国科大杭州高等研究院
Hangzhou Institute for Advanced Study, UCAS

Workshop on the Standard Model
and Beyond, Corfu · 2024

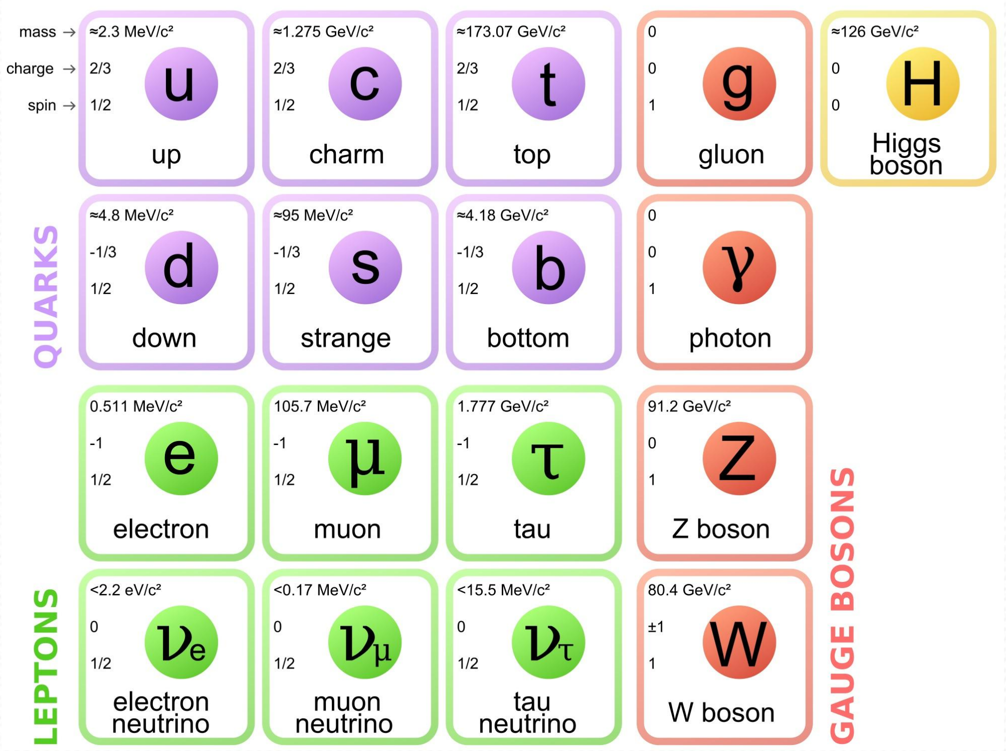
W -Boson Mass Anomaly from $\mathrm{SU}(2)_L$ Scalar Multiplets

Jia-Jun Wu

Corfu · August 29, 2024

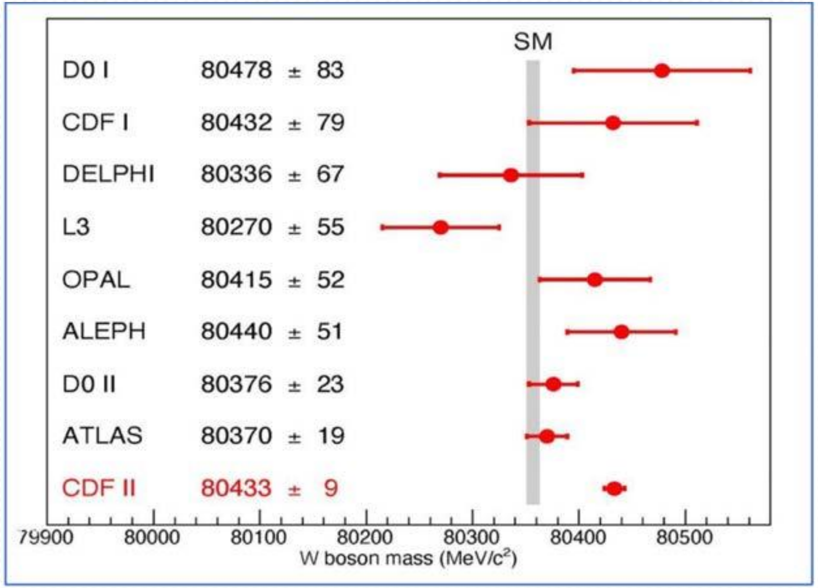
- Jiajun Wu, Chao-Qiang Geng, and Da Huang, *Physics Letters B* 2024, 852 (2024): 138637.
- Jiajun Wu, Da Huang, and Chao-Qiang Geng, *Chinese Physics C* 2023, 47(6):063103.

The Standard Model and Beyond



- The Nature of Dark Matter
- Muon *g*-2 Anomaly
- The Origin of Neutrino Mass
- *W*-boson Mass Anomaly
- … … … …

W-boson Mass Anomaly:



