Hierarchies and conformal UV Completions

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The Standard Model and beyond

The SM is the endpoint of a very successful development: d=4 renormalizable gauge theory

QED ⇒	QCD ⇒	SM
$U(1)_{em} \Rightarrow$	$SU(3)_c \Rightarrow$	$SU(3)_c \times SU(2)_L \times U(1)_Y$

➔ **excellent agreement of theory and experiment**

Theoretical problems:

SM does not exist without cutoff
(triviality, vacuum stability)
Gauge hierarchy problem
Gauge unification &charge quantization
Strong CP problem
Unification with gravity
3 generations, reps., d=4, many parameters

Exper. facts, hints, problems:

- Electro-weak scale $\ll$ Planck scale
- Gauge couplings almost unify
- Neutrino masses & large mixings
- Flavour: Patterns of masses & mixings
- Baryon asymmetry of the Universe
- Dark Matter
- Dark Energy

Hierarchy Problems

Emerge from scalars upon embedding / connecting to other vastly different scales

Solutions within d=4 QFT:

➔ **an additional symmetry**

- supersymmetry, conformal

➔ **a scale Λ where the scalar sector is composite**

- technicolor, other composite ideas

both:
symmetry breaking
Goldstone Bosons
TeV-ish new physics

Experiment:

Neither SUSY nor TeV-ish compositeness observed (so far)

➔ **little hierarchy problem (LHP)** ←→ BSM scale is too far away...

➔ **amplifies the old hierarchy problem (HP)**

Must solve both LHP and HP ➔ LHP first: parameter tuning *or* systematically?

➔ **symmetry: all scalars dof (including the Higgs particle) GBs or PGBs**

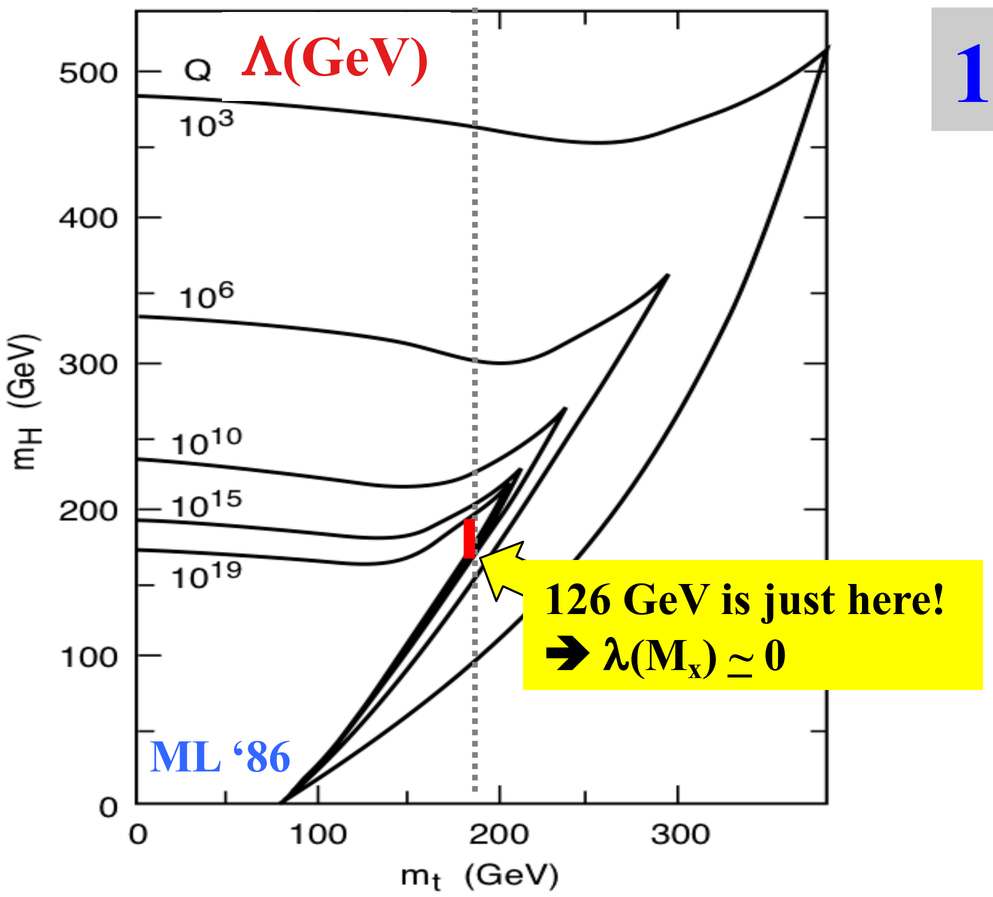
- problem: GB decay constant ←→ Λ
- relaxed in **little Higgs models** ←→ natural explanation of LHP

