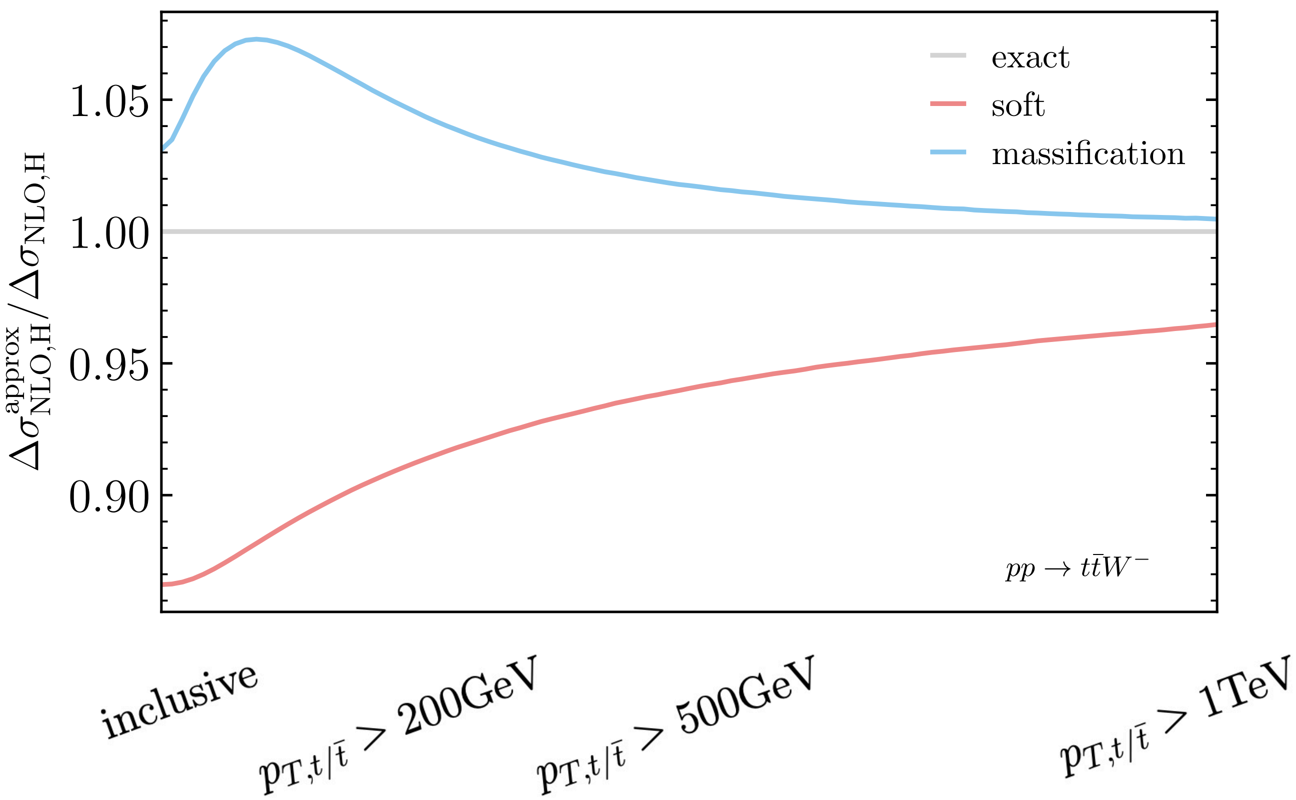
IR subtraction

For $n = 2$ this definition allows us to single out the only missing ingredient in the NNLO calculation, that is, the coefficient $H^{(2)}$

Note that all the remaining terms are computed exactly (including $|\mathcal{M}^{(1)}_{\rm fin}|^2$)

ttW

Buonocore, Devoto, Kallweit,
Mazzitelli,Rottoli, Savoini, MG (2023)



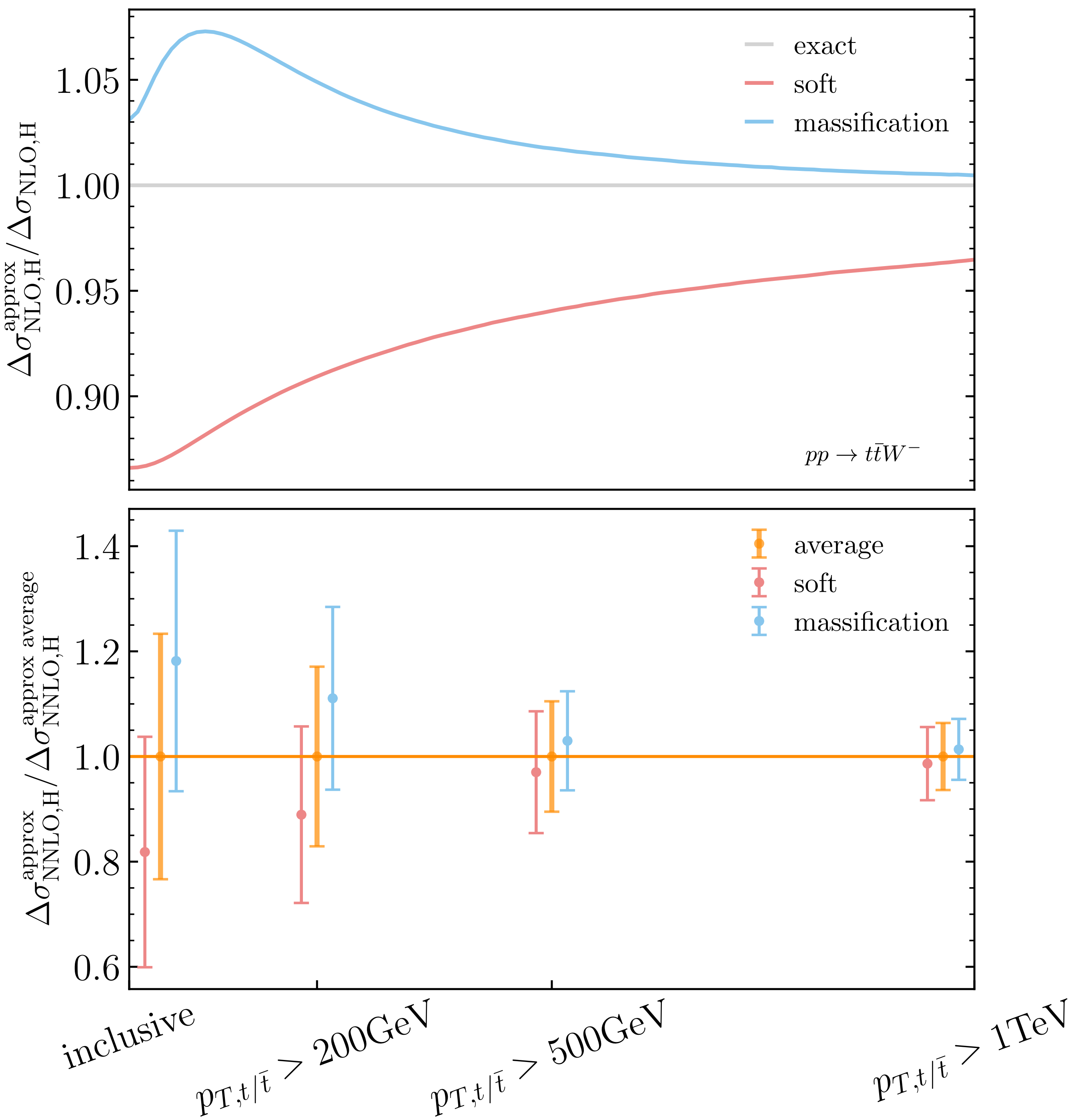
Both approximations provide a good estimate of the exact one-loop contribution

Soft approximation undershoots the exact results while massification tends to overshoot it

Clear asymptotic behaviour towards exact result for high p_T of the top quarks where both approximations are expected to work

ttW

Buonocore, Devoto, Kallweit,
Mazzitelli,Rottoli, Savoini, MG (2023)



The pattern is preserved at NNLO: massified result systematically higher than soft approximation

We define the uncertainty of each approximation as the maximum between what we obtain varying the subtraction scale $1/2 \leq \mu_{\rm IR}/Q \leq 2$ and twice the NLO deviation

➡ Our best prediction obtained as average of the two with linear combination of uncertainties

Final uncertainty on two-loop contribution about 25% and similar to what obtained in recent $2 \to 3$ calculations in leading color approximation

Abreu et al (2023)

Impact of two-loop virtual contribution: 6-7% of NNLO cross section

ttW

	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
$\mathrm{LO_{QCD}}$	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.2^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
$\mathrm{NLO_{QCD}}$	$416.9^{+12.5\%}_{-11.4\%}$	$205.1^{+13.2\%}_{-11.7\%}$	$622.0^{+12.7\%}_{-11.5\%}$	$2.033^{+3.0\%}_{-3.4\%}$
$\mathrm{NNLO_{QCD}}$	$475.2^{+4.8\%}_{-6.4\%} \pm 1.9\%$	$235.5^{+5.1\%}_{-6.6\%} \pm 1.9\%$	$710.7^{+4.9\%}_{-6.5\%} \pm 1.9\%$	$2.018^{+1.6\%}_{-1.2\%}$
$\mathrm{NNLO_{QCD}+NLO_{EW}}$	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\%$	$745.3^{+6.7\%}_{-6.7\%} \pm 1.8\%$	$2.007^{+2.1\%}_{-2.1\%}$
ATLAS [11]	$585^{+6.0\%+8.0\%}_{-5.8\%-7.5\%}$	$301^{+9.3\%+11.6\%}_{-9.0\%-10.3\%}$	$890^{+5.6\%+7.9\%}_{-5.6\%-7.9\%}$	$1.95^{+10.8\%+8.2\%}_{-9.2\%\ -6.7\%}$
CMS [10]	$553^{+5.4\%+5.4\%}_{-5.4\%-5.4\%}$	$343^{+7.6\%+7.3\%}_{-7.6\%-7.3\%}$	$868^{+4.6\%+5.9\%}_{-4.6\%-5.9\%}$	$1.61^{+9.3\%+4.3\%}_{-9.3\%-3.1\%}$

Conservative estimate of uncertainty from missing exact two-loop amplitudes

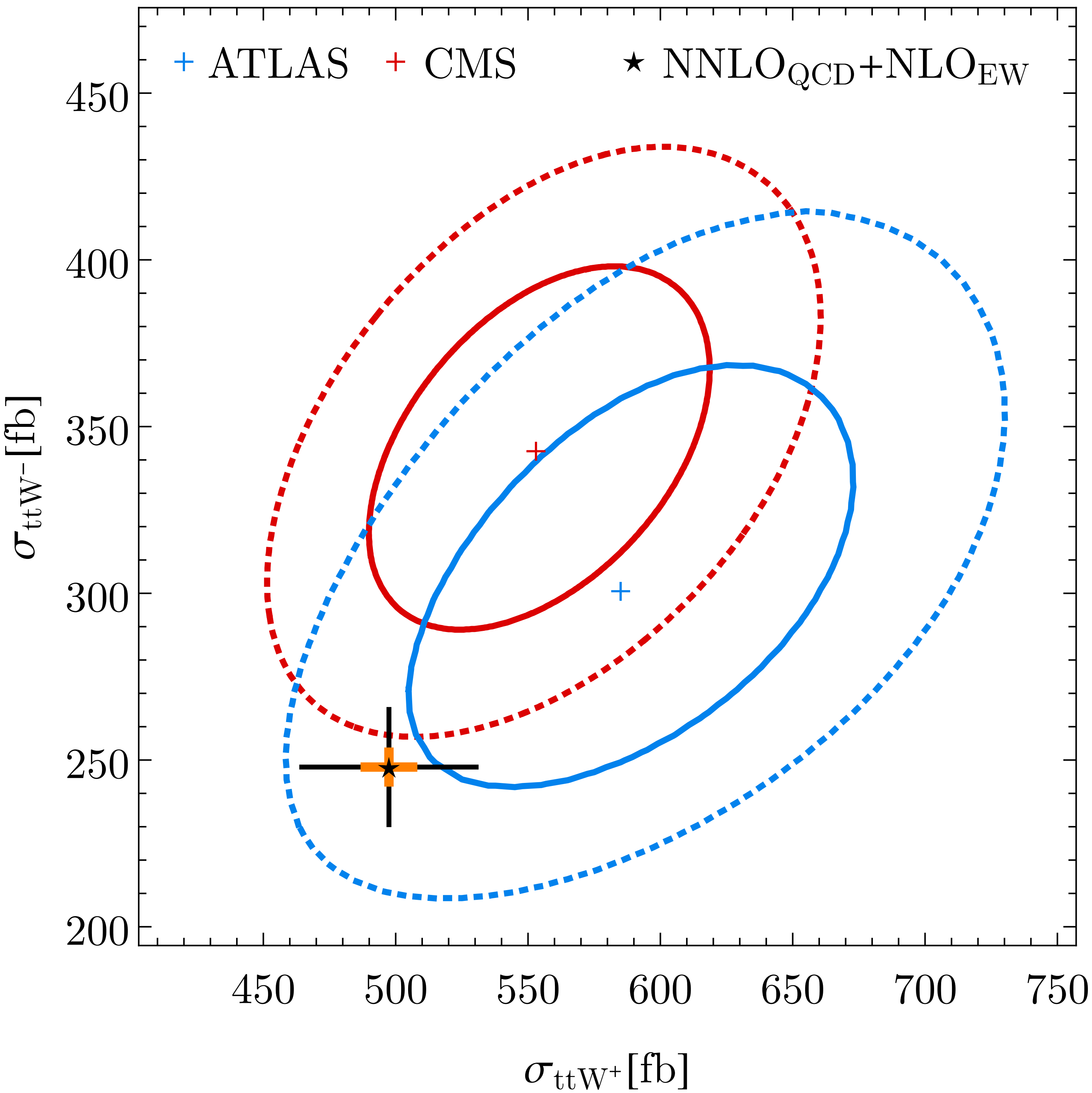
Large NLO QCD corrections (+50%)

Moderate NNLO corrections (+14-15%)

All subdominant LO and NLO contributions at $\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha_S^2\alpha^2)$, $\mathcal{O}(\alpha_S\alpha^3)$, $\mathcal{O}(\alpha^4)$ consistently included and denoted as NLO EW: effect is +5%

$\sigma(t\bar{t}W^+)/\sigma(t\bar{t}W^-)$ only slightly decreases increasing the perturbative order

ttW



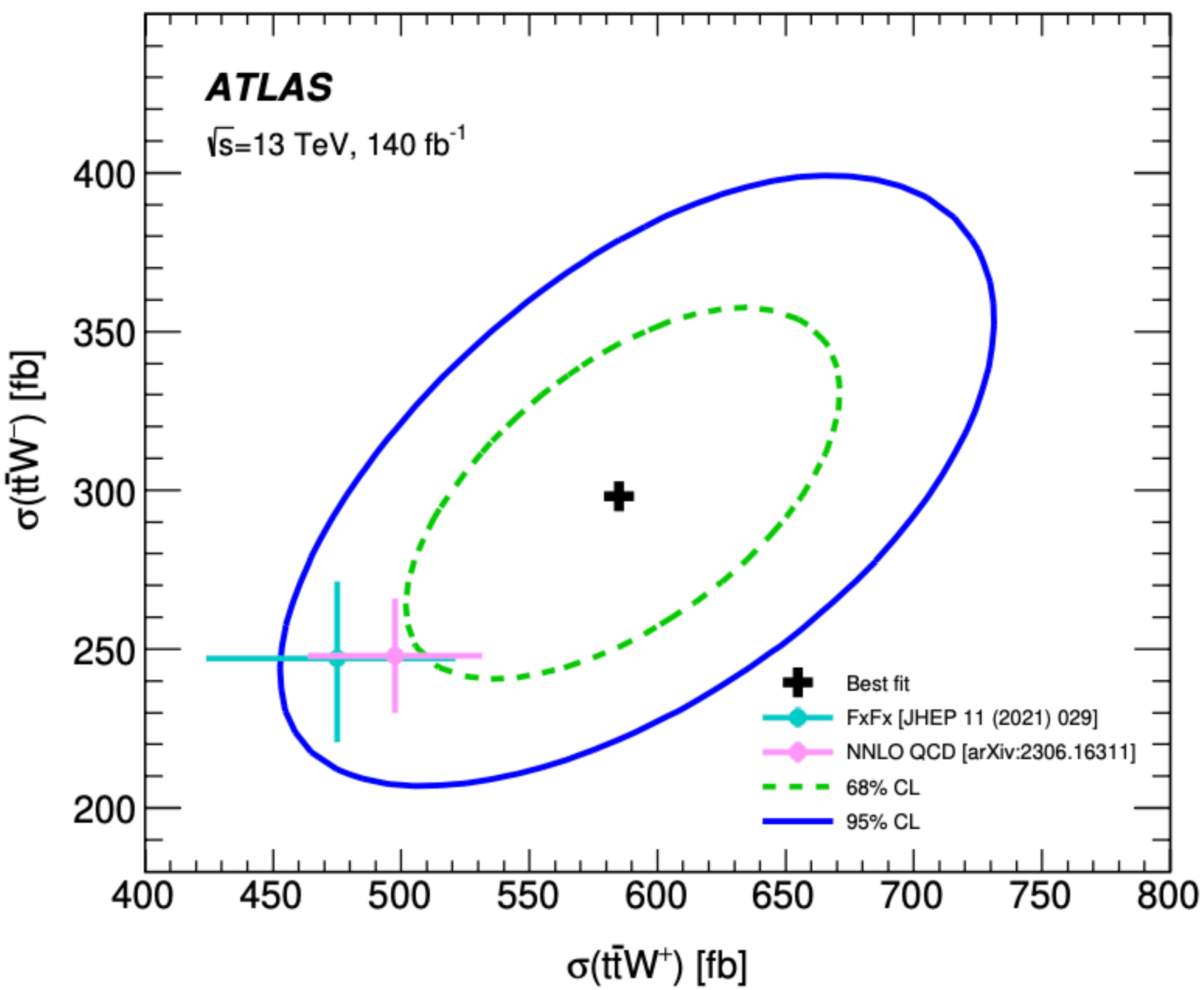
The comparison with the ATLAS and CMS results shows that discrepancy remains at the 1-2σ level