April 2022

The deviation from the prediction of the standard model reaches 7σ

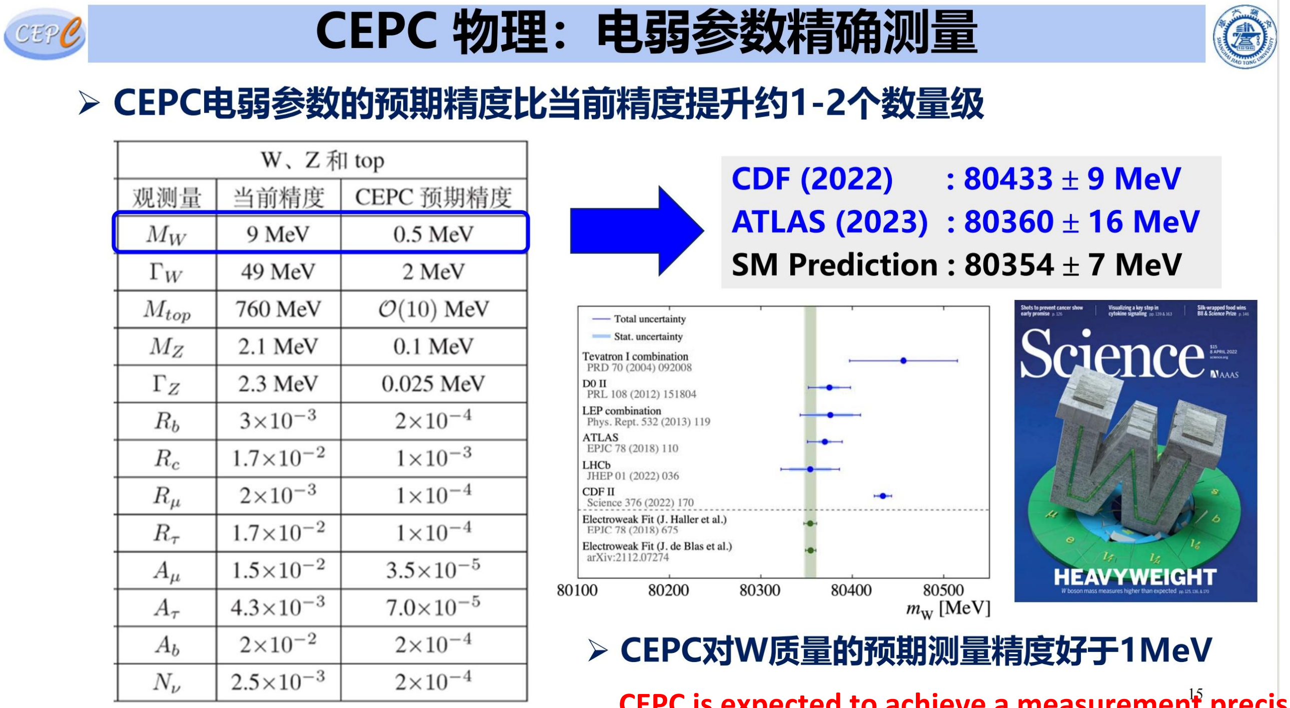
A possible signal of new physics

CDF-II Results: $M_{W,CDF} = 80.4335\pm0.0094\text{GeV}$

The SM Prediction: $M_{W,SM} = 80.3570\pm0.006\text{ GeV}$

- CDF Collaboration, Aaltonen T, Amerio S, et al. High-precision measurement of the W boson mass with the CDF II detector[J]. Science, 2022, 376(6589): 170-176.

W-boson Mass Anomaly:



CEPC 物理：电弱参数精确测量

- ➢ CEPC电弱参数的预期精度比当前精度提升约1-2个数量级

W、Z和 top		
观测量	当前精度	CEPC 预期精度
M_W	9 MeV	0.5 MeV
Γ_W	49 MeV	2 MeV
M_{top}	760 MeV	$\mathcal{O}(10)$ MeV
M_Z	2.1 MeV	0.1 MeV
Γ_Z	2.3 MeV	0.025 MeV
R_b	3×10^{-3}	2×10^{-4}
R_c	1.7×10^{-2}	1×10^{-3}
R_μ	2×10^{-3}	1×10^{-4}
R_τ	1.7×10^{-2}	1×10^{-4}
A_μ	1.5×10^{-2}	3.5×10^{-5}
A_τ	4.3×10^{-3}	7.0×10^{-5}
A_b	2×10^{-2}	2×10^{-4}
N_ν	2.5×10^{-3}	2×10^{-4}

CDF (2022) : 80433 ± 9 MeV
ATLAS (2023) : 80360 ± 16 MeV
SM Prediction : 80354 ± 7 MeV

- ➢ CEPC对W质量的预期测量精度好于1MeV

15

CEPC is expected to achieve a measurement precision for the W boson mass better than 1 MeV

Future electron-positron colliders, such as CEPC, will provide more precise measurements.

New Physics Perspectives: Introduction of New Particles (primarily)

◆ New Gauge Bosons:

e.g.

- Kai-Yu Zhang, Wan-Zhe Feng, *CPC* 2023.
- Yu-Pan Zeng , Chengfeng Cai, et al., *PRD* 2023.
- Mingxuan Du , Zuowei Liu, Pran Nath, *PLB* 2022.
- Y.Cheng, X.G.He, et al., *PRD* 2022.
- George N. Wojcik, *PRD* 2023.
- A.W. Thomas, X.G. Wang, *PRD* 2022.
- Faraggi, Alon E. and Guzzi, Marco, *EPJC* 2022.
- …… …… ……

◆ New Fermions:

e.g.

- Hyun Min Lee, Kimiko Yamashita, *EPJC* 2022.
- Mattias Blennow, Pilar Coloma, et al., *PRD* 2022.
- Kingman Cheung, Wai-Yee Keung, et al., *PRD* 2022.
- A. Crivellin, M. Kirk, et al., *PRD* 2022.
- J. Kawamura, S. Okawa, and Y. Omura, *PRD* 2022 .
- O. Popov and R. Srivastava, *PLB* 2023.
- R. Dermisek, J. Kawamura, et al., *JHEP* 2022.
- …… …… ……

New Physics Perspectives: Introducing New Particles (primarily)

◆ SUSY, EFT, and different combinations of the new particles:

e.g.

