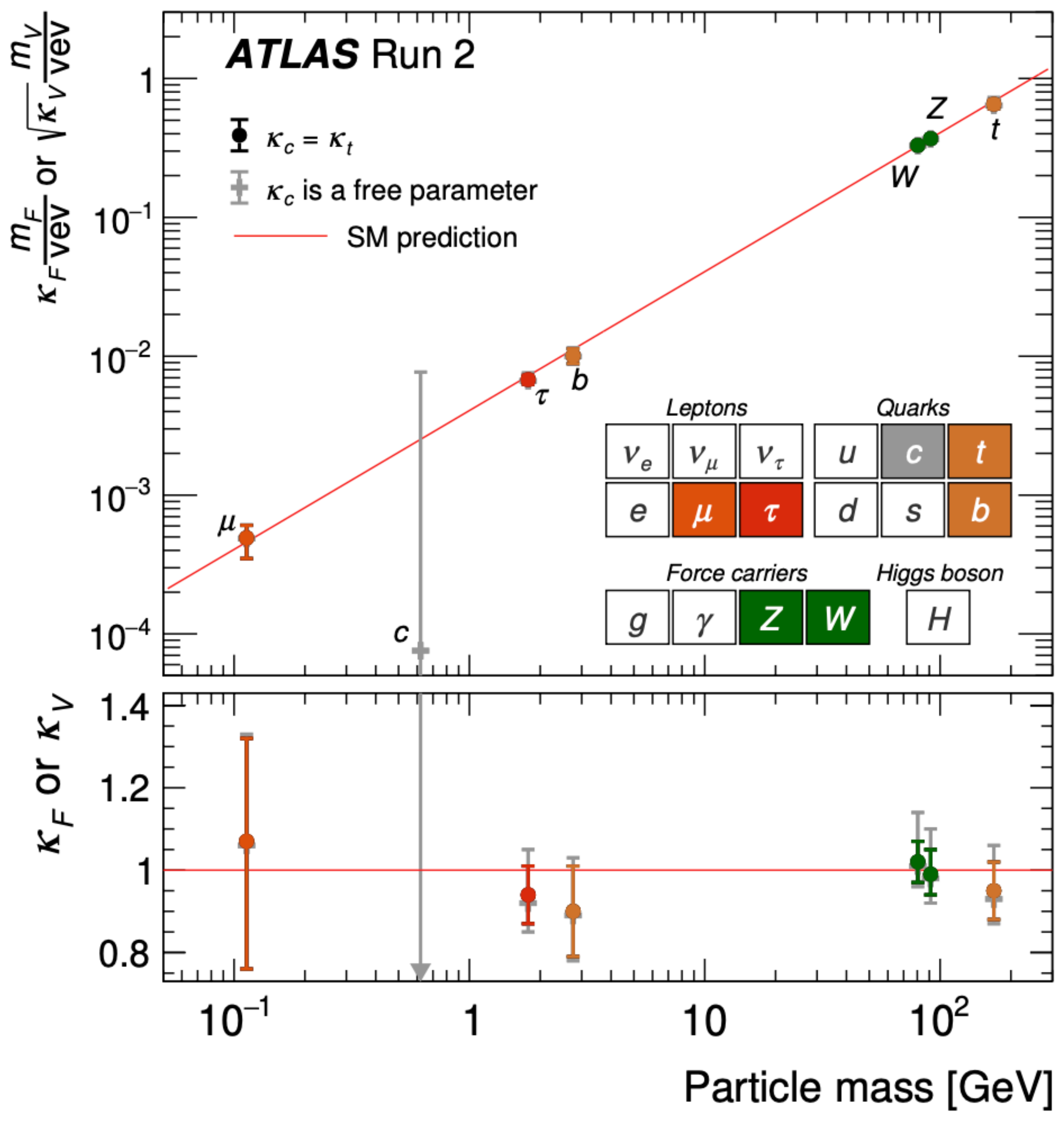
EFTs and their anomalous dimensions

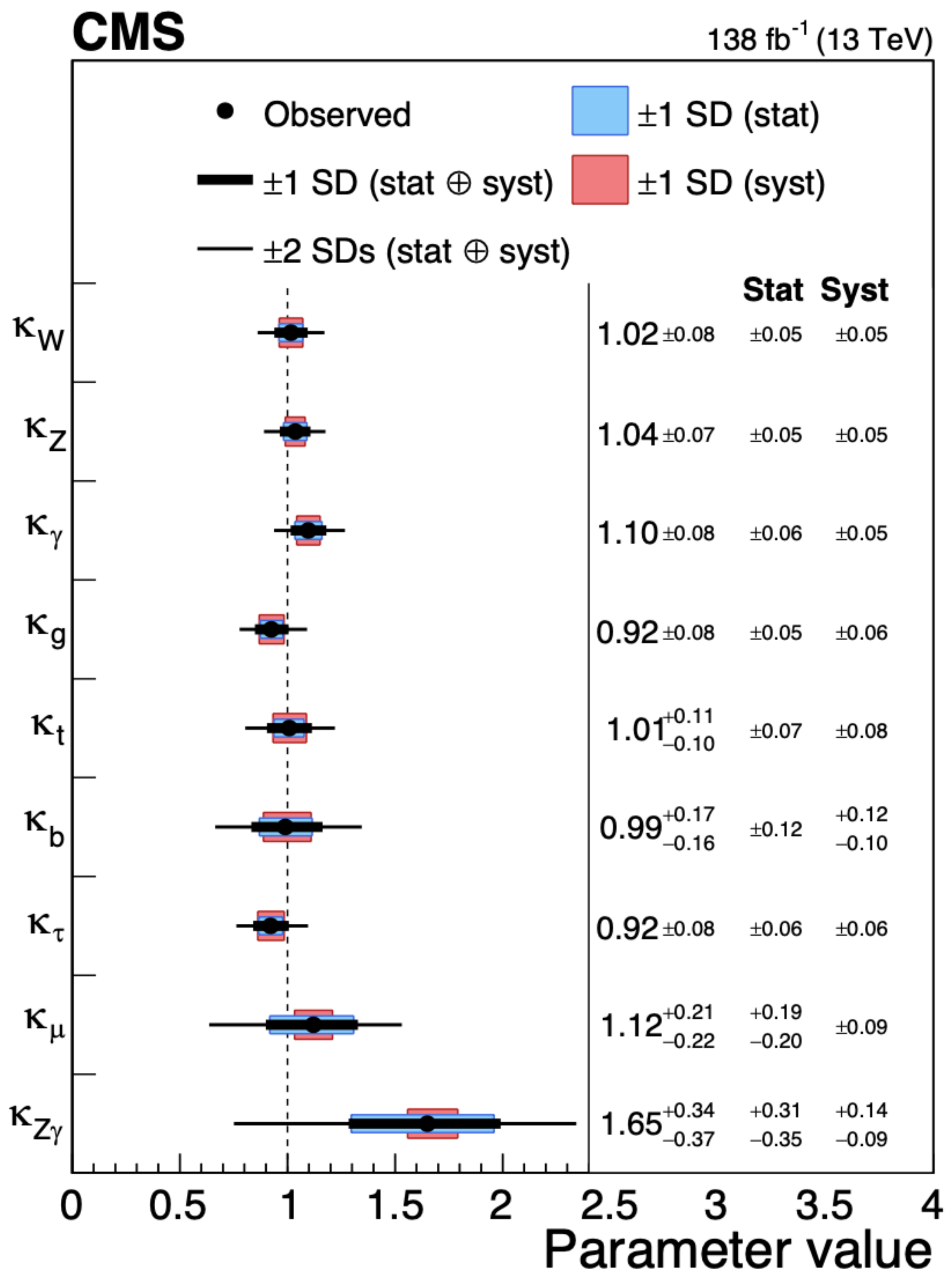
Matching and RGEs for EFTs at one loop and beyond.

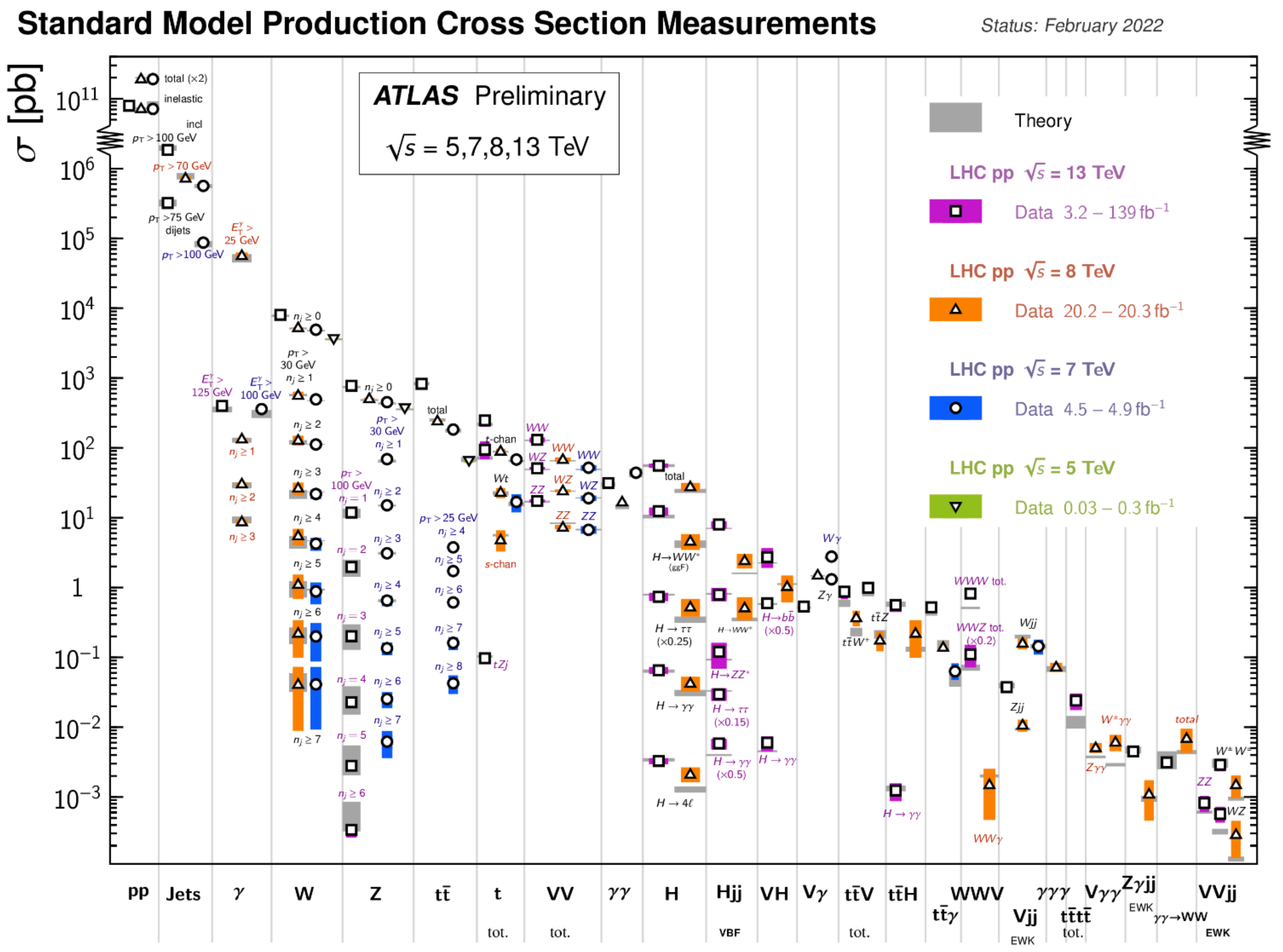
Achilleas Lazopoulos, 1 September 2024, Corfu Workshop on SM and Beyond