Inclusion of NNLO corrections significantly reduces perturbative uncertainties

Our result is fully consistent with FxFx prediction but with smaller uncertainties

$$\sigma_{t\bar{t}W}^{\rm FxFx} = 722.4^{+9.7\%}_{-10.8\%}\ {\rm fb}$$

ttW



Similar situation with the new ATLAS measurement

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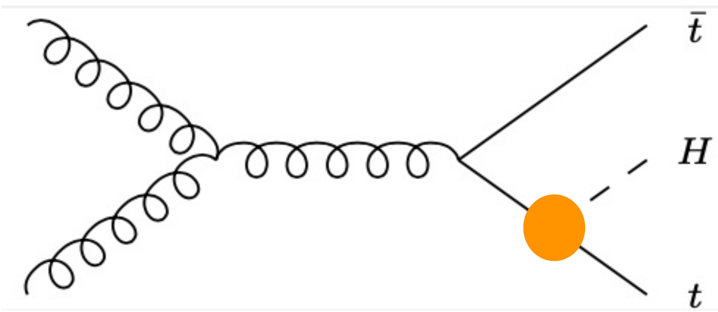
$$\sigma_{t\bar{t}W}^{\mathrm{FxFx}} = 722.4^{+9.7\%}_{-10.8\%}\ \mathrm{fb}$$

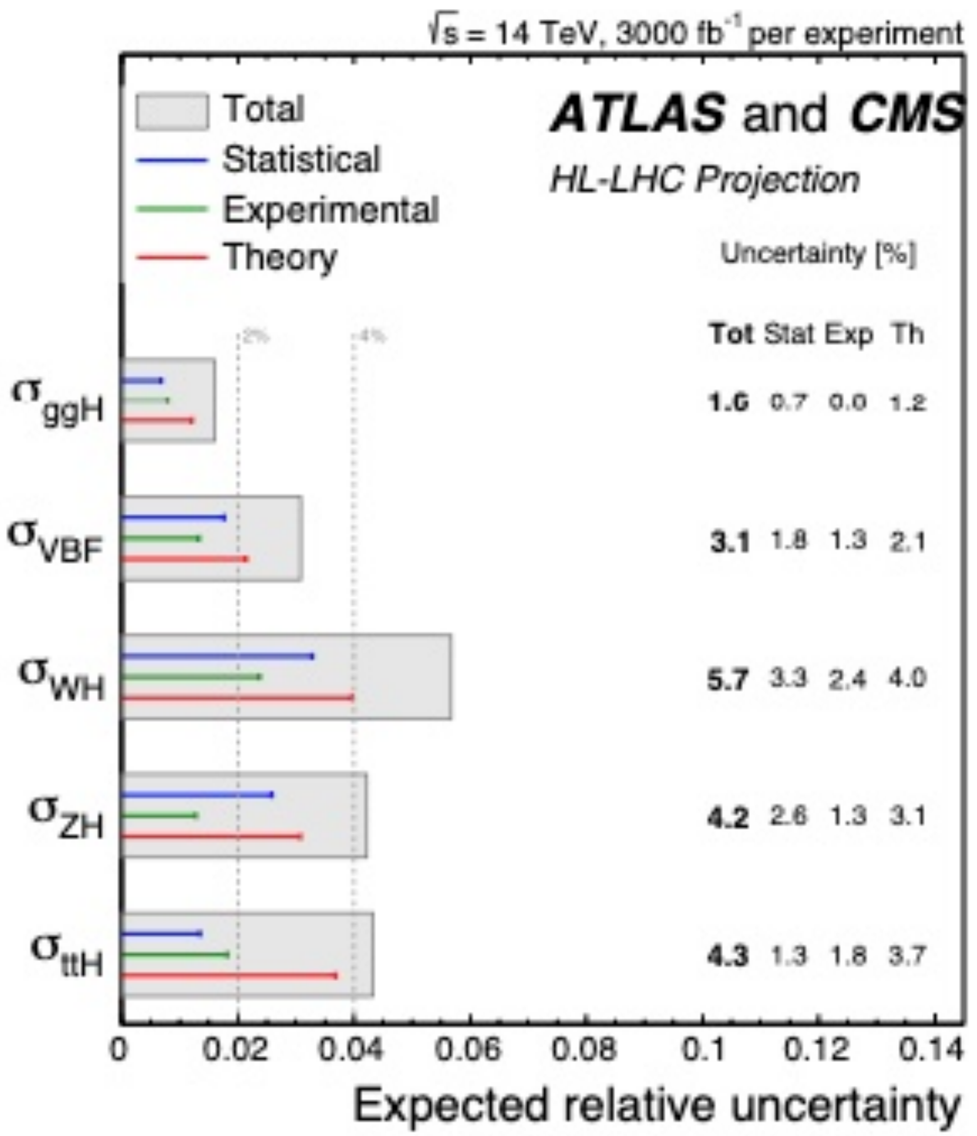
ttH

Catani, Devoto, Kallweit, Mazzitelli,
Savoini, MG (2022)



The associated production of the Higgs boson with a top-quark pair is a crucial process at the LHC

It allows a direct extraction of the top Yukawa



Experimental uncertainties are now at the $\mathcal{O}(20\%)$ level but expected to go down to the 2% level at the end of the HL-LHC

Predictions based on NLO QCD+EW (+ resummations) affected by $\mathcal{O}(10\%)$ uncertainty

Missing ingredients for NNLO are the two-loop $gg \to t\bar{t}H$ and $q\bar{q} \to t\bar{t}H$ amplitudes

Recent progress:

- one-loop at $\mathcal{O}(\epsilon^2)$ — Tancredi et al (2023)
- some master integrals — Reina et al (2023)
- boosted limit — Wang, Xia, Yang, Ye (2024)
- $q\bar{q}$ n_F part — Heinrich et al (2024)

ttH

The idea: use soft approximation for the missing two-loop amplitude

Tree-level soft-Higgs current

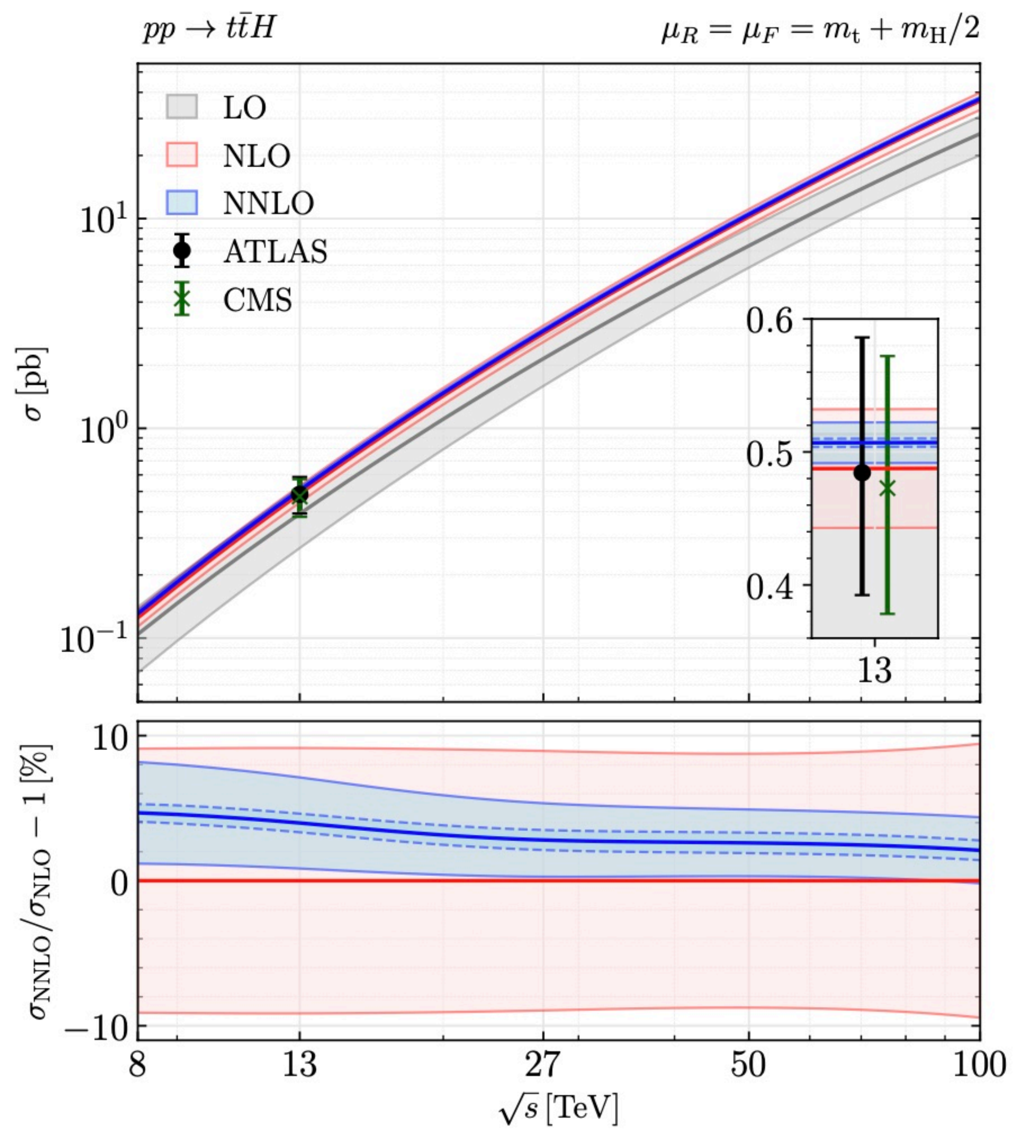
$$\mathcal{M}(\{p_i\},k) \simeq F(\alpha_S(\mu_R); m/\mu_R)\, J(k) \mathcal{M}(\{p_i\})$$

Soft limit of the scalar heavy-quark form factor

Bernreuther et al (2005); Blümlein et al (2017)
Fael, Lange, Schönwald, Steinhauser (2022)

Approximated term has very small impact

	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\mathrm{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\mathrm{NLO,H}}\vert_{\mathrm{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\mathrm{NNLO,H}}\vert_{\mathrm{soft}}$	−2.980(3)	2.622(0)	−239.4(4)	65.45(1)



Estimate uncertainty by starting from relative deviation at NLO and multiplying by a factor of 3

ttH

The idea: use soft approximation for the missing two-loop amplitude

Tree-level soft-Higgs current

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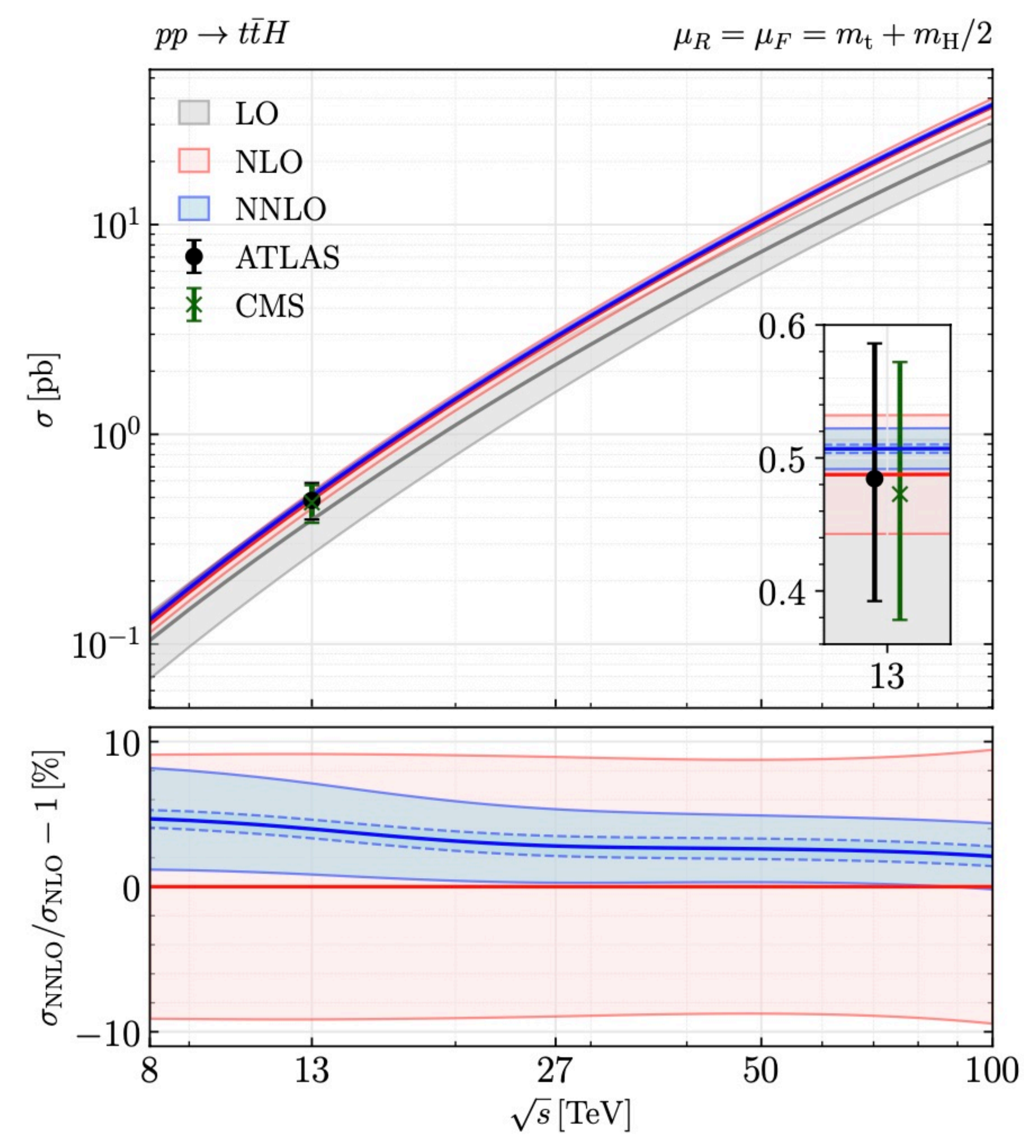
Bernreuther et al (2005); Blümlein et al (2017)
Fael, Lange, Schönwald, Steinhauser (2022)

Approximated term has very small impact

σ [pb]	$\sqrt{s} = 13$ TeV	$\sqrt{s} = 100$ TeV
σ_{LO}	$0.3910\,^{+31.3\%}_{-22.2\%}$	$25.38\,^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875\,^{+5.6\%}_{-9.1\%}$	$36.43\,^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070\,(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)\,^{+0.1\%}_{-2.2\%}$

NNLO effect is about $+4\,\%$ at 13 TeV and $+2\,\%$ at 100 TeV

Catani, Devoto, Mazzitelli, Kallweit, Savoini, MG (2022)



ttH

Devoto, Kallweit, Mazzitelli, Savoini, MG (to appear)

Recently we have been working to extend our calculation to differential distributions and to consolidate our approximation of the two-loop virtual contribution