- J. M. Yang and Y. Zhang, *Sci.Bull 2022.*
- P. Athron, M. Bach, PRD 2022.
- J. de Blas, M. Pierini, L. Reina, et al., PRL 2022.
- J. Fan, L. Li, T. Liu and K. F. Lyu, PRD 2022.
- E. Bagnaschi, J. Ellis, et al., JHEP 2022.
- A. Paul and M. Valli, PRD 2022.
- R. Balkin, E. Madge, et al., JHEP 2022.
- V. Cirigliano, W. Dekens, et al., PRD 2022.
- G. Guedes and P. Olgoso, JHEP 2022.
- Y. Liu, Y. Wang, et al., CPC 2022.
- A. Strumia, JHEP 2022.
- G. Arcadi and A. Djouadi, PRD 2022.
- T. A. Chowdhury, J. Heeck, et al., PRD 2022.

- …… …… ……

New Physics Perspectives: Introducing New Particles (primarily)

◆ New Scalars: Extension of the Higgs sector

e.g.

- K. Sakurai, F. Takahashi and W. Yin, *PLB* 2022.
- Y. Z. Fan, T. P. Tang, Y. L. S.Tsai and L. Wu, *PRL* 2022.
- K. S. Babu, S. Jana and V. P. K., *PRL* 2022.
- X. K. Du, Z. Li, F. Wang and Y. K. Zhang, *EPJC* 2023.
- T. Appelquist, J. Ingoldby and M. Piai, *NPB* 2022.
- N. D. Barrie, C. Han and H. Murayama, *JHEP* 2022.
- J. L. Evans, T. T. Yanagida and N. Yokozaki, *PLB* 2022.
- …… …… ……
- N. D. Barrie, C. Han and H. Murayama, *JHEP* 2022.
- E. Ma, *PLB* 2022.
- W. Abdallah, R. Gandhi and S. Roy, *PLB* 2022.
- A. Addazi, A. Marciano, et al., *EPJC* 2023.
- S. Kanemura and K. Yagyu, *PLB* 2022.
- T. K. Chen, C. W. Chiang and K. Yagyu, *PRD* 2022.
- H. Bahl, W. H. Chiu, C. Gao, L. T. Wang, et al., *EPJC* 2022.
- …… …… ……

Our work

- Jiajun Wu, Chao-Qiang Geng, and Da Huang, *PLB* 2024.
- Jiajun Wu, Da Huang, and Chao-Qiang Geng, *CPC* 2023.

➢ A comprehensive study of the $\mathrm{SU}(2)_L$ scalar multiplet models, as well as their extensions to higher-dimensional cases;

➢ Consider the tree-level and one-loop-level corrections, respectively.

The Model：

The $SU(2)_L$ scalar multiplet: $\Phi_{JY} = \begin{pmatrix} \cdots \\ \mathit{\Phi}_I^Q \\ \cdots \end{pmatrix}$

The Potential:

$$\begin{aligned} V\left(H,\Phi_{JY}\right) = & -\mu_H^2 H^\dagger H + \lambda_H \left(H^\dagger H\right)^2 + \mu_{\Phi_{JY}}^2 \Phi_{JY}^\dagger \Phi_{JY} + \lambda_1 \left(\Phi_{JY}^\dagger \Phi_{JY}\right)^2 \\ & + \lambda_2 \left(\Phi_{JY}^\dagger T_\Phi^a \Phi_{JY}\right)^2 + \lambda_3 \left(\Phi_{JY}^\dagger \Phi_{JY}\right)\left(H^\dagger H\right) \\ & + \lambda_4 \left(\Phi_{JY}^\dagger T_{\Phi_{JY}}^a \Phi_{JY}\right)\left(H^\dagger T_H^a H\right) + \lambda_5 \left(\Phi_{JY}^\dagger T_{\Phi_{JY}}^a T_{\Phi_{JY}}^b \Phi_{JY}\right)^2 \end{aligned}$$

The general potential terms with U(1) symmetry

Only tree-level corrections: Introducing the VEV of additional scalar fields

Odd-dimensional multiplets with Y=0 （*J*≥1）:

$$D_\mu = \partial_\mu + ieQA_\mu + i\frac{g}{c_W}\underline{\left(T_3 - Qs_W^2\right)}\, Z_\mu + ig\left(W_\mu^+ T_+ + W_\mu^- T_-\right)$$

This term automatically disappears when Y=0

$$D^\mu H^\dagger D_\mu H + D^\mu \Phi^\dagger D_\mu \Phi \supset -\left(\frac{1}{4}g^2 v_H^2 + \frac{1}{2}k(1+k)v_\Phi^2\right) W_\mu^+ W^{-\,\mu} - \frac{1}{8}(g^2 + g'^{\,2})v_H^2 Z_\mu Z^\mu$$

$$\Longrightarrow \quad m_W = \frac{1}{2}g\sqrt{v_H^2 + 2k(1+k)v_\Phi^2}, \qquad m_Z = \frac{v_H}{2}\sqrt{g^2 + g'^{\,2}}$$

The required range of VEV of the additional scalar field:

$$\frac{(\Delta v)^2}{v_H^2} \equiv \frac{2k(1+k)v_\Phi^2}{v_H^2} = \left(\frac{m_W^{\text{CDF}}}{m_W^{\text{SM}}}\right)^2 - 1 \sim [0.00090, 0.00201]\,, \quad \text{at } 2\sigma \text{ C.L.} \qquad v_H = 246.22 \text{ GeV}$$

