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EW CORRECTIONS TO BELL INEQUALITIES AND ENTANGLEMENT IN $H \to VV$ PROCESSES AT LHC

(WORK IN PROGRESS)

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Content

- Motivation
- Define irreducible tensor operator parameterization to define quantum tomography
- Observables and density matrix at LO for SM for $H \to VV \to 4f$
- NLO effect on density matrix and on observables
- New physics effects on spin density matrix
- Conclusion

Motivation:

It is challenging to see entanglement at High Energy Colliders(HEC) and it is interesting to check the sensitivity of HEC to probe quantum correlations

We saw it at LHC!!!!!

Observation of quantum entanglement in top-quark pairs using the ATLAS detector

ATLAS Collaboration

We report the highest-energy observation of entanglement, in top—antitop quark events produced at the Large Hadron Collider, using a proton—proton collision data set with a center-of-mass energy of $\sqrt{s} = 13$ TeV and an integrated luminosity of 140 fb^{-1} recorded with the ATLAS experiment. Spin entanglement is detected from the measurement of a single observable D, inferred from the angle between the charged leptons in their parent top- and antitop-quark rest frames. The observable is measured in a narrow interval around the top—antitop quark production threshold, where the entanglement detection is expected to be significant. It is reported in a fiducial phase space defined with stable particles to minimize the uncertainties that stem from limitations of the Monte Carlo event generators and the parton shower model in modelling top-quark pair production. The entanglement marker is measured to be $D = -0.547 \pm 0.002\ \text{(stat.)} \pm 0.021\ \text{(syst.)}$ for $340 < m_{t\bar{t}} < 380$ GeV. The observed result is more than five standard deviations from a scenario without entanglement and hence constitutes both the first observation of entanglement in a pair of quarks and the highest-energy observation of entanglement to date.

Observation of quantum entanglement in top quark pair production in proton-proton collisions at $\sqrt{s} = 13$ TeV

CMS Collaboration

6 June 2024

Submitted to Reports on Progress in Physics

Abstract: Entanglement is an intrinsic property of quantum mechanics and is predicted to be exhibited in the particles produced at the Large Hadron Collider. A measurement of the extent of entanglement in top quark-antiquark ($t\bar{t}$) events produced in proton-proton collisions at a center-of-mass energy of 13 TeV is performed with the data recorded by the CMS experiment at the CERN LHC in 2016, and corresponding to an integrated luminosity of 36.3 fb^{-1}. The events are selected based on the presence of two leptons with opposite charges and high transverse momentum. An entanglement-sensitive observable D is derived from the top quark spin-dependent parts of the $t\bar{t}$ production density matrix and measured in the region of the $t\bar{t}$ production threshold. Values of $D < -1/3$ are evidence of entanglement and D is observed (expected) to be $-0.480^{+0.026}_{-0.029}$ ($-0.467^{+0.026}_{-0.029}$) at the parton level. With an observed significance of 5.1 standard deviations with respect to the non-entangled hypothesis, this provides observation of quantum mechanical entanglement within $t\bar{t}$ pairs in this phase space. This measurement provides a new probe of quantum mechanics at the highest energies ever produced.

Motivation:

Find new quantum observables and check if they are Sensitive to new physics e.g. CP-phases in Yukawa, contribution from other SMEFT operators depending on channels

Why $H \to VV$?

- Higgs as a scalar pure state.
- Massive vector boson decay is chiral in SM that mean there decay product have spin polarization information

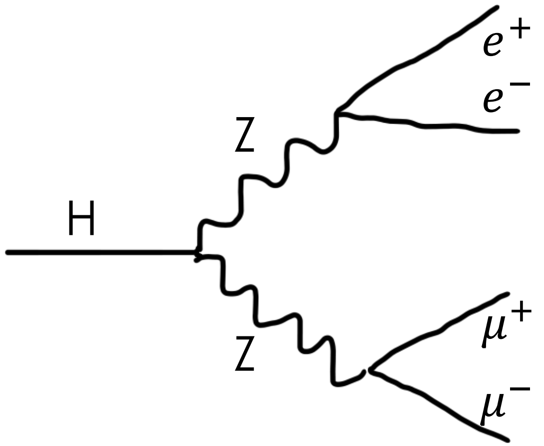
List of work on VV spin correlation at colliders

- Testing entanglement and Bell inequalities in H → ZZ by J. A. Aguilar-Saavedra , A. Bernal , J. A. Casas , and J. M. Moreno
- Entanglement and Bell inequalities violation in H → ZZ with anomalous coupling by Alexander Bernal, Pawel Caban and Jakub Rembielinski
- Quantum state tomography, entanglement detection and Bell violation prospects in weak decays of massive particles: Rachel Ashby-Pickering, Alan J. Barr, Agnieszka Wierzchucka
- Bell inequalities and quantum entanglement in weak gauge boson production at the LHC and future colliders by Marco Fabbrichesi, Roberto Floreanini, Emidio Gabrielli, Luca Marzola
- Spin Correlations in Decay Chains Involving W Bosons⋆ by Jennifer M. Smillie
- Stringent bounds on HWW and HZZ anomalous couplings with quantum tomography at the LHC by M. Fabbrichesia, R. Floreaninia, E. Gabriellib,a,c,d and L. Marzolad
- Bell-type inequalities for systems of relativistic vector bosons by Alan J. Barr, Paweł Caban, and Jakub Rembieliński
- Breaking down the entire W boson spin observables from its decay by J. A. Aguilar-Saavedra, J. Bernabéu
- Testing Bell inequalities in Higgs boson decays by Alan J. Barr
- The Z boson spin observables as messengers of new physics by J. A. Aguilar-Saavedra, J. Bernabéu, V. A. Mitsou, A. Segarra

- Talk on three-particle entanglement in particle decay and scattering by Kazuki sakurai
- Talk on Entanglment in QED scattering processes by Bruno Micciola
- Talk on Entanglement in flavored scalar scattering by Enrico Maria Sessolo

Quantum tomography

- We can't measure spin polarization of particles at colliders. We observe angular distribution of decay particles, In our case we observe angular distribution of 4 lepton.



Reconstruct angular distribution and ensemble of decays

Quantum state tomography →

Spin density matrix

↓

Look for entanglement and bell nonlocality observables

←

Look for sensitivity of these observables for new physics

Quantum tomography

The Polarization operator basis parameterization/ irreducible tensor parameterization

$$\rho = \frac{1}{9}[\mathbf{1}_3 \otimes \mathbf{1}_3 + A^a_{LM}\hat{T}^{LM} \otimes \mathbf{1}_3 + A^b_{LM}\mathbf{1}_3 \otimes \hat{T}^{LM} + C_{L_1,M_1,L_2,M_2}\hat{T}^{L_1M_1} \otimes \hat{T}^{L_2M_2}]$$

For spin-1, the spin operator S and polarization operator are relates as

$$T_{00} = \frac{1}{\sqrt{3}}\hat{I}, \qquad T_{1M} = \frac{1}{\sqrt{2}}\hat{S}_M, \qquad T_{2M} = \sum_{\mu\nu} C^{2M}_{1\mu 1\nu}\hat{S}_\mu\hat{S}_\nu$$

Constraint on A and C coefficient in spherical basis

$$\begin{aligned}(A^j_{L,M})^* &= (-1)^M A^j_{L,-M}, \qquad j = 1,2 \\ C_{L_1,M_1,L_2,M_2} &= (-1)^{M_1+M_2}(C_{L_1,-M_1,L_2,-M_2})^*\end{aligned}$$

Quantum tomography

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We know how the angular differential cross section is related to density matrix:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} = \frac{2S_a+1}{4\pi}\frac{2S_b+1}{4\pi}\sum_{\lambda_a,\lambda_a',\lambda_b,\lambda_b'} \rho(\lambda_a,\lambda_a',\lambda_b,\lambda_b')\Gamma_{a(2S_a)(\lambda_a,\lambda_a')}\Gamma_{b(2S_b)(\lambda_b,\lambda_b')}$$