→ Combine soft approximation with "massification" as done in ttW calculation

Exploit massless amplitudes from Badger et al (2021)

New result for inclusive cross section in nice agreement with the previous one based only on the soft approximation

More conservative estimate of uncertainty

σ [fb]	$\sqrt{s} = 13.6\,\mathrm{TeV}$
$\sigma_{\mathrm{LO_{QCD}}}$	$423.438\,^{+30.7\%}_{-21.8\%}$
$\sigma_{\mathrm{NLO_{QCD}}}$	$528.665\,^{+5.7\%}_{-9.0\%}$
$\sigma^{\mathrm{SA}}_{\mathrm{NNLO_{QCD}}}$	$548.8\,(3.4)\,^{+0.8\%}_{-3.0\%}$
$\sigma^{\mathrm{best}}_{\mathrm{NNLO_{QCD}}}$	$550.1\,(4.7)\,^{+0.9\%}_{-3.0\%}$

ttH

Devoto, Kallweit, Mazzitelli, Savoini, MG (to appear)

- NLO based error

$$\varepsilon_{\rm soft} = 2 \times \left| \frac{H^{(1)}|_{\rm soft}}{H^{(1)}} - 1 \right| \times \max\left(\left| H^{(2)}|_{\rm soft} \right|, \left| H^{(2)}|_{\rm MA} \right| \right) ,$$

$$\varepsilon_{\rm MA} = 2 \times \max\left(\left| \frac{H^{(1)}|_{\rm MA,fcfc}}{H^{(1)}} - 1 \right|, \left| \frac{H^{(1)}|_{\rm MA,lcfc}}{H^{(1)}} - 1 \right| \right) \times \max\left(\left| H^{(2)}|_{\rm soft} \right|, \left| H^{(2)}|_{\rm MA} \right| \right)$$

- Subtraction-scale based error

$$\varsigma_{\rm soft} = \max\left(\left| H^{(2)}|_{\rm soft}(\widetilde{Q}/2) + (Q/2 \to Q) - H^{(2)}|_{\rm soft} \right|, \left| H^{(2)}|_{\rm soft}(2\widetilde{Q}) + (2Q \to Q) - H^{(2)}|_{\rm soft} \right| \right)$$

$$\varsigma_{\rm MA} = \max\left(\left| H^{(2)}|_{\rm MA}(\widetilde{Q}/2) + (Q/2 \to Q) - H^{(2)}|_{\rm MA} \right|, \left| H^{(2)}|_{\rm MA}(2\widetilde{Q}) + (2Q \to Q) - H^{(2)}|_{\rm MA} \right| \right)$$

- For each approximation and each partonic channel take the maximum between the two

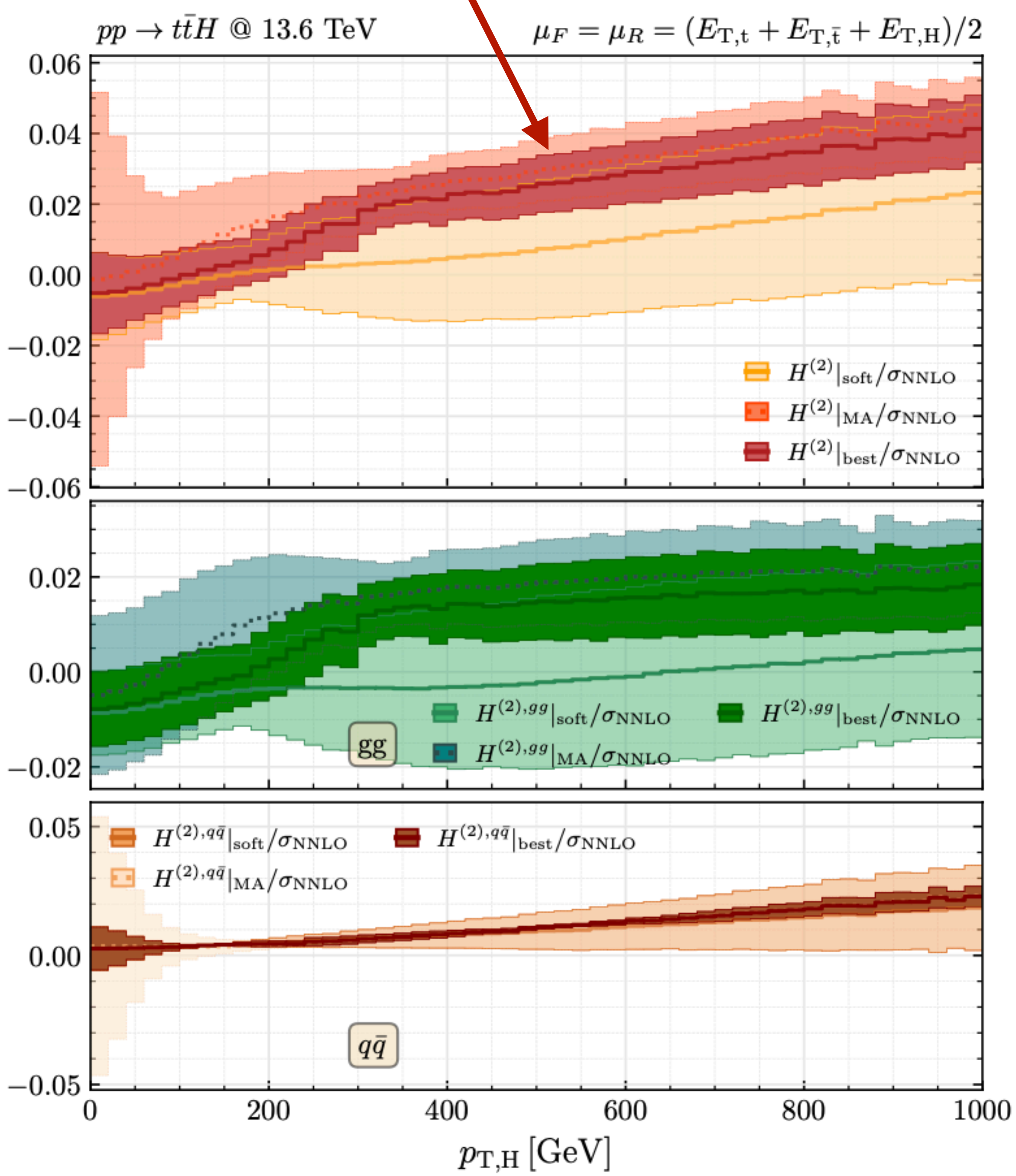
- Eventually separately combine for $q\bar{q}$ and gg partonic channel

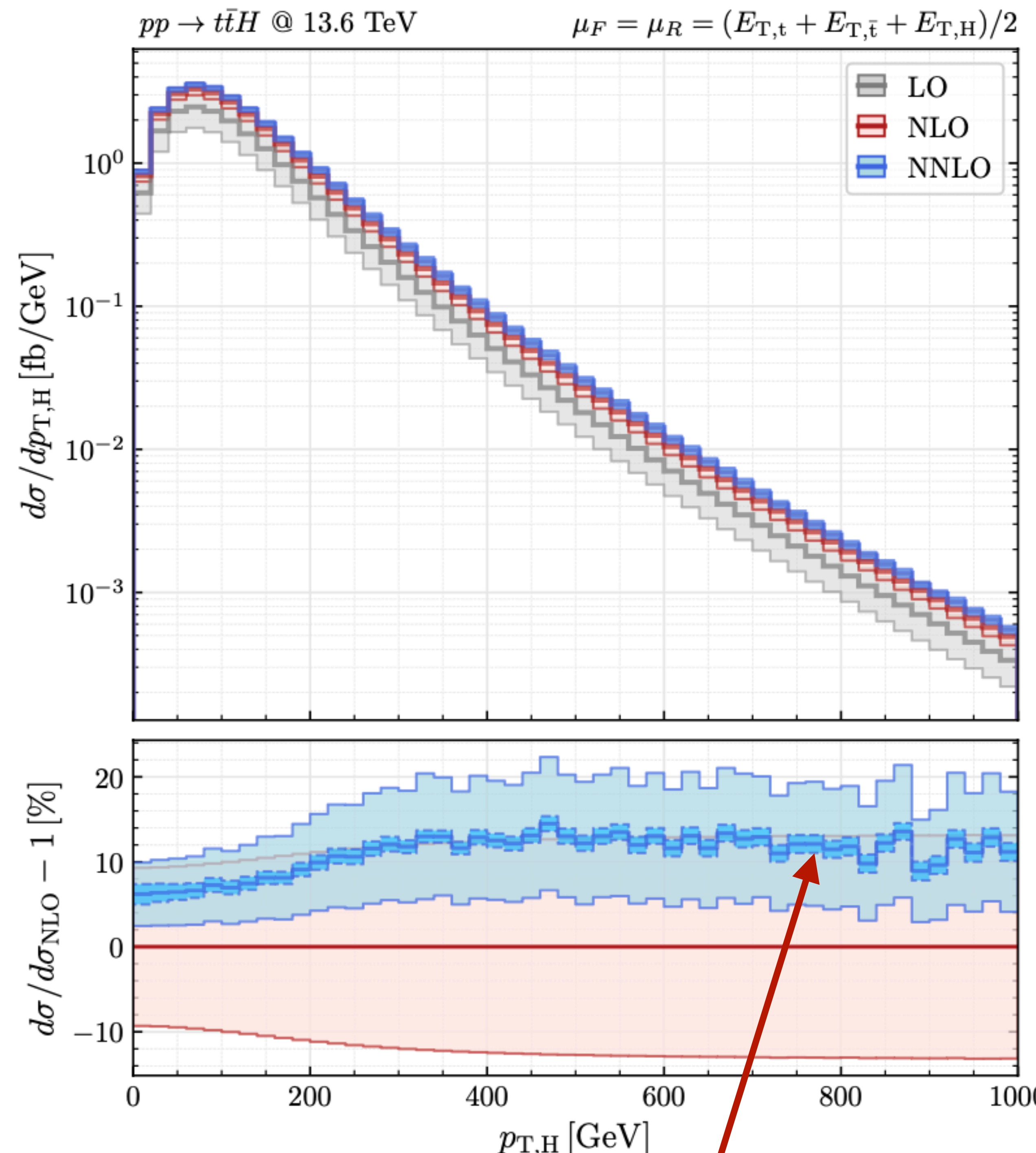
$$H^{(2)}|_{\rm best} = \frac{1}{\omega_{\rm soft} + \omega_{\rm MA}} \left(\omega_{\rm soft} H^{(2)}|_{\rm soft} + \omega_{\rm MA} H^{(2)}|_{\rm MA} \right) \qquad \omega_{\rm soft} = \frac{1}{\xi^2_{\rm soft}} \qquad \omega_{\rm MA} = \frac{1}{\xi^2_{\rm MA}}$$

ttH

Devoto, Kallweit, Mazzitelli, Savoini, MG (to appear)

Best prediction for $H^{(2)}$ nicely interpolates between soft and MA approximations





Final uncertainty significantly smaller than scale uncertainty over the whole range of $p_{T,H}$

NNLO matching

Deployment of NNLO precision in experimental analyses requires extension of available NLO matching schemes to the next order

NNLOPS: MiNLO+reweighting

[Hamilton, Nason, Oleari, Zanderighi '12, + Re '13], [Karlberg, Re, Zanderighi '14]

- LL accuracy (+ simple NLL terms) from PS
- no new unphysical scale (i.e. physically sound)
- numerically very intensive
- applied beyond 2→1 processes

Geneva

[Alioli, Bauer, Berggren, Tackmann, Walsh '15 + Zuberi '13]

- LL accuracy from PS (at most! no NNLL nonesense!)
- slicing cutoff (missing power corrections)
- numerical cancellations in slicing parameter
- applied beyond 2→1 processes

MiNNLO$_{PS}$

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

- LL accuracy (+ simple NLL terms) from PS
- no new unphysical scale (i.e. physically sound)
- numerically efficient
- applied beyond 2→1 and even beyond colour singlet

UNNLOPS

[Höche, Prestel '14 '15]

- extension of UNLOPS merging of event samples
- two-loop corrections entirely in 0-jet bin
- only applied to 2→1 processes

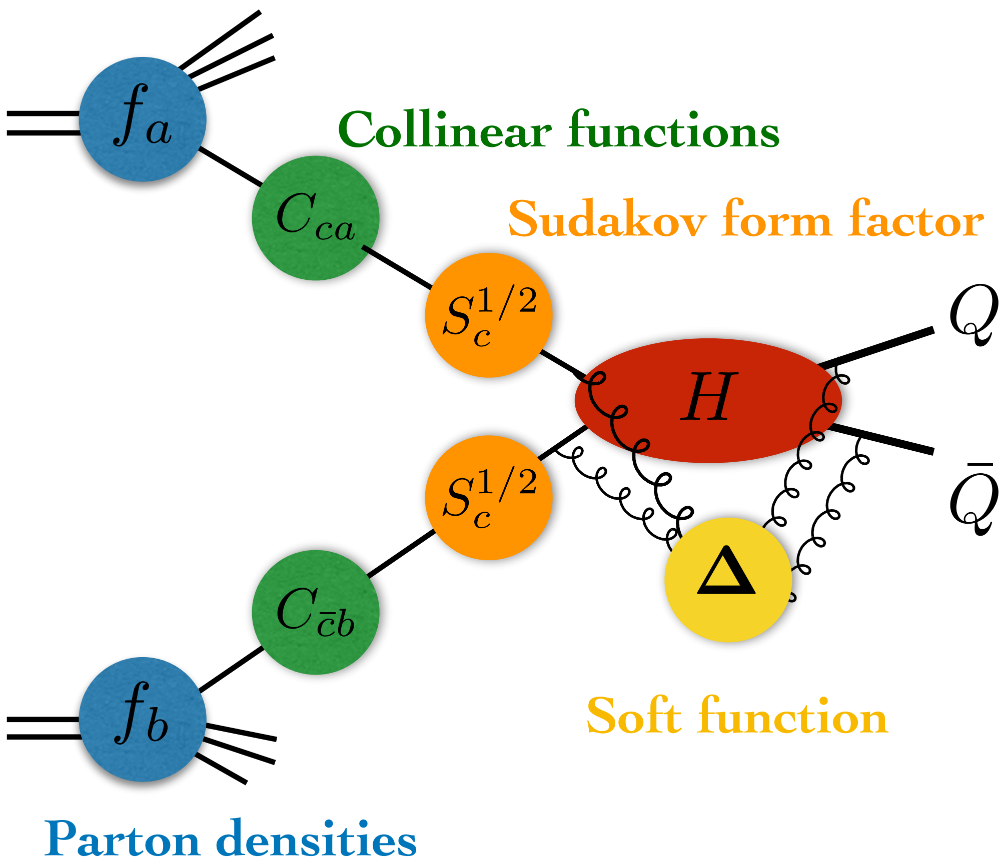
MiNNLOPS

Exploit available knowledge of transverse-momentum resummation

Catani, de Florian, MG(2000)
Bozzi et al (2005), Catani, MG (2010)

Recently extended to heavy-quark production

Mazzitelli et al (2020,2021)



Catani, Torre, MG (2014)
Catani, Devoto, Mazzitelli, MG (2023)