Limitations of the ρ and T parameters？ No

Only one-loop corrections:

Definitions of oblique parameters at the one-loop level:

$$S \equiv \frac{4s_W^2 c_W^2}{\alpha}\left[A'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{c_W s_W} A'_{Z\gamma}(0) - A'_{\gamma\gamma}(0)\right],$$

$$T \equiv \frac{1}{\alpha m_Z^2}\left[\frac{A_{WW}(0)}{c_W^2} - A_{ZZ}(0)\right],$$

$$U \equiv \frac{4s_W^2}{\alpha}\left[A'_{WW}(0) - \frac{c_W}{s_W} A'_{Z\gamma}(0) - A'_{\gamma\gamma}(0)\right] - S,$$

W-boson mass represented by oblique parameters:

$$M_W = M_{W,SM}\left(1 - \frac{\alpha\left(M_Z^2\right)}{4\left(c_W^2 - s_W^2\right)}\left(S - 2c_W^2 T\right) + \frac{\alpha\left(M_Z^2\right)}{8s_W^2}U\right)$$

Vacuum polarization of the electroweak gauge fields:

$$\Pi_{VV'}^{\mu\nu}(q) = g^{\mu\nu}A_{VV'}\left(q^2\right) + q^\mu q^\nu B_{VV'}\left(q^2\right)$$

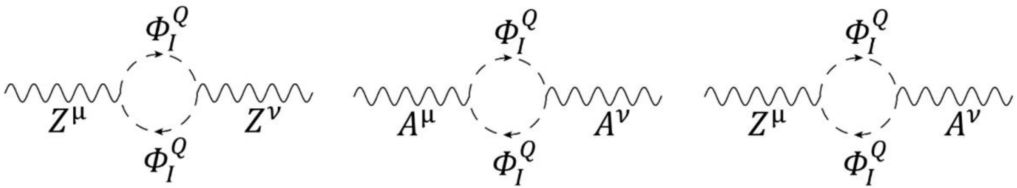
Kinetic terms：

$$
\begin{aligned}
\mathcal{L}_{W\Phi} = & \sum_{I=-J}^{J} \partial_\mu \left(\Phi_I^Q\right)^* \partial^\mu \Phi_I^Q \\
& + \sum_{I=-J}^{J} i g W_\mu^+ \left[N_I \Phi_{I-1}^Q \partial_\mu \left(\Phi_I^Q\right)^* - N_{I+1} \left(\Phi_{I+1}^Q\right)^* \partial^\mu \Phi_I^Q\right] \\
& + \sum_{I=-J}^{J} i g W_\mu^- \left[N_{I+1} \Phi_{I+1}^Q \partial_\mu \left(\Phi_I^Q\right)^* - N_I \left(\Phi_{I-1}^Q\right)^* \partial_\mu \Phi_I^Q\right] \\
& + \sum_{I=-J}^{J} g^2 N_{I+1}^2 W_\mu^+ W_\mu^- \left(\Phi_{I+1}^Q\right)^* \Phi_{I+1}^Q \\
& + \sum_{I=-J}^{J} g^2 N_I^2 W_\mu^+ W_\mu^- \left(\Phi_{I-1}^Q\right)^* \Phi_{I-1}^Q ,
\end{aligned}
$$

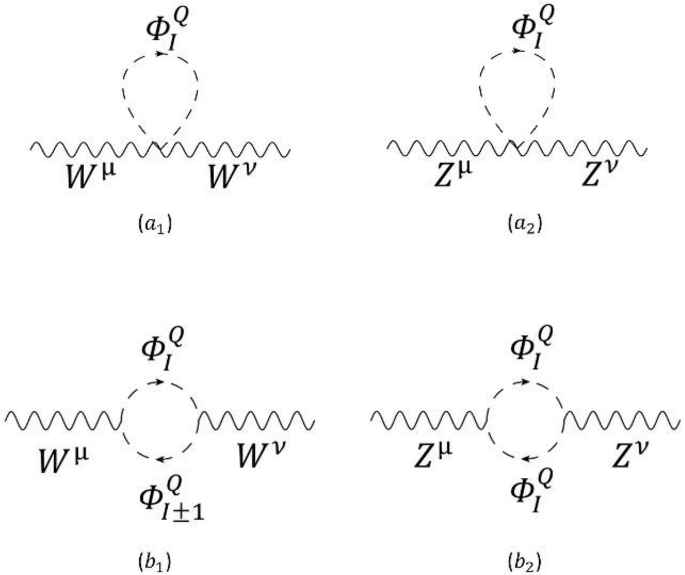
$$
N_I = \sqrt{(J+I)(J-I+1)/2}
$$

$$
\begin{aligned}
\mathcal{L}_{Z\Phi} = & \sum_{I=-J}^{J} \partial_\mu \left(\Phi_I^Q\right)^* \partial^\mu \Phi_I^Q \\
& + \sum_{I=-J}^{J} i \frac{g}{c_W} \left(I - Q s_W^2\right) Z_\mu \left[\partial_\mu \left(\Phi_I^Q\right)^* \Phi_I^Q - \partial^\mu \Phi_I^Q \left(\Phi_I^Q\right)^*\right] \\
& + \sum_{I=-J}^{J} \frac{g^2}{c_W^2} \left(I - Q s_W^2\right)^2 Z_\mu Z^\mu \Phi_I^Q \left(\Phi_I^Q\right)^* ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{A\Phi} = & \sum_{I=-J}^{J} \partial_\mu \left(\Phi_I^Q\right)^* \partial^\mu \Phi_I^Q \\
& + \sum_{I=-J}^{J} i e Q A_\mu \left[\Phi_I^Q \partial_\mu \left(\Phi_I^Q\right)^* - \left(\Phi_I^Q\right)^* \partial_\mu \Phi_I^Q\right] .
\end{aligned}
$$



One-loop diagrams contributing to S



(a_1) (a_2)

(b_1) (b_2)

One-loop diagrams contributing to T

S and T are dominant, while U is suppressed in most cases;

Since U is derived from high-dimensional operators.

