Neutrino mixing beyond Pontecorvo, entangled vacuum and all of that!

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References

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- M. Blasone, G. Lambiase, **G. L.** and L. Petruzziello [arXiv:1803.05695].
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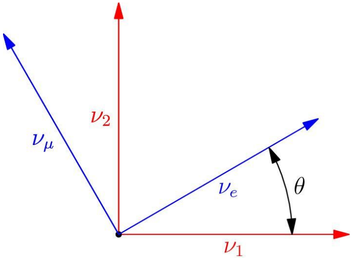
The "magic" of neutrinos

- **QM** (Simp. 2-flavors) [*B. Pontecorvo (1978)*]

$$|\nu_e\rangle = |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta$$

$$|\nu_\mu\rangle = -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta$$

- **Unitary equivalence** between flavor and mass representations



$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \hat{U} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

$$\hat{U} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \implies \hat{U}^\dagger = \hat{U}^{-1}$$

The "magic" of neutrinos

- **Time evolution**

$$|\nu_e(t)\rangle = \cos\theta e^{-iE_1 t}|\nu_1\rangle + \sin\theta e^{-iE_2 t}|\nu_2\rangle$$

- **Flavor oscillations:**

$$P_{\nu_e\to\nu_\mu}(t) = |\langle\nu_e|\nu_\mu(t)\rangle|^2 = \sin^2 2\theta \sin^2\left(\frac{\Delta E}{2}t\right) = 1 - P_{\nu_e\to\nu_e}(t)$$

- **Flavor conservation:**

$$|\langle\nu_e|\nu_e(t)\rangle|^2 + |\langle\nu_e|\nu_\mu(t)\rangle|^2 = 1$$

QM vs QFT

- **QFT** [*M.Blasone and G.Vitiello (1995)*]

$$\nu_e(x) = \nu_1(x)\,\cos\theta + \nu_2(x)\,\sin\theta$$

$$\nu_\mu(x) = -\nu_1(x)\,\sin\theta + \nu_2(x)\,\cos\theta$$

- **Mass** (Dirac free) fields

$$\nu_i(x) = \sum_{\mathbf{k},\sigma} \frac{1}{\sqrt{V}} \left[\alpha^{\sigma}_{\mathbf{k},i}\, u^{\sigma}_{\mathbf{k},i}\, e^{-ik\cdot x} + \beta^{\sigma\,\dagger}_{\mathbf{k},i}\, v^{\sigma}_{\mathbf{k},i}\, e^{+ik\cdot x}\right], \quad i = 1, 2,$$

$$\left\{\alpha^{\sigma}_{\mathbf{k},i}, \alpha^{\sigma'\,\dagger}_{\mathbf{q},j}\right\} = \left\{\beta^{\sigma}_{\mathbf{k},i}, \beta^{\sigma'\,\dagger}_{\mathbf{q},j}\right\} = \delta_{\mathbf{k},\mathbf{q}}\,\delta_{\sigma\sigma'}\delta_{ij}$$

- **Flavor** (free-like) fields

$$\nu_\chi(x) = \sum_{\mathbf{k},\sigma} \frac{1}{\sqrt{V}} \left[\alpha^{\sigma}_{\mathbf{k},\chi}(t)\, u^{\sigma}_{\mathbf{k},j}\, e^{-ik\cdot x} + \beta^{\sigma\,\dagger}_{\mathbf{k},\chi}(t)\, v^{\sigma}_{\mathbf{k},j}\, e^{+ik\cdot x}\right], \quad (\chi, j) = (e, 1), (\mu, 2)$$

QM vs QFT

- Flavor ladder operators

$$\alpha^{\sigma}_{\mathbf{k},e} = \underbrace{\cos\theta\ \alpha^{\sigma}_{\mathbf{k},1}\ +\ \sin\theta}_{\textit{Pontecorvo rotation}}\ \underbrace{\left(\rho^{\mathbf{k}\,*}_{12}(t)\ \alpha^{\sigma}_{\mathbf{k},2}\ +\ \varepsilon^{\sigma}\ \lambda^{\mathbf{k}}_{12}(t)\ \beta^{\sigma\dagger}_{-\mathbf{k},2}\right)}_{\textit{Bogoliubov transformation}}$$

- **Bogoliubov** coefficients

$$\rho^{\mathbf{k}}_{12}(t)\ \equiv\ u^{\sigma\dagger}_{\mathbf{k},2}(t)\, u^{\sigma}_{\mathbf{k},1}(t)\ =\ v^{\sigma\dagger}_{-\mathbf{k},1}(t)\, v^{\sigma}_{-\mathbf{k},2}(t)$$

$$\lambda^{\mathbf{k}}_{12}(t)\ \equiv\ \varepsilon^{\sigma}\ u^{\sigma\dagger}_{\mathbf{k},1}(t)\, v^{\sigma}_{-\mathbf{k},2}(t)\ =\ -\varepsilon^{\sigma}\ u^{\sigma\dagger}_{\mathbf{k},2}(t)\, v^{\sigma}_{-\mathbf{k},1}(t)$$

$$|\rho^{\mathbf{k}}_{12}|^2 + |\lambda^{\mathbf{k}}_{12}|^2 = 1\ \Longrightarrow\ \left\{\alpha^{\sigma}_{\mathbf{k},e}(t), \alpha^{\sigma\dagger}_{\mathbf{k},e}(t)\right\} = \cos\theta + \left(|\rho^{\mathbf{k}}_{12}|^2 + |\lambda^{\mathbf{k}}_{12}|^2\right)\sin^2\theta = 1$$

- Inequivalent Fock spaces ($|0\rangle_{e,\mu} \neq |0\rangle_{1,2} = |0\rangle_1 \otimes |0\rangle_2$)

$$\lim_{V\to\infty}\ {}_{1,2}\langle 0|0(\theta)\rangle_{e,\mu} = 0$$

QM vs QFT

- Quantum Mechanics:

- **Finite** $\sharp$ of degrees of freedom

- Unitary equivalence of the representations of the canonical commutation relations (**Stone-von Neumann theorem**)

- Quantum Field Theory:

- **Infinite** $\sharp$ of degrees of freedom

- ∞ many unitarily inequivalent representations of the field algebra $\Leftrightarrow$ many vacua

- Examples: theories with spontaneous symmetry breaking, particle creation (Hawking-Unruh effect), etc.

QM vs QFT

- Algebraic structure of **mixing transformations**:

$$\nu_e^\alpha(x) = G_\theta^{-1}(t)\ \nu_1^\alpha(x)\ G_\theta(t)$$

$$\nu_\mu^\alpha(x) = G_\theta^{-1}(t)\ \nu_2^\alpha(x)\ G_\theta(t)$$

- **Mixing generator**:

$$G_\theta(t) = \exp\left[\theta \int d^3\mathbf{x} \left(\nu_1^\dagger(x)\,\nu_2(x) - \nu_2^\dagger(x)\,\nu_1(x)\right)\right]$$

Flavor vacuum

- Flavor vacuum: **Perelomov-like coherent state**

$$|0(\theta)\rangle_{e,\mu} = G_\theta^{-1}|0\rangle_{1,2}$$

$$= \prod_{\mathbf{k},r}\Big[(1-\sin^2\theta\,|\lambda^{\mathbf{k}}_{12}|^2) - \epsilon^r \sin\theta\,\cos\theta\,|\lambda^{\mathbf{k}}_{12}|\,(\alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},2} + \alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},1})$$

$$+\epsilon^r \sin^2\theta\,|\lambda^{\mathbf{k}}_{12}||\rho^{\mathbf{k}}_{12}|\,(\alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},1} - \alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},2}) + \sin^2\theta\,|\lambda^{\mathbf{k}}_{12}|^2\,\alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},2}\alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},1}\Big]|0\rangle_{1,2}$$

- **Condensation density:**

$${}_{e,\mu}\langle 0(\theta)|\alpha^{r\dagger}_{\mathbf{k},i}\alpha^{r}_{\mathbf{k},i}|0(\theta)\rangle_{e,\mu} = {}_{e,\mu}\langle 0(\theta)|\beta^{r\dagger}_{\mathbf{k},i}\beta^{r}_{\mathbf{k},i}|0(\theta)\rangle_{e,\mu} = \sin^2\theta\,|\lambda^{\mathbf{k}}_{12}|^2$$

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$$|0(\theta)\rangle_{e,\mu} = G_\theta^{-1}|0\rangle_{1,2}$$

$$= \prod_{\mathbf{k},r} \Big[(1-\sin^2\theta\,|\lambda_{12}^{\mathbf{k}}|^2) - \epsilon^r \sin\theta\,\cos\theta\,|\lambda_{12}^{\mathbf{k}}|\,(\alpha_{\mathbf{k},1}^{r\dagger}\beta_{-\mathbf{k},2}^{r\dagger} + \alpha_{\mathbf{k},2}^{r\dagger}\beta_{-\mathbf{k},1}^{r\dagger})$$

$$+\epsilon^r \sin^2\theta\,|\lambda_{12}^{\mathbf{k}}||\rho_{12}^{\mathbf{k}}|\,(\alpha_{\mathbf{k},1}^{r\dagger}\beta_{-\mathbf{k},1}^{r\dagger} - \alpha_{\mathbf{k},2}^{r\dagger}\beta_{-\mathbf{k},2}^{r\dagger}) + \sin^2\theta\,|\lambda_{12}^{\mathbf{k}}|^2\,\alpha_{\mathbf{k},1}^{r\dagger}\beta_{-\mathbf{k},2}^{r\dagger}\alpha_{\mathbf{k},2}^{r\dagger}\beta_{-\mathbf{k},1}^{r\dagger}\Big]|0\rangle_{1,2}$$

- **Condensation density:**

$${}_{e,\mu}\langle 0(\theta)|\alpha_{\mathbf{k},i}^{r\dagger}\alpha_{\mathbf{k},i}^{r}|0(\theta)\rangle_{e,\mu} = {}_{e,\mu}\langle 0(\theta)|\beta_{\mathbf{k},i}^{r\dagger}\beta_{\mathbf{k},i}^{r}|0(\theta)\rangle_{e,\mu} = \sin^2\theta\,|\lambda_{12}^{\mathbf{k}}|^2$$