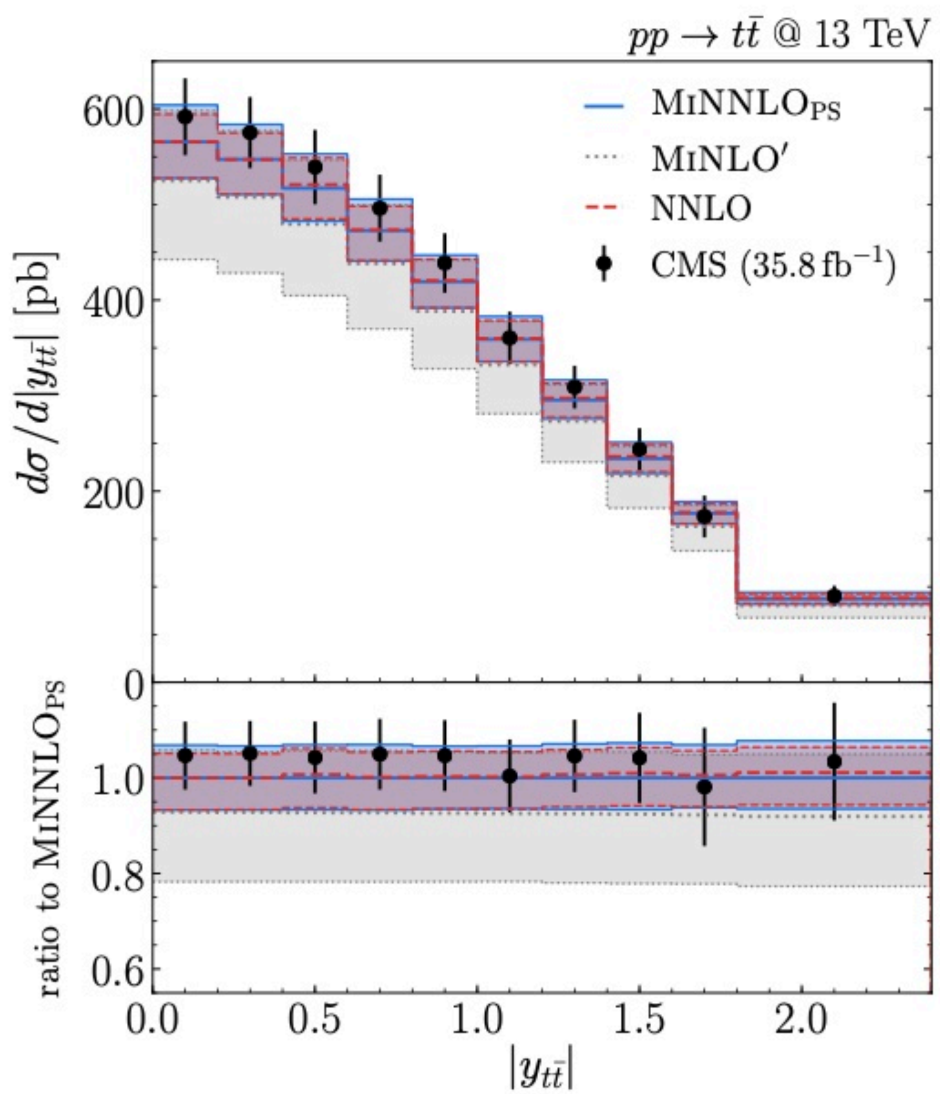
NNLO matching for colourless production well established

$t\bar{t}$ production first example of coloured final state with non trivial soft-radiation pattern

Allows to directly deploy NNLO precision into $t\bar{t}$ experimental analyses

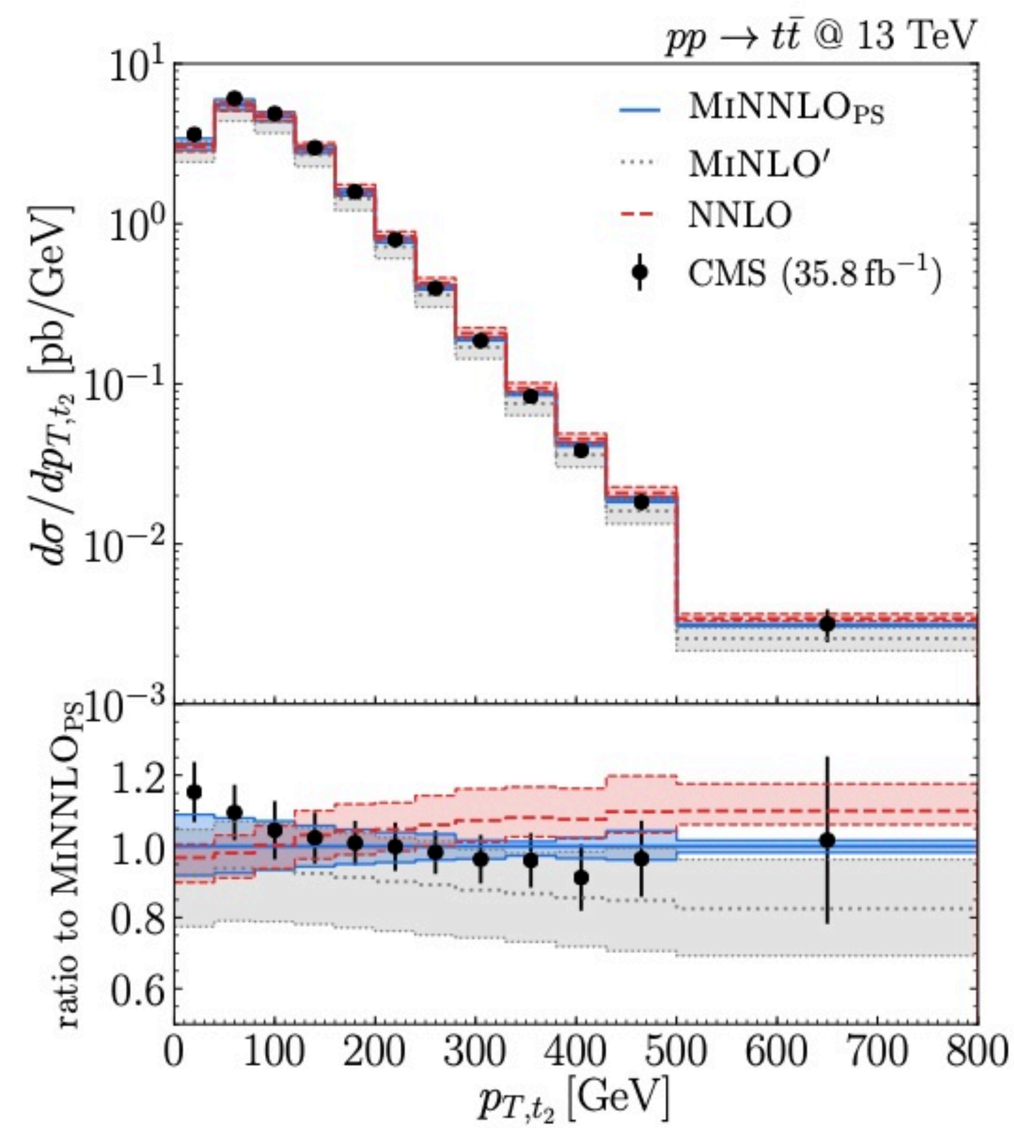
MiNNLOPS

Mazzitelli et al (2020,2021)



Excellent agreement with NNLO prediction, with differences only at the permille level

Excellent description of the data

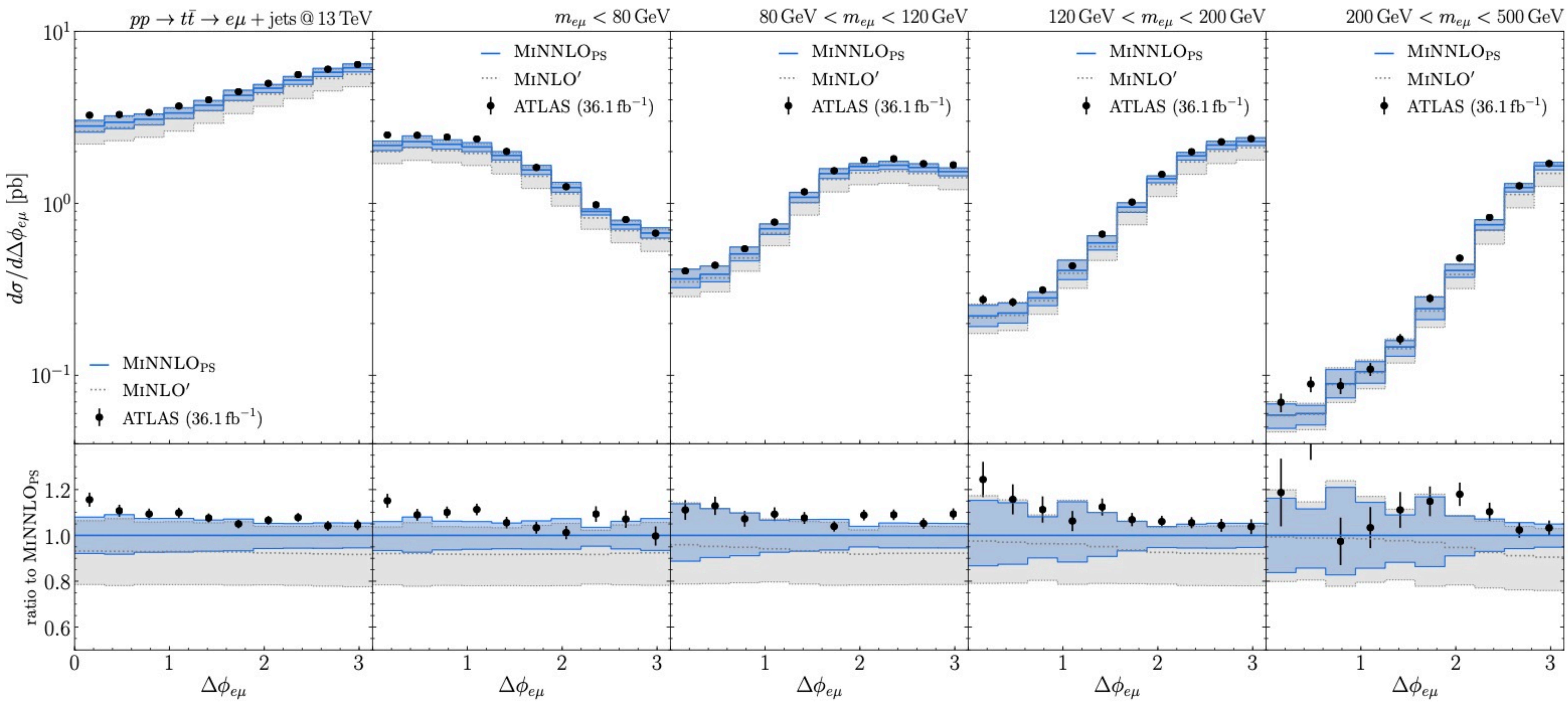


Improves where fixed order NNLO has problems (like p_T of softer top quark)

MiNNLOPS

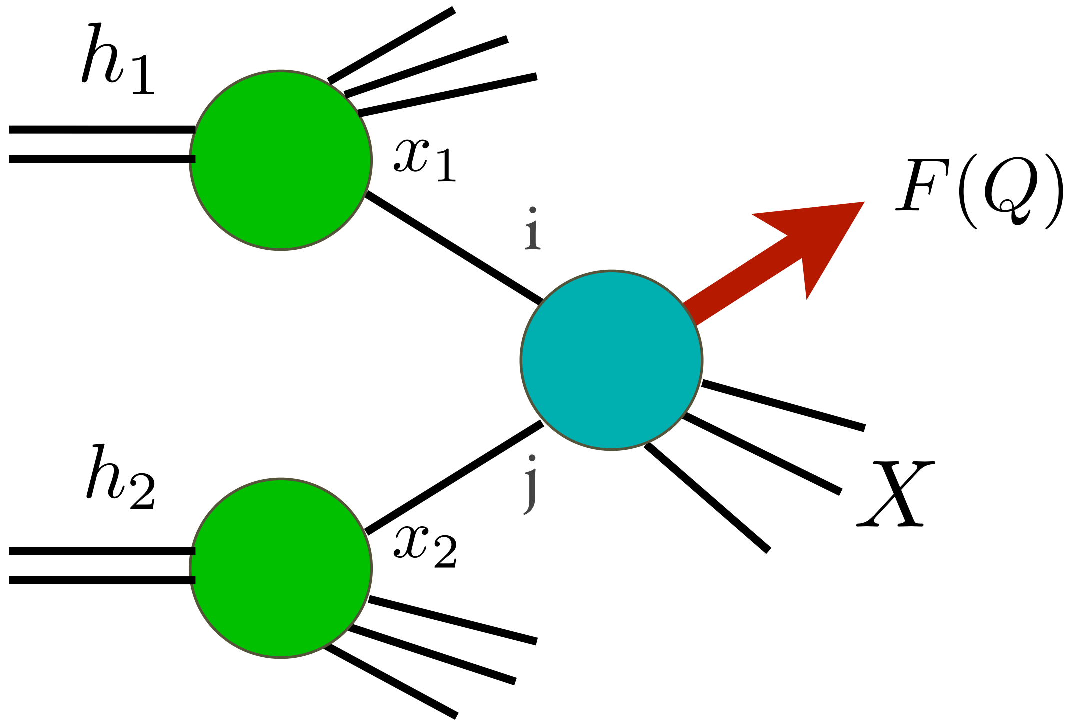
Mazzitelli et al (2020,2021)

Top decay and spin correlations included at LO only



Still good description of the data

Our starting point



High-p_T interactions are characterised by the presence of a hard scale Q (invariant mass of a lepton pair, high-p_T jet, heavy-quark mass…)

→ **Can be controlled through the factorisation theorem**

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2)\, \hat{\sigma}_{ij}(x_1 P_1, x_2 P_2, Q^2, \alpha_S(\mu_R); \mu_F^2, \mu_R^2) + \mathcal{O}\left(\left(\frac{\Lambda_{\rm QCD}}{Q}\right)^p\right)$$

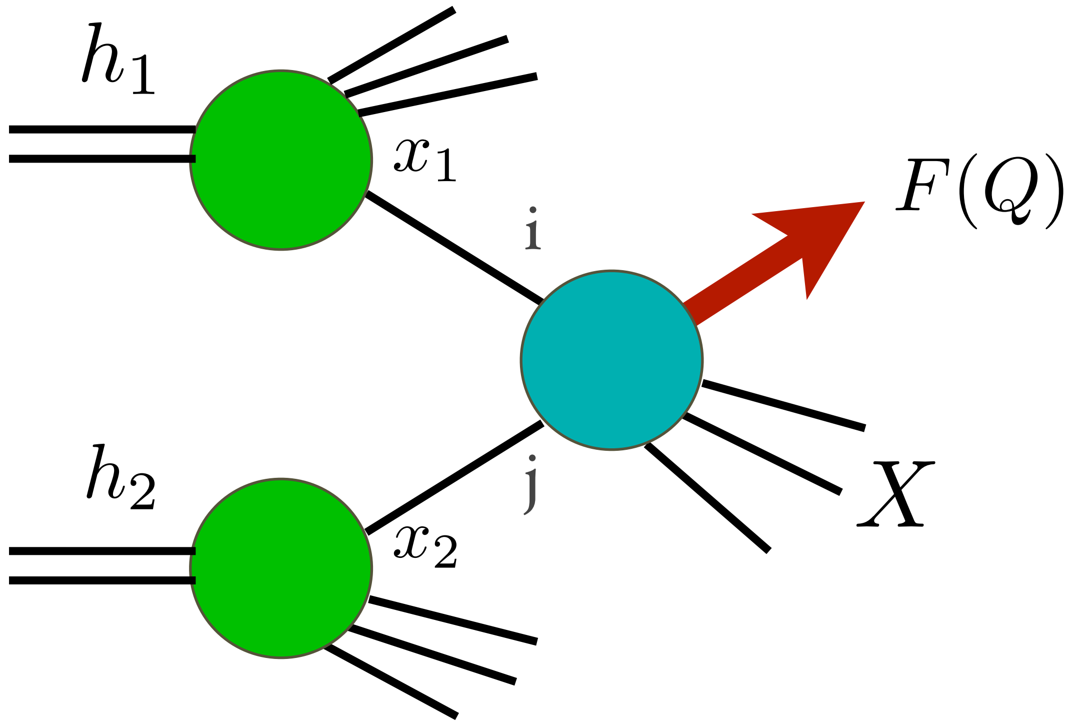
Parton distributions: universal but not perturbatively computable

Hard partonic cross section: process dependent but computable in perturbation theory

Power-suppressed contributions

The factorisation picture is systematically improvable (until the power-suppressed contributions become quantitative relevant…)

Our starting point



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Parton distributions: universal but not perturbatively computable