BUT: These models have scalars and scales ➔ only shifting problems?

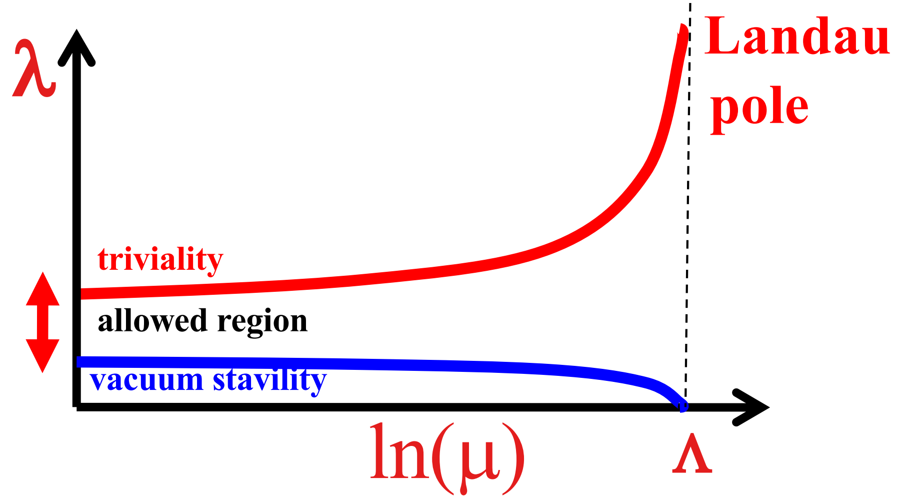
Another experimentally driven Observation

➔ SM is a renormalizable QFT like QED w/o hierarchy problem

➔ **Cutoff "Λ" has no meaning ➔ triviality, vacuum stability**



126 GeV $< m_H <$ 174 GeV



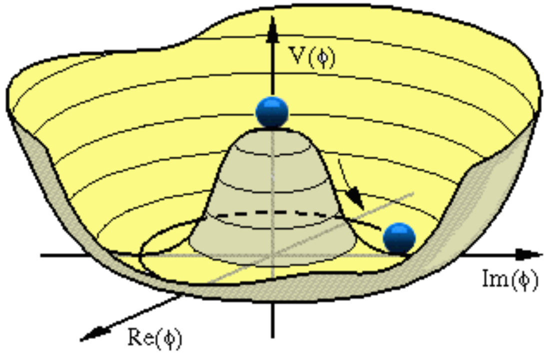
➔ SM quantum corrections OK over large scals distances

Important observation:

- **a remarkable relation between the weak scale, m_t, m_H, gauge couplings and Λ**
- **connected to log divergences – not to quadartic divergences ←➔ HP**

Is there a Message?

- $\lambda(M_X) \simeq 0$? ➔ remarkable log cancellations
- **remember: μ is the only single scale of the SM ➔ special role**
- if in addition $\mu^2 = 0$ ➔ $V(M_X) \simeq 0$
  ➔ Mexican hat becomes flat due to conspiring quantum effects