Condensation density

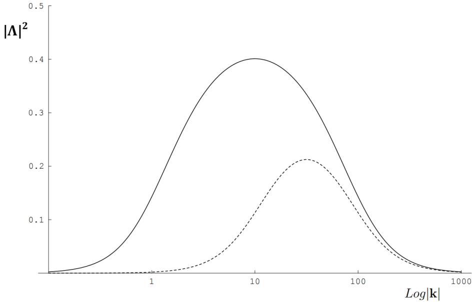

Fig.: Solid line: $m_1 = 1$, $m_2 = 100$; Dashed line: $m_1 = 10$, $m_2 = 100$

- Maximum at $k = \sqrt{m_1 m_2}$
- $|\lambda_{12}^{\mathbf{k}}|^2 \simeq \frac{(m_2 - m_1)^2}{4k^2}$ for $k \gg \sqrt{m_1 m_2}$
- $\lambda_{12}^{\mathbf{k}} = 0$ for $m_1 = m_2$ and/or $\theta = 0$ (no mixing)
- $\lambda_{12}^{\mathbf{k}} \to 0$ for $k \to \infty$ (ultrarelativistic limit = Pontecorvo limit)

Vacuum entanglement

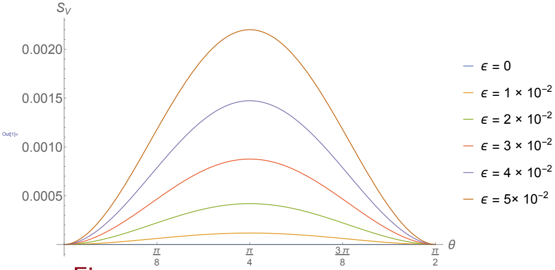

Fig.: von Neumann entropy versus θ for different ϵ

- **von Neumann entropy** $(\epsilon \propto \sqrt{m_1 m_2}/k)$

$$S_V = -\left[1 - \frac{\varepsilon^2}{32}(1-\cos 4\theta)\right] \lg_2 \left[1 - \frac{\varepsilon^2}{32}(1-\cos 4\theta)\right] - \frac{\varepsilon^2}{16}\sin^2 2\theta \lg_2 \left(\frac{\varepsilon^2}{16}\sin^2 2\theta\right)$$

- Maximum at $\theta = \pi/4$
- $S_V \to 0$ for $\epsilon \to 0$ (Pontecorvo limit)

QM vs QFT

In the spotlight

Physical neutrinos: flavor or mass?

$$\mathcal{A} = {}_{\text{out}}\langle \bar{\ell}, \nu, \ldots \,|\, \hat{S}_I \,|\, \ldots \rangle_{\text{in}}$$

$\searrow$

$|\nu_{e,\mu}\rangle$ or $|\nu_{1,2}\rangle$?

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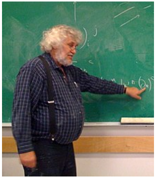
Unruh effect (UE)

[...] *the behavior of particle detectors under acceleration a is investigated, where it is shown that an* ***accelerated detector even in flat spacetime will detect particles in the vacuum*** [...]

[...] *This result is exactly what one would expect of a detector in a* ***thermal bath of temperature*** [*W. Unruh (1976)*]

$$T_{\mathrm{U}} = \frac{\hbar a}{2\pi c k_B}$$



Unruh effect (UE)

- **Rindler** coordinates

$$x^0 = \xi \sinh\eta, \quad x^1 = \xi \cosh\eta$$

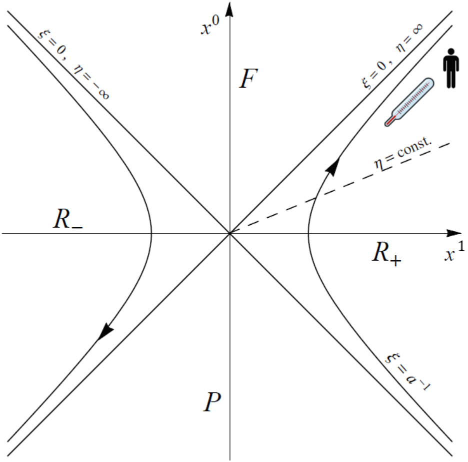
- Rindler vs Minkowski metrics

$$ds^2 = \left(dx^0\right)^2 - \left(dx^1\right)^2 - \left(d\vec{x}\right)^2 \rightarrow$$
$$\rightarrow ds^2 = \xi^2 d\eta^2 - d\xi^2 - \left(d\vec{x}\right)^2$$

- Rindler worldline

$$\eta = a\tau, \quad \xi = \mathrm{const} \equiv a^{-1}, \quad \vec{x} = \mathrm{const}$$

a: proper acceleration.



Unruh effect (UE)

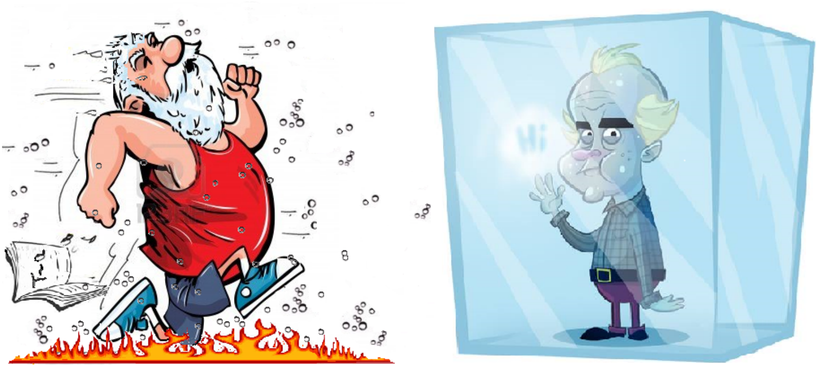


Fig.: Pictorial representation of UE: on the left the uniformly accelerated ("burned") observer in Minkowski vacuum, on the right the ("frozen") inertial observer

Decay of accelerated particles

Non-universality of **decay properties** [*R. Muller (1997)*]

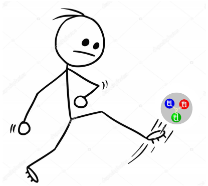
Proton decay:

$$\tau^{(obs)}_{proton} \gg \tau_{universe} \implies \text{inertial protons are } \textbf{stable} \text{ (in the SM)}$$

Fig.: Credit to Sandbox Studio, Chicago with Reidar Hahn, Fermilab

Decay of accelerated particles

However, if we "kick" the proton..



acceleration	lifetime
a_{LHC}	$\tau_p \sim 10^{3\times 10^8}\,\mathrm{yr}$
a_{pulsar}	$\tau_p \sim 10^{-1}\,\mathrm{s}$

$$\mathbf{p} \xrightarrow{\mathbf{a}} \mathbf{n} + \mathbf{e}^+ + \nu_{\mathbf{e}}$$

Decay of accelerated particles

$$\mathbf{p} \xrightarrow{\mathbf{a}} \mathbf{n} + \mathbf{e}^{+} + \nu_{\mathbf{e}}$$

Fig.: Credit to Sandbox Studio, Chicago with Reidar Hahn, Fermilab

Accelerated proton decay

Laboratory frame

$$p \rightarrow n + e^{+} + \nu_e$$

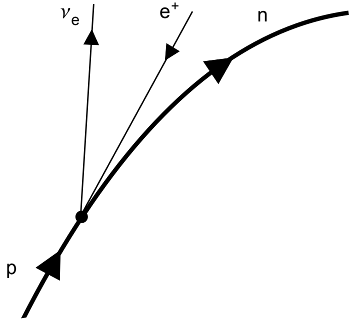


Fig.: Proton decay (laboratory frame)

Comoving frame

$$p + e \rightarrow n + \nu_e \qquad p + \bar{\nu}_e \rightarrow n + e^+ \qquad p + e + \bar{\nu}_e \rightarrow n$$

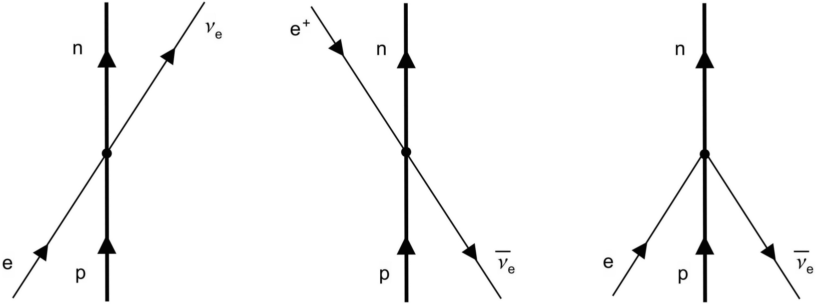


Fig.: Proton decay (comoving frame)

Accelerated proton decay

[*D. Vanzella and G. Matsas (1999)*]

- **Massless** neutrinos
- $|\mathbf{k}_e| \sim |\mathbf{k}_{\nu_e}| \ll M_{p,n}$ (**no-recoil** approximation)
- $a \ll m_{Z^0}, m_{W^\pm}$ (**Fermi-like** effective theory)

$$\widehat{S}_I = \int d^4x\sqrt{-g}\,\widehat{j}_\mu\left(\widehat{\overline{\Psi}}_\nu\gamma^\mu\widehat{\Psi}_e + \widehat{\overline{\Psi}}_e\gamma^\mu\widehat{\Psi}_\nu\right)$$

$$\widehat{j}^\mu = \widehat{q}(\tau)\,u^\mu\,\delta\left(u - a^{-1}\right), \quad \widehat{q}(\tau) = e^{i\widehat{H}\tau}\,\widehat{q}_0\,e^{-i\widehat{H}\tau}$$

$$\widehat{H}\,|n\rangle = m_n\,|n\rangle\,, \quad \widehat{H}\,|p\rangle = m_p\,|p\rangle\,, \quad \Delta m = m_n - m_p \simeq 1.7\,\mathrm{MeV}, \quad G_F = |\langle p|\,\hat{q}_0\,|n\rangle|$$