Set U=0

The general expression of S and T:

- L. Lavoura, Ling-Fong Li , *PRD* 1994.

$$S_{\Phi_{JY}} = -\frac{Y}{3\pi}\sum_{I=-J}^{J} I \ln m^2_{\Phi_I^Q}$$

$$T_{\Phi_{JY}} = \frac{1}{4\pi s_w^2 m_W^2}\sum_{I=-J}^{J-1} N_{I+1}^2 F\left(m^2_{\Phi_I^Q}, m^2_{\Phi_{I+1}^Q}\right)$$

$$F(A,B) \equiv \begin{cases} \frac{A+B}{2} - \frac{AB}{A-B}\ln\frac{A}{B} & A \neq B, \\ 0 & A = B. \end{cases}$$

The mass difference of the components is the key point!

The potential：

$$\begin{aligned} V\left(H, \Phi_{JY}\right) = & -\mu_H^2 H^\dagger H + \lambda_H \left(H^\dagger H\right)^2 + \mu^2_{\Phi_{JY}} \Phi^\dagger_{JY}\Phi_{JY} + \lambda_1\left(\Phi^\dagger_{JY}\Phi_{JY}\right)^2 \\ & + \lambda_2\left(\Phi^\dagger_{JY} T^a_\Phi \Phi_{JY}\right)^2 + \lambda_3\left(\Phi^\dagger_{JY}\Phi_{JY}\right)\left(H^\dagger H\right) \\ & + \underline{\lambda_4\left(\Phi^\dagger_{JY} T^a_{\Phi_{JY}}\Phi_{JY}\right)\left(H^\dagger T^a_H H\right)} + \lambda_5\left(\Phi^\dagger_{JY} T^a_{\Phi_{JY}} T^b_{\Phi_{JY}}\Phi_{JY}\right)^2 \end{aligned}$$

Phenomenological Study

1. SU(2) real representation:

$$
\begin{aligned}
O_4 =&\lambda_4\left(\Phi_{JY}^{\dagger}T^a_{\Phi_{JY}}\Phi_{JY}\right)\left(H^{\dagger}T^a_H H\right)\\
=&\lambda_4\left(\Phi_{JY}^{\dagger}T^+_{\Phi_{JY}}\Phi_{JY}\right)\left(H^{\dagger}T^-_H H\right)+\lambda_4\left(\Phi_{JY}^{\dagger}T^-_{\Phi_{JY}}\Phi_{JY}\right)\left(H^{\dagger}T^+_H H\right)\\
&+\lambda_4\left(\Phi_{JY}^{\dagger}T^3_{\Phi_{JY}}\Phi_{JY}\right)\left(H^{\dagger}T^3_H H\right)\\
=&-\frac{\lambda_4}{4}(h+v)^2\sum_{I=-J}^{J} I\varPhi_I^Q\left(\varPhi_I^Q\right)^*\\
\supset&-\frac{\lambda_4}{4}v^2\sum_{I=-J}^{J} I\varPhi_I^Q\left(\varPhi_I^Q\right)^*.
\end{aligned}
\quad\longrightarrow\quad
\begin{cases}
-\frac{\lambda_4}{4}v^2\left[I\varPhi_I^Q\left(\varPhi_I^Q\right)^*-I\varPhi_{-I}^Q\left(\varPhi_{-I}^Q\right)^*\right]=0 & I>0,\\
-\frac{\lambda_4}{4}v^2 I\varPhi_I^Q\left(\varPhi_I^Q\right)^*=0 & I=0.
\end{cases}
$$

Unable to produce mass splitting, hence unable to explain the W-boson mass anomaly

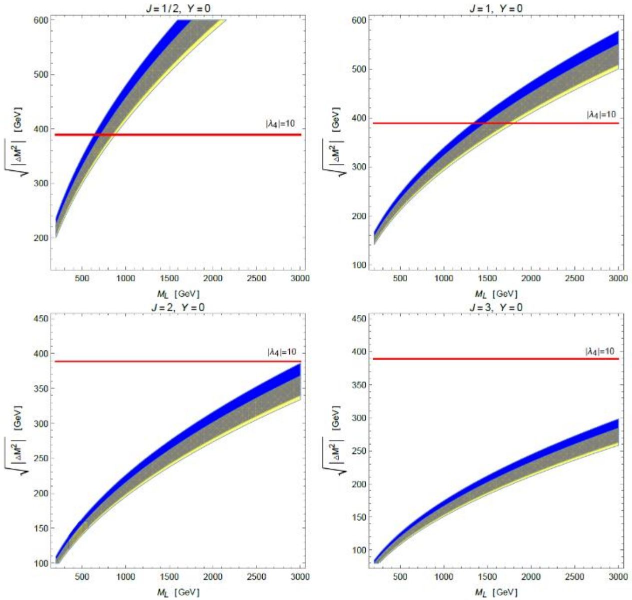
2. SU(2) complex representation:

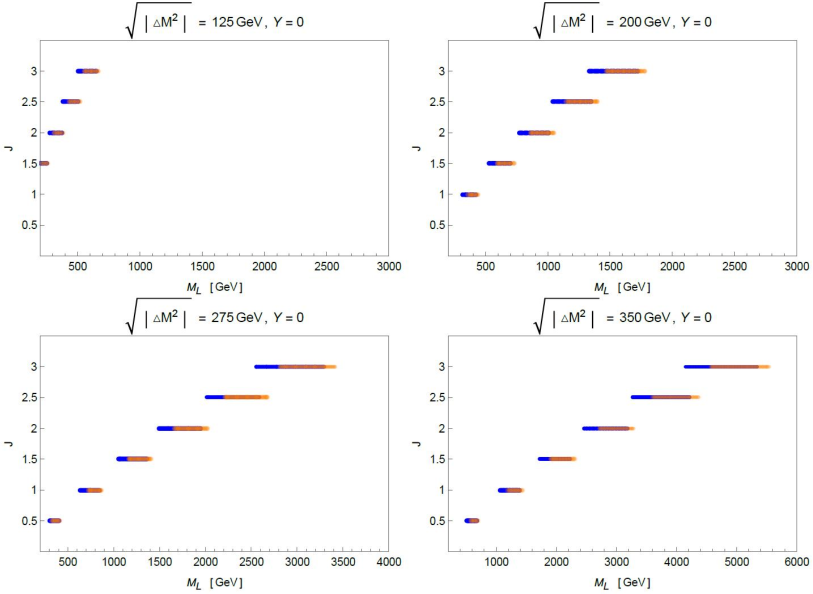
2.1 Y=0:

S=U=0

$$S_{\Phi_{JY}} = -\frac{Y}{3\pi}\sum_{I=-J}^{J} I \ln m^2_{\Phi^Q_I}$$

Perturbativity bound





Incompatible with dark matter candidate

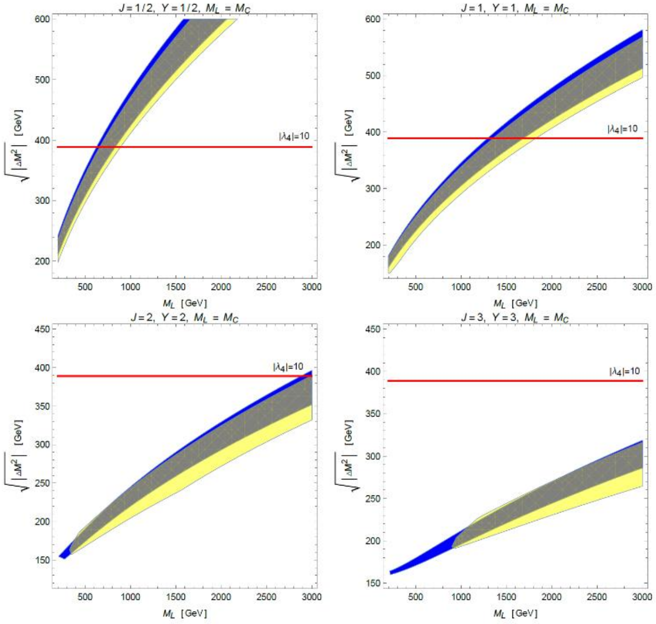
EW Global Fits: Y. Cheng, X. G. He, F. Huang, J. Sun and Z. P. Xing, [arXiv:2208.06760 [hep-ph]].

2. SU(2) complex representation:

2.2 Y=J： U=0

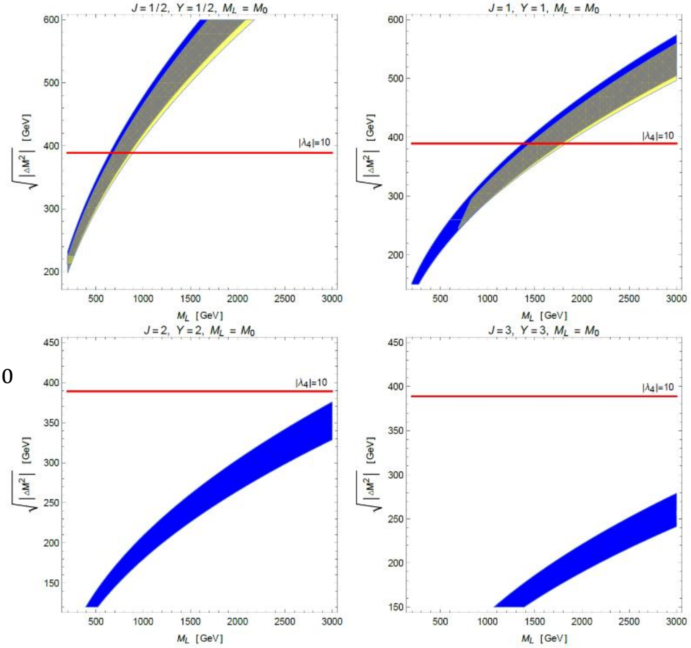
Type-A:

$M_L = M_C$



Type-B:

$M_L = M_0$



EW Global Fits: P. Asadi, C. Cesarotti, K. Fraser, S. Homiller and A. Parikh, [arXiv:2204.05283 [hep-ph]].