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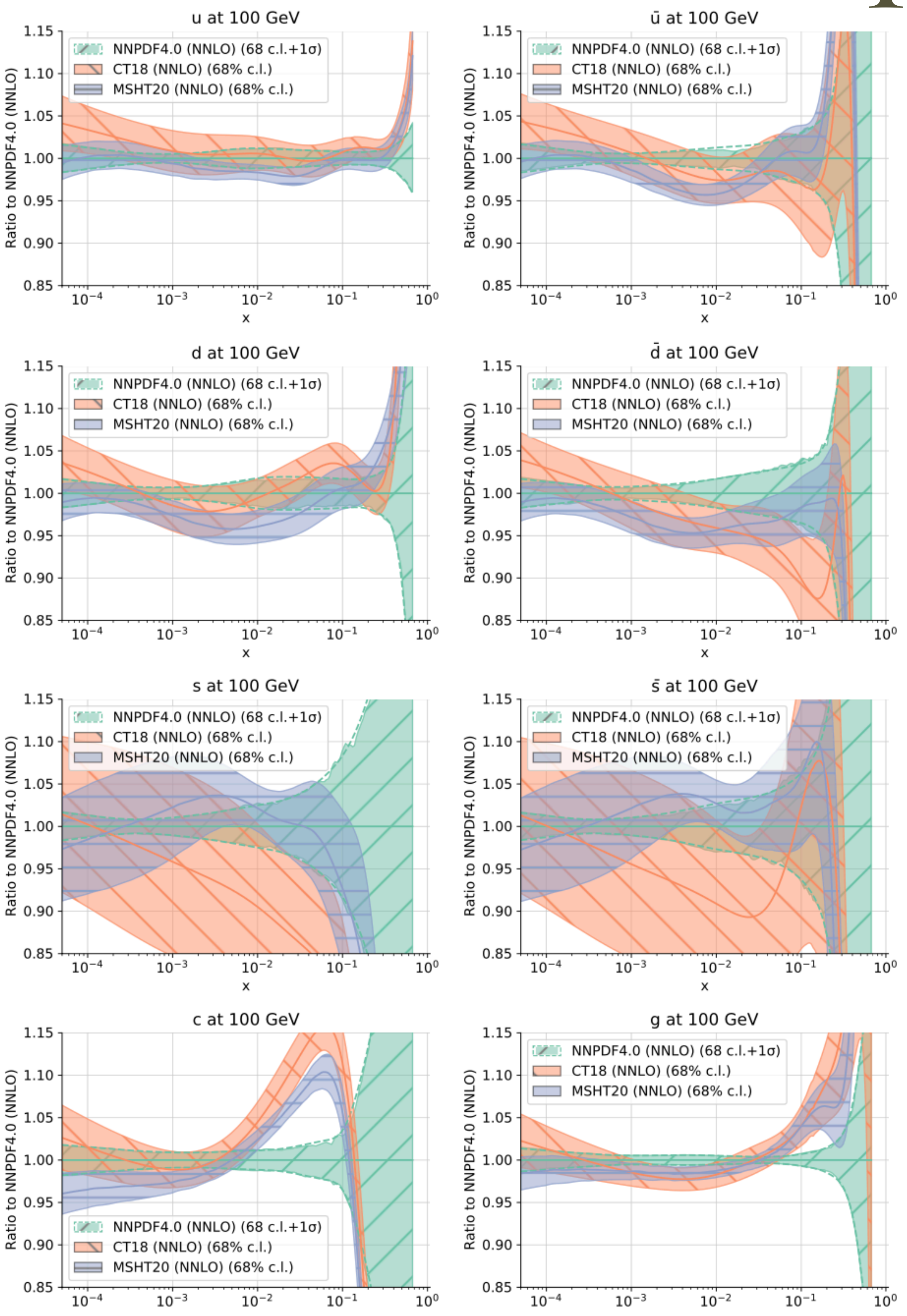
The factorisation picture is systematically improvable (until the power-suppressed contributions become quantitative relevant…)

PDFs



PDF precision now approaching the percent level

Overall fair agreement between three NNLO global sets with some differences

As for charm differences originate from the different input (perturbative vs fitted)

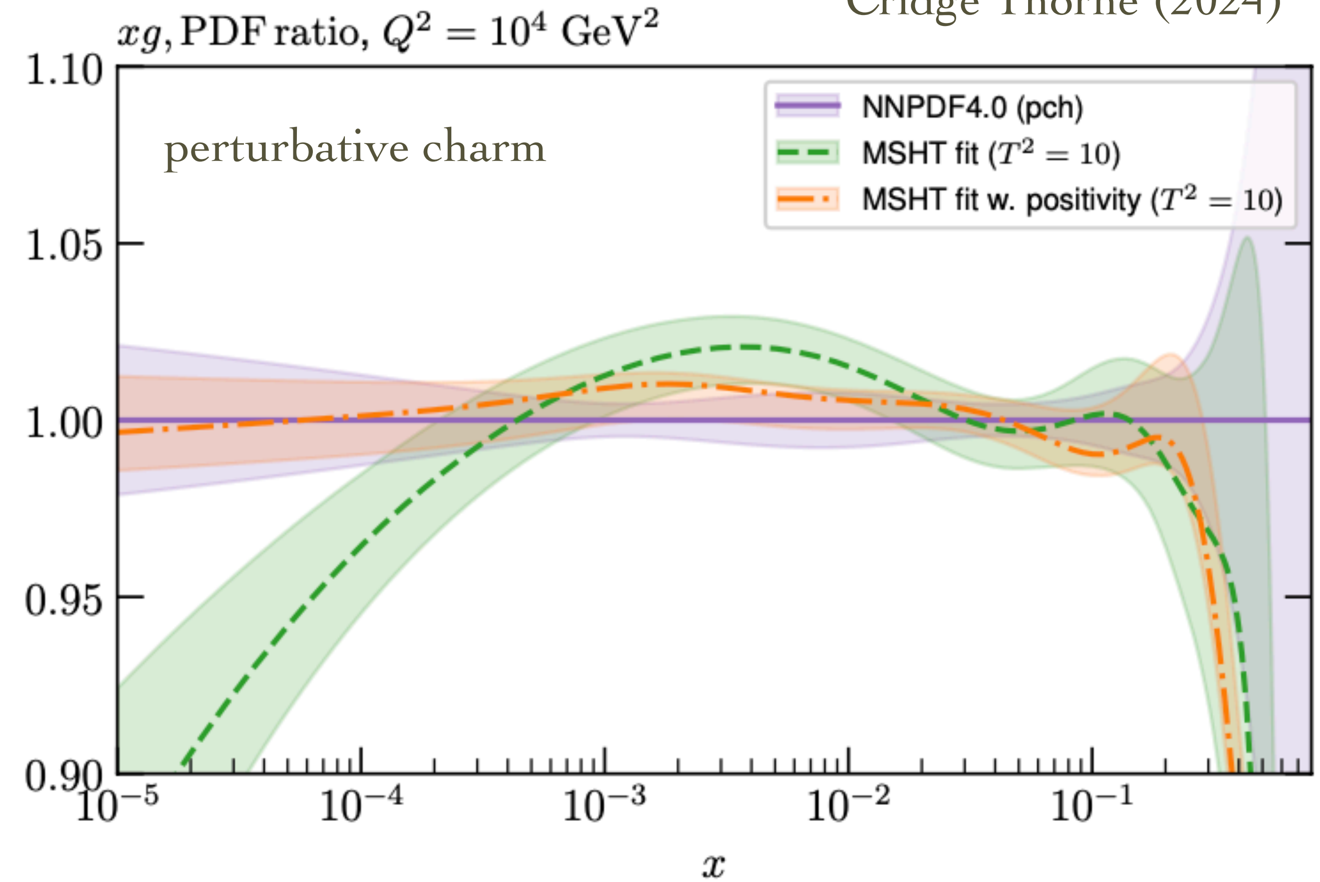
NNPDF gluon density has somewhat different shape wrt CT18 and MSHT20

NNPDF vs MSHT

Harland-Lang, Cridge Thorne (2024)

Fit of NNPDF4 data and theory input with MSHT parametrisation

	NNPDF4.0 pch	MSHT fit	MSHT fit (w positivity)
NMC $\sigma^{NC,p}$ (204) [49]	349.2 (1.71)	317.1 (1.55)	337.2 (1.65)
BCDMS F_2^p (333) [50]	497.6 (1.49)	471.6 (1.42)	483.2 (1.45)
NuTeV $\sigma_{CC}^{\bar{\nu}}$ (37) [51]	28.2 (0.76)	32.5 (0.88)	37.2 (1.00)
DIS Fixed–Target (1881)	**2076.1 (1.10)**	**2029.2 (1.08)**	**2063.3 (1.09)**
E886 σ^p (NuSea) (89) [52]	109.0 (1.23)	120.7 (1.36)	118.4 (1.33)
E906 $\sigma^d/2\sigma^p$ (SeaQuest) (6) [53]	5.55 (0.93)	4.37 (0.73)	3.54 (0.59)
DY Fixed–Target (195)	**190.4 (0.98)**	**201.8 (1.04)**	**198.6 (1.02)**
NC e^-p 575 GeV (254) [15]	265.5 (1.05)	254.5 (1.00)	259.0 (1.02)
NC e^+p 820 GeV (70) [15]	83.7 (1.20)	87.0 (1.24)	76.1 (1.09)
NC e^+p 920 GeV (377) [15]	576.5 (1.53)	536.1 (1.42)	574.0 (1.52)
NC, c (47) [54]	125.8 (2.68)	130.4 (2.77)	120.8 (2.57)
NC, b (26) [54]	71.0 (2.73)	64.1 (2.47)	67.9 (2.61)
HERA DIS (1145)	**1698.4 (1.48)**	**1650.2 (1.44)**	**1671.5 (1.46)**
D0 W muon asymmetry (9) [55]	16.2 (1.79)	14.2 (1.58)	13.8 (1.53)
ATLAS W, Z 7 TeV ($\mathcal{L} = 4.6\,\text{fb}^{-1}$) (61) [56]	119.9 (1.97)	101.1 (1.66)	100.2 (1.64)
ATLAS high–mass DY 2D 8 TeV (48) [57]	54.4 (1.13)	59.3 (1.24)	59.8 (1.25)
CMS electron asymmetry 7 TeV (11) [58]	7.67 (0.70)	11.0 (1.00)	11.1 (1.01)
CMS DY 2D 7 TeV (110) [59]	156.3 (1.42)	143.0 (1.30)	145.4 (1.32)
CMS W rapidity 8 TeV (22) [60]	26.1 (1.18)	22.2 (1.01)	22.1 (1.00)
LHCb $Z \to ee$ (17) [61]	28.8 (1.70)	25.2 (1.48)	25.7 (1.51)
LHCb $W, Z \to \mu$ 7 TeV (29) [62]	45.7 (1.58)	40.6 (1.40)	43.9 (1.51)
LHCb $W, Z \to \mu$ 8 TeV (30) [63]	43.7 (1.46)	34.3 (1.14)	36.1 (1.20)
LHCb $Z \to \mu\mu$ 13 TeV (16) [64]	22.7 (1.42)	17.7 (1.11)	18.9 (1.18)
LHCb $Z \to ee$ 13 TeV (15) [64]	28.8 (1.92)	24.4 (1.63)	26.6 (1.78)
Collider DY (576)	**794.7 (1.38)**	**727.5 (1.26)**	**741.6 (1.29)**
ATLAS incl. jets 8 TeV, $R = 0.6$ (171) [65]	137.7 (0.81)	129.8 (0.76)	137.8 (0.81)
ATLAS dijets 7 TeV (90) [66]	242.5 (2.69)	235.2 (2.61)	240.6 (2.67)
LHC Jets (500)	**823.9 (1.65)**	**813.2 (1.63)**	**826.5 (1.65)**
ATLAS $W^\pm$+ jet 8 TeV (30) [67]	43.0 (1.47)	48.4 (1.61)	47.8 (1.59)
LHC V+ Jets (122)	**136.6 (1.12)**	**137.7 (1.13)**	**139.5 (1.14)**
Isolated Photon (53)	**39.3 (0.74)**	**41.3 (0.78)**	**40.7 (0.77)**
Top quark (81)	**82.7 (1.02)**	**82.0 (0.78)**	**83.9 (1.04)**
Global, t_0 (4626)	**5928.3 (1.282)**	**5736.7 (1.240)**	**5837.8 (1.262)**
Global, exp. (4626)	**5543.7 (1.198)**	**5380.0 (1.163)**	**5470.7 (1.18)**



Differences with respect to nominal NNPDF4.0 partons, especially with perturbative charm

Differences in benchmark cross sections

Slightly better fit quality

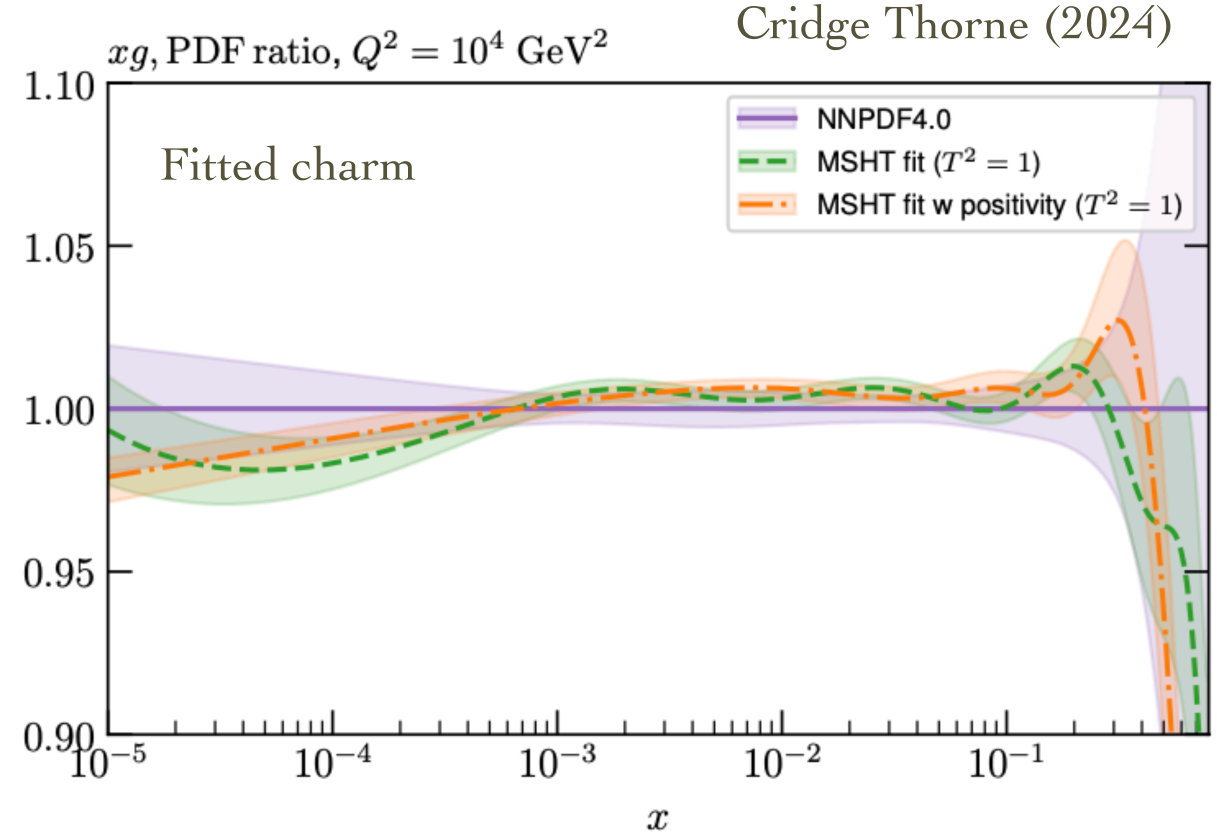
$T^2 = 1$ criterion not applicable ?

NNPDF vs MSHT

Fit of NNPDF4 data and theory input with MSHT parametrisation

	NNPDF4.0	MSHT fit	MSHT fit (w positivity)
BCDMS F_2^p (333) [50]	473.6 (1.42)	451.8 (1.36)	453.9 (1.36)
NuTeV $\sigma_{CC}^{\bar{\nu}}$ (37) [51]	21.1 (0.57)	33.9 (0.92)	35.0 (0.95)
DIS Fixed–Target (1881)	**2011.6 (1.07)**	**2018.6 (1.07)**	**2015.3 (1.07)**
E886 σ^p (NuSea) (89) [52]	105.3 (1.18)	112.5 (1.26)	110.0 (1.24)
E906 $\sigma^d/2\sigma^p$ (SeaQuest) (6) [53]	5.72 (0.95)	3.33 (0.56)	3.69 (0.62)
DY Fixed–Target (195)	**185.6 (0.95)**	**192.1 (0.99)**	**190.2 (0.98)**
NC e^+p 920 GeV (377) [15]	518.6 (1.38)	506.0 (1.34)	506.0 (1.34)
CC e^+p (39) [15]	47.5 (1.22)	42.9 (1.10)	44.5 (1.14)
NC, c (37) [54]	82.8 (2.24)	82.7 (2.24)	91.1 (2.46)
HERA DIS (1145)	**1575.6 (1.38)**	**1557.6 (1.36)**	**1565.8 (1.38)**
D0 W muon asymmtery (9) [55]	17.9 (1.99)	15.2 (1.69)	15.4 (1.72)
CMS DY 2D 7 TeV (110) [59]	146.2 (1.33)	138.6 (1.26)	140.7 (1.28)
CMS W rapidity 8 TeV (22) [60]	26.2 (1.19)	22.7 (1.03)	23.3 (1.06)
LHCb $W, Z \to \mu$ 7 TeV (29) [62]	56.3 (1.94)	51.8 (1.78)	53.6 (1.85)
Collider DY (576)	**767.8 (1.33)**	**743.1 (1.29)**	**754.4 (1.31)**
LHC Jets (500)	**804.8 (1.61)**	**796.7 (1.59)**	**797.6 (1.60)**
ATLAS $W^{\pm}$+ jet 8 TeV (30) [67]	43.9 (1.46)	48.2 (1.61)	47.3 (1.58)
LHC V+ Jets (122)	**136.1 (1.12)**	**140.4 (1.15)**	**139.2 (1.14)**
Isolated Photon (53)	**41.9 (0.79)**	**40.5 (0.76)**	**40.6 (0.77)**
ATLAS $t\bar{t}$ l+ jets 8 TeV (8) [83]	25.9 (3.24)	30.6 (3.82)	26.1 (3.26)
Top quark (81)	**85.0 (1.05)**	**87.4 (1.08)**	**83.0 (1.02)**
Global, t_0 (4616)	**5692.1 (1.233)**	**5645.2 (1.222)**	**5651.0 (1.224)**
Global, exp. (4616)	**5354.1 (1.160)**	**5322.5 (1.153)**	**5341.5 (1.155)**

Harland-Lang, Cridge Thorne (2024)



Differences with respect to nominal NNPDF4.0 partons, especially with perturbative charm

Differences in benchmark cross sections

Slightly better fit quality

$T^2 = 1$ criterion not applicable ?