The traces of decay density matrix can be written in term of spherical harmonics as

$$\mathrm{Tr}\left[\mathbf{1}_3\Gamma^T\right] = 2\sqrt{\pi}Y_0^0(\theta,\phi), \quad \mathrm{Tr}\left[T^1_M\Gamma^T\right] = B_1Y_1^M(\theta,\phi), \quad \mathrm{Tr}\left[T^2_M\Gamma^T\right] = B_2Y_2^M(\theta,\phi)$$

These traces of decay matrix is same for all spin-1 particle decay except B_1 coefficient, which depends on decay products

$B_1 = \sqrt{2\pi}\alpha$ and $B_2 = \sqrt{\frac{2\pi}{5}}$

Spin analyzing power

Quantum tomography

Now we have normalized joint angular distribution in term of spherical harmonics and function

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} = \frac{1}{(4\pi)^2}[1 + A^a_{LM} B^a_L Y^M_L(\theta_a,\phi_a) + A^b_{LM} B^b_L Y^M_L(\theta_b,\phi_b) + C_{L_1M_1L_2M_2} B^a_{L_1} B^b_{L_2} Y^{M_1}_{L_1}(\theta_a,\phi_a) Y^{M_2}_{L_2}(\theta_b,\phi_b)]$$

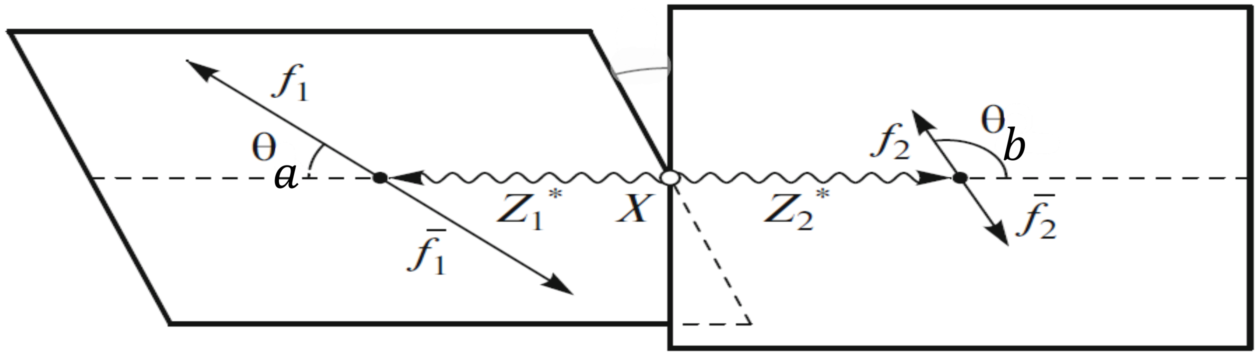
We can compute full spin density matrix using experimental data by using following tomographic reconstruction:

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y^{*M}_L(\Omega_j)\, d\Omega_a d\Omega_b = \frac{B^j_L}{4\pi} A^j_{LM} \quad j = a, b$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y^{*M_1}_{L_1}(\Omega_a) Y^{*M_2}_{L_2}(\Omega_b)\, d\Omega_a d\Omega_b = \frac{B^a_{L_1} B^b_{L_2}}{(4\pi)^2} C_{L_1M_1L_2M_2}$$

Total 80 parameters

$$d\Omega_a = \sin\theta_a d\theta_a d\phi_a$$



Observables and density matrix

Lets compute amplitude square for generic vector and pseudo-vector currents

$$\mathcal{A}_V = c_V\ \bar{u}_1\gamma_\mu(c_LP_L + c_RP_R)v_2\ \bar{u}_3\gamma_\mu(d_LP_L + d_RP_R)v_4$$

$$|\mathcal{M}(H \to aa'bb')|^2 = \sum_s \mathcal{A}_V^*\mathcal{A}_V = 16|c_V|^2\left[\left(c_L^2d_L^2 + c_R^2d_R^2\right)\Pi_1 + \left(c_L^2d_R^2 + c_R^2d_L^2\right)\Pi_2\right]$$

$$B_1^a = \sqrt{2\pi}\frac{c_R^2 - c_L^2}{c_R^2 + c_L^2}$$

$$B_1^b = \sqrt{2\pi}\frac{d_R^2 - d_L^2}{d_R^2 + d_L^2}$$

$$\Pi_1 = (p_1.p_3)(p_2.p_4)$$
$$\Pi_2 = (p_1.p_4)(p_2.p_3)$$

Non-zero A and C coefficients for vector and vector-axial couplings

$$A_{2,0}^a = A_{2,0}^b \neq 0$$

$$\frac{A_{2,0}^a}{\sqrt{2}} + 1 = C_{2,2,2,-2} \neq 0$$

$$C_{1,-1,1,1} = C_{1,1,1,-1} = -C_{2,-1,2,1} = -C_{2,1,2,-1} \neq 0$$

$$C_{2,2,2,-2} = -C_{1,0,1,0} = 2 - C_{2,0,2,0} \neq 0$$

All A and C coefficient are real in this case

$$\rho = \frac{1}{9}[\mathbf{1}_3 \otimes \mathbf{1}_3 + A_{LM}^a\hat{T}^{LM} \otimes \mathbf{1}_3 + A_{LM}^b\mathbf{1}_3 \otimes \hat{T}^{LM} + C_{L_1,M_1,L_2,M_2}\hat{T}^{L_1M_1} \otimes \hat{T}^{L_2M_2}]$$

Observables and density matrix

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$$|\mathcal{M}(H \to aa'bb')|^2 = \sum_s \mathcal{A}_V^* \mathcal{A}_V = 16 |c_V|^2 \left[\left(c_L^2 d_L^2 + c_R^2 d_R^2 \right) \Pi_1 + \left(c_L^2 d_R^2 + c_R^2 d_L^2 \right) \Pi_2 \right]$$

$$B_1^a = \sqrt{2\pi} \frac{c_R^2 - c_L^2}{c_R^2 + c_L^2}$$

$$B_1^b = \sqrt{2\pi} \frac{d_R^2 - d_L^2}{d_R^2 + d_L^2}$$

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$$\frac{A_{2,0}^a}{\sqrt{2}} + 1 = C_{2,2,2,-2} \neq 0$$

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$$C_{2,2,2,-2} = -C_{1,0,1,0} = 2 - C_{2,0,2,0} \neq 0$$

All A and C coefficient are real in this case

$$\rho = \frac{1}{3} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 + \frac{1}{\sqrt{2}} A_{2,0}^1 & 0 & C_{2,1,2,-1} & 0 & C_{2,2,2,-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{2,-1,2,1} & 0 & 1 - \sqrt{2} A_{2,0}^1 & 0 & C_{2,1,2,-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{2,-2,2,2} & 0 & C_{2,-1,2,1} & 0 & 1 + \frac{1}{\sqrt{2}} A_{2,0}^1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Observables and density matrix

First task: reconstruct the quantum state: easy in this case but not always

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & y & 0 & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y & 0 & z & 0 & y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & y & 0 & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$a_+ = a_-$

$$\frac{A^a_{2,0}}{\sqrt{2}} + 1 = C_{2,2,2,-2}$$

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_+a_+^* & 0 & a_+a_0^* & 0 & a_+a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_0a_+^* & 0 & a_0a_0^* & 0 & a_0a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_-a_+^* & 0 & a_-a_0^* & 0 & a_-a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

By looking this we can directly write the helicity state.

