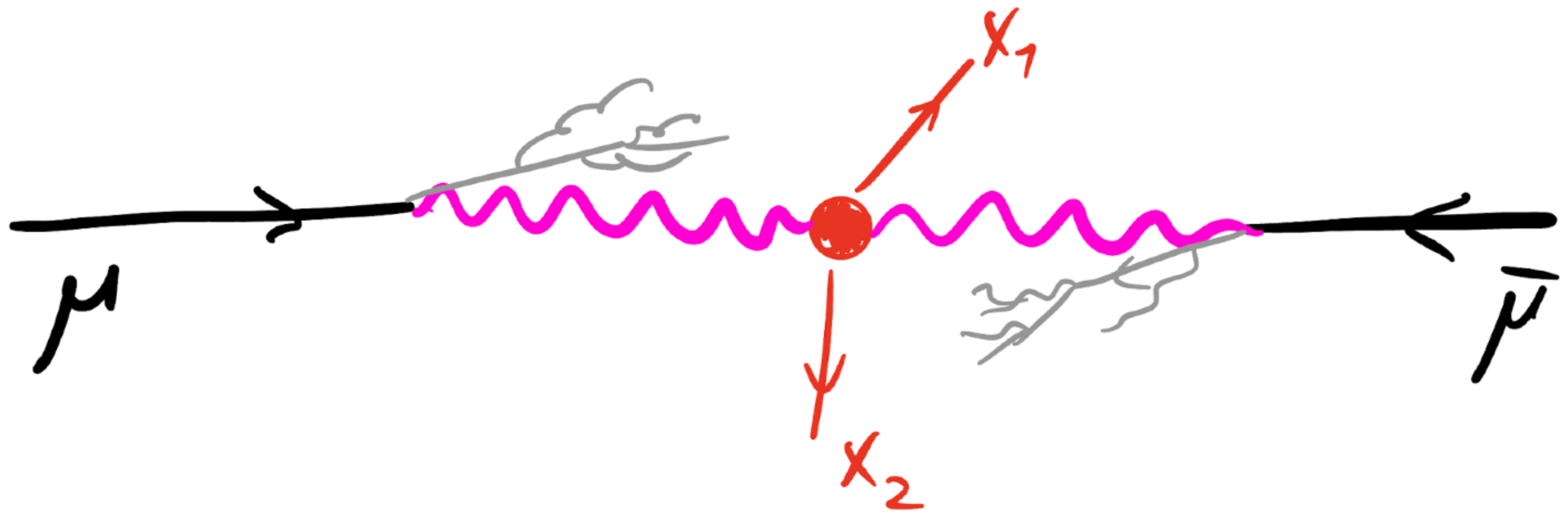
Impact of EW PDFs on muon collider phenomenology



David Marzocca

INFN Trieste

Based on:

- **Francesco Garosi**, D.M., **Sokratis Trifinopoulos** *JHEP 09 (2023) 107* [2303.16964]
- D.M. and **Alfredo Stanzione** [2408.13191]
- **F. Garosi**, **R. Capdevilla**, D.M. and **B. Stechauner** *[in progress]*

LePDF

Source + Downloads available at
https://github.com/DavidMarzocca/LePDF

Workshop on the Standard Model and Beyond, Corfu, 27/08/2024

Electroweak interactions @ multi-TeV

Future colliders will be built to **explore physics at the multi-TeV scale**.

This energy regime is **exciting**, not only for the **possibility of uncovering New Physics**, but also because it contains **many Standard Model effects never observed before**.

Electroweak interactions @ multi-TeV

Future colliders will be built to **explore physics at the multi-TeV scale**.

This energy regime is **exciting**, not only for the **possibility of uncovering New Physics**, but also because it contains **many Standard Model effects never observed before**.

The effects of EW symmetry breaking diminish at large energies: **EW symmetry restoration**.

New "exotic" SM effects will be extremely important to be studied and understood in detail, both theoretically and experimentally:

Sudakov double-logarithms, EW radiation, EW collinear splittings and PDFs, WW scattering unitarization, etc..

Electroweak interactions @ multi-TeV

Future colliders will be built to **explore physics at the multi-TeV scale**.

This energy regime is **exciting**, not only for the **possibility of uncovering New Physics**, but also because it contains **many Standard Model effects never observed before**.

The effects of EW symmetry breaking diminish at large energies: **EW symmetry restoration**.

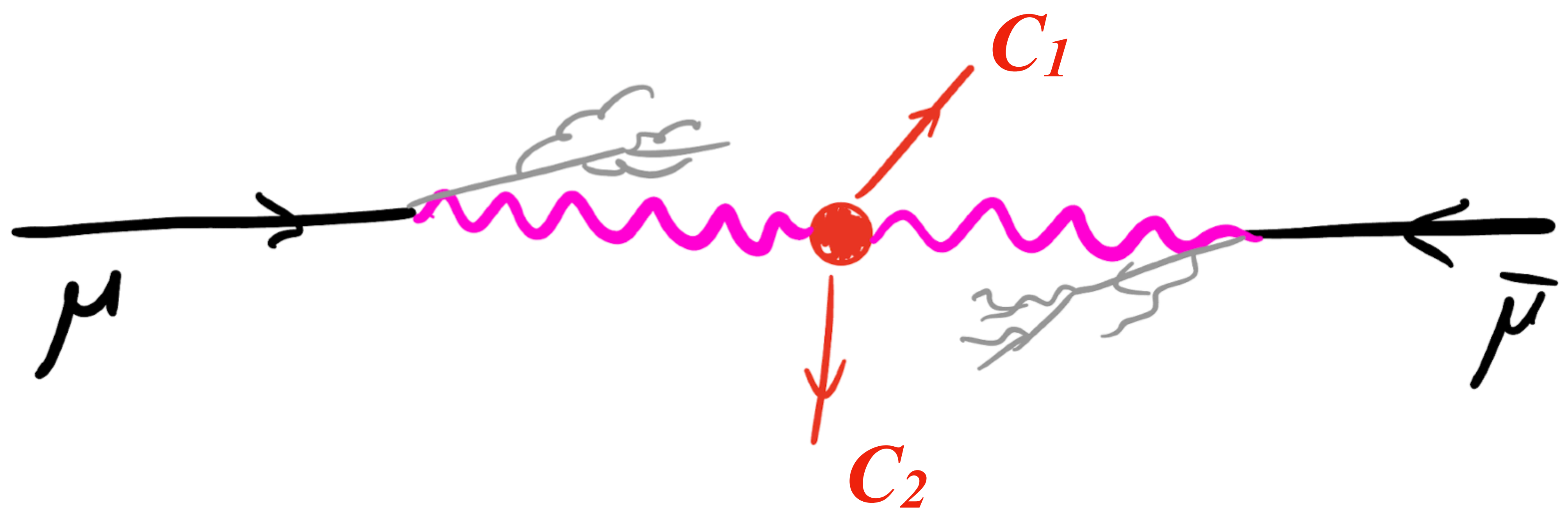
New "exotic" SM effects will be extremely important to be studied and understood in detail, both theoretically and experimentally:

Sudakov double-logarithms, EW radiation, EW collinear splittings and PDFs, WW scattering unitarization, etc..

Muon Colliders are the ideal environment to study this physics with high precision!

In this talk I will focus on 2 effects related to EW Parton Distribution Functions and their application to Muon Collider processes.

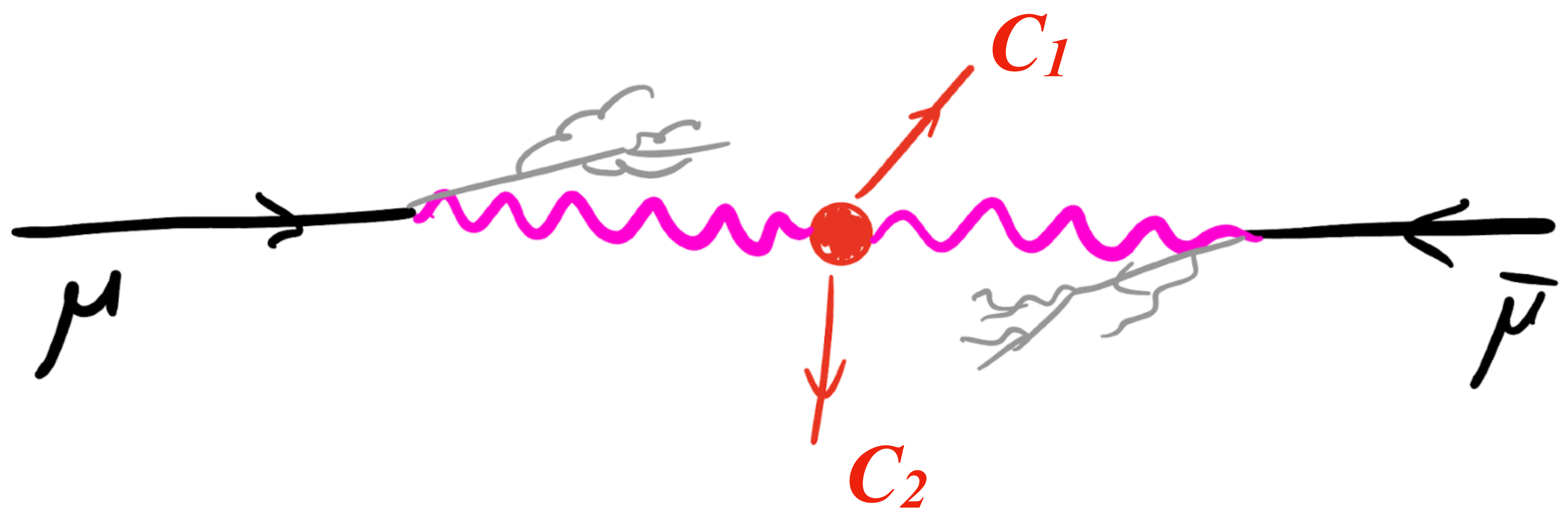
PDFs of a muon



For processes well above threshold, the **contribution from collinear virtual bosons** emitted from the muons can become **dominant**.

"The muon collider is a weak boson collider"

PDFs of a muon

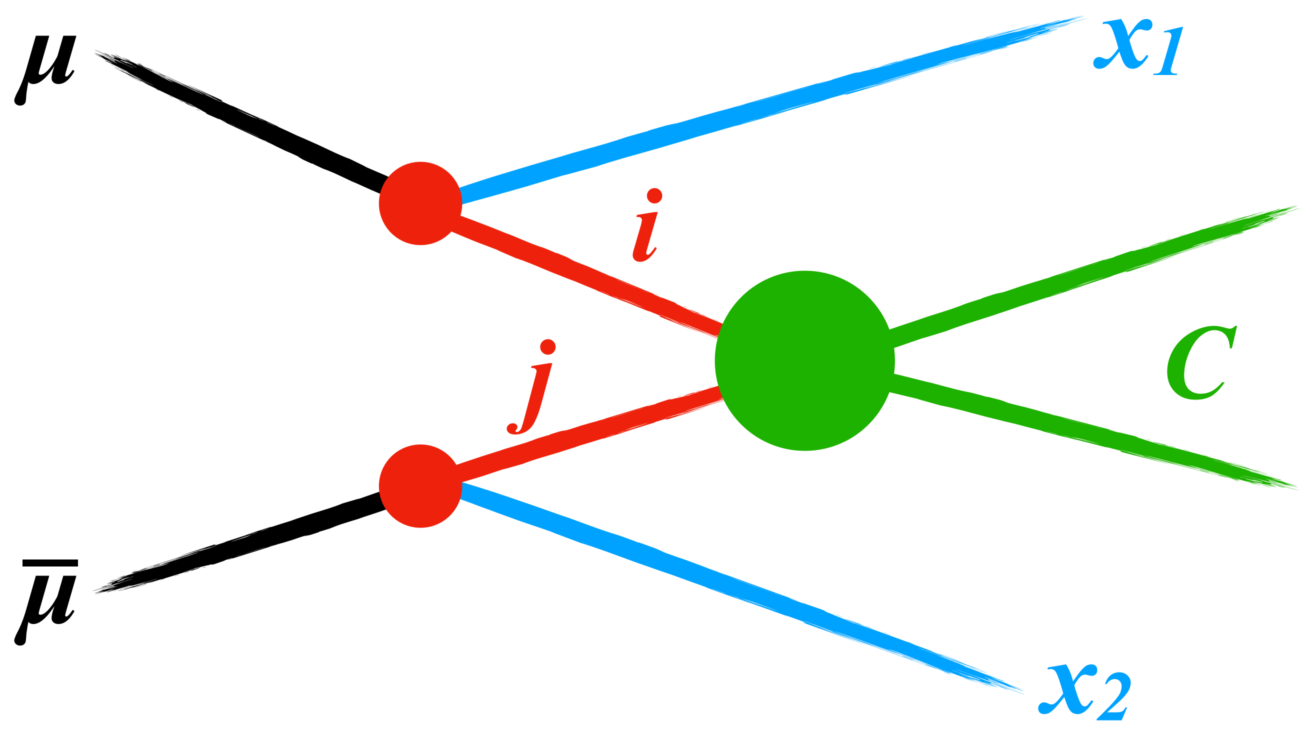


For processes well above threshold, the **contribution from collinear virtual bosons** emitted from the muons can become **dominant**.

"The muon collider is a weak boson collider"

Collinear Factorization: The amplitudes for collinear splitting and hard scattering can be factorised if the p_T of the emitted radiation is small compared to the hard scattering energy.

This can be described in terms of **generalised Parton Distribution Functions**, like for proton colliders:



$$\sigma(\mu\bar{\mu} \to C + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{ij} f_i(x_1, \mu)\, f_j(x_2, \mu)\, \hat{\sigma}(ij \to C)(\hat{s})$$

PDFs of a muon

Unlike for protons, since the muon is elementary this can be done **from first principles**.

PDFs of a muon

Unlike for protons, since the muon is elementary this can be done **from first principles**.

The **boundary condition** is set by $f_\mu(x,\ m_\mu) = \delta(1\text{-}x) + O(\alpha), \quad f_{i\neq\mu}(x,\ m_\mu) = 0 + O(\alpha)$

NLO corrections in Frixione [1909.03886]

PDFs of a muon

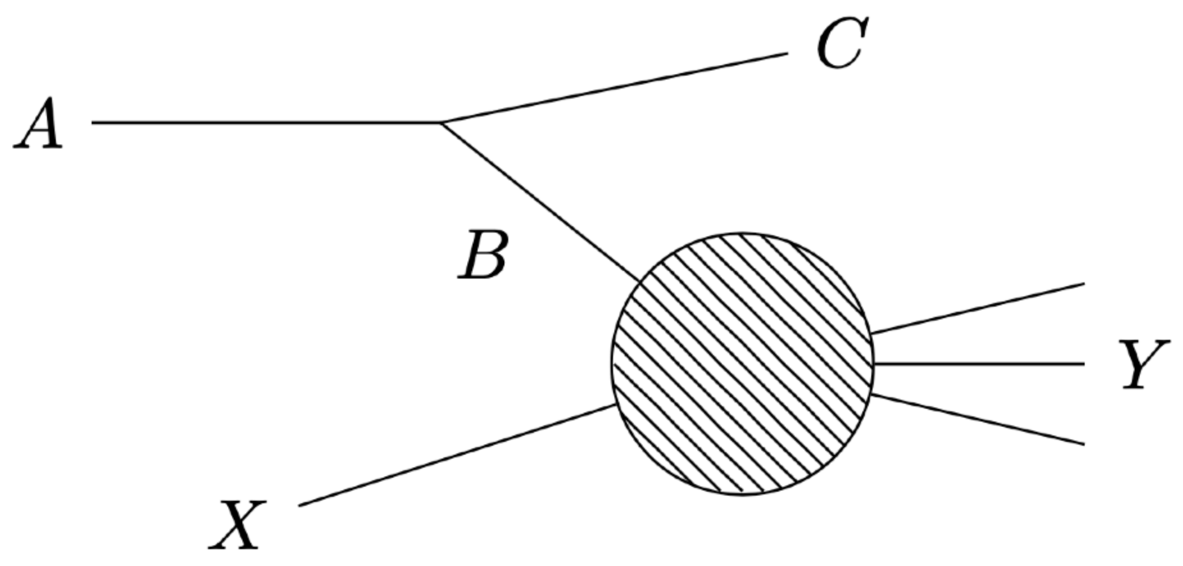
Unlike for protons, since the muon is elementary this can be done **from first principles**.

The **boundary condition** is set by $f_\mu(x, m_\mu) = \delta(1\text{-}x) + O(\alpha)$, $f_{i\neq\mu}(x, m_\mu) = 0 + O(\alpha)$

NLO corrections in Frixione [1909.03886]

The **DGLAP equations** describe the evolution of the PDFs

M. Ciafaloni, P. Ciafaloni, D. Comelli hep-ph/0111109, hep-ph/0505047]



$$Q^2 \frac{df_B(x,Q^2)}{dQ^2} = P_B^v\, f_B(x,Q^2) + \sum_{A,C} \frac{\alpha_{ABC}}{2\pi} \widetilde{P}_{BA}^C \otimes f_A + \frac{v^2}{16\pi^2 Q^2} \sum_{A,C} \widetilde{U}_{BA}^C \otimes f_A$$

Virtual corrections **Real emission** **ultra-collinear terms (EWSB)**

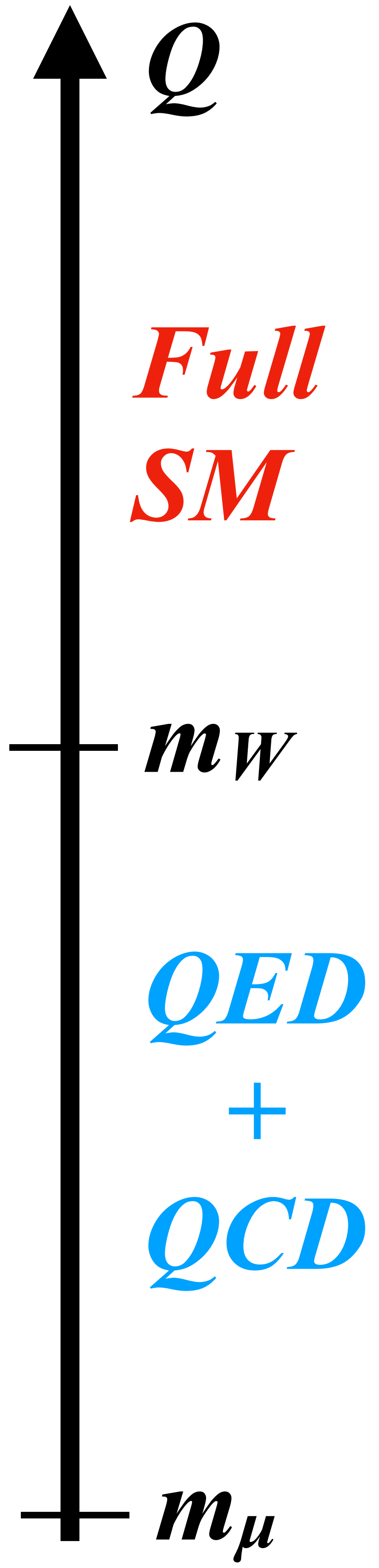
Chen, Han, Tweedie [1611.00788]

PDFs of a muon

Unlike for protons, since the muon is elementary this can be done **from first principles**.

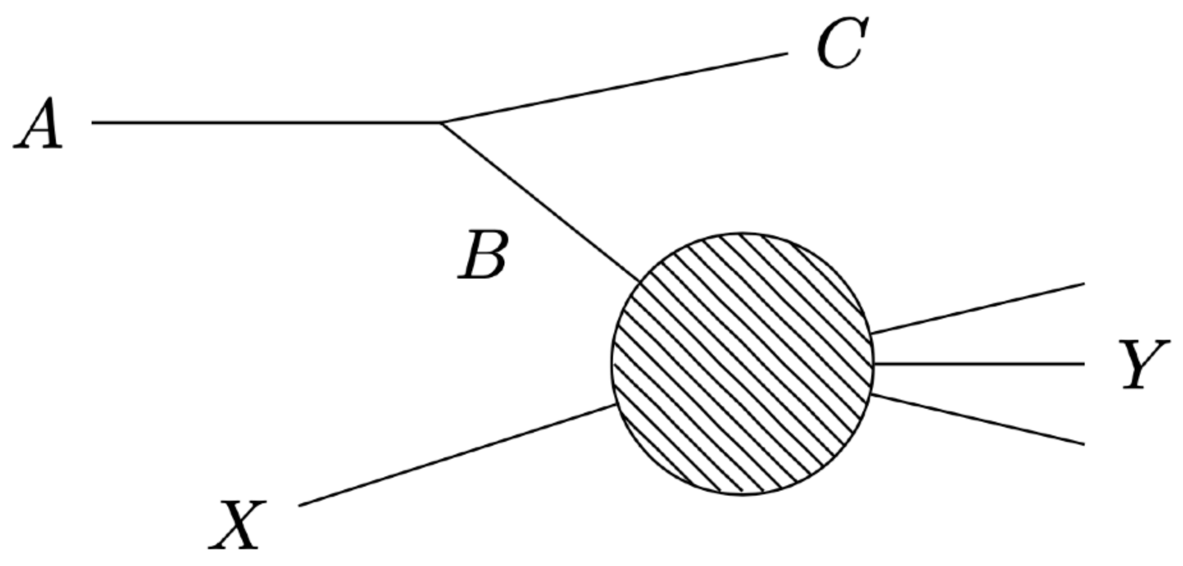
The **boundary condition** is set by $f_\mu(x, m_\mu) = \delta(1\text{-}x) + O(\alpha), \quad f_{i\neq\mu}(x, m_\mu) = 0 + O(\alpha)$

NLO corrections in Frixione [1909.03886]



The **DGLAP equations** describe the evolution of the PDFs

M. Ciafaloni, P. Ciafaloni, D. Comelli hep-ph/0111109, hep-ph/0505047]



$$Q^2 \frac{df_B(x,Q^2)}{dQ^2} = P_B^v f_B(x,Q^2) + \sum_{A,C} \frac{\alpha_{ABC}}{2\pi} \widetilde{P}_{BA}^C \otimes f_A + \frac{v^2}{16\pi^2 Q^2} \sum_{A,C} \widetilde{U}_{BA}^C \otimes f_A$$

Virtual corrections **Real emission** **ultra-collinear terms (EWSB)**

Chen, Han, Tweedie [1611.00788]

Use only **QED+QCD below EW** scale, **full SM above**.

Above the EW scale

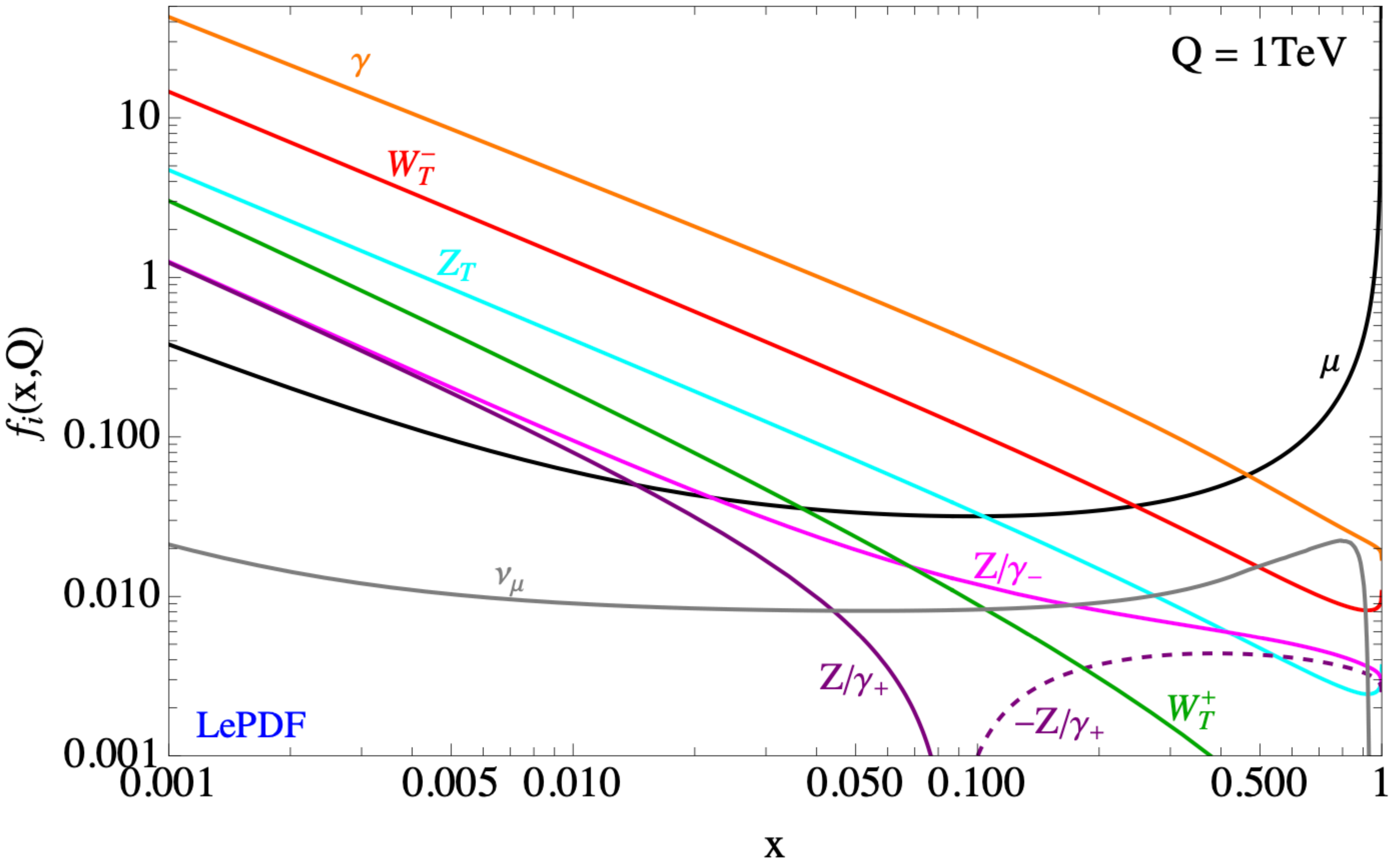
All SM interactions and fields must be considered and several new effects must be taken into account:

- **PDFs become polarised**, since EW interactions are chiral. Bauer, Webber [1808.08831]
- At high energies **EW Sudakov double logarithms** are generated. P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0001142, hep-ph/0505047], Bauer, Webber [1703.08562, 1808.08831], Chen, Han, Tweedie [1611.00788], Han, Ma, Xie [2103.09844], F. Garosi, D.M., S. Trifinopoulos [2303.16964]
- **Neutral bosons interfere with each other: Z/γ and h/Z_L PDFs mix**. P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0505047] Chen, Han, Tweedie [1611.00788]
- **Mass effects** of partons with EW masses (W, Z, h, t) become relevant and some remain so even at multi-TeV scale.
- EW symmetry is broken. Another set of splitting functions, proportional to v^2 instead of p_T^2, arise: **ultra-collinear splitting functions**. Chen, Han, Tweedie [1611.00788]

LePDF

We work in the **mass eigenstate basis** and **solve the DGLAP numerically** in **x-space**, discretising the [10^{-6}, -1] interval

All EW & SM interactions are implemented, including all features listed in the previous slide.



- **Sizeable PDFs of EW gauge bosons**
- **Large muon-neutrino PDF for $x \gtrsim 0.5$**

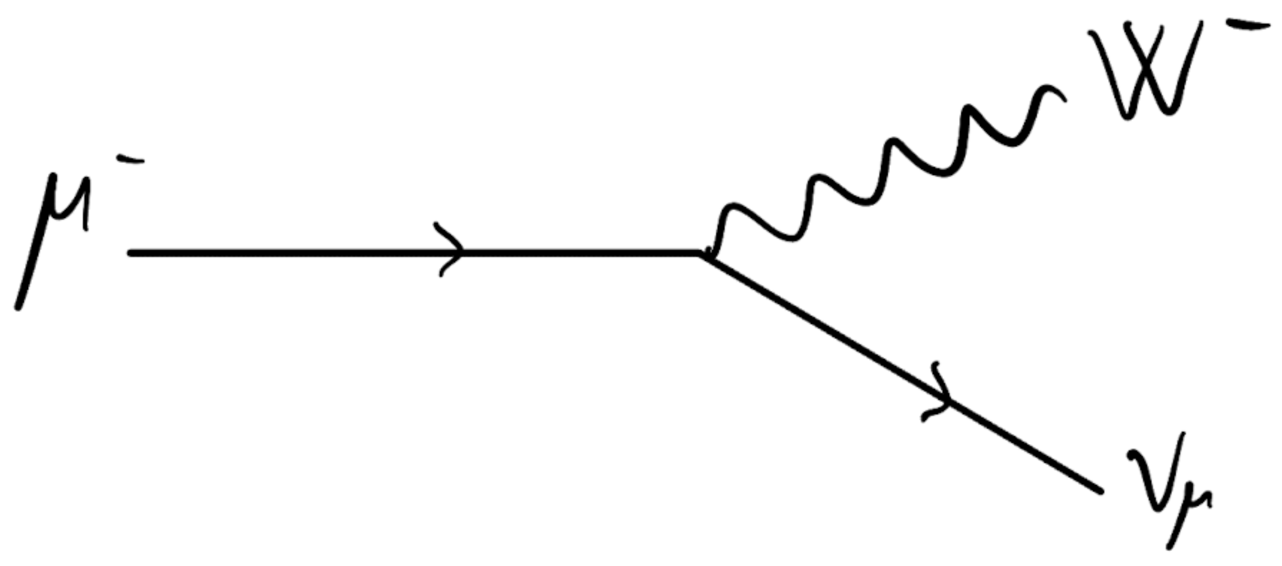
Francesco Garosi, D.M., Sokratis Trifinopoulos
JHEP 09 (2023) 107 [**2303.16964**]

Source + Downloads available at
https://github.com/DavidMarzocca/LePDF

Effective Vector Boson Approximation

At energies **above the EW scale**, **collinear emission of EW gauge bosons** can be described at LO with the **Effective Vector Boson Approximation**

Fermi ('24) Weizsacker, Williams ('34) Landau, Lifschitz ('34) Kane, Repko, Rolnik; Dawson; Chanowitz, Gaillard '84, See also Borel et al. [1202.1904], Costantini et al. [2005.10289] Ruiz et al. [2111.02442], etc…



Including W-mass effects:

$$f_{W^-_\pm}^{(\alpha)}(x,Q^2) = \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x) \left(\log \frac{Q^2+(1-x)m_W^2}{m_\mu^2+(1-x)m_W^2} - \frac{Q^2}{Q^2+(1-x)m_W^2} \right)$$

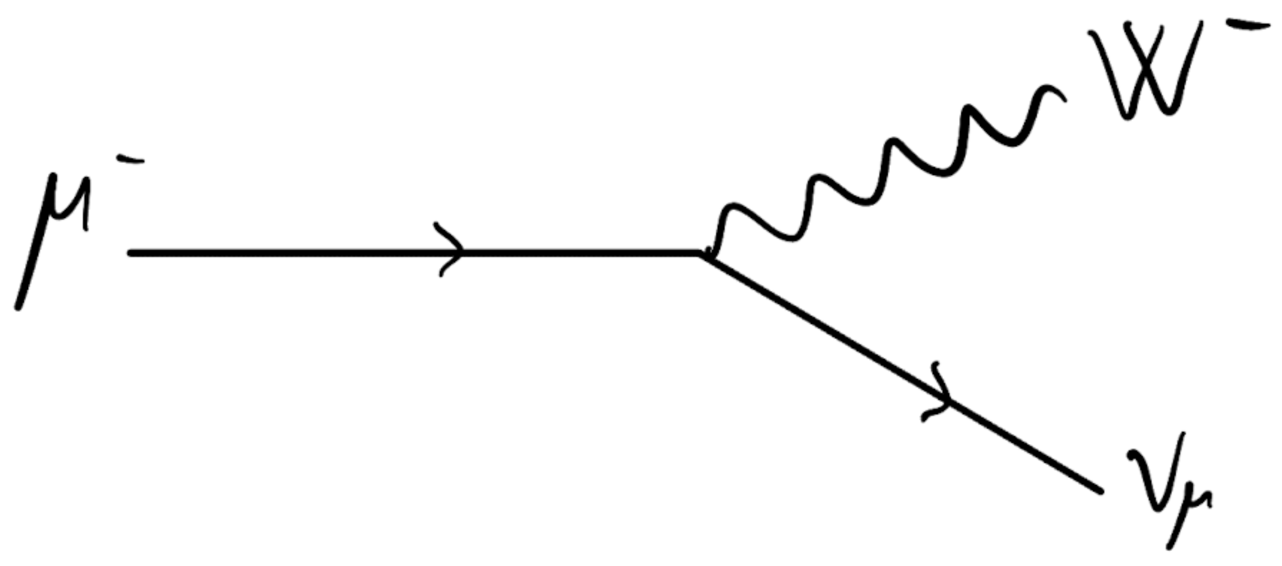
$$f_{W^-_L}^{(\alpha)}(x,Q^2) = \frac{\alpha_2}{4\pi} \frac{1-x}{x} \frac{Q^2}{Q^2+(1-x)m_W^2}$$

