Constraints on GUT model building and its' phenomenological implications

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Outline

1. Introduction
2. First principle model building
3. Constraints from unification
4. Conclusions

1. Introduction

The Standard Model (SM)

- SM is the most general renormalizable quantum field theory with the gauge group
$$\mathcal{G}_{\rm SM} = {\rm SU}(3)_C \times {\rm SU}(2)_L \times {\rm U}(1)_Y\,,$$
and three generations of fermions and a scalar transforming under the representations
$$(\mathbf{3},\mathbf{2})_{1/6} + (\bar{\mathbf{3}},\mathbf{1})_{-2/3} + (\bar{\mathbf{3}},\mathbf{1})_{1/3} + (\mathbf{1},\mathbf{2})_{-1/2} + (\mathbf{1},\mathbf{1})_{1} \quad \textbf{and} \quad (\mathbf{1},\mathbf{2})_{1/2}\,.$$

- SM is formulated by the Lagrangian density:
$$\mathcal{L}_{\rm SM} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_Y + \mathcal{L}_H\,.$$

- SM contains 19 parameters: 3 gauge couplings in $\mathcal{L}_G + \mathcal{L}_F$, 13 parameters in $\mathcal{L}_Y$ (12 real and 1 phase), 1 scalar coupling λ and 1 dimensionful parameter m_h in $\mathcal{L}_H$.

- There is an additional parameter $\bar{\theta}_{QCD} \lesssim 10^{-10}$ coming from non-trivial topological configuration of gauge field localized in the spacetime (the instantons). (See e.g. Mario Reig,)
$$\mathcal{L} \supset \theta\ {\rm tr} F \wedge F \sim \theta\ \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} \sim \theta\ {\rm tr}\ F\tilde{F}\,.$$

Facts about the SM

- SM is not complete, it must be an Effective Field Theory (EFT) parameterized by the cut-off scale Λ and a few dimensionless coefficients c_i (e.g. $g_i, Y_{ij}, \theta, \ldots$)

Big questions in BSM

- Two big questions naturally arise concerning:
- 1. What is the origin of parameters in the Standard Model?
- 2. What is the cutoff scale of the Standard Model?

  To certain extent, these answers can be found in Grand Unified Theory

2. First Principle model building

Common strategies for BSM model building

Top down

Integrating out:

$$e^{iS_{\text{IR}}[\phi]} = \int \mathscr{D}\Phi e^{iS_{\text{UV}}[\Phi,\phi]}$$

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{ren}} + \sum_{n=d}^{\infty} \frac{c_n \mathcal{O}_n}{\Lambda^{n-d}}$$

Bottom up

Integrating in: adding new degrees of freedom above Λ_{SM} and writing down every local operator consistent with symmetries. Theoretical or experimental constraints must be imposed to ensure the consistency of the model.

The more constraints added for model building, the less free-parameters are allowed which rendering the model more predictive!

Principles behind EFT approach

Agmon, Bedroya, Kang, Vafa ’22

- Symmetry principle: all terms allowed by symmetries are allowed. Renormalizability is certainly not required. The symmetry $\mathscr{G}_{\rm Lorentz} \times \mathscr{G}_{\rm Gauge}$ is a free parameter.

- UV/IR decoupling principle: low-energy physics can be effectively described independently of high-energy physics within the EFT framework. (Philosophy of Wilson’s Renormalization group)

- Naturalness principle: coupling constants in a theory are of order one in the appropriate mass scale. Therefore, if any parameter is unusually small or large, a good explanation, such as an underlying symmetry, is require.

An example: non-SUSY SO(10) GUT

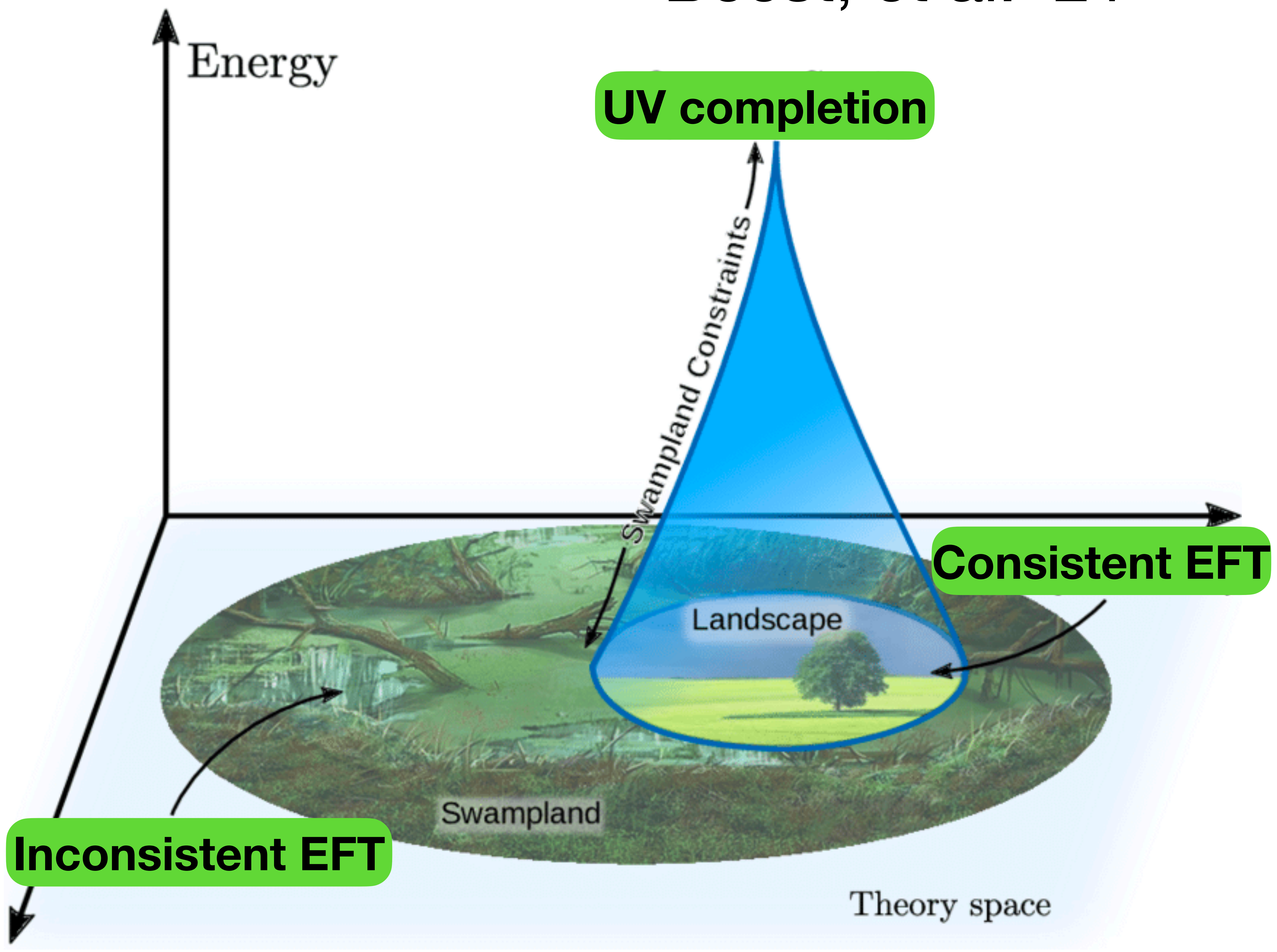
- There are a few other motivations to consider the non-SUSY SO(10) GUT explicitly, such as the charge quantization, parity violation, absence of low energy supersymmetry, etc.
- A minimal non-SUSY SO(10) model usually contains the following additional particles: right-handed neutrinos, axions, and heavy Higgses.
- The exact matter representation needed should be considered together with the constraints.