$$|\psi> = a_+|+-> + a_0|0\,0> + a_-|-+>$$

$a_+ = a_-$ Also CP conserving condition

Observables and density matrix

1. Entanglement

$$\rho = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1+\frac{1}{\sqrt{2}}A^1_{2,0} & 0 & C_{2,1,2,-1} & 0 & C_{2,2,2,-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{2,-1,2,1} & 0 & 1-\sqrt{2}A^1_{2,0} & 0 & C_{2,1,2,-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{2,-2,2,2} & 0 & C_{2,-1,2,1} & 0 & 1+\frac{1}{\sqrt{2}}A^1_{2,0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Sufficient condition for Entanglement

$C_{2,2,2,-2} \neq 0$ or $C_{2,1,2,-1} \neq 0$

$$|\psi> = a_+|+-> + a_0|0\ 0> + a_-|-+>$$

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_+a_+^* & 0 & a_+a_0^* & 0 & a_+a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_0a_+^* & 0 & a_0a_0^* & 0 & a_0a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_-a_+^* & 0 & a_-a_0^* & 0 & a_-a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Bell nonlocality for the qutrit system

CGLMP inequality

$$I_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) - P(A_1 = B_1 - 1) - P(B_1 = A_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1) \leq 2.$$

D. Collins, N. Gisin, N. Linden, S. Massar and S. Popescu, Phys. Rev. Lett. 88, 040404 (2002)

P$(A_i = B_j + k)$ are the probability that the outcomes for party A and B, measuring A_i *and* B_j, differ by k modulo 3 .

For maximal entangled state $I_3 \approx 2.8729$
Upper value in QM $I_3 \approx 2.9149$

A. Ac´ın, T. Durt, N. Gisin, and J. I. Latorre, "Quantum nonlocality in two three-level systems,"Phys. Rev. A, vol. 65, p. 052325, May 2002

$$B = \frac{4}{3\sqrt{3}}(T_1^1 \otimes T_1^1 + T_{-1}^1 \otimes T_{-1}^1) + \frac{2}{3}(T_2^2 \otimes T_2^2 + T_{-2}^2 \otimes T_{-2}^2)$$
$$= \frac{2}{\sqrt{3}}(S_x^T \otimes S_x + S_y^T \otimes S_y) + \lambda_4^T \otimes \lambda_4 + \lambda_5^T \otimes \lambda_5$$

How we can measure it?

As we know we can compute expectation value of any operator in QM if we know density matrix

$$B' = (V \otimes U)^T B (V \otimes U)$$

$$I_3 = Tr[\rho B']$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 \\ \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Where B' is bell operator.

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$$I_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1)$$
$$- P(A_1 = B_1 - 1) - P(B_1 = A_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1) \leq 2.$$

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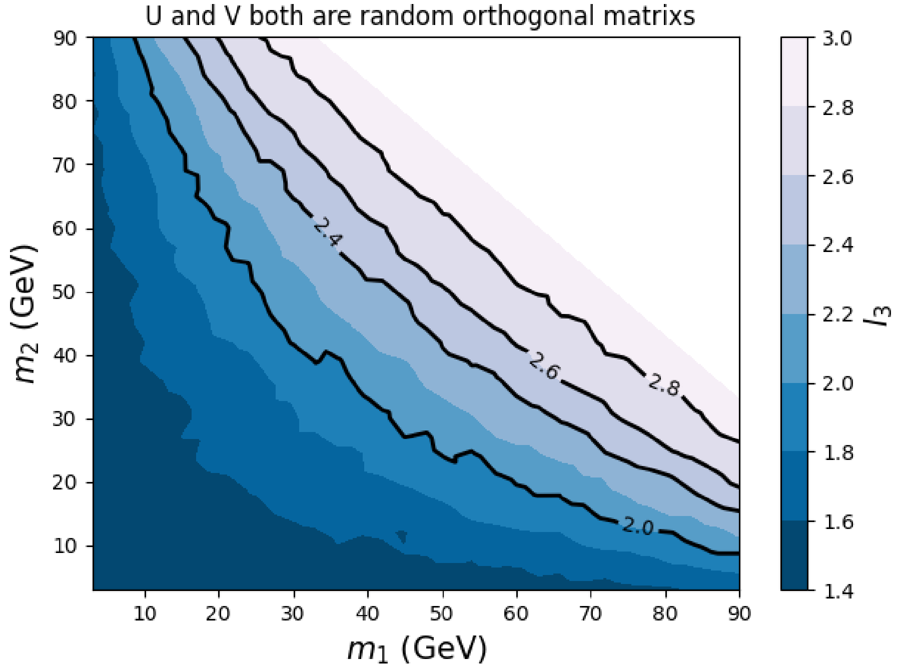

Numerical simulation

- Generate event for $H \to e^+e^-\mu^+\mu^-$ with Madgraph5 aMC@NLO at NLO EW accuracy, label large invariant mass is $Z_{1/a}$ and other one is $Z_{2/b}$.

- Define Helicity basis, $\hat{z}$-axis is taken in the direction of the Z_1 three-momentum in the H rest frame.

$$\hat{x} = sign(\cos\theta)(\hat{p} - \cos\theta\hat{z})/\sin\theta,\ \hat{y} = \hat{z} \times \hat{x}$$

- The angles $(\theta_{1/a}, \phi_{1/a})$ are the polar coordinates of the 3-momentum of negatively charge lepton from the $Z_{1/a}$, in the $Z_{1/a}$ rest frame.

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y_L^{*M}(\Omega_j)\, d\Omega_a d\Omega_b = \frac{B_L^j}{4\pi} A_{LM}^j \quad j = a, b$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y_{L_1}^{*M_1}(\Omega_a) Y_{L_2}^{*M_2}(\Omega_b)\, d\Omega_a d\Omega_b = \frac{B_{L_1}^a B_{L_2}^b}{(4\pi)^2} C_{L_1 M_1 L_2 M_2}$$

Observables at LO level

Sufficient condition for Entanglement

$C_{2,2,2,-2} \neq 0$ or $C_{2,1,2,-1} \neq 0$

	no cuts	$m_{Z_2} > 10$ GeV	> 20 GeV	> 30 GeV
$C_{2,2,2,-2}$, LO	0.58	0.63	0.71	0.78
$A^1_{2,0}/\sqrt{2}+1$, LO	0.58	0.62	0.71	0.77
$C_{2,1,2,-1}$, LO	-0.94	-0.97	-1.01	-1.02
I_3, LO	2.60	2.66	2.77	2.80

Bell nonlocality condition $I_3 > 2$
For maximal entangled state $I_3 \approx 2.8729$

Observables at LO level

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$\frac{A^{a}_{2,0}}{\sqrt{2}} + 1 = C_{2,2,2,-2}$

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Observables at LO level

$$A^a_{2,0} = A^b_{2,0} \neq 0$$

$$\frac{A^a_{2,0}}{\sqrt{2}} + 1 = C_{2,2,2,-2} \neq 0$$

$$C_{1,-1,1,1} = C_{1,1,1,-1} = -C_{2,-1,2,1} = -C_{2,1,2,-1} \neq 0$$

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$$\rho = \begin{pmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&x&0&y&0&x&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&y&0&z&0&y&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&x&0&y&0&x&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

No cuts

$$\rho_{\mathrm{LO}} = \begin{pmatrix} 0.00&0.00&0.00&0.00&0.01&0.00&0.00&0.00&0.00\\ 0.00&0.00&0.00&0.00&0.00&0.01&0.00&0.00&0.00\\ 0.00&0.00&0.19&0.00&-0.31&0.00&0.20&0.00&0.00\\ 0.00&0.00&0.00&0.00&0.00&0.00&0.00&0.01&0.01\\ 0.01&0.00&-0.31&0.00&0.61&0.00&-0.31&0.00&0.01\\ 0.01&0.01&0.00&0.00&0.00&0.00&0.01&0.01&0.00\\ 0.00&0.00&0.19&0.00&-0.31&0.01&0.20&0.00&0.00\\ 0.00&0.00&0.00&0.01&0.00&0.01&0.00&0.00&0.00\\ 0.00&0.00&0.00&0.01&0.01&0.00&0.00&0.00&0.00 \end{pmatrix}$$