On EFTs and the SMEFT

The frustrating success of the Standard model



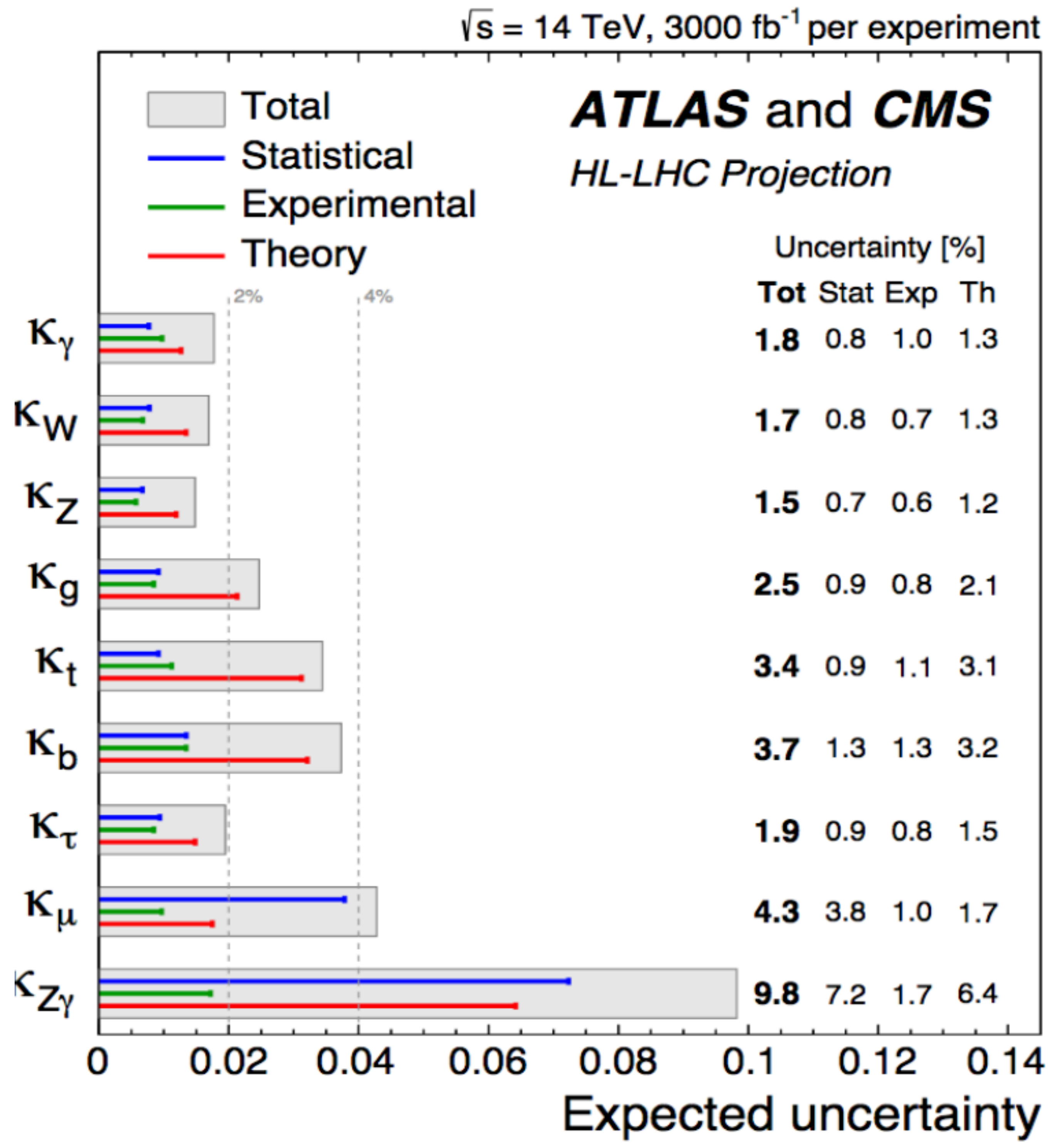




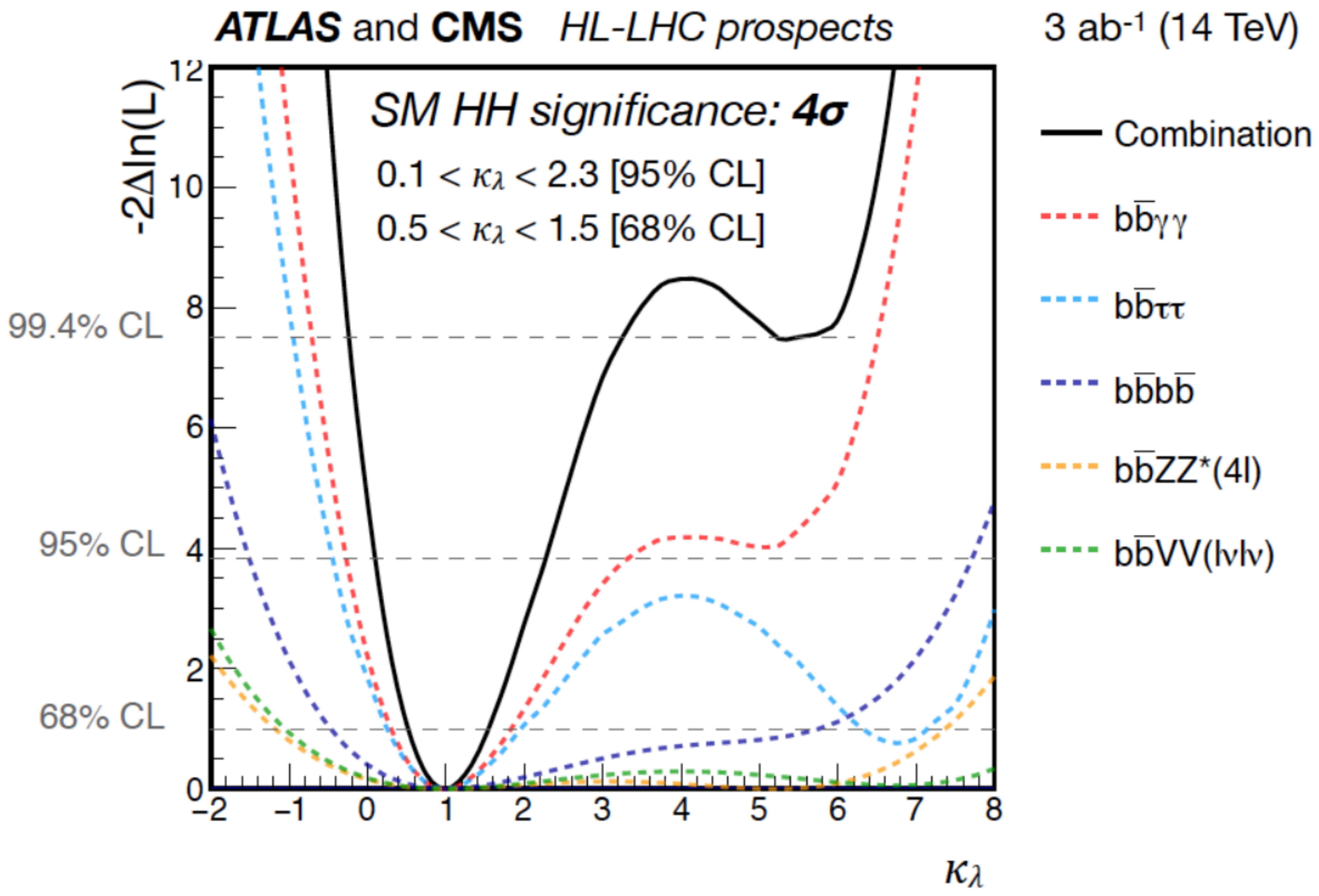
The foreseeable future: LHC as a precision machine

Precision on Higgs coupling strength modifiers κ_i

(assuming no BSM particles in Higgs boson decays)



Higgs boson self-coupling?



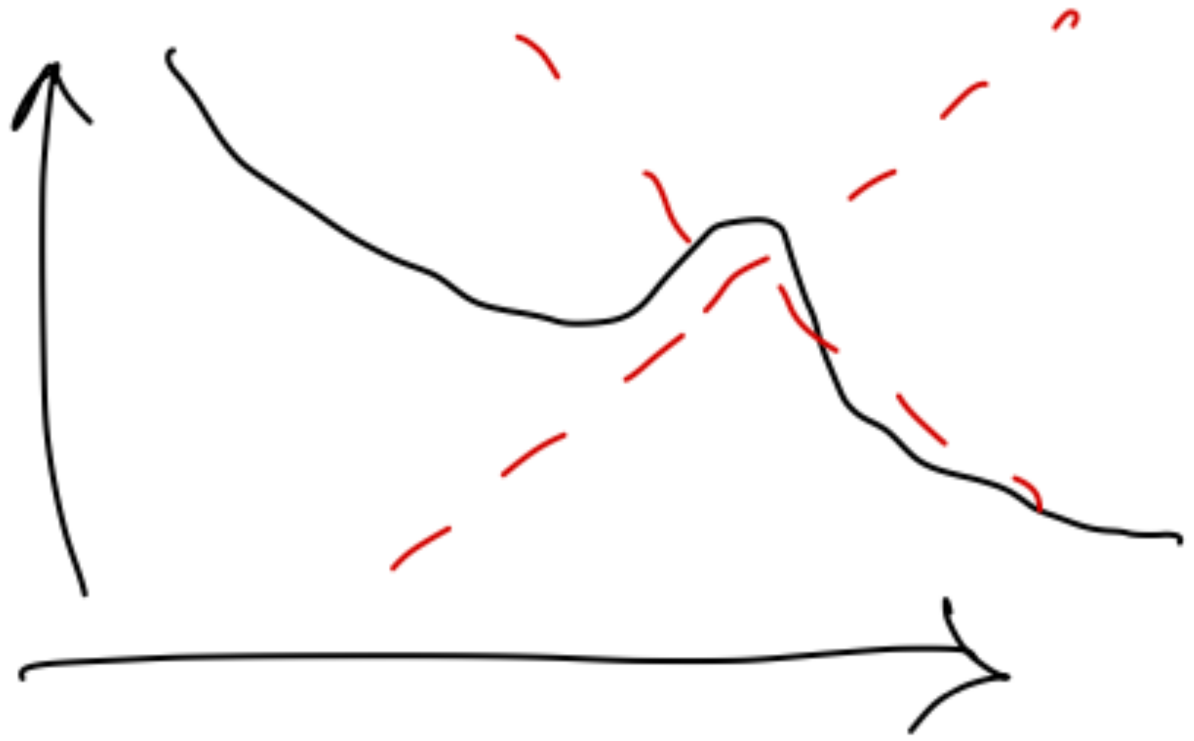
HL-LHC:

- Very significant improvement of the precision on the Higgs boson couplings (reach level of few %)
- First sensitivity on the Higgs boson self coupling (±50% uncertainty)

Slide by Karl Jakobs, Santander 2023

No energy upgrade till FCC ~2050

(and that’s assuming a next generation collider will be build)



No resonance production foreseeable

New physics might appear only as deviations
in Standard Model couplings due to New Physics.