2. SU(2) complex representation:

$$\Phi^\dagger \Phi \rightarrow H^\dagger H$$

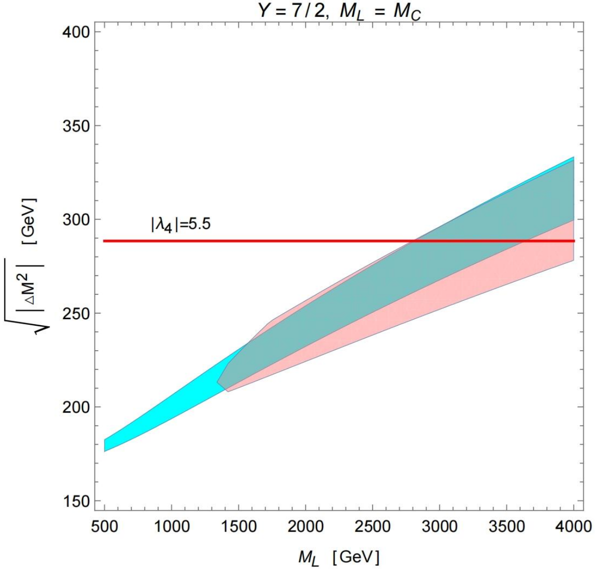
2.2 Y=J： U=0

The unitarity gives stronger constraints

$$|\lambda_4| \le 8\pi/\sqrt{21} \approx 5.5$$

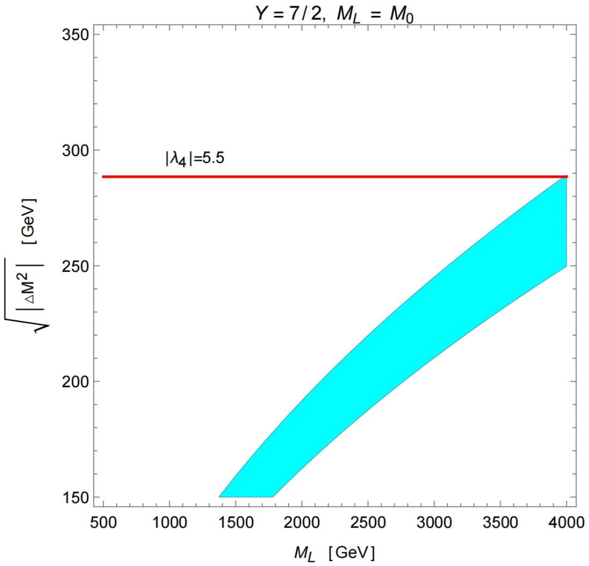
Type-A:

$M_L = M_C$



Type-B:

$M_L = M_0$

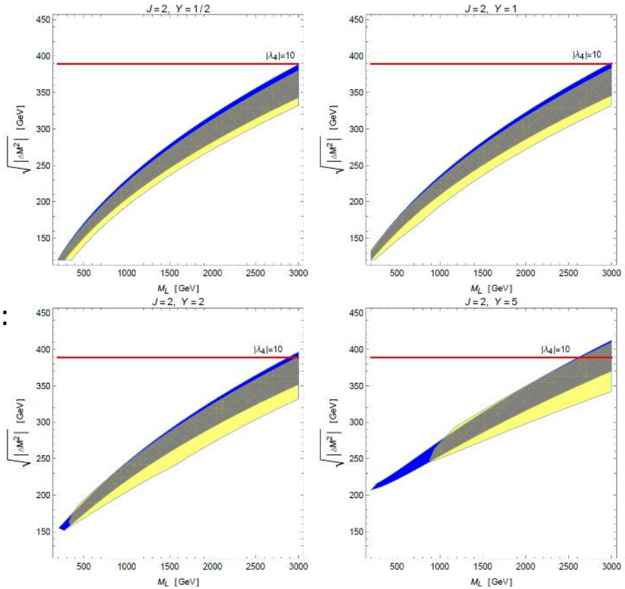


- K. Hally, H. E. Logan, and T. Pilkington, *PRD* 2012.
- Darius Jurčiukonis, Luís Lavoura, arXiv:2404.07897 [hep-ph].

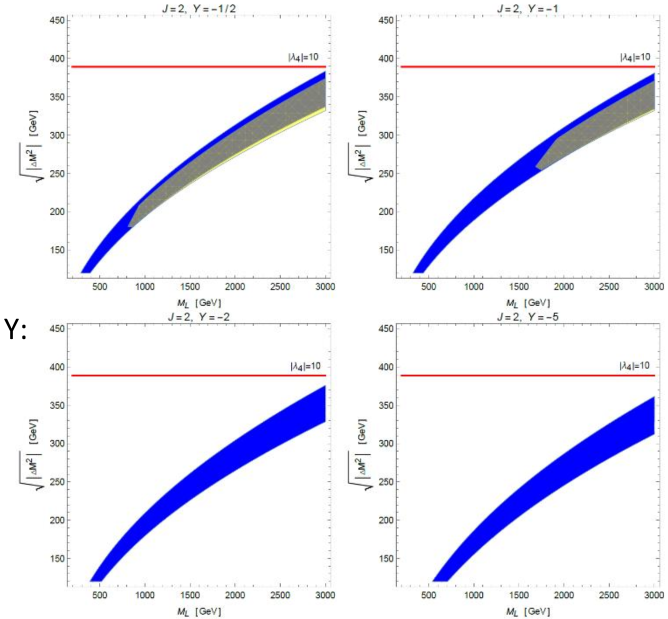
2. SU(2) complex representation:

2.3 Arbitrary Y

Tpye-A model with positive Y:



Tpye-A model with negative Y:



EW Global Fits: P. Asadi, C. Cesarotti, K. Fraser, S. Homiller and A. Parikh, [arXiv:2204.05283 [hep-ph]].