Constraints on GUT model building

Consider building a BSM model where the symmetry group and representations are free parameters. For the model to be phenomenologically-consistent, it must satisfy certain constraints, for examples:

1. Anomaly cancellation
2. Vacuum structure (alignment) (talk by A. Pilaftsis)
3. Stable (long-lived) vacuum (talk by K. Kowalska)
4. UV completion: asymptotic free/safe
5. Non-trivial constraints from gravity
6. Unification of fundamental couplings

Beest, et al. ’21



3. Constraints from unification

Example 1: unification of gauge couplings

Example 2: unification of Yukawa couplings

Ex1: Unification of gauge couplings

- The unification of fundamental couplings is a specific type of "Reduction" (see talks by G. Patellis and M. Mondragon).
- The RGEs are a set of differential equations that takes the form:

$$\frac{\mathrm{d}\alpha_i^{-1}(\mu)}{\mathrm{d}\ln\mu} = -\frac{a_i}{2\pi} - \sum_j \frac{b_{ij}}{8\pi^2\alpha_j^{-1}(\mu)}$$

- It has an approximate solution:

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) - \frac{a_i}{2\pi}\ln\frac{\mu}{\mu_0} \underbrace{\boxed{-\frac{1}{4\pi}\sum_j \frac{b_{ij}}{a_j}\ln\frac{\alpha_j(\mu)}{\alpha_j(\mu_0)}}}_{\gamma_i} + \Delta_Y^i$$

- All three gauge couplings unify at a scale implies:

$$\alpha_1^{-1}(\Lambda_G) = \alpha_2^{-1}(\Lambda_G) = \alpha_3^{-1}(\Lambda_G) = \alpha_U^{-1}(\Lambda_G)$$

Ex1: Unification of gauge couplings

- The two-loop corrections can be approximated by: [Langecker & Polonsky ’92]

$$\gamma_i^{\mathcal{G}} = -\frac{1}{4\pi}\sum_j \frac{b_{ij}^{\mathcal{G}}}{a_j^{\mathcal{G}}} \ln\frac{\alpha_{j,\mathcal{G}}(\mu)}{\alpha_{j,\mathcal{G}}(\mu_0)} \approx -\frac{\alpha_U}{8\pi^2}\theta_i^{\mathcal{G}} \ln\frac{\mu}{\mu_0}$$

$$\theta_i^{\mathcal{G}} \equiv \sum_j b_{ij}^{\mathcal{G}} \frac{\ln(1 + a_j^{\mathcal{G}}\alpha_U t)}{a_j^{\mathcal{G}}\alpha_U t} \quad \text{and} \quad t = \frac{1}{2\pi}\ln\frac{\mu}{\mu_0}$$

- The original 2-loop RGEs becomes: [Djouadi, Fonseca, RO, Raidal, ’22] (Appendix A2)

$$\alpha_{i,\mathcal{G}}^{-1}(\mu) = \alpha_{i,\mathcal{G}}^{-1}(\mu_0) - \boxed{\left(\frac{a_i^{\mathcal{G}}}{2\pi} + \frac{\theta_i^{\mathcal{G}}}{8\pi^2}\alpha_U\right)} \ln\frac{\mu}{\mu_0}$$

- With initial conditions ($\alpha_i(M_W)$) and the boundary conditions:

$$\alpha_1^{-1}(\Lambda_G) = \alpha_2^{-1}(\Lambda_G) = \alpha_3^{-1}(\Lambda_G) = \alpha_U^{-1}(\Lambda_G)$$

Ex1: Unification of gauge couplings

- In particular in non-SUSY SO(10) with only one intermediate scale, this can be approximately solvable: [Djouadi, Fonseca, RO, Raidal, '22]

$$\ln\left(\frac{M_I}{M_Z}\right) = \frac{(\alpha^{-1}_{1_{\rm EW}} - \alpha^{-1}_{3_{\rm EW}}) - C_{\mathcal{G}_I}(\alpha^{-1}_{2_{\rm EW}} - \alpha^{-1}_{3_{\rm EW}}) + D_{\mathcal{G}_I}}{C_{\mathcal{G}_I}\Delta^{\mathcal{G}_{321}}_{32} - \Delta^{\mathcal{G}_{321}}_{31}}$$

$$\ln\left(\frac{M_U}{M_I}\right) = -\frac{\alpha^{-1}_{2_{\rm EW}} - \alpha^{-1}_{3_{\rm EW}}}{\Delta^{\mathcal{G}_I}_{3_I 2L_I}} - \frac{\Delta^{\mathcal{G}_{321}}_{32}}{\Delta^{\mathcal{G}_I}_{3_I 2L_I}}\ln\left(\frac{M_I}{M_Z}\right) - \frac{D'_{\mathcal{G}_I}}{\Delta^{\mathcal{G}_I}_{3_I 2L_I}}$$

$$\Delta^{\mathcal{G}}_{ij} = \frac{a^{\mathcal{G}}_i - a^{\mathcal{G}}_j}{2\pi} + \frac{\theta^{\mathcal{G}}_i - \theta^{\mathcal{G}}_j}{8\pi^2}\alpha_U\,.$$

$$C_{\mathcal{G}_{422}} = 3\Delta^{\mathcal{G}_{422}}_{42_R}/(5\Delta^{\mathcal{G}_{422}}_{42_L})\,,\quad C_{\mathcal{G}_{3221}} = (3\Delta^{\mathcal{G}_{3221}}_{32_R} + 2\Delta^{\mathcal{G}_{3221}}_{3B\text{-}L})/(5\Delta^{\mathcal{G}_{3221}}_{32_L})\,,$$