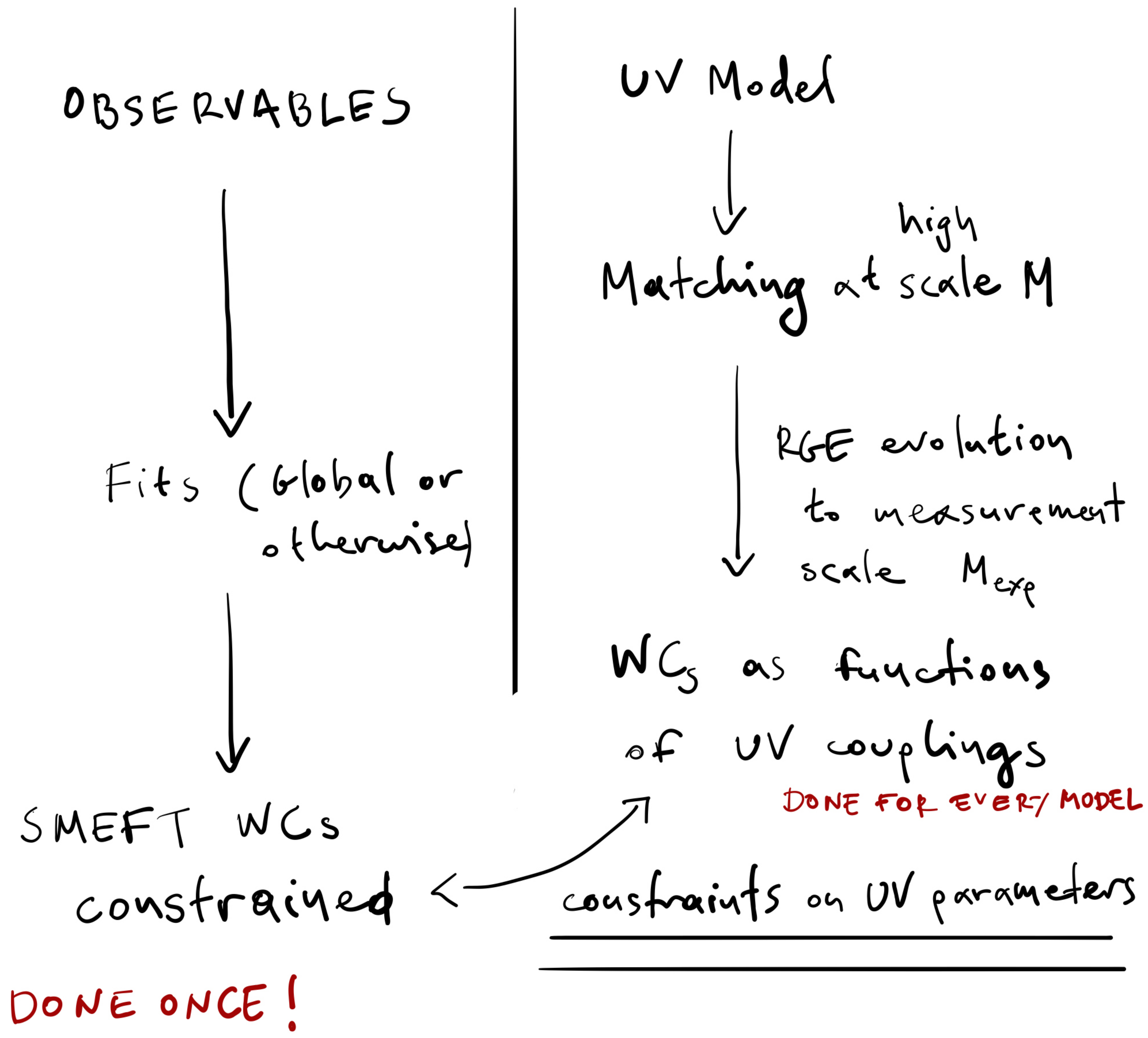
If there are resonances in the multi-Tev scale,
the most efficient way to parametrize deviations
and put bounds to UV models:

Effective field theories with the SMEFT as the main framework

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{C_i O_i}{\Lambda^2} + \sum_j \frac{C_j O_j}{\Lambda^4} + \dots$$

The SMEFT program

- Observables are computed within the SMEFT and a fit of the Wilson coefficients is performed to experimental data.
- Any specific UV model is matched to the SMEFT and their parameters are constrained via the WC constraints of the fit.
- The matching is done at a high scale and SMEFT RGEs are used down to the electroweak scale.



SMEFT

Matching and running at one loop necessary

- There are UV models for which important SMEFT operators do not receive contributions from tree level matching, so their parameters cannot be constraint by SMEFT fits.
- These operators receive contributions with one loop matching.
- Example: 2HDM where none of the dim 6 operators contributing to EW observables get tree-level matching contributions, see Dawson et al., 2401.12279
- Actually a whole set of SMEFT operators cannot be generated at tree level by any weakly coupled extension of the SM!

X^3	X^2H^2	$\psi^2 XH + \text{h.c.}$
$\mathcal{O}_{3G} = f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_{HG} = G^A_{\mu\nu} G^{A\mu\nu} H^\dagger H$	$\mathcal{O}_{uG} = (\bar{q} T^A \sigma^{\mu\nu} u) \widetilde{H} G^A_{\mu\nu}$
$\mathcal{O}_{\widetilde{3G}} = f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_{H\widetilde{G}} = \widetilde{G}^A_{\mu\nu} G^{A\mu\nu} H^\dagger H$	$\mathcal{O}_{uW} = (\bar{q} \sigma^{\mu\nu} u) \sigma^I \widetilde{H} W^I_{\mu\nu}$
$\mathcal{O}_{3W} = \epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathcal{O}_{HW} = W^I_{\mu\nu} W^{I\mu\nu} H^\dagger H$	$\mathcal{O}_{uB} = (\bar{q} \sigma^{\mu\nu} u) \widetilde{H} B_{\mu\nu}$
$\mathcal{O}_{\widetilde{3W}} = \epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathcal{O}_{H\widetilde{W}} = \widetilde{W}^I_{\mu\nu} W^{I\mu\nu} H^\dagger H$	$\mathcal{O}_{dG} = (\bar{q} T^A \sigma^{\mu\nu} d) H G^A_{\mu\nu}$
	$\mathcal{O}_{HB} = B_{\mu\nu} B^{\mu\nu} H^\dagger H$	$\mathcal{O}_{dW} = (\bar{q} \sigma^{\mu\nu} d) \sigma^I H W^I_{\mu\nu}$
	$\mathcal{O}_{H\widetilde{B}} = \widetilde{B}_{\mu\nu} B^{\mu\nu} H^\dagger H$	$\mathcal{O}_{dB} = (\bar{q} \sigma^{\mu\nu} d) H B_{\mu\nu}$
	$\mathcal{O}_{HWB} = W^I_{\mu\nu} B^{\mu\nu} H^\dagger \sigma^I H$	$\mathcal{O}_{eW} = (\bar{\ell} \sigma^{\mu\nu} e) \sigma^I H W^I_{\mu\nu}$
	$\mathcal{O}_{H\widetilde{W}B} = \widetilde{W}^I_{\mu\nu} B^{\mu\nu} H^\dagger \sigma^I H$	$\mathcal{O}_{eB} = (\bar{\ell} \sigma^{\mu\nu} e) H B_{\mu\nu}$

One Loop Matching and RGEs

One loop matching and RGEs

Overview

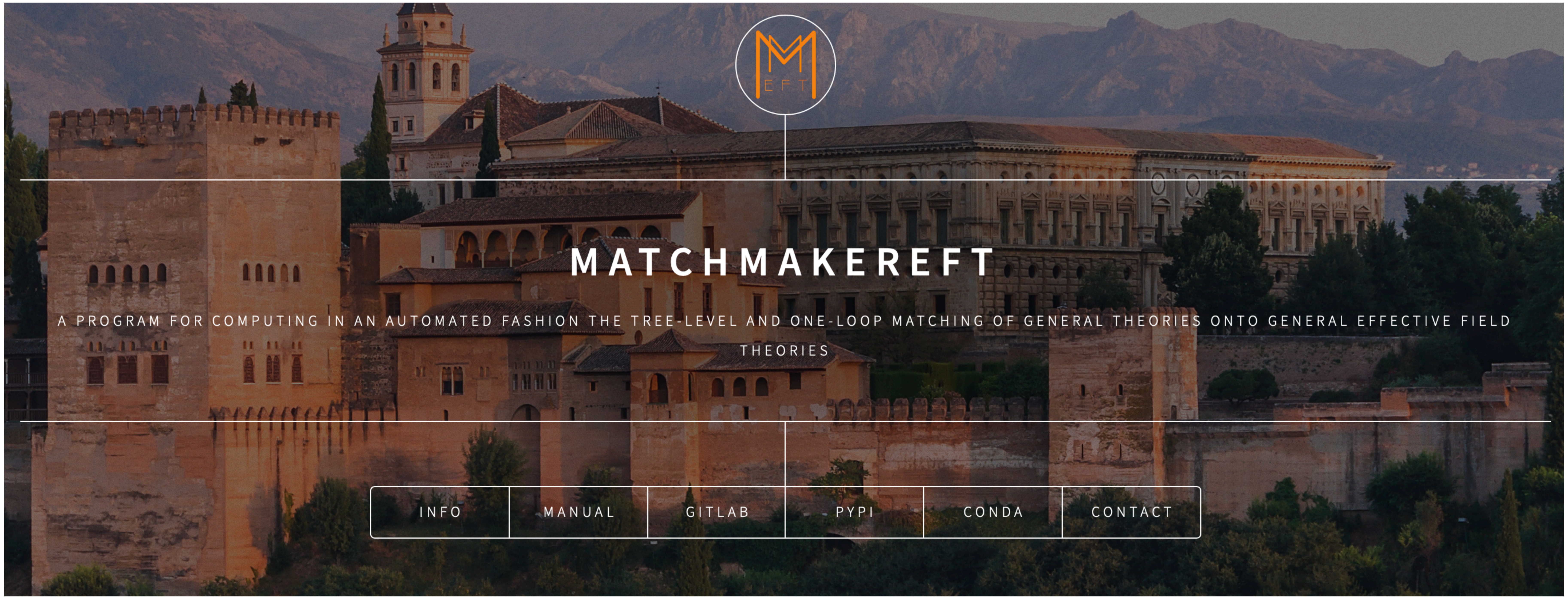
- Diagrammatic approach: compute the hard region of every one loop diagram, as a polynomial in masses and momenta, and match to the WC coefficients of the EFT. Carmona, Olgoso, AL, Santiago: 2112.10787 [hep-ph]