NLO effect on rho martix

$$\rho = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1+\frac{1}{\sqrt{2}}A^1_{2,0} & 0 & C_{2,1,2,-1} & 0 & C_{2,2,2,-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{2,-1,2,1} & 0 & 1-\sqrt{2}A^1_{2,0} & 0 & C_{2,1,2,-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{2,-2,2,2} & 0 & C_{2,-1,2,1} & 0 & 1+\frac{1}{\sqrt{2}}A^1_{2,0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rho_{\mathrm{LO}} = \begin{pmatrix} 0.00 & 0.00 & 0.00 & 0.00 & 0.01 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.01 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.19 & 0.00 & -0.31 & 0.00 & 0.20 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.01 & 0.01 \\ 0.01 & 0.00 & -0.31 & 0.00 & 0.61 & 0.00 & -0.31 & 0.00 & 0.01 \\ 0.01 & 0.01 & 0.00 & 0.00 & 0.00 & 0.00 & 0.01 & 0.01 & 0.00 \\ 0.00 & 0.00 & 0.19 & 0.00 & -0.31 & 0.01 & 0.20 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.01 & 0.00 & 0.01 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.01 & 0.01 & 0.00 & 0.00 & 0.00 & 0.00 \end{pmatrix}$$

$$A^a_{2,0} = A^b_{2,0} \neq 0$$

$$\frac{A^a_{2,0}}{\sqrt{2}} + 1 = C_{2,2,2,-2} \neq 0$$

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$$\rho_{\mathrm{NLO}} = \begin{pmatrix} 0.08 & 0.00 & 0.00 & 0.01 & -0.01 & 0.01 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.12 & 0.01 & -0.02 & 0.00 & 0.00 & 0.01 \\ 0.00 & 0.00 & 0.12 & 0.01 & -0.18 & 0.00 & 0.19 & 0.00 & 0.00 \\ 0.00 & 0.12 & 0.01 & 0.00 & 0.01 & 0.00 & -0.01 & 0.01 & 0.01 \\ 0.01 & 0.00 & -0.18 & 0.01 & 0.59 & 0.00 & -0.18 & 0.00 & -0.01 \\ 0.00 & 0.02 & 0.00 & 0.00 & 0.00 & -0.01 & 0.01 & 0.14 & 0.00 \\ 0.00 & 0.00 & 0.19 & 0.01 & -0.18 & 0.01 & 0.12 & 0.01 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.01 & 0.00 & 0.14 & 0.01 & 0.01 & 0.00 \\ 0.00 & 0.01 & 0.00 & 0.01 & 0.02 & 0.00 & 0.00 & 0.01 & 0.09 \end{pmatrix}$$

NLO effect on rho martix

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At NLO level all relation are broken.

	LO	NLO
$A^1_{2,0}$	-0.59	-0.51
$A^2_{2,0}$	-0.58	-0.56
$C_{2,1,2,-1}$	-0.94	-0.95
$C_{1,1,1,-1}$	0.91	0.14
$A^1_{2,0}/\sqrt{2}+1$	0.58	0.64
$C_{2,2,2,-2}$	0.59	0.57
$C_{1,0,1,0}$	-0.61	-0.10
$C_{2,0,2,0}$	1.41	1.38
$C_{1,0,1,0}+2$	1.39	1.90

NLO effect on rho martix

$$A^a_{2,0} = A^b_{2,0} \neq 0$$

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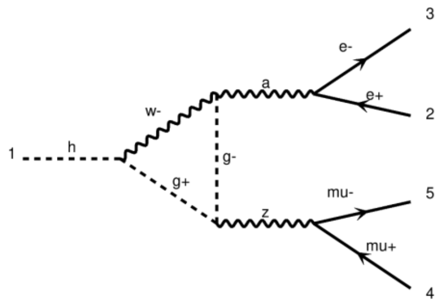

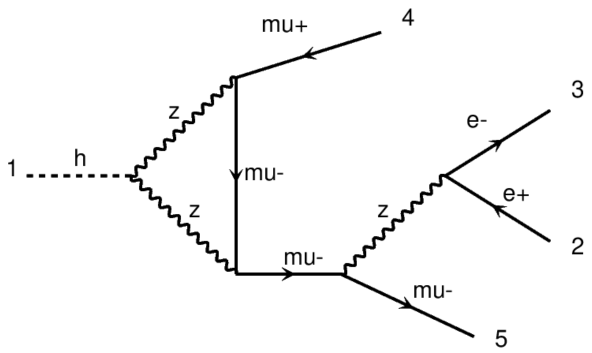

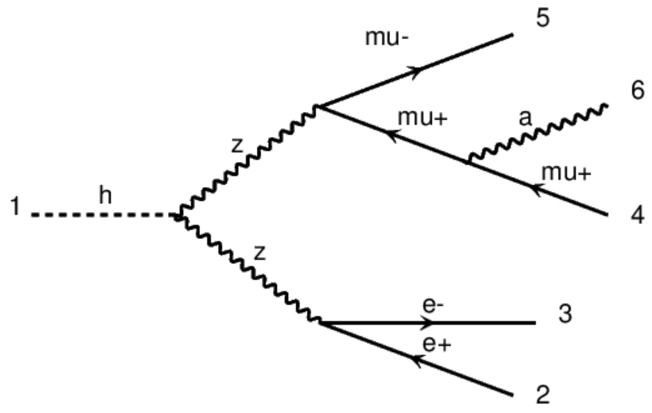

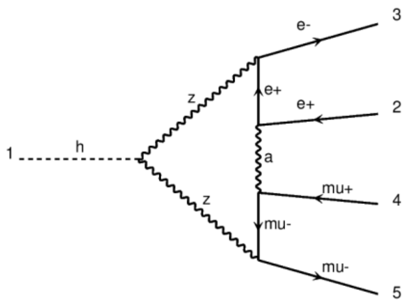

Till now we have to keep in mind

- ✓ Fing quantum tomographic state so that instead of one or two parameters you can produce whole density matrix. As we see at LO our spin density matrix was corresponding to pure state but at NLO level it is no more pure state it is correspond to mix state.
- ✓ Due to change in spin density matrix our definition of both Entanglement and Bell-non-locality condition will modify.
- ✓ We can check how NLO correction can misinterpreted with new physics e.g.
- ✓ Spin density matrix can get modified if instead of V we have some other e.g. scalar or tensor intermediate state.
- ✓ Or for modified H to VV couplings due to EFT contributions.

How NLO correction can be misinterpreted as new physics?

1. What we are measuring at collider? -> four fermion angular momentum distribution generated from Higgs decay. Lets write a generic current for H-> 4f

$$\mathcal{L}^7_{\rm EFT} = \frac{h}{\Lambda^3}\sum_i a_i \bar{\psi}_1 \Gamma^i \psi_2 \; \bar{\psi}_3 \Gamma^i \psi_4,$$

With $\Gamma^i = \{1, \gamma_5, \sigma_{\mu\nu}, \gamma_\mu, \gamma_\mu\gamma_5\}\,,$
$a_i = \{a_S, a_5, a_T, a_V, a_A\}$

This is similar to using simplified models with resonant intermediate states.

$$\begin{aligned}
\mathcal{A}_S &= c_S \bar{u}_1 (a + ib\gamma_5) v_2 \; \bar{u}_3 (a' + ib'\gamma_5) v_4 \\
\mathcal{A}_V &= c_V \; \bar{u}_1 \gamma_\mu (c_L P_L + c_R P_R) v_2 \; \bar{u}_3 \gamma_\mu (d_L P_L + d_R P_R) v_4 \\
\mathcal{A}_T &= c_T \bar{u}_1 \sigma_{\mu\nu} v_2 \; \bar{u}_3 \sigma^{\mu\nu} v_4
\end{aligned}$$

As we know the A and C coefficient are proportional to amplitude square due to following equation

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y_L^{*M}(\Omega_j)\, d\Omega_a d\Omega_b = \frac{B_L^j}{4\pi} A^j_{LM} \quad j = a, b$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y_{L_1}^{*M_1}(\Omega_a) Y_{L_2}^{*M_2}(\Omega_b)\, d\Omega_a d\Omega_b = \frac{B^a_{L_1} B^b_{L_2}}{(4\pi)^2} C_{L_1 M_1 L_2 M_2}$$

How NLO effect can be misinterpreted as new physics?