Predictions for gauge unification

Breaking chains	$C_{\mathcal{G}_I}$	$\Delta_{31}^{\mathcal{G}_{321}}$	$\Delta_{32}^{\mathcal{G}_{321}}$	$\Delta_{3_I 2L_I}^{\mathcal{G}_I}$
$\mathcal{G}_{422} \to \mathcal{G}_{321}$(SM)	$\frac{21}{13}$	-$\frac{111}{20\pi}$	-$\frac{23}{12\pi}$	$-\frac{13}{6\pi}$
$\mathcal{G}_{422} \to \mathcal{G}_{321}$(2HDM)	$\frac{21}{13}$	-$\frac{28}{5\pi}$	-$\frac{2}{\pi}$	$-\frac{13}{6\pi}$
$\mathcal{G}_{3221} \to \mathcal{G}_{321}$(SM)	$\frac{24}{13}$	-$\frac{111}{20\pi}$	-$\frac{23}{12\pi}$	$-\frac{13}{6\pi}$
$\mathcal{G}_{3221} \to \mathcal{G}_{321}$(2HDM)	$\frac{24}{13}$	-$\frac{28}{5\pi}$	-$\frac{2}{\pi}$	$-\frac{13}{6\pi}$

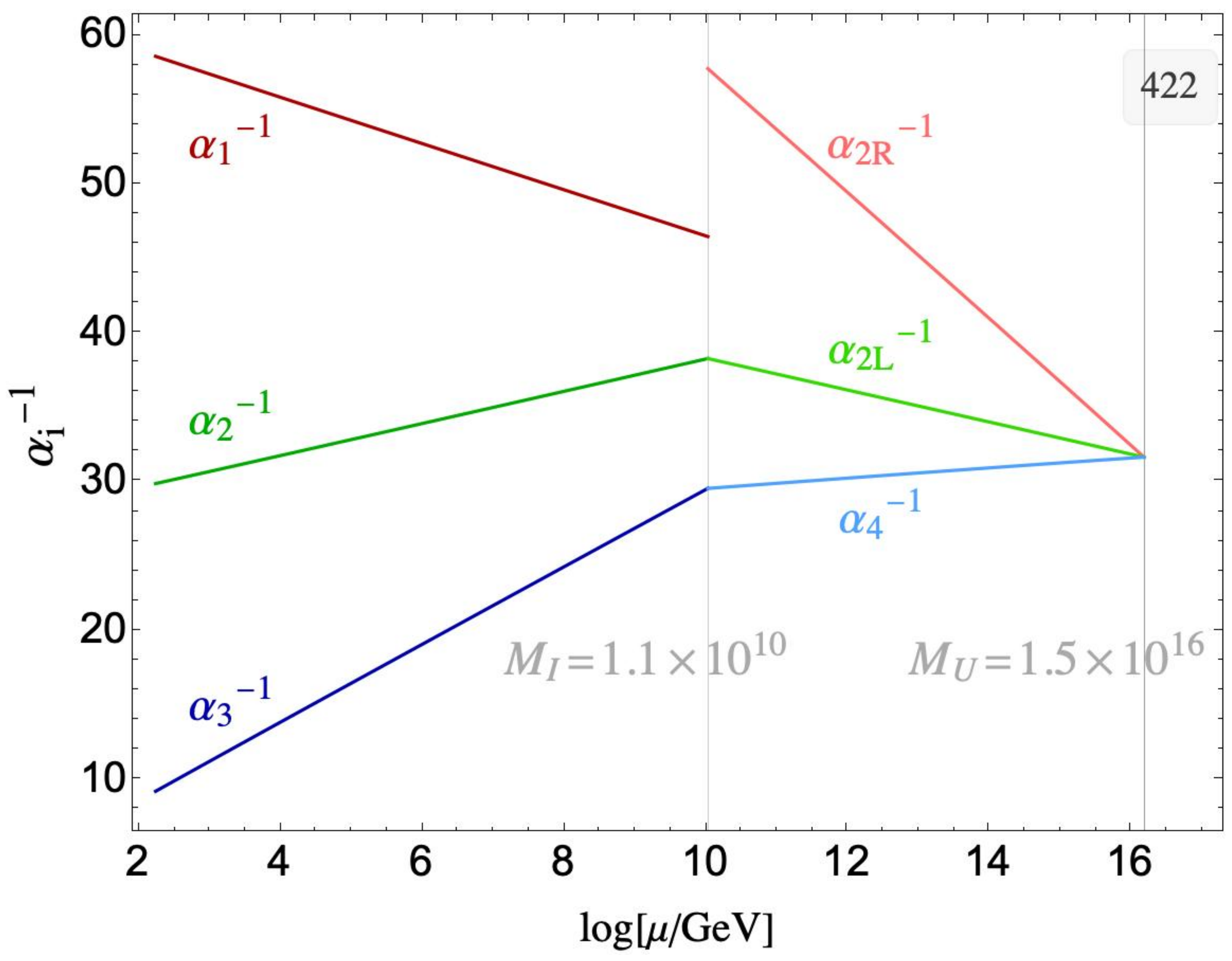
$\mathcal{G}_{321}$	$\mathcal{G}_I$	$\log\left(\frac{M_{I2}}{\text{GeV}}\right)$	$\log\left(\frac{M_{U2}}{\text{GeV}}\right)$	$\alpha_U^{2-\text{loop}}$
SM	$\mathcal{G}_{422}$	9.627	16.718	0.0313
SM	$\mathcal{G}_{3221}$	9.942	15.929	0.0262
2HDM	$\mathcal{G}_{422}$	10.133	16.346	0.0304
2HDM	$\mathcal{G}_{3221}$	10.398	15.652	0.0230

$$\tau(p \to e^+\pi^0) \simeq (7.47 \times 10^{35}\text{yr})\left(\frac{M_U}{10^{16}\text{ GeV}}\right)^4 \left(\frac{0.03}{\alpha_U}\right)^2$$