W mass and PDFs

PDF set	p_T^ℓ fit m_W	p_T^ℓ fit σ_{tot}	p_T^ℓ fit σ_{PDF}	p_T^ℓ fit χ^2/n.d.f.	m_T fit m_W	m_T fit σ_{tot}	m_T fit σ_{PDF}	m_T fit χ^2/n.d.f.
CT14	80358.3	$^{+16.1}_{-16.2}$	4.6	543.3/558	80401.3	$^{+24.3}_{-24.5}$	11.6	557.4/558
CT18	80362.0	$^{+16.2}_{-16.2}$	4.9	529.7/558	80394.9	$^{+24.3}_{-24.5}$	11.7	549.2/558
CT18A	80353.2	$^{+15.9}_{-15.8}$	4.8	525.3/558	80384.8	$^{+23.5}_{-23.8}$	10.9	548.4/558
MMHT2014	80361.6	$^{+16.0}_{-16.0}$	4.5	539.8/558	80399.1	$^{+23.2}_{-23.5}$	10.0	561.5/558
MSHT20	80359.0	$^{+13.8}_{-15.4}$	4.3	550.2/558	80391.4	$^{+23.6}_{-24.1}$	10.0	557.3/558
ATLASpdf21	80362.1	$^{+16.9}_{-16.9}$	4.2	526.9/558	80405.5	$^{+28.2}_{-27.7}$	13.2	544.9/558
NNPDF3.1	80347.5	$^{+15.2}_{-15.7}$	4.8	523.1/558	80368.9	$^{+22.7}_{-22.9}$	9.7	556.6/558
NNPDF4.0	80343.7	$^{+15.0}_{-15.0}$	4.2	539.2/558	80363.1	$^{+21.4}_{-22.1}$	7.7	558.8/558

18.3 MeV

(total error 15.9 MeV)

ATLAS 7 TeV update 2403.15085

NEW CMS measurement

PDF set	Extracted m_W (MeV) Original σ_{PDF}	Extracted m_W (MeV) Scaled σ_{PDF}
CT18Z	$80\,360.2 \pm 9.9$	
CT18	$80\,361.8 \pm 10.0$	
PDF4LHC21	$80\,363.2 \pm 9.9$	
MSHT20	$80\,361.4 \pm 10.0$	$80\,361.7 \pm 10.4$
MSHT20aN3LO	$80\,359.9 \pm 9.9$	$80\,359.8 \pm 10.3$
NNPDF3.1	$80\,359.3 \pm 9.5$	$80\,361.3 \pm 10.4$
NNPDF4.0	$80\,355.1 \pm 9.3$	$80\,357.0 \pm 10.8$

6.7 MeV

(total error 9.9 MeV)

…going beyond….

Mixed QCD-EW corrections

$\mathcal{O}(\alpha\alpha_S) \sim \mathcal{O}(\alpha_S^3)$ → mixed QCD-EW corrections expected to be of the same order as N³LO QCD

Such calculations are technically within current possibilities (provided relevant two-loop amplitudes are available) since they can rely on existing NNLO QCD methods

Two exact independent computations for the neutral current Drell-Yan process

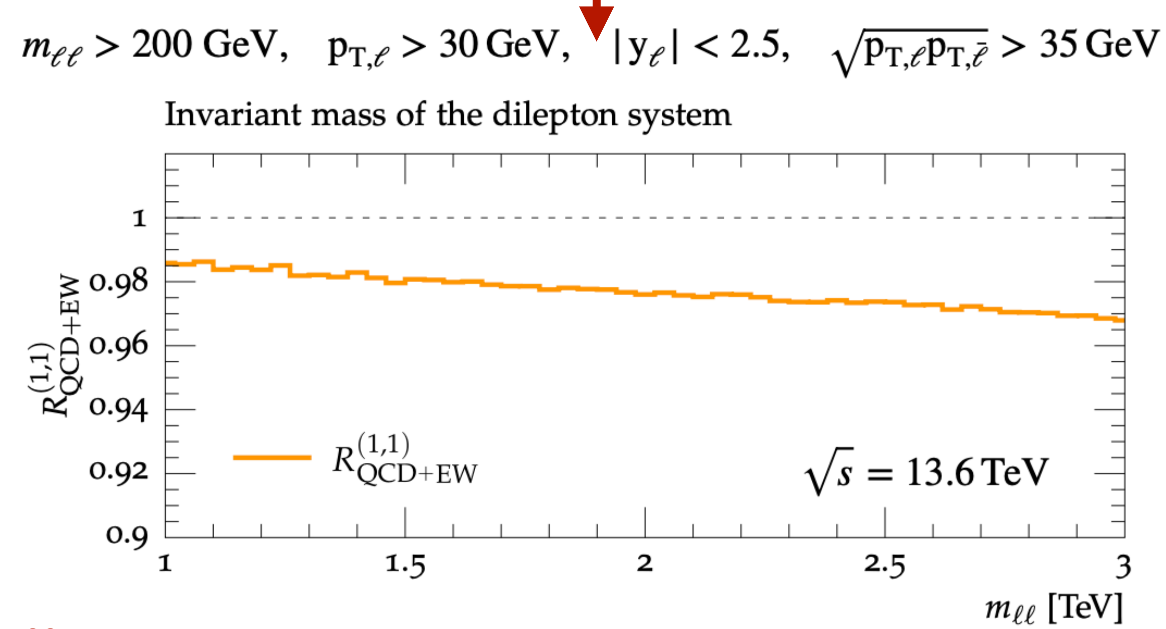
Massive bare muons

σ [pb]	$\sigma_{\rm LO}$	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	809.56(1)	191.85(1)	−33.76(1)	49.9(7)	−4.8(3)
qg	—	−158.08(2)	—	−74.8(5)	8.6(1)
$q(g)\gamma$	—	—	−0.839(2)	—	0.084(3)
$q(\bar{q})q'$	—	—	—	6.3(1)	0.19(0)
gg	—	—	—	18.1(2)	—
$\gamma\gamma$	1.42(0)	—	−0.0117(4)	—	—
tot	810.98(1)	33.77(2)	−34.61(1)	−0.5(9)	4.0(3)

$\mathcal{O}(0.5\%)$ effect

Buonocore et al (2021)

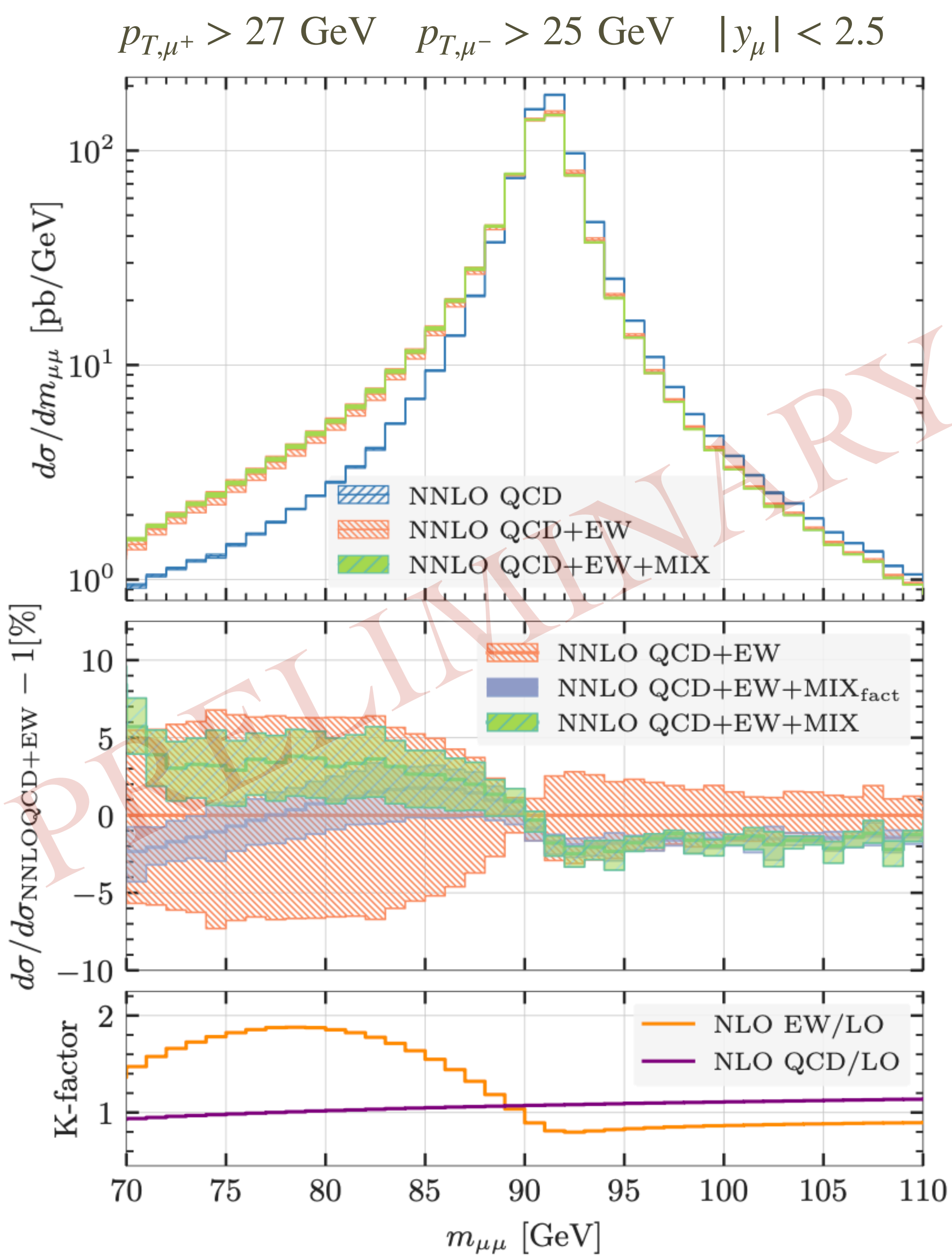
Massless dressed leptons

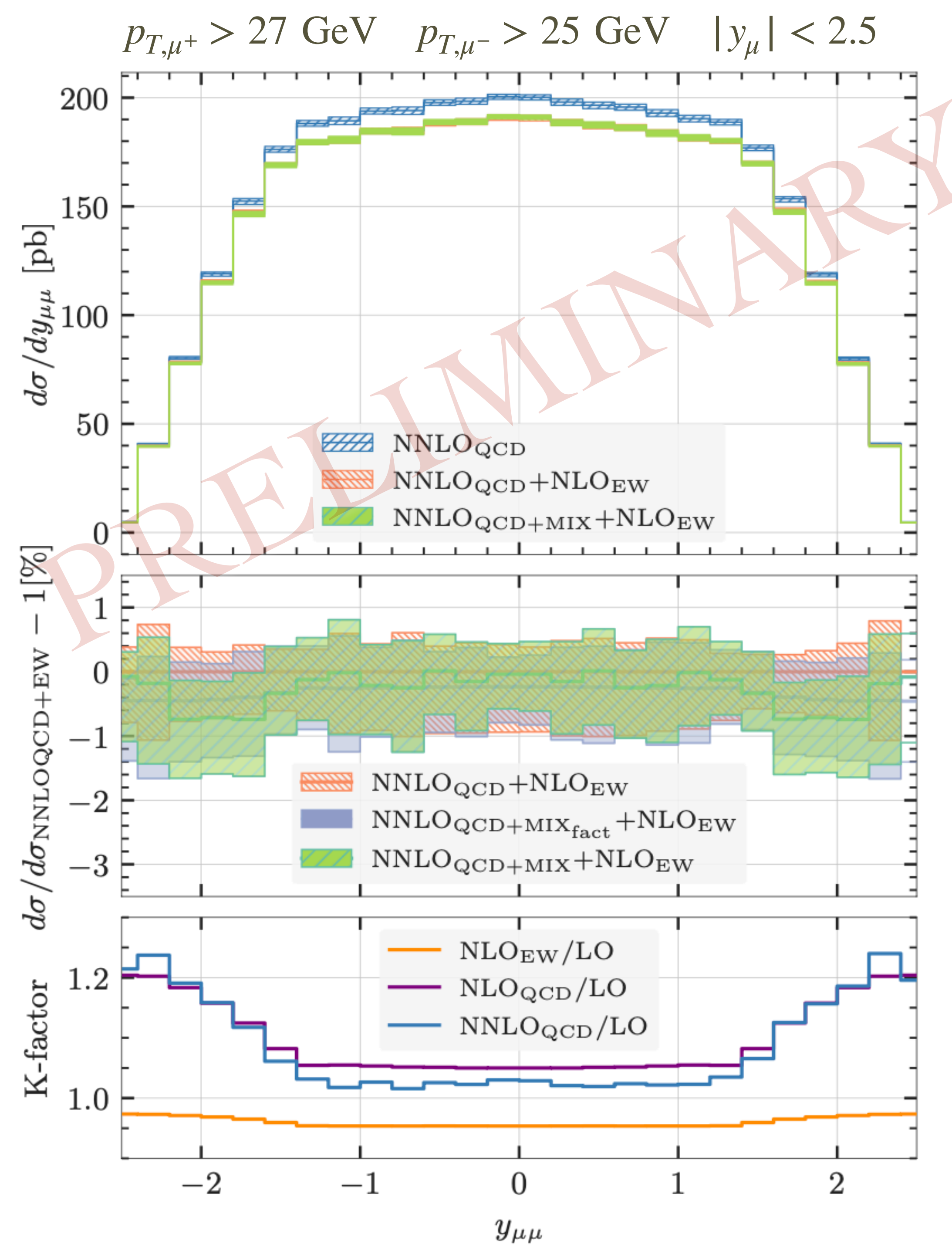


Buccioni et al (2022)