- Path integral approach: Compute the one loop correction to the Effective Action directly from the path integral, based on the concept of a covariant derivative expansion. Murayama et al, Ellis et al., Matchete collaboration (J. Fuentes et al. 2212.04510)

- For RGEs: Complete one loop RGEs in SMEFT dim 6, see Alonso, Jenkins, Manohar, Trott 1308.2627, 1310.4838, 1312.2014 and LEFT RGEs see Manohar, Jenkins, Stoffer, 1711.05270. Also recently dim 8 one loop RGEs by Boughezal et al. see 2408.15378.

One loop matching

Matchmakereft



Carmona, Olgoso, AL, Santiago: 2112.10787 [hep-ph]

A program to match an arbitrary UV model to an arbitrary EFT.

One loop matching

Matchmakereft

Carmona, Olgoso, AL, Santiago: 2112.10787 [hep-ph]

Approach:

- The UV model and the EFT model are arbitrary, an input in the form of Feynrules files.
- Matching performed off-shell: we match one-light-particle-irreducible functions
- We use Dimensional Regularization to control the singularities.
- We use the hard region expansion to isolate the UV part of 1LPI functions
- A workflow that involves QGRAF, Mathematica, Form and Python

Off-shell matching

We also need redundant operators, i.e. operators that can be related to the Physical Operators of the SMEFT Lagrangian by the equations of motion. We set up an extended, Green's basis, we perform the matching and only then use the equations of motion to relate the contributions of redundant operators to the physical ones.

X^2D^2	
$\mathcal{R}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$
$\mathcal{R}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$
$\mathcal{R}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$

H^2XD^2	
$\mathcal{R}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger \mathrm{i}\overleftrightarrow{D}^I_\mu H)$
$\mathcal{R}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger \mathrm{i}\overleftrightarrow{D}_\mu H)$

H^2D^4	
$\mathcal{R}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$

H^4D^2	
$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
$\mathcal{R}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{R}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger \mathrm{i}\overleftrightarrow{D}^\mu H)$

ψ^2D^3	
$\mathcal{R}_{qD}$	$\frac{\mathrm{i}}{2}\bar q\{D_\mu D^\mu, \not{D}\}q$
$\mathcal{R}_{uD}$	$\frac{\mathrm{i}}{2}\bar u\{D_\mu D^\mu, \not{D}\}u$
$\mathcal{R}_{dD}$	$\frac{\mathrm{i}}{2}\bar d\{D_\mu D^\mu, \not{D}\}d$
$\mathcal{R}_{\ell D}$	$\frac{\mathrm{i}}{2}\bar \ell\{D_\mu D^\mu, \not{D}\}\ell$
$\mathcal{R}_{eD}$	$\frac{\mathrm{i}}{2}\bar e\{D_\mu D^\mu, \not{D}\}e$

$\psi^2HD^2 + \mathrm{h.c.}$	
$\mathcal{R}_{uHD1}$	$(\bar q u)D_\mu D^\mu \widetilde{H}$
$\mathcal{R}_{uHD2}$	$(\bar q\,\mathrm{i}\sigma_{\mu\nu}D^\mu u)D^\nu\widetilde{H}$
$\mathcal{R}_{uHD3}$	$(\bar q D_\mu D^\mu u)\widetilde{H}$
$\mathcal{R}_{uHD4}$	$(\bar q D_\mu u)D^\mu\widetilde{H}$
$\mathcal{R}_{dHD1}$	$(\bar q d)D_\mu D^\mu H$
$\mathcal{R}_{dHD2}$	$(\bar q\,\mathrm{i}\sigma_{\mu\nu}D^\mu d)D^\nu H$
$\mathcal{R}_{dHD3}$	$(\bar q D_\mu D^\mu d)H$
$\mathcal{R}_{dHD4}$	$(\bar q D_\mu d)D^\mu H$
$\mathcal{R}_{eHD1}$	$(\bar\ell e)D_\mu D^\mu H$
$\mathcal{R}_{eHD2}$	$(\bar\ell\,\mathrm{i}\sigma_{\mu\nu}D^\mu e)D^\nu H$
$\mathcal{R}_{eHD3}$	$(\bar\ell D_\mu D^\mu e)H$
$\mathcal{R}_{eHD4}$	$(\bar\ell D_\mu e)D^\mu H$

$\psi^2XH + \mathrm{h.c.}$	
$\mathcal{O}_{uG}$	$(\bar q T^A\sigma^{\mu\nu}u)\widetilde{H}G^A_{\mu\nu}$
$\mathcal{O}_{uW}$	$(\bar q\sigma^{\mu\nu}u)\sigma^I\widetilde{H}W^I_{\mu\nu}$
$\mathcal{O}_{uB}$	$(\bar q\sigma^{\mu\nu}u)\widetilde{H}B_{\mu\nu}$
$\mathcal{O}_{dG}$	$(\bar q T^A\sigma^{\mu\nu}d)HG^A_{\mu\nu}$
$\mathcal{O}_{dW}$	$(\bar q\sigma^{\mu\nu}d)\sigma^I HW^I_{\mu\nu}$
$\mathcal{O}_{dB}$	$(\bar q\sigma^{\mu\nu}d)HB_{\mu\nu}$
$\mathcal{O}_{eW}$	$(\bar\ell\sigma^{\mu\nu}e)\sigma^I HW^I_{\mu\nu}$
$\mathcal{O}_{eB}$	$(\bar\ell\sigma^{\mu\nu}e)HB_{\mu\nu}$

ψ^2XD	
$\mathcal{R}_{Gq}$	$(\bar q T^A\gamma^\mu q)D^\nu G^A_{\mu\nu}$
$\mathcal{R}'_{Gq}$	$\frac{1}{2}(\bar q T^A\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu q)G^A_{\mu\nu}$
$\mathcal{R}'_{\widetilde{G}q}$	$\frac{1}{2}(\bar q T^A\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu q)\widetilde{G}^A_{\mu\nu}$
$\mathcal{R}_{Wq}$	$(\bar q\sigma^I\gamma^\mu q)D^\nu W^I_{\mu\nu}$
$\mathcal{R}'_{Wq}$	$\frac{1}{2}(\bar q\sigma^I\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu q)W^I_{\mu\nu}$
$\mathcal{R}'_{\widetilde{W}q}$	$\frac{1}{2}(\bar q\sigma^I\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu q)\widetilde{W}^I_{\mu\nu}$
$\mathcal{R}_{Bq}$	$(\bar q\gamma^\mu q)\partial^\nu B_{\mu\nu}$
$\mathcal{R}'_{Bq}$	$\frac{1}{2}(\bar q\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu q)B_{\mu\nu}$
$\mathcal{R}'_{\widetilde{B}q}$	$\frac{1}{2}(\bar q\gamma^\mu i\overleftrightarrow{D}^\nu q)\widetilde{B}_{\mu\nu}$
$\mathcal{R}_{Gu}$	$(\bar u T^A\gamma^\mu u)D^\nu G^A_{\mu\nu}$
$\mathcal{R}'_{Gu}$	$\frac{1}{2}(\bar u T^A\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu u)G^A_{\mu\nu}$
$\mathcal{R}'_{\widetilde{G}u}$	$\frac{1}{2}(\bar u T^A\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu u)\widetilde{G}^A_{\mu\nu}$
$\mathcal{R}_{Bu}$	$(\bar u\gamma^\mu u)\partial^\nu B_{\mu\nu}$
$\mathcal{R}'_{Bu}$	$\frac{1}{2}(\bar u\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu u)B_{\mu\nu}$
$\mathcal{R}'_{\widetilde{B}u}$	$\frac{1}{2}(\bar u\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu u)\widetilde{B}_{\mu\nu}$
$\mathcal{R}_{Gd}$	$(\bar d T^A\gamma^\mu d)D^\nu G^A_{\mu\nu}$
$\mathcal{R}'_{Gd}$	$\frac{1}{2}(\bar d T^A\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu d)G^A_{\mu\nu}$
$\mathcal{R}'_{\widetilde{G}d}$	$\frac{1}{2}(\bar d T^A\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu d)\widetilde{G}^A_{\mu\nu}$
$\mathcal{R}_{Bd}$	$(\bar d\gamma^\mu d)\partial^\nu B_{\mu\nu}$
$\mathcal{R}'_{Bd}$	$\frac{1}{2}(\bar d\gamma^\mu i\overleftrightarrow{D}^\nu d)B_{\mu\nu}$
$\mathcal{R}'_{\widetilde{B}d}$	$\frac{1}{2}(\bar d\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu d)\widetilde{B}_{\mu\nu}$
$\mathcal{R}_{W\ell}$	$(\bar\ell\sigma^I\gamma^\mu\ell)D^\nu W^I_{\mu\nu}$
$\mathcal{R}'_{W\ell}$	$\frac{1}{2}(\bar\ell\sigma^I\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu\ell)W^I_{\mu\nu}$
$\mathcal{R}'_{\widetilde{W}\ell}$	$\frac{1}{2}(\bar\ell\sigma^I\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu\ell)\widetilde{W}^I_{\mu\nu}$
$\mathcal{R}_{B\ell}$	$(\bar\ell\gamma^\mu\ell)\partial^\nu B_{\mu\nu}$
$\mathcal{R}'_{B\ell}$	$\frac{1}{2}(\bar\ell\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu\ell)B_{\mu\nu}$
$\mathcal{R}'_{\widetilde{B}\ell}$	$\frac{1}{2}(\bar\ell\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu\ell)\widetilde{B}_{\mu\nu}$
$\mathcal{R}_{Be}$	$(\bar e\gamma^\mu e)\partial^\nu B_{\mu\nu}$
$\mathcal{R}'_{Be}$	$\frac{1}{2}(\bar e\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu e)B_{\mu\nu}$
$\mathcal{R}'_{\widetilde{B}e}$	$\frac{1}{2}(\bar e\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu e)\widetilde{B}_{\mu\nu}$