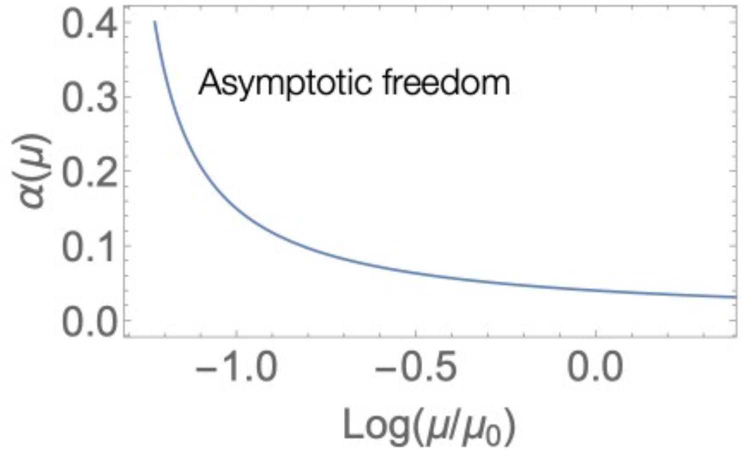
➔

- alternatively: All scalar and Yukawa couplings dissolve
  i.e. composite scalars ➔ potential dissolves (no metastability issues)

- **In both cases tempting: conformal (or shift) symmetry ←➔ HP?**

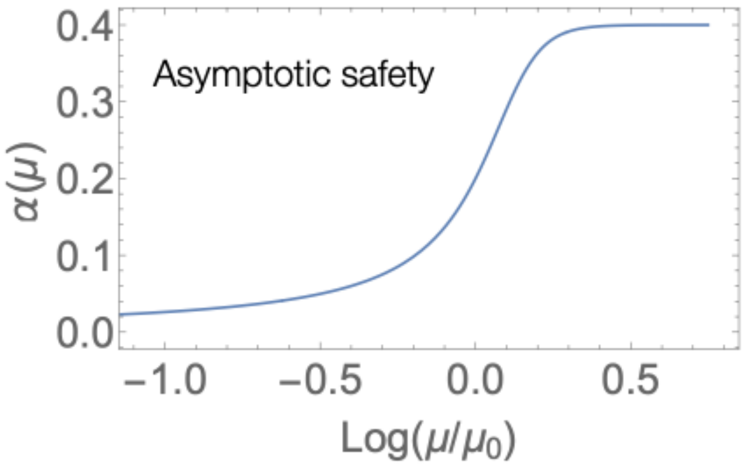
UV-Completion & Conformal Symmetry

Successful theories should have a meaningful UV-completion
➔ vanishing β-functions (UV fixedpoints) ←→ restored scale symmetry



Interacting UV-fixedpoint ➔

⇐ trivial fixedpoint



Interacting UV-fixedpoints:

- scalar and Yukawa couplings tend to have Landau poles, instability...
- requires carefully selected particle content ➔ explanation?

Trivial fixedpoints:

- no fundamental scalars
- no Yukawa couplings
- asymptotically free non-abelian gauge theories w/o scalars ➔ easy