[Meloni, Ohlsson, Pernow, '20]

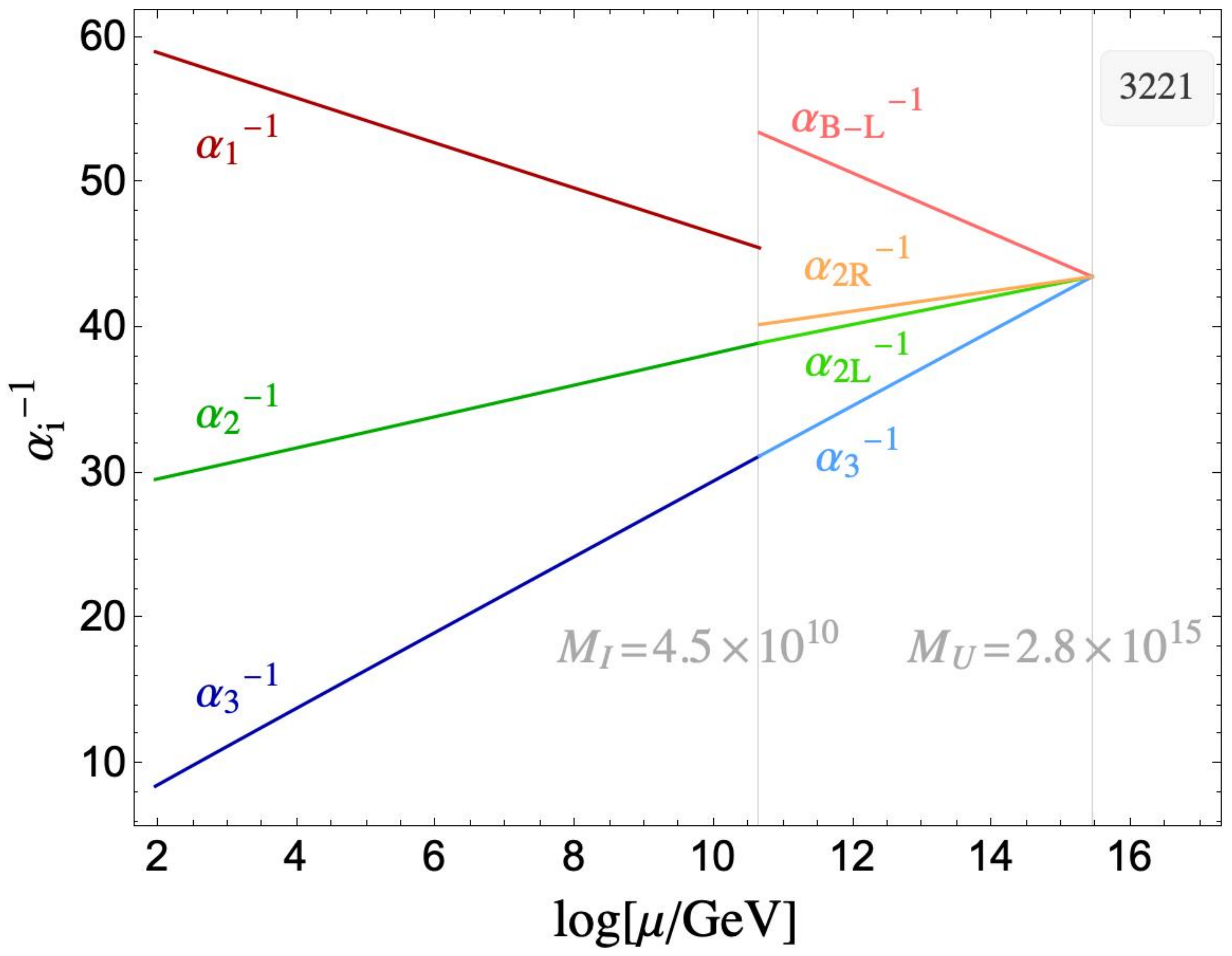
Breaking chain	$\log\left(\frac{M_{Ic}}{\text{GeV}}\right)^{2-\text{loop}}$	$\log\left(\frac{M_{Uc}}{\text{GeV}}\right)^{2-\text{loop}}$	$\alpha_U^{2-\text{loop}}$	$\tau(p \to e^+\pi^0)/\text{yr}$
422	10.03	16.19	0.032	3.82×10^{36}
3221	10.66	15.45	0.023	7.84×10^{33}

Unification of gauge couplings in non-SUSY SO(10)



Pati-Salam (422)

$\mathscr{G}_{422} = \mathrm{SU}(4)\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$



Left-Right Symmetry (3221)

$\mathscr{G}_{3221} = \mathrm{SU}(3)\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}$

Ex2: Unification of Yukawa couplings

- Yukawa unification are regarded as boundary conditions for the RGEs.
- Yukawa couplings flows to different values in IR because of RGEs.