(similar expressions also for Z_T, Z_L, Z/γ)

Effective Vector Boson Approximation

At energies **above the EW scale**, **collinear emission of EW gauge bosons** can be described at LO with the **Effective Vector Boson Approximation**

Fermi ('24) Weizsacker, Williams ('34) Landau, Lifschitz ('34) Kane, Repko, Rolnik; Dawson; Chanowitz, Gaillard '84, See also Borel et al. [1202.1904], Costantini et al. [2005.10289] Ruiz et al. [2111.02442], etc…



Including W-mass effects:

$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) = \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x) \left(\log \frac{Q^2 + (1-x)m_W^2}{m_\mu^2 + (1-x)m_W^2} - \frac{Q^2}{Q^2 + (1-x)m_W^2} \right)$$

$$f^{(\alpha)}_{W^-_L}(x,Q^2) = \frac{\alpha_2}{4\pi} \frac{1-x}{x} \frac{Q^2}{Q^2 + (1-x)m_W^2}$$

(similar expressions also for Z_T, Z_L, Z/γ)

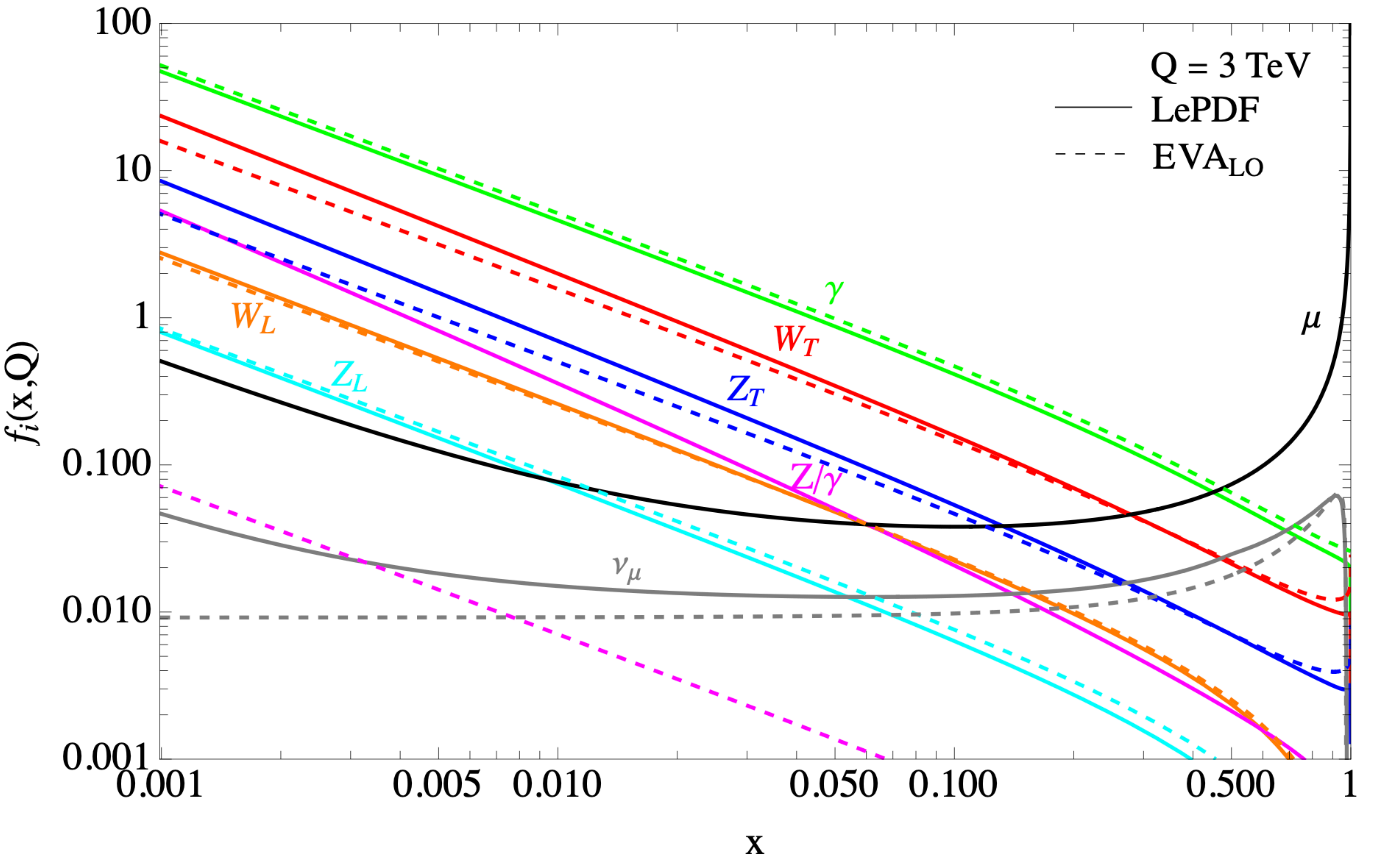
For $Q \gg m_W$:

$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) \approx \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x) \log \frac{Q^2}{m_W^2}$$

This one is now implemented in `MadGraph5_aMC@NLO`
[Ruiz, Costantini, Maltoni, Mattelaer 2111.02442]

NOTE: mass effects remain of O(1) also at TeV scale! Chen, Han, Tweedie [1611.00788]

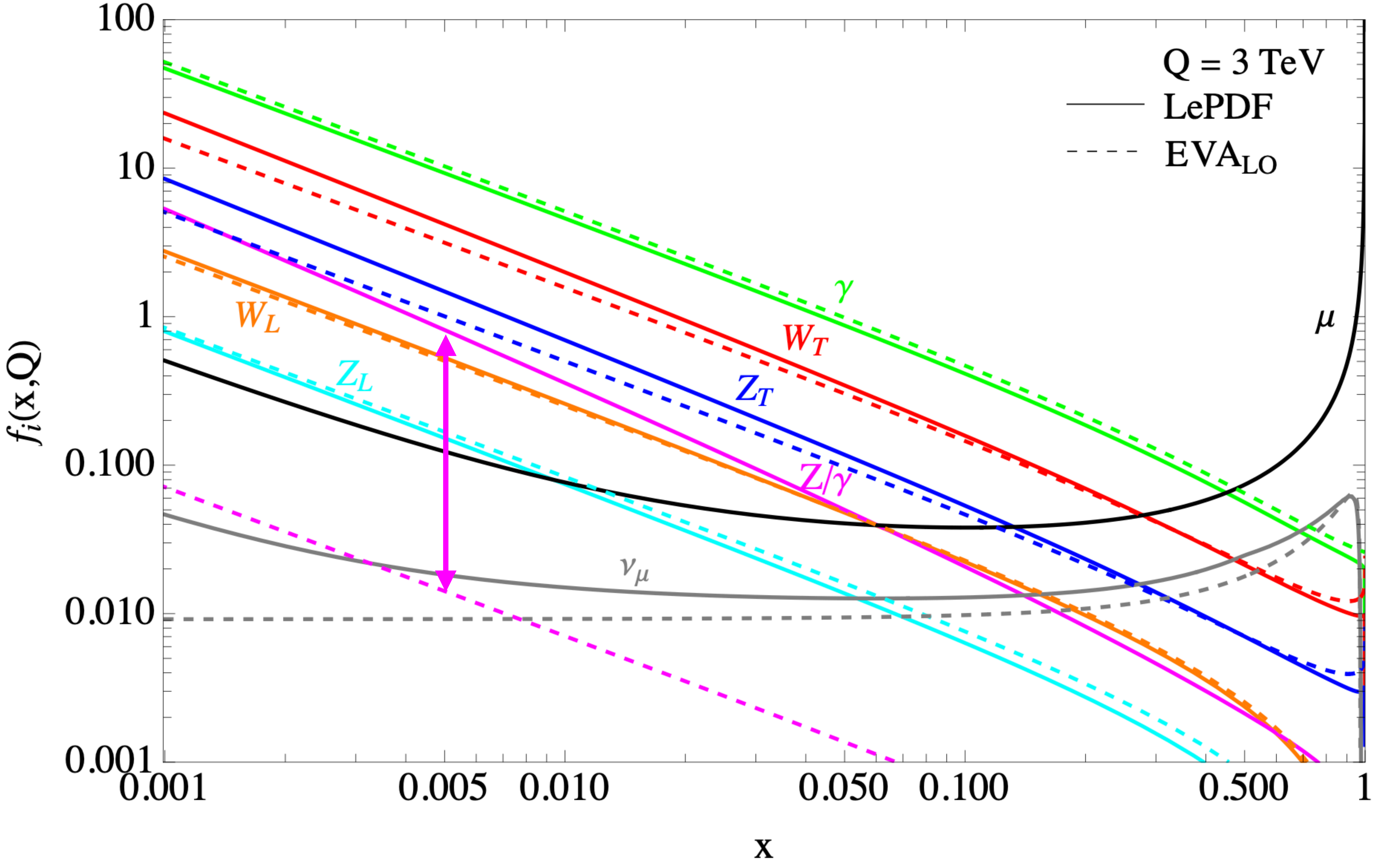
LePDF vs. EVA



Q = 3 TeV
LePDF
EVA$_{LO}$
γ
W_T
Z_T
W_L
Z_L
Z/γ
μ
ν_μ
$f_i(x,Q)$
100
10
1
0.100
0.010
0.001
0.001
0.005
0.010
0.050
0.100
0.500
1
x

LePDF vs. EVA

$$f^{(\alpha)}_{Z/\gamma_\pm}(x,Q^2) = -\frac{\sqrt{\alpha_\gamma\alpha_2}}{2\pi c_W}\left(P^f_{V_\pm f_L}(x)Q^Z_{\mu_L} + P^f_{V_\pm f_R}(x)Q^Z_{\mu_R}\right)\log\frac{Q^2+(1-x)m_Z^2}{m_\mu^2+(1-x)m_Z^2}$$

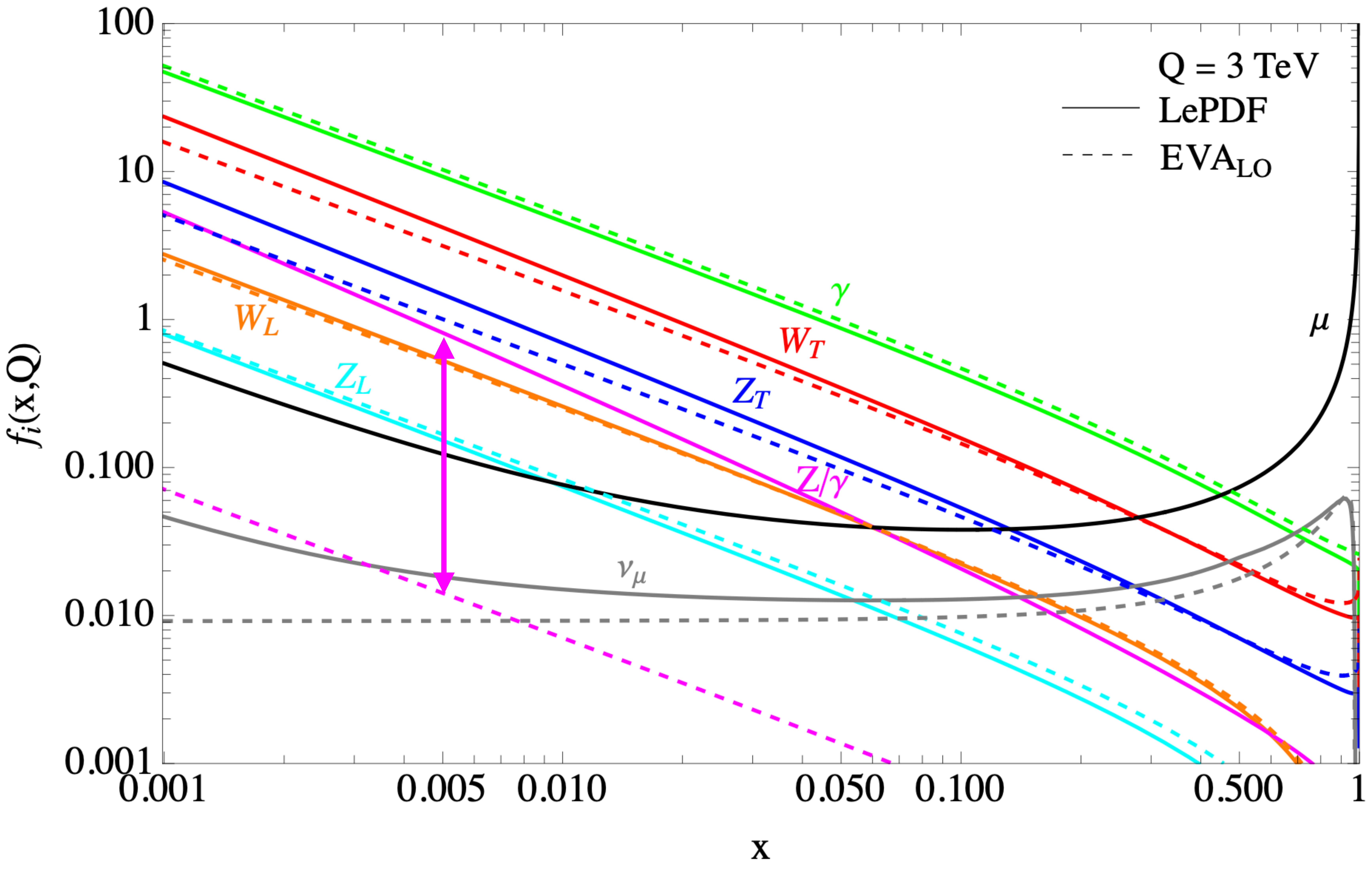


★

The **EVA Z/γ PDF is off by ~10^2**,
Will focus on this in a few slides.

LePDF vs. EVA

$$f^{(\alpha)}_{Z/\gamma\pm}(x,Q^2) = -\frac{\sqrt{\alpha_\gamma\alpha_2}}{2\pi c_W}\left(P^f_{V_\pm f_L}(x)Q^Z_{\mu_L} + P^f_{V_\pm f_R}(x)Q^Z_{\mu_R}\right)\log\frac{Q^2+(1-x)m_Z^2}{m_\mu^2+(1-x)m_Z^2}$$



★

The **EVA Z/γ PDF is off by ~10^2**,
Will focus on this in a few slides.

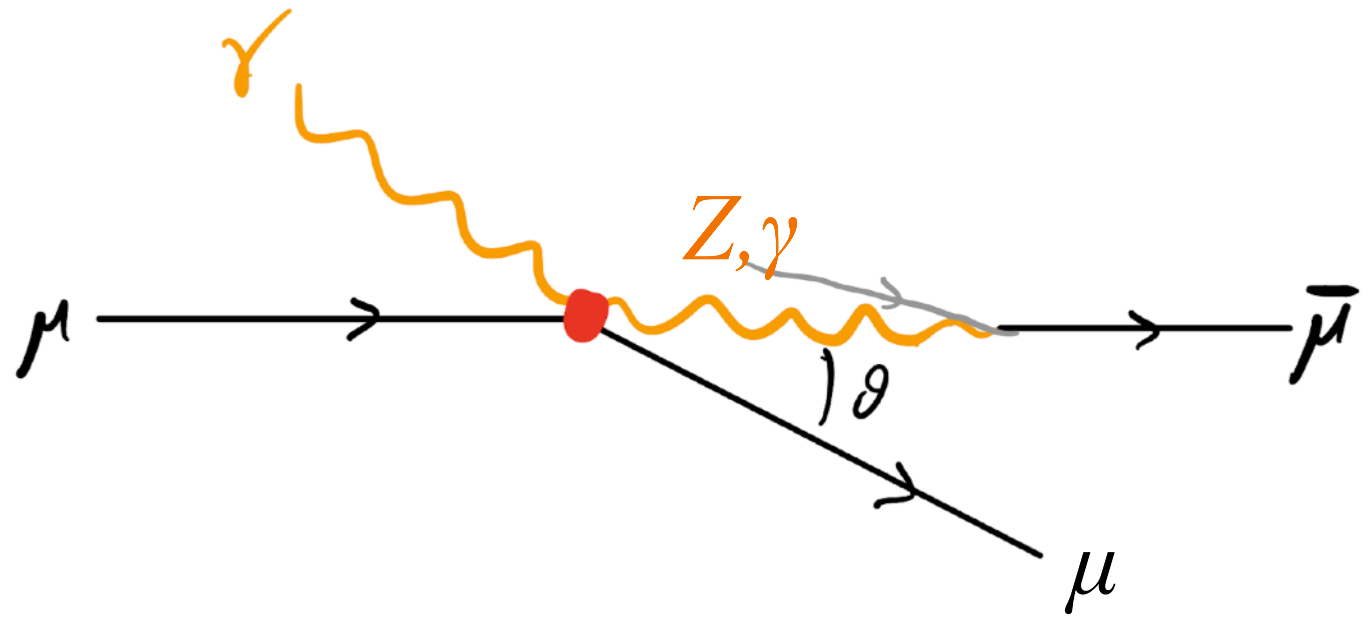
★

We can also see a **sizeable deviation** (in this log-log plot) for the **W_T** and **Z_T** PDF.
Mostly due to the double-log arising at $O(\alpha^2)$ from VVV interactions.

More details in [2303.16964]

Pheno of EW PDF effects (1) Mixed Z/γ PDF

[D.M. and A. Stanzione 2408.13191]



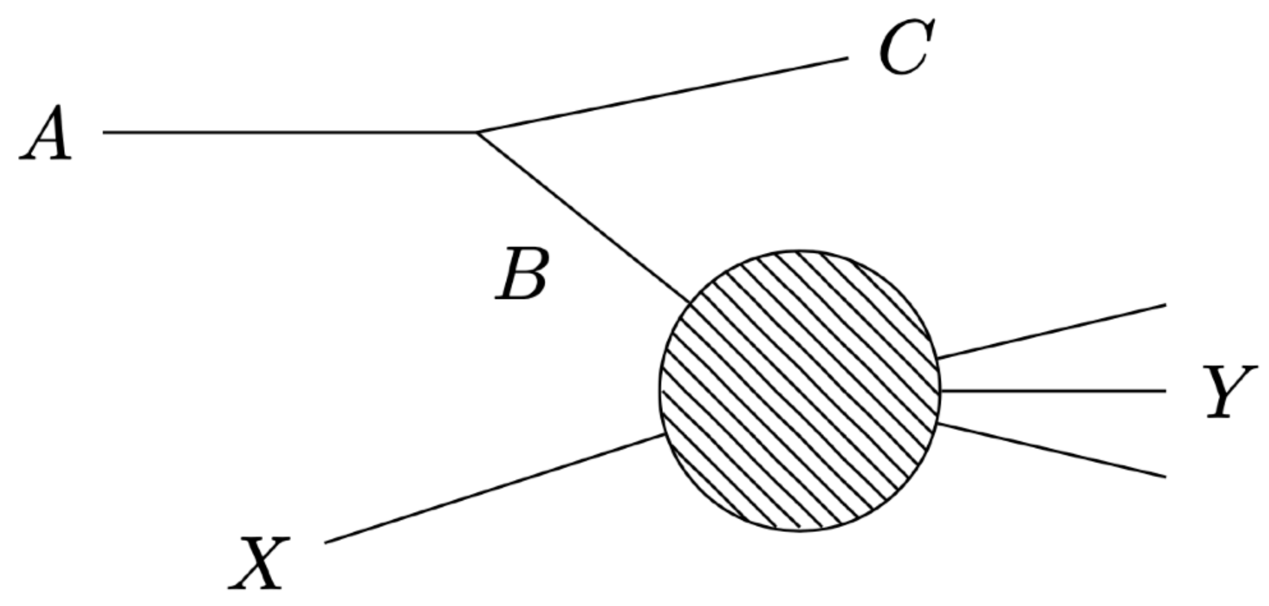
Photon - Z mixing PDF

Factorisation takes place at the amplitude level:

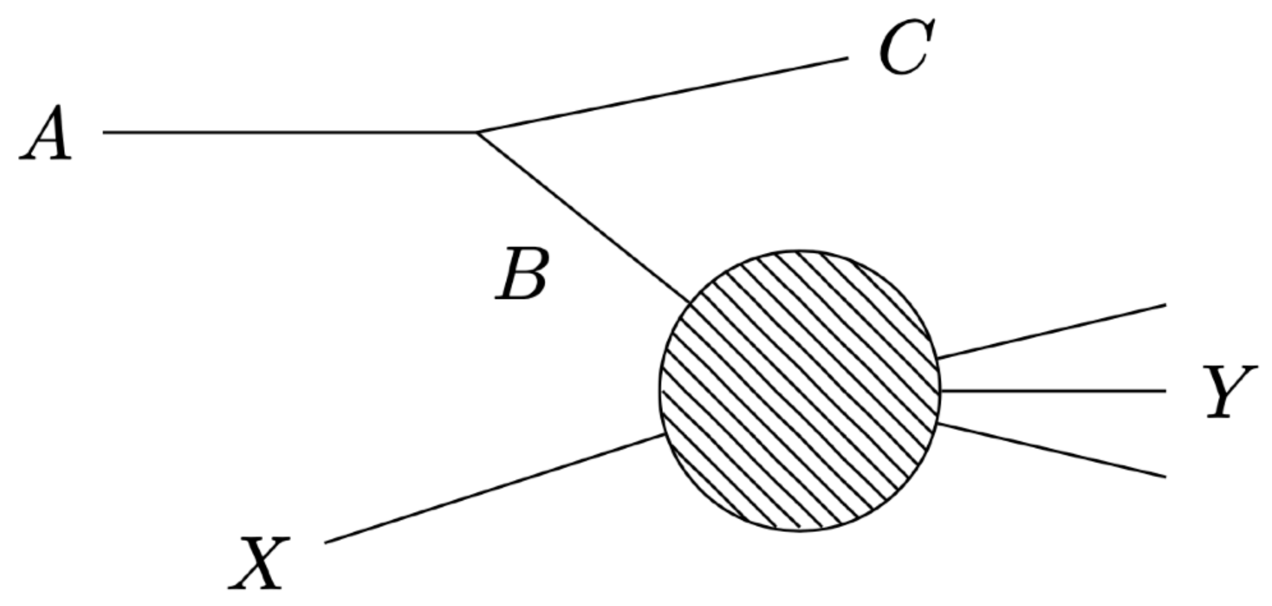
$$i\mathcal{M}(AX \to CY) = \sum_B i\mathcal{M}^{\text{split}}(A \to CB^*) \frac{i}{Q^2} i\mathcal{M}^{\text{hard}}(BX \to Y)\,(1 + \mathcal{O}(\delta_{m,\perp}))$$

[Cuomo, Vecchi, Wulzer 1911.12366, …]

$$\delta_\perp = |\mathbf{k}_\perp|/E \ll 1$$
$$\delta_m = m/E \ll 1$$



Photon - Z mixing PDF



Factorisation takes place at the amplitude level:

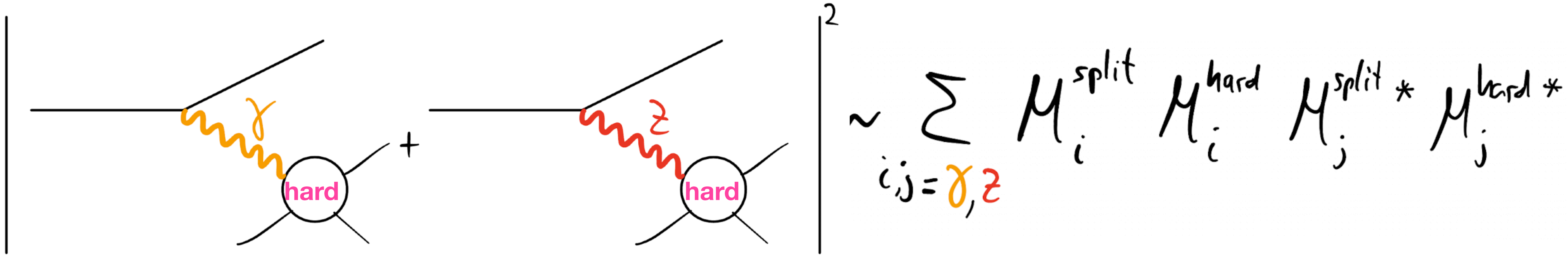
$$i\mathcal{M}(AX \to CY) = \sum_B i\mathcal{M}^{\text{split}}(A \to CB^*)\frac{i}{Q^2} i\mathcal{M}^{\text{hard}}(BX \to Y)\,(1 + \mathcal{O}(\delta_{m,\perp}))$$

[Cuomo, Vecchi, Wulzer 1911.12366, …]

$$\delta_\perp = |\mathbf{k}_\perp|/E \ll 1$$
$$\delta_m = m/E \ll 1$$

If two different states B and B' can enter in the same splitting and hard **processes**, they **can interfere:**

In the SM this can happen between: Z_T **and** γ $\quad$ Z_L and H



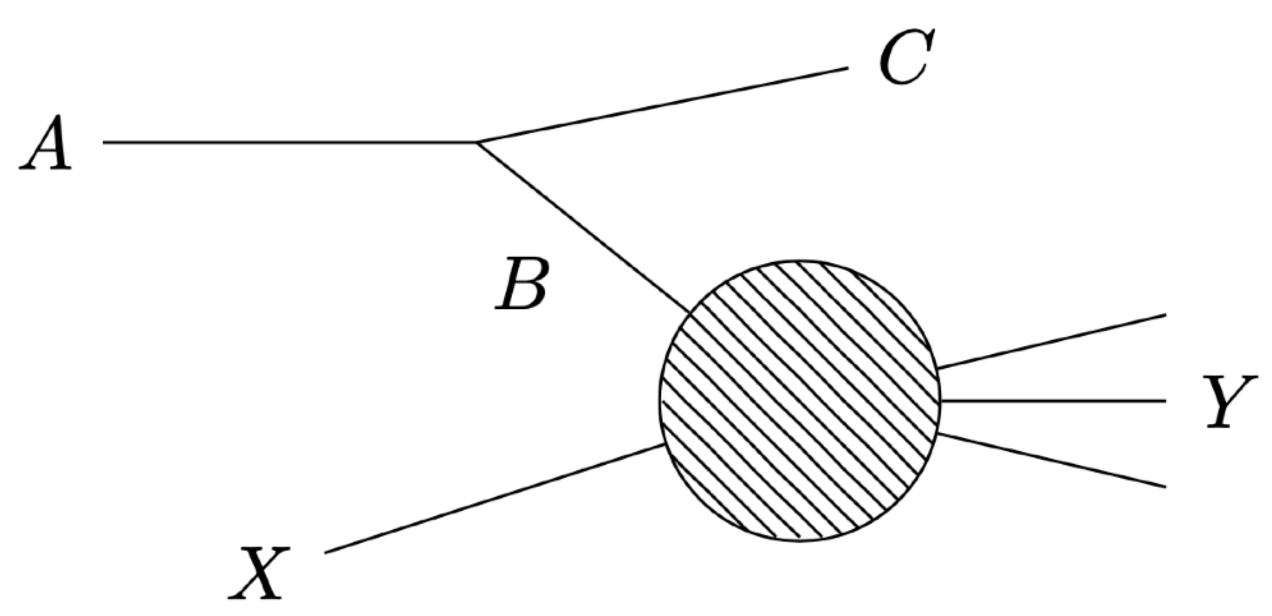
Photon - Z mixing PDF

Factorisation takes place at the amplitude level:

$$i\mathcal{M}(AX \to CY) = \sum_B i\mathcal{M}^{\text{split}}(A \to CB^*)\frac{i}{Q^2}i\mathcal{M}^{\text{hard}}(BX \to Y)\,(1 + \mathcal{O}(\delta_{m,\perp}))$$

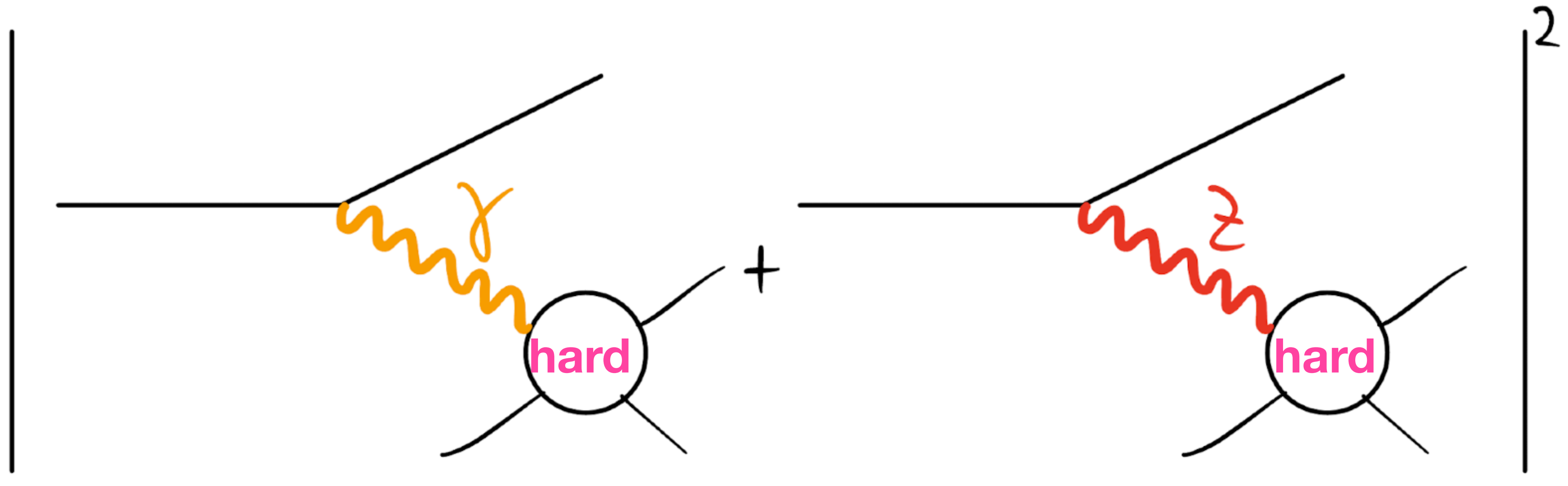
[Cuomo, Vecchi, Wulzer 1911.12366, …]

$$\delta_\perp = |\mathbf{k}_\perp|/E \ll 1$$
$$\delta_m = m/E \ll 1$$



If two different states B and B' can enter in the same splitting and hard **processes**, they **can interfere:**

In the SM this can happen between: Z_T **and** γ $\quad$ Z_L and H



$$\sim \sum_{i,j=\gamma,Z} \mathcal{M}_i^{\text{split}}\,\mathcal{M}_i^{\text{hard}}\,\mathcal{M}_j^{\text{split}\,*}\,\mathcal{M}_j^{\text{hard}\,*} \sim \sum_{i,j=\gamma,Z} d\rho_{ij}^{\text{split}}\,d\rho_{ji}^{\text{hard}}$$

