The role of Entanglement and CCR in QED scattering Processes

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[*] M. Blasone, G. Lambiase and B. M., "Entanglement distribution in Bhabha scattering with entangled spectator particle", Phys. Rev. D 109 (2024).

[†] M. Blasone, S. De Siena, G. Lambiase, C. Matrella and B. M., "Complete complementarity relations in tree level QED processes", [arXiv:2402.09195 [quant-ph]].

Summary

1. Entanglement in QED scattering processes

2. Complete Complementarity Relations in QED scattering processes

Motivations

- Investigation of fundamental properties of (elementary) particles via quantum correlations[‡]
- Entanglement in relativistic systems
- A possible resource for quantum information
- Entanglement in high energy processes as a probe for new physics beyond Standard Model

[‡] In this conference see related talks by **P. Lamba**, **E.M. Sessolo**, **K. Sakurai**

Entanglement in QED scattering processes

Entanglement generation in QED processes*

- How entanglement (in helicity) is generated at the fundamental level in QED

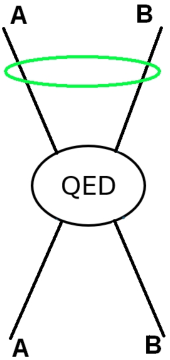
$e^- e^+ \to \mu^- \mu^+$

$e^- e^+ \to e^- e^+$

$e^- \mu^- \to e^- \mu^-$

$e^- \gamma \to e^- \gamma$

...



*A. Cervera-Lierta et al., SciPost Phys. (2017).
S. Fedida, A. Serafini, Phys.Rev.D (2023).

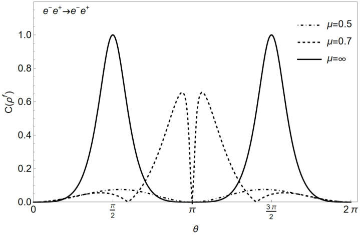
Entanglement Generation in QED processes

Concurrence:

$C(\rho) = max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$

$\mathcal{R} = \rho_{s_1 s_2} \tilde{\rho}_{s_1 s_2}$

$\tilde{\rho}_{s_1 s_2} = (\sigma_y \otimes \sigma_y) \rho^*_{s_1 s_2} (\sigma_y \otimes \sigma_y)$



Initial and final helicity states:

$|i\rangle = |R\rangle_A \otimes |L\rangle_B$

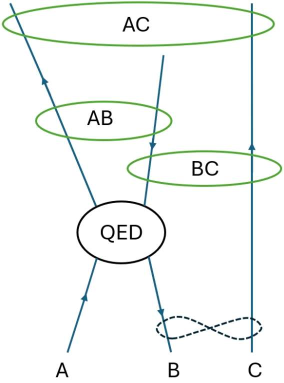
$|f\rangle = \sum_{r,s=R,L} \mathcal{M}(RL; rs) \, |r\rangle_A \, |s\rangle_B$

QED processes with spectator particles†

Initial and final states

$$|i\rangle = |R\rangle_A \otimes \Big(\cos\eta\, |R\rangle_B\, |R\rangle_C + e^{i\beta} \sin\eta\, |L\rangle_B\, |L\rangle_C \Big)$$

$$|f\rangle = \sum_{r,s=R,L} \Big[\cos\eta \mathcal{M}(RR;rs)\, |r\rangle_A\, |s\rangle_B\, |R\rangle_C + e^{i\beta} \sin\eta \mathcal{M}(RL;rs)\, |r\rangle_A\, |s\rangle_B\, |L\rangle_C \Big]$$



† J.B. Araujo et al., Phys. Rev.D (2019).
M. Blasone, G. Lambiase and B. M., Phys. Rev. D 109 (2024).

