Natural Alignment and CP Violation Beyond the Standard Model

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Based on:

- P.S.B. Dev, AP, JHEP1412 (2014) 024
- AP, PRD93 (2016) 075012; arXiv:2408.04511
- N. Darvishi, AP, PRD99 (2019) 115014; PRD101 (2020) 095008 . . .
- N. Darvishi, AP, J.-H. Yu, JHEP05 (2024) 233

Outline:

- **The Standard Theory of Electroweak Symmetry Breaking**

- **Brief history of symmetries for natural SM alignment**

- **SM Alignment in the 2HDM and Beyond**

- **CP Violation in 2HDM: Explicit, Spontaneous and Mixed**

- **Phenomenology at the LHC**

- **Summary**

Higgs Mechanism in the SM: $SU(3)_{\text{colour}} \otimes SU(2)_L \otimes U(1)_Y$

[P. W. Higgs '64; F. Englert, R. Brout '64.]
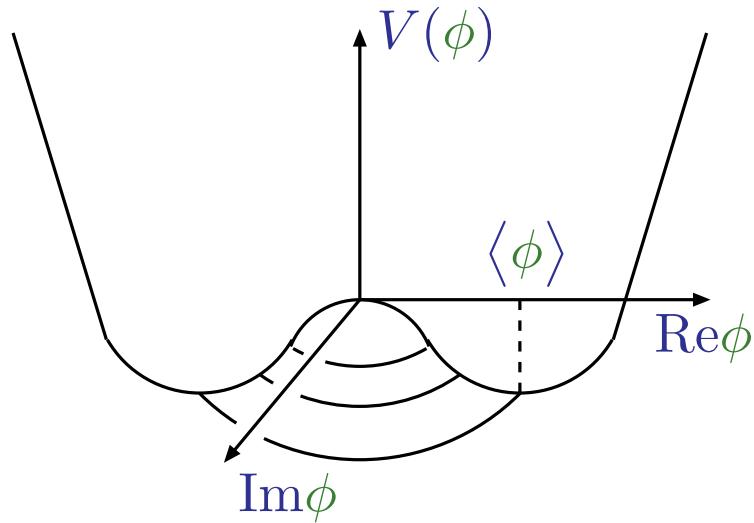
Higgs potential $V(\phi)$

$$V(\phi) = -m^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \ .$$

Ground state:

$$\langle \phi \rangle = \sqrt{\frac{m^2}{2\lambda}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

carries weak charge, but no electric charge and colour.

Custodial Symmetry of the SM with $g' = Y_f = 0$ and $V(\phi)$:

[P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov '80.]

$$\boldsymbol{\Phi} \equiv \left(\phi \, , \, i\sigma^2 \phi^* \right) \quad \mapsto \quad \boldsymbol{\Phi}' \equiv U_L \, \boldsymbol{\Phi} \, U_{\text{C}} \, ,$$

with $U_L \in SU(2)_L$ and $U_{\text{C}} \in SU(2)_{\text{C}}$, and $SU(2)_L \otimes SU(2)_{\text{C}}/\mathbb{Z}_2 \simeq S\mathbb{O}(4)$.

On the SM Higgs-Boson at the LHC

ATLAS & CMS Results 2019:
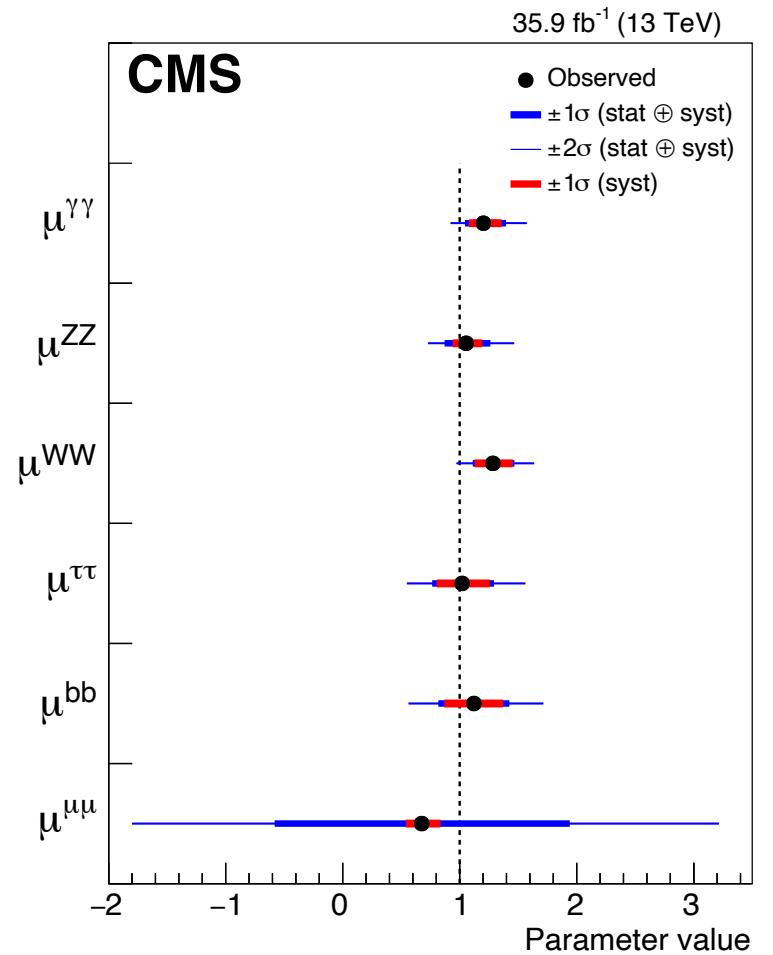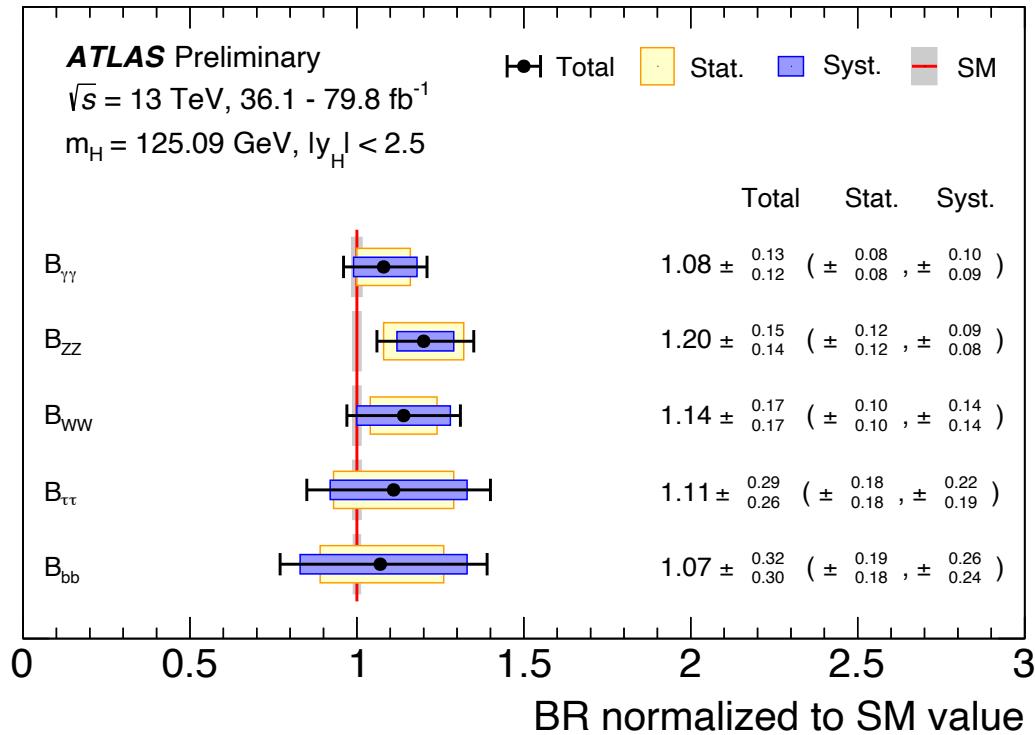
• Brief history of symmetries for natural SM alignment

– Flavour unitarity of the CKM mixing matrix
[Gell-Mann, Levy '60; Cabbibo '63; Kobayashi, Maskawa '73]

– GIM mechanism to suppress s-changing interactions at the quantum level
(requires the existence of the c-quark) [Glashow, Iliopoulos, Maiani '70]

– Conditions for diagonal neutral currents in Z-boson interactions to quarks
[Paschos '77]

– Natural diagonal neutral currents in Z- & multi-Higgs-boson interactions
to quarks [Glashow, Weinberg '77; Peccei, Quinn '77]

– Renormalizable models with partial flavour non-conservation at tree level
(GIM suppressed). [Branco, Grimus, Lavoura '96]

– Yukawa alignment in the 2HDM broken by RG effects
(induced by non-diagonal PQ charges of Higgs doublets) [Pich, Tuzon '09]

– Natural Alignment Beyond the Standard Model [Dev, Pilaftsis '14, AP '16
(for all Higgs couplings to gauge bosons and fermions) & this talk]

• 2HDM potential

[TD Lee '73; AP, C Wagner '99;

Review: Branco, Ferreira, Lavoura, Rebelo, Sher, Silva '12.]

$$
\begin{aligned}
V = &-\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) - m_{12}^2(\phi_1^\dagger\phi_2) - m_{12}^{*2}(\phi_2^\dagger\phi_1) \\
&+ \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
&+ \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) \\
&+ \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1) \,.
\end{aligned}
$$

• Physical (CP-conserving) spectrum:

CP-even Higgs bosons H and h;   CP-odd scalar a;   charged scalars $h^\pm$.

• Higgs coupling to gauge bosons $V = W, Z$:

$$
g_{HVV} = \cos(\beta - \alpha)\,, \qquad g_{hVV} = \sin(\beta - \alpha)\,,
$$

where $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle$ and α diagonalizes the CP-even mass matrix.

- **Global fit to SM mis-alignment**
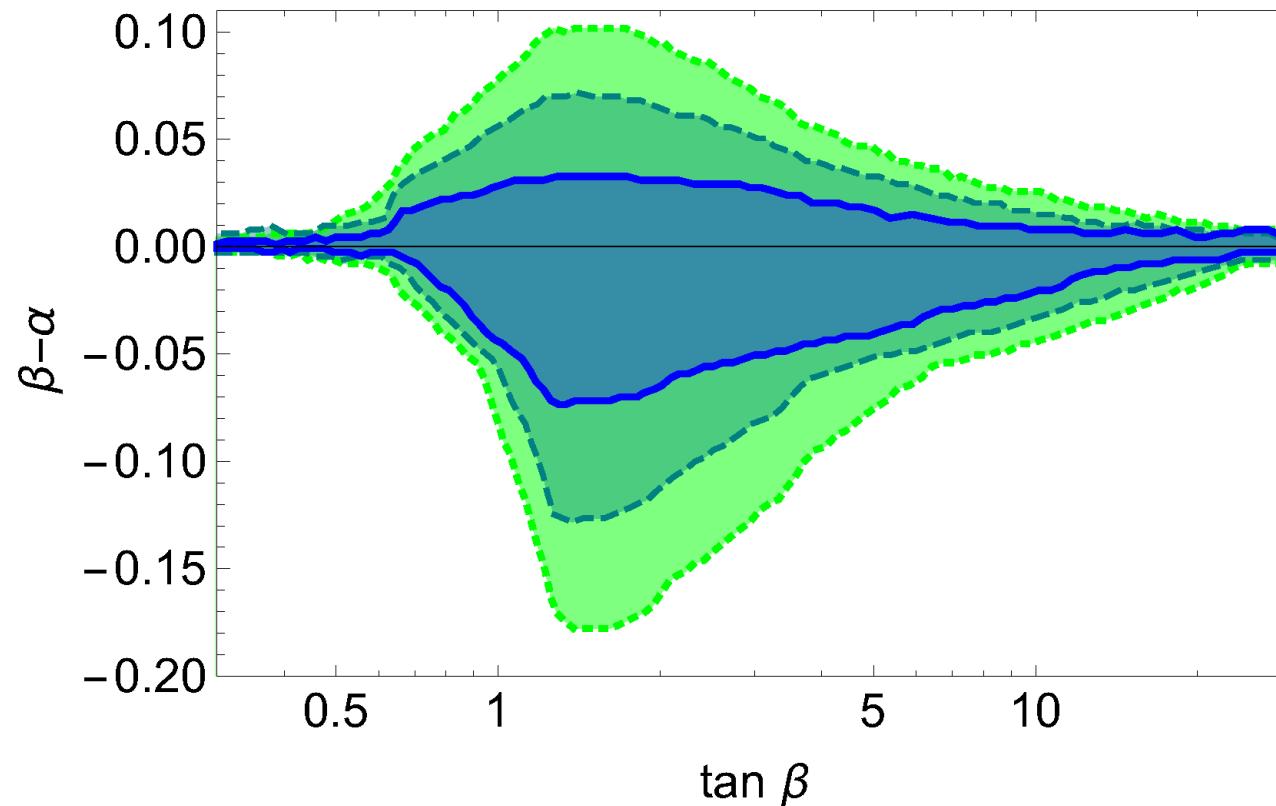
Pheno favoured limit $\beta \to \alpha$: $g_{HVV} = \cos(\beta - \alpha) \;\to\; g_{H_{\text{SM}}VV} = 1.$