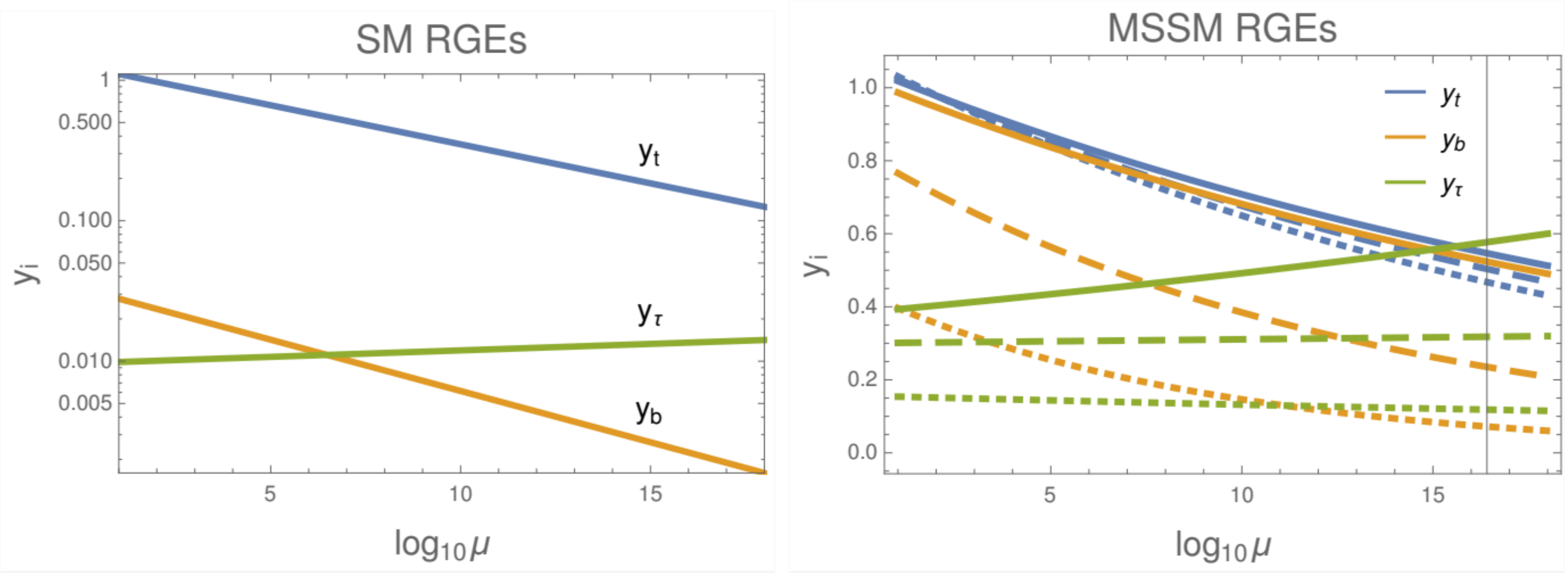


Figure 5: *One loop renormalisation group flow of the SM (left) and MSSM (right) Yukawa couplings, with* $m_0 = 2$ *TeV,* $m_{1/2} = 3$ *TeV,* $A_0 = 0$ *and* $\tan\beta = 40$ *(solid),* $\tan\beta = 30$ *(dashed) and* $\tan\beta = 15$ *(dotted).*

[Croon, Gonzalo, Graf, Košnik, White '19]

The idea of Yukawa unification has been extended to non-supersymmetric case

[Djouadi, RO, Raidal, '21]

Ex2: Unification of Yukawa couplings

- Yukawa unification are regarded as boundary conditions for the RGEs.
- Yukawa couplings flows to different values in IR because of RGEs.

What is the implication of Yukawa unification? ← How to motivate the Yukawa unification? ← The idea of Yukawa unification has been extended to non-supersymmetric case

[Djouadi, RO, Raidal, '21]

What is the implication of Yukawa unification? ↓

There is a common origin for Yukawa hierarchy for a single generation.

How to motivate the Yukawa unification? ↓

The original motivation of GUT:

Unification of matter representation:

Fermions: $\mathbf{16} \longrightarrow \mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1}\ (E_6)$

Scalars: $\mathbf{10} + \overline{\mathbf{126}} \longrightarrow\ ?\ (E_6)$

[Djouadi, Fonseca, RO, Raidal, '22]

Common origin of Yukawas in minimal SO(10)

- In E_6, we calculate the CG decomposition of spinor product $\mathbf{27} \times \mathbf{27}$ and found:

$$\mathbf{351}' \supset \mathbf{10} + \overline{\mathbf{126}} + \cdots$$

$$Y \times \mathbf{27}_F \cdot \mathbf{27}_F \cdot \mathbf{351}'_H \supset c_{10} Y \times \mathbf{16}_F \cdot \mathbf{16}_F \cdot \mathbf{10}_H + c_{126} Y \times \mathbf{16}_F \cdot \mathbf{16}_F \cdot \overline{\mathbf{126}}_H + \cdots$$

- As $\mathbf{351}'$ is a complex representation, $\mathbf{10}_H$ must be associated to a complex field.
- An E_6-symmetric Yukawa section does not involve the coupling $\mathbf{16_F 16_F 10}^*$, hence, there is no such an interaction at leading order. Its absence can be understood by the fact that E_6 contains an extra U(1) subgroup which commutes with SO(10).
- After CG decomposition, the SO(10) Yukawa couplings are unified by:

$$\frac{Y_{10}}{Y_{126}} = \frac{c_{10} Y}{c_{126} Y} = \frac{c_{10}}{c_{126}} = \sqrt{\frac{3}{5}}$$

[Fonseca, '21]
[Babu, Bajc, Susič, '15]

What happens at the intermediate scale?

- The mass should be continuous at the intermediate scale M_I. Therefore some matching conditions can be deduced for Yukawa couplings in both EFTs above or below M_I.