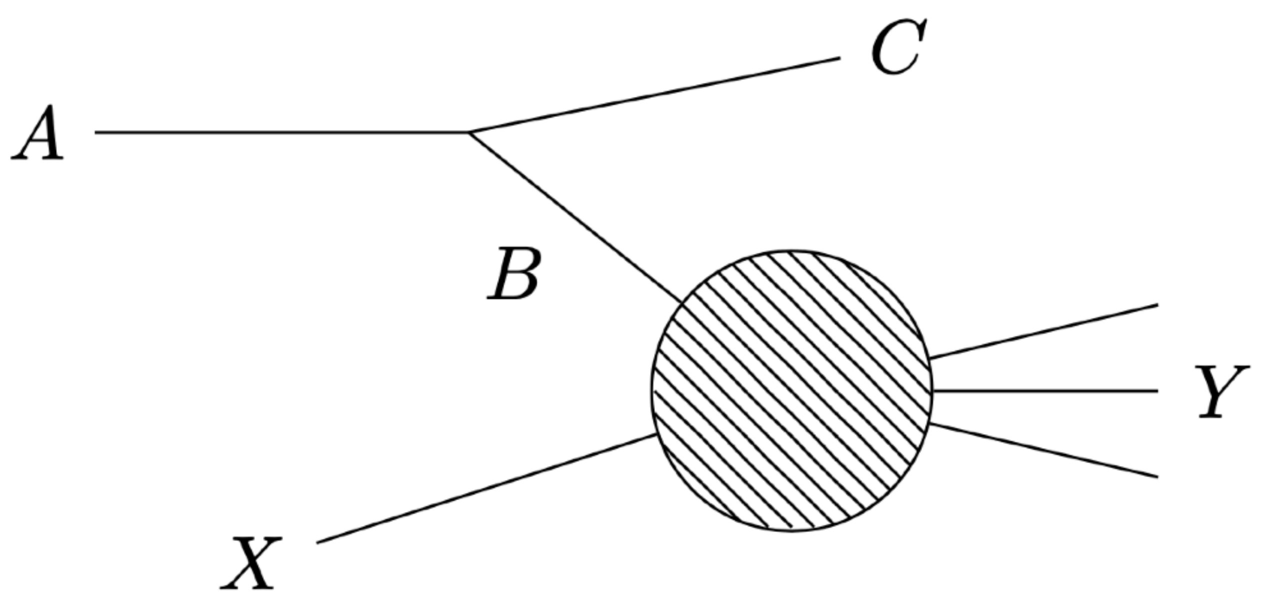
Photon - Z mixing PDF

Factorisation takes place at the amplitude level:

$$i\mathcal{M}(AX \to CY) = \sum_B i\mathcal{M}^{\rm split}(A \to CB^*)\frac{i}{Q^2} i\mathcal{M}^{\rm hard}(BX \to Y)\,(1 + \mathcal{O}(\delta_{m,\perp}))$$

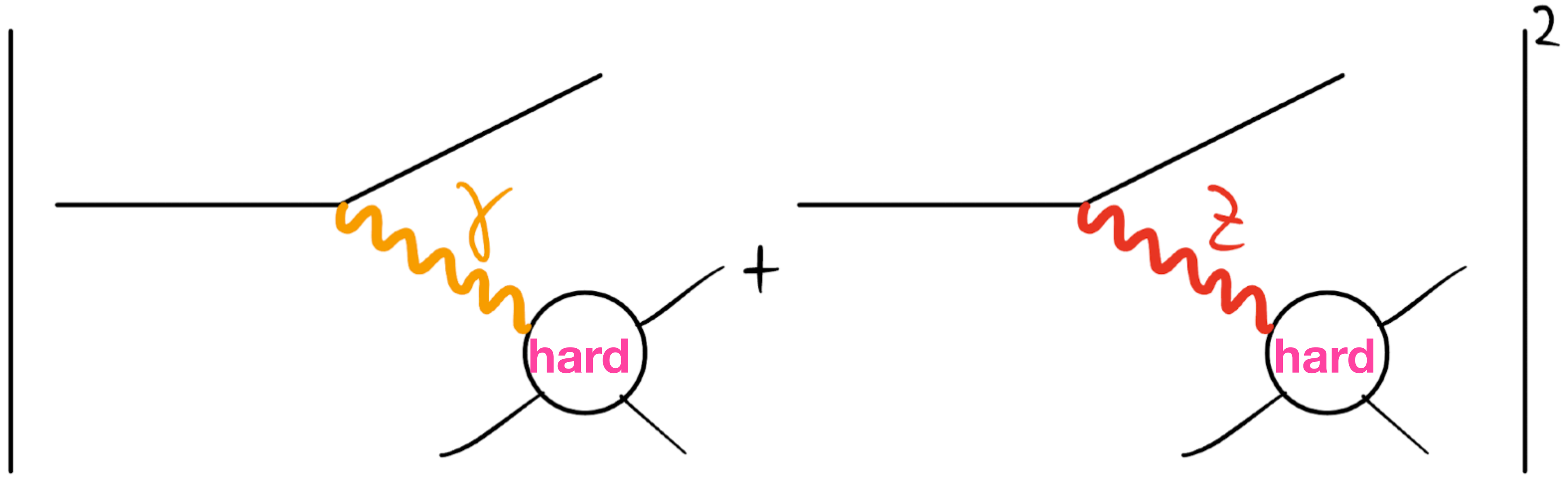
[Cuomo, Vecchi, Wulzer 1911.12366, …]

$$\delta_\perp = |\mathbf{k}_\perp|/E \ll 1$$
$$\delta_m = m/E \ll 1$$



If two different states B and B' can enter in the same splitting and hard **processes**, they **can interfere:**

In the SM this can happen between: Z_T **and** γ Z_L **and** H



$$\sim \sum_{i,j=\gamma,Z} \mathcal{M}_i^{\rm split}\,\mathcal{M}_i^{\rm hard}\,\mathcal{M}_j^{\rm split\,*}\,\mathcal{M}_j^{\rm hard\,*} \sim \sum_{i,j=\gamma,Z} d\rho_{ij}^{\rm split}\,d\rho_{ji}^{\rm hard}$$

$$d\sigma \sim \mathrm{Tr}\left[\underbrace{\begin{pmatrix} f_\gamma & f_{Z\gamma} \\ f_{Z\gamma}^* & f_Z \end{pmatrix}}_{d\rho^{\rm split}} \cdot \underbrace{\begin{pmatrix} |\mathcal{M}_\gamma^h|^2 & \mathcal{M}_Z^h \mathcal{M}_\gamma^{h*} \\ \mathcal{M}_\gamma^h \mathcal{M}_Z^{h*} & |\mathcal{M}_Z^h|^2 \end{pmatrix}}_{d\rho^{\rm hard}}\right]$$

up to $O(k_T^2/E^2, m^2/E^2)$

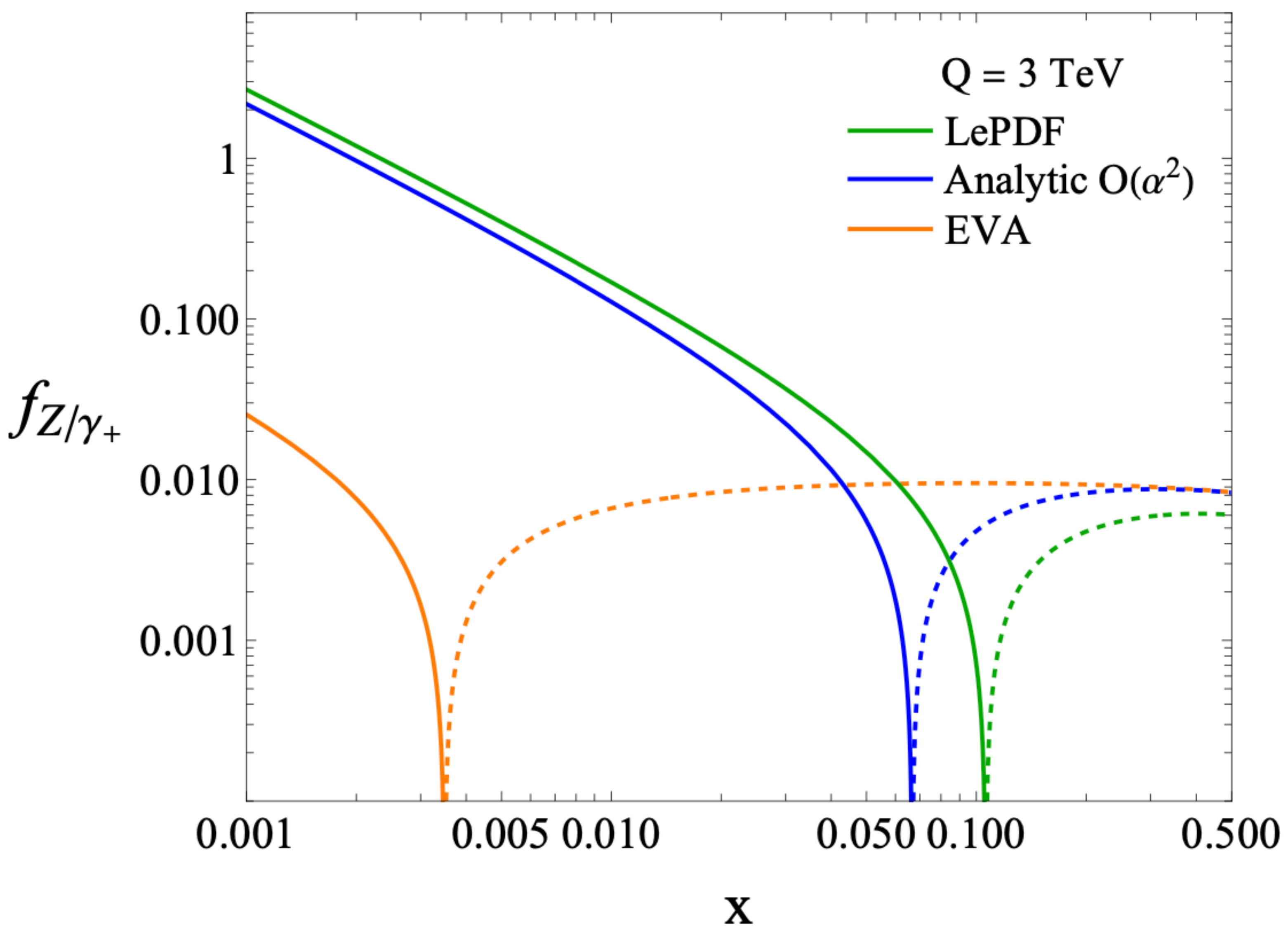
To describe the interference in the splitting one introduces the **mixed Z/γ PDF**.

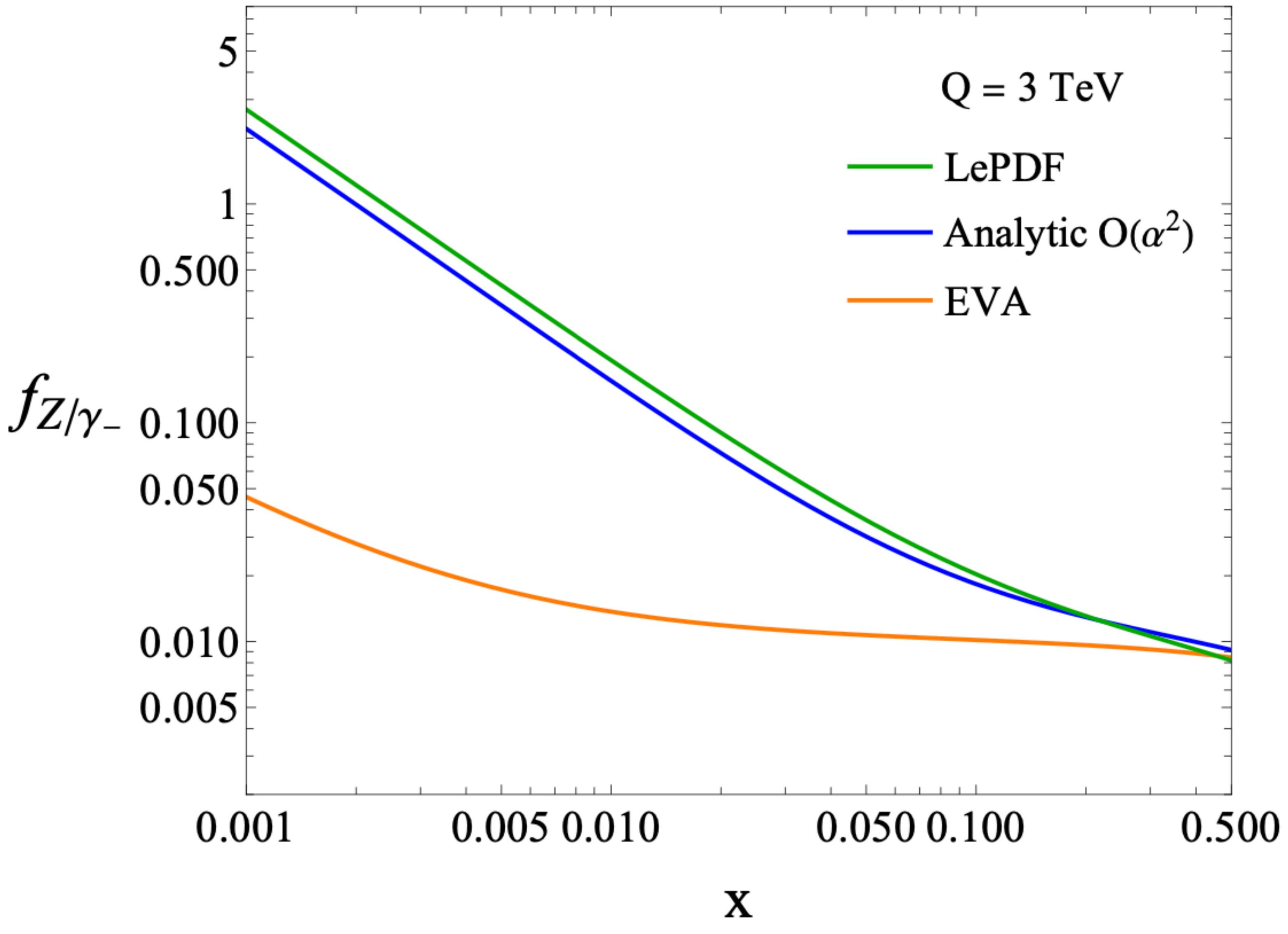
(Similarly also for Z_L and H)

P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0505047]
Chen, Han, Tweedie [1611.00788]

The *different virtuality* due to the different masses is an effect of $O(\delta_m^2)$.

Comparison with EVA





Solving iteratively the DGLAP equations at O(α) one can derive the **LO EVA for the Z/γ PDF**:

$$f^{(\alpha)}_{Z/\gamma_\pm}(x,Q^2) = -\int_{m_\mu^2}^{Q^2} dp_T^2 \, \frac{\alpha_{\gamma 2}}{2\pi c_W} \frac{1}{(p_T^2 + (1-x)m_Z^2)} \left(P^f_{V_\pm f_L}(x) Q^Z_{\mu_L} + P^f_{V_\pm f_R}(x) Q^Z_{\mu_R} \right) =$$

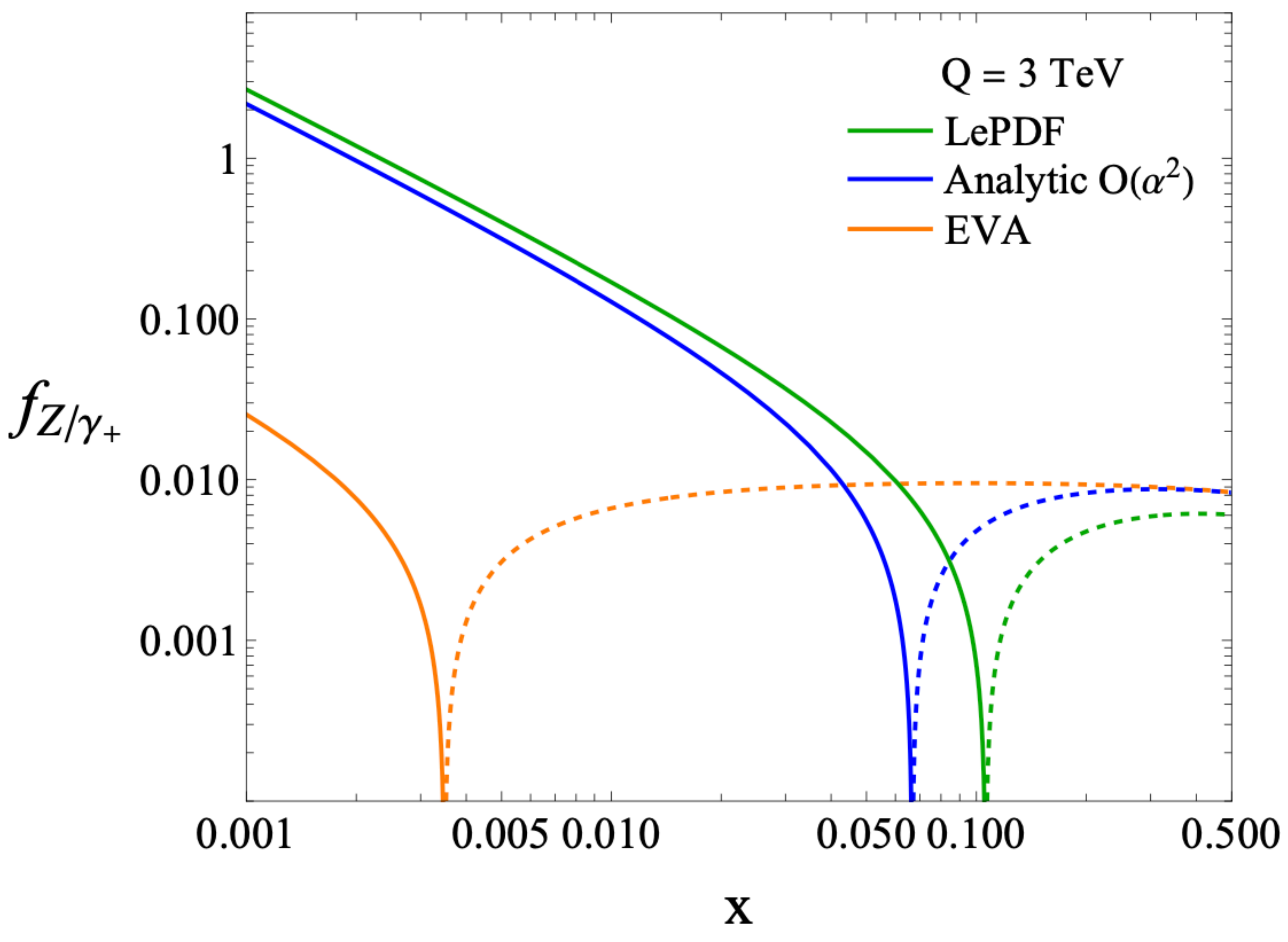
$$= -\frac{\alpha_{\gamma 2}}{2\pi c_W} \left(P^f_{V_\pm f_L}(x) Q^Z_{\mu_L} + P^f_{V_\pm f_R}(x) Q^Z_{\mu_R} \right) \log \frac{Q^2 + (1-x)m_Z^2}{m_\mu^2 + (1-x)m_Z^2} \, ,$$

$$Q^Z_{\mu_L} = -\frac{1}{2} + s_W^2 \qquad Q^Z_{\mu_R} = s_W^2$$

$$P^f_{V_+ f_L}(x) = P^f_{V_- f_R}(x) = \frac{(1-x)^2}{x}$$

$$P^f_{V_- f_L}(x) = P^f_{V_+ f_R}(x) = \frac{1}{x}$$

Comparison with EVA



Solving iteratively the DGLAP equations at O(α) one can derive the **LO EVA for the Z/γ PDF**:

$$f^{(\alpha)}_{Z/\gamma\pm}(x,Q^2) = -\int_{m_\mu^2}^{Q^2} dp_T^2 \, \frac{\alpha_{\gamma 2}}{2\pi c_W} \frac{1}{(p_T^2+(1-x)m_Z^2)} \left(P^f_{V\pm f_L}(x) Q^Z_{\mu_L} + P^f_{V\pm f_R}(x) Q^Z_{\mu_R} \right) =$$

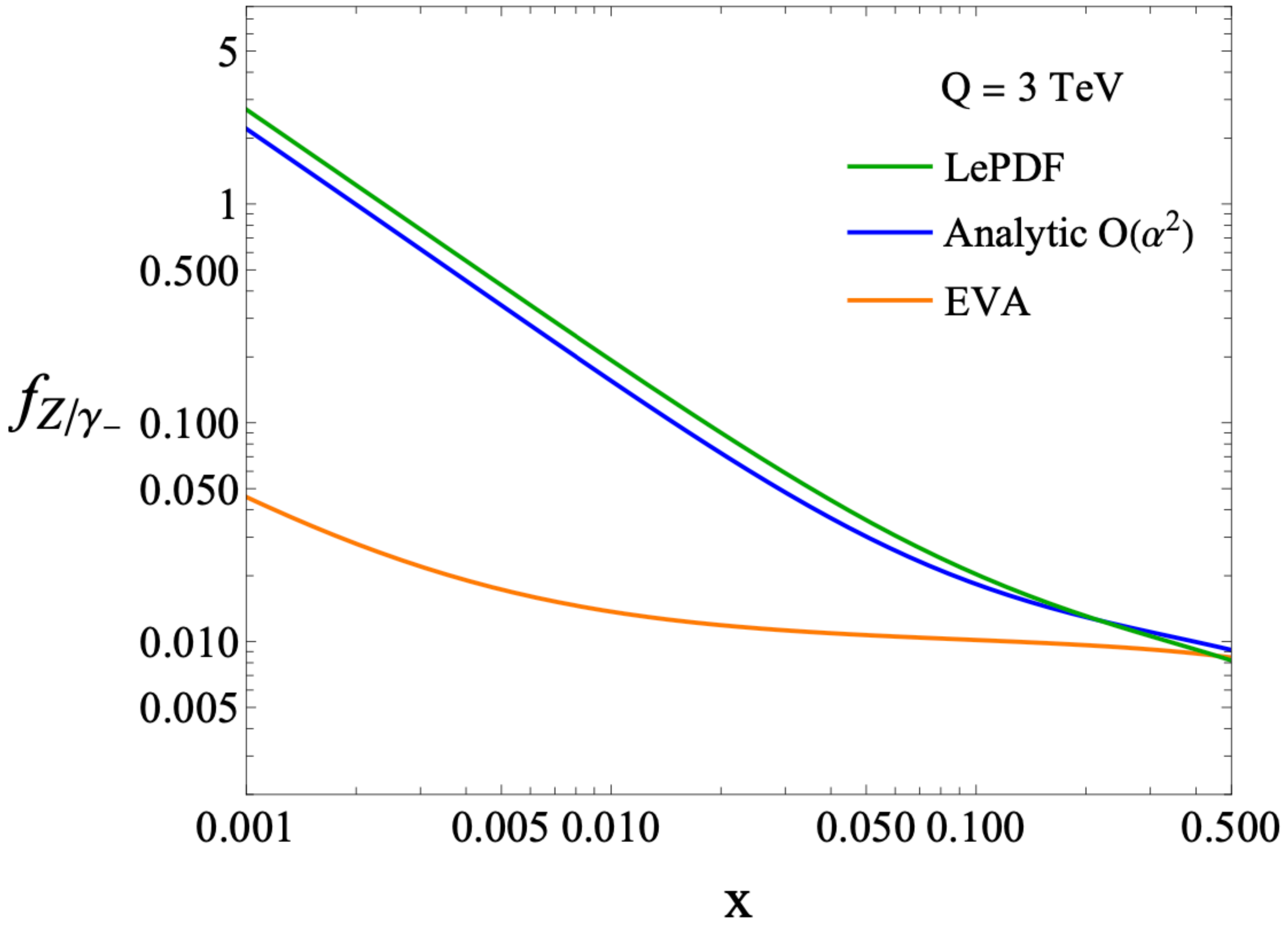
$$= -\frac{\alpha_{\gamma 2}}{2\pi c_W} \left(P^f_{V\pm f_L}(x) Q^Z_{\mu_L} + P^f_{V\pm f_R}(x) Q^Z_{\mu_R} \right) \log \frac{Q^2+(1-x)m_Z^2}{m_\mu^2+(1-x)m_Z^2} ,$$

$$Q^Z_{\mu_L} = -\tfrac{1}{2} + s_W^2 \qquad Q^Z_{\mu_R} = s_W^2$$

$$P^f_{V_+ f_L}(x) = P^f_{V_- f_R}(x) = \frac{(1-x)^2}{x}$$

$$P^f_{V_- f_L}(x) = P^f_{V_+ f_R}(x) = \frac{1}{x}$$

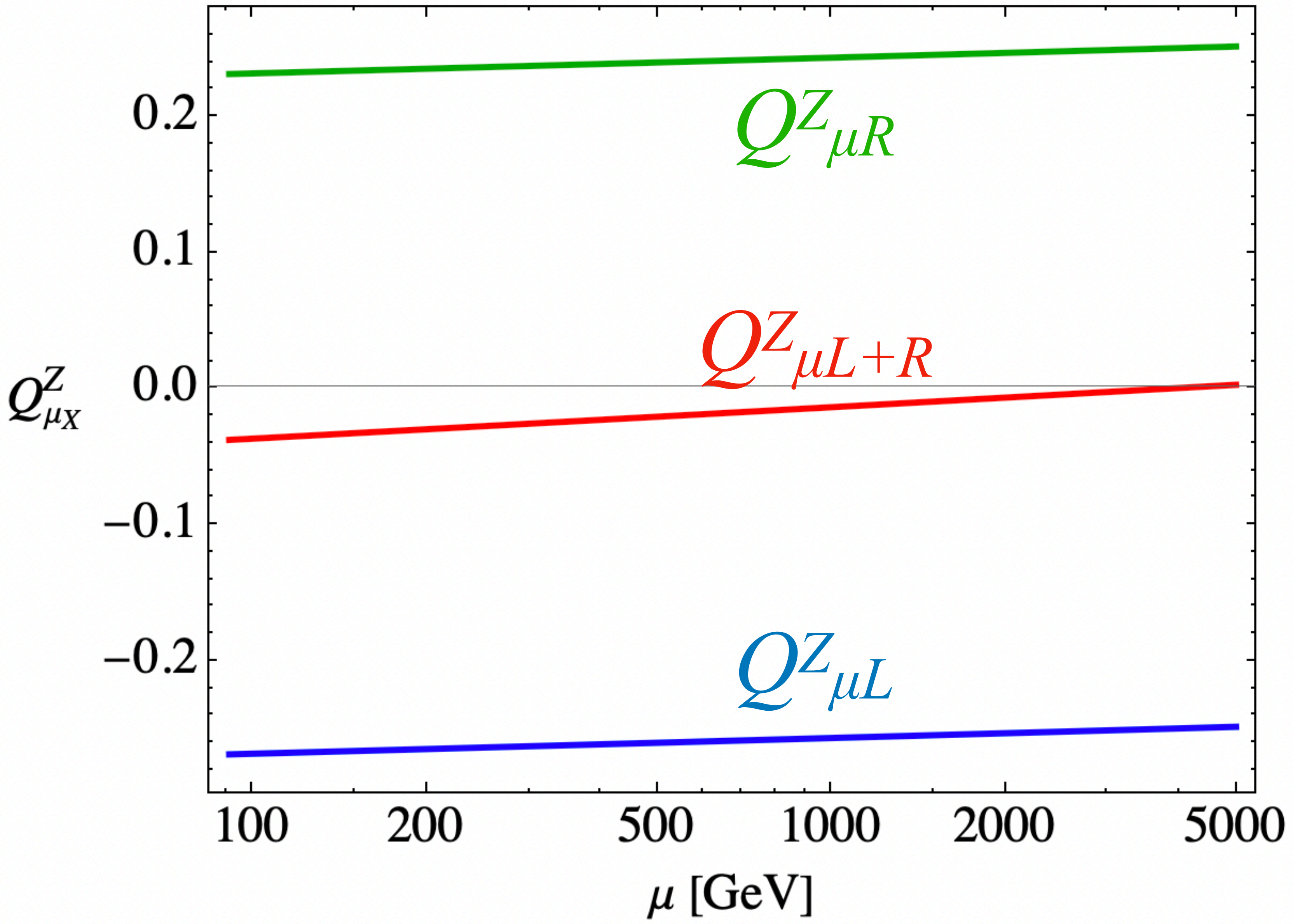
For $x \ll 1$ this becomes proportional to the vector-like muon coupling to the Z boson:



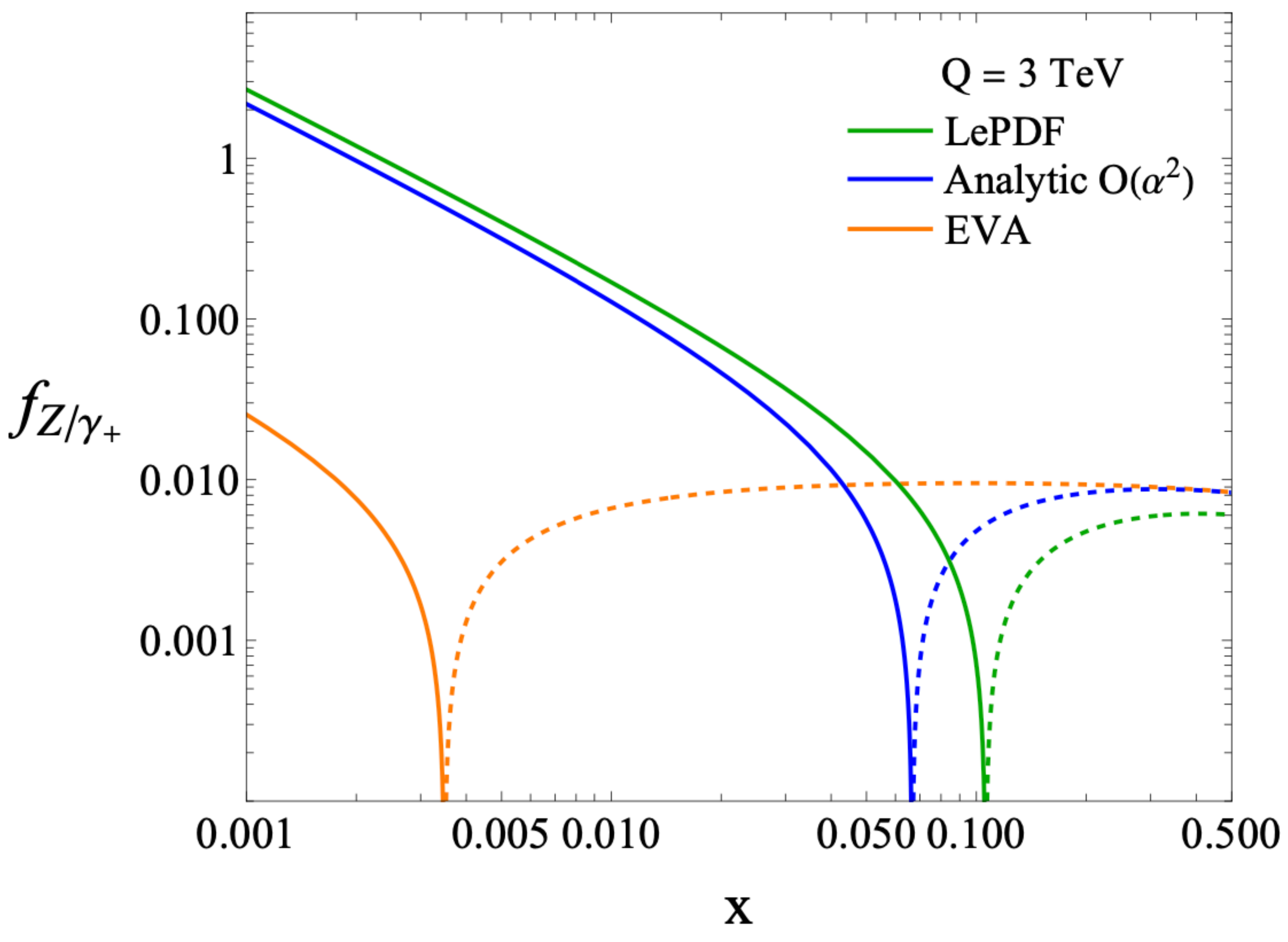
ACCIDENTAL SUPPRESSION !