Analysis:

Mainly limited by the range of values of S： $-0.024 \leq S \leq 0.364$

Take J=2 for example:

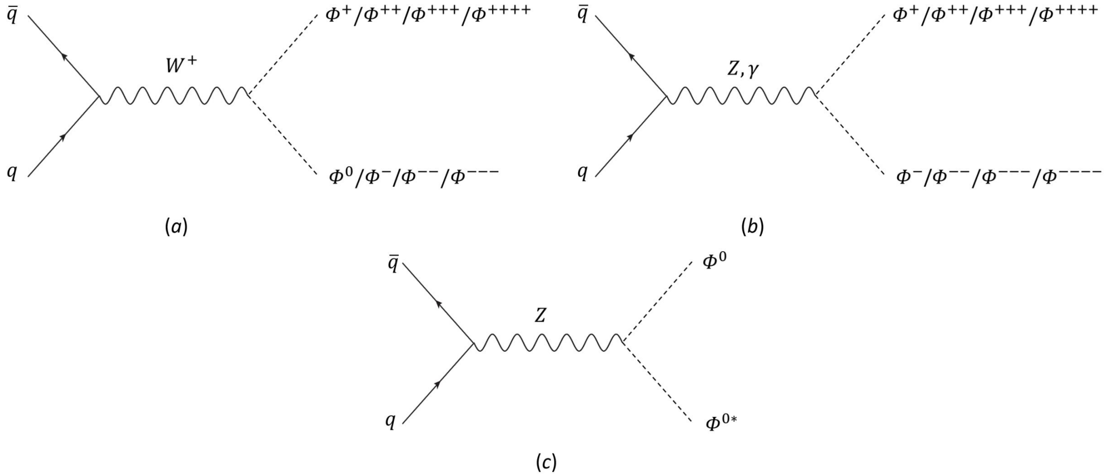
$$
\begin{aligned}
S_{\Phi_{2Y}} &= -\frac{Y}{3\pi}\left(2\ln m^2_{\Phi^Q_2} + \ln m^2_{\Phi^Q_1} - \ln m^2_{\Phi^Q_{-1}} - 2\ln m^2_{\Phi^Q_{-2}}\right) \\
&= -\frac{Y}{3\pi}\left[2\ln\left(\frac{m^2_{\Phi^Q_{-2}} - \lambda_4 v^2}{m^2_{\Phi^Q_{-2}}}\right) + \ln\left(\frac{m^2_{\Phi^Q_{-1}} - \frac{\lambda_4 v^2}{2}}{m^2_{\Phi^Q_{-1}}}\right)\right] \\
&\cong \lambda_4 Y\left(\frac{2v^2}{3\pi m^2_{\Phi^Q_{-2}}} + \frac{v^2}{6\pi m^2_{\Phi^Q_{-1}}}\right).
\end{aligned}
$$

Assuming $\lambda_4 v^2 \ll m^2_{\Phi}$

S is proportional to $\lambda_4 Y$

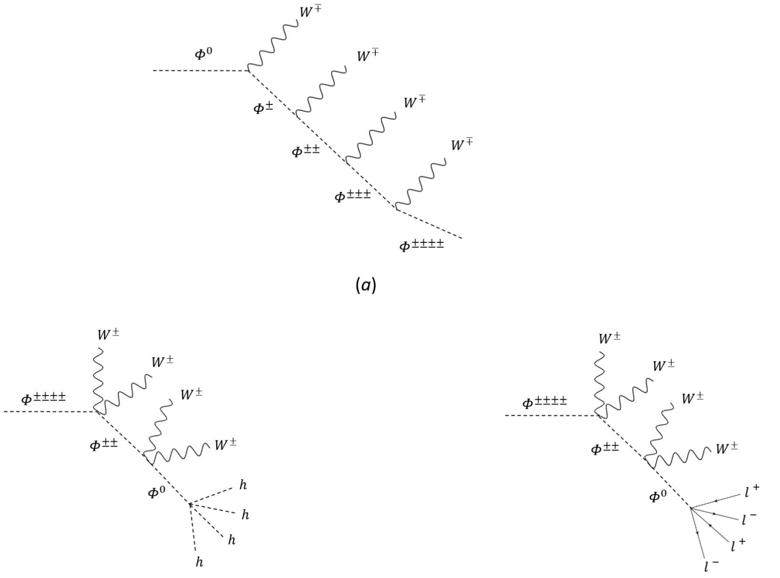
Collider analysis, taking the case of J=Y=2 Type-A as an example:

Production：



Drell-Yan Process

Collider Signals: multi-W or multi-higgs or multi-leptons



(*a*)

(*b*)

$\Phi_{abcd}(H^*)^a(H^*)^b(H^*)^c(H^*)^d$

(*c*)

$\Phi_{abcd}\,(\bar{L}^C)^a\,(\hat{L})^b\,(\bar{L}^C)^c\,(\hat{L})^d$

Introduction of high-dimensional operators

Summary:

- **The *W*-boson mass anomaly has significant physical implications. If CDF collaboration's result is confirmed by further experiments, it will be a clear signal of new physics.**
- **For the scalar multiplet models:**
  - ☐ At tree-level:
    - Odd-dimensional multiplets with Y=0 (J≥1), introducing an extra VEV can explain the W-boson mass anomaly.
  - ☐ At one-loop-level:
    - The SU(2) real representation model cannot generate a mass difference between components, so it cannot explain the W-boson mass anomaly.
    - The SU(2) complex representation model (Type-A) has significant parameter space that can explain the W-boson mass anomaly.
    - The unitarity gives stronger constraints than the perturbativity.
    - The sign of the hypercharge Y significantly impacts the parameter space.

THANKS!