Entanglement distribution in Bhabha scattering [‡]

- In the same framework we have studied the generation and distribution of the entanglement in Bhabha scattering for the three bipartite channel AB, AC and BC.
- For incoming momenta of the order of the mass, the entanglement has a non trivial distribution in the three output channels.
- In the relativistic regime, analytic expressions for the concurrences:

$$\lim_{\mu\to\infty} C(\rho^f_{AB}) = \frac{2\sin^2\eta\sin^4(\theta/2)\cos^4(\theta/2)}{1-(1-\frac{1}{8}\sin^2\theta)\sin^2\theta\sin^2\eta}$$

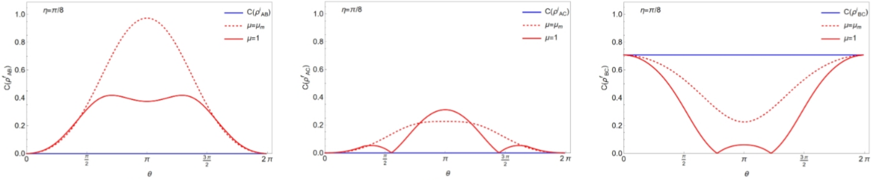
$$\lim_{\mu\to\infty} C(\rho^f_{AC}) = \frac{\sin(2\eta)\sin^4(\theta/2)}{1-(1-\frac{1}{8}\sin^2\theta)\sin^2\theta\sin^2\eta}$$

$$\lim_{\mu\to\infty} C(\rho^f_{BC}) = \frac{\sin(2\eta)\cos^4(\theta/2)}{1-(1-\frac{1}{8}\sin^2\theta)\sin^2\theta\sin^2\eta}$$

[‡] M. Blasone, G. Lambiase and B. M., Phys. Rev. D 109 (2024).

Entanglement distribution ($\eta = \pi/8$)

AB AC BC



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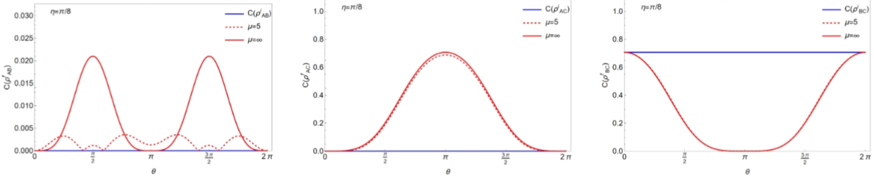
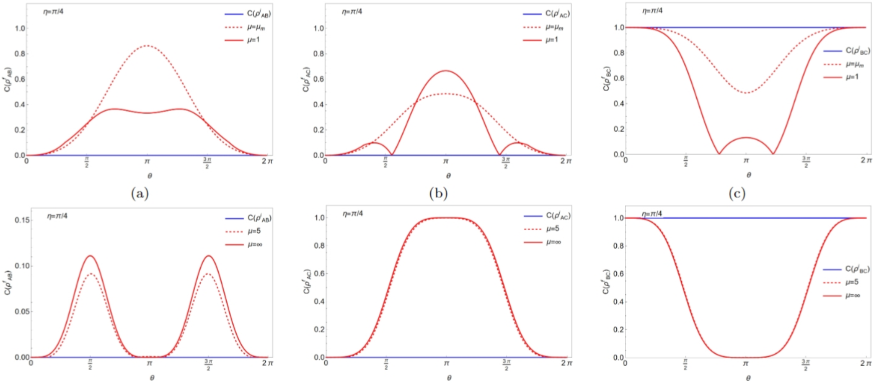


(d) (e) (f)

$$\mu = \frac{|\vec{p}|}{m_e}, \quad \mu_m = \frac{1}{2}\sqrt{-3+\sqrt{17}}$$

Entanglement distribution ($\eta = \pi/4$)

AB AC BC



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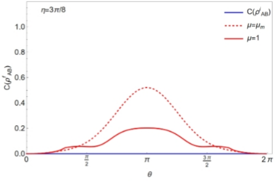
Entanglement distribution ($\eta = 3\pi/8$)

AB | AC | BC



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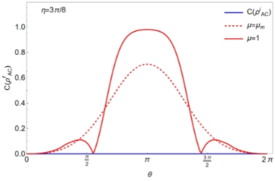

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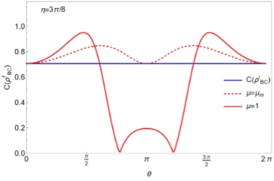

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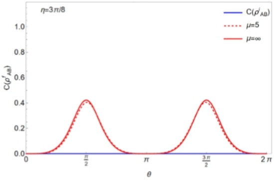