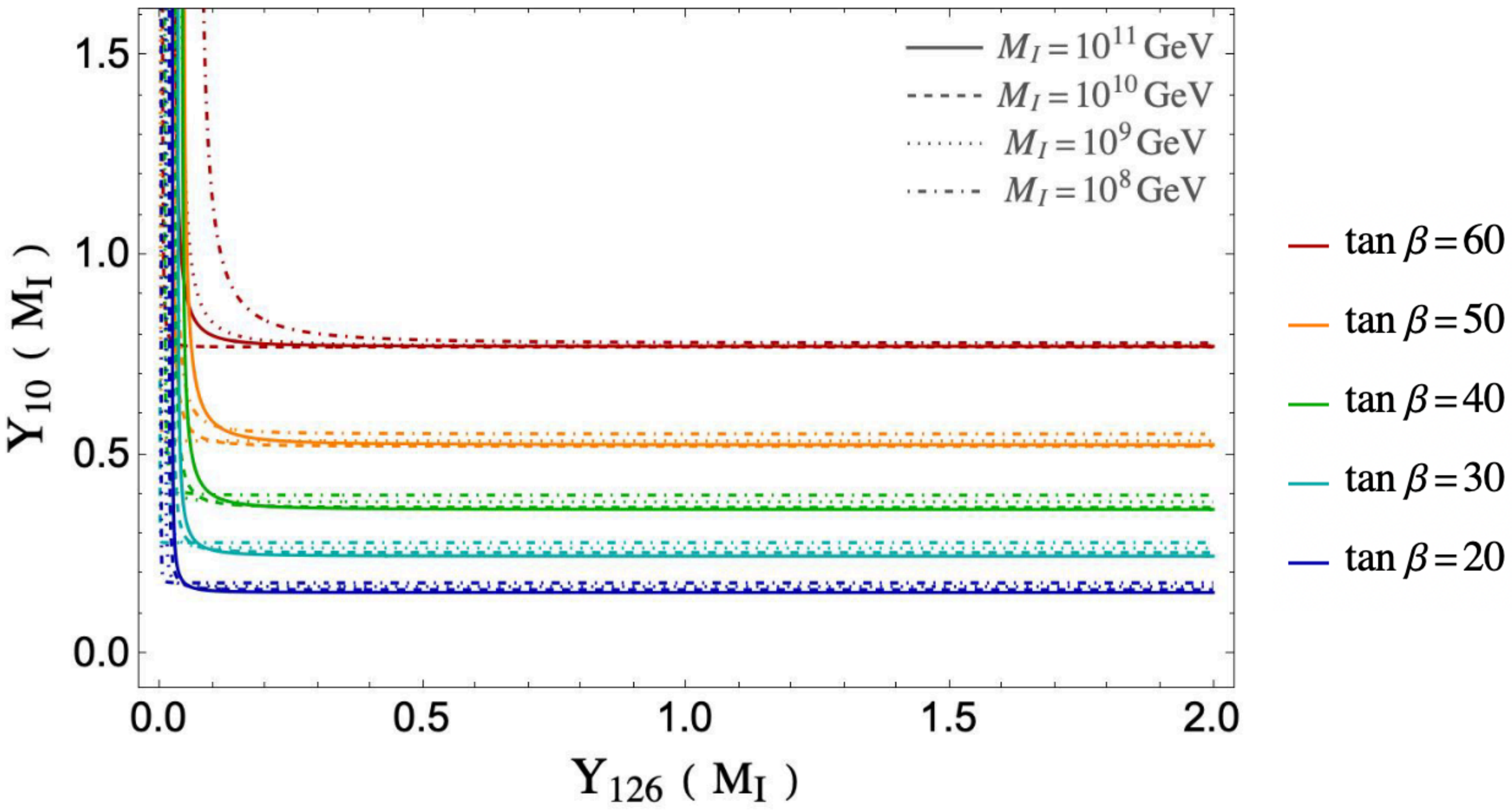
- From 422 model: $m_t = \frac{v_{10}^u}{\sqrt{2}} \boxed{Y_{10}^{422}} + \frac{v_{126}^u}{4\sqrt{2}} \boxed{Y_{126}^{422}}, \; m_b = \frac{v_{10}^d}{\sqrt{2}} \boxed{Y_{10}^{422}} + \frac{v_{126}^d}{4\sqrt{2}} \boxed{Y_{126}^{422}}, \; m_\tau = \frac{v_{10}^d}{\sqrt{2}} \boxed{Y_{10}^{422}} - \frac{3v_{126}^d}{4\sqrt{2}} \boxed{Y_{126}^{422}}.$

- From 2HDM: $m_t = \frac{1}{\sqrt{2}} \boxed{Y_t} v_u\,, \quad m_b = \frac{1}{\sqrt{2}} \boxed{Y_b} v_d\,, \quad m_\tau = \frac{1}{\sqrt{2}} \boxed{Y_\tau} v_d\,.$

- These relations can be simplified to be (assuming no tree-level FCNCs):

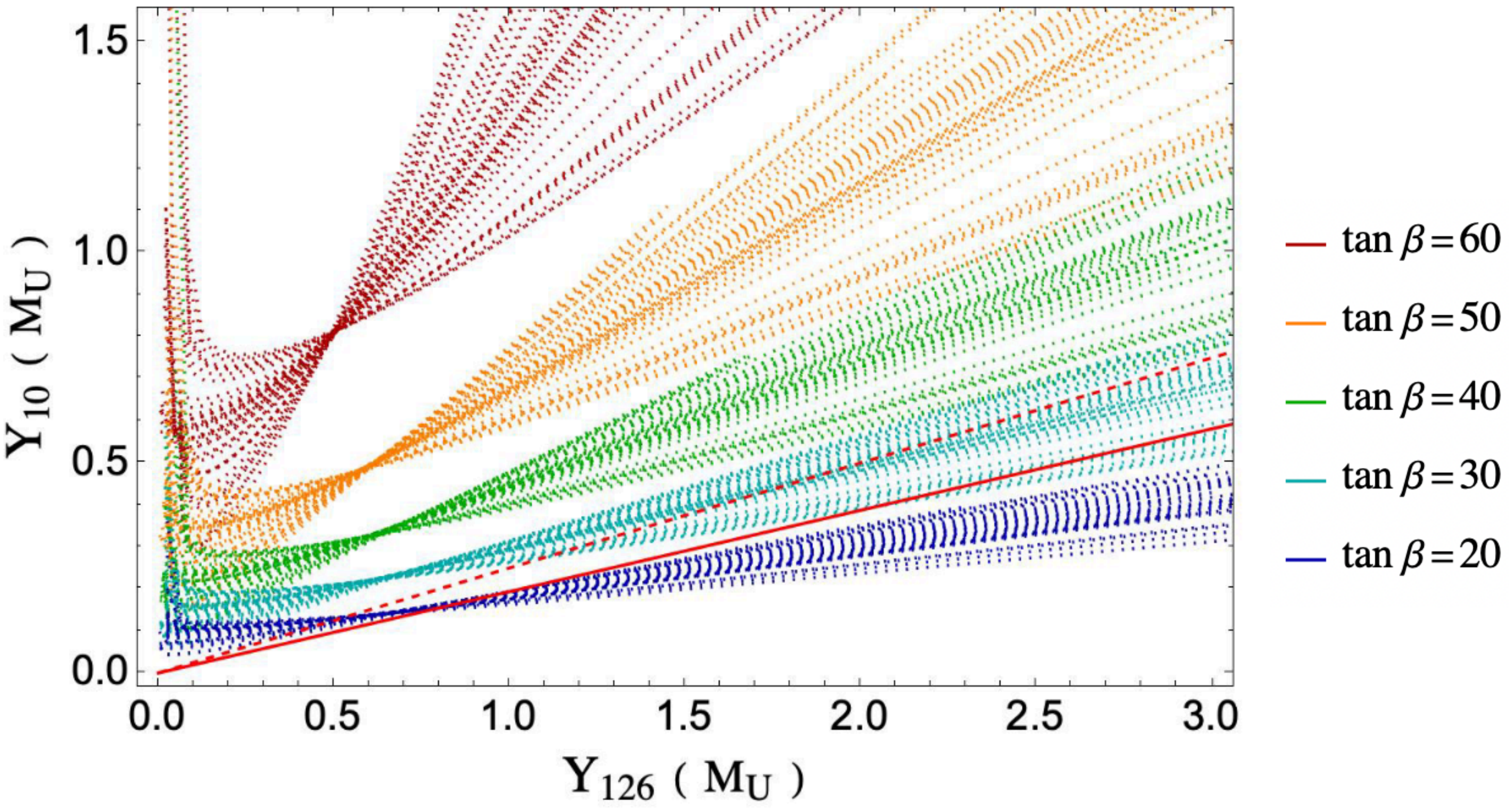
$$\left(\boxed{Y_{10}^{422}(M_I)}\right)^2 = \frac{\left(\boxed{Y_{126}^{422}(M_I)}\right)^2 \left(3Y_b(M_I) + Y_\tau(M_I)\right)^2}{16\left[\left(\boxed{Y_{126}^{422}(M_I)}\right)^2 - \left(Y_b(M_I) - Y_\tau(M_I)\right)^2\right]}$$