Mixed QCD-EW corrections

Buonocore et al to appear

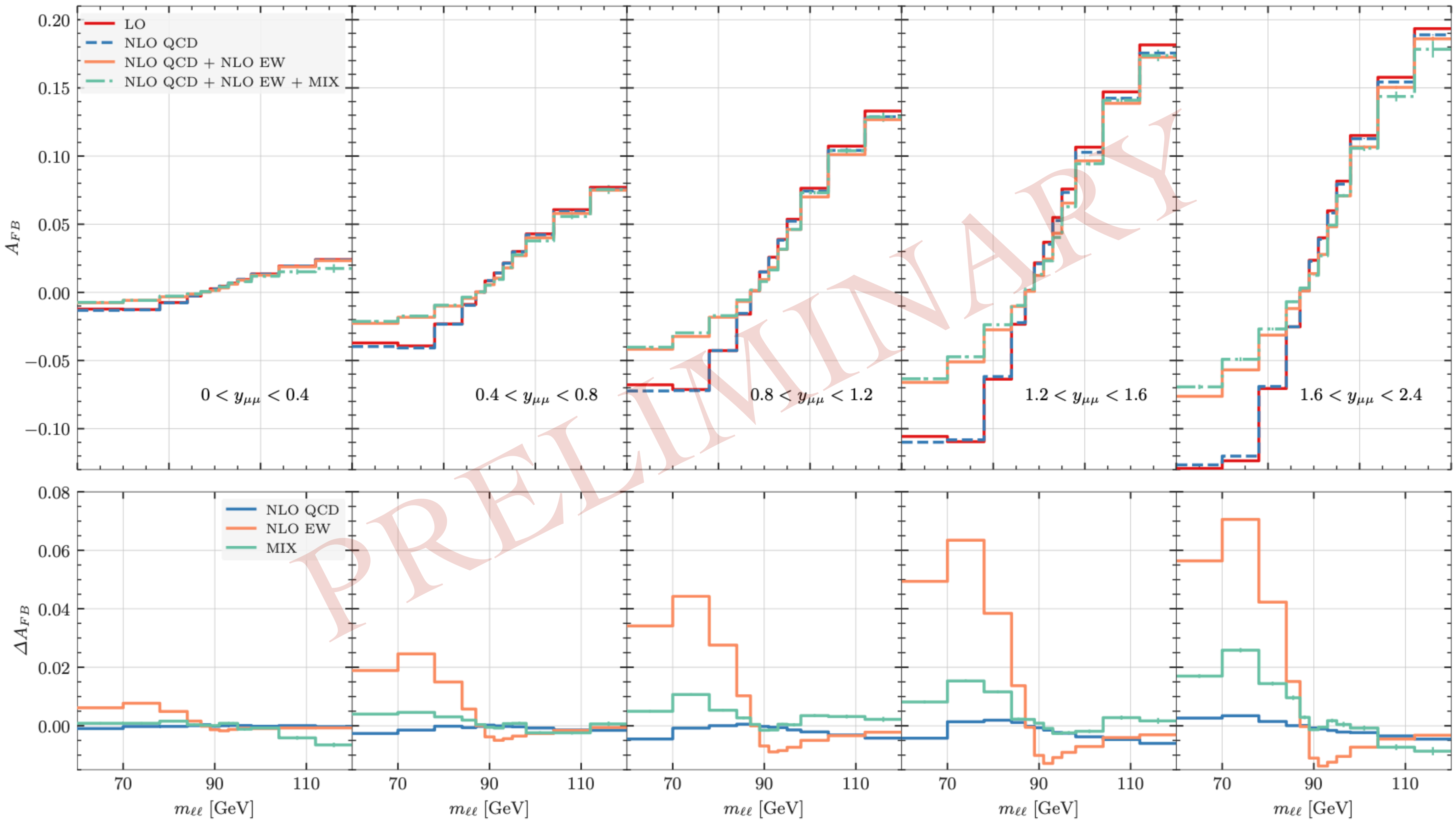




Mixed QCD-EW corrections

Buonocore et al to appear

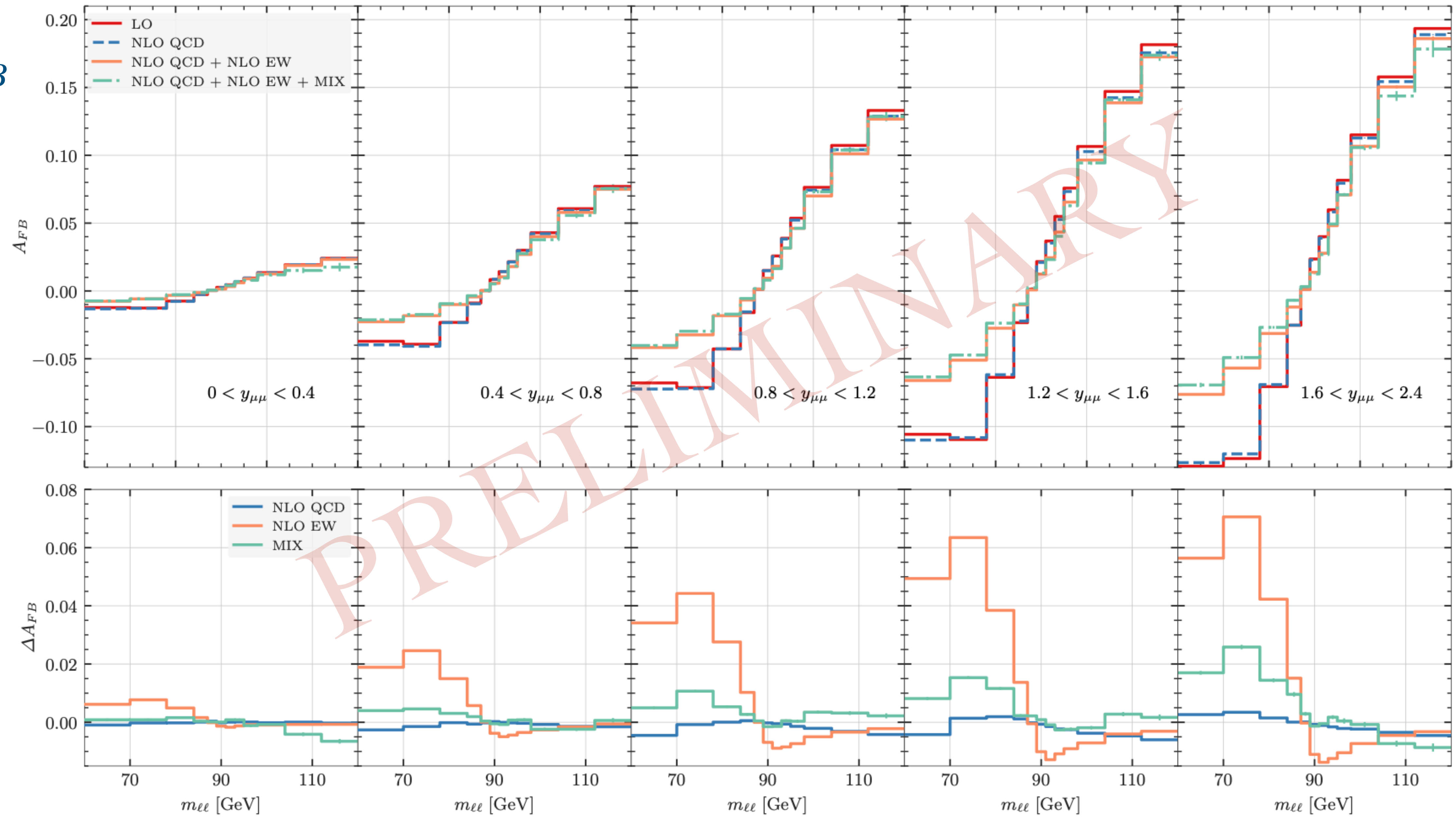
A_{FB}



Mixed QCD-EW corrections

Buonocore et al to appear

A_{FB}



More to come:

Two-loop amplitude for charged current process now available making the corresponding exactcalculation possible

Bonciani et al (2024)

Partial results for (on shell) Z + jet production

Bargiela et al (2023)

N3LO: the frontier

For some benchmark processes NNLO QCD may not be enough….

N^3LO corrections for some $2 \to 1$ processes now available: total cross sections

	Q [GeV]	$\delta\sigma^{N^3LO}$	$\delta\sigma^{NNLO}$	δ(scale)	$\delta(\text{PDF}+\alpha_S)$	δ(PDF-TH)
$gg \to$ Higgs	m_H	3.5%	30%	$^{+0.21\%}_{-2.37\%}$	$\pm3.2\%$	$\pm1.2\%$
$b\bar{b} \to$ Higgs	m_H	-2.3%	2.1%	$^{+3.0\%}_{-4.8\%}$	$\pm8.4\%$	$\pm2.5\%$
NCDY	30	-4.8%	-0.34%	$^{+1.53\%}_{-2.54\%}$	$^{+3.7\%}_{-3.8\%}$	$\pm2.8\%$
	100	-2.1%	-2.3%	$^{+0.66\%}_{-0.79\%}$	$^{+1.8\%}_{-1.9\%}$	$\pm2.5\%$
CCDY(W^+)	30	-4.7%	-0.1%	$^{+2.5\%}_{-1.7\%}$	$\pm3.95\%$	$\pm3.2\%$
	150	-2.0%	-0.1%	$^{+0.5\%}_{-0.5\%}$	$\pm1.9\%$	$\pm2.1\%$
CCDY(W^-)	30	-5.0%	-0.1%	$^{+2.6\%}_{-1.6\%}$	$\pm3.7\%$	$\pm3.2\%$
	150	-2.1%	-0.6%	$^{+0.6\%}_{-0.5\%}$	$\pm2\%$	$\pm2.13\%$

Baglio et al (2022)

Small but significant impact of N3LO corrections, sometimes outside NNLO scale uncertainties

N3LO: the frontier

For some benchmark processes NNLO may not be enough….

N^3LO corrections for some $2 \to 1$ processes now available: fully differential results

- Projection to Born

  Jet production in DIS — Currie, Gehrmann, Glover, Huss Niehues (2018)

  Higgs production in gluon fusion — Gehrmann et al (2021)

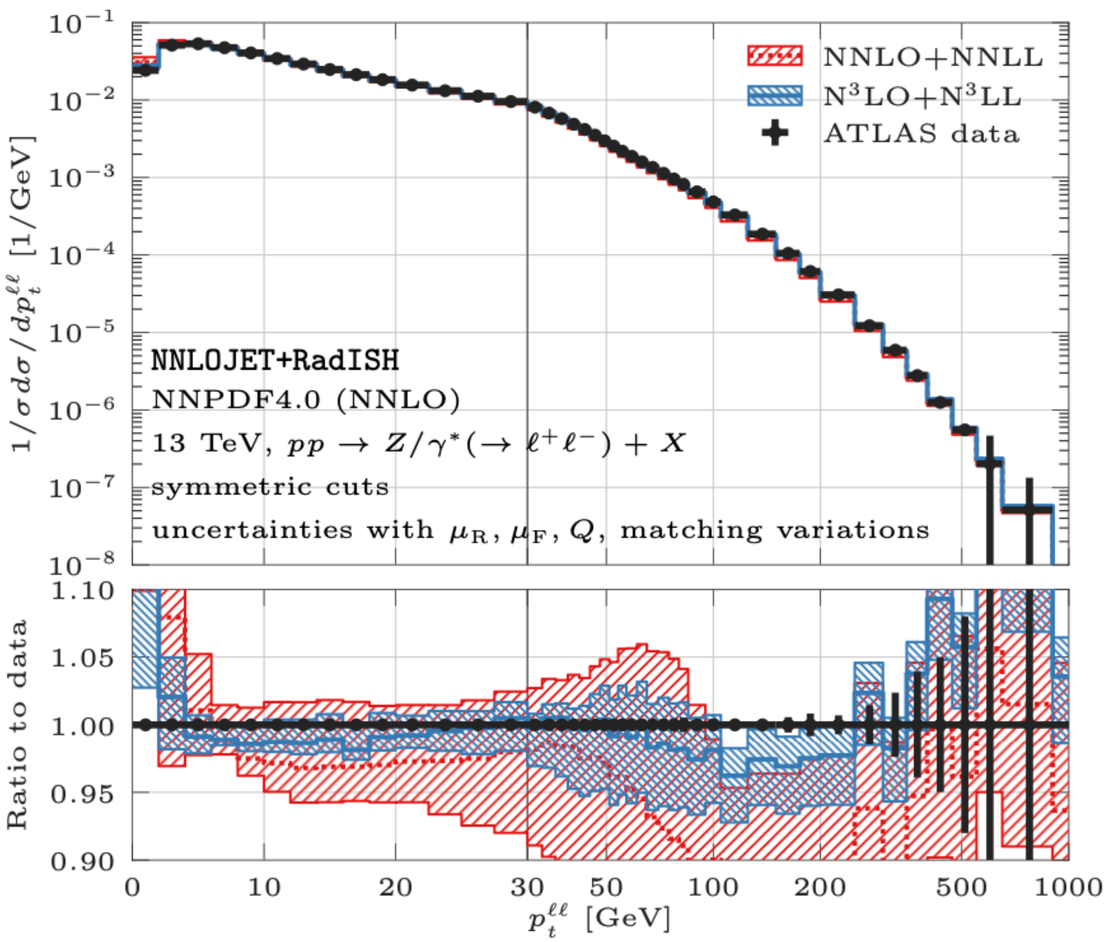
  $H \to b\bar{b}$ — Mondini, Schiavi, Williams (2019)

- q_T subtraction

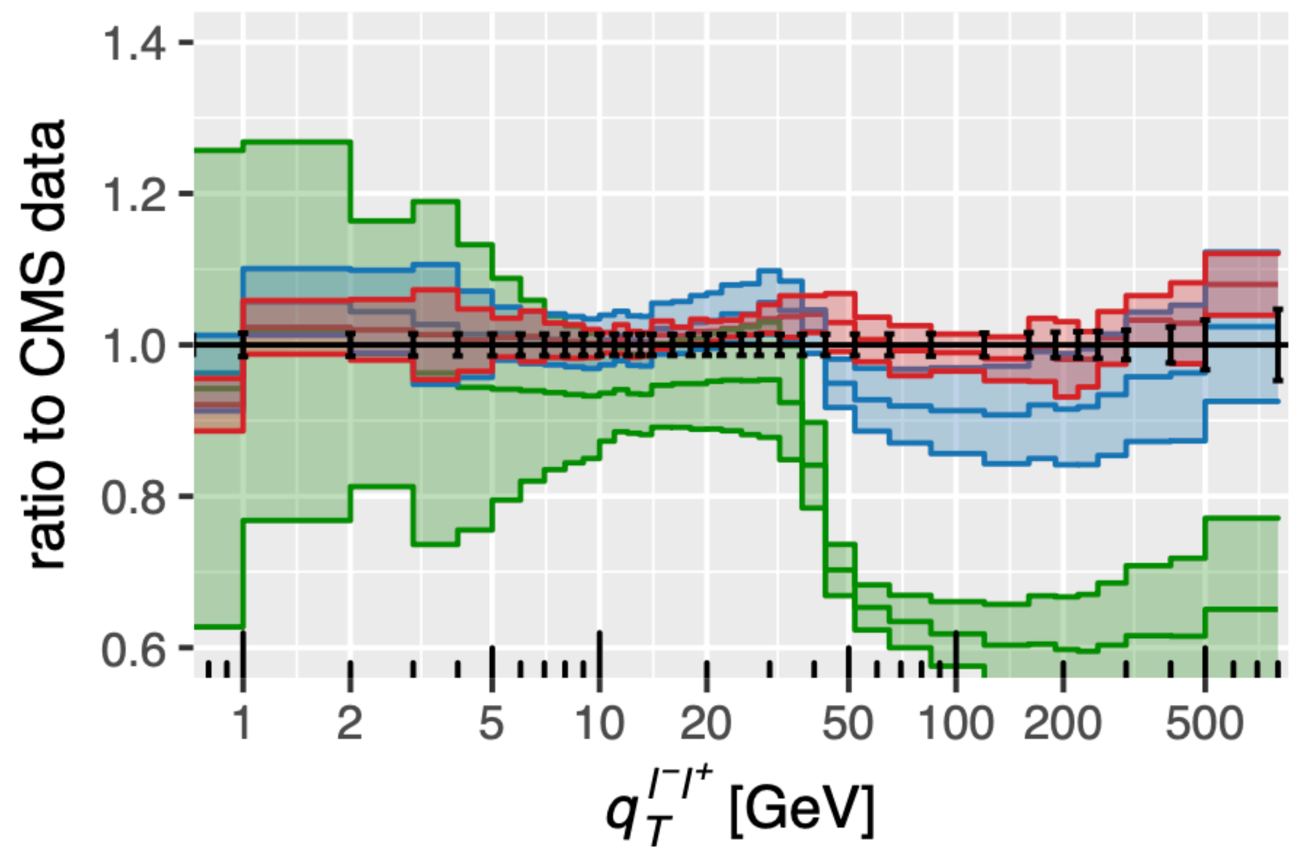
  Higgs production in gluon fusion — Cieri et al (2018), Gehrmann et al (2018)

  Drell-Yan — Camarda, Cieri, Ferrera (2021-2023); Gehrmann et al (2022), Campbell, Neumann (2022,23)

N3LO: the frontier



Rottoli et al. (2022)



Campbell, Neumann (2023)