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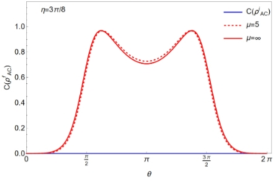

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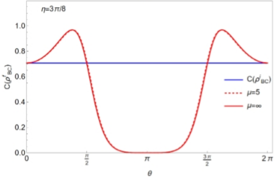


(f)

Main results

- The distribution and generation of entanglement tend to concentrate in some bipartitions.
- Complete transfer from BC channel to the AC channel in $\theta = \pi$ in relativistic regim: interaction as a *quantum gate* $\Rightarrow$ possible application in quantum information protocols.
- For $\frac{\pi}{4} < \eta < \frac{3\pi}{4}$ the interaction generates entanglement in the BC channel, where it was present before the interaction.

Complete Complementarity Relations in QED scattering processes

Complementarity

Complementarity[§]: the two aspect of duality, corpuscular and undulatory, are equally real but mutually exclusive.

The first quantitative version[¶] of the wave-particle duality was summarized by a simple complementarity relation

$$P^2 + V^2 \leq 1.$$

where P is the **predictability**, a measure of path information and V is the **visibility** of the interference pattern.

[§] N. Bohr, *The quantum postulate and the recent development of atomic theory*, Nature (1928)

[¶] W. K. Wootters, W. H. Zurek, Phys. Rev. D. (1979), B-G. Englert, Phys. Rev. Lett. (1996)

Complete Complementarity Relations

For bipartite states[‖] we have to consider a **triality relation** formed by two quantities generating *local*, single-partite realities and the correlation between subsystems, which generates an exclusive bipartite *non-local* reality:

$$P_k^2 + V_k^2 + C^2 = 1, \qquad k = A, B$$

where P_k and V_k are the predictability and visibility for the single-partite systems. C is a measure of entanglement.

[‖] M. Jakob, J. A. Bergou, Opt. Commun. (2010)

CCR for general bipartite states

We can write a general bipartite state in terms of two qubits as

$$|\Theta\rangle = a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle,$$

and express the three terms of the previous triality relation as:

$$C(\Theta) = 2|ad - bc|$$

$$P_A = \Big|(|c|^2 + |d|^2) - (|a|^2 + |b|^2)\Big|$$

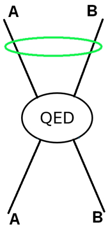
$$P_B = \Big|(|b|^2 + |d|^2) - (|a|^2 + |c|^2)\Big|$$

$$V_A = 2\Big|ac^* + bd^*\Big|$$

$$V_B = 2\Big|ab^* + cd^*\Big|$$

CCR for Bhabha scattering: initial states

We have analyzed C, V and P before and after the QED interaction for three different states:



$$|i\rangle_I \quad = \quad |R\rangle_A \, |L\rangle_B$$

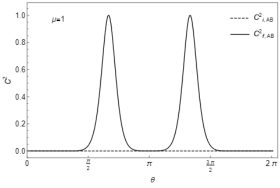
$$|i\rangle_{II} \quad = \quad \big(\cos\alpha \, |R\rangle_A + e^{i\xi} \sin\alpha \, |L\rangle_A \big) \otimes \big(\cos\beta \, |R\rangle_B + e^{i\eta} \sin\beta \, |L\rangle_B \big)$$

$$\begin{aligned} |i\rangle_{III} \quad = \quad & \cos\alpha \, |R\rangle_A \, |R\rangle_B + e^{i\xi} \sin\alpha \cos\beta \, |R\rangle_A \, |L\rangle_B \\ & + e^{i\eta} \sin\alpha \sin\beta \cos\chi \, |L\rangle_A \, |R\rangle_B + e^{i\tau} \sin\alpha \sin\beta \sin\chi \, |L\rangle_A \, |L\rangle_B \end{aligned}$$

‖ M. Blasone, S. De Siena, G. Lambiase, C. Matrella and B. M., [arXiv:2402.09195 [quant-ph]].