- **SM Alignment** $\beta \to \alpha$:

(i) (Non-)Decoupling: [Appelquist, Carazzone '74; Georgi, Nanopoulos '79; Donoghue, Li '79;
Gunion, Haber '03; Ginzburg, Krawczyk '05.]

$$M_h^2 \;\simeq\; M_a^2 + \lambda_5 v^2 \;\simeq\; M_{h^\pm}^2 \;\gg\; v_{\mathsf{SM}}^2$$

$$M_H^2 \;\simeq\; 2\lambda_{\mathsf{SM}} v^2 - \frac{v^4 s_\beta^2 c_\beta^2}{M_a^2 + \lambda_5 v^2} \left[s_\beta^2 \left(2\lambda_2 - \lambda_{345}\right) - c_\beta^2 \left(2\lambda_1 - \lambda_{345}\right) \,+\, \ldots \right]^2$$

(ii) Fine-tuning: [Krawczyk et al. '99; Carena, Low, Shah, Wagner '14; Dev, AP '14.]

$$\lambda_7 t_\beta^4 \;-\; (2\lambda_2 - \lambda_{345}) t_\beta^3 \;+\; 3(\lambda_6 - \lambda_7)\, t_\beta^2 \;+\; (2\lambda_1 - \lambda_{345}) t_\beta - \lambda_6 \;=\; 0$$

(iii)   Natural SM Alignment (independent of $M_{h\pm}$ and t_β):   [PSB Dev, AP '14

N Darvishi, AP, J-H Yu, arXiv:2312.00882]

$$\lambda_1 = \lambda_2 = \frac{\lambda_{345}}{2} \quad (\text{with } \lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5), \qquad \lambda_6 = \lambda_7 = 0$$

Symmetries:

1.	Sp(4):	$\lambda_4 = \lambda_5 = 0$
2.	$\text{SU(2)}_{\text{HF}} \times \text{U(1)}_Y$:	$\lambda_5 = 0$
3.	$\mathcal{CP} \times \text{SO(2)}_{\text{HF}} \times \text{U(1)}_Y$:	$\lambda_{3,4,5} \neq 0$
4*	$\mathcal{CP} \times \text{Sp(2)}_{\phi_1 \widetilde{\phi}_2} \times \text{Sp(2)}_{\phi_2 \widetilde{\phi}_1}$:	$\lambda_4 = -\lambda_5$
5.	$\text{SO(2)}_{\text{HF}} \times \text{Sp(2)}_{\phi_1 + \phi_2}$:	$\lambda_4 = \lambda_5$

*Noted by Aiko, Kanemura '21

[Battye, Brawn, AP, '11; AP, '12

AP '24]

Sp(4) **Majorana** field multiplet:

$$\boldsymbol{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2 \phi_1^* \\ i\sigma^2 \phi_2^* \end{pmatrix}, \qquad \boldsymbol{\Phi} = C\boldsymbol{\Phi}^*$$

$\Rightarrow$ $\boldsymbol{\Phi}$ as a Dirac fermion: arXiv:2408.04511

References (*an incomplete list on SM Alignment in the 2HDM*)

- **On the SM Higgs basis (also Decoupling of FCNC Effects):**
  H. Georgi and D. V. Nanopoulos, Phys. Lett. B82 (1979) 95.

- **Alignment via Decoupling:**
  - J. F. Gunion, H. E. Haber, Phys. Rev. D67 (2003) 075019.
  - I. F. Ginzburg, M. Krawczyk, Phys. Rev. D72 (2005) 115013.

- **Alignment via Fine-tuning:**
  - P. H. Chankowski, T. Farris, B. Grzadkowski, J. F. Gunion, J. Kalinowski, M. Krawczyk, Phys. Lett. B496 (2000) 195.
  - A. Delgado, G. Nardini, M. Quiros, JHEP1307 (2013) 054.
  - M. Carena, I. Low, N. R. Shah, C. E. M. Wagner, JHEP1404 (2014) 015.

- **Natural Alignment without Decoupling and without Fine-tuning:**
  - P.S.B. Dev, AP, JHEP1412 (2014) 024.
  - B. Grzadkowski, O. M. Ogreid, P. Osland, Phys. Rev. D94 (2016) 115002.

References (*an incomplete list on symmetries in the 2HDM*)

- **Spontaneous CP Violation:** T. D. Lee, Phys. Rev. D8 (1973) 1226;
  G.C. Branco, Phys. Rev. D22 (1980) 2901;
  G.C. Branco, M.N. Rebelo, Phys. Lett. B160 (1985) 117.

- **Z_2 symmetry:** S. L. Glashow, S. Weinberg, Phys. Rev. D15 (1977) 1958.

- **PQ U(1) symmetry:** R. D. Peccei, H. R. Quinn, Phys. Rev. Lett. 38 (1977) 1440.

- **Custodial $SU(2)_L$-preserving symmetry:**
  P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov, Nucl. Phys. B173 (1980) 189.

- **Bilinear formalism:**
  M. Maniatis, A. von Manteuffel, O. Nachtmann, F. Nagel, EPJC48 (2006) 805;
  C. C. Nishi, Phys. Rev. D74 (2006) 036003.

- **$SU(2)_L \otimes U(1)_Y$-preserving symmetries:** **<u>6</u>**
  I. P. Ivanov, Phys. Rev. D75 (2007) 035001;
  P. M. Ferreira, H. E. Haber, J. P. Silva, Phys. Rev. D79 (2009) 116004.

- **Custodial $SU(2)_L$-preserving symmetries:** **<u>7</u>**
  R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.

- **On completeness and uniqueness of classification: <u>6</u> + <u>7</u> = <u>13</u>**
  AP, Phys. Lett. B706 (2012) 465.

- **Maximally Symmetric Two Higgs Doublet Model**

$G_{\boldsymbol{\Phi}} = \mathsf{SU(2)}_L \otimes \mathsf{Sp(4)}/\mathsf{Z}_2 \simeq \mathsf{SU(2)}_L \otimes \mathsf{SO(5)}.$

$$V = -\mu^2\left(|\Phi_1|^2 + |\Phi_2|^2\right) + \lambda\left(|\Phi_1|^2 + |\Phi_2|^2\right)^2 = -\frac{\mu^2}{2}\boldsymbol{\Phi}^\dagger\boldsymbol{\Phi} + \frac{\lambda}{4}\left(\boldsymbol{\Phi}^\dagger\boldsymbol{\Phi}\right)^2,$$

where

$$\boldsymbol{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix}, \quad \text{with } U_L \in \mathsf{SU(2)}_L : \boldsymbol{\Phi} \mapsto \boldsymbol{\Phi}' = U_L\boldsymbol{\Phi},$$

so that under **global field transformations,**

$$\mathsf{Sp}(4): \boldsymbol{\Phi} \mapsto \boldsymbol{\Phi}' = U\boldsymbol{\Phi}, \quad \text{with } U \in \mathsf{U(4)} \ \& \ U\mathsf{C}U^\mathsf{T} = \mathsf{C} \equiv i\sigma^2 \otimes \sigma^0$$

$\mathsf{SU(2)}_L$ gauge kinetic terms remain invariant.

Breaking Effects: $-m_{12}^2\,\phi_1^\dagger\phi_2$, $\mathsf{U(1)}_Y$ coupling g', Yukawa couplings $\mathbf{Y}^{u,d}$.
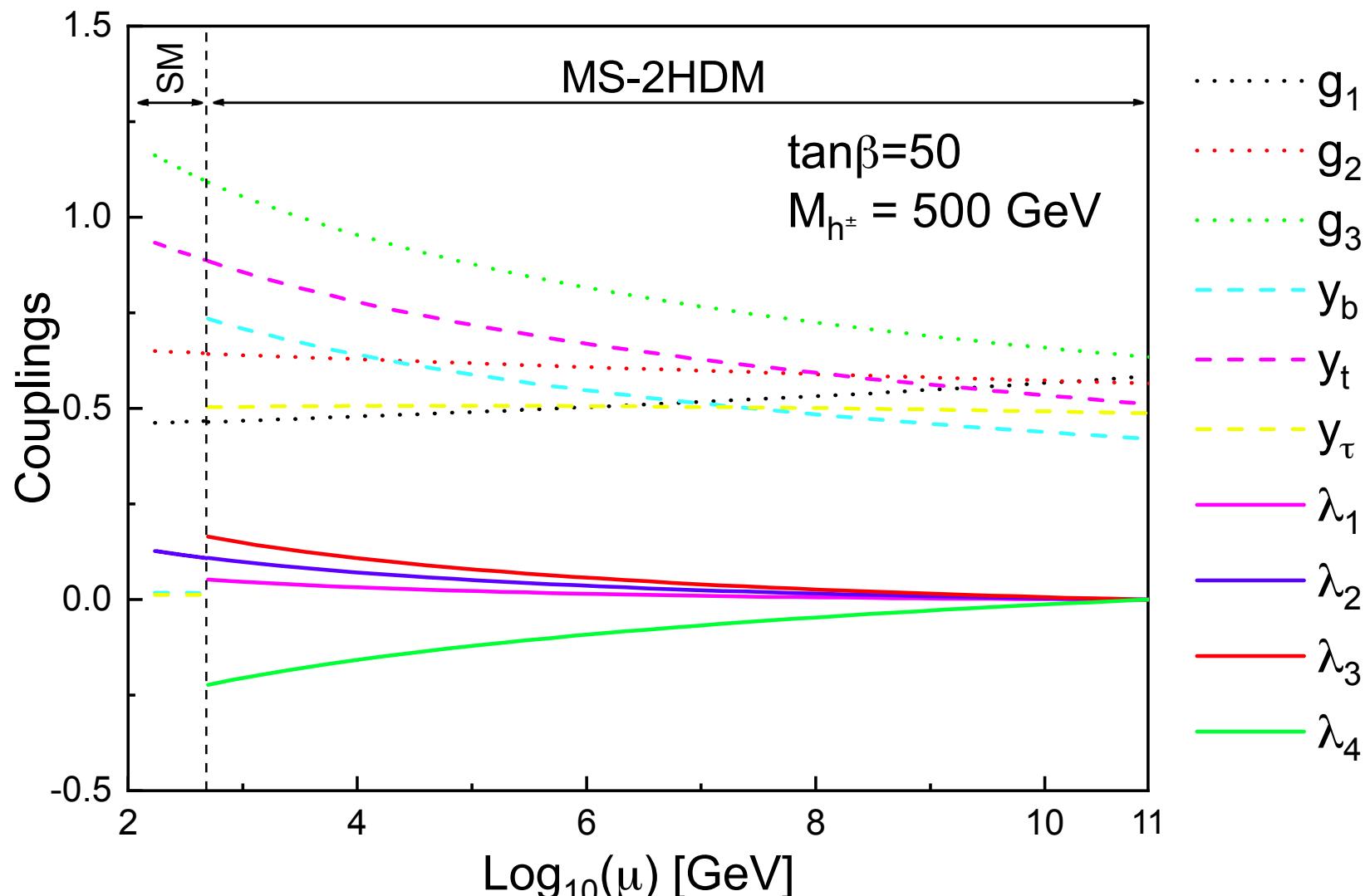
First conformal unification point: $\mu_X^{(1)} \sim 10^{11}$ GeV (of order PQ scale)