Laboratory frame

- (Tree-level) Transition amplitude

$$\mathcal{A}^{p\to n} = \langle n| \otimes \langle e^{+}_{k_e\,\sigma_e}, \nu_{k_\nu\,\sigma_\nu}| \widehat{S}_I \, |0\rangle \otimes |p\rangle$$

- Differential transition rate

$$\frac{d^2\mathcal{P}^{p\to n}_{lab}}{dk_e dk_\nu} = \frac{1}{2}\sum_{\sigma_e=\pm}\sum_{\sigma_\nu=\pm} |\mathcal{A}^{p\to n}|^2$$

- <u>Mean proper lifetime</u> (**scalar** decay rate)

$$\Gamma^{p\to n}_{lab} \equiv \frac{\mathcal{P}^{p\to n}_{lab}}{T} = \frac{4G_F^2 a}{\pi^2 e^{\pi\Delta m/a}} \int_0^\infty d\tilde{k}_e \int_0^\infty d\tilde{k}_\nu K_{2i\Delta m/a}\left[2\left(\tilde{\omega}_e + \tilde{\omega}_\nu\right)\right]$$

Comoving frame

$$\Gamma^{p\to n}_{com} = \frac{G_F^2\, m_e}{a\,\pi^2\, e^{\pi\Delta m/a}} \int_{-\infty}^{+\infty} d\omega\, \frac{K_{i\omega/a+1/2}(m_e/a) K_{i\omega/a-1/2}(m_e/a)}{\cosh\left[\pi\left(\omega - \Delta m\right)/a\right]}$$

Result

$$\Gamma^{p\to n}_{lab} = \Gamma^{p\to n}_{com}$$

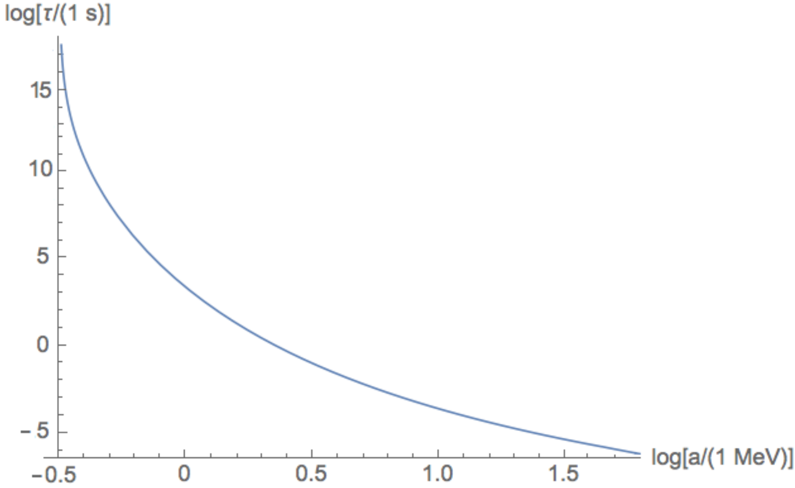


Fig.: Mean proper lifetime τ of the proton versus proper acceleration a.

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UE and accelerated proton decay

[*D. Ahluwalia, L. Labun and G. Torrieri (2016)*]

"*In the laboratory frame, the interaction is the electroweak vertex, hence neutrinos are flavor eigenstates.*

In the comoving frame, the proton interacts with neutrinos in Rindler states, which display a thermal weight and are mass eigenstates [...]

[...] *we conclude that the* ***rates in the two frames disagree*** *when taking into account neutrino mixings*".

Violation of General Covariance of QFT!

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~~**Violation of General Covariance of QFT!**~~

UE and accelerated proton decay

The "paladins" of General Covariance

UE and accelerated proton decay

$$\mathcal{A}^{p\to n} = \langle n| \otimes \langle e^{+}_{k_e\,\sigma_e}, \nu_{k_\nu\,\sigma_\nu}| \widehat{S}_I\,|0\rangle \otimes |p\rangle$$

Flavor states: **Laboratory frame** [G. L. *et al.* 2018]

$$\Gamma^{p\to n}_{lab} \;=\; \cos^4\theta\,\Gamma^{p\to n}_{1} + \sin^4\theta\,\Gamma^{p\to n}_{2} + \cos^2\theta\sin^2\theta\,\Gamma^{p\to n}_{12}$$

$$\Gamma^{p\to n}_{i} \;\equiv\; \frac{1}{T}\sum_{\sigma_\nu,\sigma_e} {G_F}^2 \int d^3k_\nu \int d^3k_e\,\left|\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_i},\omega_e)\right|^2, \quad i=1,2,$$

$$\Gamma^{p\to n}_{12} \;\equiv\; \frac{1}{T}\sum_{\sigma_\nu,\sigma_e} {G_F}^2 \int d^3k_\nu \int d^3k_e\left[\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_{\mathbf{1}}},\omega_e)\,\mathcal{I}^{*}_{\sigma_\nu\sigma_e}(\omega_{\nu_{\mathbf{2}}},\omega_e) + \mathrm{c.c.}\right]$$

Flavor states: **Comoving frame**

$$\Gamma_{com}^{p\to n} = \cos^4\theta\,\widetilde{\Gamma}_1^{p\to n} + \sin^4\theta\,\widetilde{\Gamma}_2^{p\to n} + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}_{12}^{p\to n}$$

$$\begin{aligned}
\widetilde{\Gamma}_{12}^{p\to n} \;=\;& \frac{2\,G_F^2}{a^2\,\pi^7\,\sqrt{l_{\nu_1} l_{\nu_2}}\,e^{\pi\Delta m/a}}\int_{-\infty}^{+\infty} d\omega\left\{\int d^2k_e\, l_e\left|K_{i\omega/a+1/2}\left(\frac{l_e}{a}\right)\right|^2\right.\\
&\times\int d^2k_\nu\,\left(\kappa_\nu^2 + m_{\nu_1}m_{\nu_2} + l_{\nu_1}l_{\nu_2}\right)\\
&\times\mathrm{Re}\left\{K_{i(\omega-\Delta m)/a+1/2}\left(\frac{l_{\nu_1}}{a}\right)K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_2}}{a}\right)\right\}\\
&+m_e\int d^2k_e\int d^2k_\nu\left(l_{\nu_1}m_{\nu_2} + l_{\nu_2}m_{\nu_1}\right)\\
&\times\mathrm{Re}\left\{K^2_{i\omega/a+1/2}\left(\frac{l_e}{a}\right)K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_1}}{a}\right)\right.\\
&\left.\left.\times\,K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_2}}{a}\right)\right\}\right\},\qquad \kappa_\nu\equiv(k_\nu^x, k_\nu^y)
\end{aligned}$$

UE and accelerated proton decay

Laboratory vs comoving decay rates

$$\Gamma_{lab}^{p\to n} \; = \; \cos^4\theta\,\Gamma_1^{p\to n} + \sin^4\theta\,\Gamma_2^{p\to n} + \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n},$$

$$\Gamma_{com}^{p\to n} = \cos^4\theta\,\widetilde{\Gamma}_1^{p\to n} + \sin^4\theta\,\widetilde{\Gamma}_2^{p\to n} + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}_{12}^{p\to n}$$

$$\Gamma_i^{p\to n} = \widetilde{\Gamma}_i^{p\to n}, \qquad i = 1, 2$$

Interference terms?

$$\Gamma_{12}^{p\to n} \stackrel{?}{=} \widetilde{\Gamma}_{12}^{p\to n}$$

Comparing the decay rates

Non-trivial calculations. . .

. . . but for $\delta m/m_{1,2} \ll 1$

$$\Gamma_{12}^{p\to n} = \widetilde{\Gamma}_{12}^{p\to n} \quad \text{up to } \mathcal{O}\left(\frac{\delta m}{m_{1,2}}\right)$$

Result

$$\Gamma_{lab}^{p\to n} = \Gamma_{com}^{p\to n} \quad \text{up to } \mathcal{O}\left(\frac{\delta m}{m_{1,2}}\right)$$

General Covariance and mass eigenstates?

[*G. Cozzella et al (2018)*]

“ [. . .] *a physical Fock space for flavor neutrinos cannot be constructed. Flavor states are only phenomenological since their definition depends on the specific process.*”

“ *We should view neutrino states with well defined mass as fundamental.* [. . .] *The decay rates calculated in this way are perfectly in agreement*”.

Why not mass eigenstates?

- Non-phenom. definition of **flavor space** [*M. Blasone et al. (1995)*]

- **Violation of flavor charge** in the vertex [*M. Blasone et al. (2010)*]

$$p \to n + \bar{\ell}_\alpha + \nu_i \,, \quad i = 1, 2$$

- No **quantum interference**

$$\Gamma^{p \to n + \bar{\ell}_\alpha + \nu_i} = |U_{\alpha,i}|^2 \Gamma_i, \qquad i = 1, 2$$

- Inconsistency with **CP-violation** in neutrino oscillations

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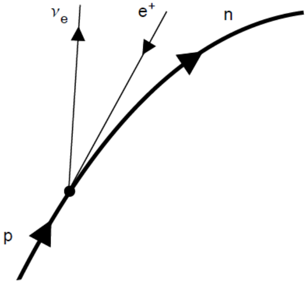
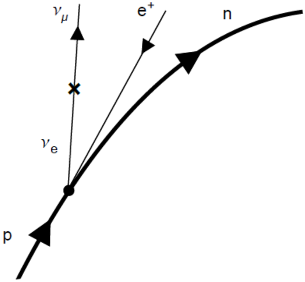
- Inconsistency with **CP-violation** in neutrino oscillations

Neutrino oscillations (laboratory frame)



No oscillations

$$\Gamma_{lab}^{(\nu_e)} = c_\theta^4\,\Gamma_1^{p\to n} + s_\theta^4\,\Gamma_2^{p\to n} + c_\theta^2\,s_\theta^2\,\Gamma_{12}^{p\to n}$$