Examples: initial factorized state $|i\rangle_I$

$$P_i = 1, \quad V_i = 0, \quad C = 0.$$



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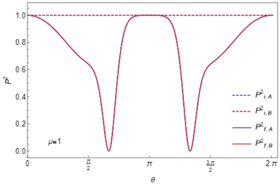

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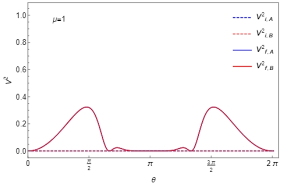

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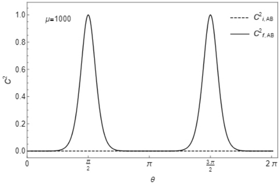

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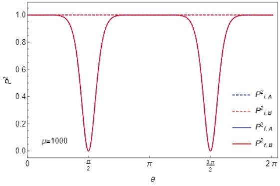

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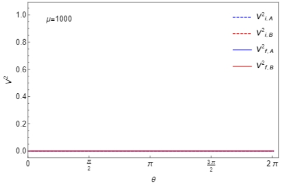

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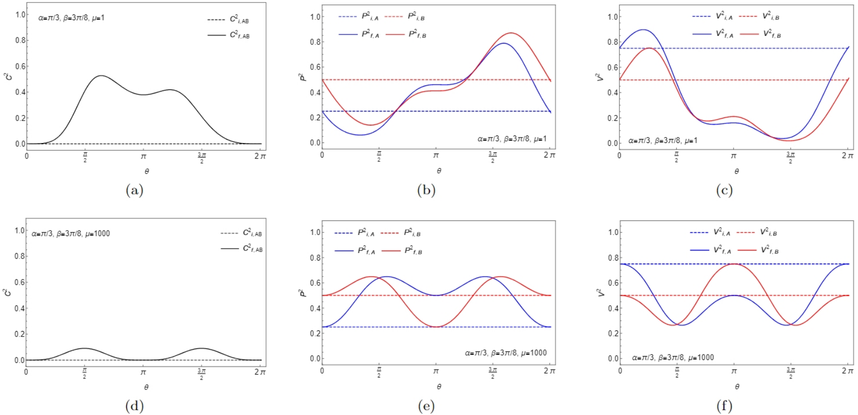
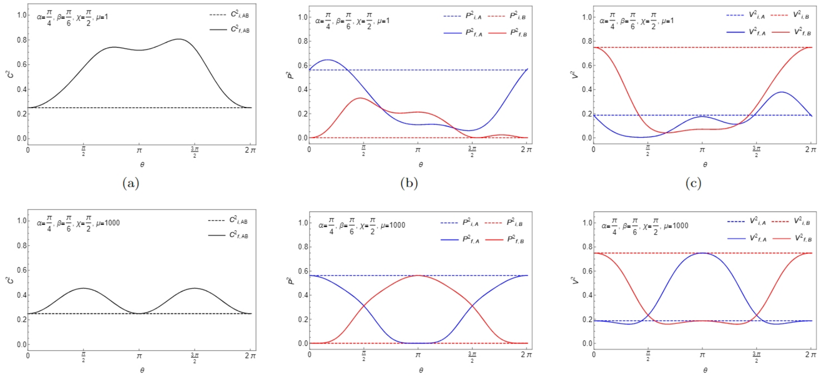
Examples: initial factorized state $|i\rangle_{II}$

$$P_i \neq 0, \quad V_i \neq 0, \quad C = 0.$$



Examples: initial entangled states $|i\rangle_{III}$

Entanglophilus regime

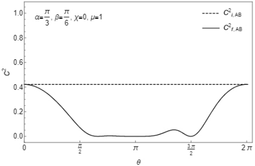


(a) (b) (c)

(d) (e) (f)

Examples: initial entangled states III

Entanglophobous regime



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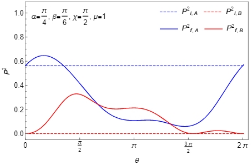

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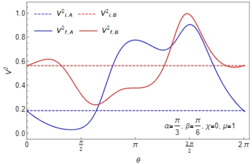

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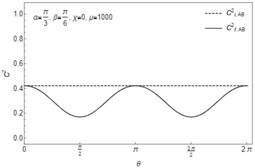