– Low- and high-scale quartic coupling unification: $\tan\beta$ **vs** $\mu_X^{(1,2)}$
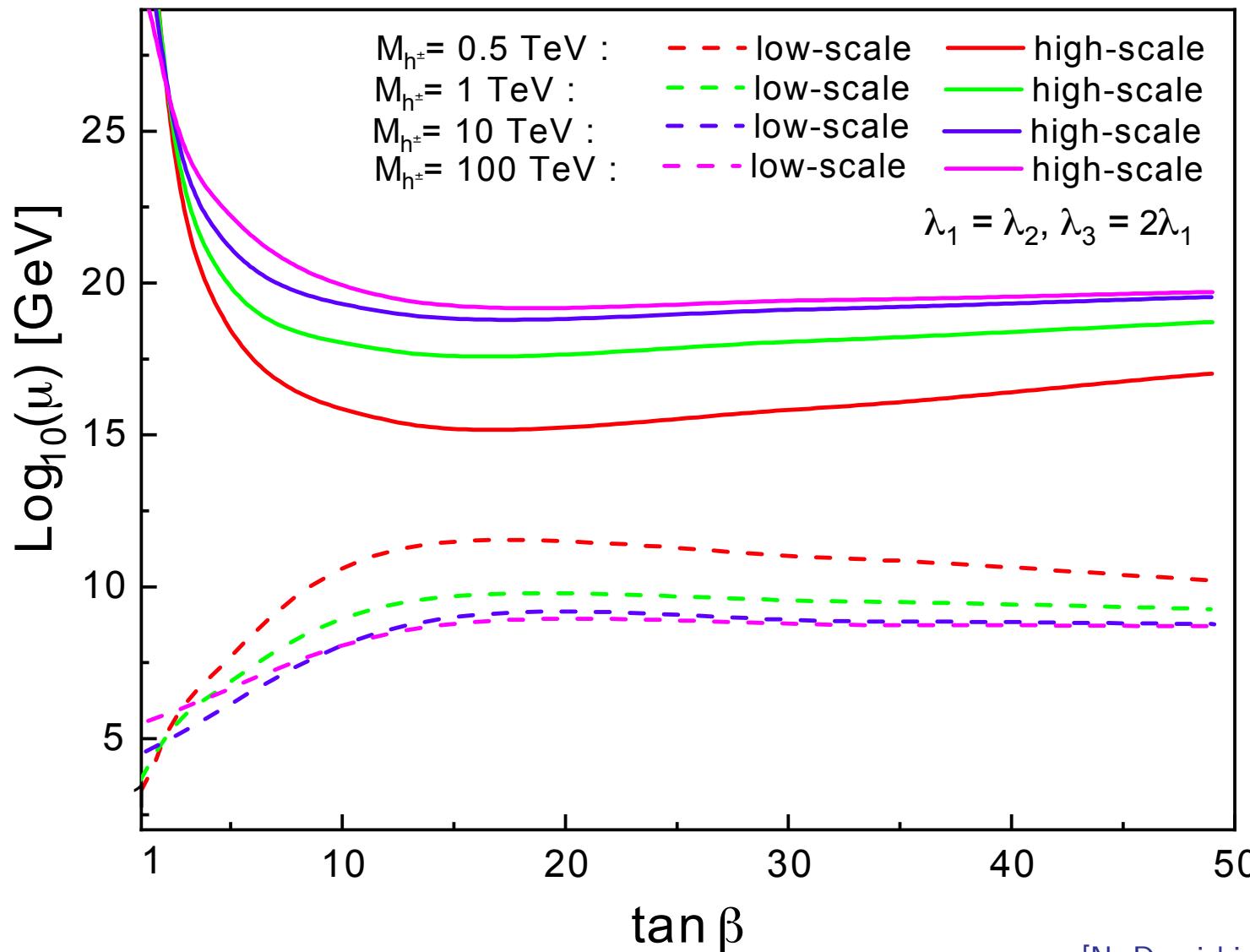
[N. Darvishi, AP '19]

• Natural Alignment Beyond the 2HDM

For nHDM with $m < n$ inert scalar doublets, there are still **3** continuous alignment symmetries in the **field space of the *non*-inert sector**:

$$\text{(i)}\ \ \mathsf{Sp}(2N_H) \times \mathcal{D}; \qquad \text{(ii)}\ \ \mathsf{SU}(N_H) \times \mathcal{D}; \qquad \text{(iii)}\ \ \mathsf{SO}(N_H) \times \mathcal{CP} \times \mathcal{D},$$

where $N_H = n - m$, $\mathcal{D}$ acts on the inert sector *only*, and $\mathcal{CP}$ is the canonical CP: $\Phi_i(t, \mathbf{x}) \to \Phi_i^*(t, -\mathbf{x})$ (with $i = 1, 2, \ldots, N_H$).

Symmetry invariants:

$$\text{(i)} \quad S \;=\; \Phi_1^\dagger \Phi_1 \;+\; \Phi_2^\dagger \Phi_2 \;+\; \ldots \;=\; \tfrac{1}{2}\, \mathbf{\Phi}^\dagger\, \mathbf{\Phi}$$

$$\text{(ii)} \quad D^a \;=\; \Phi_1^\dagger \sigma^a \Phi_1 \;+\; \Phi_2^\dagger \sigma^a \Phi_2 \;+\; \ldots$$

$$\text{(iii)} \quad T \;=\; \Phi_1 \Phi_1^\mathsf{T} \;+\; \Phi_2 \Phi_2^\mathsf{T} \;+\; \ldots$$

Symmetric part of the scalar potential:

$$V_{\mathsf{sym}} \;=\; -\mu^2\, S \;+\; \lambda_S\, S^2 \;+\; \lambda_D\, D^a D^a \;+\; \lambda_T\, \mathsf{Tr}\,(T T^*)\,.$$

Minimal Symmetry of Alignment: $\mathsf{Z}_2^{\mathsf{EW}} \times \mathsf{Z}_2^{\mathsf{I}}$.

– Quartic Coupling Unification in the MS-3HDM

Input parameters: $\tan\beta_1 = v_2/v_1$, $\tan\beta_2 = v_3/\sqrt{v_1^2 + v_2^2}$,

$M_{h_{1,2}^\pm}$ and $h_1^\pm h_2^\mp$-mixing angle: σ

Low-Scale Quartic Coupling Unification

$M_{h_1^\pm}$=500 GeV

$\tan\beta_1$=50

$\tan\beta_2$=0.018

λ_{11} λ_{1122}
λ_{22} λ_{1221}
λ_{33} λ_{1133}
λ_{1331}
λ_{2233}
λ_{2332}

Quartic Couplings

$\text{Log}_{10}(\mu)$ [GeV]

[N. Darvishi, M. Masouminia, AP '21]

$M_{h_1^\pm}=500$ GeV

$\tan\beta_1=50$

$\tan\beta_2=0.018$

Legend:

High-Scale:
- $1 - g_{HVV}^2$
- $1 - g_{Ht\bar{t}}^2$
- $1 - g_{Hb\bar{b}}^2$
- $1 - g_{H\tau\bar{\tau}}^2$

Low-Scale:
- $1 - g_{HVV}^2$
- $1 - g_{Ht\bar{t}}^2$
- $1 - g_{Hb\bar{b}}^2$
- $1 - g_{H\tau\bar{\tau}}^2$

Axes: $\text{Log}_{10}(\mu)$ [GeV] (horizontal), Misalignment (vertical)

[N. Darvishi, AP, J.-H. Yu '2023]
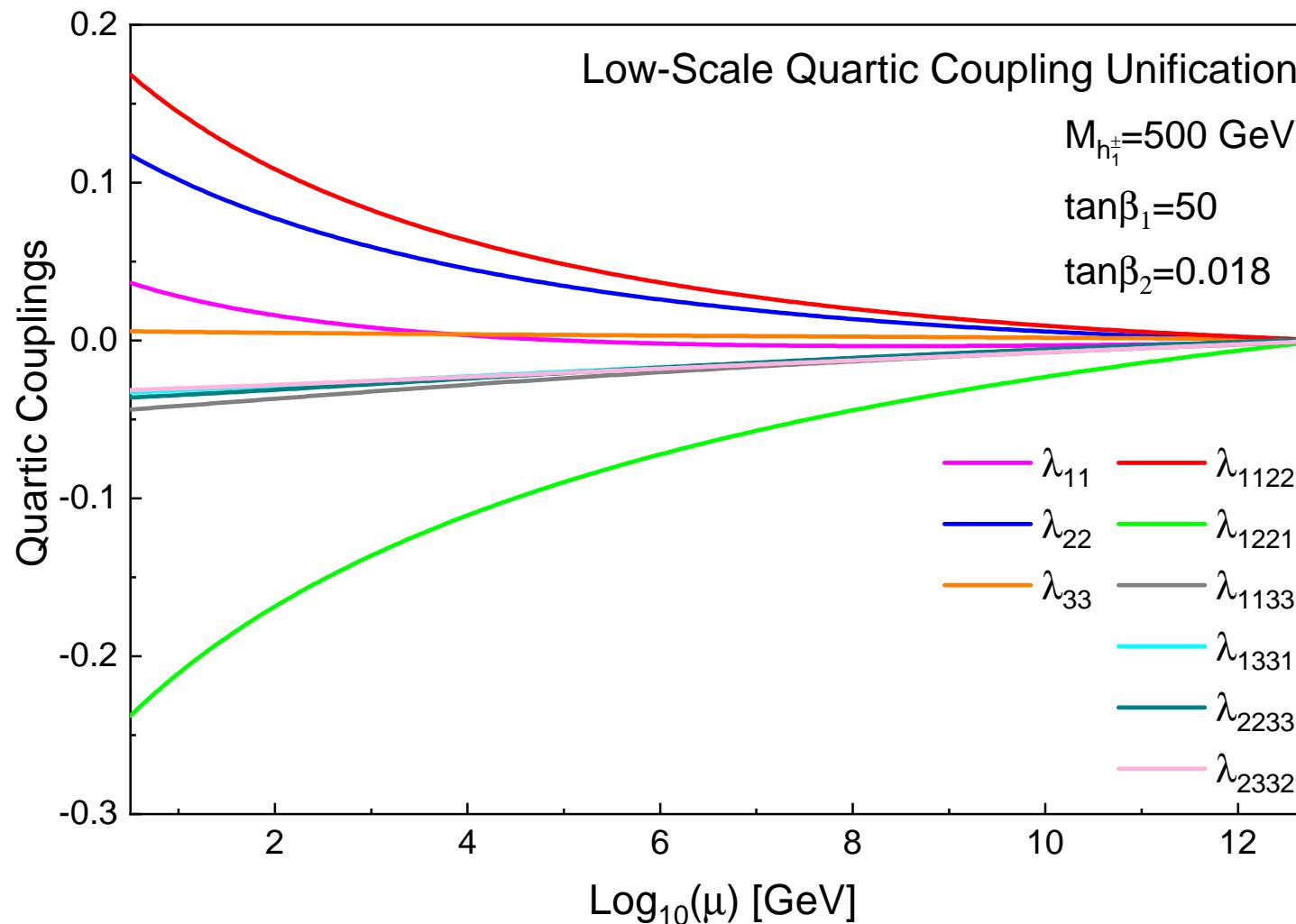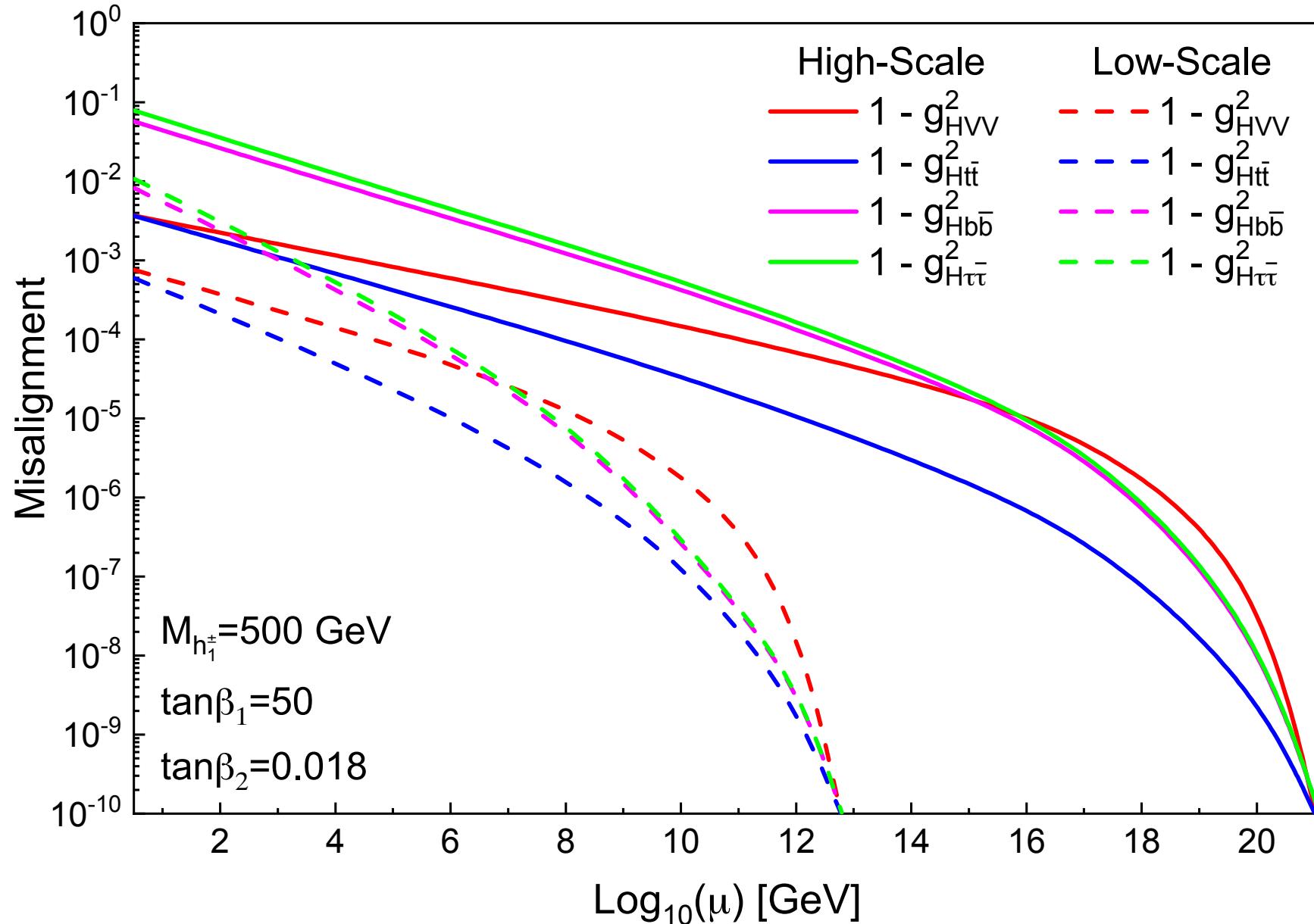
$$R_\xi = \frac{\mathcal{V}_\xi(v_1, v_2 e^{i\xi})}{|\mathcal{V}_{\xi=0}|}$$