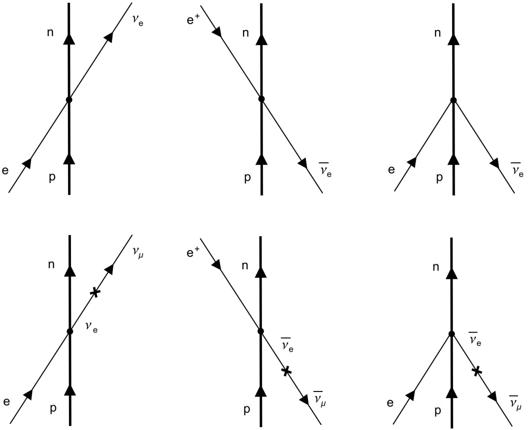
Oscillations

$$\Gamma_{lab}^{(\nu_\mu)} = c_\theta^2\,s_\theta^2\left(\Gamma_1^{p\to n} + \Gamma_2^{p\to n} - \Gamma_{12}^{p\to n}\right)$$

Total decay rate [*G.L. et al. (2020)*]

$$\Gamma_{lab}^{tot} \equiv \Gamma_{lab}^{(\nu_e)} + \Gamma_{lab}^{(\nu_\mu)} = \cos^2\theta\,\Gamma_1^{p\to n} + \sin^2\theta\,\Gamma_2^{p\to n}$$

Neutrino oscillations (comoving frame)



No oscillations

$$\Gamma^{(\nu_e)}_{com} = c^4_\theta\, \widetilde{\Gamma}^{p\to n}_1 + s^4_\theta\, \widetilde{\Gamma}^{p\to n}_2 + c^2_\theta\, s^2_\theta\, \widetilde{\Gamma}^{p\to n}_{12}$$

Oscillations

$$\Gamma^{(\nu_\mu)}_{com} = c^2_\theta\, s^2_\theta \left(\widetilde{\Gamma}^{p\to n}_1 + \widetilde{\Gamma}^{p\to n}_2 - \widetilde{\Gamma}^{p\to n}_{12}\right)$$

Total decay rate [*G.L. et al. (2020)*]

$$\Gamma^{tot}_{com} = \cos^2\theta\, \widetilde{\Gamma}^{p\to n}_1 + \sin^2\theta\, \widetilde{\Gamma}^{p\to n}_2 = \Gamma^{tot}_{lab}$$

Three generations and CP-violation

- PMNS matrix

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix}$$

- **Jarlskog invariant**

$$\mathrm{Im}\left[U_{\delta i}\,U^*_{\gamma_i}\,U^*_{\delta j}\,U_{\gamma j}\right] \equiv J \sum_{\lambda,k} \varepsilon_{\delta\gamma\lambda}\,\varepsilon_{ijk}$$

$$\delta, \gamma, \lambda = \{e, \mu, \tau\}, \quad i, j, k = \{1, 2, 3\}$$

- Physics cannot depend on PMNS matrix parameterization $\Longrightarrow$ **Observables** $\propto J$

Three generations and CP-violation

[G. L. et al. (2020)]

- Mass states:

$$A_{CP}^{(e,j)} = \Gamma_{\nu_e,\nu_j} - \Gamma_{\bar{\nu}_e,\bar{\nu}_j} = |U_{ej}|^2 |\mathcal{A}_j|^2 - |U_{ej}^*|^2 |\mathcal{A}_j|^2 = 0\,, \qquad j = 1,2,3$$

- **Flavor states**:

$$\mathcal{A}_{CP}^{(e,\mu)} \equiv \Gamma_{\nu_e,\nu_\mu} - \Gamma_{\bar{\nu}_e,\bar{\nu}_\mu} = 4J\Big\{-\mathrm{Im}\,[\mathcal{A}_1\mathcal{A}_2^*] + \mathrm{Im}\,[\mathcal{A}_1\mathcal{A}_3^*] - \mathrm{Im}\,[\mathcal{A}_2\mathcal{A}_3^*]\Big\}$$

Three generations and CP-violation

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Take-home message

	Hybrid	Mass-mass	Flavor-flavor
Physical ν_s (Inertial f.)	Flavor	Mass	Flavor
Physical ν_s (Comoving f.)	Mass	Mass	Flavor
Agreement btw decay rates	✗	✓	✓
Lepton charge conservation	✗	✗	✓
Neutrino oscillations	✗	?	✓
CP-violation	✗	✗	✓

Open issues

- Beyond the linear approximation
- Full quantum treatment
- Extension to **curved spacetime**
- Theoretical test of **quantum equivalence principle**

$$\vdots$$

Fig.: Mon Repos. Corfu Painting by Danish School - Fine Art America

Thank you!

BACKUP slides

- Structure of the annihilation operators for $|0(t)\rangle_{e,\mu}$:

$$
\begin{aligned}
\alpha^r_{\mathbf{k},e}(t) &= \cos\theta\,\alpha^r_{\mathbf{k},1} + \sin\theta\left(\rho^{\mathbf{k}*}_{12}(t)\,\alpha^r_{\mathbf{k},2} + \epsilon^r\lambda^{\mathbf{k}}_{12}(t)\,\beta^{r\dagger}_{-\mathbf{k},2}\right)\\
\alpha^r_{\mathbf{k},\mu}(t) &= \cos\theta\,\alpha^r_{\mathbf{k},2} - \sin\theta\left(\rho^{\mathbf{k}}_{12}(t)\,\alpha^r_{\mathbf{k},1} - \epsilon^r\lambda^{\mathbf{k}}_{12}(t)\,\beta^{r\dagger}_{-\mathbf{k},1}\right)\\
\beta^r_{-\mathbf{k},e}(t) &= \cos\theta\,\beta^r_{-\mathbf{k},1} + \sin\theta\left(\rho^{\mathbf{k}*}_{12}(t)\,\beta^r_{-\mathbf{k},2} - \epsilon^r\lambda^{\mathbf{k}}_{12}(t)\,\alpha^{r\dagger}_{\mathbf{k},2}\right)\\
\beta^r_{-\mathbf{k},\mu}(t) &= \cos\theta\,\beta^r_{-\mathbf{k},2} - \sin\theta\left(\rho^{\mathbf{k}}_{12}(t)\,\beta^r_{-\mathbf{k},1} + \epsilon^r\lambda^{\mathbf{k}}_{12}(t)\,\alpha^{r\dagger}_{\mathbf{k},1}\right)
\end{aligned}
$$

- Mixing transformation = Rotation + Bogoliubov transformation

- Bogoliubov coefficients:

$$
\rho^{\mathbf{k}}_{12}(t) = u^{r\dagger}_{\mathbf{k},2}u^r_{\mathbf{k},1}\; e^{i(\omega_{k,2}-\omega_{k,1})t} \quad , \qquad \lambda^{\mathbf{k}}_{12}(t) = \epsilon^r\; u^{r\dagger}_{\mathbf{k},1}v^r_{-\mathbf{k},2}\; e^{i(\omega_{k,2}+\omega_{k,1})t}
$$

$$
|\rho^{\mathbf{k}}_{12}|^2 + |\lambda^{\mathbf{k}}_{12}|^2 = 1
$$

Currents and charges for mixed fermions

– Lagrangian in the **mass basis**:

$$\mathcal{L} = \bar{\nu}_m (i \not{\partial} - M_d) \nu_m$$

where $\nu_m^T = (\nu_1, \nu_2)$ and $M_d = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$.

- $\mathcal{L}$ invariant under global $U(1)$ with conserved charge $Q =$ total charge.

– Consider now the $SU(2)$ transformation:

$$\nu'_m = e^{i\alpha_j \tau_j} \nu_m \qquad ; \qquad j = 1, 2, 3.$$

with $\tau_j = \sigma_j/2$ and σ_j being the Pauli matrices.

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Associated currents :

$$\delta\mathcal{L} \;=\; i\alpha_j\,\bar{\nu}_m\,[\tau_j, M_d]\,\nu_m \;=\; -\alpha_j\,\partial_\mu J^\mu_{m,j}$$

$$J^\mu_{m,j} \;=\; \bar{\nu}_m\,\gamma^\mu\,\tau_j\,\nu_m$$

– The **Charges** $Q_{m,j}(t) \equiv \int d^3\mathbf{x}\, J^0_{m,j}(x)$ satisfy the $su(2)$ algebra:

$$[Q_{m,j}(t), Q_{m,k}(t)] \;=\; i\,\epsilon_{jkl}\,Q_{m,l}(t)\,.$$

– Casimir operator proportional to the total charge: $C_m \;=\; \frac{1}{2}Q$.

- $Q_{m,3}$ is conserved $\Rightarrow$ charge conserved separately for ν_1 and ν_2:

$$Q_1 \;=\; \frac{1}{2}Q \,+\, Q_{m,3} \;=\; \int d^3\mathbf{x}\,\nu_1^\dagger(x)\,\nu_1(x)$$

$$Q_2 \;=\; \frac{1}{2}Q \,-\, Q_{m,3} \;=\; \int d^3\mathbf{x}\,\nu_2^\dagger(x)\,\nu_2(x).$$

These are the flavor charges in the absence of mixing.

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$$\mathcal{L} = \bar{\nu}_f \left(i \not\partial - M\right) \nu_f$$

where $\nu_f^T = (\nu_e, \nu_\mu)$ and $M = \left(\begin{array}{cc} m_e & m_{e\mu} \\ m_{e\mu} & m_\mu \end{array} \right)$.

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- $Q_{f,3}$ is *not* conserved $\Rightarrow$ exchange of charge between ν_e and ν_μ.

Define the flavor charges as:

$$Q_e(t) \equiv \frac{1}{2}Q + Q_{f,3}(t) = \int d^3\mathbf{x}\, \nu_e^\dagger(x)\, \nu_e(x)$$

$$Q_\mu(t) \equiv \frac{1}{2}Q - Q_{f,3}(t) = \int d^3\mathbf{x}\, \nu_\mu^\dagger(x)\, \nu_\mu(x)$$

where $Q_e(t) + Q_\mu(t) = Q$.