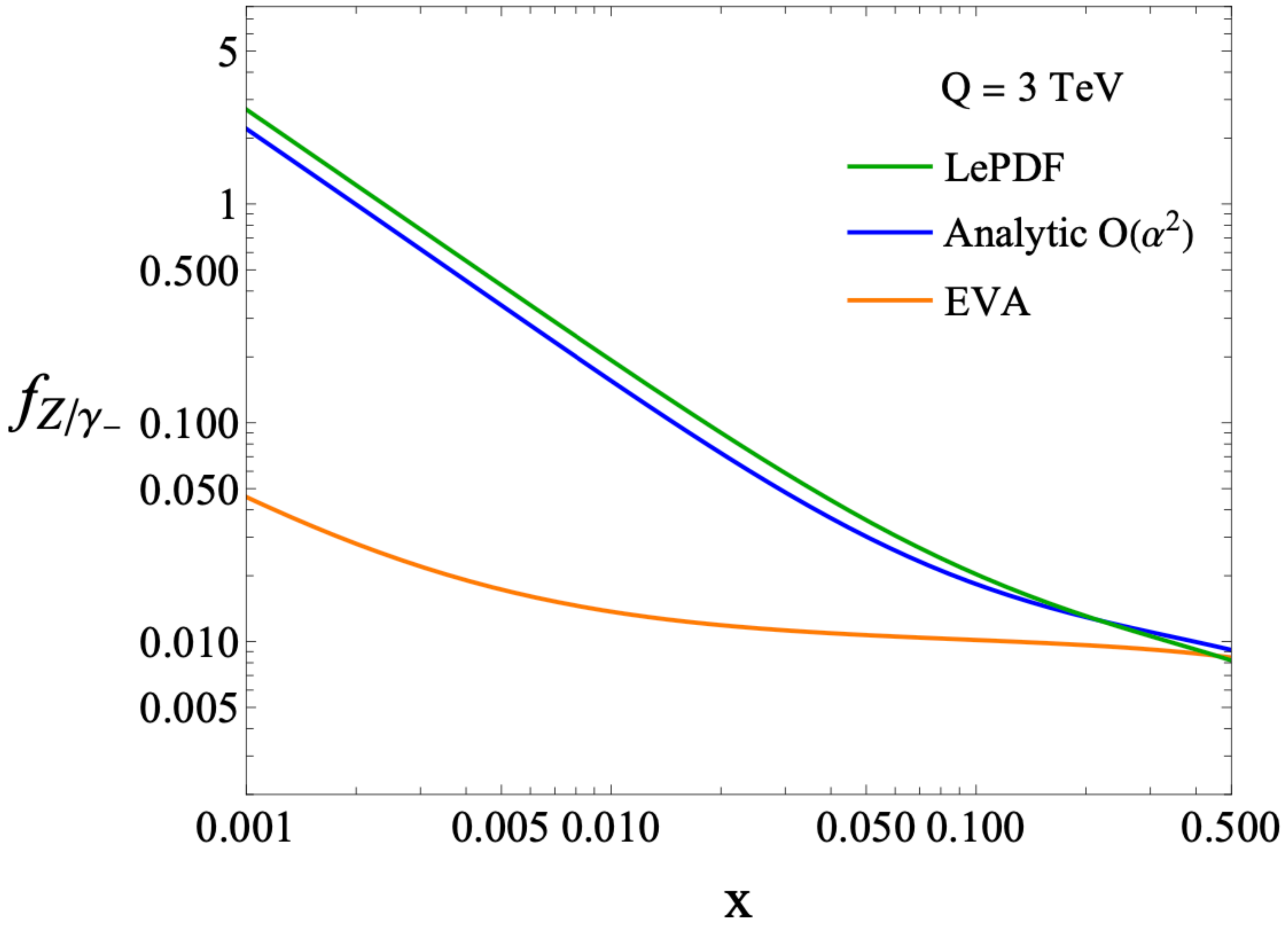
- possible because **at LO, LH and RH μ PDFs are equal**.

In the **full result** a O(1) polarisation arises, which lifts the cancellation.
Also, at O(α²) other contributions become dominant, due to Sudakov logs.



Extending EVA to O(α²)





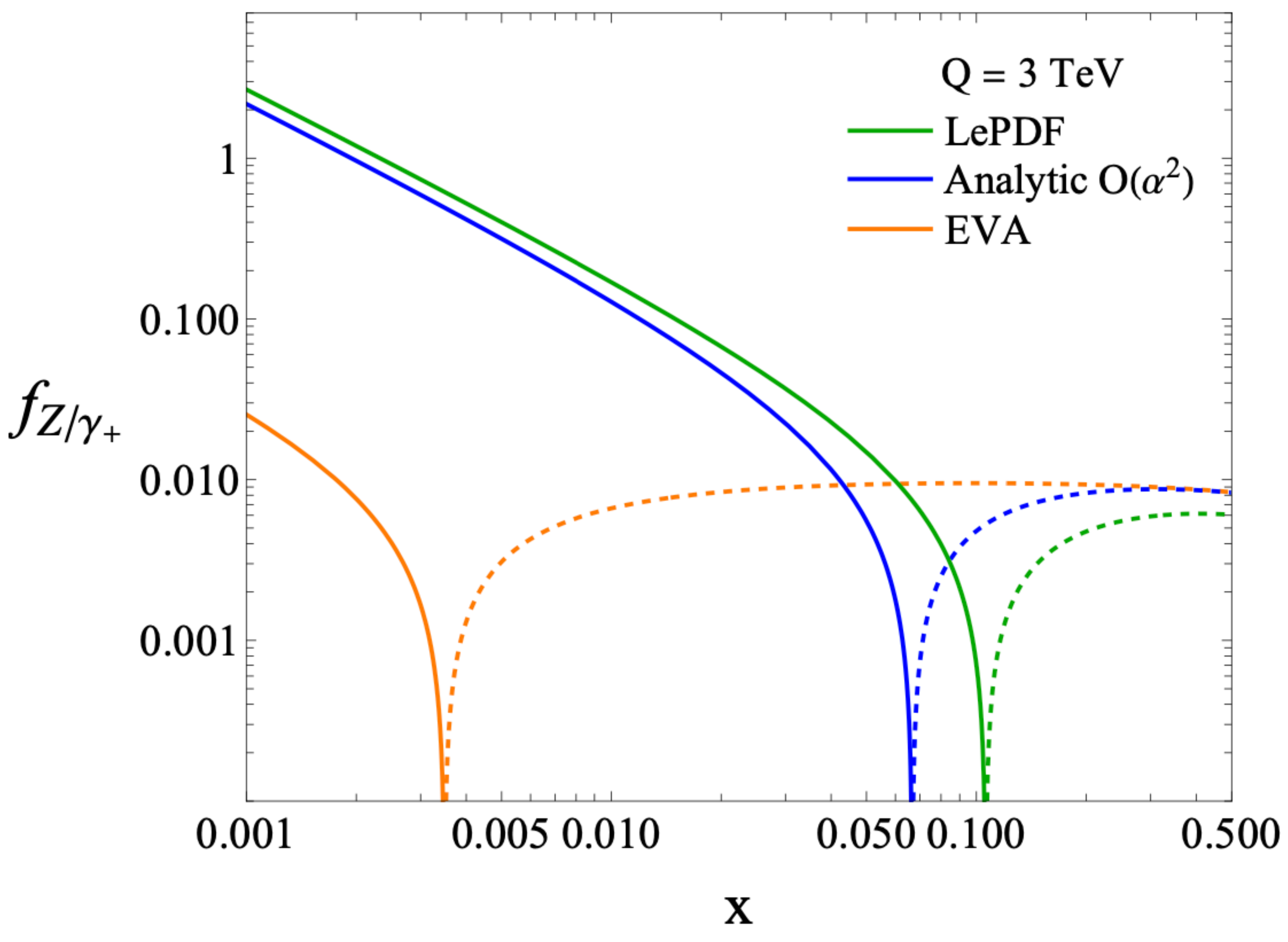
We can go **one order higher** by using the O(α) EVA expressions in the RHS of the DGLAP equation:

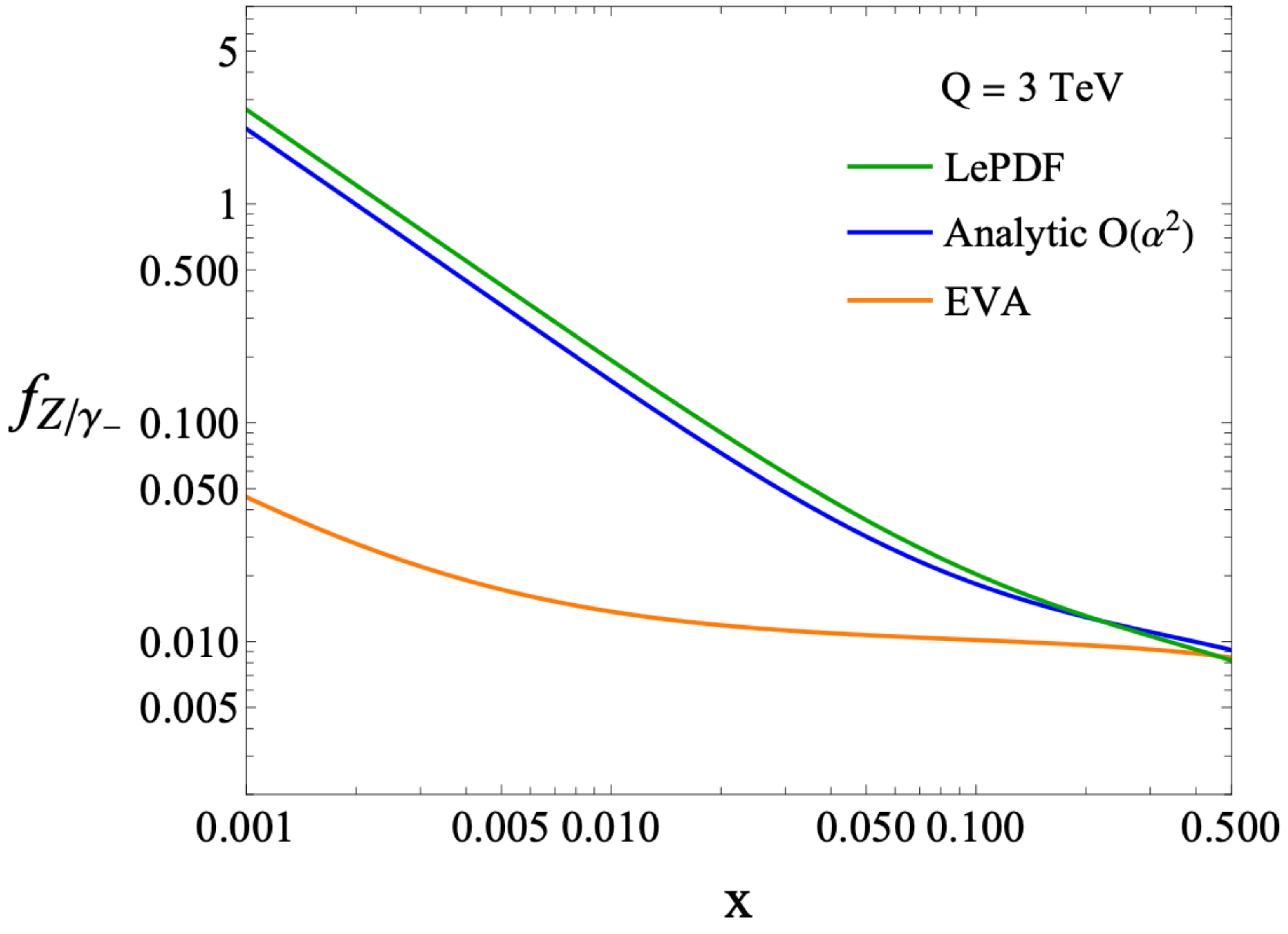
$$\frac{df^{(\alpha^2)}_{Z/\gamma_+}(x,Q^2)}{dt} = \frac{\alpha_{\gamma 2}(t)}{2\pi} 2c_W P^V_{V_+V_\pm} \otimes f^{(\alpha)}_{W^-_\pm} + \frac{\alpha_{\gamma 2}(t)}{2\pi}\frac{c_{2W}(t)}{c_W(t)} P^h_{V_+h} \otimes f^{(\alpha)}_{W^-_L} + $$

$$+ \frac{\alpha_{\gamma 2}(t)}{2\pi}\frac{2}{c_W(t)} \sum_f Q_f \left[Q^Z_{f_L} P^f_{V_+f_L} \otimes f^{(\alpha)}_{f_L} + Q^Z_{f_R} P^f_{V_-f_L} \otimes f^{(\alpha)}_{f_R} \right]$$

$$t = \log(Q^2/m_\mu^2)$$

Extending EVA to O(α²)





We can go **one order higher** by using the O(α) EVA expressions in the RHS of the DGLAP equation:

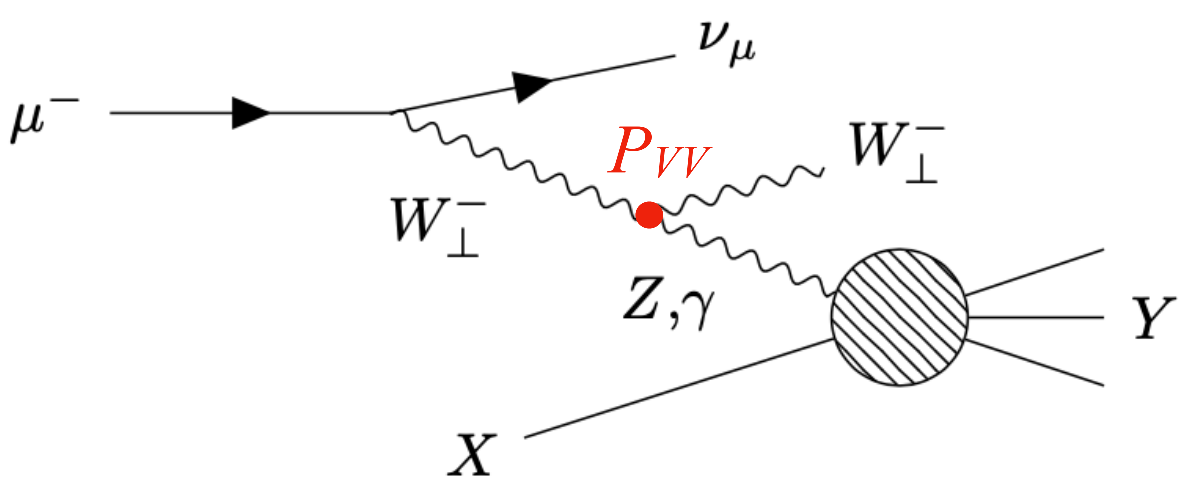
$$\frac{df^{(\alpha^2)}_{Z/\gamma_+}(x,Q^2)}{dt} = \frac{\alpha_{\gamma 2}(t)}{2\pi} 2c_W P^V_{V_+V_\pm} \otimes f^{(\alpha)}_{W^-_\pm} + \frac{\alpha_{\gamma 2}(t)}{2\pi}\frac{c_{2W}(t)}{c_W(t)} P^h_{V_+h} \otimes f^{(\alpha)}_{W^-_L} + \\ + \frac{\alpha_{\gamma 2}(t)}{2\pi}\frac{2}{c_W(t)}\sum_f Q_f \left[Q^Z_{f_L} P^f_{V_+f_L} \otimes f^{(\alpha)}_{f_L} + Q^Z_{f_R} P^f_{V_-f_L} \otimes f^{(\alpha)}_{f_R}\right]$$

$$t = \log(Q^2/m_\mu^2)$$

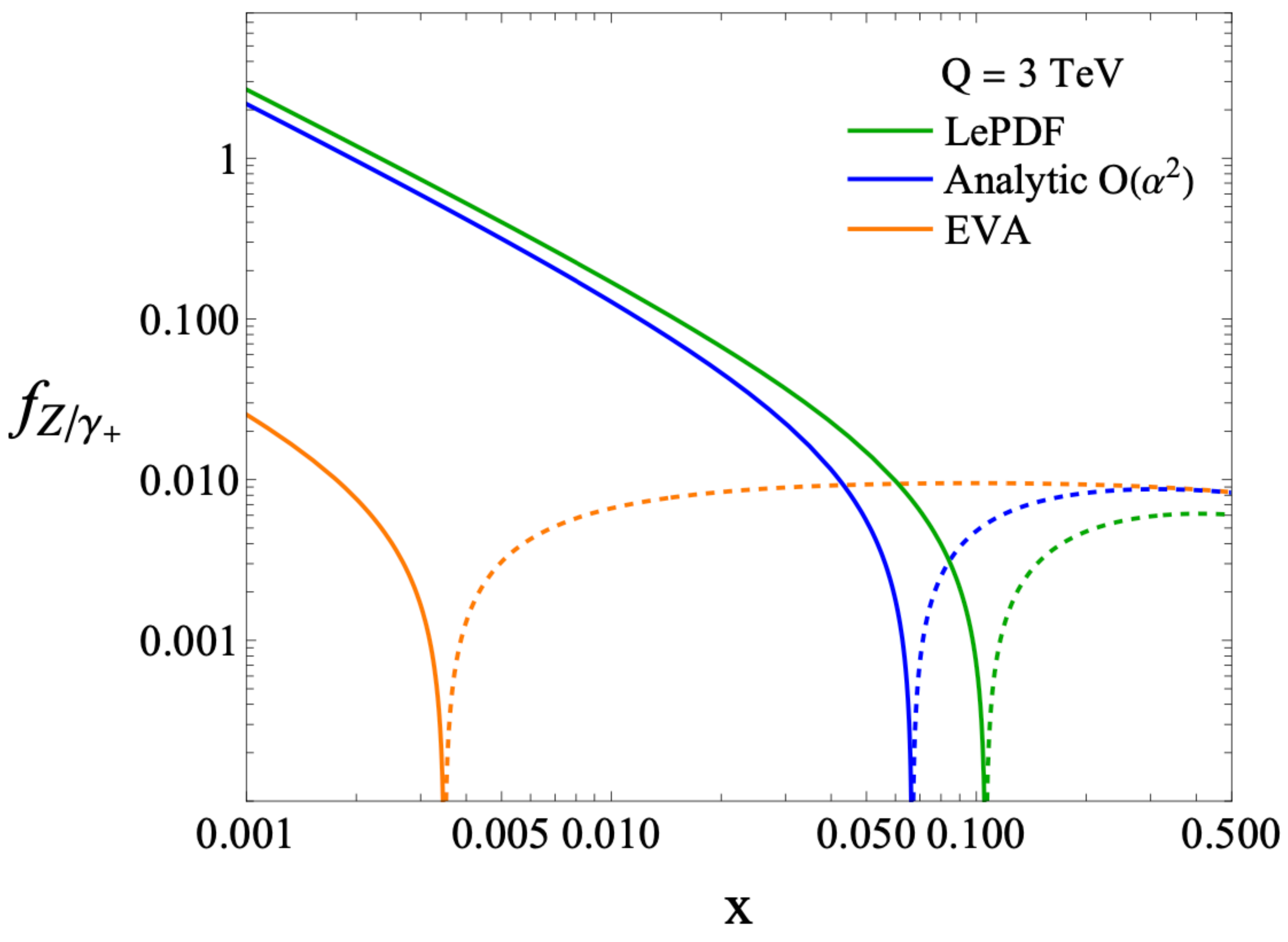
Let us focus on the first term, where

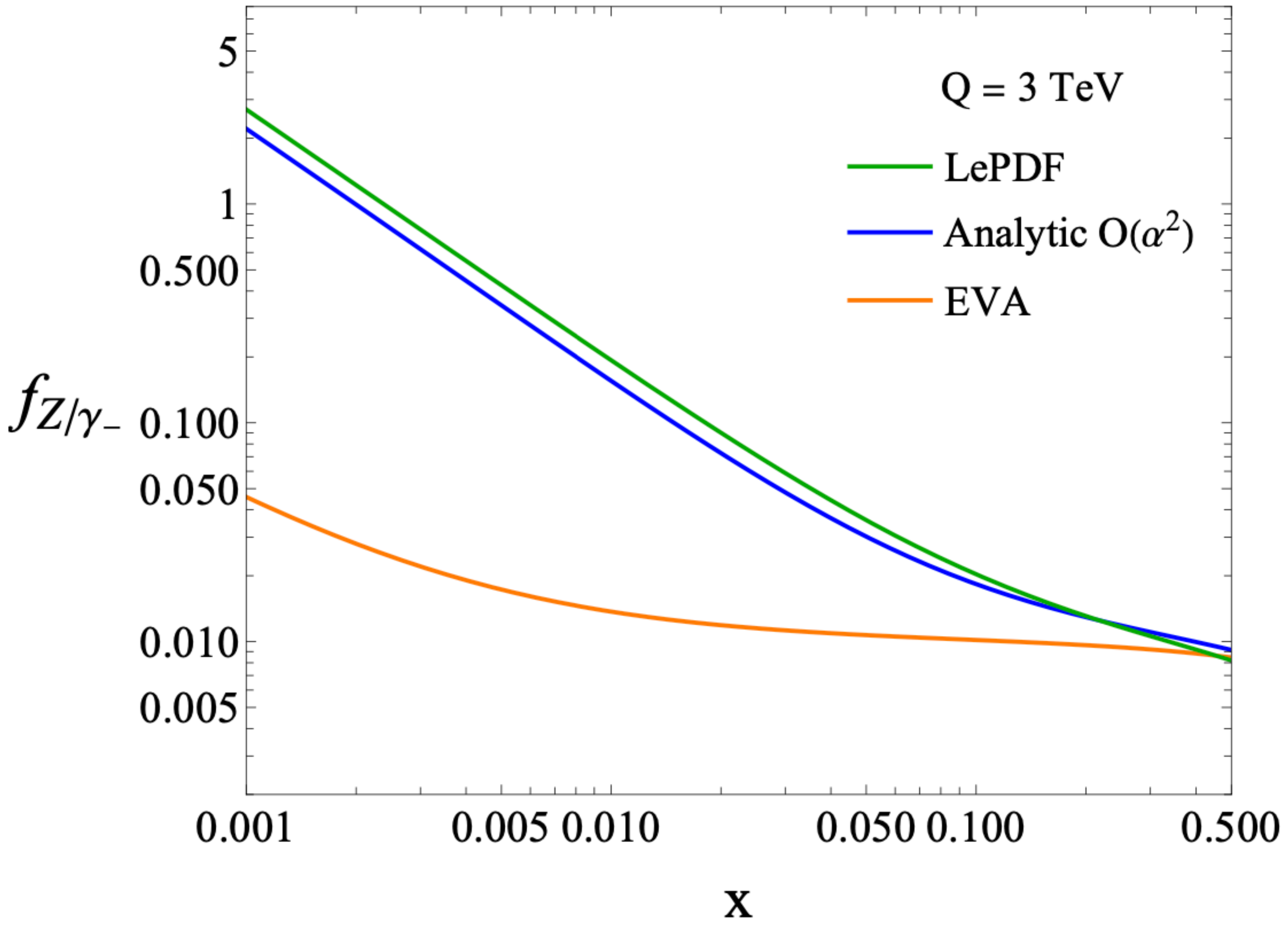
$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) \approx \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x) \log\frac{Q^2}{m_Z^2}$$

Corresponds to a **double-emission**



Extending EVA to O(α²)





We can go **one order higher** by using the O(α) EVA expressions in the RHS of the DGLAP equation:

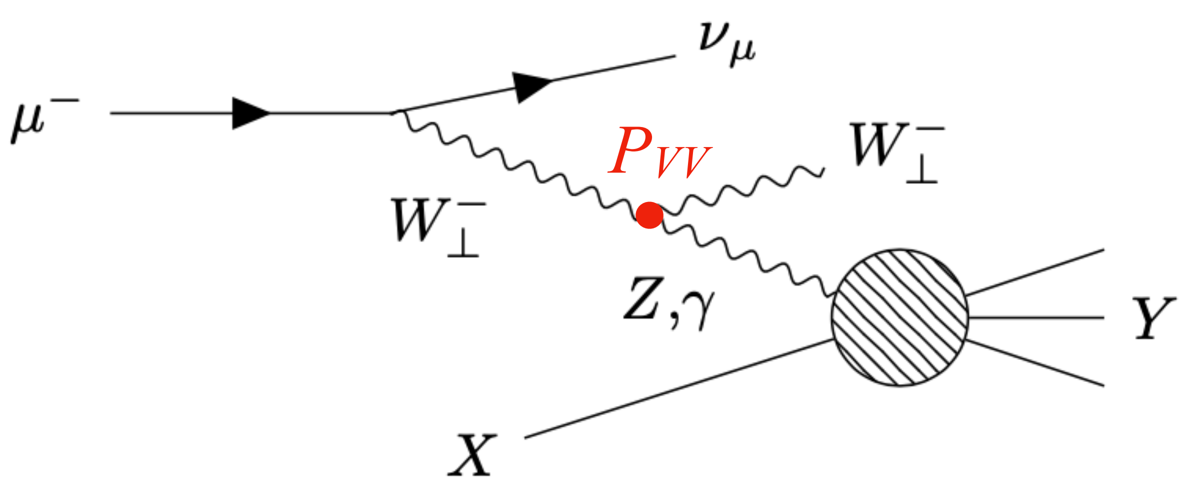
$$\frac{df^{(\alpha^2)}_{Z/\gamma_+}(x,Q^2)}{dt} = \frac{\alpha_{\gamma 2}(t)}{2\pi} 2c_W P^V_{V_+V_\pm} \otimes f^{(\alpha)}_{W^-_\pm} + \frac{\alpha_{\gamma 2}(t)}{2\pi}\frac{c_{2W}(t)}{c_W(t)} P^h_{V_+h} \otimes f^{(\alpha)}_{W^-_L} + \frac{\alpha_{\gamma 2}(t)}{2\pi}\frac{2}{c_W(t)}\sum_f Q_f \left[Q^Z_{f_L} P^f_{V_+f_L} \otimes f^{(\alpha)}_{f_L} + Q^Z_{f_R} P^f_{V_-f_L} \otimes f^{(\alpha)}_{f_R}\right]$$

$$t = \log(Q^2/m_\mu^2)$$

Let us focus on the first term, where

$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) \approx \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x) \log\frac{Q^2}{m_Z^2}$$

Corresponds to a **double-emission**



The result for that term is:

$$f^{(\alpha^2)P_{VV}}_{Z/\gamma_+}(x,Q) = \frac{\alpha_2\alpha_{\gamma 2}}{96\pi^2 x}(t-t_Z)^2\, 2c_W(x-1)^2 \cdot \left[(t-t_Z) + J(x)\right]$$

$$f^{(\alpha^2)P_{VV}}_{Z/\gamma_-}(x,Q) = \frac{\alpha_2\alpha_{\gamma 2}}{96\pi^2 x}(t-t_Z)^2\, 8 \cdot \left[(t-t_Z) + K(x)\right],$$

J(x) and *K(x)* are O(1) functions of x.

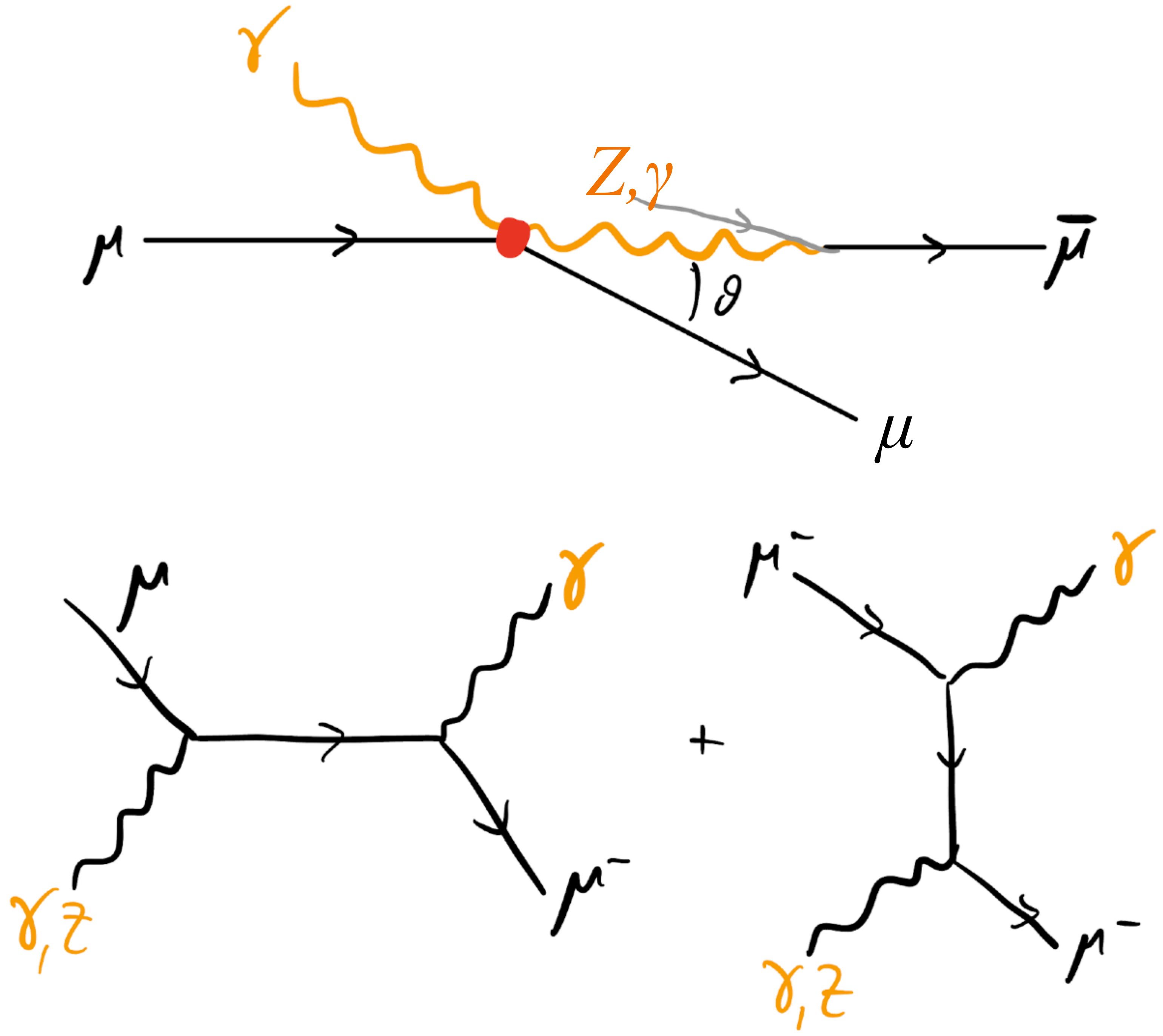
A **Sudakov double-log** appears:

$$\alpha^2(t-t_Z)^3 = \alpha^2 \log^3(Q^2/m_Z^2)$$

The full O(α²) expression gives a much more accurate approximation to the numerical result.

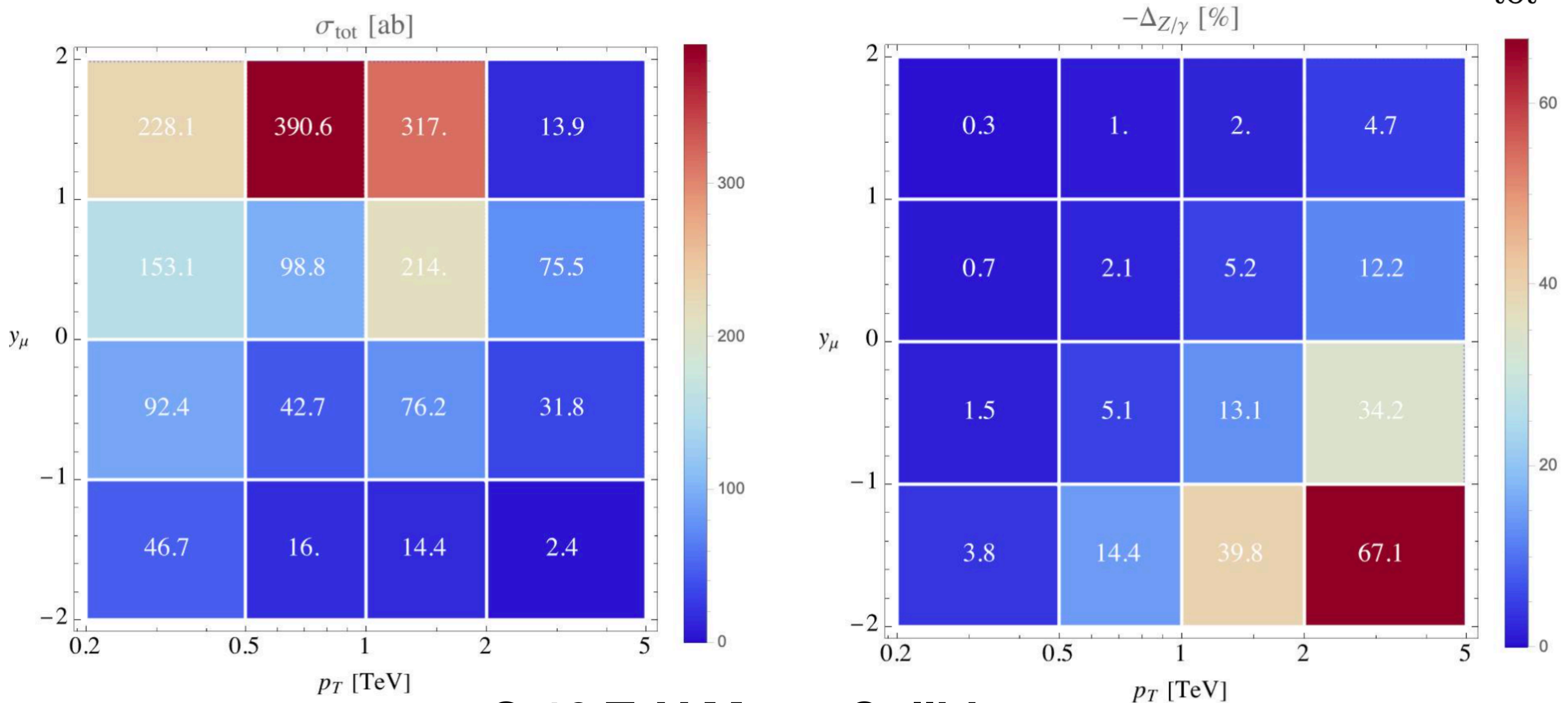
Compton Scattering @ MuC

No large new physics effect is expected in this process, since muon couplings to photon and Z boson are well tested. It is thus **perfect to study this EW SM effect**.