ψ^2DH^2	
$\mathcal{O}^{(1)}_{Hq}$	$(\bar q\gamma^\mu q)(H^\dagger \mathrm{i}\overleftrightarrow{D}_\mu H)$
$\mathcal{R}'^{(1)}_{Hq}$	$(\bar q\,\mathrm{i}\overleftrightarrow{\not{D}}\,q)(H^\dagger H)$
$\mathcal{R}''^{(1)}_{Hq}$	$(\bar q\gamma^\mu q)\partial_\mu(H^\dagger H)$
$\mathcal{O}^{(3)}_{Hq}$	$(\bar q\sigma^I\gamma^\mu q)(H^\dagger \mathrm{i}\overleftrightarrow{D}^I_\mu H)$
$\mathcal{R}'^{(3)}_{Hq}$	$(\bar q\,\mathrm{i}\overleftrightarrow{\not{D}}^I q)(H^\dagger\sigma^I H)$
$\mathcal{R}''^{(3)}_{Hq}$	$(\bar q\sigma^I\gamma^\mu q)D_\mu(H^\dagger\sigma^I H)$
$\mathcal{O}_{Hu}$	$(\bar u\gamma^\mu u)(H^\dagger \mathrm{i}\overleftrightarrow{D}_\mu H)$
$\mathcal{R}'_{Hu}$	$(\bar u\,\mathrm{i}\overleftrightarrow{\not{D}}\,u)(H^\dagger H)$
$\mathcal{R}''_{Hu}$	$(\bar u\gamma^\mu u)\partial_\mu(H^\dagger H)$
$\mathcal{O}_{Hd}$	$(\bar d\gamma^\mu d)(H^\dagger \mathrm{i}\overleftrightarrow{D}_\mu H)$
$\mathcal{R}'_{Hd}$	$(\bar d\,\mathrm{i}\overleftrightarrow{\not{D}}\,d)(H^\dagger H)$
$\mathcal{R}''_{Hd}$	$(\bar d\gamma^\mu d)\partial_\mu(H^\dagger H)$
$\mathcal{O}_{Hud}$	$(\bar u\gamma^\mu d)(\widetilde{H}^\dagger \mathrm{i}D_\mu H)$
$\mathcal{O}^{(1)}_{H\ell}$	$(\bar\ell\gamma^\mu\ell)(H^\dagger \mathrm{i}\overleftrightarrow{D}_\mu H)$
$\mathcal{R}'^{(1)}_{H\ell}$	$(\bar\ell i\overleftrightarrow{\not{D}}\,\ell)(H^\dagger H)$
$\mathcal{R}''^{(1)}_{H\ell}$	$(\bar\ell\gamma^\mu\ell)\partial_\mu(H^\dagger H)$
$\mathcal{O}^{(3)}_{H\ell}$	$(\bar\ell\sigma^I\gamma^\mu\ell)(H^\dagger \mathrm{i}\overleftrightarrow{D}^I_\mu H)$
$\mathcal{R}'^{(3)}_{H\ell}$	$(\bar\ell\mathrm{i}\overleftrightarrow{\not{D}}^I\ell)(H^\dagger\sigma^I H)$
$\mathcal{R}''^{(3)}_{H\ell}$	$(\bar\ell\sigma^I\gamma^\mu\ell)D_\mu(H^\dagger\sigma^I H)$
$\mathcal{O}_{He}$	$(\bar e\gamma^\mu e)(H^\dagger \mathrm{i}\overleftrightarrow{D}_\mu H)$
$\mathcal{R}'_{He}$	$(\bar e\,\mathrm{i}\overleftrightarrow{\not{D}}\,e)(H^\dagger H)$
$\mathcal{R}''_{He}$	$(\bar e\gamma^\mu e)\partial_\mu(H^\dagger H)$

$\psi^2H^3 + \mathrm{h.c.}$	
$\mathcal{O}_{uH}$	$(H^\dagger H)\bar q\widetilde{H}u$
$\mathcal{O}_{dH}$	$(H^\dagger H)\bar q Hd$
$\mathcal{O}_{eH}$	$(H^\dagger H)\bar\ell He$

D-dimensional computation

Since we compute at one-loop, we need a UV regulator. We use dimensional regularization, and therefore, relations between operators that are strictly valid in 4 dimensions (gamma chain reduction and Fierz), now leave remnant contributions, captured by Evanescent operators.

$\Psi^2 XH + \text{h.c.}$		$\Psi^2 XD$			
$\mathcal{E}_{uG}$	$\bar{q}T^A\sigma^{\mu\nu}u\widetilde{H}\widetilde{G}^A_{\mu\nu}$	$\mathcal{E}_{Gq}$	$\bar{q}T^A(\sigma^{\mu\nu}\gamma^\rho+\gamma^\rho\sigma^{\mu\nu})qD_\rho\widetilde{G}^A_{\mu\nu}$	$\mathcal{E}_{Gd}$	$\bar{d}T^A(\sigma^{\mu\nu}\gamma^\rho+\gamma^\rho\sigma^{\mu\nu})dD_\rho\widetilde{G}^A_{\mu\nu}$
$\mathcal{E}_{uW}$	$\bar{q}\sigma^I\sigma^{\mu\nu}u\widetilde{H}\widetilde{W}^I_{\mu\nu}$	$\mathcal{E}'_{Gq}$	$\mathrm{i}\bar{q}(T^A\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu}T^A)qG^A_{\mu\nu}$	$\mathcal{E}'_{Gd}$	$\mathrm{i}\bar{d}(T^A\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu}T^A)dG^A_{\mu\nu}$
$\mathcal{E}_{uB}$	$\bar{q}\sigma^{\mu\nu}u\widetilde{H}\widetilde{B}_{\mu\nu}$	$\mathcal{E}'_{\widetilde{G}q}$	$\mathrm{i}\bar{q}(T^A\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu}T^A)q\widetilde{G}^A_{\mu\nu}$	$\mathcal{E}'_{\widetilde{G}d}$	$\mathrm{i}\bar{d}(T^A\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu}T^A)d\widetilde{G}^A_{\mu\nu}$
$\mathcal{E}_{dG}$	$\bar{q}T^A\sigma^{\mu\nu}dH\widetilde{G}^A_{\mu\nu}$	$\mathcal{E}_{Wq}$	$\bar{q}\sigma^I(\sigma^{\mu\nu}\gamma^\rho+\gamma^\rho\sigma^{\mu\nu})qD_\rho\widetilde{W}^I_{\mu\nu}$	$\mathcal{E}_{Bd}$	$\bar{d}(\sigma^{\mu\nu}\gamma^\rho+\gamma^\rho\sigma^{\mu\nu})d\partial_\rho\widetilde{B}_{\mu\nu}$
$\mathcal{E}_{dW}$	$\bar{q}\sigma^I\sigma^{\mu\nu}dH\widetilde{W}^I_{\mu\nu}$	$\mathcal{E}'_{Wq}$	$\mathrm{i}\bar{q}(\sigma^I\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu}\sigma^I)qW^I_{\mu\nu}$	$\mathcal{E}'_{Bd}$	$\mathrm{i}\bar{d}(\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu})dB^A_{\mu\nu}$
$\mathcal{E}_{dB}$	$\bar{q}\sigma^{\mu\nu}dH\widetilde{B}_{\mu\nu}$	$\mathcal{E}'_{\widetilde{W}q}$	$\mathrm{i}\bar{q}(\sigma^I\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu}\sigma^I)q\widetilde{W}^I_{\mu\nu}$	$\mathcal{E}'_{\widetilde{B}d}$	$\mathrm{i}\bar{d}(\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu})d\widetilde{B}_{\mu\nu}$
$\mathcal{E}_{eW}$	$\bar{\ell}\sigma^I\sigma^{\mu\nu}eH\widetilde{W}^I_{\mu\nu}$	$\mathcal{E}_{Bq}$	$\bar{q}(\sigma^{\mu\nu}\gamma^\rho+\gamma^\rho\sigma^{\mu\nu})q\partial_\rho\widetilde{B}_{\mu\nu}$	$\mathcal{E}_{W\ell}$	$\bar{\ell}\sigma^I(\sigma^{\mu\nu}\gamma^\rho+\gamma^\rho\sigma^{\mu\nu})\ell D_\rho\widetilde{W}^I_{\mu\nu}$
$\mathcal{E}_{eB}$	$\bar{\ell}\sigma^{\mu\nu}eH\widetilde{B}_{\mu\nu}$	$\mathcal{E}'_{Bq}$	$\mathrm{i}\bar{q}(\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu})qB_{\mu\nu}$	$\mathcal{E}'_{W\ell}$	$\mathrm{i}\bar{\ell}(\sigma^I\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu}\sigma^I)\ell W^I_{\mu\nu}$
$\psi^2 HD^2 + \text{h.c.}$		$\mathcal{E}'_{\widetilde{B}q}$	$\mathrm{i}\bar{q}(\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu})q\widetilde{B}_{\mu\nu}$	$\mathcal{E}'_{\widetilde{W}\ell}$	$\mathrm{i}\bar{\ell}(\sigma^I\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu}\sigma^I)\ell\widetilde{W}^I_{\mu\nu}$
$\mathcal{E}_{uH}$	$\bar{q}\sigma^{\mu\nu}D^\rho uD^\sigma\widetilde{H}\epsilon_{\mu\nu\rho\sigma}$	$\mathcal{E}_{Gu}$	$\bar{u}T^A(\sigma^{\mu\nu}\gamma^\rho+\gamma^\rho\sigma^{\mu\nu})uD_\rho\widetilde{G}^A_{\mu\nu}$	$\mathcal{E}_{B\ell}$	$\bar{\ell}(\sigma^{\mu\nu}\gamma^\rho+\gamma^\rho\sigma^{\mu\nu})\ell\partial_\rho\widetilde{B}_{\mu\nu}$
$\mathcal{E}_{dH}$	$\bar{q}\sigma^{\mu\nu}D^\rho dD^\sigma H\epsilon_{\mu\nu\rho\sigma}$	$\mathcal{E}'_{Gu}$	$\mathrm{i}\bar{u}(T^A\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu}T^A)uG^A_{\mu\nu}$	$\mathcal{E}'_{B\ell}$	$\mathrm{i}\bar{\ell}(\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu})\ell B_{\mu\nu}$
$\mathcal{E}_{eH}$	$\bar{\ell}\sigma^{\mu\nu}D^\rho eD^\sigma H\epsilon_{\mu\nu\rho\sigma}$	$\mathcal{E}'_{\widetilde{G}u}$	$\mathrm{i}\bar{u}(T^A\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu}T^A)u\widetilde{G}^A_{\mu\nu}$	$\mathcal{E}'_{\widetilde{B}\ell}$	$\mathrm{i}\bar{\ell}(\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu})\ell\widetilde{B}_{\mu\nu}$
		$\mathcal{E}_{Bu}$	$\bar{u}(\sigma^{\mu\nu}\gamma^\rho+\gamma^\rho\sigma^{\mu\nu})u\partial_\rho\widetilde{B}_{\mu\nu}$	$\mathcal{E}_{Be}$	$\bar{e}(\sigma^{\mu\nu}\gamma^\rho+\gamma^\rho\sigma^{\mu\nu})e\partial_\rho\widetilde{B}_{\mu\nu}$
		$\mathcal{E}'_{Bu}$	$\mathrm{i}\bar{u}(\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu})uB_{\mu\nu}$	$\mathcal{E}'_{Be}$	$\mathrm{i}\bar{e}(\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu})eB_{\mu\nu}$
		$\mathcal{E}'_{\widetilde{B}u}$	$\mathrm{i}\bar{u}(\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu})u\widetilde{B}_{\mu\nu}$	$\mathcal{E}'_{\widetilde{B}e}$	$\mathrm{i}\bar{e}(\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu})e\widetilde{B}_{\mu\nu}$

Table 4: Evanescent operators with two fermions.