– We have:

$$Q_e(t) = \cos^2\theta\, Q_1 + \sin^2\theta\, Q_2 + \sin\theta\cos\theta \int d^3\mathbf{x} \left[\nu_1^\dagger\nu_2 + \nu_2^\dagger\nu_1\right]$$

$$Q_\mu(t) = \sin^2\theta\, Q_1 + \cos^2\theta\, Q_2 - \sin\theta\cos\theta \int d^3\mathbf{x} \left[\nu_1^\dagger\nu_2 + \nu_2^\dagger\nu_1\right]$$

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To summarize:

– In presence of mixing, neutrino flavor charges are defined as

$$Q_e(t) \equiv \int d^3\mathbf{x}\, \nu_e^\dagger(x)\, \nu_e(x) \quad ; \qquad Q_\mu(t) \equiv \int d^3\mathbf{x}\, \nu_\mu^\dagger(x)\, \nu_\mu(x)$$

– They are *not* conserved charges $\Rightarrow$ flavor oscillations.

– They are still (approximately) conserved in the vertex $\Rightarrow$ define flavor neutrinos as their eigenstates

- Problem: find the eigenstates of the above charges.

Charge operator, flavor state and oscillations

- **Charge** operator and observable (relative, e.g., to $|\nu_e\rangle$):

$$Q_\chi(t) = \int d^3\mathbf{x}\, \nu_\chi^\dagger(x)\nu_\chi(x)\,, \qquad \mathcal{Q}_{\mathbf{k},\chi}(t) \equiv \langle \nu^r_{\mathbf{k},e}|\, Q_\chi(t)\, |\nu^r_{\mathbf{k},e}\rangle$$

- **Flavor state:**

$$|\nu^r_{\mathbf{k},\chi}\rangle \equiv \alpha^{r\dagger}_{\mathbf{k},\chi}(0)|0_{e,\mu}\rangle \implies Q_\chi(t=0)|\nu^r_{\mathbf{k},\chi}\rangle = |\nu^r_{\mathbf{k},\chi}\rangle$$

- **Oscillation** probability [*P.Henning et al. (1999)*]

$$\mathcal{Q}_{\mathbf{k},\mu}(t) = |\rho^{\mathbf{k}}_{12}|^2 \sin^2(2\theta)\, \sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}\, t\right) + |\lambda^{\mathbf{k}}_{12}|^2 \sin^2(2\theta)\, \sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}\, t\right)$$

Lepton charge violation for Pontecorvo states

- Pontecorvo states:

$$
\begin{aligned}
|\nu^r_{\mathbf{k},e}\rangle_P &= \cos\theta\,|\nu^r_{\mathbf{k},1}\rangle + \sin\theta\,|\nu^r_{\mathbf{k},2}\rangle \\
|\nu^r_{\mathbf{k},\mu}\rangle_P &= -\sin\theta\,|\nu^r_{\mathbf{k},1}\rangle + \cos\theta\,|\nu^r_{\mathbf{k},2}\rangle\,,
\end{aligned}
$$

are *not* eigenstates of the flavor charges.

$\Rightarrow$ *violation of lepton charge conservation* in the production/detection vertices (at tree level)

$$
{}_P\langle\nu^r_{\mathbf{k},e}|\,Q_e(0)\,|\nu^r_{\mathbf{k},e}\rangle_P = \cos^4\theta + \sin^4\theta + 2|\rho^{\mathbf{k}}_{12}|\sin^2\theta\cos^2\theta < 1
$$

for any $\theta \neq 0$, $\mathbf{k} \neq 0$ and for $m_1 \neq m_2$.

- Pontecorvo states are (approximate) flavor eigenstates in the ultrarelativistic regime ($|\rho^{\mathbf{k}}_{12}| \to 1$).

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Setting the stage

In 2D with **massless** neutrino ($a \ll M_{Z^0}, M_{W^\pm} \approx 10^{36} cm/s^2$)

$$\widehat{j}^\mu = \widehat{q}(\tau)\, u^\mu\, \delta\left(u - a^{-1}\right), \quad \widehat{q}(\tau) = e^{i\widehat{H}\tau}\, \widehat{q}_0\, e^{-i\widehat{H}\tau}$$

$$\widehat{H}\,|n\rangle = m_n\,|n\rangle\,, \quad \widehat{H}\,|p\rangle = m_p\,|p\rangle\,, \quad G_F = |\langle p|\,\hat{q}_0\,|n\rangle|$$

In this regime a **Fermi current-current interaction** can be considered

$$\widehat{S}_I = \int d^2x\sqrt{-g}\;\widehat{j}_\mu\left(\widehat{\overline{\Psi}}_\nu\gamma^\mu\widehat{\Psi}_e + \widehat{\overline{\Psi}}_e\gamma^\mu\widehat{\Psi}_\nu\right)$$

Inertial frame calculation

Field quantization:

$$\widehat{\Psi} = \sum_{\sigma=\pm} \int_{-\infty}^{+\infty} dk \left[\widehat{b}_{k\sigma}\, \psi_{k\sigma}^{(+\omega)} + \widehat{d}_{k\sigma}^{\dagger}\, \psi_{-k-\sigma}^{(-\omega)} \right], \quad \omega = \sqrt{m^2 + \mathbf{k}^2}$$

$$\psi_{k+}^{(\pm\omega)} = \frac{e^{i(\mp\omega x^{\mathbf{0}} + kx^{\mathbf{3}})}}{\sqrt{2\pi}} \begin{pmatrix} \pm\sqrt{(\omega \pm m)/2\omega} \\ 0 \\ k/\sqrt{2\omega(\omega \pm m)} \\ 0 \end{pmatrix}$$

$$\psi_{k-}^{(\pm\omega)} = \frac{e^{i(\mp\omega x^{\mathbf{0}} + kx^{\mathbf{3}})}}{\sqrt{2\pi}} \begin{pmatrix} 0 \\ \pm\sqrt{(\omega \pm m)/2\omega} \\ 0 \\ -k/\sqrt{2\omega(\omega \pm m)} \end{pmatrix}$$

Inertial frame calculation

The tree-level transition amplitude...

$$\mathcal{A}^{p\to n} = \langle n| \otimes \langle e^{+}_{k_e\,\sigma_e}, \nu_{k_\nu\,\sigma_\nu}| \widehat{S}_I \,|0\rangle \otimes |p\rangle$$

... and the related differential transition rate...

$$\frac{d^2\mathcal{P}^{p\to n}_{in}}{dk_e dk_\nu} = \sum_{\sigma_e=\pm}\sum_{\sigma_\nu=\pm} |\mathcal{A}^{p\to n}|^2 , \qquad \frac{\mathcal{P}^{p\to n}}{T} = \Gamma^{p\to n}$$

... give the **inertial decay rate**

$$\Gamma^{p\to n}_{in} = \frac{4G_F^2 a}{\pi^2 e^{\pi\Delta m/a}} \int_0^\infty d\tilde{k}_e \int_0^\infty d\tilde{k}_\nu K_{2i\Delta m/a}\left[2\left(\tilde{\omega}_e + \tilde{\omega}_\nu\right)\right]$$

Comoving frame calculation

Field quantization:

$$\widehat{\Psi} = \sum_{\sigma=\pm} \int_0^{+\infty} d\omega \left[\widehat{b}_{\omega\sigma}\, \psi_{\omega\sigma} \;+\; \widehat{d}^{\dagger}_{\omega\sigma}\, \psi_{-\omega-\sigma} \right]$$

$$\psi_{\omega+} = \sqrt{\frac{m\cosh(\pi\omega/a)}{2\pi^2 a}} \begin{pmatrix} K_{i\omega/a+1/2}(m\,\xi) + iK_{i\omega/a-1/2}(m\,\xi) \\ 0 \\ -K_{i\omega/a+1/2}(m\,\xi) + iK_{i\omega/a-1/2}(m\,\xi) \\ 0 \end{pmatrix} e^{-i\omega\eta/a}$$

$$\psi_{\omega-} = \sqrt{\frac{m\cosh(\pi\omega/a)}{2\pi^2 a}} \begin{pmatrix} 0 \\ K_{i\omega/a+1/2}(m\,\xi) + iK_{i\omega/a-1/2}(m\,\xi) \\ 0 \\ K_{i\omega/a+1/2}(m\,\xi) - iK_{i\omega/a-1/2}(m\,\xi) \end{pmatrix} e^{-i\omega\eta/a}$$

Comoving frame calculation

The tree-level transition amplitude for each process...

$$\mathcal{A}^{p\to n}_{(\mathcal{I})} \quad = \quad \langle n| \otimes \langle\, emit \,|\widehat{S}_I|\, abs \,\rangle \otimes |p\rangle\,, \quad \mathcal{I} = i, ii, iii$$

... and the respective differential transition rates...

$$\frac{d^2\mathcal{P}^{p\to n}_{\mathcal{I}}}{d\omega_e d\omega_\nu} = \sum_{\sigma_e=\pm}\sum_{\sigma_\nu=\pm} |\mathcal{A}^{p\to n}_{\mathcal{I}}|^2\, n_F^{(abs)}(\omega_{e(\nu)})[1 - n_F^{(emit)}(\omega_{\nu(e)})],$$

$$n_F(\omega) = \frac{1}{1 + e^{2\pi\omega/a}}$$

Comoving frame calculation

... give the **total (comoving) decay rate**

$$
\begin{aligned}
\Gamma^{p\to n}_{com} &= \Gamma^{p\to n}_{(i)} + \Gamma^{p\to n}_{(ii)} + \Gamma^{p\to n}_{(iii)} \\
&= \frac{G_F^2\, m_e}{a\,\pi^2\, e^{\pi\Delta m/a}} \int_{-\infty}^{+\infty} d\omega\, \frac{K_{i\omega/a+1/2}(m_e/a) K_{i\omega/a-1/2}(m_e/a)}{\cosh\left[\pi\,(\omega-\Delta m)/a\right]}.
\end{aligned}
$$