Little Higgs + conformal UV Completion

Conformal little Higgs: **Ahmed, ML, Saake, 2309.07845, PRD 109.075041**

1) All scalars (including Higgs) are GBs or PGBs

→ scale **$\Lambda \simeq$ multi-TeV little Higgs model**

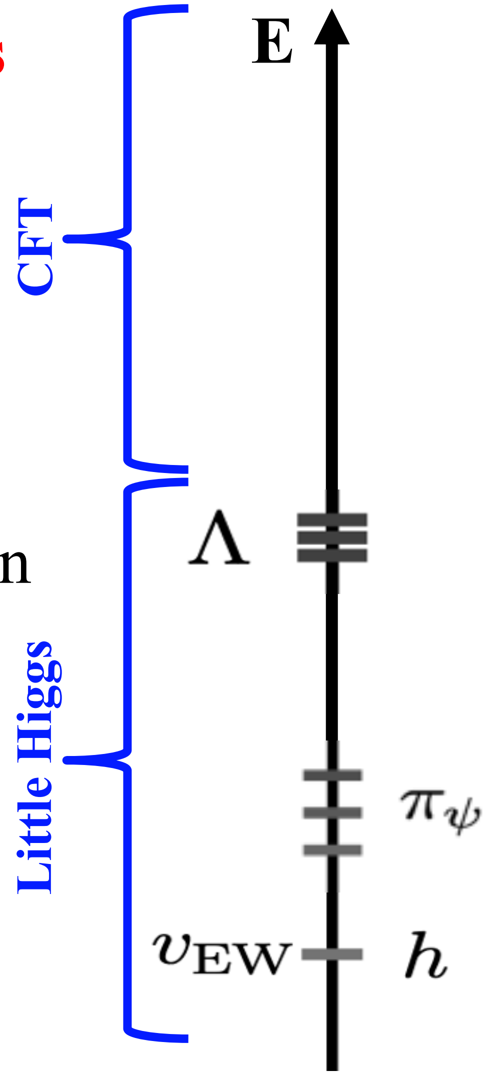
→ symmetry explanation of the LHP

→ all λ's and Yukawa couplings dissolve at Λ

2) Conformal non-abelian UV completion

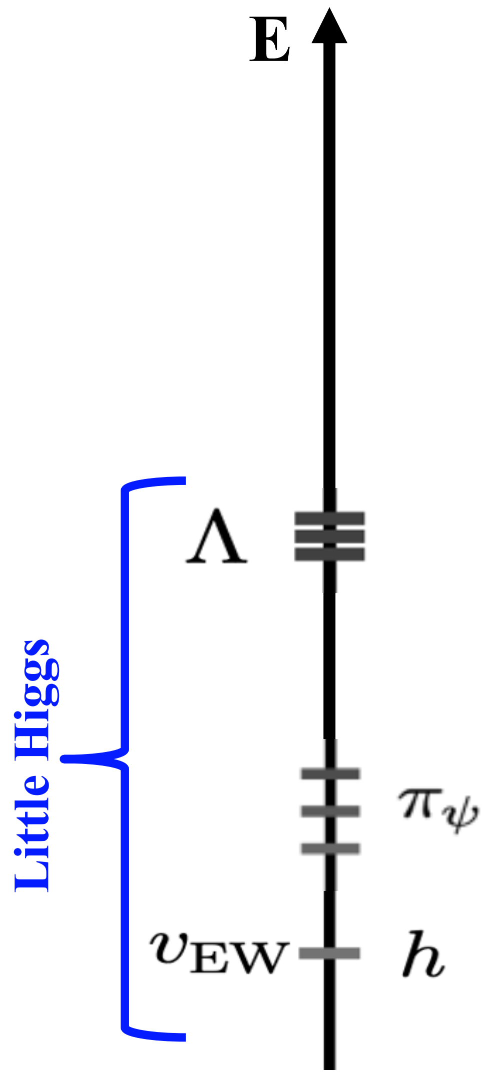
→ Λ becomes scale of a dimensional transmutation

→ no new scalars or scales ←→ HP



Remarks:
- realized for SM = CW, but works only for extended Higgs sectors
- can be combined with neutrino masses, DM, BAU, ...
- gravity – comments if time allows

A little Higgs reminder



Λ = scale of compositeness dynamics

- condensates generate GBs, PGBs

  $\mathcal{L}_{\rm kin} = f^2\,\partial_\mu\Sigma^\dagger\partial^\mu\Sigma$ → radiative: M_W, potential:

$$\mu^2 = c\frac{g^2}{16\pi^2}\Lambda^2 \sim c g^2 f^2, \quad \lambda = c'\frac{g^2}{f^2}\frac{1}{16\pi^2}\Lambda^2 \sim c' g^2$$

- f = 200-300 GeV ←→ correct EW scale (M_W)

  → Λ at most 2-3 TeV: exp. excluded operators

  → spectrum may contain lower lying states?