1. What we are measuring at collider? -> four fermion angular momentum distribution generated from Higgs decay. Lets write a generic current for H-> 4f

$$\mathcal{L}^7_{\rm EFT} = \frac{h}{\Lambda^3}\sum_i a_i \bar\psi_1 \Gamma^i \psi_2\, \bar\psi_3 \Gamma^i \psi_4,$$

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This is similar to using simplified models with resonant intermediate states.

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\mathcal{A}_S &= c_S \bar u_1 (a + ib\gamma_5) v_2\, \bar u_3 (a' + ib'\gamma_5) v_4 \\
\mathcal{A}_V &= c_V\, \bar u_1 \gamma_\mu (c_L P_L + c_R P_R) v_2\, \bar u_3 \gamma_\mu (d_L P_L + d_R P_R) v_4 \\
\mathcal{A}_T &= c_T \bar u_1 \sigma_{\mu\nu} v_2\, \bar u_3 \sigma^{\mu\nu} v_4
\end{aligned}$$

amplitude square for different currents

$$\begin{aligned}
\sum_s \mathcal{A}_S^* \mathcal{A}_S &= 16|c_S|^2 \Pi_0 \,(a^2 + b^2)\,(a'^2 + b'^2) \\
\sum_s \mathcal{A}_V^* \mathcal{A}_V &= 16|c_V|^2 \left[(c_L^2 d_L^2 + c_R^2 d_R^2)\,\Pi_1 + (c_L^2 d_R^2 + c_R^2 d_L^2)\,\Pi_2\right] \\
\sum_s \mathcal{A}_T^* \mathcal{A}_T &= 128|c_T|^2 (2\Pi_1 + 2\Pi_2 - \Pi_0) \\
\sum_s \mathcal{A}_S^* \mathcal{A}_T + \mathcal{A}_S \mathcal{A}_T^* &= -64\,\mathrm{Re}(c_S c_T^*)\left[(ab' + a'b)\Pi_\epsilon + (aa' - bb')(\Pi_1 - \Pi_2)\right] \\
\sum_s \mathcal{A}_V^* \mathcal{A}_T &= 0 \\
\sum_s \mathcal{A}_V^* \mathcal{A}_S &= 0
\end{aligned}$$

$$\begin{aligned}
\Pi_0 &= (p_1.p_2)(p_3.p_4) = \frac{m_a^2 m_b^2}{4} \\
\Pi_1 &= (p_1.p_3)(p_2.p_4) \\
\Pi_2 &= (p_1.p_4)(p_2.p_3) \\
\Pi_e &= (\epsilon^{p_1 p_2 p_3 p_4})
\end{aligned}$$

For SS current all A and C coefficient zero, as amplitude square of SS in independent on any angle.

V V

$$A^a_{2,0} = A^b_{2,0} \neq 0$$

$$\frac{A^a_{2,0}}{\sqrt{2}} + 1 = C_{2,2,2,-2} \neq 0$$

$$C_{1,-1,1,1} = C_{1,1,1,-1} = -C_{2,-1,2,1} = -C_{2,1,2,-1} \neq 0$$

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$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & y & 0 & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y & 0 & z & 0 & y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & y & 0 & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

T T

$$\begin{aligned} A^1_{2,0} &= A^2_{2,0} \neq 0, \qquad C_{2,0,2,0} \neq 0 \\ C_{2,-1,2,1} &= C_{2,1,2,-1} \neq 0 \qquad C_{2,-2,2,2} = C_{2,2,2,-2} \neq 0 \end{aligned}$$

$$\rho = \begin{pmatrix} x_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -x_1 & 0 & -y_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x_1 & 0 & y_1 & 0 & 4x_1 & 0 & 0 \\ 0 & -y_1 & 0 & -x_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y_1 & 0 & 1 & 0 & y_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -x_1 & 0 & -y_1 & 0 \\ 0 & 0 & 4x_1 & 0 & y_1 & 0 & x_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -y_1 & 0 & -x_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1 \end{pmatrix}$$

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T T

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$$\rho_{\mathrm{NLO}} = \begin{pmatrix} 0.08 & 0.00 & 0.00 & 0.01 & -0.01 & 0.01 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.12 & 0.01 & -0.02 & 0.00 & 0.00 & 0.01 \\ 0.00 & 0.00 & 0.12 & 0.01 & -0.18 & 0.00 & 0.19 & 0.00 & 0.00 \\ 0.00 & 0.12 & 0.01 & 0.00 & 0.01 & 0.00 & -0.01 & 0.01 & 0.01 \\ 0.01 & 0.00 & -0.18 & 0.01 & 0.59 & 0.00 & -0.18 & 0.00 & -0.01 \\ 0.00 & 0.02 & 0.00 & 0.00 & 0.00 & -0.01 & 0.01 & 0.14 & 0.00 \\ 0.00 & 0.00 & 0.19 & 0.01 & -0.18 & 0.01 & 0.12 & 0.01 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.01 & 0.00 & 0.14 & 0.01 & 0.01 & 0.00 \\ 0.00 & 0.01 & 0.00 & 0.01 & 0.02 & 0.00 & 0.00 & 0.01 & 0.09 \end{pmatrix}$$

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Effect of modified H to ZZ vertex on spin density matrix

$$A_V(EFT) = \frac{1}{v}\left(a_1 g_{\mu\nu} m_V^2 + a_2(g_{\mu\nu}p_a.p_b - p_{a\nu}p_{b\mu}) + a_3\epsilon_{\mu\nu\alpha\beta}p_a^\alpha p_b^\beta\right) * \bar{u}(p_1)\gamma_\mu(c_L P_L + c_R P_R)v(p_2)\ \bar{u}(p_3)\gamma_\nu(d_L P_L + d_R P_R)v(p_4)$$

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_+a_+^* & 0 & a_+a_0^* & 0 & a_+a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_0a_+^* & 0 & a_0a_0^* & 0 & a_0a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_-a_+^* & 0 & a_-a_0^* & 0 & a_-a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Higher Dim Operators

$$\mathcal{L}_{hZZ} = \frac{M_Z^2}{v} a_1 Z_\mu Z^\mu h + \frac{a_2}{4v} h Z_{\mu\nu} Z^{\mu\nu} + \frac{a_3}{4v} h Z_{\mu\nu}\tilde{Z}^{\mu\nu}$$

Again we got pure density matrix of pure state $|\psi> = a_+|+-> + a_0|0\ 0> + a_-|-+>$

$a_+ \neq a_-$

$$a_\pm = \frac{4}{3}\pi m_a m_b \sqrt{(c_L^2 + c_R^2)(d_L^2 + d_R^2)}\left(-2a_1^* m_V^2 + a_2^*(m_a^2 + m_b^2 - m_h^2) \mp i a_3^* \lambda^{1/2}(m_h^2, m_a^2, m_b^2)\right)$$

$$a_0 = \frac{4}{3}\pi \sqrt{(c_L^2 + c_R^2)(d_L^2 + d_R^2)}\left(2a_2^* m_a^2 m_b^2 - a_1^* m_V^2(m_a^2 + m_b^2 - m_h^2)\right)$$