Result

At tree level

$$\Gamma^{p\to n}_{in} = \Gamma^{p\to n}_{com}$$

Remarks

The equality of the two decay rates confirms:

- the necessity of Unruh effect in QFT
- the General Covariance of QFT in curved background

Generalizing to 4D with **massive neutrino** [*H. Suzuki et al. (2003)*]

$$\hat{j}^{\mu} = \hat{q}(\tau) u^{\mu} \delta\left(u - a^{-1}\right) \delta(x^1) \delta(x^2), \quad \hat{q}(\tau) = e^{i\hat{H}\tau} \hat{q}_0 e^{-i\hat{H}\tau}$$

$$\hat{H} \left| n \right\rangle = m_n \left| n \right\rangle, \quad \hat{H} \left| p \right\rangle = m_p \left| p \right\rangle, \quad G_F = \left| \left\langle p \right| \hat{q}_0 \left| n \right\rangle \right|$$

$$\hat{S}_I = \int d^4x \sqrt{-g}\, \hat{j}_{\mu} \left(\hat{\bar{\Psi}}_{\nu} \gamma^{\mu} \hat{\Psi}_e + \hat{\bar{\Psi}}_e \gamma^{\mu} \hat{\Psi}_{\nu} \right)$$

Inertial frame calculation

Field quantization:

$$\widehat{\Psi} = \sum_{\sigma=\pm} \int d^3k \left[\widehat{b}_{\mathbf{k}\sigma} \psi^{(+\omega)}_{\mathbf{k}\sigma} + \widehat{d}^{\dagger}_{\mathbf{k}\sigma} \psi^{(-\omega)}_{-\mathbf{k}-\sigma} \right]$$

$$\psi^{(\pm\omega)}_{\mathbf{k}+}(x^0, \mathbf{x}) = \frac{e^{i(\mp\omega x^0 + \mathbf{k}\cdot\mathbf{x})}}{2^2 \pi^{\frac{3}{2}}} \frac{1}{\sqrt{\omega(\omega \pm m)}} \begin{pmatrix} m \pm \omega \\ 0 \\ k^3 \\ k^1 + ik^2 \end{pmatrix}$$

$$\psi^{(\pm\omega)}_{\mathbf{k}-}(x^0, \mathbf{x}) = \frac{e^{i(\mp\omega x^0 + \mathbf{k}\cdot\mathbf{x})}}{2^2 \pi^{\frac{3}{2}}} \frac{1}{\sqrt{\omega(\omega \pm m)}} \begin{pmatrix} 0 \\ m \pm \omega \\ k^1 - ik^2 \\ -k^3 \end{pmatrix}$$

Using the integral representation of the Bessel function

$$K_\mu(z) \;=\; \frac{1}{2}\int_{C_1}\frac{ds}{2\pi i}\Gamma(-s)\Gamma(-s-\mu)\left(\frac{z}{2}\right)^{2s+\mu}$$

together with the expansion formula...

$$(A+B)^z = \int_C \frac{dt}{2\pi i}\,\frac{\Gamma(-t)\,\Gamma(t-z)}{\Gamma(-z)}\,A^{t+z}\,B^t$$

Inertial frame calculation

...the decay rate in the inertial frame becomes

$$\Gamma_{in}^{p\to n} = \frac{a^5\, G_F^2}{2^5\, \pi^{7/2}\, e^{\Delta m/a}} \int_{C_s} \frac{ds}{2\pi i} \int_{C_t} \frac{dt}{2\pi i} \frac{\left(\frac{m_e}{a}\right)^2 \left(\frac{m_\nu}{a}\right)^2}{\Gamma(-s-t+3)\,\Gamma\left(-s-t+7/2\right)}$$

$$\times \Bigg[|\Gamma(-s-t+i\Delta m/a+3)|^2\, \Gamma(-s)\,\Gamma(-t)\,\Gamma(-s+2)\,\Gamma(-t+2)$$

$$+ \mathrm{Re}\Big\{\Gamma(-s-t+i\Delta m/a+2)\,\Gamma(-s-t-i\Delta m/a+4)\Big\}$$

$$\times\, \Gamma(-s+1/2)\,\Gamma(-t+1/2)\,\Gamma(-s+3/2)\,\Gamma(-t+3/2)\Big)\Bigg]$$

where $C_{s(t)}$ picks up all poles of gamma functions in $s(t)$ complex plane.

Comoving frame calculation

Field quantization:

$$\widehat{\Psi} = \sum_{\sigma=\pm} \int_0^{+\infty} d\omega \int d^2k \left[\widehat{b}_{\mathbf{w}\sigma} \psi^{(+\omega)}_{\mathbf{w}\sigma} + \hat{d}^\dagger_{\mathbf{w}\sigma} \psi^{(-\omega)}_{\mathbf{w}-\sigma} \right], \quad \mathbf{w} \equiv (\omega, k^x, k^y)$$

$$\psi^{(\omega)}_{\mathbf{w}+} = N \frac{e^{i(-\omega\eta/a + k_x x + k_y y)}}{(2\pi)^{\frac{3}{2}}} \begin{pmatrix} i l K_{i\omega/a-1/2}(\xi l) + m K_{i\omega/a+1/2}(\xi l) \\ -(k^1 + ik^2) K_{i\omega/a+1/2}(\xi l) \\ i l K_{i\omega/a-1/2}(\xi l) - m K_{i\omega/a+1/2}(\xi l) \\ -(k^1 + ik^2) K_{i\omega/a+1/2}(\xi l) \end{pmatrix}$$

with $l = \sqrt{m^2 + (k^x)^2 + (k^y)^2}$

Comoving frame calculation

Summing up the contributions of the three processes and using

$$x^\sigma K_\nu K_\mu = \frac{\sqrt{\pi}}{2} G^{40}_{24}\left(x^2 \middle| \begin{matrix} \frac{1}{2}\sigma,\ \frac{1}{2}\sigma\ +\ \frac{1}{2} \\ \frac{1}{2}(\nu+\mu+\sigma), \frac{1}{2}(\nu-\mu+\sigma), \frac{1}{2}(-\nu+\mu+\sigma), \frac{1}{2}(-\nu-\mu+\sigma) \end{matrix}\right)$$

the total decay rate in the comoving frame becomes

$$\Gamma^{p\to n}_{com} = \frac{a^5\, G_F^2}{2^5\, \pi^{7/2}\, e^{\Delta m/a}} \int_{C_s} \frac{ds}{2\pi i} \int_{C_t} \frac{dt}{2\pi i} \frac{(\frac{m_e}{a})^2\,(\frac{m_\nu}{a})^2}{\Gamma(-s-t+3)\,\Gamma\,(-s-t+7/2)}$$

$$\times \Big[|\Gamma(-s-t+i\Delta m/a+3)|^2\, \Gamma(-s)\,\Gamma(-t)\,\Gamma(-s+2)\,\Gamma(-t+2)$$

$$+\,\mathrm{Re}\Big\{\Gamma(-s-t+i\Delta m/a+2)\,\Gamma(-s-t-i\Delta m/a+4)\Big\}$$

$$\times\,\Gamma(-s+1/2)\,\Gamma(-t+1/2)\,\Gamma(-s+3/2)\,\Gamma(-t+3/2))\Big]$$

Proton decay and neutrino mixing: a paradox?

Recently, it has been argued that **neutrino mixing** can spoil the agreement between the two decay rates *[D. V. Ahluwalia et al. (2016)]*

The *leitmotiv* is the violation of the KMS **thermal** condition for the accelerated neutrino vacuum. In particular, this would occur when one requires asymptotic (observed) neutrinos in the comoving frame to be in **flavor eigenstates**

The authors conclude by claiming that the contradiction must be solved **experimentally**

~~The authors conclude by claiming that the contradiction must be solved **experimentally**~~

Remark

No experiment can be used for checking the internal consistency of theory against a theoretical paradox.

The question must be settled at a theoretical level, in conformity with the General Covariance of QFT in curved background.

The source of the ambiguity

Under the "magnifying glass"

Are the asymptotic neutrinos in mass or flavor eigenstates?

An attempt to solve the ambiguity has been proposed by requiring asymptotic neutrinos to be in **mass eigenstates** in both the inertial and comoving frames (on the basis that *flavor eigenstates make physical sense only for* $\delta m_{ij}^2 \sim 0$) *[Cozzella et al. (2018)]*

Inverse β decay with mixed neutrinos

$$p \to n + \bar{\ell}_\alpha + \nu_i, \qquad \ell = \{e, \tau, \mu\}, \quad i = \{1, 2, 3\}$$

...but some criticism arises with reference to such a choice:

- flavor eigenstates can be rigorously defined in any regime
- if flavor states are well-defined for $\delta m_{ij}^2 \sim 0$, as those authors claim, why would they not use them, at least in that regime?
- using asymptotic mass neutrinos washes out mixing from calculations (and, indeed, in this case the same decay rates as in absence of mixing are obtained, up to a factor $\cos^2\theta$ or $\sin^2\theta$)

It is clear that some fundamental point is missing in the treatment of mixing by Cozzella *et al.*

Inertial frame calculation (our approach)

Requiring asymptotic neutrinos to be in **flavor eigenstates** as in Ahluwalia's approach, the transition amplitude in the inertial frame becomes *[G. L. et al. (2018)]*

$$\mathcal{A}_{in}^{p\to n} = G_F\Big[\cos^2\theta\,\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_1},\omega_e) + \sin^2\theta\,\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_2},\omega_e)\Big]$$

$$\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_i},\omega_e) = \int_{-\infty}^{+\infty} d\tau\, e^{i\Delta m\tau}\, u_\mu \left[\bar{\psi}_{\sigma_\nu}^{(+\omega_{\nu_i})}\,\gamma^\mu\,\psi_{-\sigma_e}^{(-\omega_e)}\right], \qquad i = 1,2$$

The decay rate thus takes the form

$$\Gamma_{in}^{p\to n} \;=\; \cos^4\theta\,\Gamma_1^{p\to n} \,+\, \sin^4\theta\,\Gamma_2^{p\to n} \,+\, \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n}$$

$$\Gamma_j^{p\to n} \;\equiv\; \frac{1}{T}\sum_{\sigma_\nu,\sigma_e} {G_F}^2 \int d^3k_\nu \int d^3k_e \left|\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_j},\omega_e)\right|^2, \quad j=1,2,$$

$$\Gamma_{12}^{p\to n} \;\equiv\; \frac{1}{T}\sum_{\sigma_\nu,\sigma_e} {G_F}^2 \int d^3k_\nu \int d^3k_e \Big[\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_1},\omega_e)\,\mathcal{I}^*_{\sigma_\nu\sigma_e}(\omega_{\nu_2},\omega_e) \,+\, \mathrm{c.c.}\Big]$$

Violating the KMS condition?