Cross section in **bins** of **muon rapidity and p_T**

$$\Delta_{Z/\gamma} \equiv \frac{\sigma_{Z/\gamma}}{\sigma_{tot}}$$



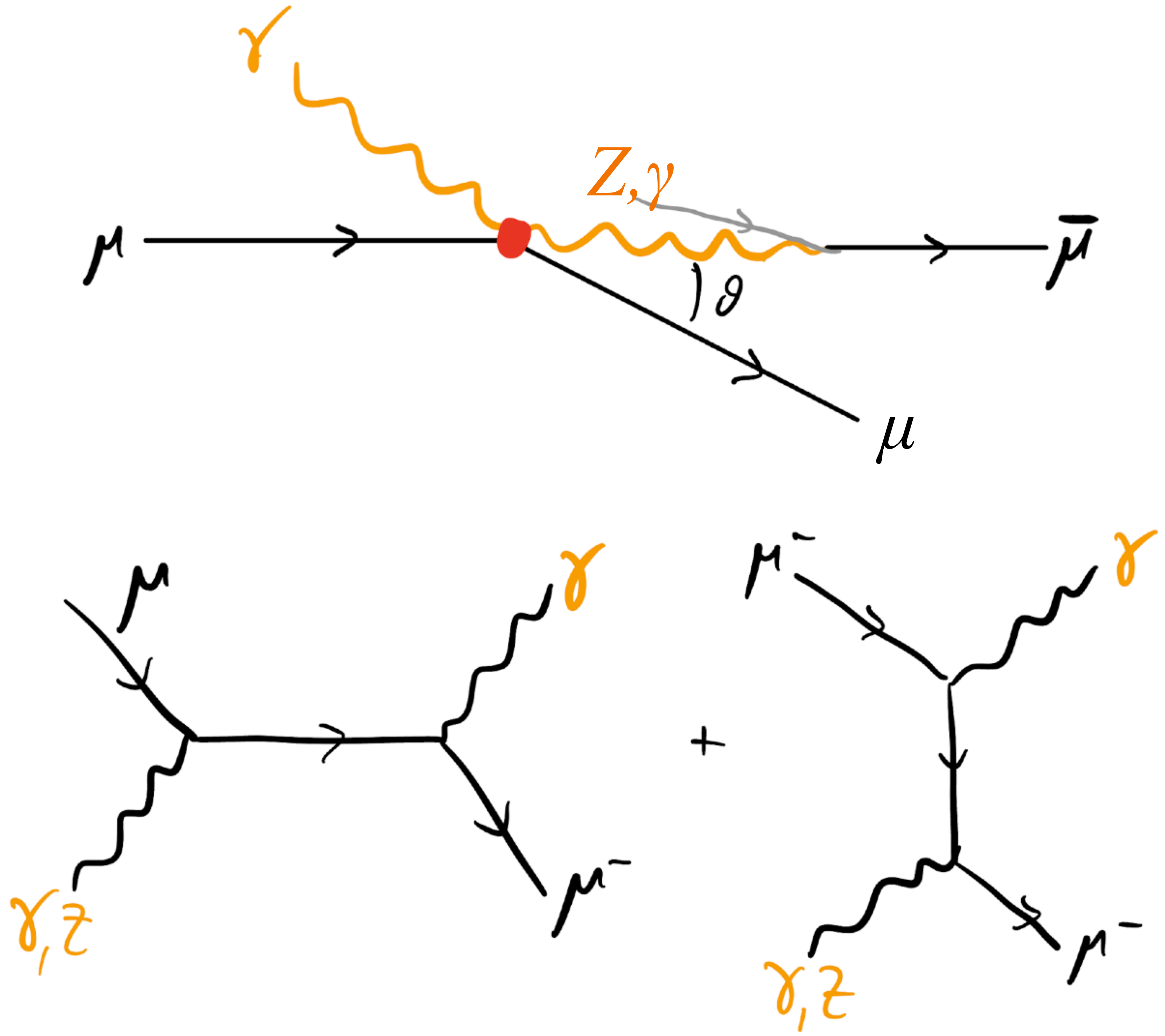
@ 10 TeV Muon Collider

We also include the background from ν_μ W- → $\mu\,\gamma$, its contribution is however marginal.

To what precision could we measure it?

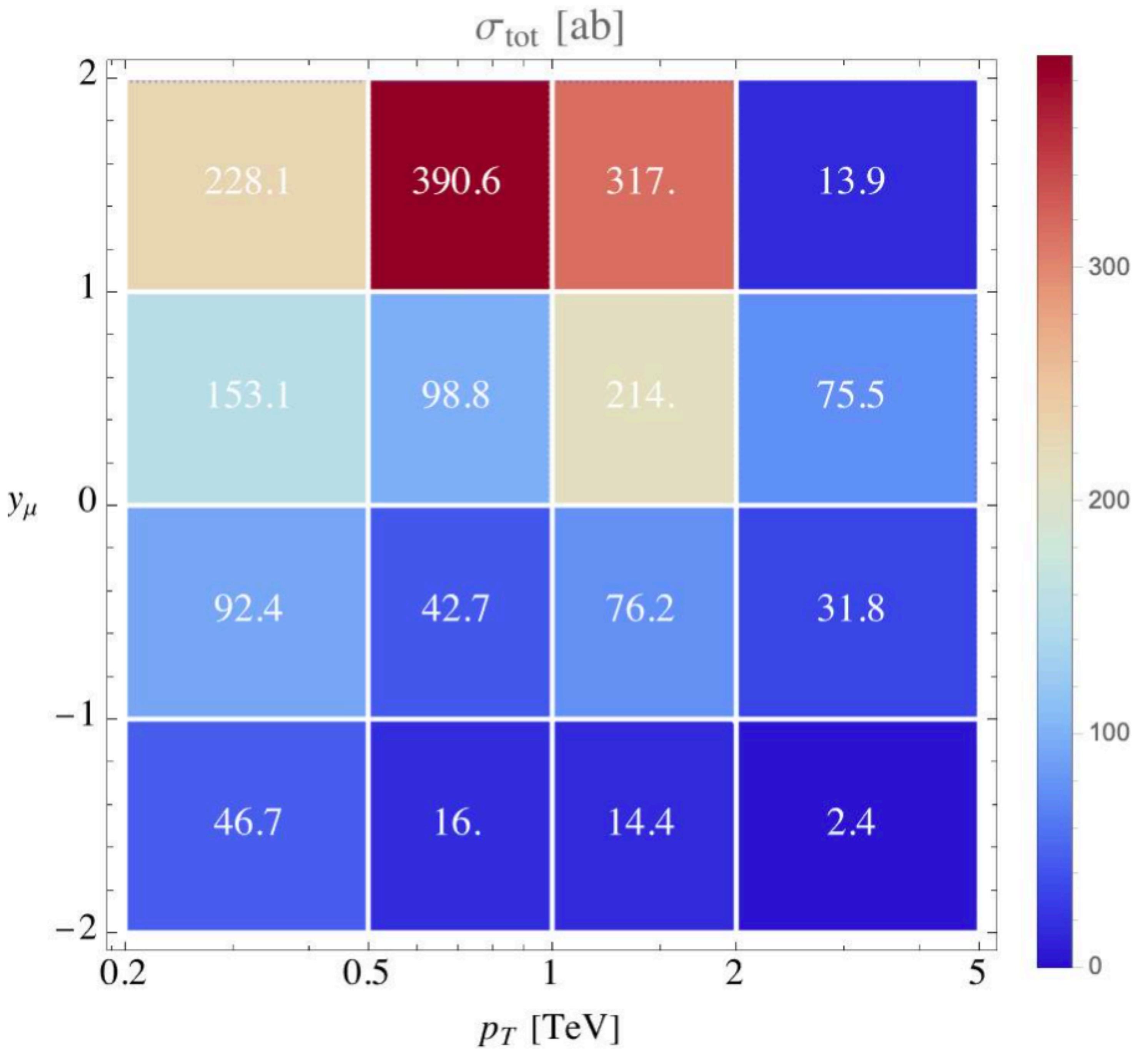
Compton Scattering @ MuC

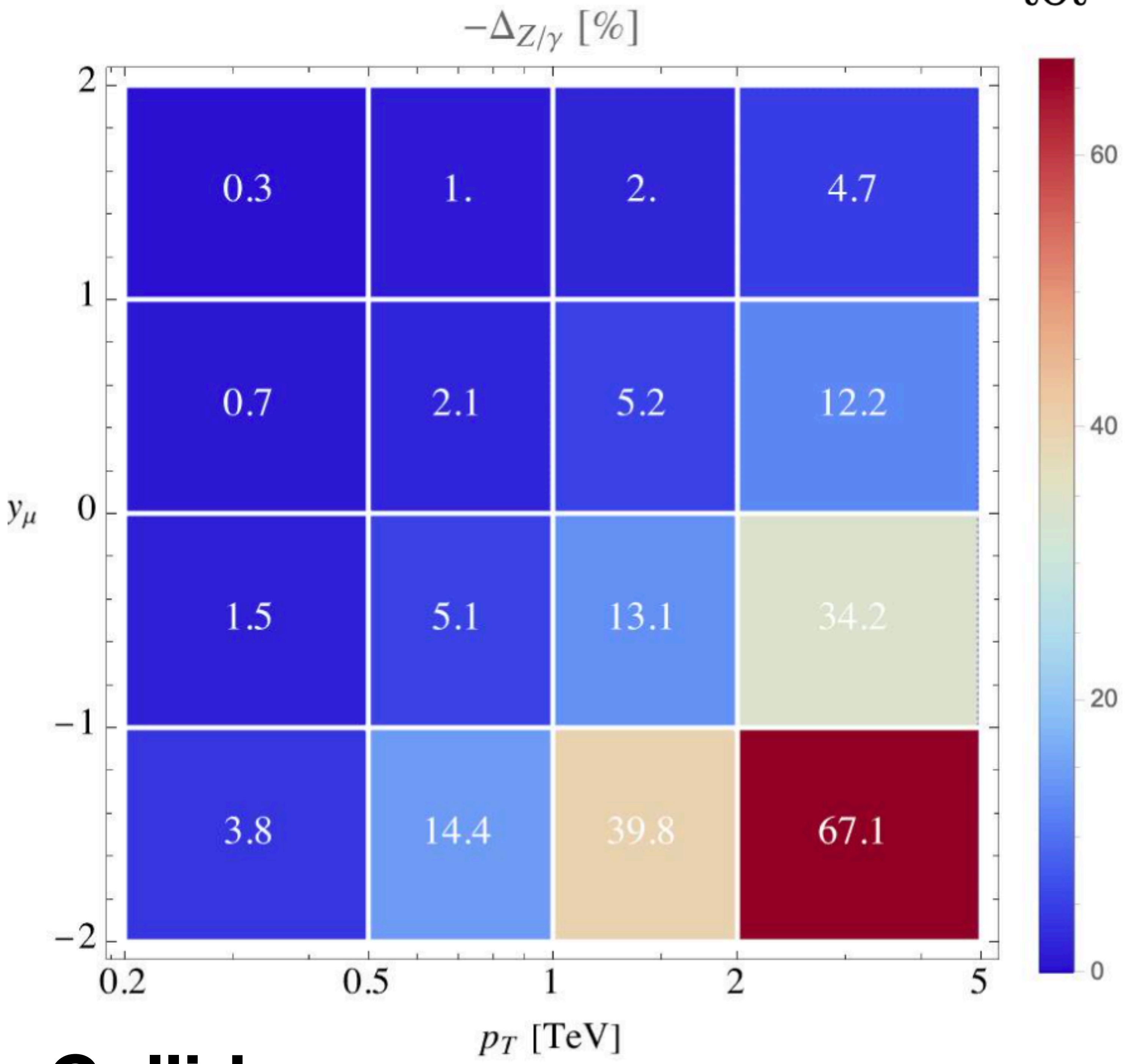
No large new physics effect is expected in this process, since muon couplings to photon and Z boson are well tested. It is thus **perfect to study this EW SM effect**.



Cross section in **bins** of **muon rapidity and p_T**

$$\Delta_{Z/\gamma} \equiv \frac{\sigma_{Z/\gamma}}{\sigma_{\rm tot}}$$





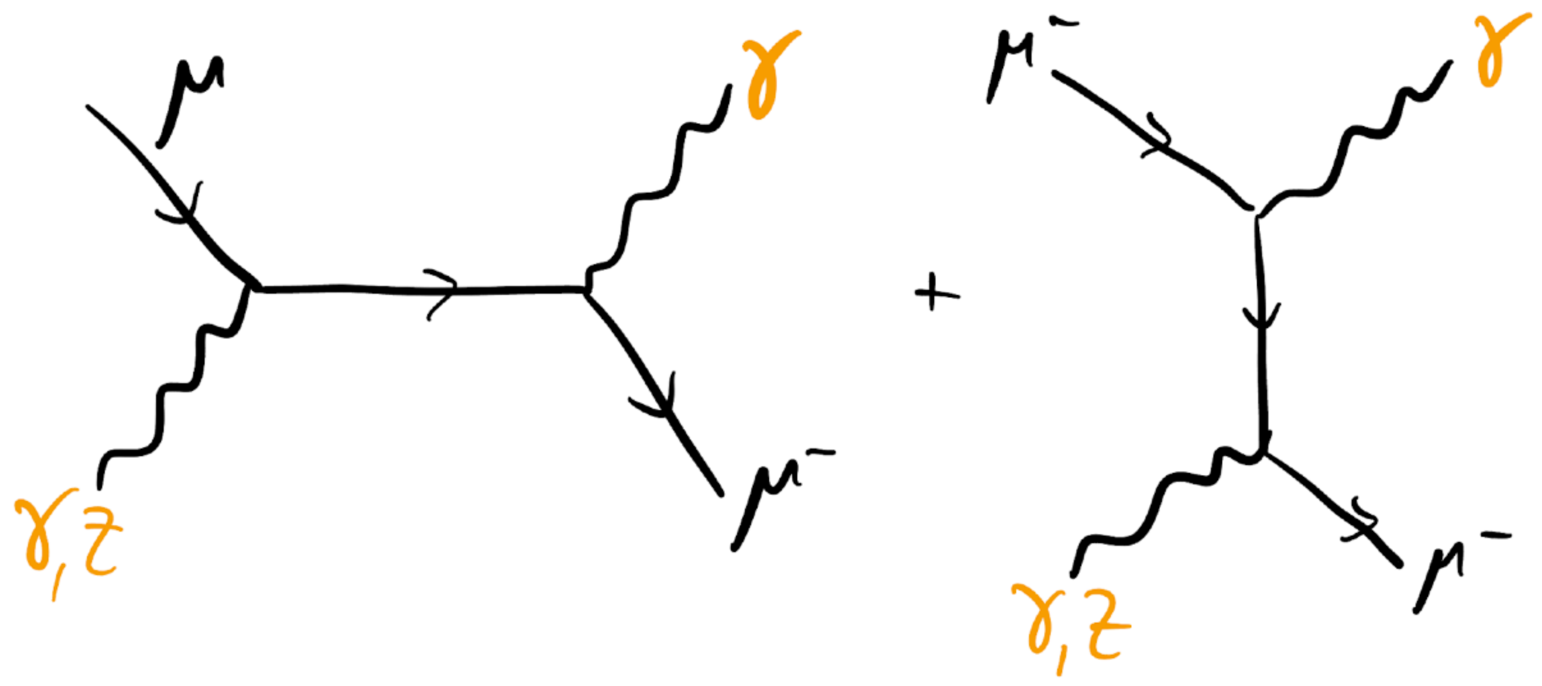
@ 10 TeV Muon Collider

We also include the background from $\nu_\mu W^- \to \mu\,\gamma$, its contribution is however marginal.

The **mixed $Z\gamma$ PDF** can **contribute from few % up to ~ 70%,** depending on the phase space region.

To what precision could we measure it?

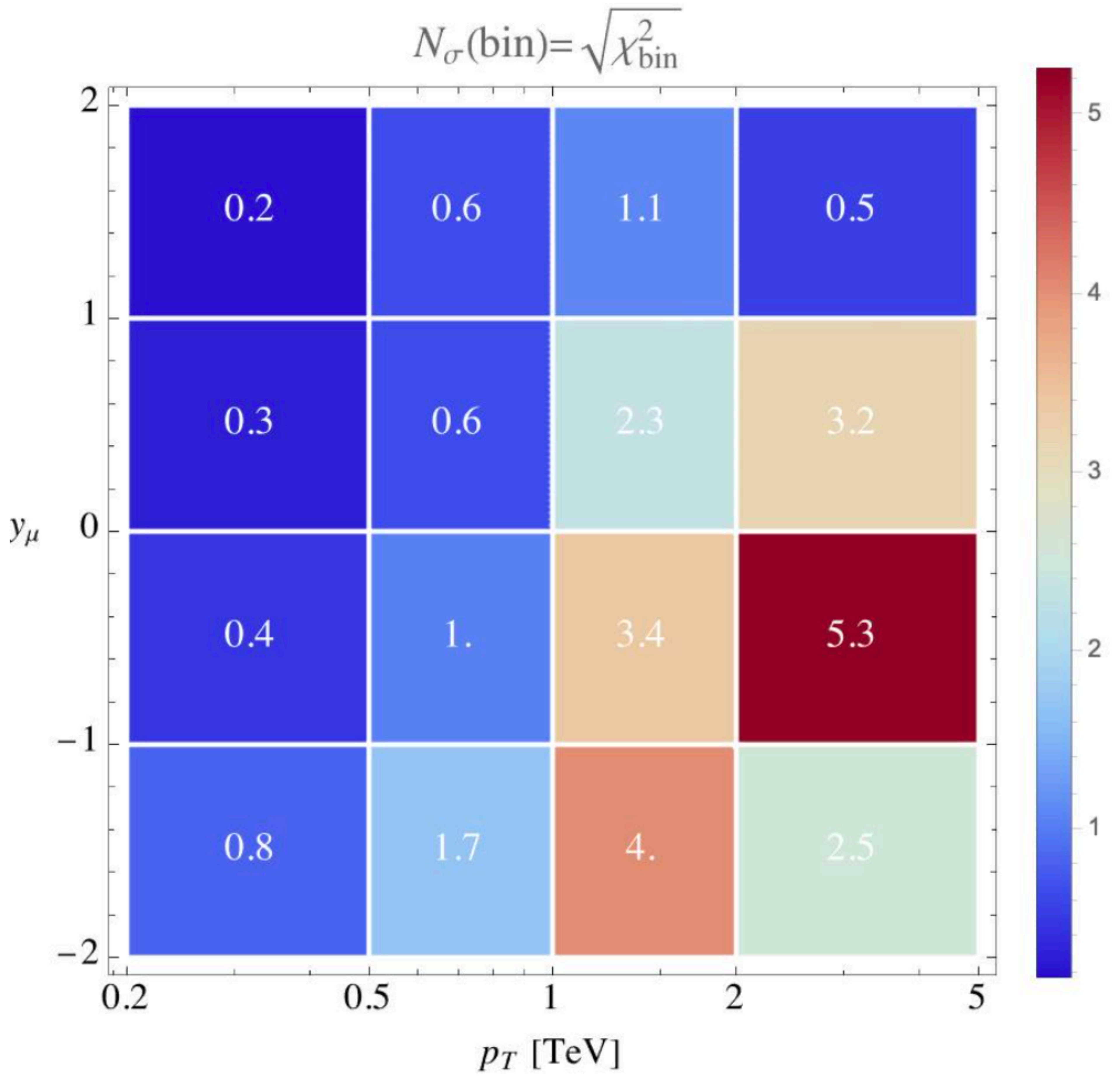
Compton Scattering @ MuC



We estimate the precision with which we can measure this effect, over the null hypothesis that it is zero, with a simple χ^2 test:

$$N_\sigma(\text{bin}) \equiv \sqrt{\chi^2_{\text{bin}}} \approx \left(\mathcal{L}\frac{(\sigma_{\text{tot}} - \hat{\sigma})^2}{\hat{\sigma}}\right)^{1/2} \qquad \text{where} \qquad \hat{\sigma} = \sigma_{\text{tot}} - \sigma_{Z/\gamma}$$

$$\mathcal{L} = 10\,\text{ab}^{-1}$$

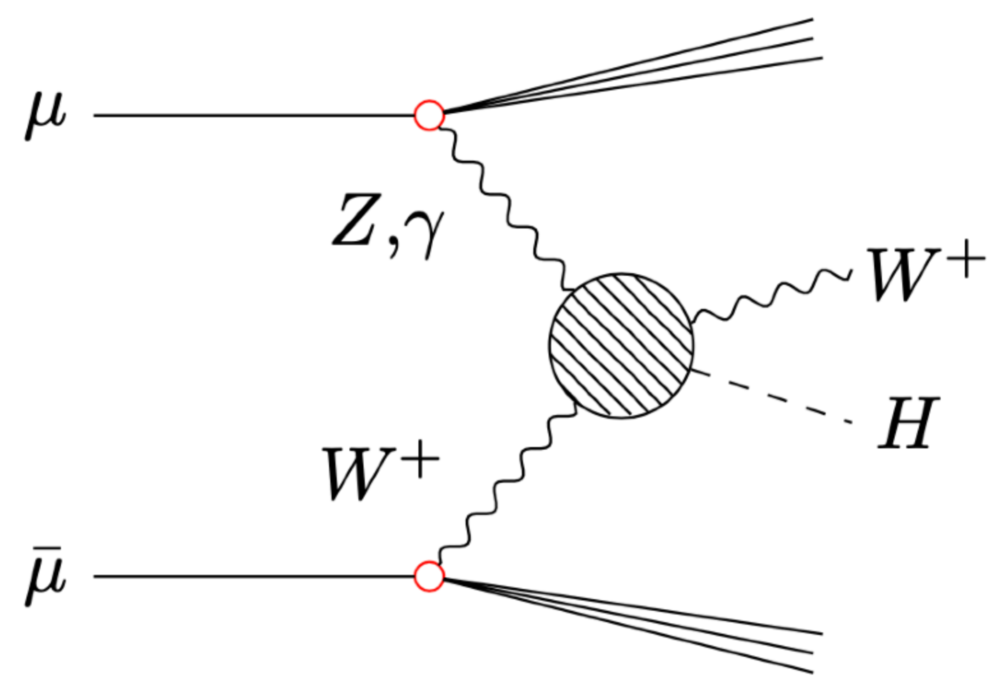


Statistical uncertainties of few % in the most sensitive bins: we neglect systematics.

The effect due to the Z/γ PDF can potentially be observed with more than 5σ precision at a future 10TeV MuC.

Impact in Higgs physics

Consider **associated W H production at a MuC**



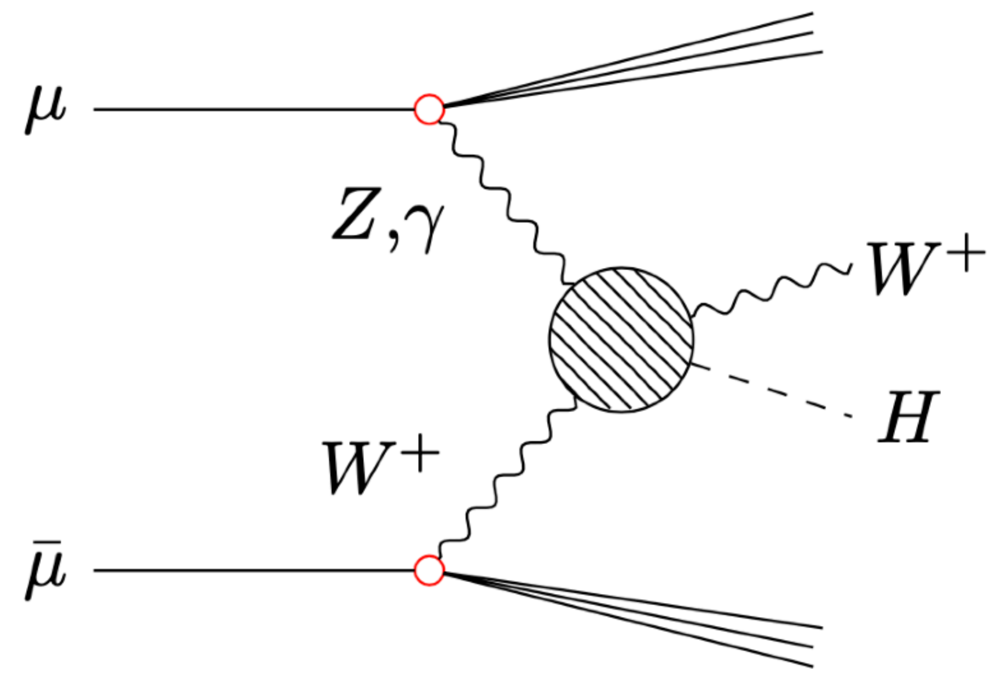
The **mixed Z/γ PDF gives a contribution. How big?**

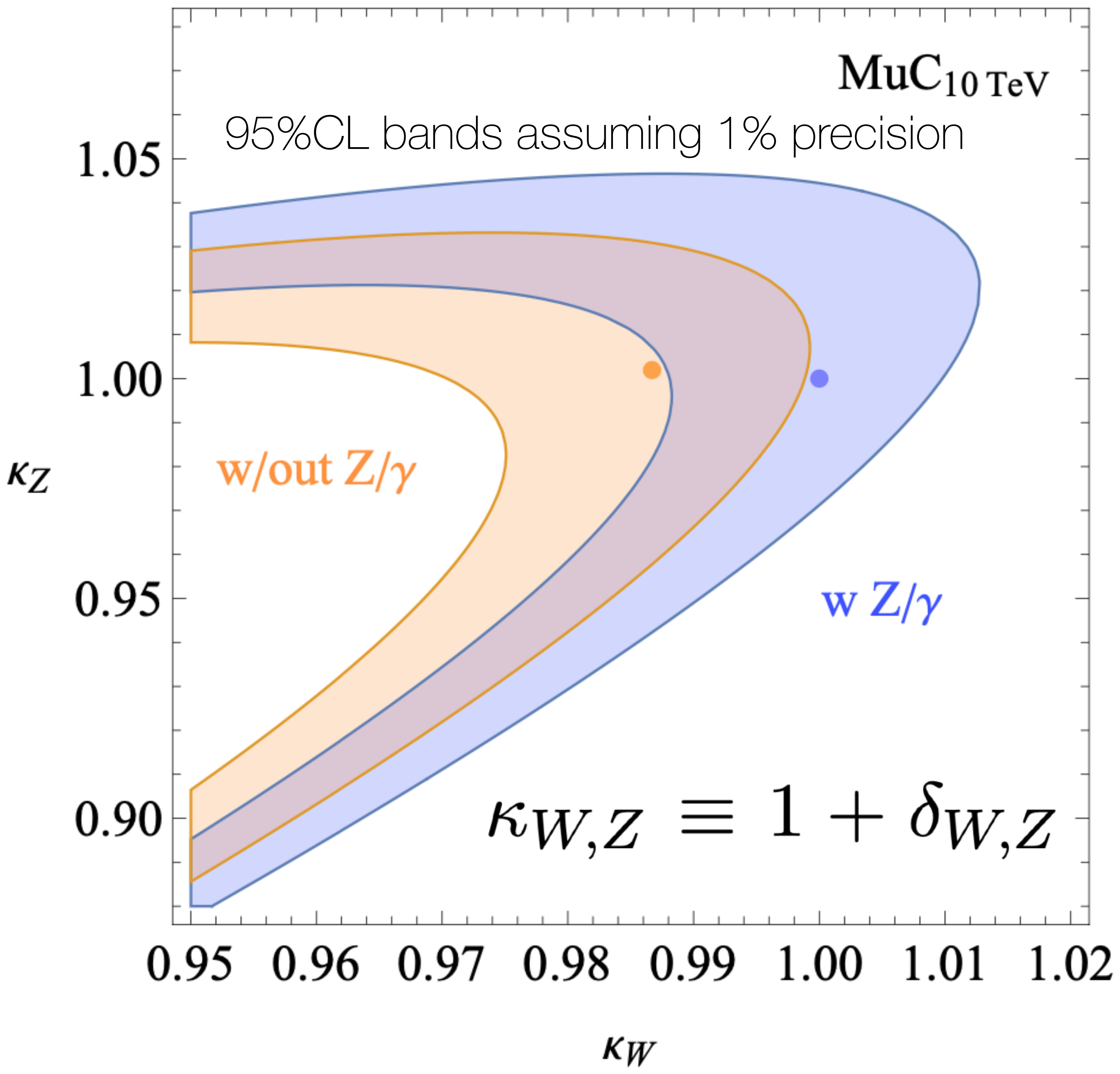
We compute the triple-differential cross section by convoluting the partonic ones with the PDFs.
Then we obtain the **total cross section with cuts**:

$$|y_W| < 2\,, \quad |y_H| < 2\,, \quad m > 0.5\,\text{TeV}$$

Impact in Higgs physics

Consider **associated W H production at a MuC**





The **mixed Z/γ PDF gives a contribution. How big?**

We compute the triple-differential cross section by convoluting the partonic ones with the PDFs.
Then we obtain the **total cross section with cuts**:

$$|y_W| < 2\,, \quad |y_H| < 2\,, \quad m > 0.5\,\text{TeV}$$

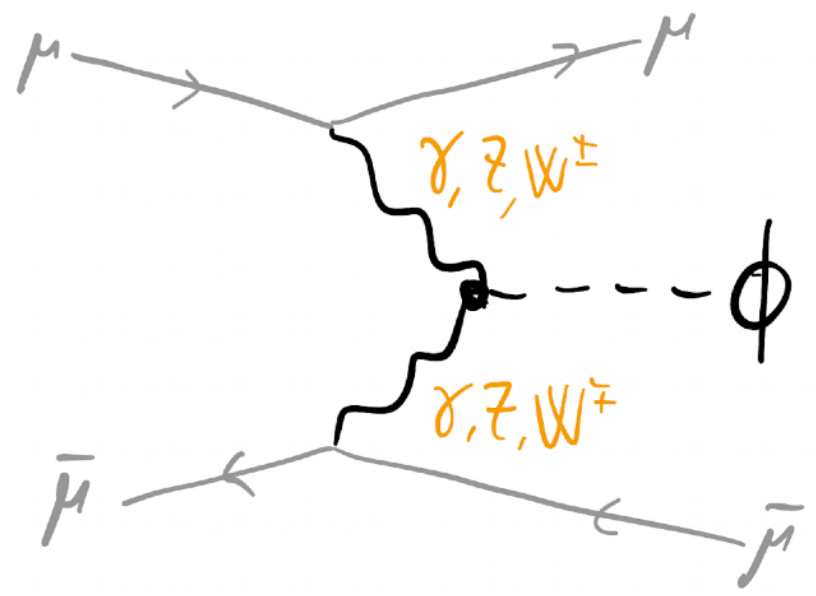
$$\sigma^{10\,\text{TeV}}_{\text{no-}Z/\gamma}\,[\text{fb}] = 6.81 + 15.58\,\delta_W - 1.96\,\delta_Z + 135.7\delta_W^2 - 255.8\delta_W\delta_Z + 126.9\delta_Z^2,$$
$$\sigma^{10\,\text{TeV}}_{\text{tot}}\,[\text{fb}] = 6.63 + 15.25\,\delta_W - 1.99\,\delta_Z + 135.6\delta_W^2 - 255.9\delta_W\delta_Z + 126.9\delta_Z^2,$$

It **modifies the SM cross section by 3%**, to be compared with an expected precision in this channel of about 1% (value used in the plot).