EFT

Complex numbers

$$A^a_{1,0} = -A^b_{1,0}$$
$$A^a_{2,0} = A^b_{2,0}$$
$$\frac{A^a_{2,0}}{\sqrt{2}} + 1 = -C_{1,0,1,0}$$
$$C_{1,-1,1,1} = C^*_{1,1,1,-1} = -C_{2,-1,2,1} = -C^*_{2,1,2,-1}$$
$$C_{2,2,2,-2} = C^*_{2,-2,2,2}$$

New non-zero coefficiients

$$C_{1,0,2,0} = -C_{2,0,1,0}$$
$$C_{2,0,2,0} = 2 + C_{1,0,1,0}$$
$$\frac{A^b_{1,0}}{\sqrt{2}} = C_{2,0,1,0}$$
$$C_{1,-1,2,1} = C^*_{1,1,2,-1} = -C_{2,-1,1,1} = -C^*_{2,1,1,-1}$$

$$\rho = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 + \frac{1}{\sqrt{2}}A^2_{2,0} - \sqrt{\frac{3}{2}}A^2_{1,0} & 0 & -C_{2,1,1,-1} + C_{2,1,2,-1} & 0 & C_{2,2,2,-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C_{2,-1,1,1} + C_{2,-1,2,1} & 0 & 1 - \sqrt{2}A^2_{2,0} & 0 & C_{2,1,1,-1} + C_{2,1,2,-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{2,-2,2,2} & 0 & C_{2,-1,1,1} + C_{2,-1,2,1} & 0 & 1 + \frac{1}{\sqrt{2}}A^2_{2,0} + \sqrt{\frac{3}{2}}A^2_{1,0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

All 9 entries are different from non-zero.

SM

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EFT

SM

Complex numbers

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$$C_{2,2,2,-2} = C^*_{2,-2,2,2}$$

New non-zero coeffi

$$C_{1,0,2,0} = -C_{2,0,1,0}$$
$$C_{2,0,2,0} = 2 + C_{1,0,1,0}$$
$$\frac{A^b_{1,0}}{\sqrt{2}} = C_{2,0,1,0}$$
$$C_{1,-1,2,1} = C^*_{1,1,2,-1} = -C_{2,-1,1,1} = -C^*_{2,1,1,-1}$$

$$\left(a_1 g_{\mu\nu} m_V^2 + a_2(g_{\mu\nu} p_a \cdot p_b - p_{a\nu} p_{b\mu}) + a_3 \epsilon_{\mu\nu\alpha\beta} p_a^\alpha p_b^\beta\right)$$

- If a_1, a_2, a_3 all are real. than all these extra new coefficient is zero. Which means $A^{a/b}_{1,0} \neq 0$ is direct signal of CP-violation.
- If a_3 is zero than $A^{a/b}_{1,0} = 0$ but $C_{1,-1,2,1} \neq 0$ if one of a_1, a_2 is complex.

$$\rho = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 + \frac{1}{\sqrt{2}}A^2_{2,0} - \sqrt{\frac{3}{2}}A^2_{1,0} & 0 & -C_{2,1,1,-1} + C_{2,1,2,-1} & 0 & C_{2,2,2,-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C_{2,-1,1,1} + C_{2,-1,2,1} & 0 & 1 - \sqrt{2}A^2_{2,0} & 0 & C_{2,1,1,-1} + C_{2,1,2,-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{2,-2,2,2} & 0 & C_{2,-1,1,1} + C_{2,-1,2,1} & 0 & 1 + \frac{1}{\sqrt{2}}A^2_{2,0} + \sqrt{\frac{3}{2}}A^2_{1,0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

All 9 entries are different from non-zero.

a_1, a_2, a_3 all are real

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & y & 0 & x_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y^* & 0 & z & 0 & y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x_1^* & 0 & y^* & 0 & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

a_3 is zero

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & y & 0 & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y^* & 0 & z & 0 & y^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & y & 0 & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Conclusion:

- As we see in top sector LHC is sensitive to probe quantum observables. So it is the time we do computation with more detail.
- It is useful to use quantum state tomography to relate spin density matrix directly to experiment data.
- NLO correction can effect quantum state which will change the entanglement and bell-nonlocality condition.
- It is still possible to look new physics but have to be careful to define the right observable which will not effect by NLO corrections.
- And we can also look for the parameter space where we can reduce NLO correction e.g if we can force one Z be on-shell then NLO corrections to the spin density matrix can reduce although it will also reduce number of events.
- Stay tuned for final paper you will find more information with more detail.

Thank you for the attention!

Back-up slides

$$
\begin{aligned}
x &= \frac{4m_a^2 m_b^2}{m_a^4 + 10m_b^2 m_a^2 + m_b^4 + m_h^4 - 2\left(m_a^2 + m_b^2\right) m_h^2} \\
y &= \frac{2m_a m_b \left(m_a^2 + m_b^2 - m_h^2\right)}{m_a^4 + 10m_b^2 m_a^2 + m_b^4 + m_h^4 - 2\left(m_a^2 + m_b^2\right) m_h^2} \\
z &= \frac{\left(m_a^2 + m_b^2 - m_h^2\right)^2}{m_a^4 + 10m_b^2 m_a^2 + m_b^4 + m_h^4 - 2\left(m_a^2 + m_b^2\right) m_h^2}
\end{aligned}
$$

$$
\begin{aligned}
x_1 &= \frac{m_a^2 m_b^2}{m_a^4 + \left(m_b^2 - 2m_h^2\right) m_a^2 + \left(m_b^2 - m_h^2\right)^2} \\
y_1 &= \frac{m_a\, m_b \left(m_a^2 + m_b^2 - m_h^2\right)}{m_a^4 + \left(m_b^2 - 2m_h^2\right) m_a^2 + \left(m_b^2 - m_h^2\right)^2}
\end{aligned}
$$

One example is the fermion-photon distance for recombination ΔR: a smaller value will correspond to more radiation appearing as real emissions.

	no cuts	$m_{Z_2} > 10$ GeV	> 20 GeV	> 30 GeV
LO	0.58	0.62	0.71	0.77
$A^1_{2,0}/\sqrt{2}+1$, LO	0.58	0.62	0.71	0.78
NLO	0.55	0.58	0.69	0.75
$A^1_{2,0}/\sqrt{2}+1$, NLO	0.70	0.73	0.83	0.89

Table 8: Values obtained for the coefficient $C_{2,2,2,-2}$.

	no cuts	$m_{Z_2} > 10$ GeV	> 20 GeV	> 30 GeV
LO	-0.93	-0.98	-1.02	-1.02
NLO	-0.95	-1.01	-1.03	-1.03

Table 9: Values obtained for the coefficient $C_{2,1,2,-1}$.

	no cuts	$m_{Z_2} > 10$ GeV	> 20 GeV	> 30 GeV
LO	-0.59	-0.53	-0.41	-0.32
NLO	-0.42	-0.37	-0.24	-0.16

Table 10: Values obtained for the coefficient $A^1_{2,0}$.

	no cuts	$m_{Z_2} > 10$ GeV	> 20 GeV	> 30 GeV
LO	2.60	2.67	2.77	2.80
NLO	2.64	2.75	2.82	2.84

Table 11: Values obtained for the observable I_3.

	no cuts	$80 < m_{Z_1} < 100$ GeV	$85 < m_{Z_1} < 95$ GeV
LO	0.58	0.61	0.61
$A^1_{2,0}/\sqrt{2}+1$, LO	0.58	0.61	0.61
NLO	0.56	0.60	0.61
$A^1_{2,0}/\sqrt{2}+1$, NLO	0.64	0.60	0.61

Table 12: Values obtained for the coefficient $C_{2,2,2,-2}$.

	no cuts	$80 < m_{Z_1} < 100$ GeV	$85 < m_{Z_1} < 95$ GeV
LO	-0.94	-0.96	-0.95
NLO	-0.94	-0.95	-0.96

Table 13: Values obtained for the coefficient $C_{2,1,2,-1}$.

	no cuts	$80 < m_{Z_1} < 100$ GeV	$85 < m_{Z_1} < 95$ GeV
LO	-0.59	-0.56	-0.56
NLO	-0.51	-0.55	-0.55

Table 14: Values obtained for the coefficient $A^1_{2,0}$.

	no cuts	$80 < m_{Z_1} < 100$ GeV	$85 < m_{Z_1} < 95$ GeV
LO	2.60	2.64	2.63
NLO	2.61	2.63	2.65

Table 15: Values obtained for the observable I_3.