Constraints from Yukawa unification



Visualizing the matching conditions

Constraints from Yukawa unification



(Numerical) Solutions of RGEs + matching conditions

Implications of Yukawa unification

- The constraint from unification of Yukawa couplings imposes non-trivial relations on the parameters of the scalar sector, which is described by the (numerical) solution of RGEs of Yukawa couplings with particular boundary conditions and matching conditions.
- The original dimesionless parameters (Yukawa couplings) will be related to the ratio of vevs ($\tan\beta$). The unification of Yukawa couplings in our model implies that $\tan\beta \lesssim 30$, which can be tested in future collider experiment. [e.g. PDG '23]
- Yukawa unification implies that the Yukawa hierarchy of a single generation can be explained dynamically by higher rank symmetry and RGEs.

Conclusions

- Many constraints can be imposed for GUT model buildings. The more constraints we have, the less free-parameters are allowed, and the more predictive the model will be!
- In particular, unification of fundamental couplings severely constrains a given GUT model. We use two explicit examples in non-SUSY SO(10) models to explain how such constraints reduce free parameters in our models by performing explicit RGEs analysis.

Thank you very much for your attention!

Fermion representations in SO(10)

- Counting SM chiral fermions of a single generation:

  8 Left-handed fermions: $u_L^{c_1}, d_L^{c_1}, u_L^{c_2}, d_L^{c_2}, u_L^{c_3}, d_L^{c_3}, \ell_L, \nu_L^{\ell}$

  7 Right-handed fermions: $u_R^{c_1}, d_R^{c_1}, u_R^{c_2}, d_R^{c_2}, u_R^{c_3}, d_R^{c_3}, \ell_R$

- All these fermion can be embedded into a single 16-dimensional spinor representation of SO(10) group: $\mathbf{16_F}$, with an additional right-handed fields identified as the right-handed neutrino: ν_R^{ℓ}

$$\mathbf{16_F} \supset \left(u_L^{c_1}, d_L^{c_1}, u_R^{c_1}, d_R^{c_1}, u_L^{c_2}, d_L^{c_2}, u_R^{c_2}, d_R^{c_2}, u_L^{c_3}, d_L^{c_3}, u_R^{c_3}, d_R^{c_3}, \ell_L, \nu_L^{\ell}, \ell_R, \nu_R^{\ell} \right)$$

Fermion masses in minimal SO(10)

- The "minimal SO(10) model" have the following Yukawa couplings:

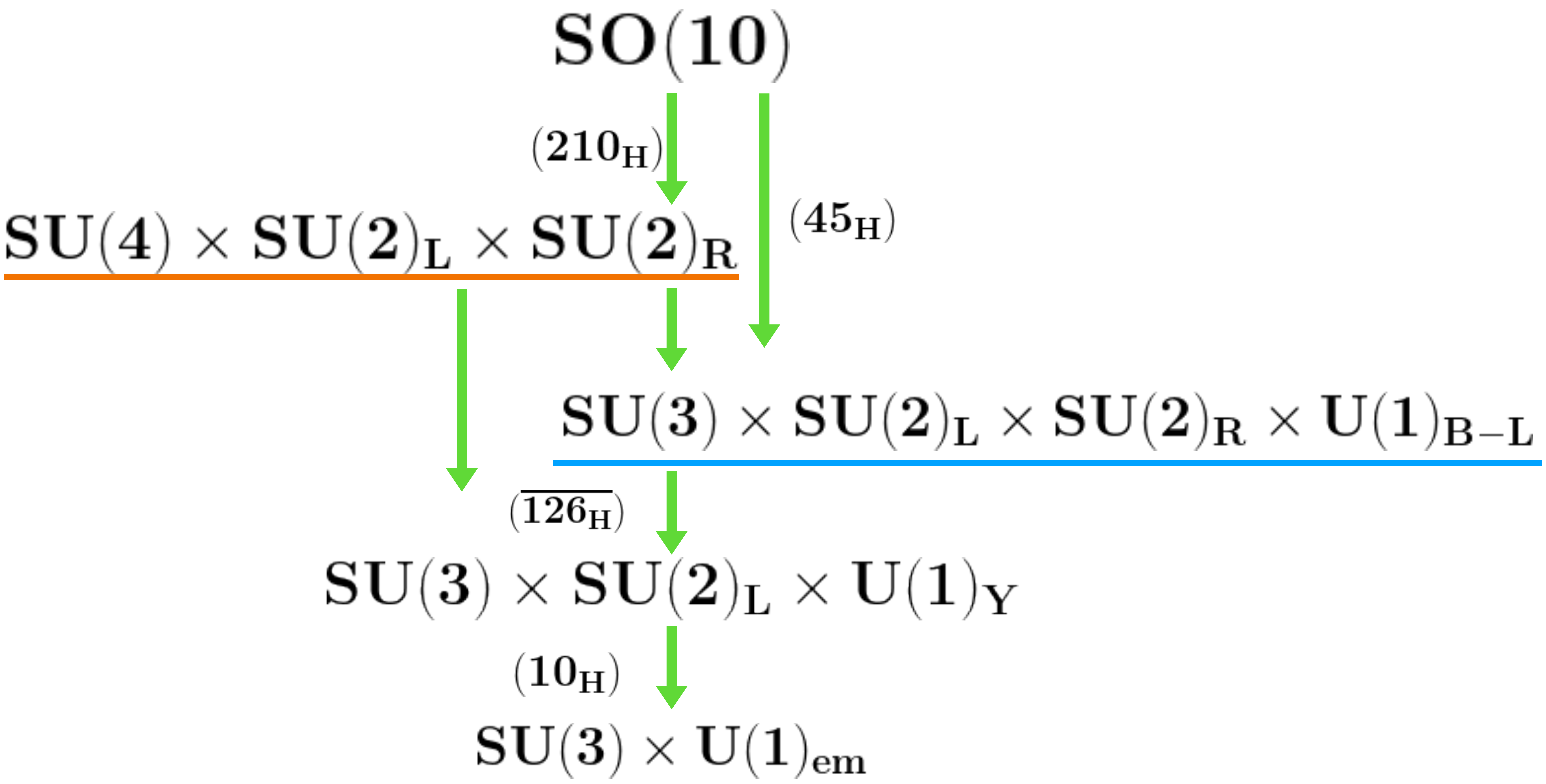
$$-\mathscr{L}_{\text{Yukawa}} = \mathbf{16}_F(Y_{10}\mathbf{10} + Y_{10*}\mathbf{10}^* + Y_{126}\overline{\mathbf{126}}_H)\mathbf{16}_F \qquad \text{(without U(1) PQ)}$$