Impressive description of data for NNLO type observables

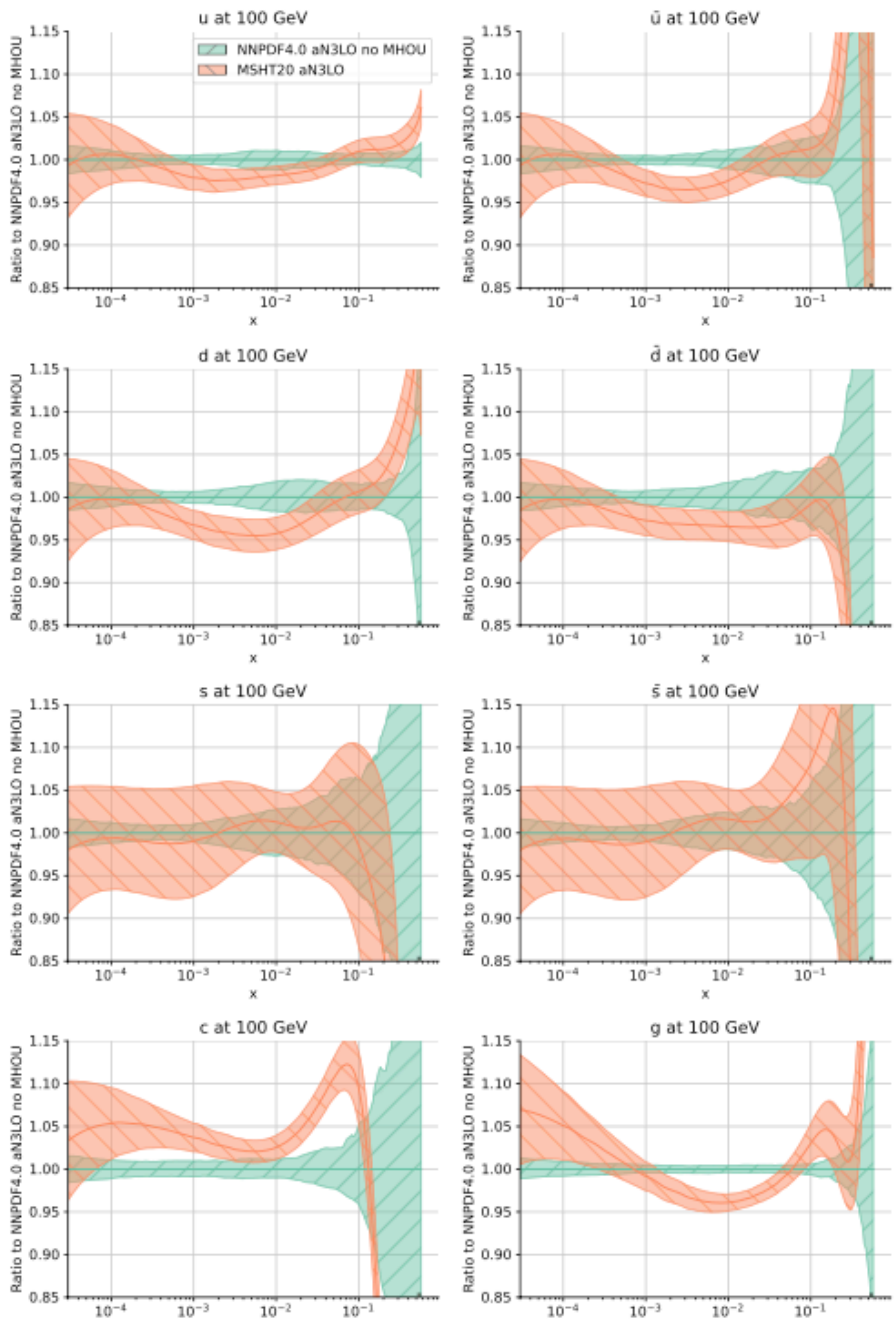
N³LO seems to improve agreement with the data for fiducial cross section

EW corrections ?

Order k	fixed-order α_s^k	res. improved α_s^k
0	694^{+85}_{-92}	—
1	732^{+19}_{-30}	$637 \pm 8_{\text{mat.}} \pm 70_{\text{sc.}}$
2	720^{+4}_{-3}	$707 \pm 3_{\text{mat.}} \pm 29_{\text{sc.}}$
3	$700^{+4}_{-6} \pm 1_{\text{slicing}}$	$702 \pm 1_{\text{mat.}} \pm 1_{\text{m.c.}} \pm 17_{\text{sc.}}$

CMS result → $699 \pm 5\,(\text{syst.}) \pm 17\,(\text{lumi.})$ (e, μ combined) [3]

N3LO: PDFs



Current approximate N3LO fits use partial available information on N3LO splitting kernels

Davies, Falcioni, Herzog, Moch, Ruijl, Soar Vermaseren, Vogt, Ueda….

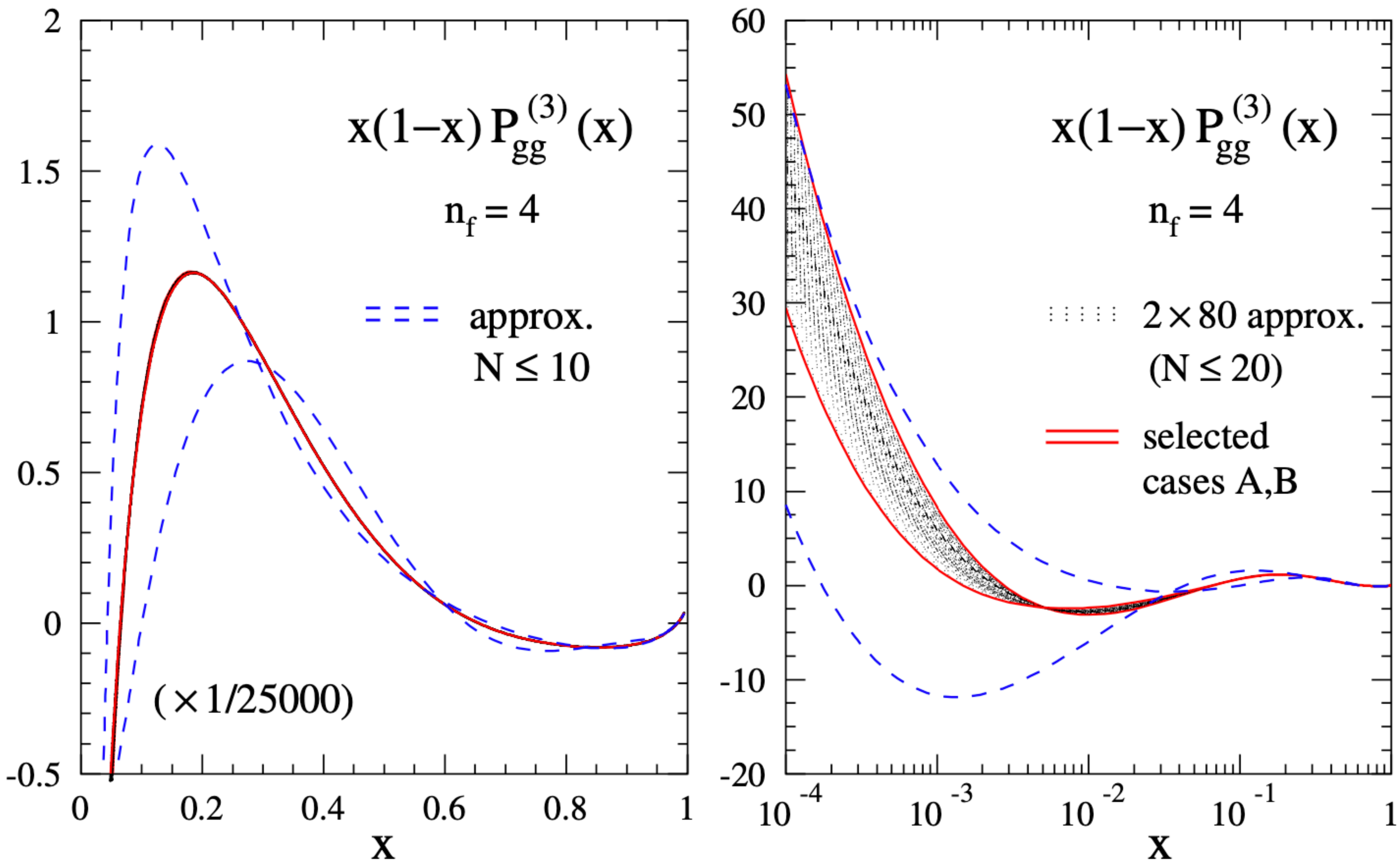
Though approximate, this information should be sufficient to obtain sufficiently accurate PDFs evolution

Still large differences between the two existing aN3LO sets mainly in the charm and gluon density

These differences are most likely due to the different approaches and fitting methodologies

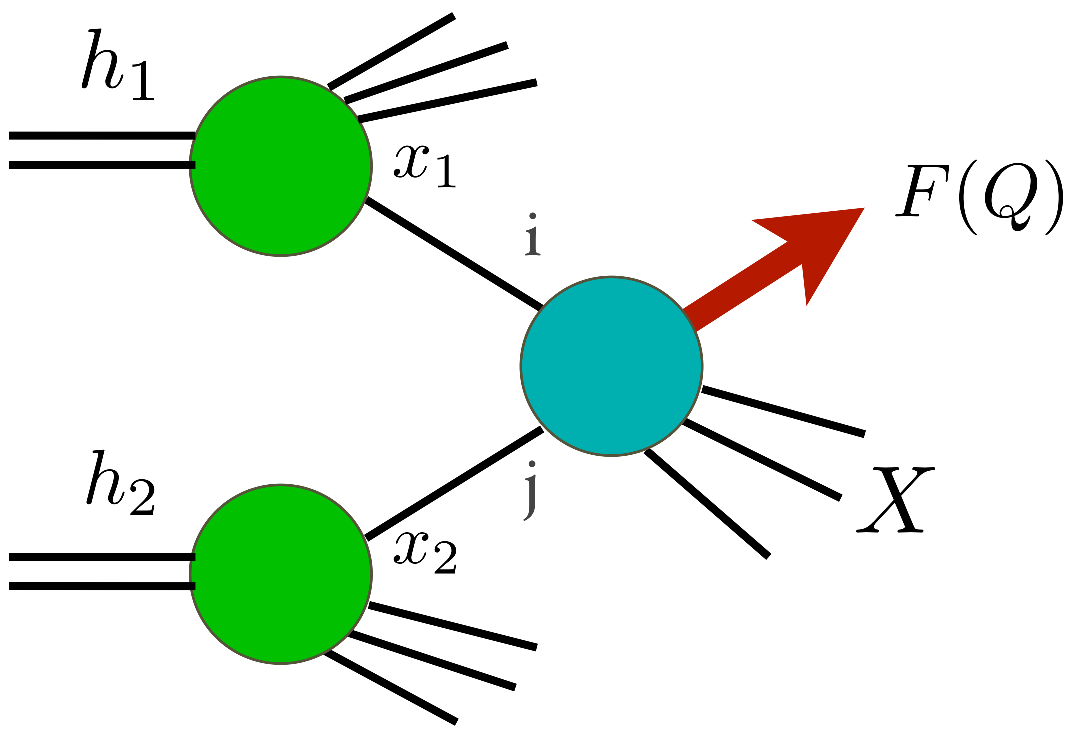
N3LO: PDFs

The recent computation of the Mellin moments up to $N = 20$ further improves the situation



G.Falcioni, talk given at HP2 2024

Our starting point



High-p_T interactions are characterised by the presence of a hard scale Q (invariant mass of a lepton pair, high-p_T jet, heavy-quark mass…)

➡ **Can be controlled through the factorisation theorem**

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2)\, \hat{\sigma}_{ij}(x_1 P_1, x_2 P_2, Q^2, \alpha_S(\mu_R); \mu_F^2, \mu_R^2) + \mathcal{O}\left(\left(\frac{\Lambda_{\rm QCD}}{Q}\right)^p\right)$$

Parton distributions: universal but not perturbatively computable

Hard partonic cross section: process dependent but computable in perturbation theory

Power-suppressed contributions

The factorisation picture is systematically improvable (until the power-suppressed contributions become quantitative relevant…)

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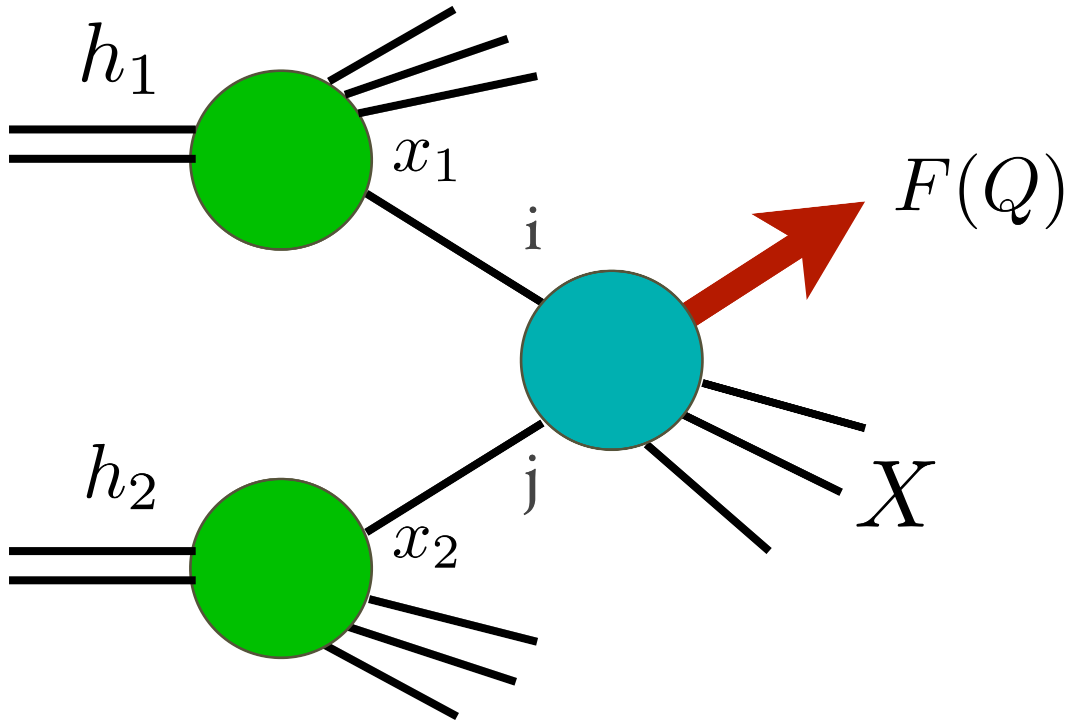

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Power corrections

0) Inclusive DIS data lead to quadratic power corrections (OPE at work)

But modern global PDF fits all heavily rely on LHC data…..

1) The "easy" case: inclusive Drell-Yan production: in this case $n = 2$

Beneke, Braun (1995)

$\Lambda_{\rm QCD} \sim 0.3$ GeV

$Q \sim 100$ GeV

→ $(\Lambda_{\rm QCD}/Q)^2 \sim 0.001\,\%$ can be safely neglected

2) Less "easy" case: Drell-Yan p_T distribution

Recent studies suggest the absence of linear power corrections

Ferrario Ravasio, Limatola, Nason (2020)

3) More "difficult" case: top production

Nason et al (2018)
Melnikov et al (2023)

Here $n = 1$ except for very special quantities (i.e. the total $t\bar{t}$ cross section expressed in terms of the $\overline{\text{MS}}$ mass) $\Lambda_{\text{QCD}}/m_t \sim 0.2\,\%$

4) Jet and photon production

In this case the situation is made more difficult not only by the fact that $n = 1$ but also by the photon and jet acceptance cuts

If $Q = p_{\text{T,min}} \sim 30$ GeV → $\Lambda_{\text{QCD}}/Q \sim 1\,\%$

5) MPI

Recent studies suggest that corrections might be $\mathcal{O}(\text{several GeV})/Q$

Rottoli et al (2023)

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Rottoli et al (2023)

→ Fully exploiting the theoretical progress in the perturbative calculations will at some point require a step forward in our understanding of a number of difficult effects

Summary & Outlook

- The lack of sufficiently precise theoretical predictions might lead to miss, or at least delay, possible discoveries

- NNLO results now available for essentially all the relevant $2 \to 1$ and $2 \to 2$ processes and lead to an improved description of the data

- Cross validation of different computations essential in consolidating the results but improvements in subtraction/slicing techniques expected/needed

- Extension to $2 \to 3$ requires facing new challenges in the computations of two-loop amplitudes: in the meanwhile approximations of the virtual allow us to achieve first NNLO accurate predictions

- NNLO computations challenging also from the point of view of computing resources

  → Only a limited subset of the results are publicly available

- Deployment of NNLO precision in MC tools still partial

Summary & Outlook

Going beyond requires progress in multiple directions

- Mixed QCD-EW corrections lead to small effects that will be relevant in selected benchmark processes

- N^3LO QCD era started with new exciting results and new challenges

  - availability of N^3LO predictions limited to inclusive $2 \rightarrow 1$ processes

  - progress in four-loop splitting functions now makes N^3LO PDF fits possible

  - Power corrections/hadronisation/MPI ?

  - Unfolding ?