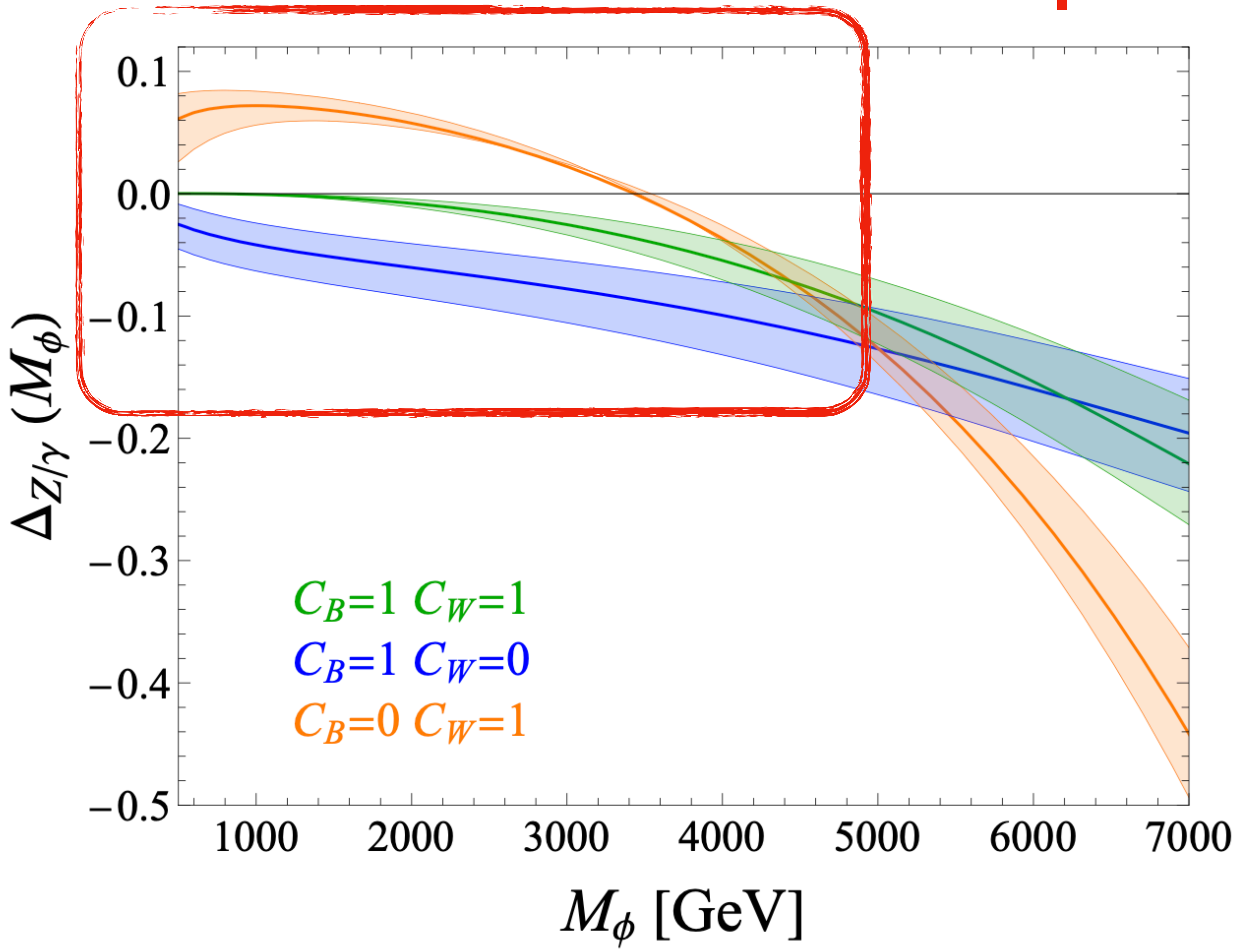
$$\sigma^{10\,\text{TeV}}_{\text{no-}Z/\gamma}\,[\text{fb}] = 135.70\,\kappa_W^2 + 126.93\,\kappa_Z^2 - 255.82\,\kappa_W\kappa_Z$$
$$\delta\sigma^{10\,\text{TeV}}_{Z/\gamma}\,[\text{fb}] = -0.15\,\kappa_W^2 - 0.030\,\kappa_W\kappa_Z\,,$$

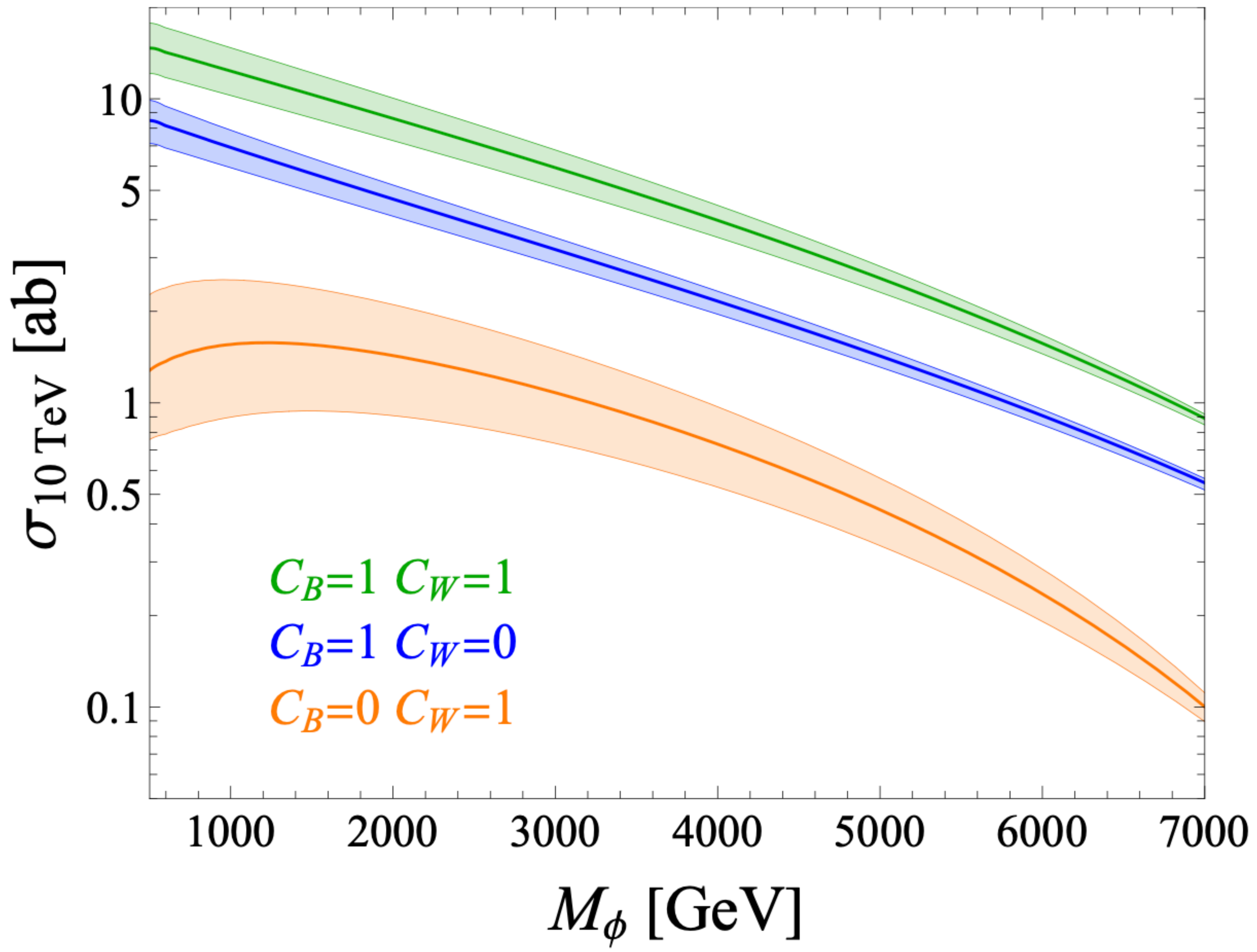
Single- ALP production @ MuC

$$\mathcal{L}_{\phi VV} = \frac{C_W}{\Lambda}\phi W^a_{\mu\nu}\widetilde{W}^{\mu\nu,a} + \frac{C_B}{\Lambda}\phi B_{\mu\nu}\widetilde{B}^{\mu\nu}$$



This ALP can be produced at muon colliders by (transverse) vector boson fusion.
What is the **impact** of the **mixed** $Z\gamma$ **PDF**?

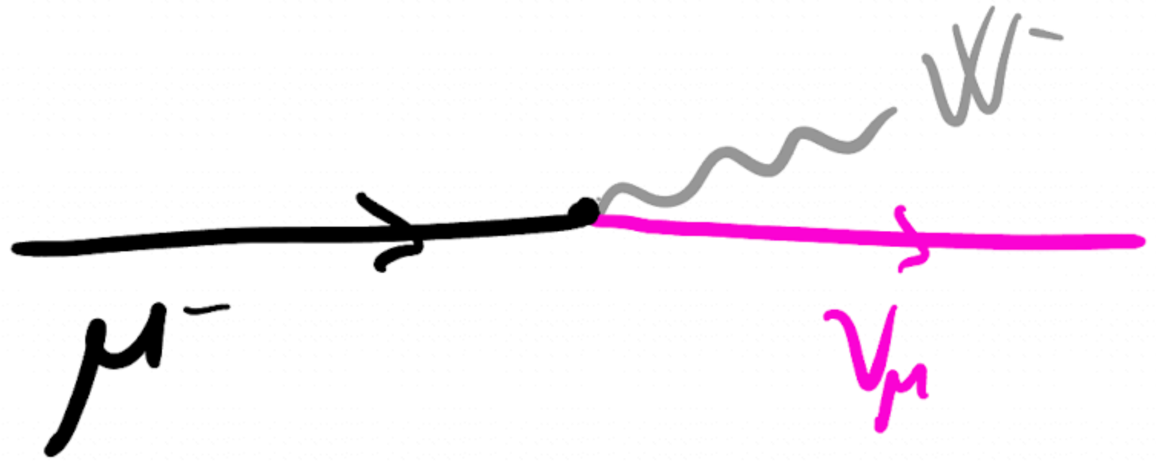




~ 10% effect in the interesting mass region!

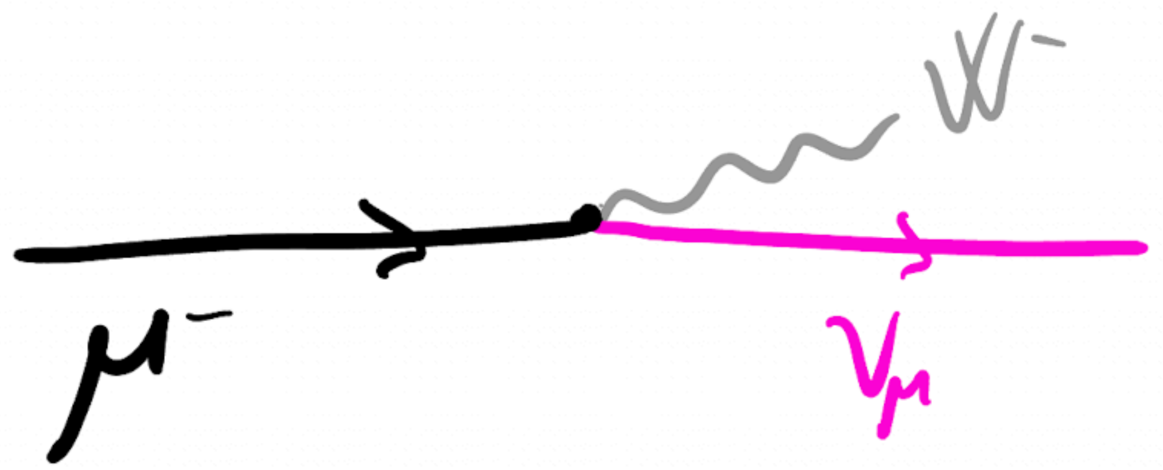
Pheno of EW PDF effects (2) Muon neutrino PDF

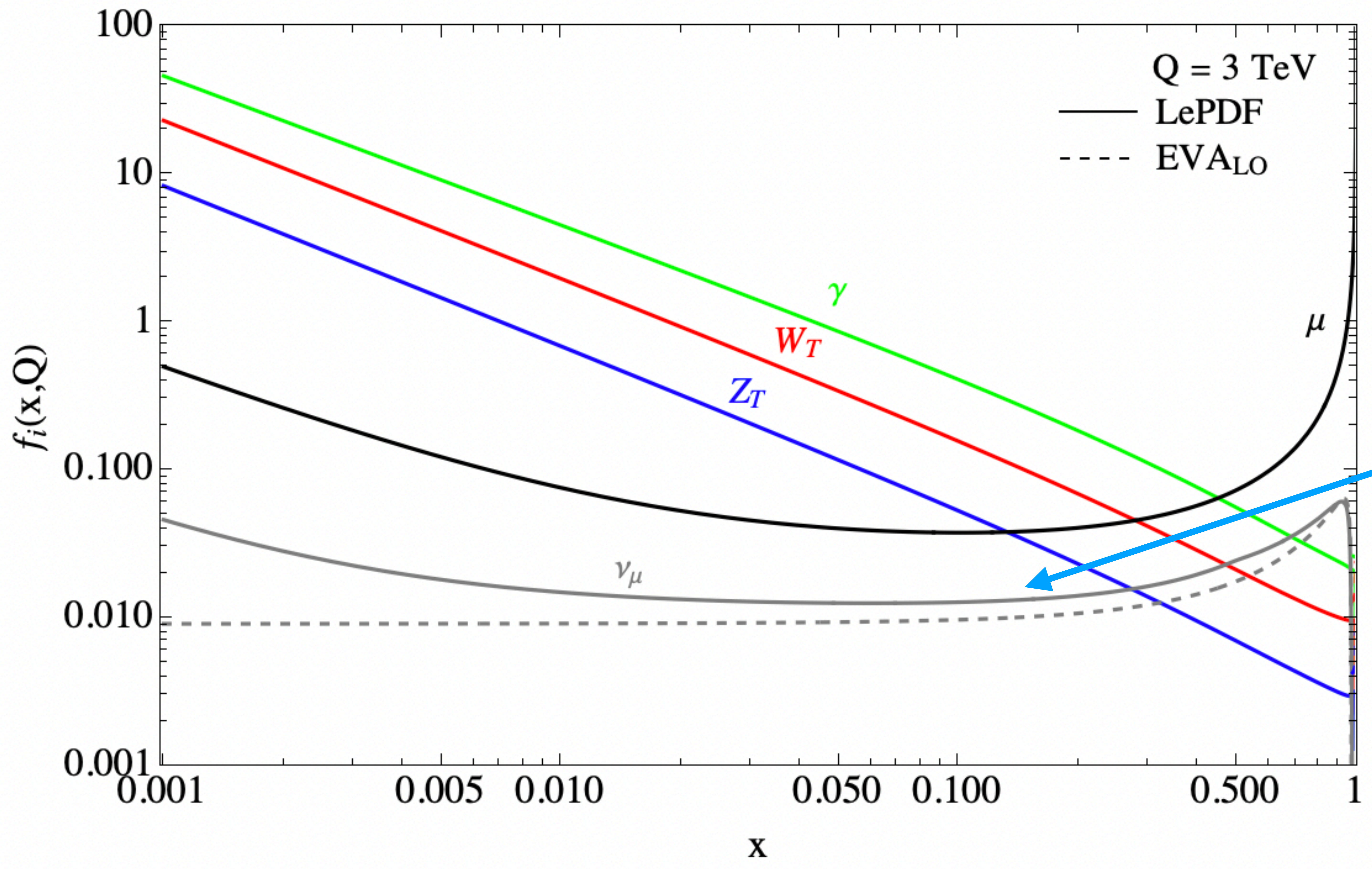
[work in progress: F. Garosi, R. Capdevilla, D.M. and B. Stechauner]



Muon Neutrino PDF

Emission of **collinear W^- from the muon** generates a **muon neutrino content inside of the muon**.



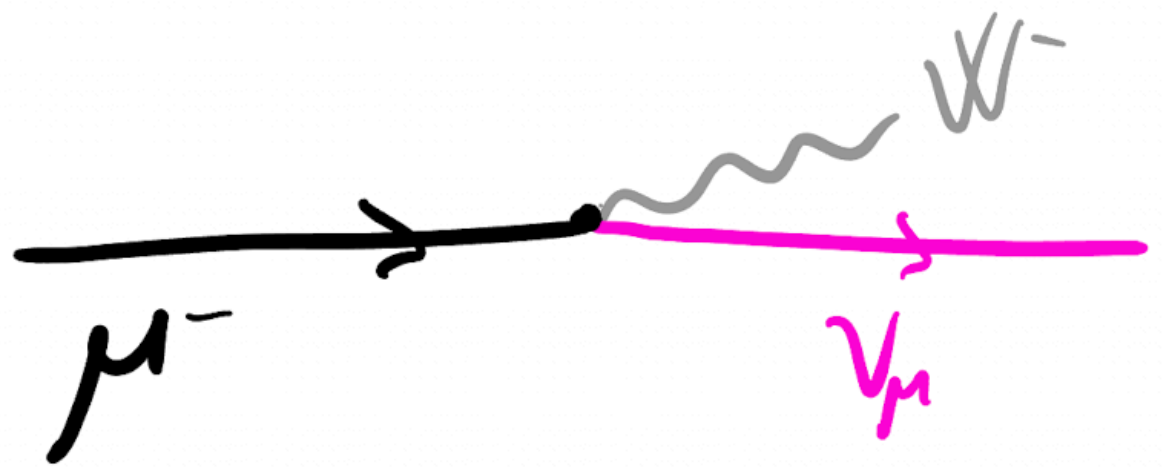


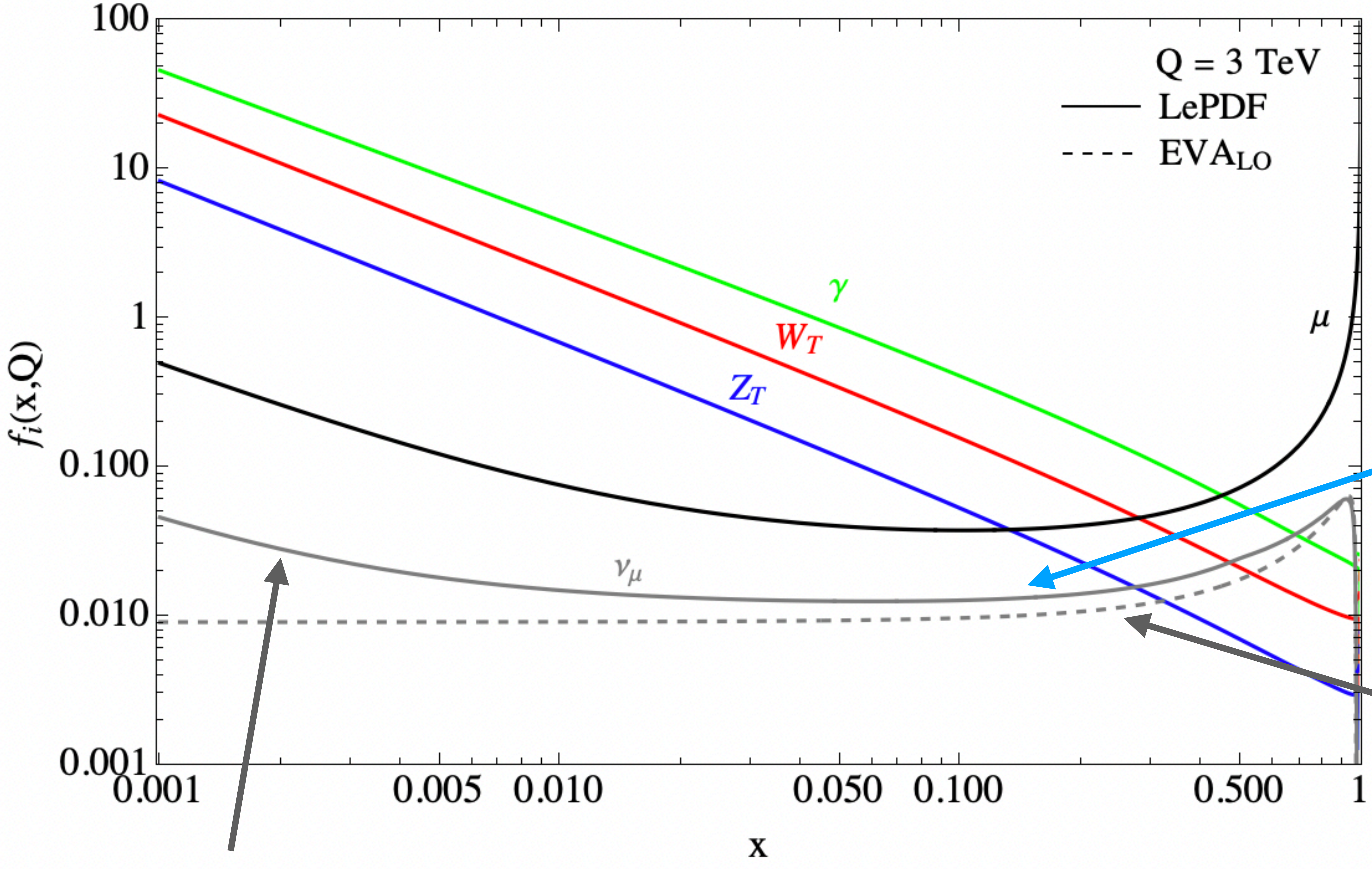
Particularly **large at x ≳ 0.3** due to the IR divergence of the $\mu \rightarrow W\,\nu_\mu$ splitting

Muon Neutrino PDF from LePDF

Muon Neutrino PDF

Emission of **collinear W^- from the muon** generates a **muon neutrino content inside of the muon**.





Particularly **large at x ≳ 0.3** due to the IR divergence of the $\mu \to W\, \nu_\mu$ splitting

Muon Neutrino PDF from LePDF

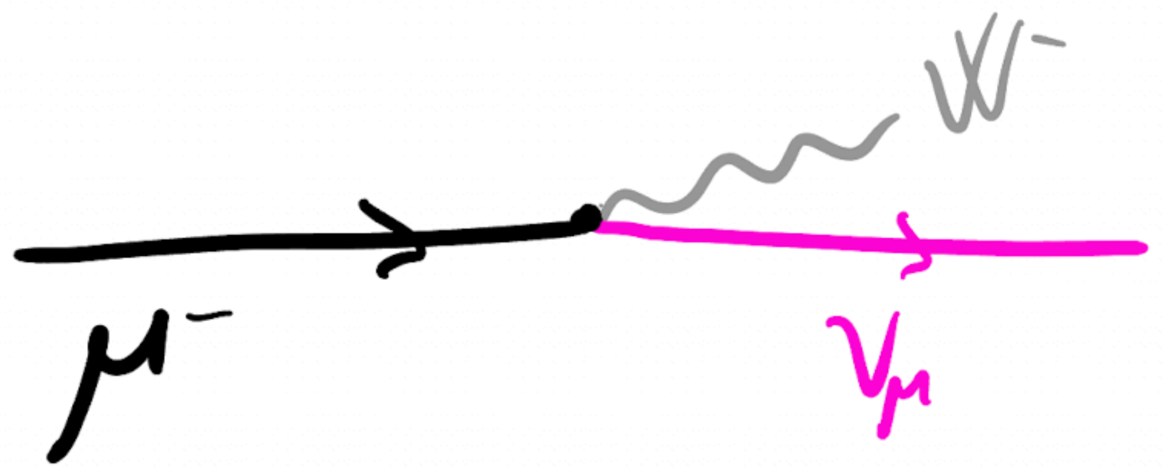
We can compute the **ν_μ PDF at $O(\alpha)$** (as for EVA)

$$f^{(\alpha)}_{\nu_\mu}(x,Q^2) = \frac{\alpha_2(Q)}{8\pi}\,\theta\!\left(Q^2 - \frac{m_W^2}{(1-x)^2}\right)\left[\frac{1+x^2}{1-x}\left(\log\frac{Q^2+xm_W^2}{m_W^2} + \log\frac{(1-x)^2}{1+x(1-x)^2} + \frac{xm_W^2}{Q^2+xm_W^2} + \frac{1}{1+x(1-x)^2} - 1\right) + \frac{2x^2(1-x)^2}{(1-x)(1+x(1-x)^2)}\frac{Q^2-m_W^2}{Q^2+xm_W^2}\right],$$

Here $Z \to \overline{\nu}_\mu\, \nu_\mu$ dominates: $O(\alpha^2)$

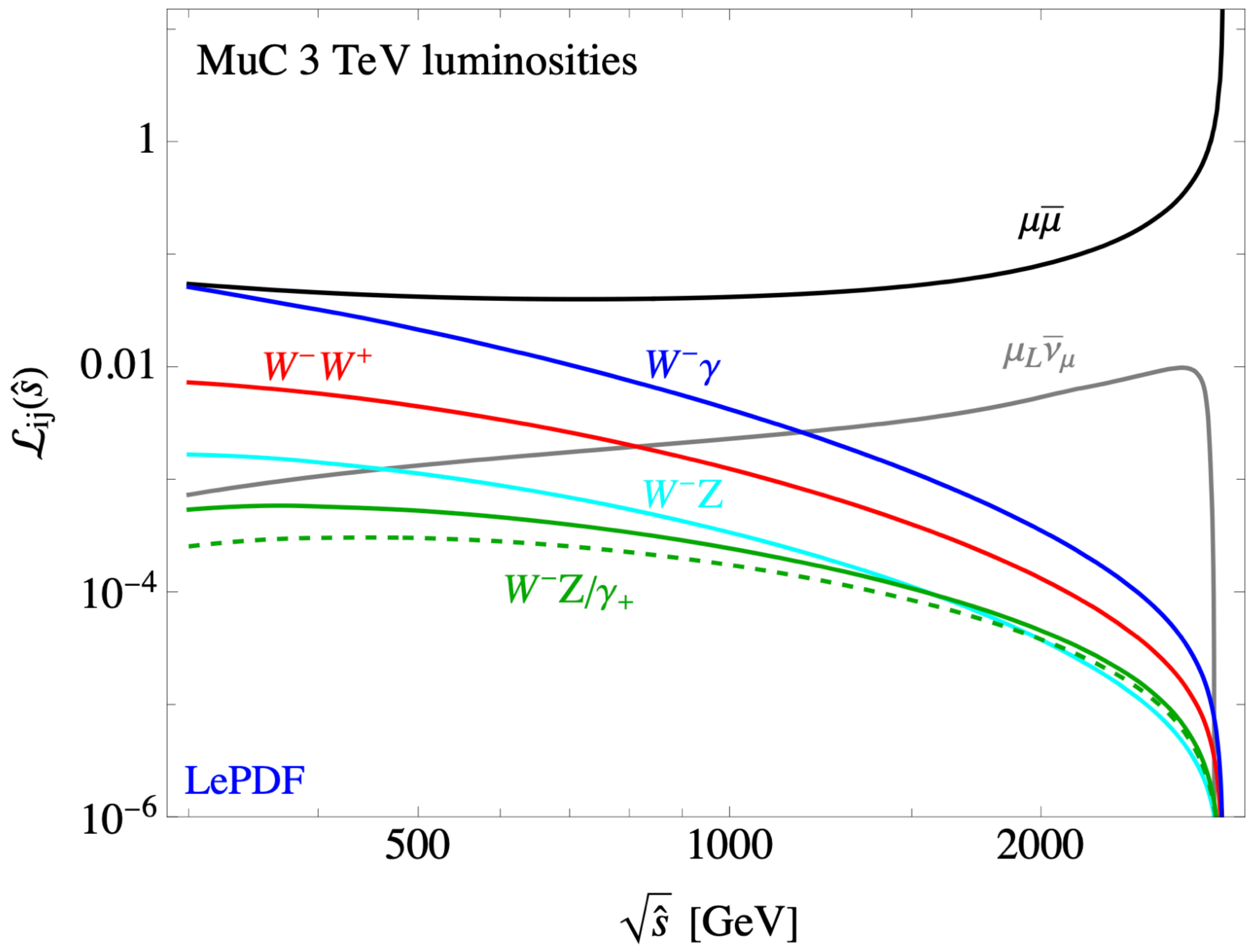
Muon Neutrino PDF

Emission of **collinear W^- from the muon** generates a **muon neutrino content inside of the muon**.

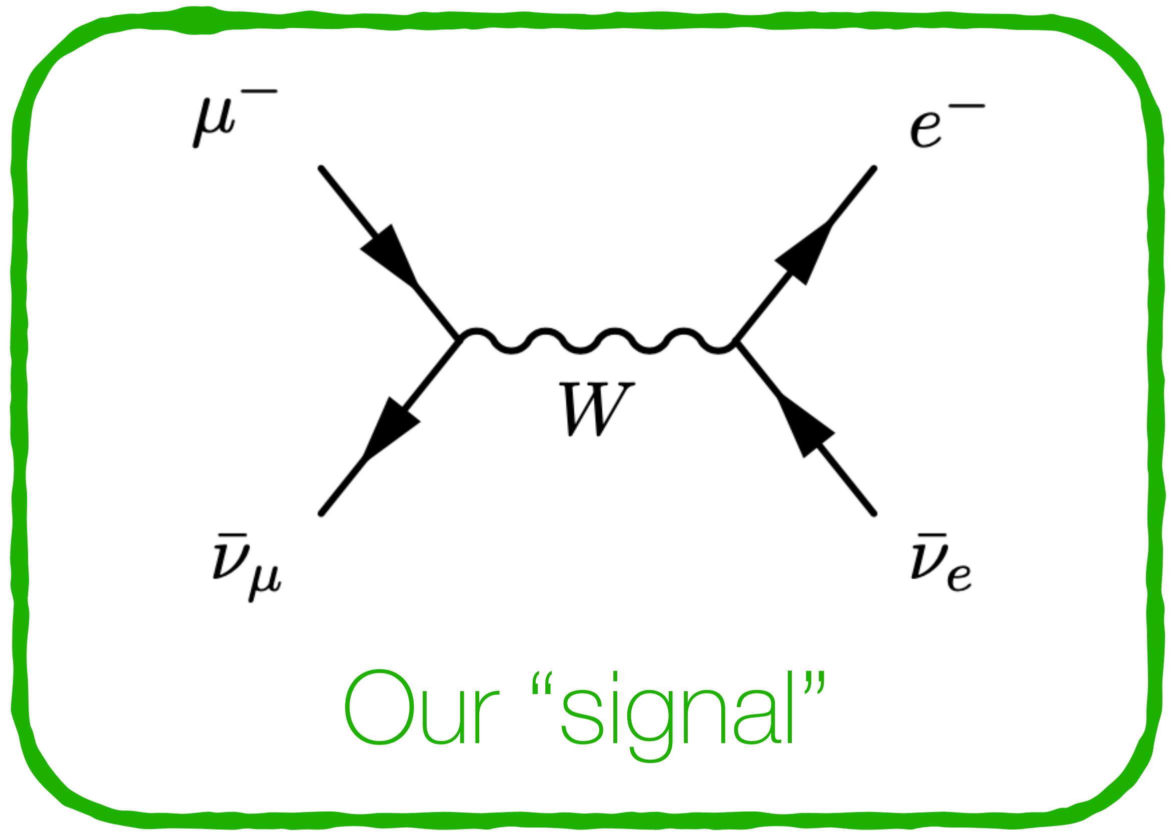


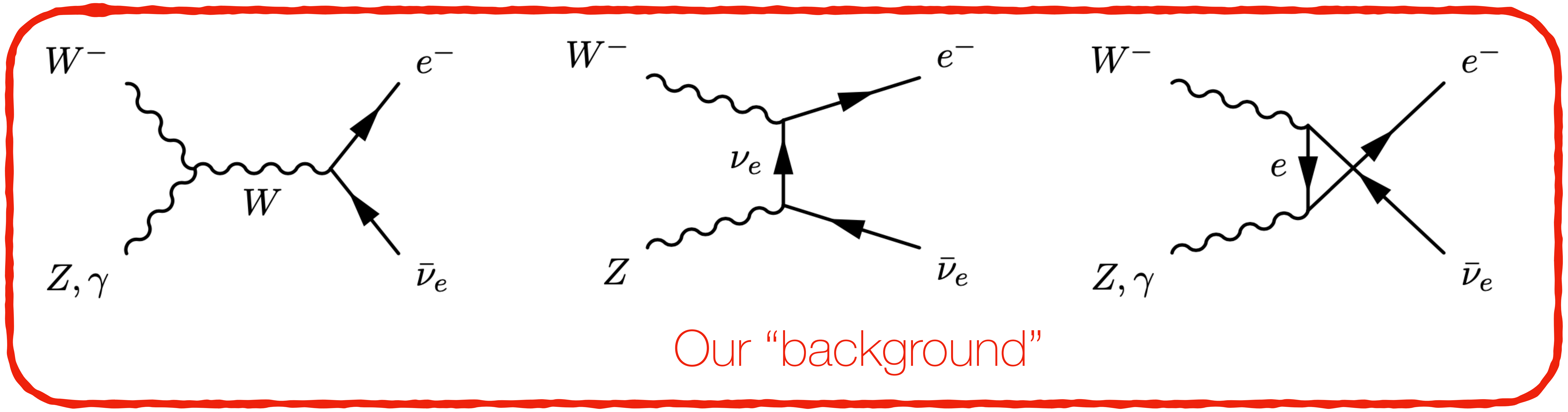
Also in terms of parton luminosities, it is clear that **the contribution from the neutrino PDF will be important in the high-energy tail of EW processes**.