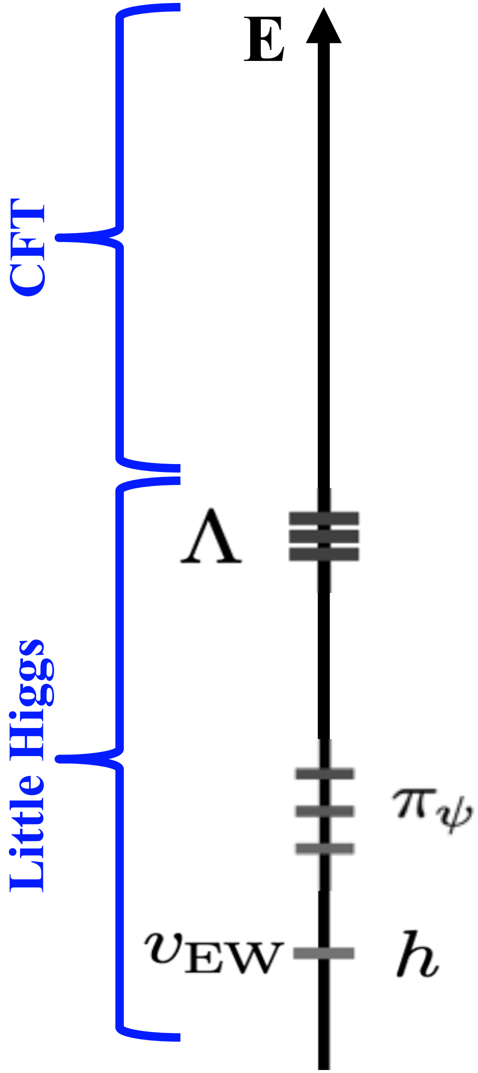
  c.f. techni-ρ in technicolor → S parameter...

- **little Higgs:** f can be O(TeV) → Λ = 5-10 TeV

$$\mu^2 \sim \frac{g^2}{16\pi^2} f^2 \log\frac{\Lambda^2}{f^2} \sim \frac{g^2}{8\pi^2} f^2 \log(4\pi)$$

- important: *all* scalar dof are GBs or PGBs
- lower lying bound states more remote

Conformal UV Completion



Suitable conformal theory:

- no fundamental scalars, no scales, χ-ral fermions
- non-abelian gauge group → asymptotically free
  → trivial UV fixedpoint
  → β=0 ←→ no conformal anomaly
  → IR dimensional transmutation like χ-ral QCD
- condensation → **little Higgs model**
- dynamical transmutation no y's or λ's beyond Λ
  → no Λ^2 corrections → no HP

Conformal Little Higgs Models

Ahmed, ML, Saake, arXiv: 2309.07845, PRD 109.075041

Exemplification for ``bested little Higgs'' model: Schmaltz, Stolarski, Thaler, 1006.1356

➔ UV completion without introducing any elementary/fundamental scalars

- confining non-abelian gauge symmetry SU(N_c) - we take $N_c = 2$
- new fermions:
  → ``technifermions''
  four light flavors

	$SU(N_c)$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\tilde{\psi} \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$	$\square$	$\mathbf{1}$	$\square$	0
$\psi' \equiv \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}$	$\square$	$\mathbf{1}$	$\begin{matrix} \mathbf{1} \\ \mathbf{1} \end{matrix}$	$\begin{matrix} -\frac{1}{2} \\ +\frac{1}{2} \end{matrix}$
$\chi \times N_m$	$\square$	$\mathbf{1}$	$\mathbf{1}$	0

- $SU(2)_L \subset SU(4)_L$ and the custodial group $SU(2)_L' \subset SU(4)_L$, respectively
- conjugate fields transform under the subgroups of $SU(4)_R$
- global symmetry breaking coset $SU(4)_L \times SU(4)_R / SU(4)_V$
- condensation ➔ flavor symmetry breaking

The Higgs Sector

- condensation ➔ 15 Goldstone bosons
- transform under the custodial symmetry SO(4) ≃ $SU(2)_L \times SU(2)_R \subset SU(4)V$ as $15_{SU(4)V}$ =(2,2)+(2,2)+(3,1)+(1,3)+(1,1)
- Goldstone matrix: $$U = \exp\left[i\Pi/\sqrt{2}f\right]$$
- where $$\Pi = \begin{pmatrix} \sigma^a \Delta_1^a + \eta/\sqrt{2} & -i\Phi_H \\ i\Phi_H^\dagger & \sigma^a \Delta_2^a - \eta/\sqrt{2} \end{pmatrix}$$
- with bi-doublet $$\Phi_H \equiv \left(\widetilde{H}_1 + i\widetilde{H}_2, \quad H_1 + iH_2\right); \quad \widetilde{H}_i \equiv i\sigma_2 H_i^*$$

  where H_i are Higgs doublets under SU(2)L
- and the triplets $$\sigma^a \Delta^a = \begin{pmatrix} \Delta^0 & \sqrt{2}\Delta^+ \\ \sqrt{2}\Delta^- & -\Delta^0 \end{pmatrix}$$