$\bar{L}R\bar{R}L$		$\bar{R}R\bar{R}R$		$\bar{L}L\bar{R}R$	
$\mathcal{E}_{qu}$	$(\bar{q}u)(\bar{u}q)$	$\mathcal{E}^{(8)}_{uu}$	$(\bar{u}\gamma^\mu T^Au)(\bar{u}\gamma_\mu T^Au)$	$\mathcal{E}^{[3]}_{qu}$	$(\bar{q}\gamma^{\mu\nu\rho}q)(\bar{u}\gamma_{\mu\nu\rho}u)$
$\mathcal{E}^{(8)}_{qu}$	$(\bar{q}T^Au)(\bar{u}T^Aq)$	$\mathcal{E}^{[3]}_{uu}$	$(\bar{u}\gamma^{\mu\nu\rho}u)(\bar{u}\gamma_{\mu\nu\rho}u)$	$\mathcal{E}^{[3](8)}_{qu}$	$(\bar{q}\gamma^{\mu\nu\rho}T^Aq)(\bar{u}\gamma_{\mu\nu\rho}T^Au)$
$\mathcal{E}_{qd}$	$(\bar{q}d)(\bar{d}q)$	$\mathcal{E}^{[3](8)}_{uu}$	$(\bar{u}\gamma^{\mu\nu\rho}T^Au)(\bar{u}\gamma_{\mu\nu\rho}T^Au)$	$\mathcal{E}^{[3]}_{qd}$	$(\bar{q}\gamma^{\mu\nu\rho}q)(\bar{d}\gamma_{\mu\nu\rho}d)$
$\mathcal{E}^{(8)}_{qd}$	$(\bar{q}T^Ad)(\bar{d}T^Aq)$	$\mathcal{E}^{(8)}_{dd}$	$(\bar{d}\gamma^\mu T^Ad)(\bar{d}\gamma_\mu T^Ad)$	$\mathcal{E}^{[3](8)}_{qd}$	$(\bar{q}\gamma^{\mu\nu\rho}T^Aq)(\bar{d}\gamma_{\mu\nu\rho}T^Ad)$
$\mathcal{E}^{[2]}_{qu}$	$(\bar{q}\gamma^{\mu\nu}u)(\bar{u}\gamma_{\mu\nu}q)$	$\mathcal{E}^{[3]}_{dd}$	$(\bar{d}\gamma^{\mu\nu\rho}d)(\bar{d}\gamma_{\mu\nu\rho}d)$	$\bar{L}L\bar{L}L$	
$\mathcal{E}^{[2](8)}_{qu}$	$(\bar{q}\gamma^{\mu\nu}T^Au)(\bar{u}\gamma_{\mu\nu}T^Aq)$	$\mathcal{E}^{[3](8)}_{dd}$	$(\bar{d}\gamma^{\mu\nu\rho}T^Ad)(\bar{d}\gamma_{\mu\nu\rho}T^Ad)$	$\mathcal{E}^{(8)}_{qq}$	$(\bar{q}\gamma^\mu T^Aq)(\bar{q}\gamma_\mu T^Aq)$
$\mathcal{E}^{[2]}_{qd}$	$(\bar{q}\gamma^{\mu\nu}d)(\bar{d}\gamma_{\mu\nu}q)$	$\mathcal{E}_{ud}$	$(\bar{u}\gamma^\mu d)(\bar{d}\gamma_\mu u)$	$\mathcal{E}^{(3,8)}_{qq}$	$(\bar{q}\gamma^\mu\sigma^IT^Aq)(\bar{q}\gamma_\mu\sigma^IT^Aq)$
$\mathcal{E}^{[2](8)}_{qd}$	$(\bar{q}\gamma^{\mu\nu}T^Ad)(\bar{d}\gamma_{\mu\nu}T^Aq)$	$\mathcal{E}^{(8)}_{ud}$	$(\bar{u}\gamma^\mu T^Ad)(\bar{d}\gamma_\mu T^Au)$	$\mathcal{E}^{[3](1)}_{qq}$	$(\bar{q}\gamma^{\mu\nu\rho}q)(\bar{q}\gamma_{\mu\nu\rho}q)$
$\bar{L}R\bar{L}R$		$\mathcal{E}^{[3]}_{ud}$	$(\bar{u}\gamma^{\mu\nu\rho}d)(\bar{d}\gamma_{\mu\nu\rho}u)$	$\mathcal{E}^{[3](3)}_{qq}$	$(\bar{q}\gamma^{\mu\nu\rho}\sigma^Iq)(\bar{q}\gamma_{\mu\nu\rho}\sigma^Iq)$
$\mathcal{E}^{[2]}_{quqd}$	$(\bar{q}_r\gamma^{\mu\nu}u)\epsilon_{rs}(\bar{q}_s\gamma_{\mu\nu}d)$	$\mathcal{E}^{[3](8)}_{ud}$	$(\bar{u}\gamma^{\mu\nu\rho}T^Ad)(\bar{d}\gamma_{\mu\nu\rho}T^Au)$	$\mathcal{E}^{[3](8)}_{qq}$	$(\bar{q}\gamma^{\mu\nu\rho}T^Aq)(\bar{q}\gamma_{\mu\nu\rho}T^Aq)$
$\mathcal{E}^{[2](8)}_{quqd}$	$(\bar{q}_r\gamma^{\mu\nu}T^Au)\epsilon_{rs}(\bar{q}_s\gamma_{\mu\nu}T^Ad)$	$\mathcal{E}'^{[3]}_{ud}$	$(\bar{u}\gamma^{\mu\nu\rho}u)(\bar{d}\gamma_{\mu\nu\rho}d)$	$\mathcal{E}^{[3](3,8)}_{qq}$	$(\bar{q}\gamma^{\mu\nu\rho}\sigma^IT^Aq)(\bar{q}\gamma_{\mu\nu\rho}\sigma^IT^Aq)$
		$\mathcal{E}'^{[3](8)}_{ud}$	$(\bar{u}\gamma^{\mu\nu\rho}T^Au)(\bar{d}\gamma_{\mu\nu\rho}T^Ad)$		

Table 5: Evanescent operators with four fermions involving only quarks.

$\bar{L}R\bar{R}L$		$\bar{R}R\bar{R}R$		$\bar{L}L\bar{R}R$	
$\mathcal{E}_{\ell u}$	$(\bar{\ell}u)(\bar{u}\ell)$	$\mathcal{E}_{eu}$	$(\bar{e}\gamma^\mu u)(\bar{u}\gamma_\mu e)$	$\mathcal{E}_{\ell qde}$	$(\bar{\ell}\gamma^\mu q)(\bar{d}\gamma_\mu e)$
$\mathcal{E}_{\ell d}$	$(\bar{\ell}d)(\bar{d}\ell)$	$\mathcal{E}_{ed}$	$(\bar{e}\gamma^\mu d)(\bar{d}\gamma_\mu e)$	$\mathcal{E}^{[3]}_{\ell u}$	$(\bar{\ell}\gamma^{\mu\nu\rho}\ell)(\bar{u}\gamma_{\mu\nu\rho}u)$
$\mathcal{E}_{qe}$	$(\bar{q}e)(\bar{e}q)$	$\mathcal{E}^{[3]}_{eu}$	$(\bar{e}\gamma^{\mu\nu\rho}u)(\bar{u}\gamma_{\mu\nu\rho}e)$	$\mathcal{E}^{[3]}_{\ell d}$	$(\bar{\ell}\gamma^{\mu\nu\rho}\ell)(\bar{d}\gamma_{\mu\nu\rho}d)$
$\mathcal{E}^{[2]}_{\ell edq}$	$(\bar{\ell}\gamma^{\mu\nu}e)(\bar{d}\gamma_{\mu\nu}q)$	$\mathcal{E}^{[3]}_{ed}$	$(\bar{e}\gamma^{\mu\nu\rho}d)(\bar{d}\gamma_{\mu\nu\rho}e)$	$\mathcal{E}^{[3]}_{qe}$	$(\bar{q}\gamma^{\mu\nu\rho}q)(\bar{e}\gamma_{\mu\nu\rho}e)$
$\mathcal{E}^{[2]}_{\ell u}$	$(\bar{\ell}\gamma^{\mu\nu}u)(\bar{u}\gamma_{\mu\nu}\ell)$	$\mathcal{E}'^{[3]}_{eu}$	$(\bar{e}\gamma^{\mu\nu\rho}e)(\bar{u}\gamma_{\mu\nu\rho}u)$	$\mathcal{E}^{[3]}_{\ell qde}$	$(\bar{\ell}\gamma^{\mu\nu\rho}q)(\bar{d}\gamma_{\mu\nu\rho}e)$
$\mathcal{E}^{[2]}_{\ell d}$	$(\bar{\ell}\gamma^{\mu\nu}d)(\bar{d}\gamma_{\mu\nu}\ell)$	$\mathcal{E}'^{[3]}_{ed}$	$(\bar{e}\gamma^{\mu\nu\rho}e)(\bar{d}\gamma_{\mu\nu\rho}d)$	$\bar{L}L\bar{L}L$	
$\mathcal{E}^{[2]}_{qe}$	$(\bar{q}\gamma^{\mu\nu}e)(\bar{e}\gamma_{\mu\nu}q)$			$\mathcal{E}_{\ell q}$	$(\bar{\ell}\gamma^\mu q)(\bar{q}\gamma_\mu\ell)$
$\bar{L}R\bar{L}R$				$\mathcal{E}^{(3)}_{\ell q}$	$(\bar{\ell}\gamma^\mu\sigma^Iq)(\bar{q}\gamma_\mu\sigma^I\ell)$
$\mathcal{E}^{[2]}_{\ell equ}$	$(\bar{\ell}_r\gamma^{\mu\nu}e)\epsilon_{rs}(\bar{q}_s\gamma_{\mu\nu}u)$			$\mathcal{E}^{[3]}_{\ell q}$	$(\bar{\ell}\gamma^{\mu\nu\rho}q)(\bar{q}\gamma_{\mu\nu\rho}\ell)$
$\mathcal{E}_{\ell uqe}$	$(\bar{\ell}_ru)\epsilon_{rs}(\bar{q}_se)$			$\mathcal{E}^{[3](3)}_{\ell q}$	$(\bar{\ell}\gamma^{\mu\nu\rho}\sigma^Iq)(\bar{q}\gamma_{\mu\nu\rho}\sigma^I\ell)$
$\mathcal{E}^{[2]}_{\ell uqe}$	$(\bar{\ell}_r\gamma^{\mu\nu}u)\epsilon_{rs}(\bar{q}_s\gamma_{\mu\nu}e)$			$\mathcal{E}'^{[3]}_{\ell q}$	$(\bar{\ell}\gamma^{\mu\nu\rho}\ell)(\bar{q}\gamma_{\mu\nu\rho}q)$
				$\mathcal{E}'^{[3](3)}_{\ell q}$	$(\bar{\ell}\gamma^{\mu\nu\rho}\sigma^I\ell)(\bar{q}\gamma_{\mu\nu\rho}\sigma^Iq)$