Assuming asymptotic neutrinos to be in **flavor eigenstates** *would violate the KMS definition of a thermal state of a quantum system by adding coherent, off-diagonal correlations in the density matrix. Consequently, the accelerated neutrino vacuum state would not be thermal, contradicting the essential characteristic of the Unruh effect [D. V. Ahluwalia et al. (2016)]*

Non-thermal Unruh effect for mixed neutrinos

Two Bogoliubov transformations involved [*G. L. et al. (2017)*]

$$\phi_{\rm R} \quad \xrightarrow{\text{thermal Bogol. } (a)} \quad \phi_{\rm M} \;\Rightarrow\; \text{condensate in } |0_{\rm M}\rangle$$

$$\phi_1, \phi_2 \quad \xrightarrow{\text{mixing Bogol. } (\theta)} \quad \phi_e, \phi_\mu \;\Rightarrow\; \text{condensate in } |0_{e,\mu}\rangle$$

How do they combine when flavor mixing for an accelerated observer is considered?

Non-thermal Unruh effect for mixed fields

Condensation density of **Rindler mixed neutrinos** in $|0\rangle_{\mathrm{M}}$:

$$\langle 0_{\mathrm{M}}|\,\widehat{N}(\theta,\omega)\,|0_{\mathrm{M}}\rangle \;=\; \underbrace{\frac{1}{e^{a\,\omega/T_{FDU}}+1}}_{Unruh\;\; thermal\;\; spectrum} \;+\; \underbrace{\sin^2\theta\left\{\mathrm{O}\left(\frac{\delta m^2}{m^2_{\nu_1}}\right)\right\}}_{non-thermal\;\; mixing\;\; corrections}$$

Remark

Non-thermal corrections only appear at orders higher than $\mathcal{O}\left(\frac{\delta m}{m}\right)$

Comoving frame calculation (Ahluwalia's approach)

Requiring asymptotic neutrinos to be in **mass eigenstates**, calculations in the comoving frame give for the process (i)

$$\mathcal{A}^{p\to n}_{(i)} = \frac{G_F}{a}\left[\cos\theta\,\mathcal{J}^{(1)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e) + \sin\theta\,\mathcal{J}^{(2)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e)\right]$$

$$\mathcal{J}_{\sigma_\nu\sigma_e} = \int_{-\infty}^{+\infty} d\eta\, e^{i\Delta m\,\eta}\, u_\mu\left[\bar{\psi}^{(\omega_\nu)}_{\mathbf{w}_\nu\sigma_\nu}\,\gamma^\mu\,\psi^{(\omega_e)}_{\mathbf{w}_e\sigma_e}\right]$$

Similar calculations for the other two processes lead to

$$\begin{aligned}\Gamma^{p\to n}_{com} &\equiv \Gamma^{p\to n}_{(i)} + \Gamma^{p\to n}_{(ii)} + \Gamma^{p\to n}_{(iii)} \\ &= \cos^2\theta\,\widetilde{\Gamma}^{p\to n}_1 + \sin^2\theta\,\widetilde{\Gamma}^{p\to n}_2\end{aligned}$$

Comoving frame calculation (Ahluwalia's approach)

$$\Gamma^{p\to n}_{com} = \cos^2\theta\,\widetilde{\Gamma}^{p\to n}_1 + \sin^2\theta\,\widetilde{\Gamma}^{p\to n}_2$$

$$\begin{aligned}\widetilde{\Gamma}^{p\to n}_j \;=\; & \frac{2\,G_F^2}{a^2\,\pi^7\,e^{\pi\Delta m/a}}\int_{-\infty}^{+\infty} d\omega \int d^2k_\nu\, l_{\nu_j}\left|K_{i(\omega-\Delta m)/a+1/2}\left(\frac{l_{\nu_j}}{a}\right)\right|^2 \\ & \times \int d^2k_e\, l_e\left|K_{i\omega/a+1/2}\left(\frac{l_e}{a}\right)\right|^2 + m_{\nu_j}m_e \\ & \times \mathrm{Re}\left\{\int d^2k_\nu K^2_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_j}}{a}\right)\int d^2k_e K^2_{i\omega/a+1/2}\left(\frac{l_e}{a}\right)\right\}\end{aligned}$$

Comparing the rates

Inertial vs comoving rates

$$\Gamma_{in}^{p\to n} \;=\; \cos^4\theta\,\Gamma_1^{p\to n} + \sin^4\theta\,\Gamma_2^{p\to n} + \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n}$$

$$\Gamma_{com}^{p\to n} = \cos^2\theta\,\widetilde{\Gamma}_1^{p\to n} + \sin^2\theta\,\widetilde{\Gamma}_2^{p\to n}$$

Although:

$$\Gamma_j^{p\to n} = \widetilde{\Gamma}_j^{p\to n}, \quad j = 1, 2$$

- there is no counterpart of the off-diagonal term in $\Gamma_{com}^{p\to n}$
- Pontecorvo matrix elements appear with different powers

Comoving frame calculation with flavor eigenstates (our approach)

Requiring asymptotic neutrinos to be in **flavor eigenstates**, calculations in the comoving frame now give for the process (i)

$$\mathcal{A}^{p\to n}_{(i)} = \frac{G_F}{a}\Big[\cos^2\theta\,\mathcal{J}^{(1)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e) + \sin^2\theta\,\mathcal{J}^{(2)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e)\Big],$$

$$\mathcal{J}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e) = \int_{-\infty}^{+\infty} d\eta\, e^{i\Delta m\eta}\, u_\mu\big[\bar{\psi}^{(\omega_\nu)}_{\mathbf{w}_\nu\sigma_\nu}\,\gamma^\mu\,\psi^{(\omega_e)}_{\mathbf{w}_e\sigma_e}\big]$$

Analogous procedures for the other processes lead to

$$\begin{aligned}\Gamma^{p\to n}_{com} &\equiv \Gamma^{p\to n}_{(i)} + \Gamma^{p\to n}_{(ii)} + \Gamma^{p\to n}_{(iii)}\\ &= \cos^4\theta\,\widetilde{\Gamma}^{p\to n}_1 + \sin^4\theta\,\widetilde{\Gamma}^{p\to n}_2 + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}^{p\to n}_{12}\end{aligned}$$

$$\Gamma^{p\to n}_{com} = \cos^4\theta\,\widetilde{\Gamma}^{p\to n}_{1} + \sin^4\theta\,\widetilde{\Gamma}^{p\to n}_{2} + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}^{p\to n}_{12},$$

$$\begin{aligned}
\widetilde{\Gamma}^{p\to n}_{12} \;=\;& \frac{2\,G_F^2}{a^2\,\pi^7\,\sqrt{l_{\nu_1} l_{\nu_2}}\,e^{\pi\Delta m/a}}\int_{-\infty}^{+\infty} d\omega \left\{ \int d^2k_e\, l_e \left| K_{i\omega/a+1/2}\left(\frac{l_e}{a}\right)\right|^2 \right.\\
&\times \int d^2k_\nu \left(\kappa_\nu^2 + m_{\nu_1} m_{\nu_2} + l_{\nu_1} l_{\nu_2}\right)\\
&\times \mathrm{Re}\left\{ K_{i(\omega-\Delta m)/a+1/2}\left(\frac{l_{\nu_1}}{a}\right) K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_2}}{a}\right)\right\}\\
&+ m_e \int d^2k_e \int d^2k_\nu \left(l_{\nu_1} m_{\nu_2} + l_{\nu_2} m_{\nu_1}\right)\\
&\times \mathrm{Re}\left\{ K^2_{i\omega/a+1/2}\left(\frac{l_e}{a}\right) K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_1}}{a}\right)\right.\\
&\left.\left.\times\; K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_2}}{a}\right)\right\}\right\}, \quad \kappa_\nu \equiv \left(k_\nu^x, k_\nu^y\right)
\end{aligned}$$

Comparing the rates

Inertial vs comoving rates

$$\Gamma_{in}^{p\to n} \;=\; \cos^4\theta\,\Gamma_1^{p\to n} + \sin^4\theta\,\Gamma_2^{p\to n} + \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n},$$

$$\Gamma_{com}^{p\to n} = \cos^4\theta\,\widetilde{\Gamma}_1^{p\to n} + \sin^4\theta\,\widetilde{\Gamma}_2^{p\to n} + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}_{12}^{p\to n}$$

$$\Gamma_j^{p\to n} = \widetilde{\Gamma}_j^{p\to n}, \quad j = 1, 2$$

...what about the "off-diagonal" terms?

$$\Gamma_{12}^{p\to n} \stackrel{?}{=} \widetilde{\Gamma}_{12}^{p\to n}$$

Evaluating these terms is non-trivial.