Weak basis:

$\text{Im}\lambda_5 = 0$, $\lambda_6 = \lambda_7$

$|r_{\mathrm{CP}}| > 1$:
Explicit CPV, CPC

$1/2 < |r_{\mathrm{CP}}| < 1$:
Explicit, Spontaneous,
& Mixed CPV.

$0 < |r_{\mathrm{CP}}| < 1/2$:
Spontaneous
& Mixed CPV.

CP-odd
minimization
condition:

$$\boxed{\sin 2\xi = 2\,|r_{\mathrm{CP}}|\sin(\xi + \phi_{\mathrm{CP}})}\,,\ r_{\mathrm{CP}} = |r_{\mathrm{CP}}|\,e^{i\phi_{\mathrm{CP}}} = \frac{m_{12}^2 - \lambda_6 v^2}{\lambda_5 s_{2\beta} v^2}.$$

$|r_{\mathrm{CP}}|=2$
— CPC, $\phi_{\mathrm{CP}} = 0$
— ECPV, $\phi_{\mathrm{CP}} = \pi/8$
···· ECPV, $\phi_{\mathrm{CP}} = 7\pi/8$

$|r_{\mathrm{CP}}|=3/4$
— SCPV, $\phi_{\mathrm{CP}} = 0$
— ECPV, $\phi_{\mathrm{CP}} = \pi/8$
— MCPV, $\phi_{\mathrm{CP}} = \pi/50$

$|r_{\mathrm{CP}}|=1/4$
— SCPV, $\phi_{\mathrm{CP}} = 0$
— MCPV, $\phi_{\mathrm{CP}} = \pi/8$
···· MCPV, $\phi_{\mathrm{CP}} = 7\pi/8$

$|r_{\mathrm{CP}}|=0$
— CPC, $\phi_{\mathrm{CP}} = 0$
— SCPV, $\phi_{\mathrm{CP}} = \pi$
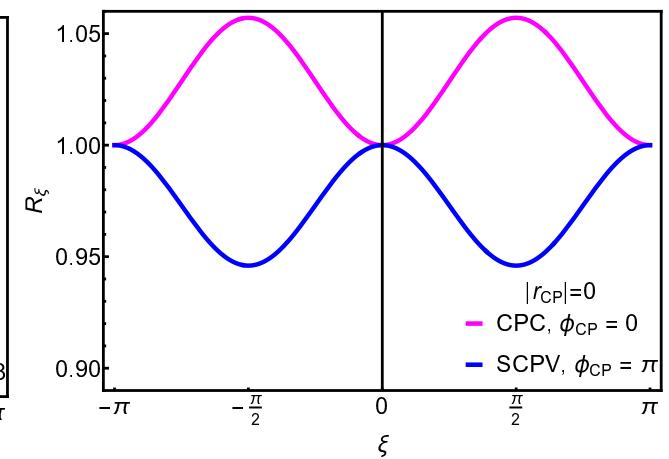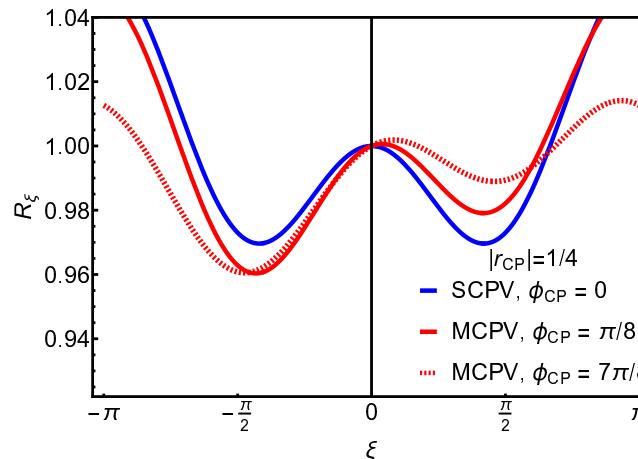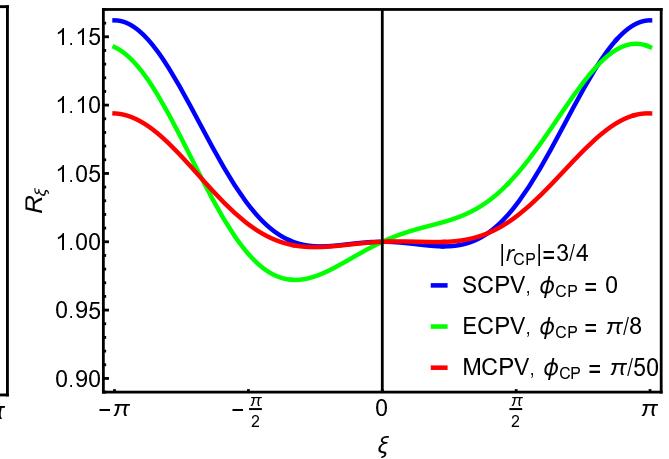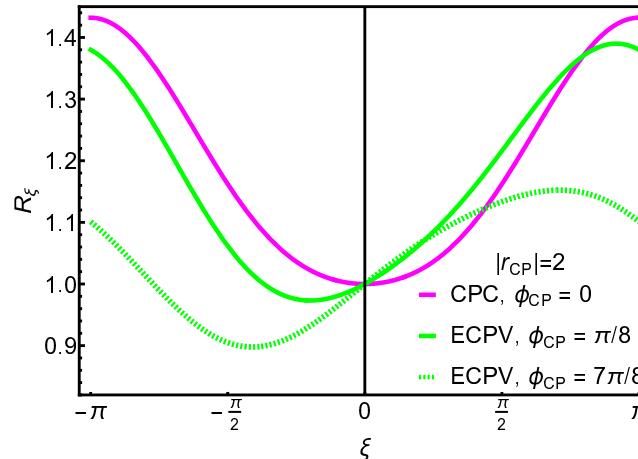
• CP Violation in Naturally Aligned (NA) 2HDM

No.	Generators	Continuous Syms	Parameters
8	T^{0-9}	SO(5)	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \lambda_3/2$.
7	$T^{0,2,5,7,8,9}$	CP1×O(4)	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \lambda_3/2$, $\lambda_4 = -\text{Re}(\lambda_5)$.
7'	$T^{0,1,4,6,8,9}$	CP2×O(4)'	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \lambda_3/2$, $\lambda_4 = \text{Re}(\lambda_5)$.
6	$T^{0,2,5,7,8,9}$	SO(4)	$m_{11}^2 = m_{22}^2$, $\text{Im}(m_{12}^2)$, $\lambda_1 = \lambda_2 = \lambda_3/2$, $\lambda_4 = -\text{Re}(\lambda_5)$, $\text{Im}(\lambda_6) = \text{Im}(\lambda_7)$.
6'	$T^{0,1,4,6,8,9}$	SO(4)'	$m_{11}^2 = m_{22}^2$, $\text{Re}(m_{12}^2)$, $\lambda_1 = \lambda_2 = \lambda_3/2$, $\lambda_4 = \text{Re}(\lambda_5)$, $\text{Re}(\lambda_6) = \text{Re}(\lambda_7)$.
5	$T^{0,2,4,6}$	O(2)× O(3)	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \lambda_{345}/2$, $\lambda_4 = \text{Re}(\lambda_5)$.
5'	$T^{0,1,5,7}$	O(2)' × O(3)	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \bar{\lambda}_{345}/2$, $\lambda_4 = -\text{Re}(\lambda_5)$.
4	T^{0-3}	O(3)× O(2)$_Y$	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \lambda_{34}/2$.
3	$T^{0,5,7}$	SO(3)	m_{11}^2, m_{22}^2, $\text{Im}(m_{12}^2)$, λ_1, λ_2, λ_3, $\lambda_4 = -\text{Re}(\lambda_5)$, $\text{Im}(\lambda_6)$, $\text{Im}(\lambda_7)$.
3'	$T^{0,4,6}$	SO(3)'	m_{11}^2, m_{22}^2, $\text{Re}(m_{12}^2)$, λ_1, λ_2, λ_3, $\lambda_4 = \text{Re}(\lambda_5)$, $\text{Re}(\lambda_6)$, $\text{Re}(\lambda_7)$.
2	$T^{0,2}$	CP1×O(2)×O(2)$_Y$	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \lambda_{345}/2$.
2'	$T^{0,1}$	CP2×O(2)'×O(2)$_Y$	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \bar{\lambda}_{345}/2$.
1	$T^{0,2}$	O(2)×O(2)$_Y$	$m_{11}^2 = m_{22}^2$, $\text{Im}(m_{12}^2)$, $\lambda_1 = \lambda_2 = \lambda_{345}/2$, $\text{Im}(\lambda_6) = \text{Im}(\lambda_7)$.
1'	$T^{0,1}$	O(2)'×O(2)$_Y$	$m_{11}^2 = m_{22}^2$, $\text{Re}(m_{12}^2)$, $\lambda_1 = \lambda_2 = \bar{\lambda}_{345}/2$, $\text{Re}(\lambda_6) = \text{Re}(\lambda_7)$.