- The real field $\mathbf{10}$ and $\mathbf{10}^*$ can be combined into a single complex field $\mathbf{10}_H$ by introducing an additional U(1) PQ symmetry, reducing the above Yukawa to:

$$-\mathscr{L}_{\text{Yukawa}} = \mathbf{16}_F(Y_{10}\mathbf{10}_H + Y_{126}\overline{\mathbf{126}}_H)\mathbf{16}_F$$

- Extensive numerical fits to fermion masses and mixings are carried out for the above model (Joshipura *et al.* '11, Dueck *et al.* '13, Altarelli *et al.* '13, Meloni *et al.* '14)

Unification of fundamental couplings



$$\text{PS}: \quad \mathrm{SO}(10)|_{M_U} \xrightarrow{\langle \mathbf{210_H} \rangle} \mathcal{G}_{422}|_{M_I} \xrightarrow{\langle \overline{\mathbf{126}}_\mathbf{H} \rangle} \mathcal{G}_{321}|_{M_Z} \xrightarrow{\langle \mathbf{10_H} \rangle} \mathcal{G}_{31}$$

$$\text{LR}: \quad \mathrm{SO}(10)|_{M_U} \xrightarrow{\langle \mathbf{45_H} \rangle} \mathcal{G}_{3221}|_{M_I} \xrightarrow{\langle \overline{\mathbf{126}}_\mathbf{H} \rangle} \mathcal{G}_{321}|_{M_Z} \xrightarrow{\langle \mathbf{10_H} \rangle} \mathcal{G}_{31}$$

The survival hypothesis

- The survival hypothesis: scalars should have masses of order 1 at the symmetry breaking scale (the GUT scale), unless there are symmetries to protect their masses. (Again motivated by Naturalness)

- Only certain scalar components from $\mathbf{10_H}$ and $\overline{\mathbf{126}}_\mathbf{H}$ representations can acquire small vevs, so they can stay light below the GUT scale;

The EFT at intermediate scale

- The EFT at the intermediate scale should be left-right symmetric in the discussed breaking chains: it is a left-right model where the left-handed and right-handed fermions are coupled via a bi-doublet scalar field as

$$\bar{F}_L(Y_{10}\Phi_{10} + Y_{126}\Sigma_{126})F_R + Y_R F_R^T C \overline{\Delta_R} F_R + \mathrm{h.c.}$$

- The $\mathrm{SU}(2)_R$ right-handed symmetry will be broken by the right-handed triplet field Δ_R, which acquires an intermediate scale masses.

- Below the intermediate scale, we can integrate out the heavy gauge bosons and decouple most scalars except for the (two) Higgs doublet fields. So we should end up with a two Higgs doublet model (2HDM) at lower energy.

SO(10) as BSM model

- SO(10) models generalize the gauge group of SM to a larger gauge symmetry. The vacuum structure is much more complicated with many different phases. We can have different intermediate breaking patterns.

- The fermion within one generation plus a right-handed neutrino can all be embedded into a single representation $\mathbf{16_F}$ of SO(10).

- The SM Higgs field, with hypercharge +1/2, come from a decomposition of the SO(10) scalar field (can be a mixing of Φ_{10} and Σ_{126}).

- At the intermediate scale, we will have a left-right model, which is broken by the vev of Δ_R. The right-handed neutrinos can thus get Majorana masses at the scale Δ_R, and triggers the seesaw mechanism in this scenario.

Proton decay

- Numerical result: proton decay only preferred the Pati-Salam (422) and Minimal Left-Right (3221) breaking chains of SO(10).

Breaking chain	$\log\left(\frac{M_{Ic}}{\mathrm{GeV}}\right)^{2-\mathrm{loop}}$	$\log\left(\frac{M_{Uc}}{\mathrm{GeV}}\right)^{2-\mathrm{loop}}$	$\alpha_U^{2-\mathrm{loop}}$	$\tau(p \to e^+\pi^0)/\mathrm{yr}$
422	10.03	16.19	0.032	3.82×10^{36}
3221	10.66	15.45	0.023	7.84×10^{33}
422D	13.65	14.66	0.026	4.22×10^{30}
3221D	11.82	14.63	0.024	3.89×10^{30}

Table 3: A summary table of the numerical results of the intermediate scale, the unification scale, and the universal gauge coupling at the two-loop level, neglecting all the threshold corrections as well as the estimated proton lifetimes obtained for each considered breaking chain with two Higgs doublets at the electroweak scale. The ratio of vevs is fixed to $\tan\beta = 65$ as the results do not change significantly for lower values of $\tan\beta$.

Scalar multiplets in different breaking chains

Intermediate symmetry	Scalar Multiplets
422	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_R \oplus \Delta_{45R}$
422D	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_L \oplus \Delta_R \oplus \Delta_{45L} \oplus \Delta_{45R}$
3221	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_R$
3221D	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_L \oplus \Delta_R \oplus \Delta_{45L} \oplus \Delta_{45R}$

Table 1: List of scalar multiplets containing light fields, for each intermediate symmetry. They are the only ones which are not integrated out below the SO(10) symmetry breaking scale mass M_U.