Quantum tomography

➢ There are at least 3-4 parameterization to define rho matrix in bipartite qutrit system e.g.

1. define your state $|\psi_i>$of system the we can construct density matrix using $\rho = \sum p_i |\psi_i >< \psi_i|$.
2. Using helicity dependent amplitude computation
3. The generalized Gell-Mann matrix basis parametrization.
4. The Weyl operator basis
5. The Polarization operator basis parametrization.

$$
\begin{aligned}
A^1_{2,0} &= A^2_{2,0} \neq 0, \qquad C_{2,0,2,0} \neq 0 \\
C_{2,-1,2,1} &= C_{2,1,2,-1} \neq 0 \qquad C_{2,-2,2,2} = C_{2,2,2,-2} \neq 0 \\
C_{1,-1,1,1} &= C^*_{1,1,1,-1} \neq 0 \qquad C_{1,0,1,0} \neq 0
\end{aligned}
$$

$$
\rho = \begin{pmatrix}
x_2+r_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & x_3 & 0 & -y_2-r_2^* & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & x_2-r_1 & 0 & y_2-r_2^* & 0 & y_3 & 0 & 0 \\
0 & -y_2-r_2 & 0 & x_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & y_2-r_2 & 0 & 1/9 & 0 & y_2-r_2^* & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & x_3 & 0 & -y_2-r_2^* & 0 \\
0 & 0 & y_3 & 0 & y_2-r_2 & 0 & x_2-r_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -y_2-r_2 & 0 & x_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_2+r_1
\end{pmatrix} \quad (6.2
$$

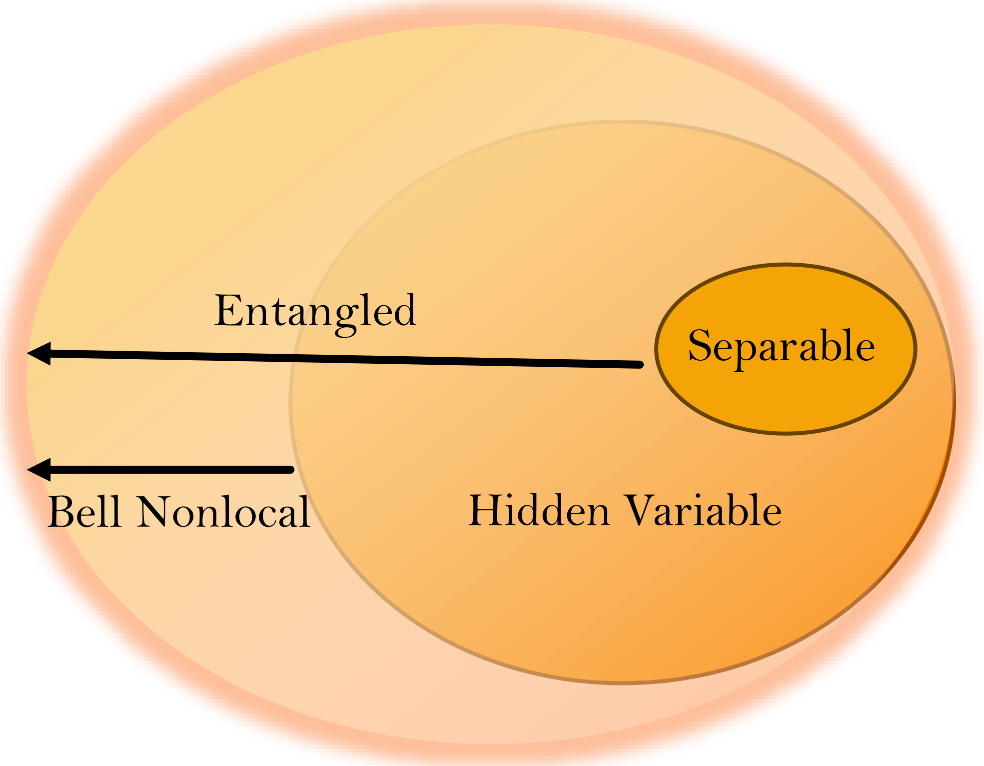
$$|\mathcal{M}(H \to aa'bb')|^2 = \sum_s \mathcal{A}_V(EFT)^* \mathcal{A}_V(EFT) = |A_1|^2 + |A_2|^2 + |A_3|^2 + A_{12} + A_{13} + A_{23} \quad (2.10)$$

where

$$\begin{aligned}
|A_1|^2 &= 16|a_1|^2 M_V^4 \left[\Pi_1(c_L^2 d_L^2 + c_R^2 d_R^2) + \Pi_2(c_L^2 d_R^2 + c_R^2 d_L^2)\right] \\
|A_2|^2 &= 16|a_2|^2 \left[H_1(c_L^2 d_L^2 + c_R^2 d_R^2) + H_2(c_L^2 d_R^2 + c_R^2 d_L^2)\right] \\
|A_3|^2 &= 16|a_3|^2 \left[H_3(c_L^2 d_L^2 + c_R^2 d_R^2) + H_4(c_L^2 d_R^2 + c_R^2 d_L^2)\right] \\
A_{12} &= A_1 A_2^* + A_1^* A_2 \\
&= 8M_V^2 \Big[(a1^* a2 + a1 a2^*)\left[(c_L^2 d_L^2 + c_R^2 d_R^2)(\Sigma_1 K_1) + (c_L^2 d_R^2 + c_R^2 d_L^2)(\Sigma_2 K_2)\right] \\
&+ i(a1^* a2 - a1 a2^*)\left[(c_L^2 d_L^2 - c_R^2 d_R^2)\Pi_e \Sigma_3 - (c_L^2 d_R^2 - c_R^2 d_L^2)\Pi_e \Sigma_4\right]\Big] \\
A_{13} &= A_1 A_3^* + A_1^* A_3 \\
&= -8iM_V^2 \Big[(a1 a3^* - a1^* a3)\left[(c_L^2 d_L^2 - c_R^2 d_R^2)(\Sigma_3 K_1) + (c_L^2 d_R^2 - c_R^2 d_L^2)(\Sigma_4 K_2)\right] \\
&- i(a1^* a3 + a1 a3^*)\left[(c_L^2 d_L^2 + c_R^2 d_R^2)\Pi_e \Sigma_1 - (c_L^2 d_R^2 + c_R^2 d_L^2)\Pi_e \Sigma_2\right]\Big] \\
A_{23} &= A_2 A_3^* + A_2^* A_3 \\
&= -16i\Big[\Pi_0(a2 a3^* - a2^* a3)\left[(c_L^2 d_L^2 - c_R^2 d_R^2)\Sigma_1 \Sigma_3 + (c_L^2 d_R^2 - c_R^2 d_L^2)\Sigma_2 \Sigma_4\right] \\
&- i(a2^* a3 + a2 a3^*)\left[(c_L^2 d_L^2 + c_R^2 d_R^2)\Pi_e K_1 - (c_L^2 d_R^2 + c_R^2 d_L^2)\Pi_e K_2\right]\Big] \qquad (2.11)
\end{aligned}$$

Quantum Information at High Energy Colliders :

- Entanglement is a quantum phenomenon where two or more particles become correlated in such a way that the quantum state of each particle cannot be described independently of the state of the others, even when they are space-like separated. It is one of the features that can't explain by classical mechanics.
- Depending on spin-1/2 pair or spin-1 pair quantum observable change. It is challenging to construct quantities which capture non-local behaviour depending on channels
- We are interested in different degree of quantum correlation
- We need a way to define observation such way that they depends on spin-correlation matrix.