Abbreviations: $\lambda_{34} \equiv \lambda_3 + \lambda_4$, $\quad \lambda_{345}\,(\bar{\lambda}_{345}) \equiv \lambda_3 + \lambda_4 + (-)\,\text{Re}(\lambda_5)$

– CP-violating breaking patterns in NA-2HDM

No.	Syms with NHAL	No.	Syms breaking of NHAL	Types of CPV after SB
8	SO(5)		—	
7	CP1×O(4)	6	SO(4)	ECPV/MCPV
7′	CP2×O(4)′	6′	SO(4)′	SCPV
5	O(2)×O(3)	6′	SO(4)′	SCPV
5′	O(2)′×O(3)	6	SO(4)	ECPV/MCPV
4	O(3)×O(2)$_Y$		—	
2	CP1×O(2)×O(2)$_Y$	1	O(2)×O(2)$_Y$	ECPV/MCPV
2′	CP2×O(2)′×O(2)$_Y$	1′	O(2)′×O(2)$_Y$	SCPV

SB: **S**oft symmetry-breaking **B**ilinear mass terms.

• EDM compatible scenarios with approximate natural alignment

Symmetries	CPV types	$\tan\beta$	ξ	ϕ_{12}	M_{H_1}	M_{H_2}	M_{H_3}	$M_{H^\pm}$	g_{H_1VV}
O(2)×O(2)$_Y$	ECPV	1.608	0.002	-0.05	125.10	682.99	787.67	640.18	0.998
	MCPV	0.804	-1.93	0.27	125.21	225.11	270.50	380.54	0.989
O(2)′×O(2)$_Y$	SCPV	0.71	0.67	0	125.52	275.23	448.01	551.38	0.957
SO(4)	ECPV	0.827	0.002	-0.02	125.28	506.93	570.58	570.59	0.995
	MCPV	0.740	-1.25	0.44	125.40	408.49	433.98	433.99	0.984
SO(4)′	SCPV	0.742	-2.42	0	125.68	216.63	265	442.21	0.948