However, for $\frac{\delta m}{m_{\nu_1}} \equiv \frac{m_{\nu_2} - m_{\nu_1}}{m_{\nu_1}} \ll 1$,

$$\Gamma_{12}^{p\to n} = 2\Gamma_{1}^{p\to n} + \frac{\delta m}{m_{\nu_1}}\Gamma_{\delta_m} + \mathcal{O}\left(\frac{\delta m^2}{m_{\nu_1}^2}\right)$$

$$\widetilde{\Gamma}_{12}^{p\to n} = 2\widetilde{\Gamma}_{1}^{p\to n} + \frac{\delta m}{m_{\nu_1}}\widetilde{\Gamma}_{\delta_m} + \mathcal{O}\left(\frac{\delta m^2}{m_{\nu_1}^2}\right)$$

Result...

$$\frac{\Gamma_{\delta_m}}{m_{\nu_1}} = \frac{\widetilde{\Gamma}_{\delta_m}}{m_{\nu_1}}$$

... and its full expression

$$\begin{aligned}
\frac{\Gamma_{\delta_m}}{m_{\nu_1}} = & \lim_{\varepsilon\to 0} \frac{G_F^2\, m_e\, a^3}{\pi^3\, e^{\pi\Delta m/a}} \int_{C_s} \frac{ds}{2\pi i} \int_{C_t} \frac{dt}{2\pi i} \left(\frac{\varepsilon}{a}\right)^{2s+2} \left(\frac{m_e}{a}\right)^{2t+2} \\
& \times \frac{\Gamma(-2s)\Gamma(-2t)\Gamma(-t-1)\Gamma(-s-1)}{\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-2s-2t)} \\
& \times \left[\Gamma\left(-s-t+1+i\frac{\Delta m}{a}\right)\Gamma\left(-s-t-1-i\frac{\Delta m}{a}\right)\right. \\
& \left. + \Gamma\left(-s-t+1-i\frac{\Delta m}{a}\right)\Gamma\left(-s-t-1+i\frac{\Delta m}{a}\right)\right]
\end{aligned}$$

$$\textbf{Tsallis statistics} \begin{cases} \nearrow \text{Long-range interactions} \\ \nearrow \text{Long-time memory} \\ \searrow \text{Entanglement} \\ \searrow \text{Non-equilibrium} \end{cases}$$

Tsallis statistics in UE for mixed fields

- **Tsallis entropy** [*C.Tsallis (1988)]*

$$S_q = \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1} = \sum_{i=1}^{W} p_i \log_q \frac{1}{p_i}$$

$$\log_q z \equiv \frac{z^{1-q} - 1}{1-q}, \quad (\log_1 z = \log z)$$

- $q \to 1$: Boltzmann-Gibbs entropy

- Generalized Planckian distribution (maximum entropy principle)

$$N_q(\epsilon_n) = \frac{1}{[1 + (q-1)\beta\epsilon_n]^{1/(q-1)} \pm 1}$$

Tsallis statistics in UE for mixed fields

- For $|q-1| \ll 1$ (bosons) [*G.L. et al. (2021)*]

$$N_q(\Omega_n) = N_{\rm BE}(\Omega_n) + \frac{\pi^2}{2}\,\Omega_n^2\,\text{csch}^2(\pi\Omega_n)\,(q-1) + \mathcal{O}\,(q-1)^2\,,$$

- UE for mixed bosons ($|\delta m^2|/k^2 \ll 1$)

$$\mathcal{N}_{\theta,\delta m}(\Omega_n) = N_{\rm BE}(\Omega_n) - \frac{|\delta m^2|}{4k^2}\sin^2\theta\,\Omega_n\,\text{csch}^2(\pi\Omega_n) + \mathcal{O}\left(\frac{|\delta m^2|}{k^2}\right)^2$$

- Mixing and Tsallis statistics

$$\mathcal{N}_{\theta,\delta m}(\Omega_n) = N_q(\Omega_n) \implies |q-1| \propto \frac{|\delta m^2|}{k^2\Omega_n}\sin^2\theta$$

Take-home message

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Review Article

Bruno Pontecorvo and Neutrino Oscillations

Samoil M. Bilenky[1,2]

Thus, if a flavor neutrino is produced, the neutrino state at a time t is a superposition of states with different energies, that is, *nonstationary state.*

Neutrinos are detected via the observation of weak processes

$$\nu_{l'} + N \longrightarrow l' + X, \text{etc.,} \quad (30)$$

in which flavor neutrinos are participating. Expanding the state $|\nu_l\rangle_t$ over the flavor neutrino states, we find

$$|\nu_l\rangle_t = \sum_{l'} |\nu_{l'}\rangle \left(\sum_i U_{l'i} e^{-iE_i t} U_{li}^{*} \right). \quad (31)$$

detector.

The expression (34) became the standard expression for the transition probability. It is commonly used in the analysis of data of experiments on the investigation of neutrino oscillations.

We know now that three flavor neutrinos exist in nature. If the number of neutrinos with definite masses is also equal to three (there are no sterile neutrino states), the neutrino transition probabilities depend on two mass-squared differences Δm_{12}^2 and Δm_{23}^2 and on parameters which characterize 3×3 unitary mixing matrix (three angles and one phase).

It follows from analysis of the experimental data that $\Delta m_{12}^2 \ll |\Delta m_{23}^2|$ and one of the mixing angle (θ_{13}) is small. It is easy to show (see, e.g., [40]) that in the leading approximation oscillations observed in atmospheric and accelerator neutrino experiments there are two-neutrino $\nu_\mu \rightleftarrows \nu_\tau$ oscillations. For the ν_μ survival probability from (34), we find the following expression:

$$P\left(\nu_\mu \longrightarrow \nu_\mu\right) \simeq 1 - \frac{1}{2}\sin^2 2\theta_{23}\left(1 - \cos\frac{\Delta m_{23}^2 L}{2E}\right). \quad (36)$$

In the leading approximation, the disappearance of $\bar{\nu}_e$'s in the reactor KamLAND experiment is due to $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$ transitions. The survival probability is given in this case by

Take-home message

Properties of Neutrinos

K. Gabathuler, Villigen

(Swiss Institute for Nuclear Research)

logy to the quark sector, where mixing occurs among the different quark flavours, neutrino mixing would appear natural. The (physical) flavour states ν_e, ν_μ, ν_τ are not necessarily pure neutrino mass eigenstates, but a mixture of the latter. This would give rise to neu-

eV/c². With $\Sigma m_\nu \cong 55$ eV/c² they would close the universe (the summation extends over all flavours of light stable neutrinos). From the presently observed expansion rate of the universe the neutrino masses are restricted to $\Sigma m_\nu <$ 200 eV/c².

now running, so the controversy will be settled rather soon. A sensitivity to the ν_e^c mass between 5 and 10 eV/c² may be expected.

Present mass limits for ν_μ and ν_τ are less stringent. From the decay of the pion at rest $\pi^+ \to \mu^+ + \nu_\mu$, $m_{\nu_\mu} < 0.26$ MeV/c² was obtained at SIN, and from the decay $\tau^\pm \to \pi^\pm \pi^+ \pi^- \nu_\tau^{(c)}$ for m_{ν_τ} a mass greater than 70 MeV/c² was excluded at DESY. If the neutrino masses scale with the squared masses of the associated charged leptons, as some grand unified theories predict, these results are roughly equally relevant in giving the ν_e^c limit. In the interpretation of the above data we have assumed that

Samoil M. Bilenky

Thus, if a flavor neutrino is produced, the neutrino state at a time t is a superposition of states with different energies, that is, *nonstationary state*.

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IOWA STATE UNIVERSITY

Neutrino

Overview Research Team

NOvA

The NuMI Off-axis ν_e Appearance (NOvA) experiment is a long-baseline neutrino oscillation experiment near Ash River, Minnesota which has been recording data since 2014.

NOvA has two detectors, separated by 810 km. In addition to the Far Detector in Ash River, MN, there is also a Near Detector at Fermilab, IL, that measures the neutrino beam before oscillations occur.

NOvA measures the 750 kW beam of muon neutrinos produced by the NUMI beam line, and the Far Detector measures both electron neutrinos and muon neutrinos due to oscillations. Neutrino oscillations are a quantum mechanical phenomenon where neutrinos created in one flavor state are observed interacting as different flavor states after travelling a given distance.

Take-home message

Properties of Neutrinos

K. Gabathuler, Villigen

(Swiss Institute for Nuclear Research)

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e standard expression for

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Neutrino

Team

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OBSERVATION OF THE TAU NEUTRINO

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Since charged-current interactions of neutrinos conserve lepton flavor, the detection of a τ lepton at a neutrino interaction vertex implies that a ν_τ was incident.

earance (NOvA) experiment is a long-baseline neutrino r Ash River, Minnesota which has been recording data

separated by 810 km. In addition to the Far Detector in so a Near Detector at Fermilab, IL, that measures the llations occur.

kW beam of muon neutrinos produced by the NUMI beam line, and the Far Detector measures both electron neutrinos and muon neutrinos due to oscillations. Neutrino oscillations are a quantum mechanical phenomenon where neutrinos created in one flavor state are observed interacting as different flavor states after travelling a given distance.

Take-home message

Properties of Neutrinos

K. Gabathuler, Villigen

(Swiss Institute for Nuclear Research)

now running, so the controversy will be settled rather soon. A sensitivity to the ν_e^c mass between 5 and 10 eV/c^2 may be expected.

Present mass limits for ν_μ and ν_τ are less stringent. From the decay of the pion at rest $\pi^+ \to \mu^+ + \nu_\mu$, $m_{\nu\mu} < 0.26$ MeV/c^2 was obtained at SIN, and from

logy to the quark sector, where mixing occurs among the different quark flavours, neutrino mixing would appear natural. The (physical) flavour states ν_e, ν_μ, ν_τ

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Review

Neutrino Mixing and Oscillations in Quantum Field Theory: A Comprehensive Introduction

Luca Smaldone [1,*] and Giuseppe Vitiello [2,*]

The use of the flavor vacuum allows to define the exact eigenstates of the flavor charges, and an exact oscillation formula can be derived by taking the expectation values of flavor charges on flavor states. It is worth remarking that such formula can also be derived in a first quantized approach (cf. Appendix C), independently of the QFT construction. However, the QFT approach gives us a deeper insight, even fixing phenomenological bounds as the TEUR (cf. Section 7), i.e., a form of Mandelstam–Tamm uncertainty relation involving flavor charges, which fixes a lower bound on neutrino energy resolution. This means that only flavor states have a physical meaning, both in weak interactions and neutrino propagation. These conclusions are also supported by the fact that contradictions and paradoxes arise by using standard QM flavor states and assuming the mass vacuum

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