Table 6: Semileptonic four-fermion evanescent operators. We use the shorthand notation $\gamma^{\mu_1\cdots\mu_n}\equiv\gamma^{\mu_1}\ldots\gamma^{\mu_n}$ with no (anti)symmetrization.

Hard region expansion

Instead of computing the one loop integrals, and then expanding in powers of momenta, we perform a hard region expansion at the integrand level. This vastly simplifies the loop computation. On the EFT side all expanded integrals vanish in DimReg, as they are scaleless.

loop momentum

Momenta and light masses

$$k^2 \sim M^2 \gg p^2 \sim m^2$$

Heavy mass in UV model

$$\frac{1}{(k+p)^2 - M^2} = \frac{1}{k^2 - M^2}\left[1 - \frac{p^2 + 2k\cdot p}{(k+p)^2 - M^2}\right],$$

$$\frac{1}{(k+p)^2 - m^2} = \frac{1}{k^2}\left[1 - \frac{p^2 + 2k\cdot p - m^2}{(k+p)^2 - m^2}\right].$$

See Manohar 2018 for a pedagogical introduction.

One loop matching

Matchmakereft

Carmona, Olgoso, AL, Santiago: 2112.10787 [hep-ph]

Cross checks:

- Scalar Singlet [Haisch, Ruhdorfer, et al. '20] previously matched to SMFET
- SMEFT RGEs [Jenkins, Manohar, et al. '13] reproduced
- Type I Seesaw [Zhang, Zhou '21] previously matched to SMEFT

Used in non-trivial computations:

- [Chala, Santiago '21] Positivity bounds dim 8
- [Crivellin, Kirk, et al. '22] W mass and B physics
- [Bakshi, Chala, et al. '22] Dim 8 SMEFT RGEs
- [Guedes, PO '22] Analysis of models contributing to the g-2 of the muon
- [Li and Zhou '23] Type II Seesaw matched to SMEFT
- [Dawson et al. 2401.12279] 2HDM constrained by EWPOs

One loop matching

Matchmakereft

Carmona, Olgoso, AL, Santiago: 2112.10787 [hep-ph]

Interfaces with the SMEFT ecosystem:

- Match2Fit interfaces Matchmakereft with SMEFiT to derive bounds on SM extensions.[J. Hoeve, G. Magni et al., ’23]
- DsixTools, a package for matching and RGE evolution down to even B-physics observables. [J. Fuentes-Martín, P. Ruiz-Femenia et al., ’20]
- SOLD, the **S**MEFT **O**ne-**L**oop **D**ictionary. [G. Guedes, PO, J. Santiago ’23]: bottom up approach - given an anomaly in a certain WC, which types of models contribute to it at one loop level?
- smelli: a package for global fits in the SMEFT [Aebischer et al. 1810.07698]. The interface is work in progress.

Towards two-loop RGEs for EFTs

Two loop RGEs for EFTs

Why going to 2 loops

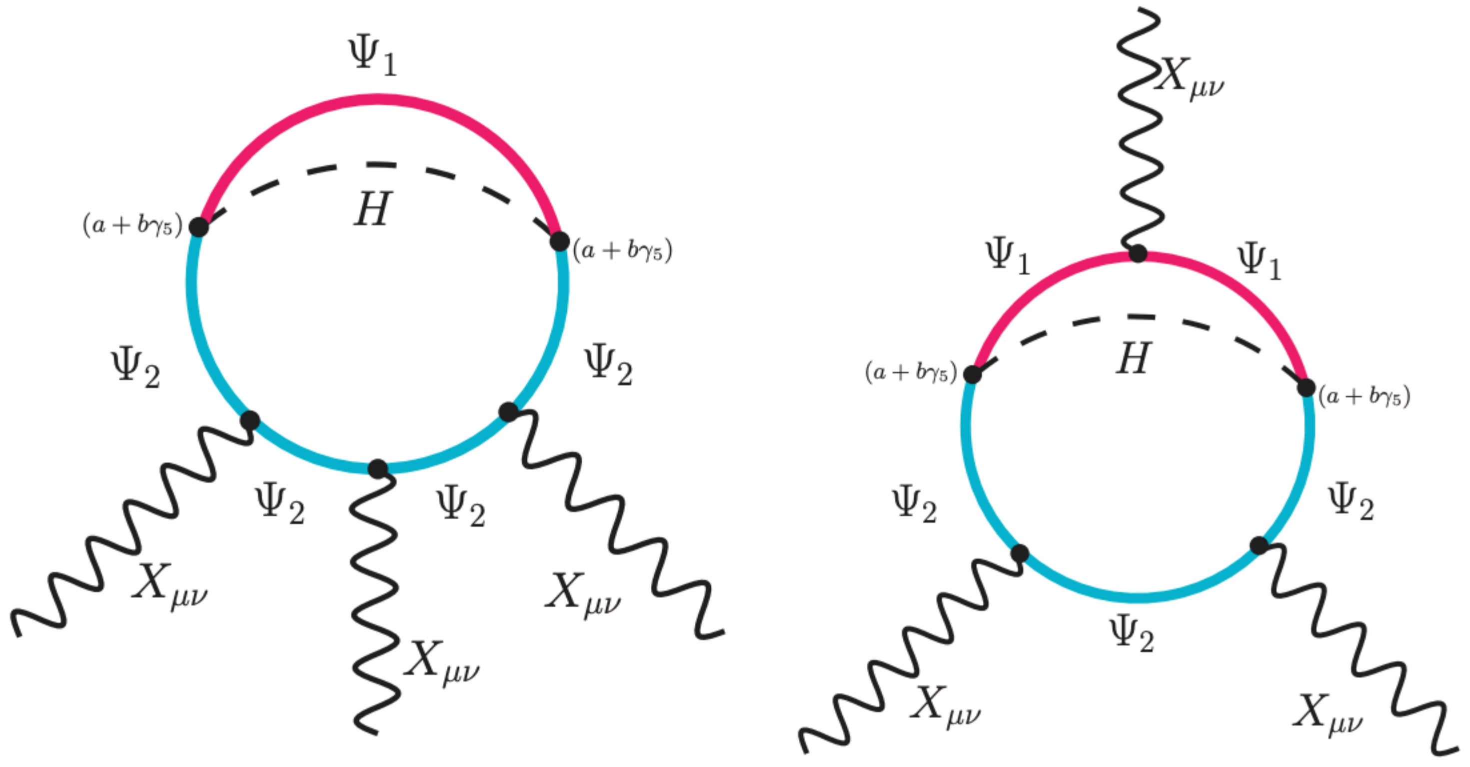
See Naskar, Prakash and Rahaman, 2205.00910

- Models for which WC only get contribution at one loop, needs two loop RGEs to run. There are even dim6 SMEFT operators that only get two loop contributions from UV models, like the CP violating operators of the pure gauge sector - see Naskar et al. 2205.00910.

- Theoretical interest: to understand the intricacies of two loop renormalization in effective theories.

- Face the gamma-5 problem in dim regularization

- Extend existing approaches to evanescent operators



$$\mathcal{O}_{\widetilde{3G}} = f^{ABC}\widetilde{G}_{\mu}^{A\nu}G_{\nu}^{B\rho}G_{\rho}^{C\mu}$$

$$\mathcal{O}_{\widetilde{3W}} = \epsilon^{IJK}\widetilde{W}_{\mu}^{I\nu}W_{\nu}^{J\rho}W_{\rho}^{K\mu}$$

Two loop RGEs for EFTs

Two-loop results

- Two loop RGEs for renormalizable theories (SM, and arbitrary models with a simple and semi-simple gauge group) exist since decades Luo, Wang Xiao hep-ph/0211440, and also Machacek and Vaughn 1983, Vladimirov and Shirkov 1979 etc.

- Extension to dim 6 EFTs with scalars recently with the geometric approach see Jenkins, Manohar, Naterop, Pages, 2310.19883

- and also with the functional approach J. Fuentes-Martin et al. 2311.13630

Two loop RGEs for EFTs

Challenges at two loops

- The presence of subdivergences requires a careful separation of UV contributions.
- The interplay with real or spurious IR singularities needs to be disentangled.
- The traditional approach is Bogoliubov’s R-operation, and its extension to include infrared divergent integrals, the R* operation (Chetyrkin, Tkachov and Smirnov ’82,’85), used for RGEs at up to 5 loops (Herzog and Ruijl 1703.03776).
- But for EFTs the number of integrals to be computed explodes, and we would like to avoid R* if possible.
- Relying on the same hard region expansion identity of one loop, employed by Misiak and Münz ’95 and Chetyrkin, Misiak and Münz ’98, but needs to be elaborated.