Phenomenology

- conformal symmetry is broken at $\Lambda \sim O(5\text{-}10)$ TeV by fermion condensate
  → spontaneous breaking of a global symmetries
  → no quadratic divergences in analogy to χ-ral QCD

- Higgs and partners emerge as pseudo-Goldstone Bosons
- low-energy phenomenology closely resembles ``bestest Little Higgs'' model
  →little hierarchy between SM and Λ explained by Little Higgs dynamics
- H_1 corresponds to the SM Higgs doublet
- H_2, scalar triplet Δ_1 and singlet η → substantial masses O(1) TeV
- heavy gauge boson partners W' and Z' → O(1) TeV
- fermionic top-partners have masses at the scale f

  → promising for future LHC runs

- The lightest stable neutral composite scalar can be a DM candidate.
- ...

Conformal Symmetry & Neutrino Masses

ML, S. Schmidt and J.Smirnov

- **No explicit scale ➔ no explicit (Dirac or Majorana) mass term**
  ➔ only Yukawa couplings ⊗ generic scales

- **Enlarge the Standard Model field spectrum**
  like in 0706.1829 - R. Foot, A. Kobakhidze, K.L. McDonald, R. Volkas

- **Consider direct product groups: SM ⊗ HS**

- **Two scales: $\boxed{\text{CS breaking scale at O(TeV) + induced EW scale}}$**

Important consequence for fermion mass terms:
➔ spectrum of Yukawa couplings ⊗TeV or ⊗EW scale
➔ interesting consequences ←→ Majorana mass terms are no longer expected at the generic L-breaking scale → anywhere

Examples

$$\mathcal{M} = \begin{pmatrix} 0 & y_D\langle H\rangle \\ y_D^T\langle H\rangle & y_M\langle\phi\rangle \end{pmatrix}$$

Yukawa seesaw:

SM + ν_R + singlet

$$\langle\phi\rangle \approx \text{TeV}$$

$$\langle H\rangle \approx 1/4\,\text{TeV}$$

➔ generically expect a TeV seesaw

BUT: y_M can be tiny

➔ wide range of sterile masses ➔ including pseudo-Dirac case

➔ suppressed $0\nu\beta\beta$

The punch line:

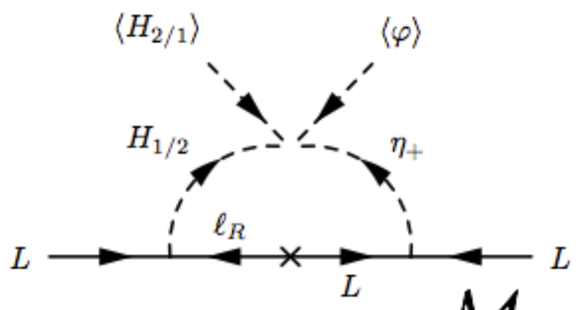
all usual neutrino mass terms can be generated

➔ suitable scalars required

➔ no explicit masses: all via Yukawa couplings

➔ different numerical expectations ←→ could easily explain keV masses

Radiative masses



$$\mathcal{M} = m_L \quad \text{or}$$

$$\mathcal{M} = \begin{pmatrix} \mu_1 & y_D\langle H\rangle \\ y_D^T\langle H\rangle & \mu_2 \end{pmatrix}$$

➔pseudo-Dirac case

The Planck Scale from CS Breaking

Conformal Gravity (CG):

- more symmetry ➔ claimed to be power counting renormalizable
- CG may have a ghost... ➔ see later
- Spontaneous generation (SG) of M_{Pl} = SG of Einstein-Hilbert theory
- most economic and simple way:

$$\frac{\xi_S}{2} S^2 R \to \frac{\xi_S}{2} \langle S \rangle^2 R \to \frac{M_{\rm Pl}^2}{2} R$$

$$M_{\rm Pl} = \sqrt{\xi_S} \langle S \rangle$$

Brans+Dicke,'61; Fujii,'74; Englert+Truffin+Grastmans,'76; Minkowsky,'77;......