$$\mathcal{L}_{ij}(\hat{s}, s_0) = \int_0^1 \frac{dz}{z} f_{i;\mu}\left(z, \frac{\hat{s}}{4}\right) f_{j;\bar{\mu}}\left(\frac{\hat{s}}{z s_0}, \frac{\hat{s}}{4}\right)$$

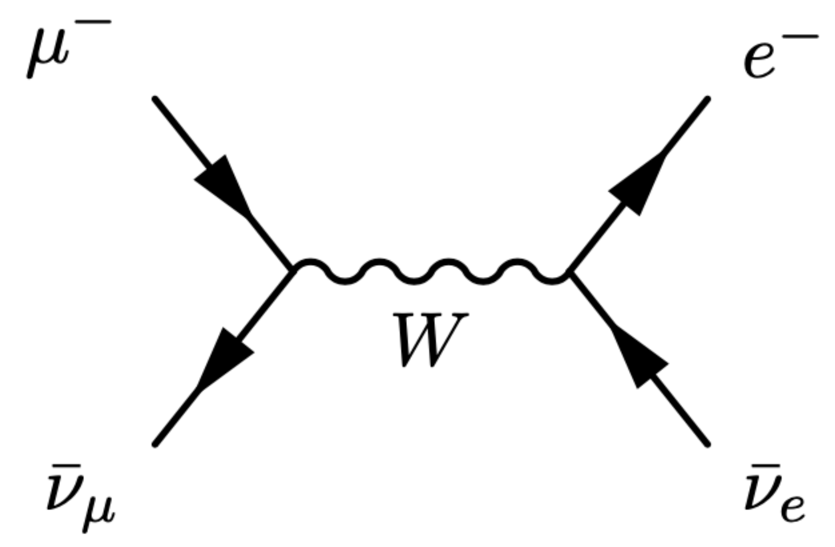


Observing f_{ν_μ} in $e^- \nu_e$ production

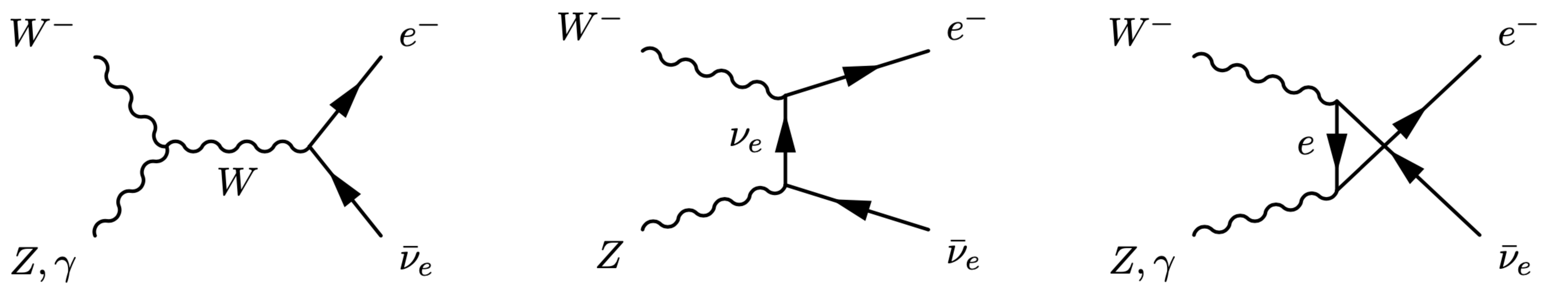




Observing f_{ν_μ} in $e^- \nu_e$ production



Our “signal”



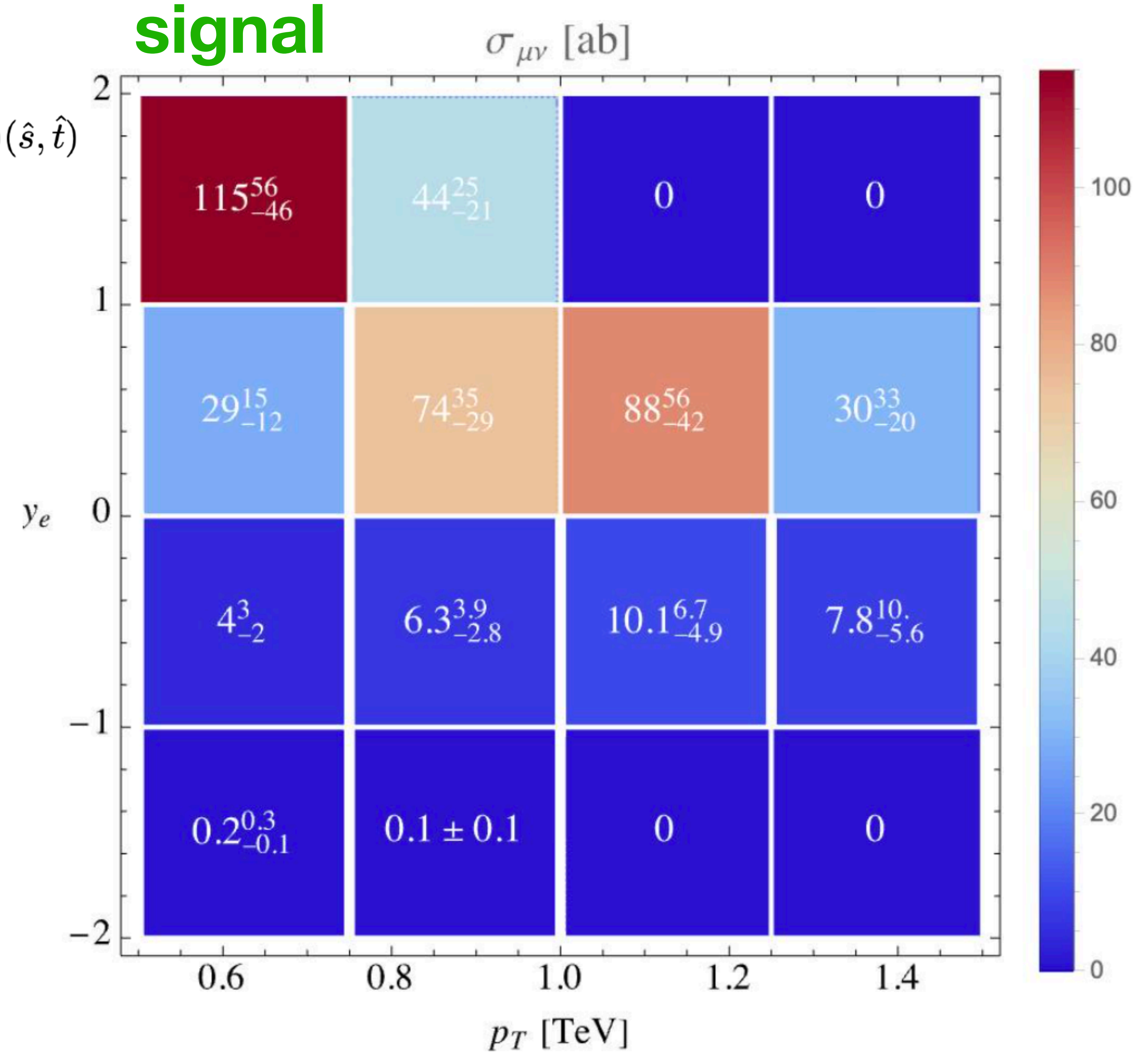
Our “background”

$$\frac{d^3\sigma(\mu\bar{\mu} \to e^-\bar{\nu}_e + X)}{dy_e dy_\nu dp_T} = \sum_{i,j} f_i^\mu(x_1) f_j^{\bar{\mu}}(x_2) \left(\frac{2p_T\hat{s}}{s_0}\right) \frac{d\hat{\sigma}}{d\hat{t}}(ij \to e^-\bar{\nu}_e)(\hat{s},\hat{t})$$

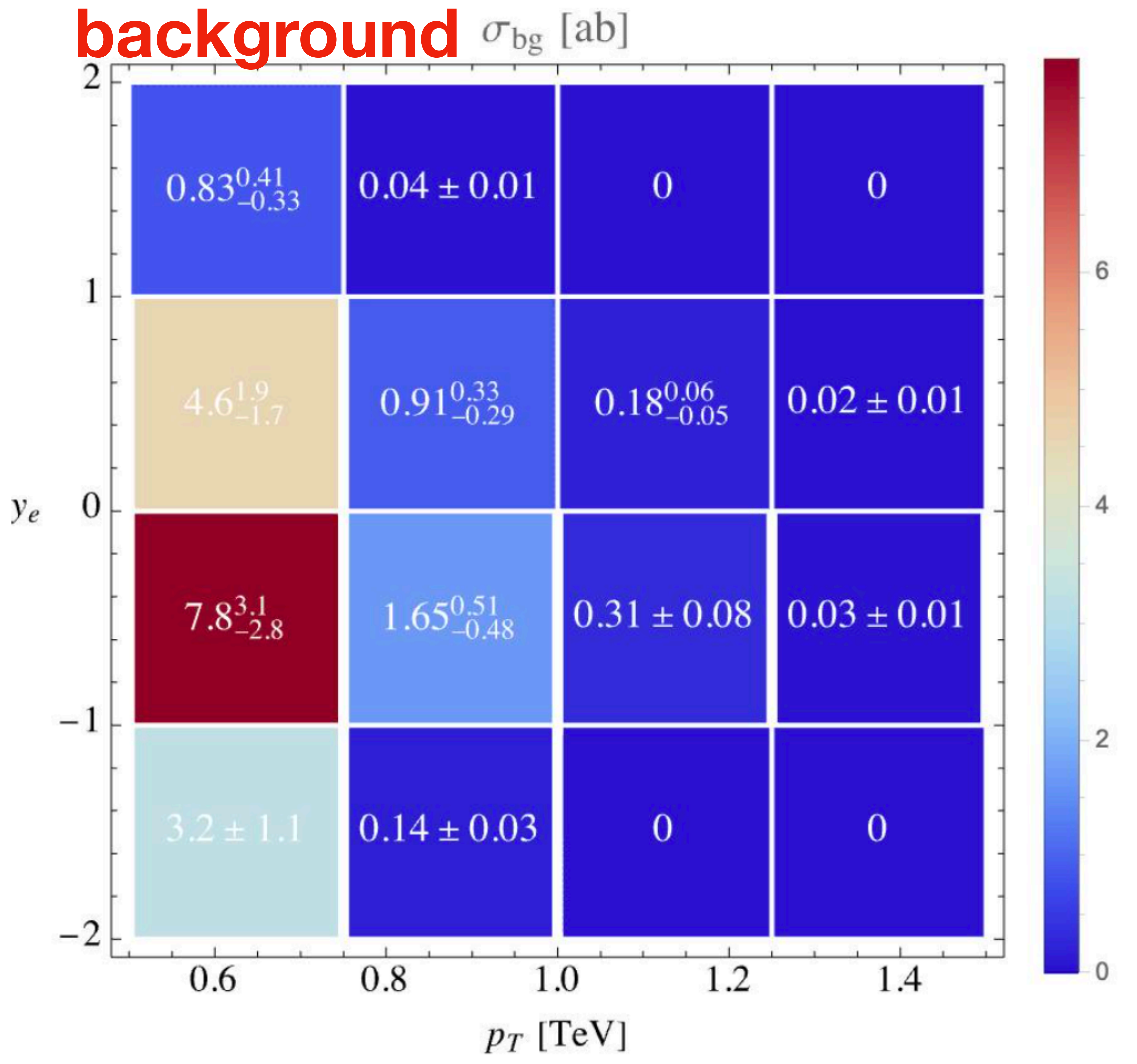
We compute **xsec in bins of electron rapidity and p_T**, for both signal and background:

Uncertainties are obtained by varying the factorisation scale Q between a factor of 1/2 and 2 around Q = p_T.

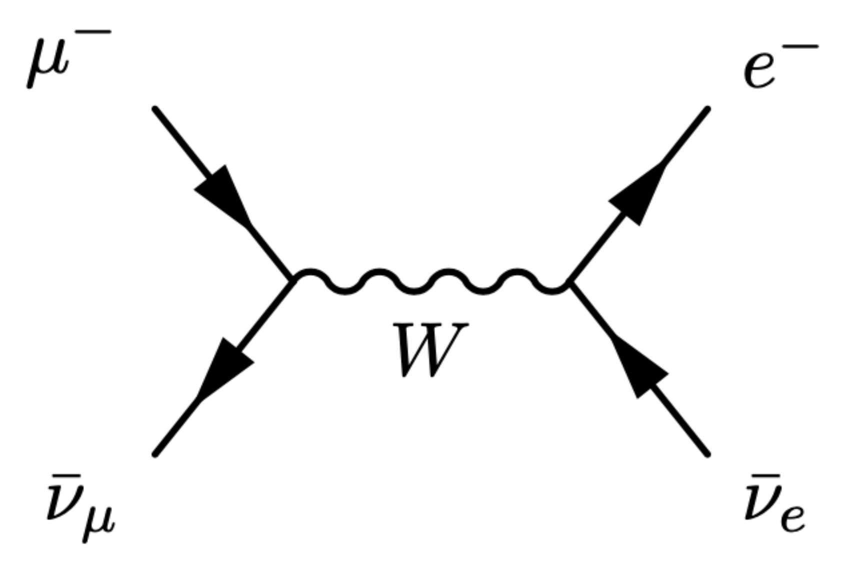
signal



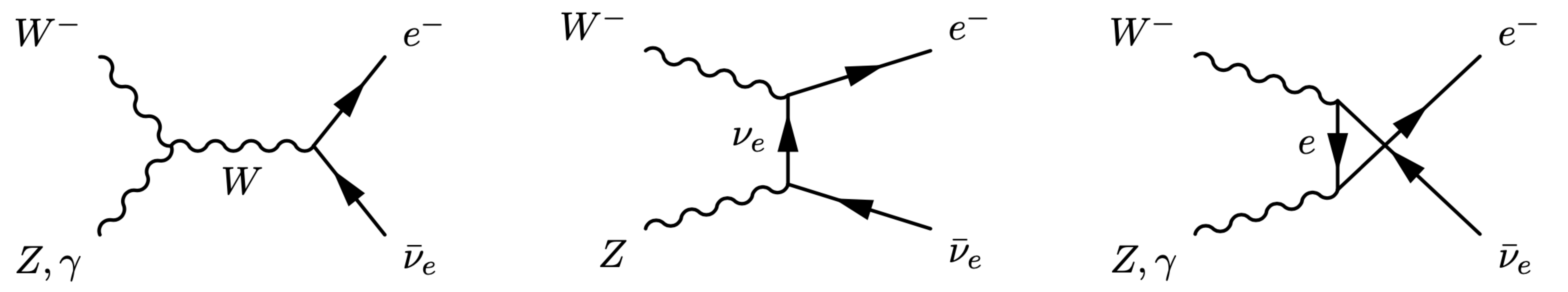
background



Observing f_{ν_μ} in $e^- \nu_e$ production



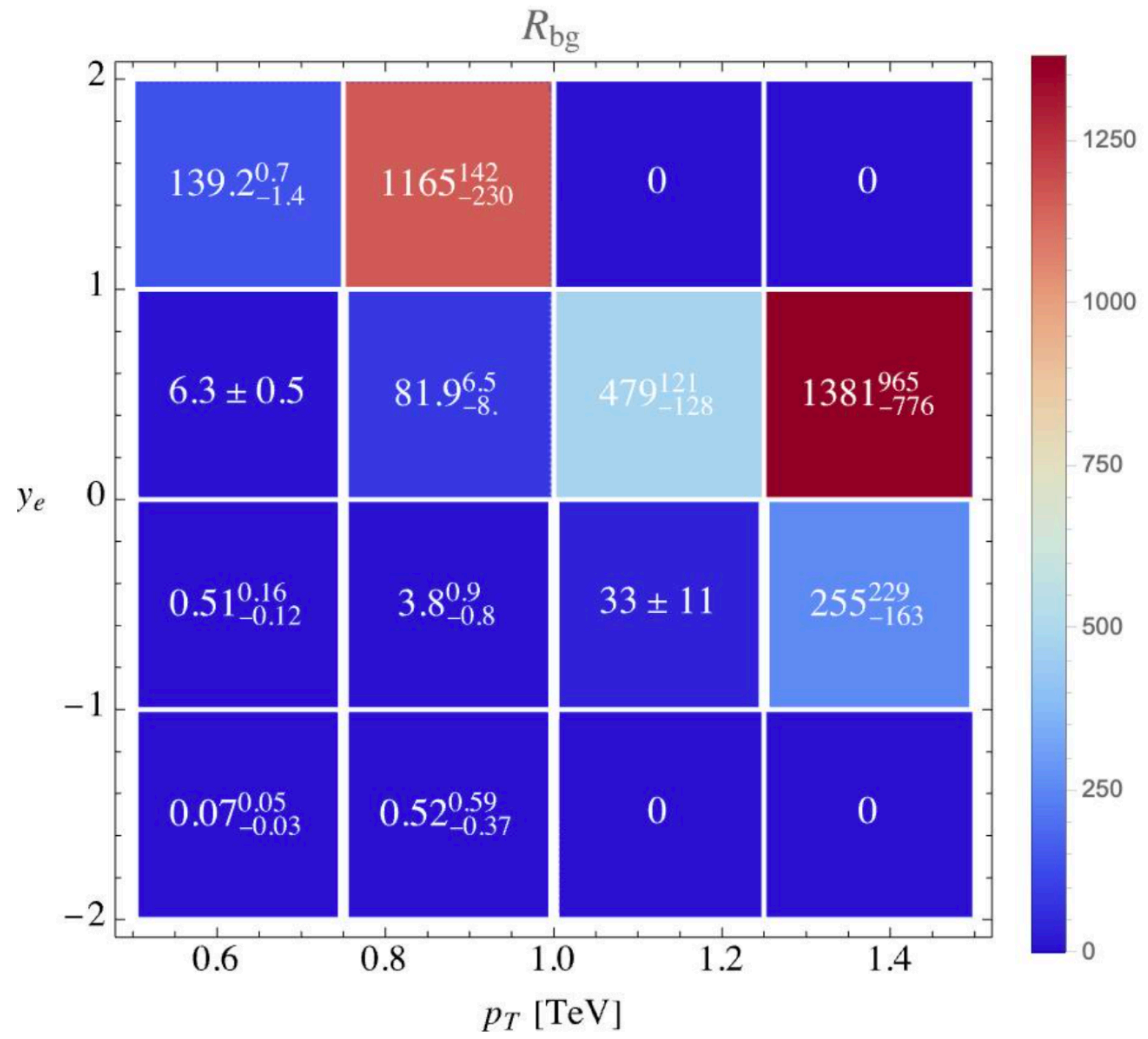
Our “signal”



Our “background”

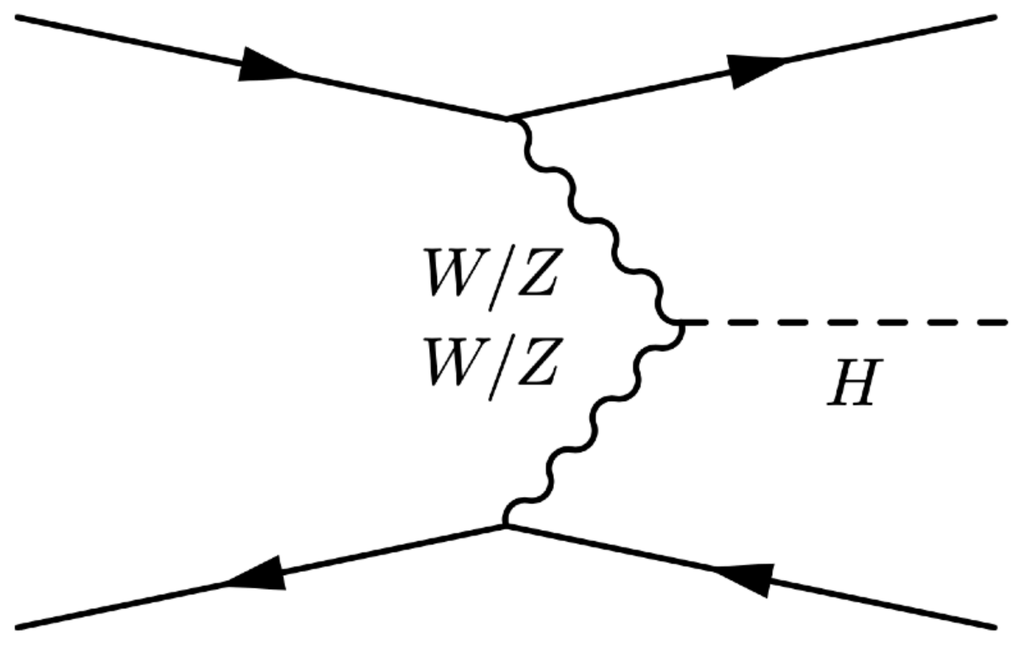
We define the **signal/background** ratio:

$$R^{e\nu}_{\text{bg}} = \frac{\sigma(\mu^- \bar{\nu}_\mu \to e \bar{\nu}_e)}{\sigma^{e\nu}_{\text{bg}}}$$



Clearly, the **contribution** from **neutrino PDF is very large and dominates** for **forward electrons** and increases with p_T.

Contribution in single-Higgs production

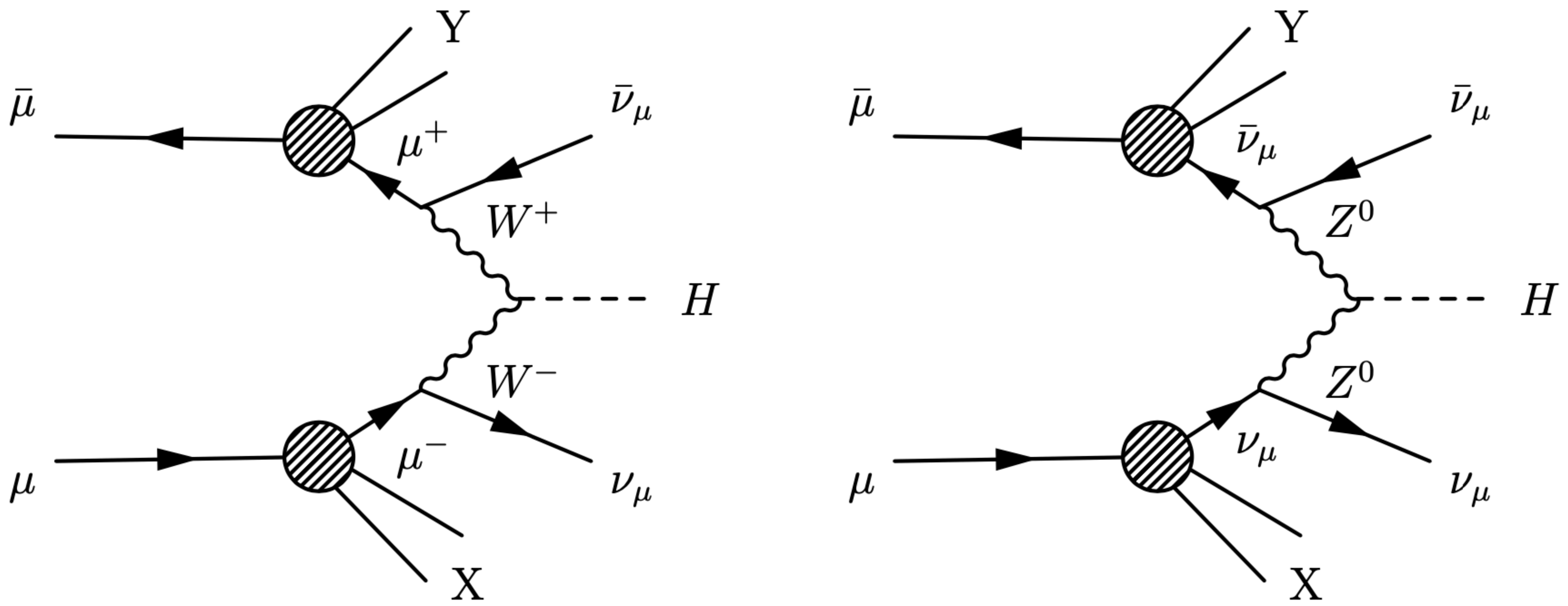


Single-H production at MuC is typically computed from initial $\mu^- \mu^+$ without ISR.

We consider 2 effects:

1) muons (and anti-muons) have their own PDF after emitting collinear radiation
2) New contributions arise from ν_μ (and $\overline{\nu}_\mu$) in the initial state

For example:



Contribution in single-Higgs production

We obtain:

Partonic process	Channel	$\sigma(3\,\text{TeV})$ [fb]	$\sigma(10\,\text{TeV})$ [fb]
$\mu^-\mu^+ \to \nu_\mu \bar{\nu}_\mu H$	CC	$480.3^{+0.8}_{-0.7}$	$820.9^{+0.6}_{-0.2}$
$\mu^-\mu^+ \to \mu^-\mu^+ H$	NC	$47.7^{+0.6}_{-0.8}$	$80.5^{+1.5}_{-1.7}$
$\mu^-\bar{\nu}_\mu \to \mu^-\bar{\nu}_\mu H$	NC	$2.4^{+1.6}_{-1.2}$	10^{+4}_{-3}
$\nu_\mu\bar{\nu}_\mu \to \mu^-\mu^+ H$	CC	$0.19^{+0.45}_{-0.15}$	$2.4^{+3.0}_{-1.5}$
$\nu_\mu\bar{\nu}_\mu \to \nu_\mu\bar{\nu}_\mu H$	NC	$0.08^{+0.17}_{-0.06}$	$1^{+1.2}_{-0.6}$

Charged current (CC) and Neutral current (NC) processes are proportional respectively to the Higgs couplings to W and Z bosons.

We can modify those with the usual **kappa parameters**, and obtain:

Contribution in single-Higgs production

We obtain:

Partonic process	Channel	$\sigma(3\,\text{TeV})$ [fb]	$\sigma(10\,\text{TeV})$ [fb]
$\mu^-\mu^+ \to \nu_\mu \bar{\nu}_\mu H$	CC	$480.3^{+0.8}_{-0.7}$	$820.9^{+0.6}_{-0.2}$
$\mu^-\mu^+ \to \mu^-\mu^+ H$	NC	$47.7^{+0.6}_{-0.8}$	$80.5^{+1.5}_{-1.7}$
$\mu^-\bar{\nu}_\mu \to \mu^-\bar{\nu}_\mu H$	NC	$2.4^{+1.6}_{-1.2}$	10^{+4}_{-3}
$\nu_\mu \bar{\nu}_\mu \to \mu^-\mu^+ H$	CC	$0.19^{+0.45}_{-0.15}$	$2.4^{+3.0}_{-1.5}$
$\nu_\mu \bar{\nu}_\mu \to \nu_\mu \bar{\nu}_\mu H$	NC	$0.08^{+0.17}_{-0.06}$	$1^{+1.2}_{-0.6}$

Charged current (CC) and Neutral current (NC) processes are proportional respectively to the Higgs couplings to W and Z bosons.

We can modify those with the usual **kappa parameters**, and obtain:

$$\sigma^{\text{N TeV}}_{\mu\bar{\mu}\to H} = \sigma^{\text{N TeV}}_{\kappa_W^2}\kappa_W^2 + \sigma^{\text{N TeV}}_{\kappa_Z^2}\kappa_Z^2$$

	Cross sections [fb]	No PDFs	Only μ PDF	Both μ and ν_μ PDF
MuC 3TeV	$\sigma^{3\text{TeV}}_{\kappa_W^2}$	498	480 (-3.6%)	480 (+0.04%)
	$\sigma^{3\text{TeV}}_{\kappa_Z^2}$	50.8	47.7 (-6.1%)	52.6 (+10%)
MuC 10TeV	$\sigma^{10\text{TeV}}_{\kappa_W^2}$	842	821 (-2.6%)	823 (+0.3%)
	$\sigma^{10\text{TeV}}_{\kappa_Z^2}$	87.4	80.5 (-7.9%)	102 (+27%)

Both the **muon PDF** and the **muon-neutrino PDF** give **large effects, up to 27%** at a 10TeV MuC.

Conclusions

The Multi-TeV scale will be an exciting frontier to explore, both with and without BSM physics!
EW symmetry becomes effectively restored and a plethora of new effects are expected to appear.

We focus on **SM PDFs for lepton colliders**. In a previous work we derived numerical LL SM-PDFs for lepton colliders: **LePDF**. https://github.com/DavidMarzocca/LePDF

The **interference-Z/γ PDF** appears when EW interactions are considered. **EVA @ LO deviates by ~10^2** due to accidental cancellation. **We provide a more accurate approximation** by a NLO computation.
This PDF can be precisely measured in **high-energy Compton scattering** and can affect Higgs and BSM physics.

The **muon neutrino PDF** inside a muon can impact physics studies. It **dominates CC lepton-neutrino production** and **affects single-Higgs production by up to ~30%**!

Thank you!

Backup

LePDF - implementation

We work in the **mass eigenstate basis** and **solve the DGLAP numerically** in **x-space**, discretising the [10^{-6}, -1] interval

After identifying PDFs which are identical because of flavour symmetry, we remain with **42 independent PDFs**:

$$
\begin{aligned}
&f_{e_L} = f_{\tau_L}\,, \quad f_{\bar{\ell}_L} = f_{\bar{e}_L} = f_{\bar{\mu}_L} = f_{\bar{\tau}_L}\,,\\
&f_{e_R} = f_{\tau_R}\,, \quad f_{\bar{\ell}_R} = f_{\bar{e}_R} = f_{\bar{\mu}_R} = f_{\bar{\tau}_R}\,,\\
&f_{\nu_e} = f_{\nu_\tau}\,, \quad f_{\bar{\nu}_\ell} = f_{\bar{\nu}_e} = f_{\bar{\nu}_\mu} = f_{\bar{\nu}_\tau}\,,\\
&f_{u_L} = f_{c_L}\,, \quad f_{\bar{u}_L} = f_{\bar{c}_L}\,, \quad f_{u_R} = f_{c_R}\,, \quad f_{\bar{u}_R} = f_{\bar{c}_R}\,,\\
&f_{d_L} = f_{s_L}\,, \quad f_{\bar{d}_L} = f_{\bar{s}_L}\,, \quad f_{d_R} = f_{s_R}\,, \quad f_{\bar{d}_R} = f_{\bar{s}_R}\,.
\end{aligned}
$$

Leptons	μ_L	μ_R	e_L	e_R	ν_μ	ν_e	$\bar{\ell}_L$	$\bar{\ell}_R$	$\bar{\nu}_\ell$
Quarks	u_L	d_L	u_R	d_R	t_L	t_R	b_L	b_R	$+$ h.c.
Gauge Bosons	$\gamma_\pm$	$Z_\pm$	$Z\gamma_\pm$	$W^\pm_\pm$	$G_\pm$				
Scalars	h	Z_L	hZ_L	$W^\pm_L$					

Starting from $Q_{\mathrm{EW}} = m_W$, heavy states are added at the corresponding mass threshold.

The uncertainties due to x and t discretisation are estimated to be of ~1% and ~0.1%, respectively.

All EW & SM interactions are implemented, including all features listed in the previous slide.

LePDF

Momentum fractions

Parton	$Q = 3$ TeV	$Q = 10$ TeV	$Q = 30$ TeV
μ_L	48.0	47.8	47.3
μ_R	45.5	43.1	40.6
ν_μ	1.75	3.58	5.89
$\bar{\nu}_\ell$	0.00201	0.00371	0.00579
ℓ	0.0164	0.0222	0.0282
q	0.125	0.180	0.240
γ	3.00	3.22	3.39
W_T^-	01.16	1.50	1.78
W_T^+	0.0926	0.196	0.333
Z_T	0.383	0.537	0.691
g	0.0187	0.0267	0.0359

Table 4. Fraction of the momentum carried by each parton at $Q = 3, 10, 30$ TeV.