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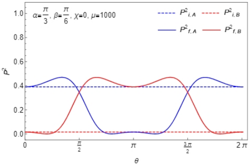

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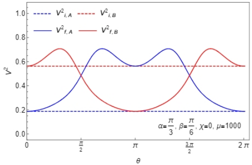
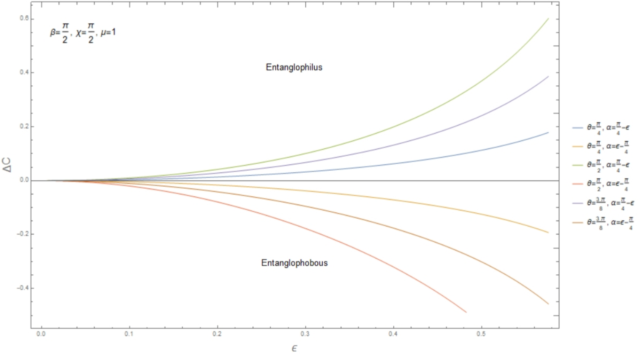


(f)

Entanglophilus, Entanglophobous and Mixed regimes

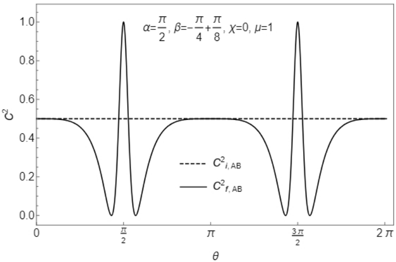
Entanglophilus, *Entanglophobous* from Bell states

$$\Delta C \equiv (C_f - C_i)/C_i \qquad \Phi^{\pm}_{\alpha} = \cos\alpha\,|RR\rangle \pm \sin\alpha\,|LL\rangle \qquad \Phi^{\pm} = \frac{1}{\sqrt{2}}(|RR\rangle \pm |LL\rangle)$$



Mixed regimes from Bell states

$$\Psi_{\alpha}^{\pm} = \cos\alpha\,|RL\rangle \pm \sin\alpha\,|LR\rangle \qquad \Psi^{\pm} = \frac{1}{\sqrt{2}}(|RL\rangle \pm |LR\rangle)$$



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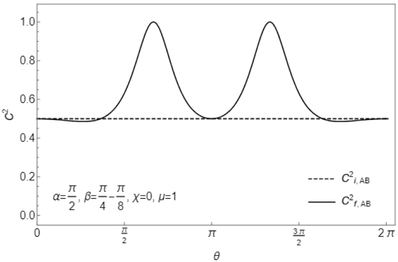


(b)

Maximal entanglement conservation

$$P_i = 0 \qquad V_i = 0 \qquad C = 1$$

- **Processes preserving maximal entanglement**
- $e^- e^+ \to e^- e^+$
- $e^- e^- \to e^- e^-$
- $e^- e^+ \to \mu^- \mu^+$
- $e^- \mu^- \to e^- \mu^-$

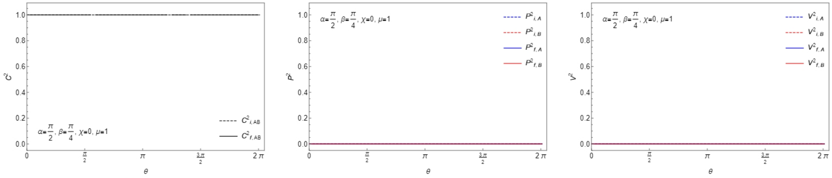
An Example: Bhabha scattering

$\Phi^+ \to \Phi^+$

$\Phi^- \to \cos s_1 \Phi^- + \sin s_1 \Psi^+$

$\Psi^+ \to \cos s_2 \Phi^- + \sin s_2 \Psi^+$

$\Psi^- \to \Psi^-$



(a) (b) (c)

QED scattering with photons

- $e^- e^+ \to 2\gamma$ **: maximal entanglement is not conserved for all the initial maximally entangled states**
- $e^- \gamma \to e^- \gamma$ **: maximal entanglement is never achieved**