Idea: Generate M_{Planck} from conformal gravity ⊗ SU(N)

➔ gauge assisted condensate via SU(N) field ➔ M_{Planck} = effective scale

Kubo, ML, Schmitz, Yamada similar ideas: **Donoghue, Menezes, ...**

$$S_C = \int d^4x \sqrt{-g} \Big[-\hat\beta\, S^\dagger S\, R + \hat\gamma\, R^2 - \frac{1}{2} \mathrm{Tr}\, F^2 + + g^{\mu\nu} (D_\mu S)^\dagger D_\nu S - \hat\lambda\, (S^\dagger S)^2 + a\, R_{\mu\nu} R^{\mu\nu} + b\, R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \Big]$$

R = Ricci curvature scalar, $R_{\mu\nu}$ = Ricci tensor, $R_{\mu\nu\alpha\beta}$ = Riemann tensor

F = field-strength tensor of the SU(N_c) gauge theory, **S = complex scalar in fund. rep. ➔ N_c**

➔ most general diffeomorphism invariance, gauge invariance, and global scale invariance

Condensation in SU(N_c) gauge sector

➔ **dimensional transmutation:** ⟨S^+S⟩ ➔ effective Planck mass

$$M_{planck} = 2\beta f_0 = \frac{N_c \beta}{16\pi^2} (2\lambda f_0) \left(1 + 2 \ln \frac{2\lambda f_0}{\Lambda^2}\right) \quad \text{with} \quad f_0 = \langle S^+ S \rangle$$

➔ Effectively normal gravity with a dynamically generated M_{Planck}

Dilaton-Scalaron Inflation

Effective Jordan-frame Lagrangian:

$$\frac{\mathcal{L}^J_{\rm eff}}{\sqrt{-g_J}} = -\frac{1}{2} B(\chi)\, M^2_{\rm Pl}\, R_J + G(\chi)\, R^2_J + \frac{1}{2} g^{\mu\nu}_J \partial_\mu \chi\, \partial_\nu \chi - U(\chi)$$ ➔ auxiliary field Ψ ➔

$$\frac{\mathcal{L}^J_{\rm eff}}{\sqrt{-g_J}} = -\left[\frac{1}{2} B(\chi)\, M^2_{\rm Pl} - 2\,G(\chi)\,\psi\right] R_J + \frac{1}{2} g^{\mu\nu}_J \partial_\mu \chi\, \partial_\nu \chi - U(\chi) - G(\chi)\,\psi^2$$

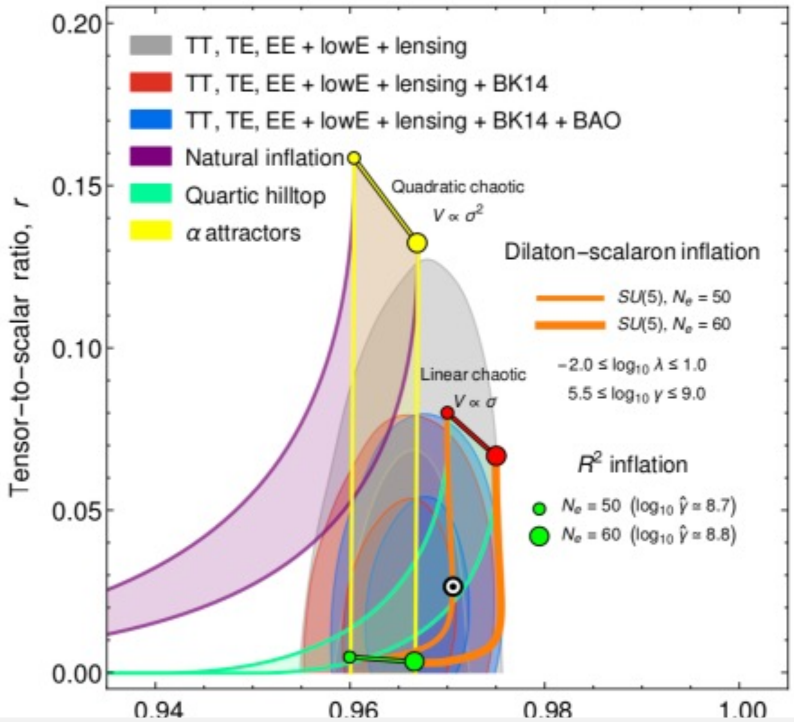
Weyl rescaling: $g_{\mu\nu} = \Omega^2\, g^J_{\mu\nu}$ $\quad \Omega^2 = e^{\Phi(\phi)}\,, \quad \Phi(\phi) = \frac{\sqrt{2}\,\phi}{\sqrt{3}\,M_{\rm Pl}}$