Momentum conservation

$$\sum_i \int dx\; x\, f_i(x, Q^2) = 1{,}0037$$

Fermion number conservation

Q = 3 TeV

$$\int dx \left(f_{\ell_L} + f_{\nu_\ell} + f_{\ell_R} - f_{\bar{\ell}_L} - f_{\bar{\nu}_\ell} - f_{\bar{\ell}_R} \right)$$

e: 6×10^{-7}

μ: 1.0018

τ: 6×10^{-7}

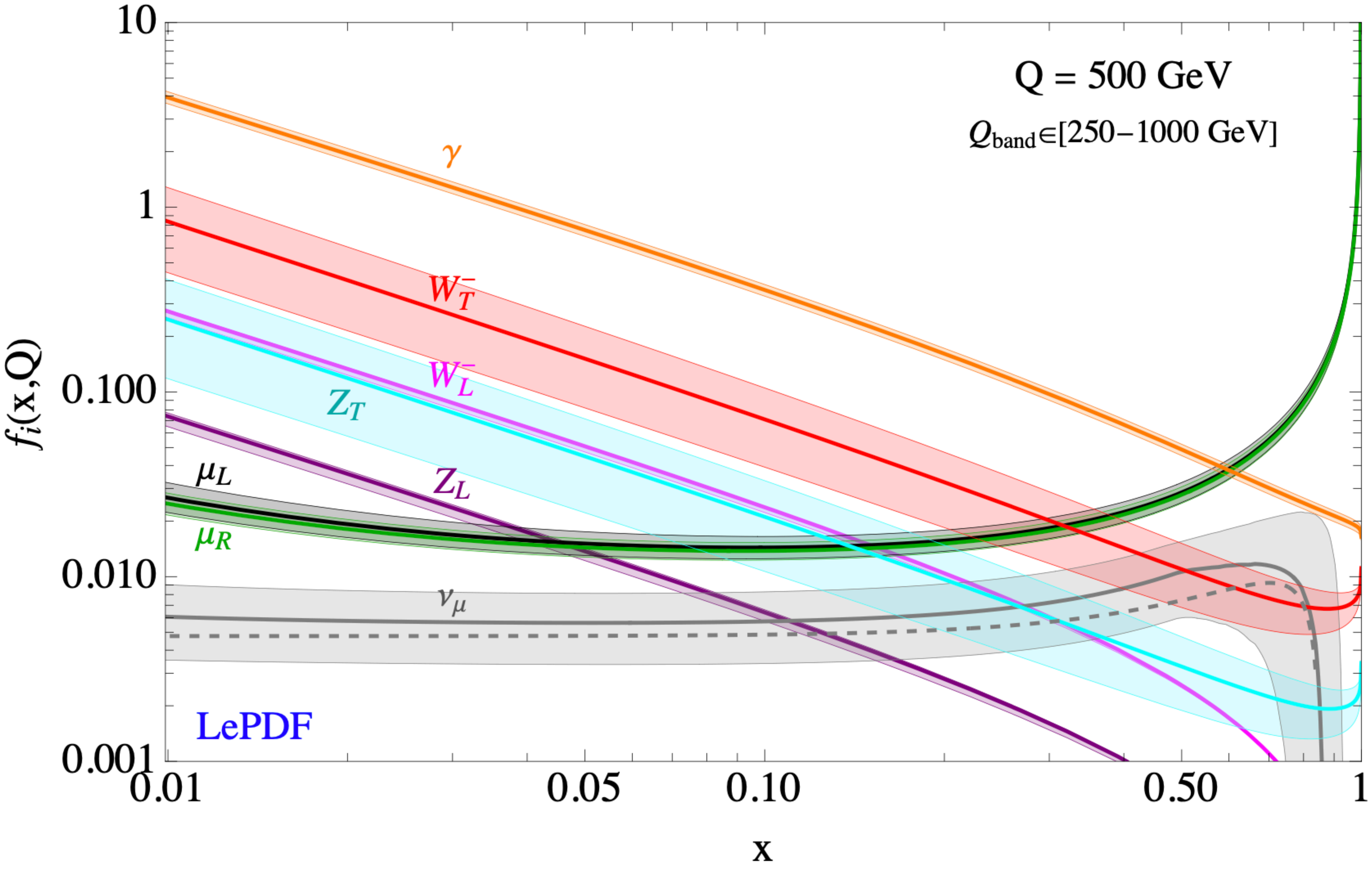
$$\int dx \left(f_{u_L^i} + f_{d_L^i} + f_{u_R^i} + f_{d_R^i} - f_{\bar{u}_L^i} - f_{\bar{d}_L^i} - f_{\bar{u}_R^i} - f_{\bar{d}_R^i} \right)$$

u,d: 1.6×10^{-7}

c,s: 1.6×10^{-7}

t,b: 4×10^{-5}

PDFs of a muon



Here we show scale uncertainties by varying the factorisation scale by a factor of 2.

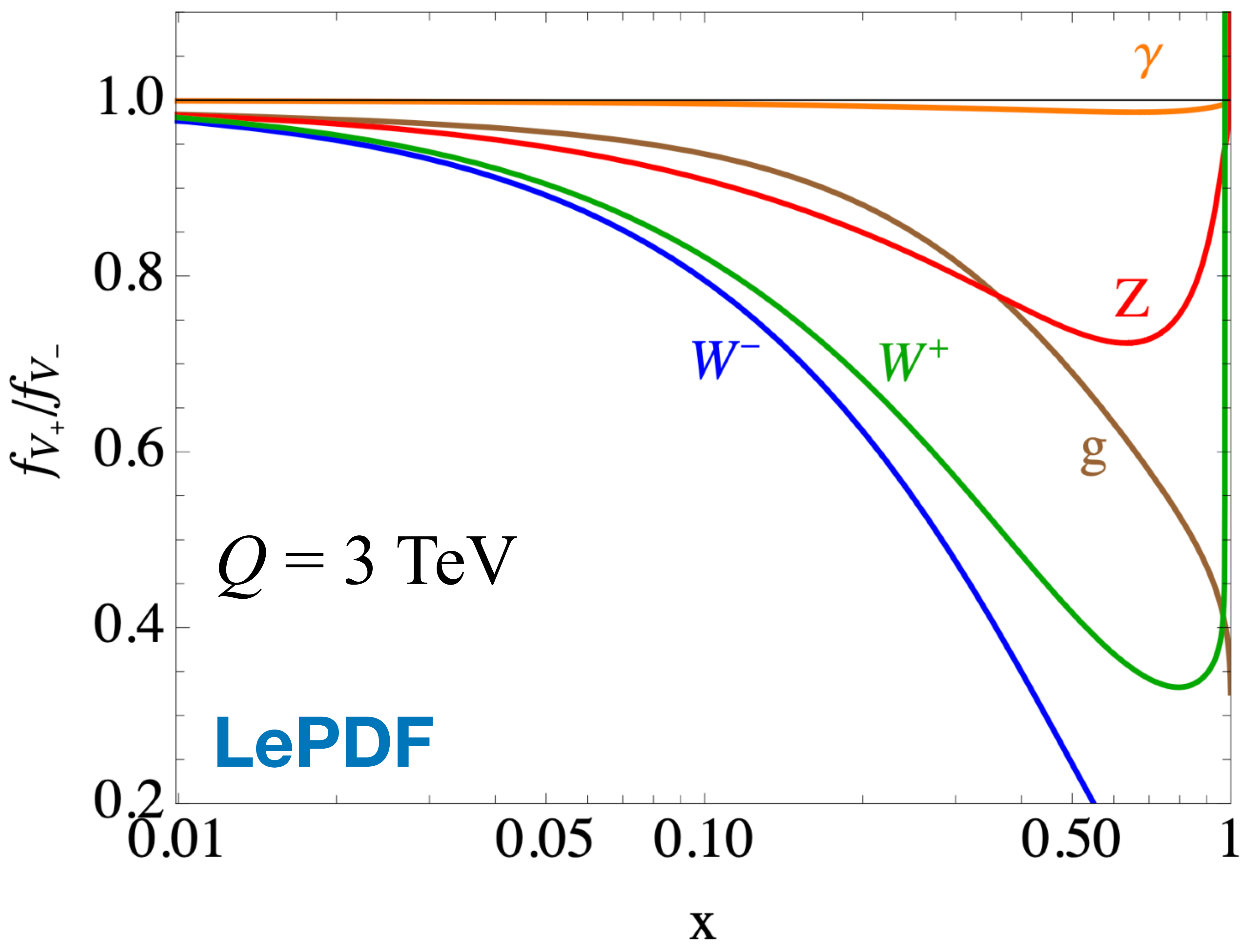
- Photon and muon PDF mainly given by QED: scale as $\alpha \log Q^2 / m_\mu^2$
  ~ 8% scale uncertainty at LL (at 500GeV)
- EW bosons and neutrino PDFs scale as $\alpha \log Q^2 / m_W^2$
  ~ 38% scale uncertainty at LL (at 500GeV)

Theory improvements are required to reduce these uncertainties down to the percent level.

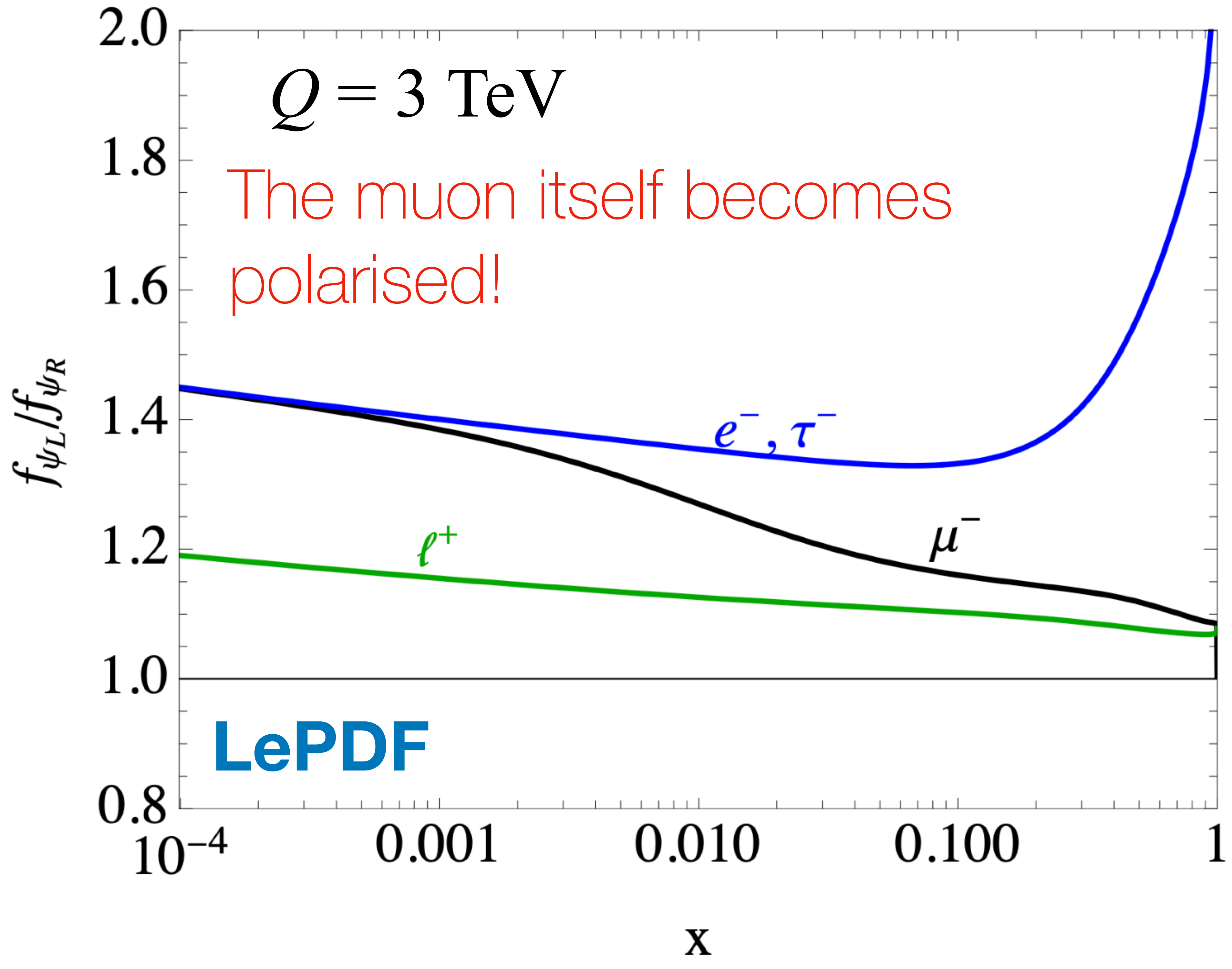
Polarisation

Since EW interactions are chiral, PDFs become polarised. Bauer, Webber [1808.08831]

Vectors polarisation: V_+ / V_-



Fermions polarisation: ψ_L / ψ_R



O(1) polarisation effects! (except for photon PDF)

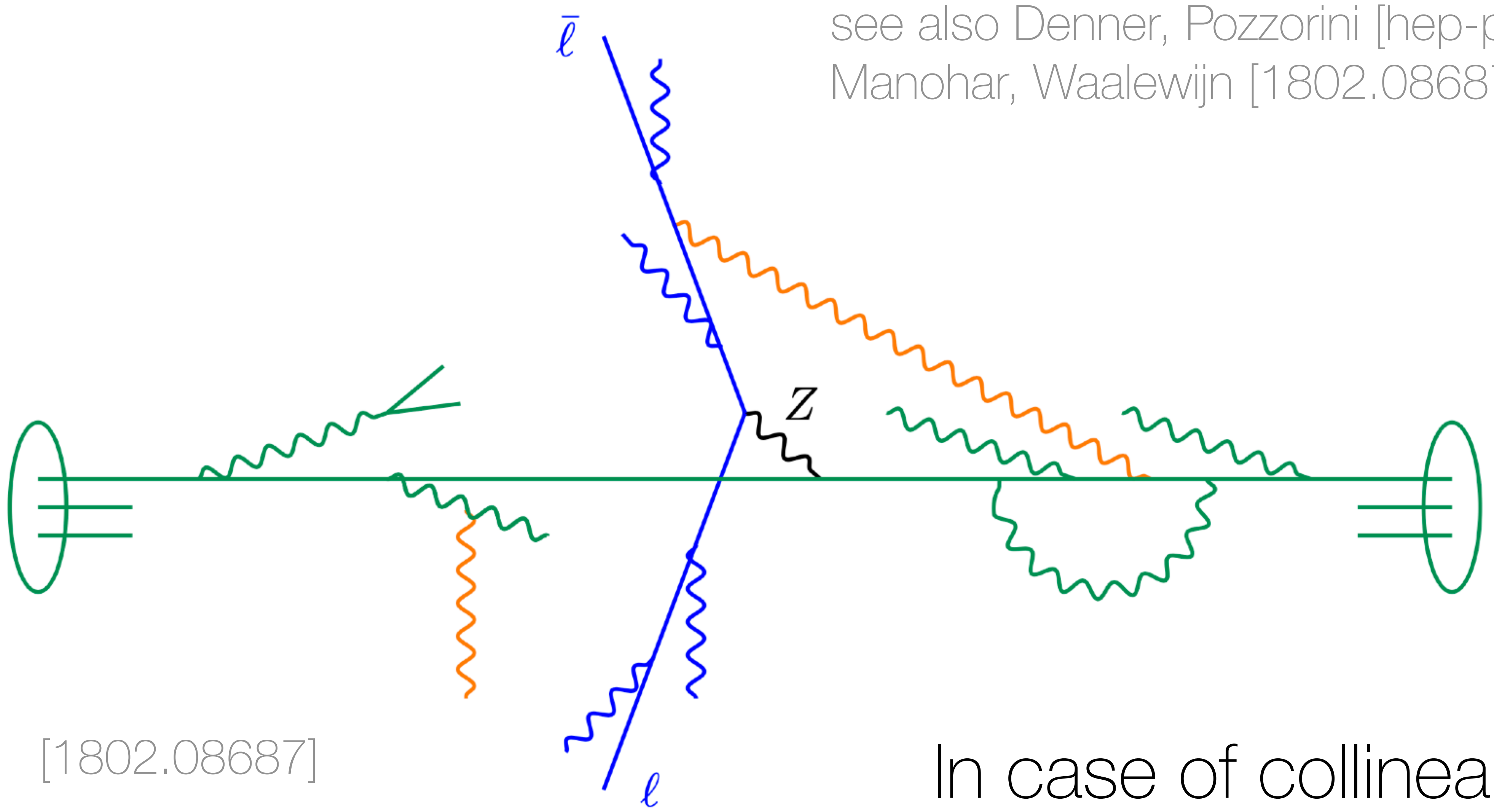
E.g. in case of $\boldsymbol{W^-}$ PDF, coupled to $\boldsymbol{\mu_L}$, the PDF for RH W's goes to zero for $x \to 1$ faster than LH W's, since $P_{V+f_L}(z) = (1-z)/z$ while $P_{V-f_L}(z) = 1/z$.

EW Sudakov double logs from ISR

The Bloch-Nordsieck theorem is violated for non-abelian gauge theories
→ IR divergencies are not cancelled in inclusive processes, since the initial state is EW non-singlet
→ We are often interested in exclusive processes, since we measure the SU(2) charge (W vs Z, t vs b, etc…)

The **EW Sudakov double logs** arises as a non-cancellation of the IR soft divergences $(z \to 1)$ between real emission and virtual corrections.

P. Ciafaloni, Comelli [hep-ph/9809321], Fadin et al. [hep-ph/9910338], M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0001142, hep-ph/0103315]
see also Denner, Pozzorini [hep-ph/0010201], Pozzorini [hep-ph/0201077], Manohar [1409.1918], Pagani, Zaro [2110.03714], …
Manohar, Waalewijn [1802.08687], Chen, Glioti, Rattazzi, Ricci, Wulzer [2202.10509]



[1802.08687]

Here I am interested in **resumming the EW double logs** related to the **initial-state radiation**.
At the leading-log level we can neglect soft radiation

Manohar, Waalewijn [1802.08687]

In case of collinear W emission they can be implemented (and resummed) at the **Leading Log** level by putting an **explicit IR cutoff** $z_{max} = 1 - Q_{EW}/Q$

M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0111109]; Bauer, Ferland, Webber [1703.08562]; Manohar, Waalewijn [1802.08687]

EW Sudakov double logs from ISR

In case of collinear W emission they can be implemented (and resummed) at he **Double Log** level equations by putting an **explicit IR cutoff** $z_{max} = 1 - Q_{EW}/Q$ $\quad (Q_{EW} = m_W)$

M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0111109]
Bauer, Ferland, Webber [1703.08562]
see Manohar, Waalewijn [1802.08687] for a different approach

$$\frac{\alpha_{ABC}(Q)}{2\pi}\int_x^1 \frac{dz}{z} P_{BA}^C(z) f_A\left(\frac{x}{z}, Q^2\right) \quad \rightarrow \quad \frac{\alpha_{ABC}(Q)}{2\pi}\int_x^{z_{\max}^{ABC}(Q)} \frac{dz}{z} P_{BA}^C(z) f_A\left(\frac{x}{z}, Q^2\right)$$

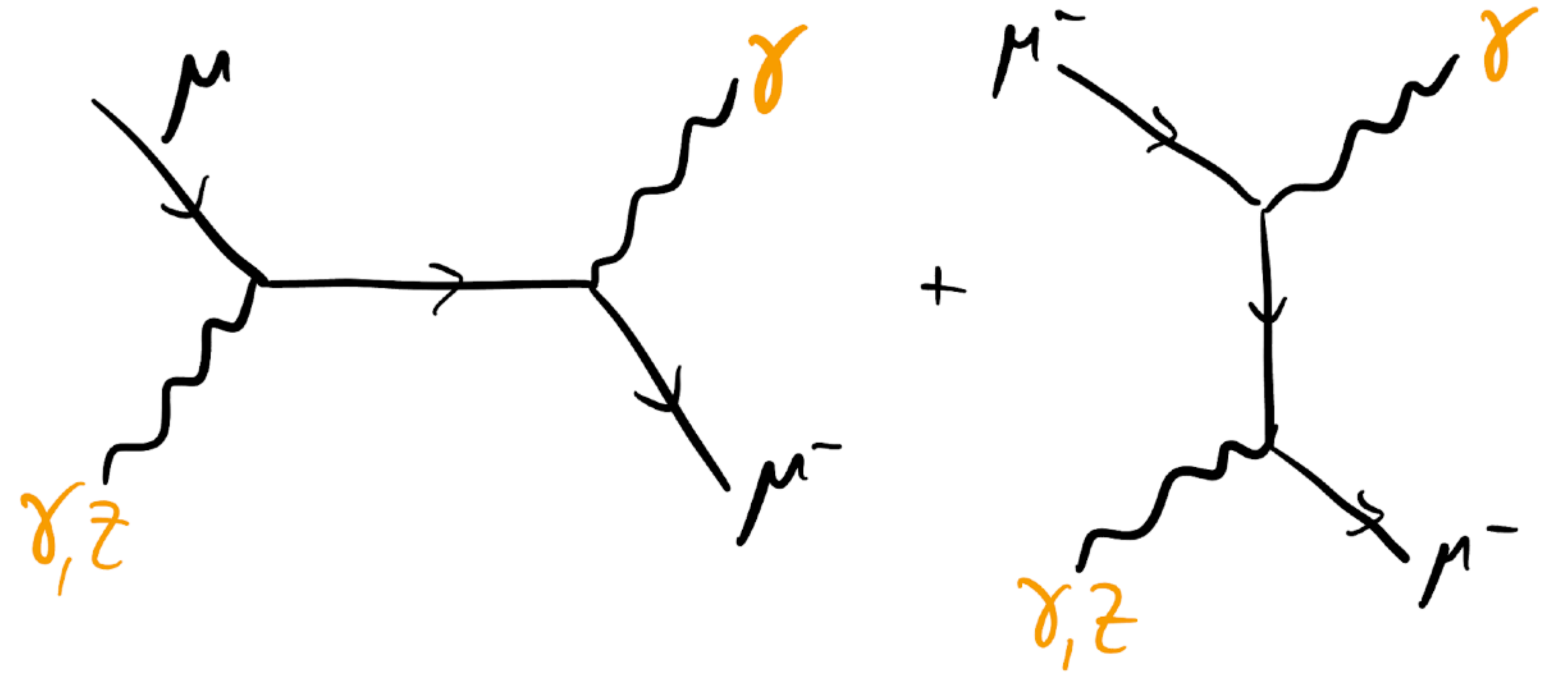
This modifies also the **virtual corrections** as:

$$P_A^v(Q) \supset -\sum_{B,C} \frac{\alpha_{ABC}(Q)}{2\pi}\int_0^{z_{\max}^{ABC}(Q)} dz\, z\, P_{BA}^C(z)$$

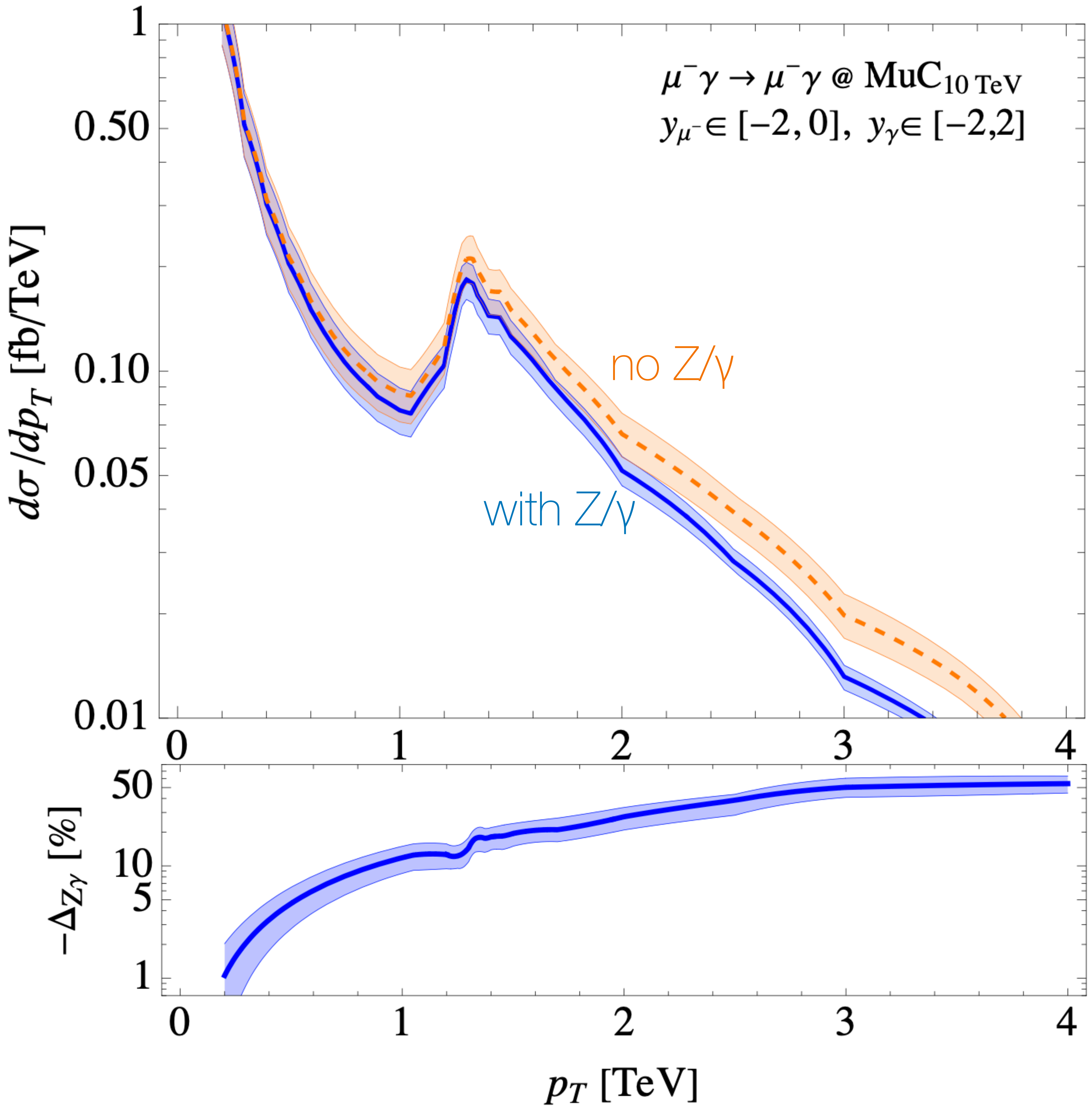
The non-cancellation of the z_{max} dependence between emission and virtual corrections generates the double logs.

This happens if $\quad P_{BA}^C,\ U_{BA}^C \propto \frac{1}{1-z}$ and $A \neq B$ $\quad$ otherwise we set $z_{max}=1$ and use the +-distribution.

Compton Scattering @ MuC

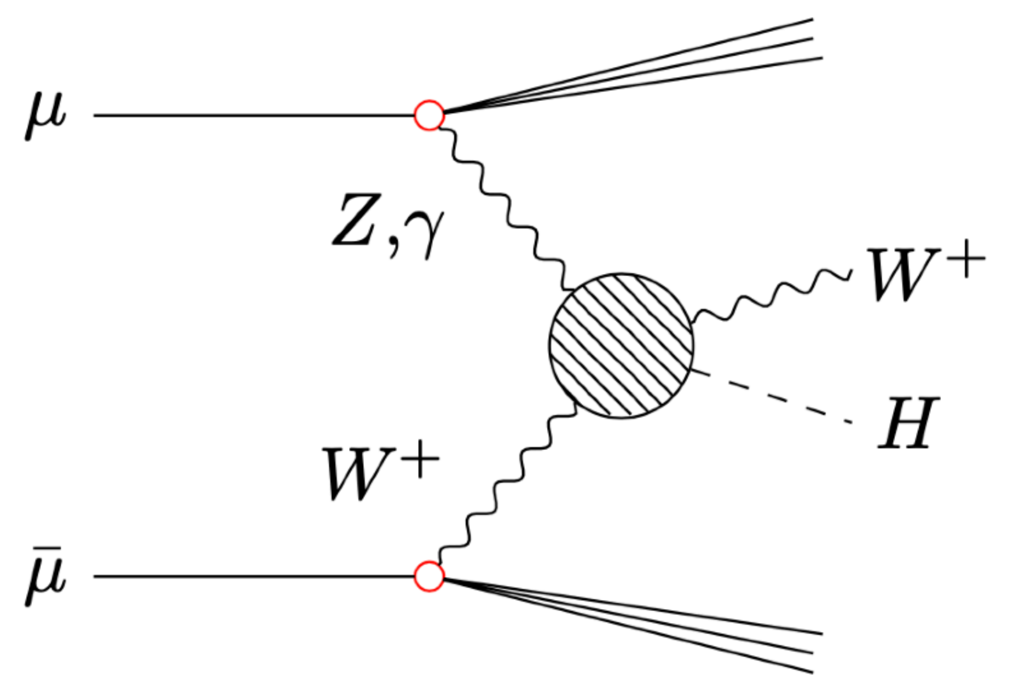


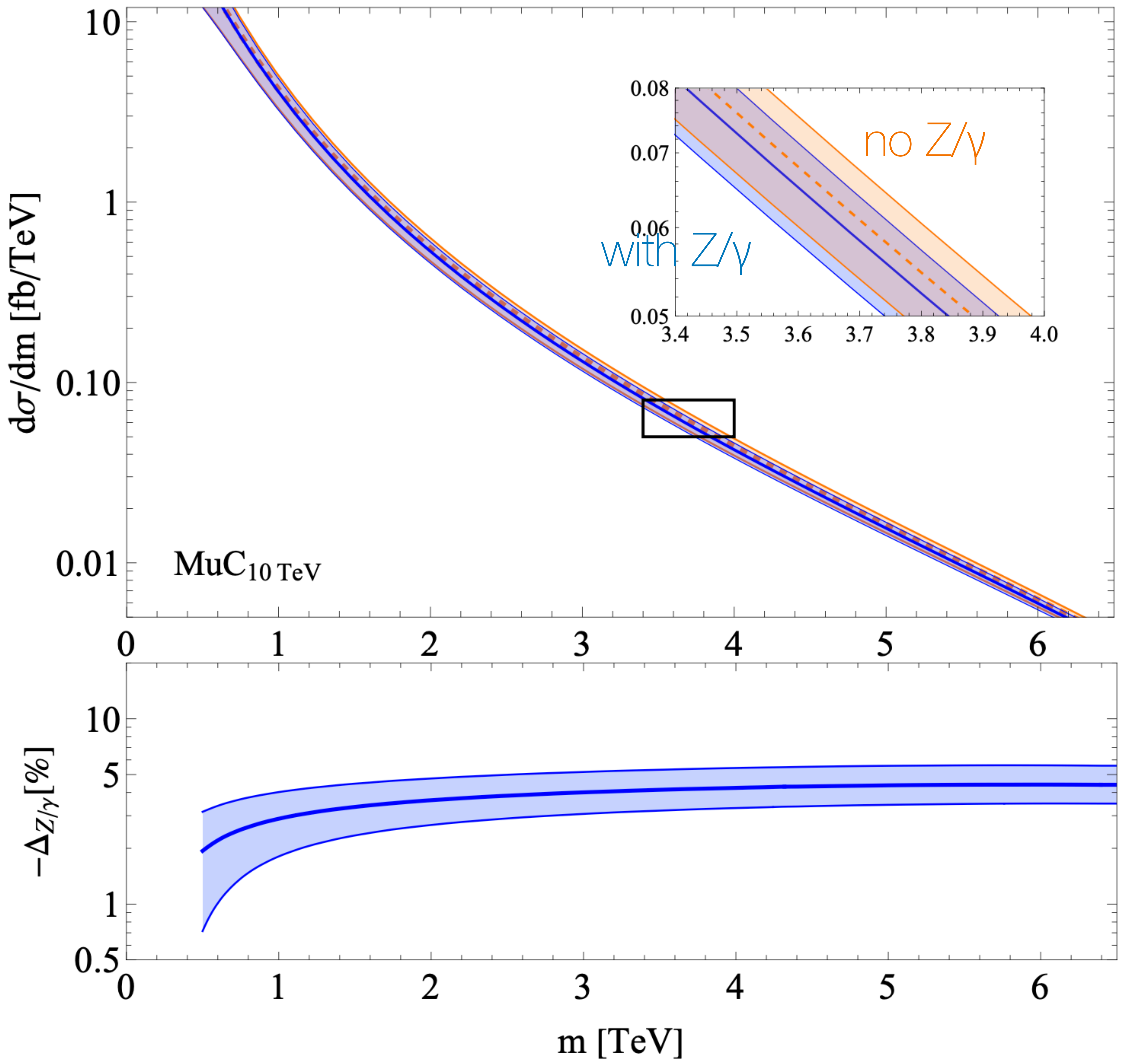
The peak at around $p_T \sim 1350$ GeV is due to the fact that, for those values of p_T the kinematical configuration with $x_1 = 1$ (x_1 being the Bjorken variable for the incoming muon) enters the range of rapidities included in the integration.
For $x_1 \approx 1$ the μ^- PDF gets the large enhancement due to it being the valence parton, remnant of the Dirac delta that describes the zeroth order PDF of the muon.



WH production @ MuC

Consider **associated W H production at a MuC**





While at present the effect is washed out by the scale uncertainties, these are expected to be reduced in the future, since one of the main goals of muon colliders is to perform measurements of EW processes at high energy with O(1%) precision.