	$e^- e^+ \to \gamma\gamma$		*		Compton	
In. State	Fin. State	Conc.	*	In. State	Fin. State	Conc.
Φ^+	$\Phi-$	1	*	Φ^+	GC	< 1
Φ^-	$\cos r\, \Phi^+ + \sin r\, \Psi^+$	$\sin(2\,r)$	*	Φ^-	GC	< 1
Ψ^+	Ψ^+	1	*	Ψ^+	GC	< 1
Ψ^-	Ψ^-	1	*	Ψ^-	GC	< 1

Discussion of results

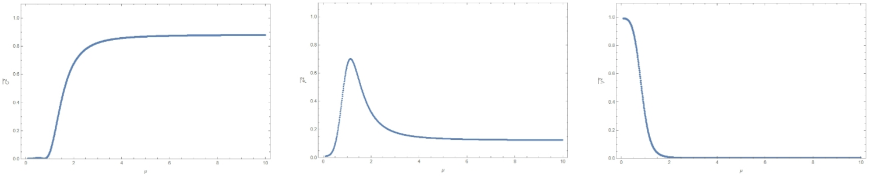
- Can we consider and quantify the QED scattering processes as a good resource in order to generate entangled particle?

- How can we interpret these results?

Entanglement resource

Weighted CCR averages for initial state $|RL\rangle$

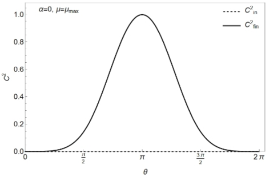
$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 E_{CM}^2} \qquad \overline{Q^2} = \frac{1}{\mathbf{N}} \sum_{rs} \int_{\mathcal{D}} |M(a,b;r,s)|^2 \, Q^2(\theta) d\theta \qquad Q = P, V, C$$



(a) (b) (c)

Interpretation

CCR vs Probabilities for initial state $|RR\rangle$



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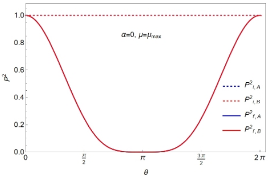

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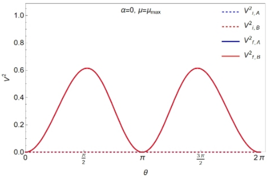

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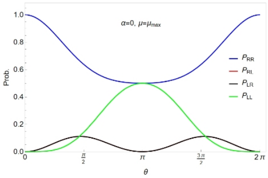

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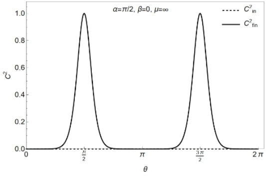
Interpretation

CCR vs Probabilieties for initial state $|RL\rangle$



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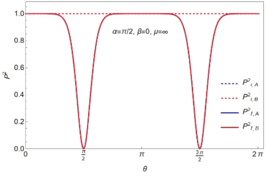

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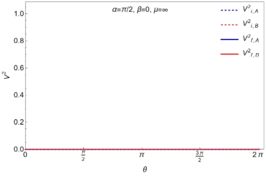

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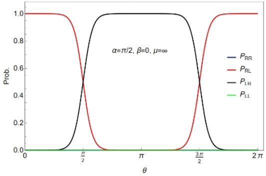

(d)

Conclusions

- We have studied entanglement generation and distribution in QED processes with and without entangled spectator particle
- With entangled spectator particle we observe a non trivial distribution of initial entanglement in the various bipartitions
- For some values of parameters we find a complete transfer of entanglement from one channel to another (quantum gate)
- By means of CCR we have widely characterized entanglement generation in elementary scattering process at tree level.
- We find a regime in which entanglement increases with respect to its initial value and another in which it decreases
- Remarkably, the maximal entanglement (Bell state) is totally preserved for processes that involve 1/2-spin massive particle

Outlook

- Extension of the analysis to other interactions
- 1 loop correction**
- Analysis of the same processes in other reference frames
- Possible implementations of our results in quantum information protocols
- Possible identification of new physics BSM

K. Kowalska and E. M. Sessolo, Entanglement in flavored scalar scattering, JHEP **07 (2024), 156.