(A)   (C)   (F)   (G)
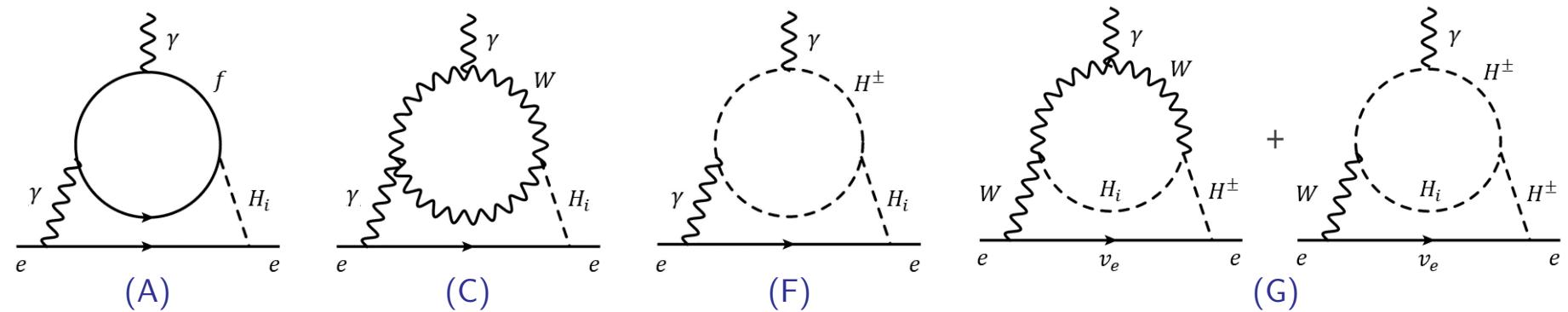
Symmetries	CPV types	The ratio of EDM contributions over the total EDM							EDM $[e \cdot cm]$
		(A)	(B)	(C)	(D)	(E)	(F)	(G)	
O(2)×O(2)$_Y$	ECPV	5.581	-0.072	-4.711	-0.308	0.0185	0.312	0.180	9.51×10^{-30}
	MCPV	2.008	-0.026	-0.942	-0.065	0.0015	0.028	-0.005	7.67×10^{-30}
O(2)′×O(2)$_Y$	SCPV	2.003	-0.026	-0.866	-0.079	0.0002	0.004	-0.037	3.86×10^{-30}
SO(4)	ECPV	1.851	-0.021	-0.832	-0.055	0.0005	0.009	0.048	2.87×10^{-30}
	MCPV	1.995	-0.021	-0.887	-0.077	-0.0013	-0.023	0.016	1.07×10^{-30}
SO(4)′	SCPV	-1.211	0.014	1.875	0.172	0.0002	0.004	0.146	3.55×10^{-30}
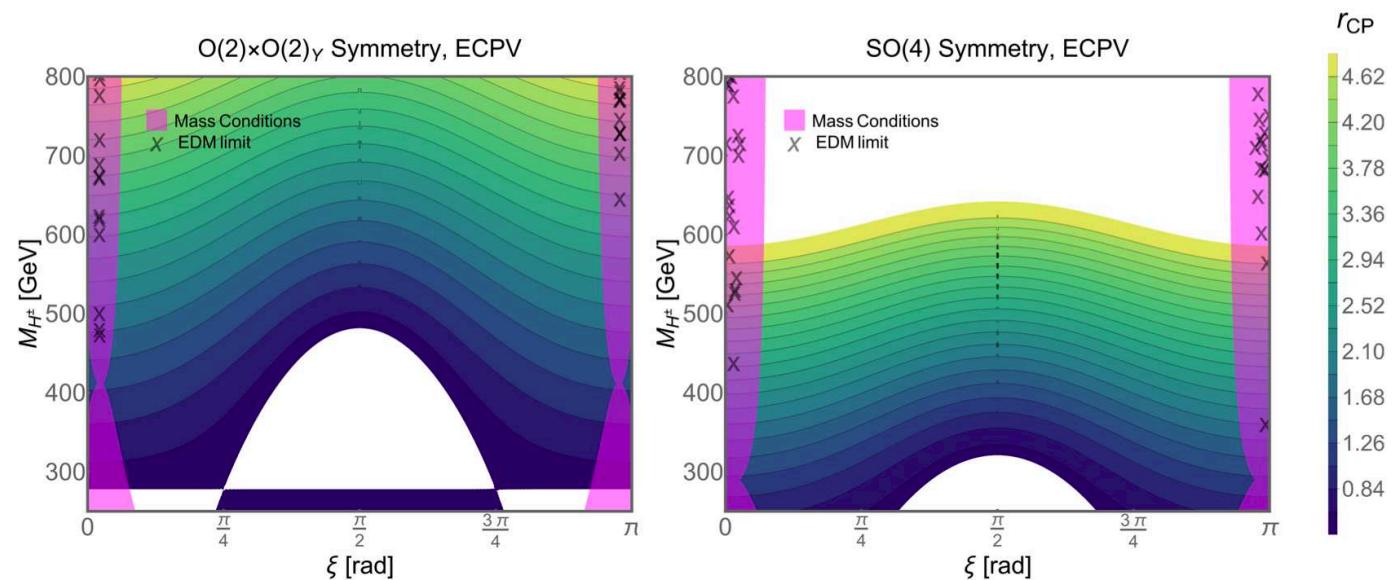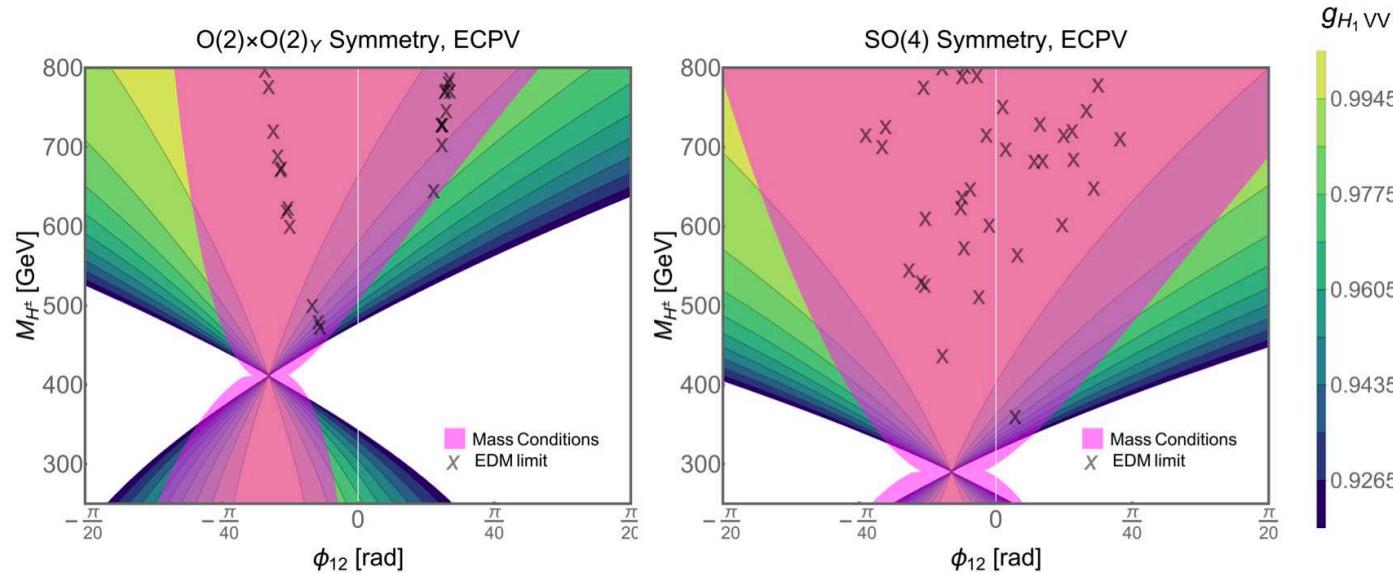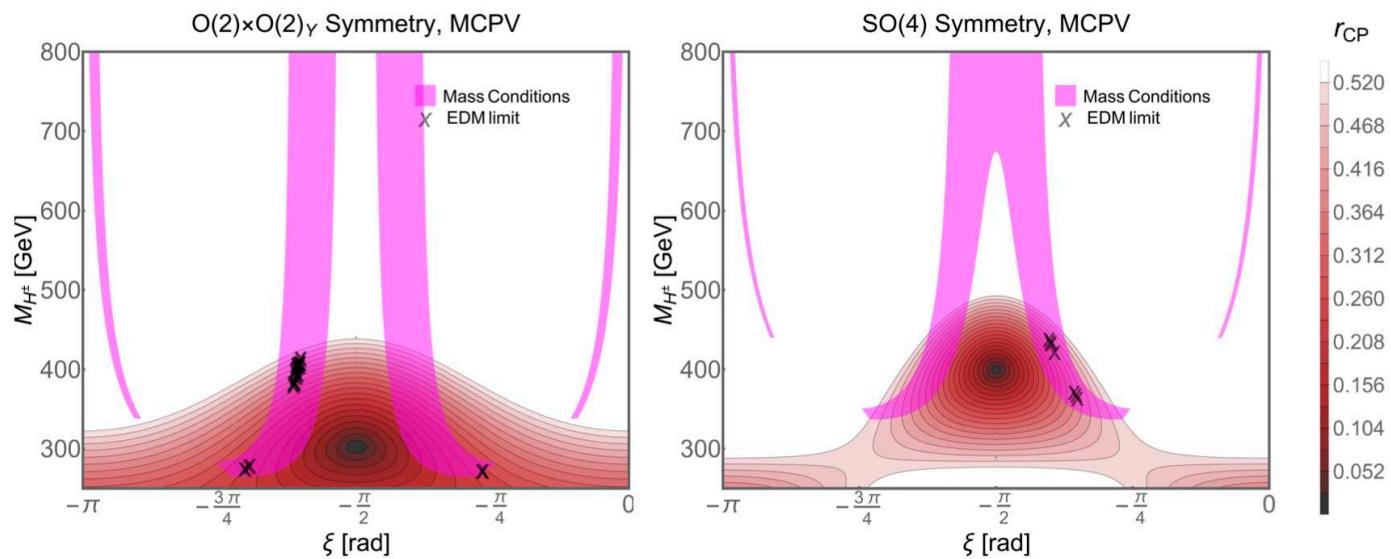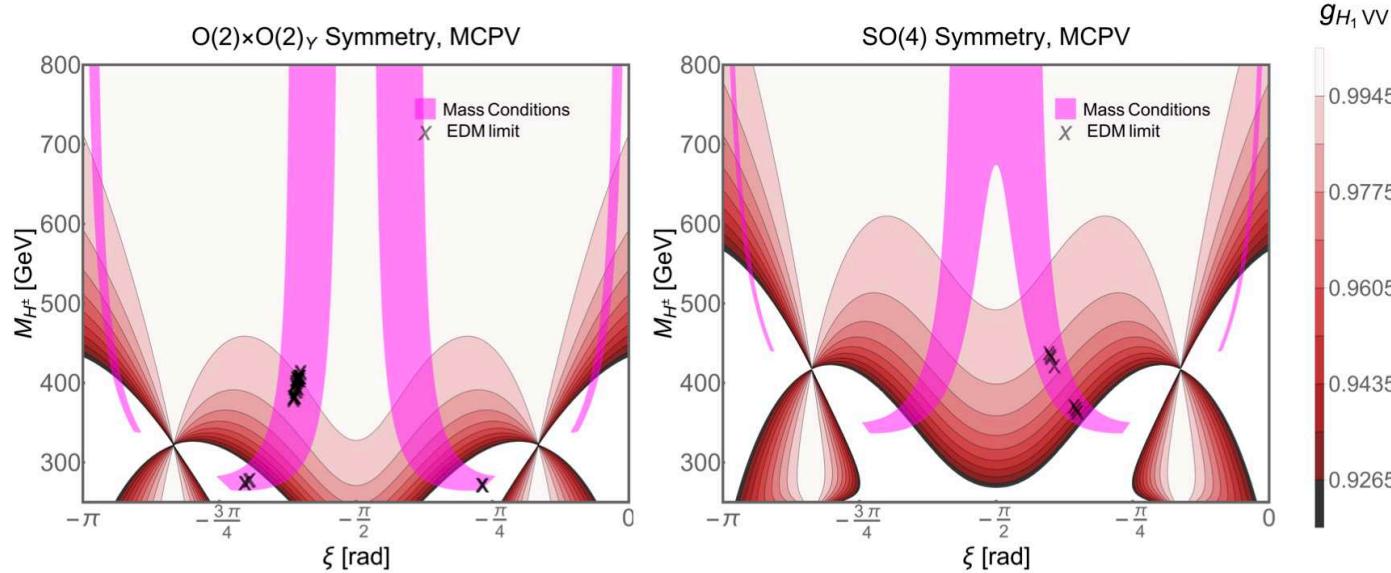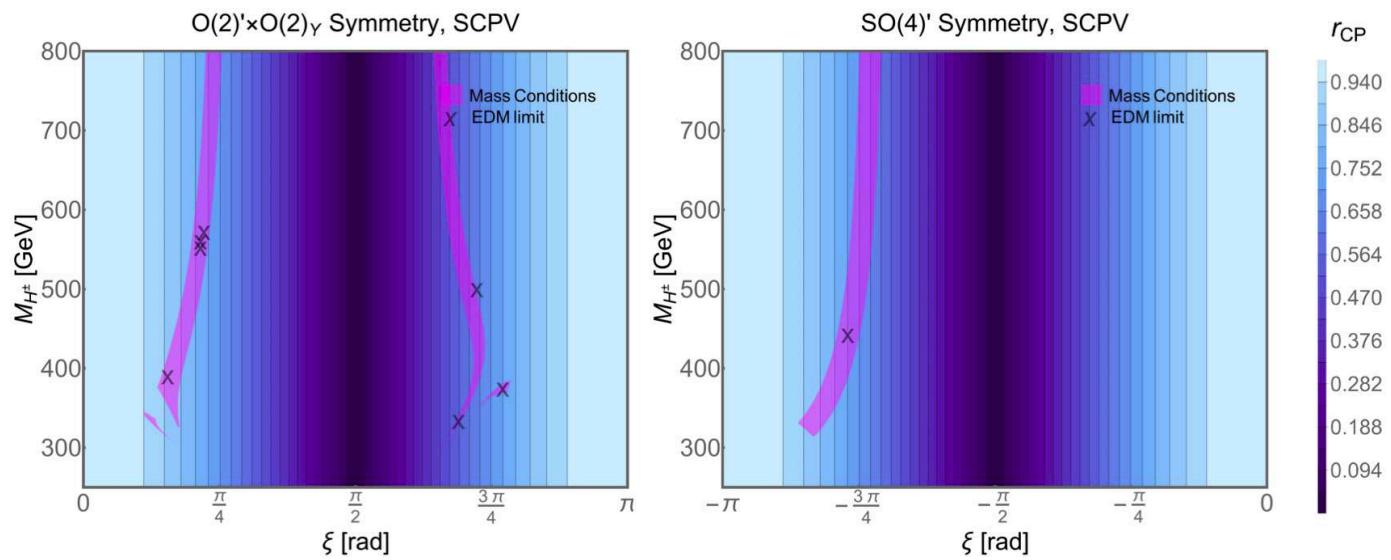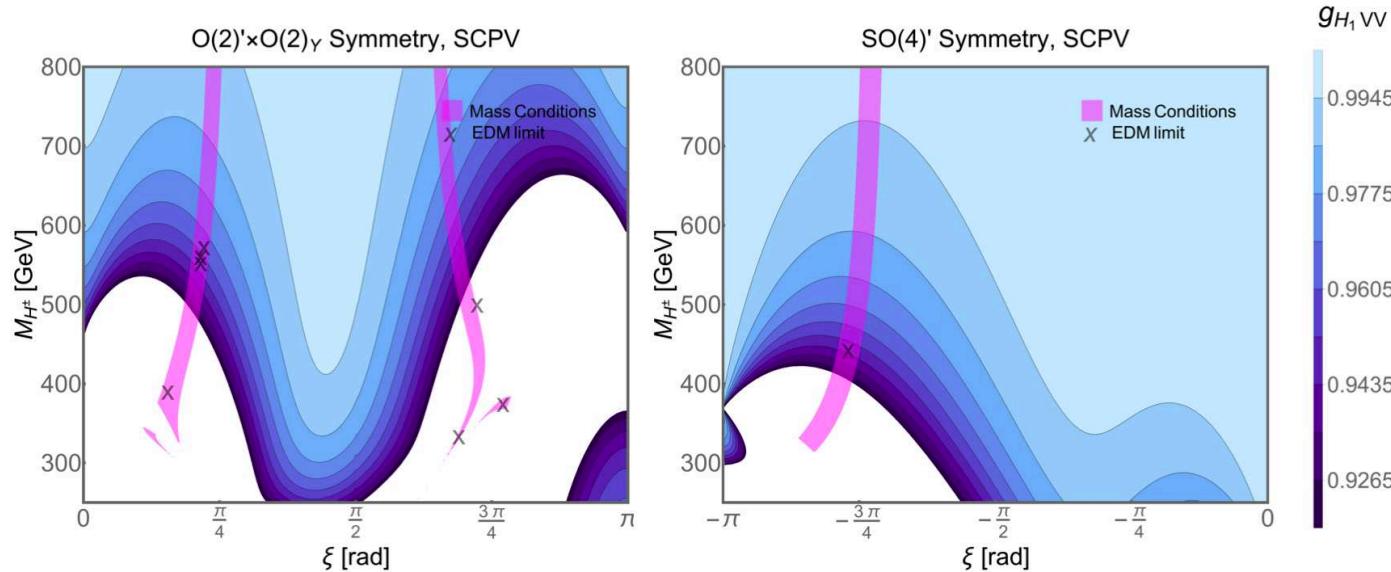
• **Branching ratios** in the **MS-2HDM**   [Dev, AP '14]
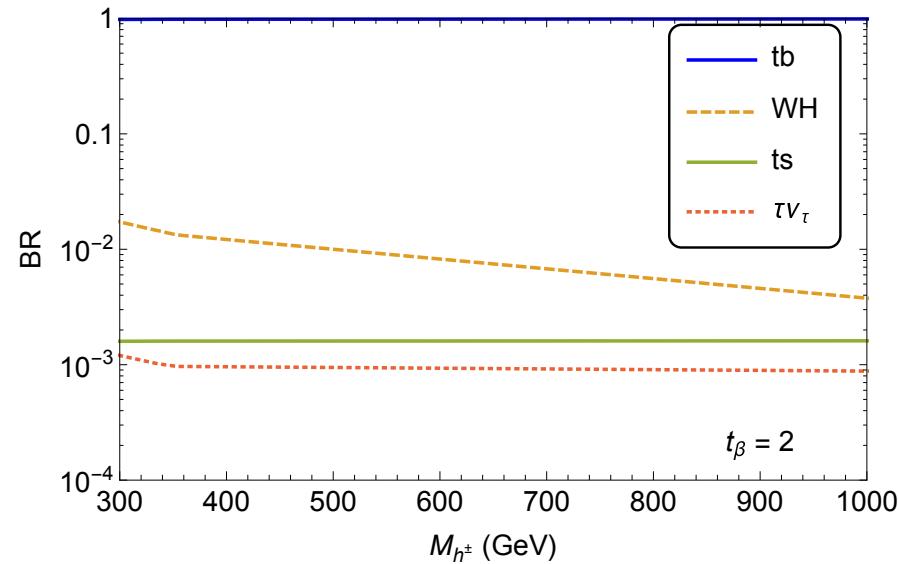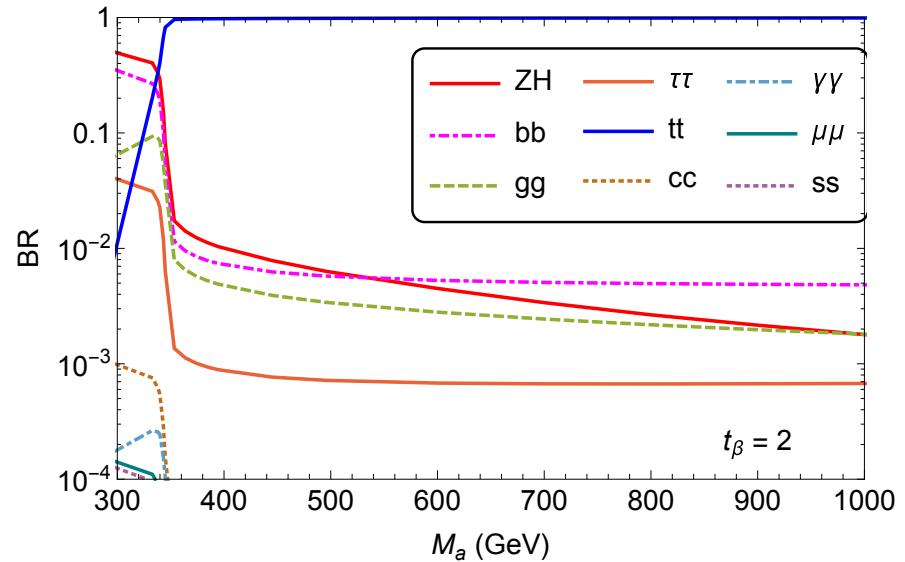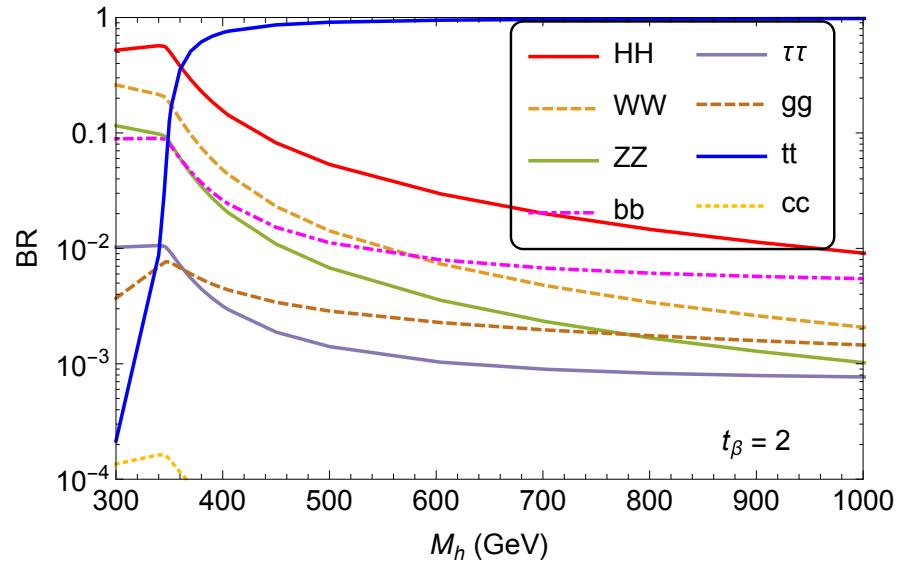
• Discovery channels for aligned Higgs doublets:

− $gg \to t\bar{b}h^- \to t\bar{b}t\bar{b}$

[Dev, AP '14;

Yvonne Peters, AP et al, PRD100 (2019) 035026]
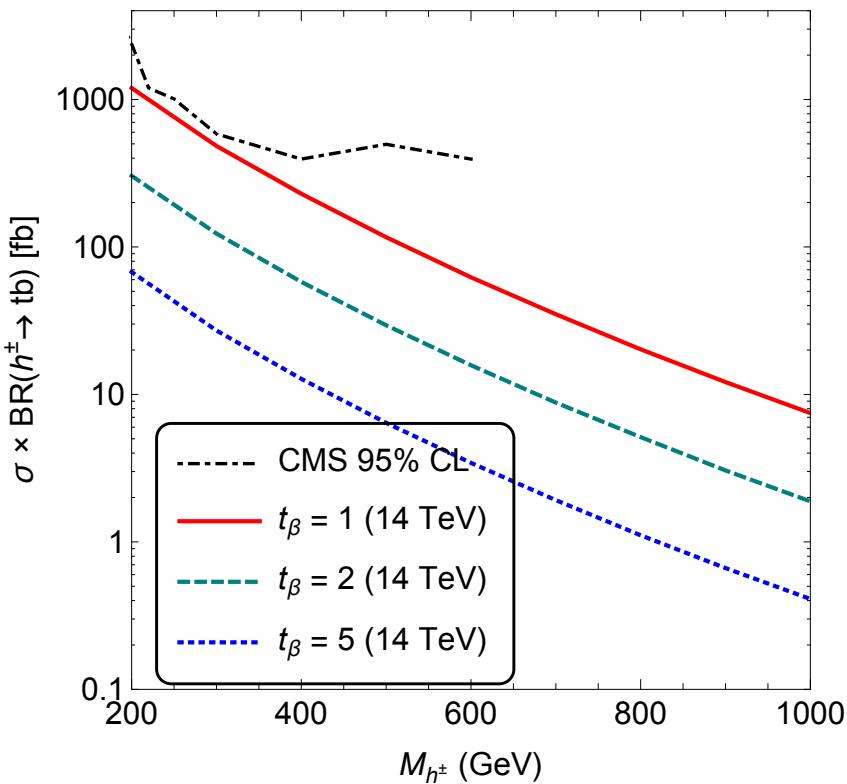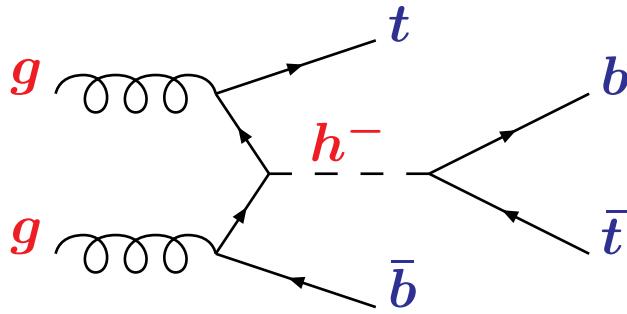
$p_T^{\ell} > 20$ GeV,

$|\eta^{\ell}| < 2.5$,

$\Delta R^{\ell\ell} > 0.4$,

$M_{\ell\ell} > 12$ GeV,

$|M_{\ell\ell} - M_Z| > 10$ GeV,

$p_T^{j} > 30$ GeV,

$|\eta^{j}| < 2.4$,

$\not{E}_T > 40$ GeV.

− $gg \to t\bar{t}(h,a) \to t\bar{t}t\bar{t}$

[Dev, AP '14]
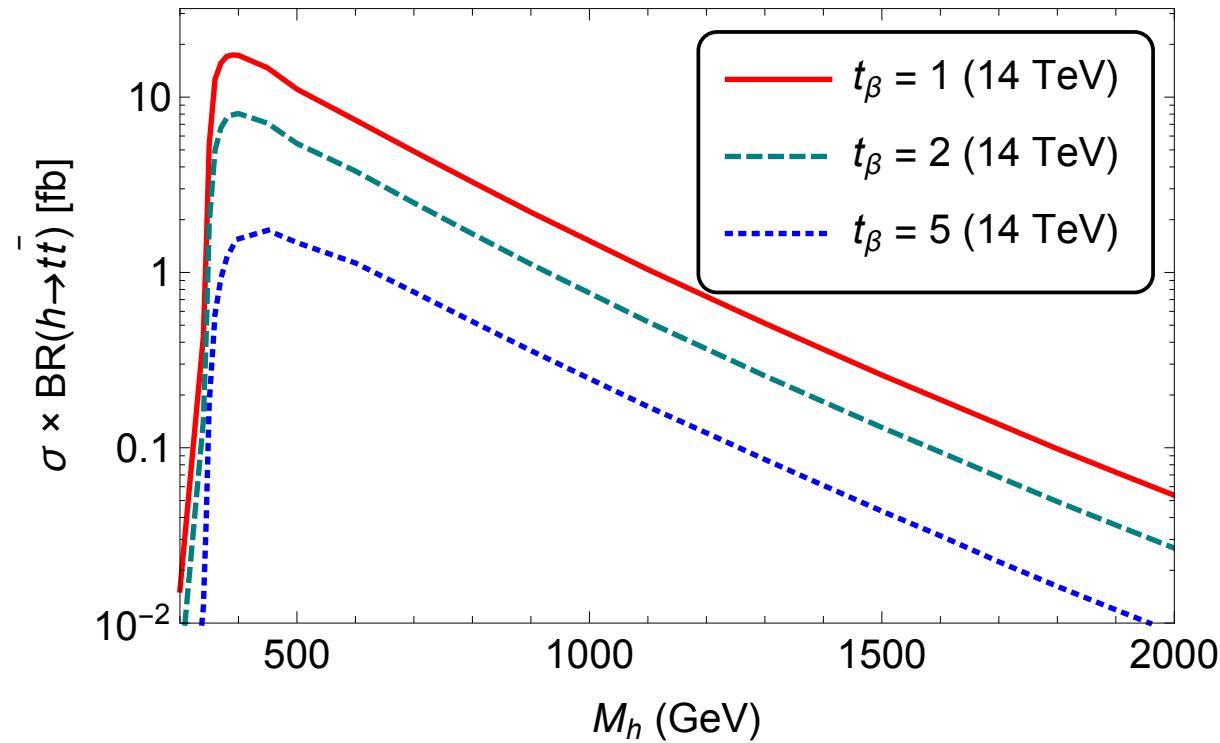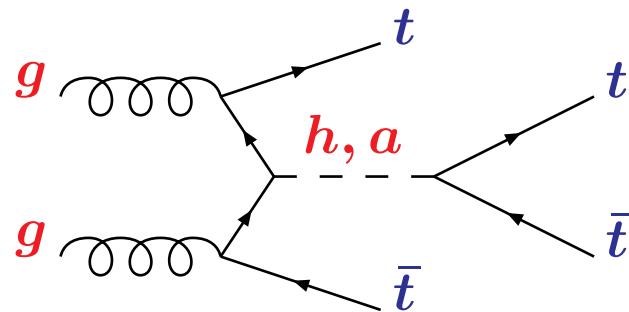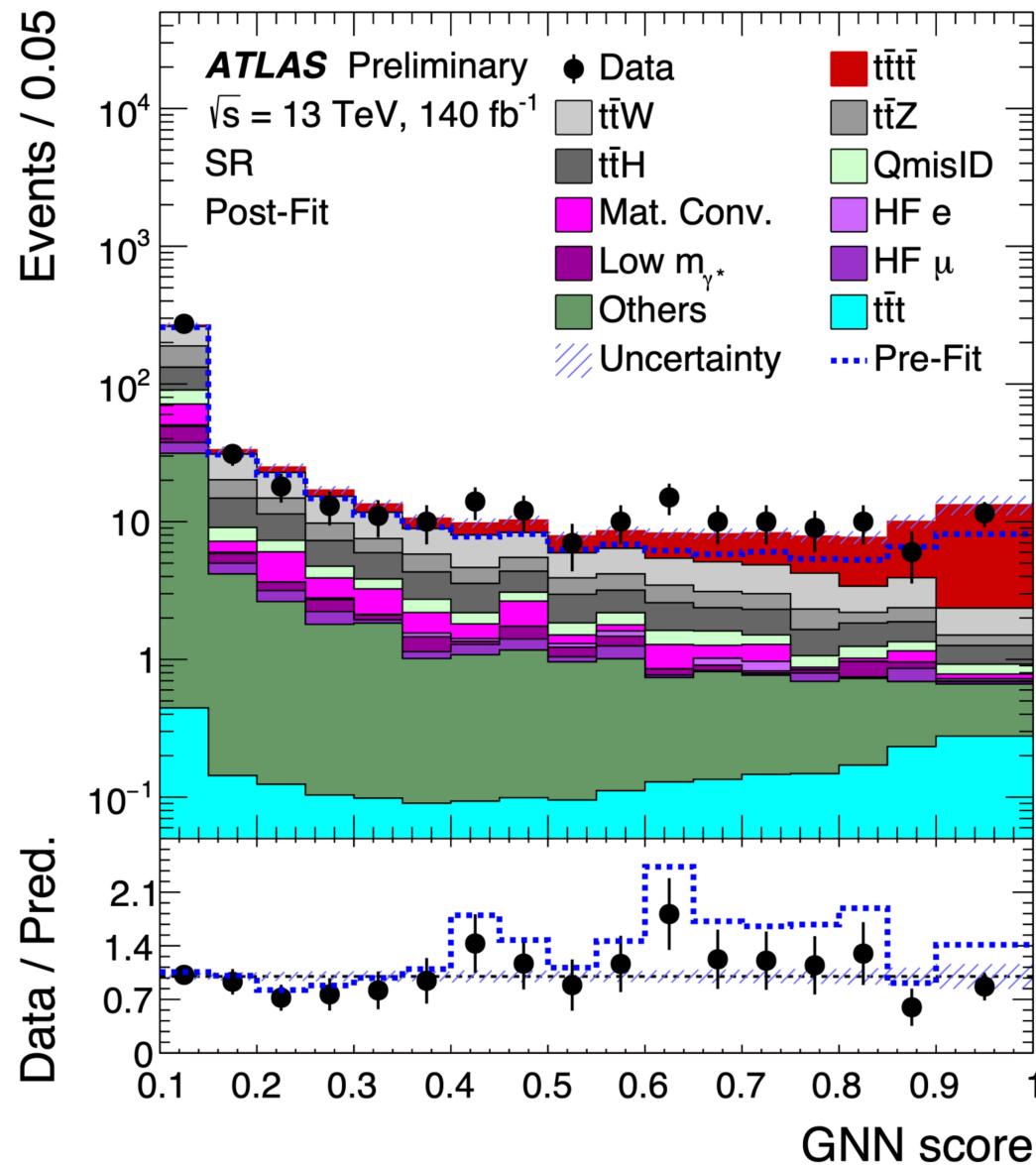
• Other production channels for CP-violating NA Higgs bosons