Einstein-frame scalar potential:

$$V(\chi,\phi) = e^{-2\,\Phi(\phi)}\left[U(\chi) + \frac{M^4_{\rm Pl}}{16\,G(\chi)}\left(B(\chi) - e^{\Phi(\phi)}\right)^2\right]$$

➔ Slow role inflation

➔ fits data very well!



The Ghost Problem in quadratic Gravity

Unlike GR, **quadratic gravity is renormalizable** thanks to four derivatives of the metric

$$\mathcal{L}_{\rm EH} = \sqrt{-g}\,\underbrace{M_{\rm pl}^2}_{\text{dimensionful}} R \qquad \mathcal{L}_{\rm QG} = \sqrt{-g}\Big(-\beta\phi^2 R + \gamma R^2 - \kappa C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\Big)$$

dimensionless

Problem: Double pole ➔ **classical Ostrogradsky instability**

$$\Delta_{hh} \sim \frac{1}{p^2} - \frac{1}{(p^2 - m_{\rm gh}^2)} \quad \Rightarrow \quad \mathcal{H} \sim c_+\pi_+^2 - c_-\pi_-^2 + \cdots$$

unbounded Hamiltonian

Leads after quantization to negative norm states ➔ **unitarity violation**

$$\begin{aligned} \left[\hat{a}_h(\mathbf{p}),\, \hat{a}_h^\dagger(\mathbf{q})\right] &= \delta^3(\mathbf{p}-\mathbf{q}) \\ \left[\hat{a}_H(\mathbf{p}),\, \hat{a}_H^\dagger(\mathbf{q})\right] &= -\delta^3(\mathbf{p}-\mathbf{q}) \end{aligned} \quad \Rightarrow \quad \sum_n \left| \langle n|S|\alpha\rangle \right|^2 \neq 1$$

breakdown of probability interpretation

Potential Solutions

- Remove ghosts from asymptotic spectrum Lee-Wick-style
  - Quantize ghosts as "fakeons" that don't appear by definition [Anselmi 1801.00915]
  - Demonstrate ghosts are unstable with nice decay products [Donoghue, Menezes 1908.02416]
- Use alternative quantization procedures
  - Define generalized QM norm [Salvio 1907.00983]
  - Employ (non-Hermitian) *PT*-symmetric QFT [Bender, Mannheim 0706.0207]

- **Unitarity OK if interaction energies are below the ghost mass**
  ➔ conformal theories OK if ghost becomes massive after SSB
  $M_{ghost} \simeq M_{Planck}$ ➔ no unitarity violation except in the early (pre-inflation) universe
  Kubo, Kuntz 2202.08298, 2208.12832

Conclusions

- **The Standard Model**
  - → works perfectly – no problems besides triviality, metastability
  - → list of unanswered questions / problems ←→ BSM
  - → lots of progress: DM, ν's, GR waves, ... + many new ideas
  - → hierarchy problem worsened due to the little hierarchy problem
  - → remarkable coincidence of parameters: flat Higgs potential @HE

- **Conformal little Higgs**
  - → a natural explanation of LHP: all scalar dof are GBs or PGBs
  - → conformal UV completion: avoid to reintroduce problems (fund. scalars)
  - → non-abelian gauge theory with fermions, gauge bosons and no scale
    - ➔ dimensional transmutation at multi Tev-ish Λ
    - ➔ GBs and PGBs explain scalar physics at EW scale
  - → generic mechanism – exemplified for ``bested little Higgs''

- **Not covered:**

  phenomenological implications, neutrino physics, dark matter, …

  combination with gravity (conformal gravity+breaking; inflation, ghosts?)