- Different inequality is sensitive for different systems(e.g. CHSH inequality for biparticle qubits, CGLMP inequality for biparticle qutrits etc.)

bipartite qutrit system:

The state is entangled if it can't be written as

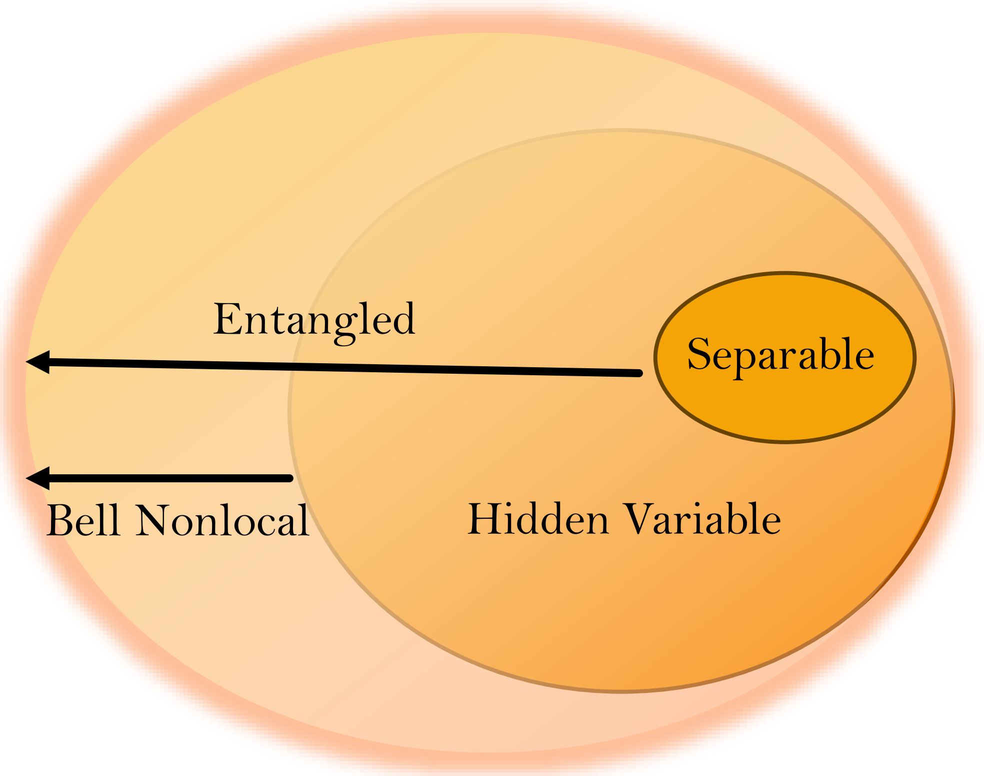
$$|\psi> = |\phi^{\alpha}> \otimes |\phi^{\beta}>$$

❖ *Entanglement* $\rho \equiv \sum_{\mathbf{k}} \mathbf{p_k} \rho_{\mathbf{k}}^{\alpha} \bigotimes \rho_{\mathbf{k}}^{\beta} \longrightarrow \rho^{\mathbf{T}_\beta} \equiv \sum_{\mathbf{k}} \mathbf{p_k} \rho_{\mathbf{k}}^{\alpha} \bigotimes [\rho_{\mathbf{k}}^{\beta}]^{\mathbf{T}}$

Peres-Horodecki (1996, 1997)

Non-positive definite → State is entangled

If it is still Physics density matrix with Tr=1 and Positive definite → State is separable



It is analytically unknown for a mix state to determining entanglement necessary and sufficient condition for bipartite system of dimension larger than 2×3, where dimension defined as d=(3s+1). But some sufficient condition is known like
A state can only have non-zero concurrence if it is entangled.
There are lower bound on square of concurrence
1.

Terminology:

- In this talk I will only discuss about bipartite system of dim $d = 2j + 1$ (also called qudit)
- I am interested spin correlation in massive vector boson pair which has spin-1 also called qutrit pair

Talk on three-particle entanglement in particle decay and scattering by Kazuki sakurai
Talk on Entanglment in QED scattering processes by Bruno Micciola
Talk on Entanglement in flavored scalar scattering by Enrico Maria Sessolo

The state is entangled if it can't be written as

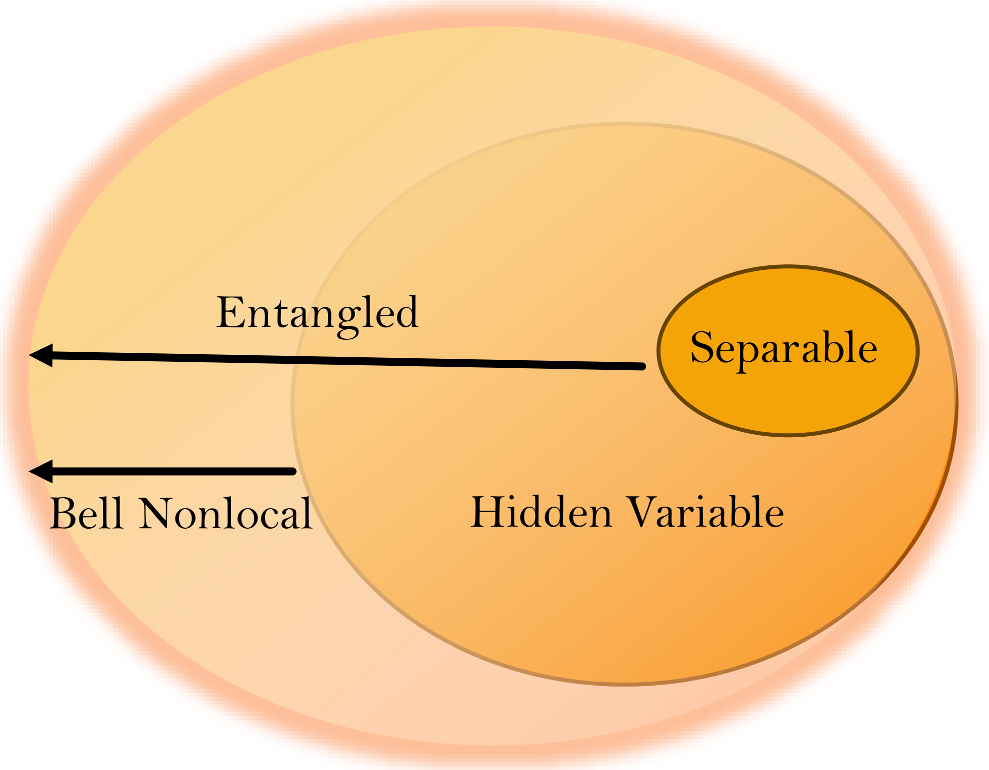
$$|\psi> = |\phi^{\alpha}> \otimes |\phi^{\beta}>$$

Once we know the state we can define density matrix as

$$\rho = |\psi><\psi|$$

With density matrix we can compute entanglement, bell nonlocality.

Although at colliders we don't know the state so we need another way to directly compute density matrix



Thanks