Two loop RGEs for EFTs

Challenges at two loops: plain scalar+fermion theory

$$\mathcal{L}_{EFT} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 + i\bar{\psi}\not{\partial}\psi - m_\psi\bar{\psi}\psi - \frac{C_3}{3!}\phi^3 - \frac{C_4}{4!}\phi^4 - C_{13}\phi\bar{\psi}\psi$$
$$+ \sum_{d\geq 5}\sum_{1}^{n_d}\frac{C_i^{(d)}}{\Lambda^{d-4}}O_i^{(d)} + \mathcal{L}_{ct}$$

$$G^{(5)} = \Big\{\phi^5, \phi^2\Box\phi, \bar{\psi}P_L\psi\phi^2, \bar{\psi}P_R\psi\phi^2, (\partial\bar{\psi})\gamma^\mu P_L\psi\phi, (\partial\bar{\psi})\gamma^\mu P_R\psi\phi,$$
$$\bar{\psi}\gamma^\mu P_L(\partial\psi)\phi, \bar{\psi}\gamma^\mu P_R(\partial\psi)\phi, \bar{\psi}P_L\Box\psi, \bar{\psi}P_R\Box\psi\Big\}$$

$$P^{(5)} = \Big\{\phi^5, \bar{\psi}\psi\phi^2\Big\}$$

$$G^{(6)} = \Big\{\phi^6, \phi^3\Box\phi, (\Box\phi)^2, \bar{\psi}P_L\psi\phi^3, \bar{\psi}P_R\psi\phi^3, \bar{\psi}\gamma^\mu P_L(\partial\psi)\phi^2, \bar{\psi}\gamma^\mu P_R(\partial\psi)\phi^2,$$
$$(\partial\bar{\psi})\gamma^\mu P_L\psi\phi^2, (\partial\bar{\psi})\gamma^\mu P_R\psi\phi^2, \bar{\psi}P_L(\Box\psi)\phi, \bar{\psi}P_R(\Box\psi)\phi,$$
$$\bar{\psi}P_L(\partial\psi)(\partial\phi), \bar{\psi}P_R(\partial\psi)(\partial\phi), \bar{\psi}P_L\psi(\Box\phi), \bar{\psi}P_R\psi(\Box\phi),$$
$$\bar{\psi}\sigma^{\mu\nu}P_L(\partial_\mu\psi)(\partial_\nu\phi), \bar{\psi}\sigma^{\mu\nu}P_R(\partial_\mu\psi)(\partial_\nu\phi), \bar{\psi}\gamma^\mu P_L(\partial_\mu\Box\psi), \bar{\psi}\gamma^\mu P_R(\partial_\mu\Box\psi),$$
$$\bar{\psi}P_L\psi\,\bar{\psi}P_L\psi, \bar{\psi}P_R\psi\,\bar{\psi}P_R\psi, \bar{\psi}P_L\psi\,\bar{\psi}P_R\psi, \bar{\psi}\gamma^\mu P_L\psi\,\bar{\psi}\gamma_\mu P_L\psi,$$
$$\bar{\psi}\gamma^\mu P_R\psi\,\bar{\psi}\gamma_\mu P_R\psi, \bar{\psi}\gamma^\mu P_L\psi\,\bar{\psi}\gamma_\mu P_R\psi, \bar{\psi}\sigma^{\mu\nu}P_L\psi\,\bar{\psi}\sigma_{\mu\nu}P_L\psi, \bar{\psi}\sigma^{\mu\nu}P_R\psi\,\bar{\psi}\sigma_{\mu\nu}P_R\psi\Big\}$$

$$P^{(6)} = \Big\{\phi^6, \bar{\psi}\psi\phi^3, \bar{\psi}\psi\bar{\psi}\psi, \bar{\psi}\gamma^\mu\psi\,\bar{\psi}\gamma_\mu\psi, \bar{\psi}\sigma^{\mu\nu}\psi\,\bar{\psi}\sigma_{\mu\nu}\psi\Big\}$$

All operators including redundant ones (which we need to absorb all poles)

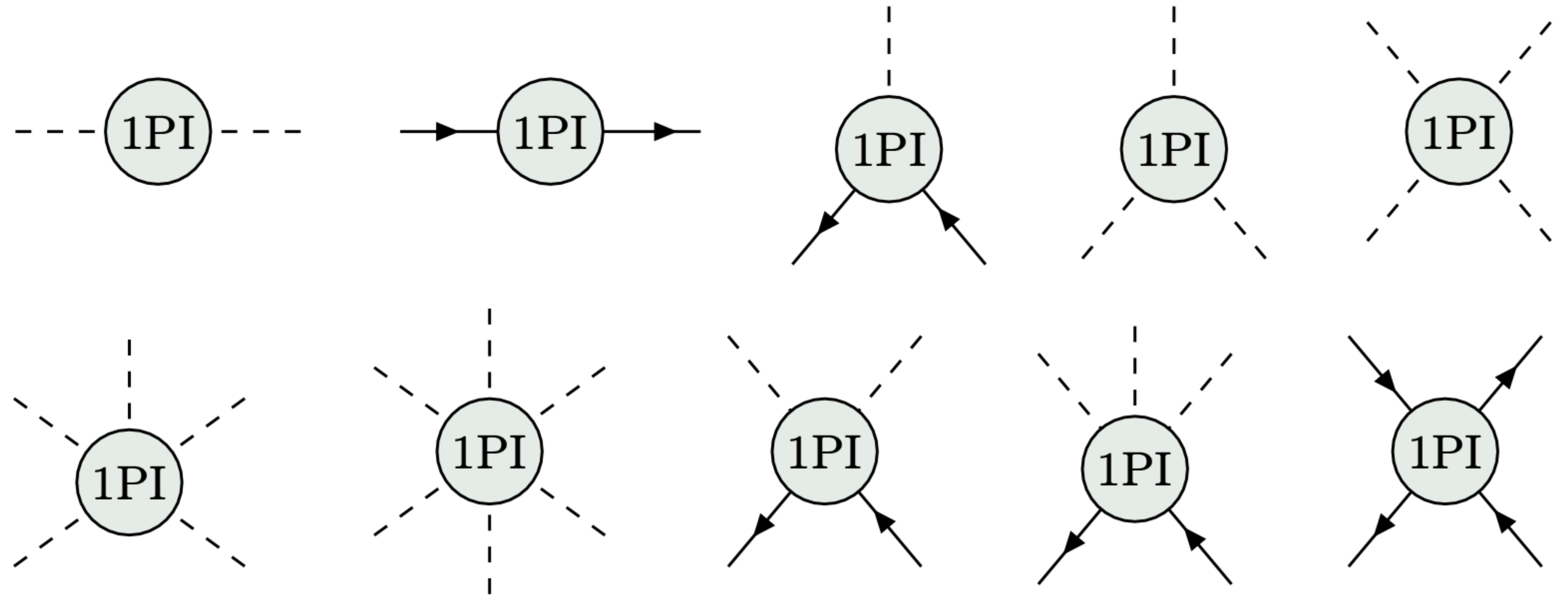
Physical operators

Two loop RGEs for EFTs

Challenges at two loops: plain scalar+fermion theory

$$\mathcal{L}_{EFT} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 + i\bar{\psi}\not{\partial}\psi - m_\psi\bar{\psi}\psi - \frac{C_3}{3!}\phi^3 - \frac{C_4}{4!}\phi^4 - C_{13}\phi\bar{\psi}\psi$$
$$+\sum_{d\geq 5}\sum_{1}^{n_d}\frac{C_i^{(d)}}{\Lambda^{d-4}}O_i^{(d)} + \mathcal{L}_{ct}$$

Divergent amplitudes



Number of diagrams

$$\Gamma_{\bar{\psi}\psi\phi\phi}\Big|_b = -2iC_{230} + [+27 \text{ diagrams}] + [+1233 \text{ diagrams}] + \mathcal{O}(3 \text{ loops})$$

$$\Gamma_{\bar{\psi}\psi\bar{\psi}\psi}\Big|_b = +[+19 \text{ diagrams}] + [+786 \text{ diagrams}] + \mathcal{O}(3 \text{ loops})$$

$$\Gamma_{\bar{\psi}\psi\phi\phi\phi}\Big|_b = -6iC_{330} + [+148 \text{ diagrams}] + [+11533 \text{ diagrams}] + \mathcal{O}(3 \text{ loops})$$

$$\Gamma_{\phi\phi\phi\phi\phi}\Big|_b = -120iC_{500} + [+226 \text{ diagrams}] + [+18090 \text{ diagrams}] + \mathcal{O}(3 \text{ loops})$$

$$\Gamma_{\phi\phi\phi\phi\phi\phi}\Big|_b = -720iC_{600} + [+1580 \text{ diagrams}] + [+197405 \text{ diagrams}] + \mathcal{O}(3 \text{ loops})$$

Two loop RGEs for EFTs

Challenges at two loops: Evanescent operators

$$\mathcal{L}_{EFT} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 + i\bar{\psi}\not{\partial}\psi - m_\psi\bar{\psi}\psi - \frac{C_3}{3!}\phi^3 - \frac{C_4}{4!}\phi^4 - C_{13}\phi\bar{\psi}\psi$$
$$+ \sum_{d\geq 5}\sum_{1}^{n_d}\frac{C_i^{(d)}}{\Lambda^{d-4}}O_i^{(d)} + \mathcal{L}_{ct}$$

Four-fermion operators reduce to a basis

$$\bar{\psi}\Gamma_i\psi\bar{\psi}\Gamma^i\psi$$

$$\{\Gamma_i\otimes\Gamma^i\} = \{P_L\otimes P_L, P_R\otimes P_R, P_L\otimes P_R, P_R\otimes P_L, \gamma^\mu P_L\otimes\gamma_\mu P_L, \gamma^\mu P_R\otimes\gamma_\mu P_R$$
$$\gamma^\mu P_L\otimes\gamma_\mu P_R, \gamma^\mu P_R\otimes\gamma_\mu P_L, \sigma^{\mu\nu}P_L\otimes\sigma_{\mu\nu}P_L, \sigma^{\mu\nu}P_R\otimes\sigma_{\mu\nu}P_R\}$$

but this is valid in strictly 4-dimensions

In D=4-2ε we get an extra, “Evanescent” contribution

$$X_1\otimes X_2 = \sum_{i=1}^{10} b_i\Gamma_i\otimes\Gamma^i + E(X_1, X_2)$$

Two loop RGEs for EFTs

Challenges at two loops: The γ5 problem

γ5 is an inherently 4-d object. There are various “schemes” to continue it to D=4-2ε

NDR scheme

$$\{\gamma_5, \gamma_\mu\} = 0$$

Maintains gauge invariance
but no cyclicity in traces!

BMHV scheme

$$\gamma^\mu = \bar{\gamma}^\mu + \hat{\gamma}^\mu$$

$$\{\bar{\gamma}^\mu, \gamma_5\} = 0$$

$$[\hat{\gamma}^\mu, \gamma_5] = 0$$

Spoils gauge invariance
which needs to be restored
by finite counter-terms
but is mathematically consistent

Various studies of both schemes
at two loop computations,
but all operate at the level of on-
shell quantities (amplitudes).

Potentially extending the
approach of J. Fuentes, M. König
et al. ’23
to two loops is the way ahead.

Summary

- Effective field theories are the ideal framework to parametrize deviations from the SM.
- One loop matching and running has been completely automated and integrated with many tools in the SMEFT ecosystem.
- Two loop RGEs for dim 6 operators is work in progress, by many different groups.

Thank you for your attention!