$$- \quad gg \to (Z^*, H_2, H_3) \to ZH_1(+b\bar{b}) \qquad \text{[N. Darvishi, AP, J.-H. Yu '23]}$$
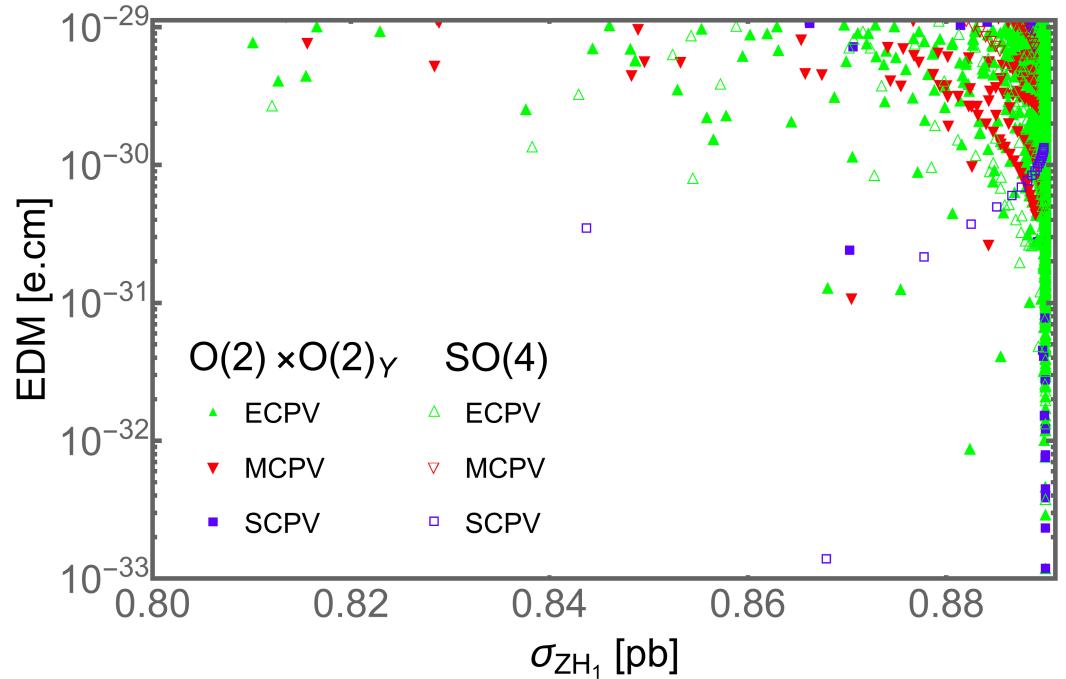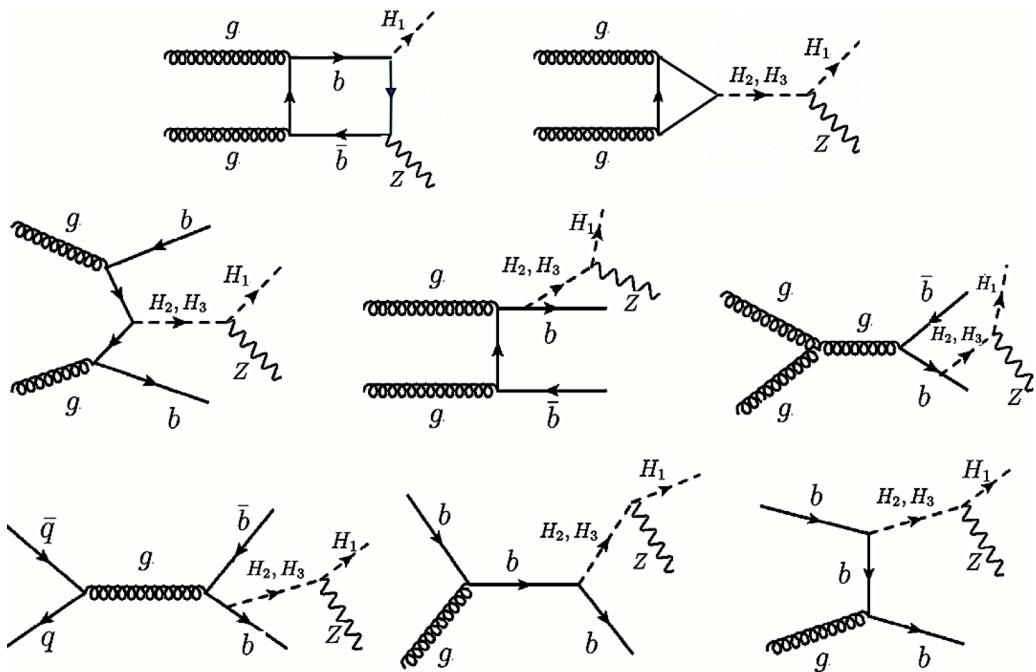
Omitted are graphs with $H_{2,3} \to Z$ and $H_{2,3}$ radiation from $\bar{b}$ quarks.

• SUMMARY

- **5 symmetries for natural alignment** *without* **decoupling in 2HDM:**

    (i) Sp(4)   (ii) SU(2)$_{\text{HF}}$ (iii) SO(2)$_{\text{HF}} \times \mathcal{CP}$

    (iv) Sp(2)$_{\phi_1 \widetilde{\phi}_2} \times$ Sp(2)$_{\phi_2 \widetilde{\phi}_1} \times \mathcal{CP}$ (v) SO(2)$_{\text{HF}} \times$ Sp(2)$_{\phi_1 + \phi_2}$

- **Soft breaking** $\longrightarrow$ **minimal alignment symmetry:** Z$_2^{\text{EW}} \times$ Z$_2^{\text{I}}$
  $\rightarrow$ **Naturally aligned (NA) heavy Higgs sector is** Z$_2^{\text{EW}}$ **odd.**

- **Three forms of CPV in 2HDM: Explicit, Spontaneous and Mixed.**
  Maximising CPV along softly-broken symmetry directions:

    (i) SO(2)$_{\text{HF}}$ (ii) Sp(2)$_{\phi_1 \widetilde{\phi}_2} \times$ Sp(2)$_{\phi_2 \widetilde{\phi}_1} \simeq$ SO(4),

  compatible with current EDM limits.

- $t\bar{t}t\bar{t}$ **channel is a powerful probe for NA-2HDMs**

- ZH_1 **channel can test CP-violating NA Higgs bosons at the LHC.**

Back-Up Slides

Symmetry-breaking of Sp(4)/$Z_2 \sim$ SO(5):

- Soft breaking (e.g. through m_{12}^2):

$$M_H^2 = 2\lambda_2 v^2, \qquad M_h^2 = M_a^2 = M_{h^\pm}^2 = \frac{\text{Re}(m_{12}^2)}{s_\beta c_\beta}$$

  Heavy Higgs spectrum is degenerate at tree level.

- Explicit breaking through RG running (two loops):

$$\text{Sp(4)}/Z_2 \otimes \text{SU(2)}_L \quad \xrightarrow{g' \neq 0} \quad \text{SU(2)}_{\text{HF}} \otimes \text{U(1)}_Y \otimes \text{SU(2)}_L$$

$$\xrightarrow{\mathbf{Y}^{u,d}} \quad \text{U(1)}_{\text{PQ}} \otimes \text{U(1)}_Y \otimes \text{SU(2)}_L$$

$$\xrightarrow[\langle \Phi_{1,2} \rangle]{m_{12}^2} \quad \text{U(1)}_{\text{em}}$$

- **Quartic Coupling Unification (two loops)**
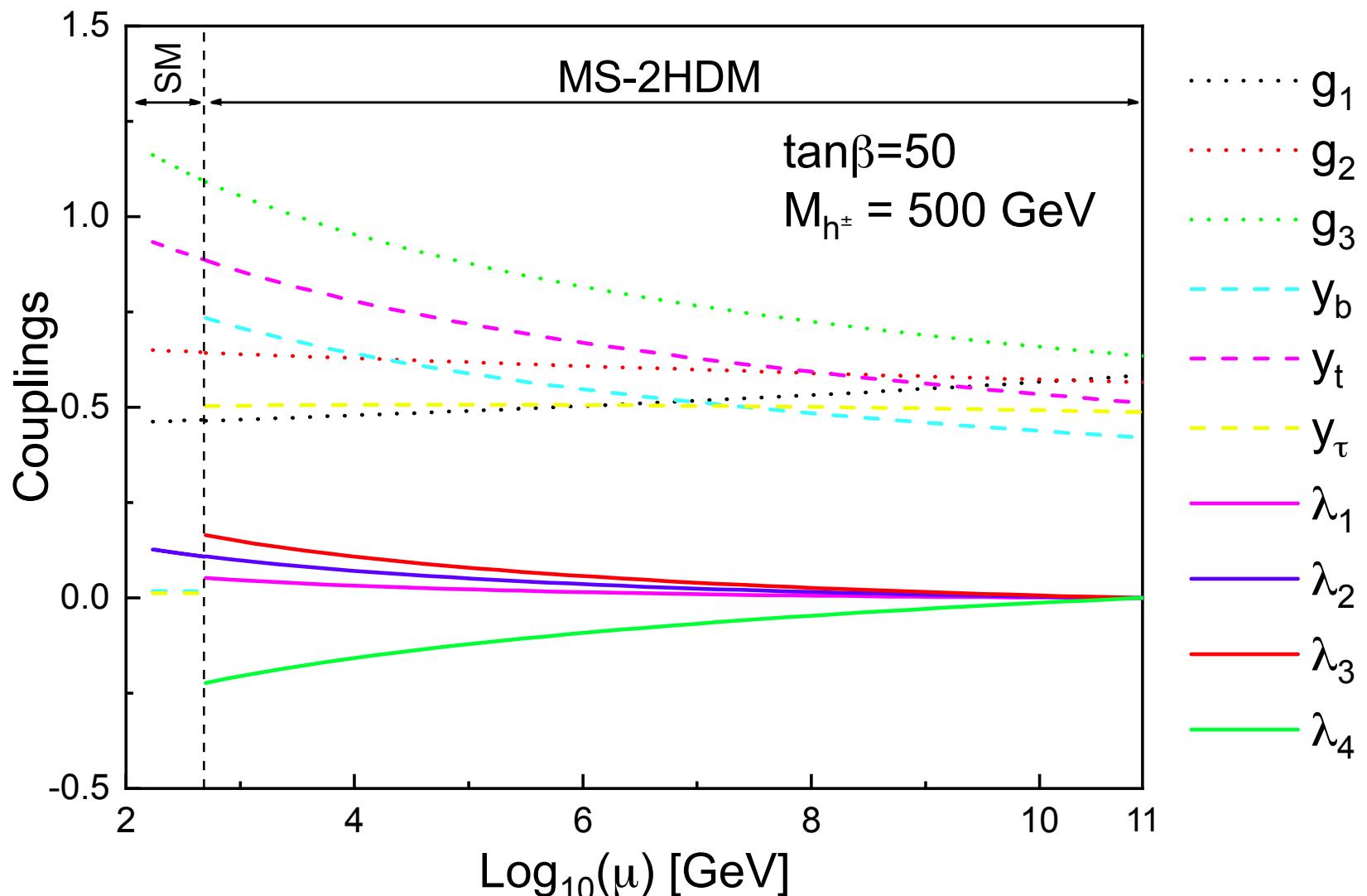
**First conformal unification point**: $\mu_X^{(1)} \sim 10^{11}$ GeV (of order PQ scale)

Second conformal unification point: $\mu_X^{(2)} \sim 10^{18}$ GeV (of order m_{Pl})

[N. Darvishi, AP '19]

– **Low- and high-scale quartic coupling unification:** $\tan\beta$ vs $\mu_X^{(1,2)}$

Legend:
- $M_{h^\pm}= 0.5$ TeV : low-scale / high-scale
- $M_{h^\pm}= 1$ TeV : low-scale / high-scale
- $M_{h^\pm}= 10$ TeV : low-scale / high-scale
- $M_{h^\pm}= 100$ TeV : low-scale / high-scale

$\lambda_1 = \lambda_2,\ \lambda_3 = 2\lambda_1$

Axes: $\mathrm{Log}_{10}(\mu)$ [GeV] vs $\tan\beta$
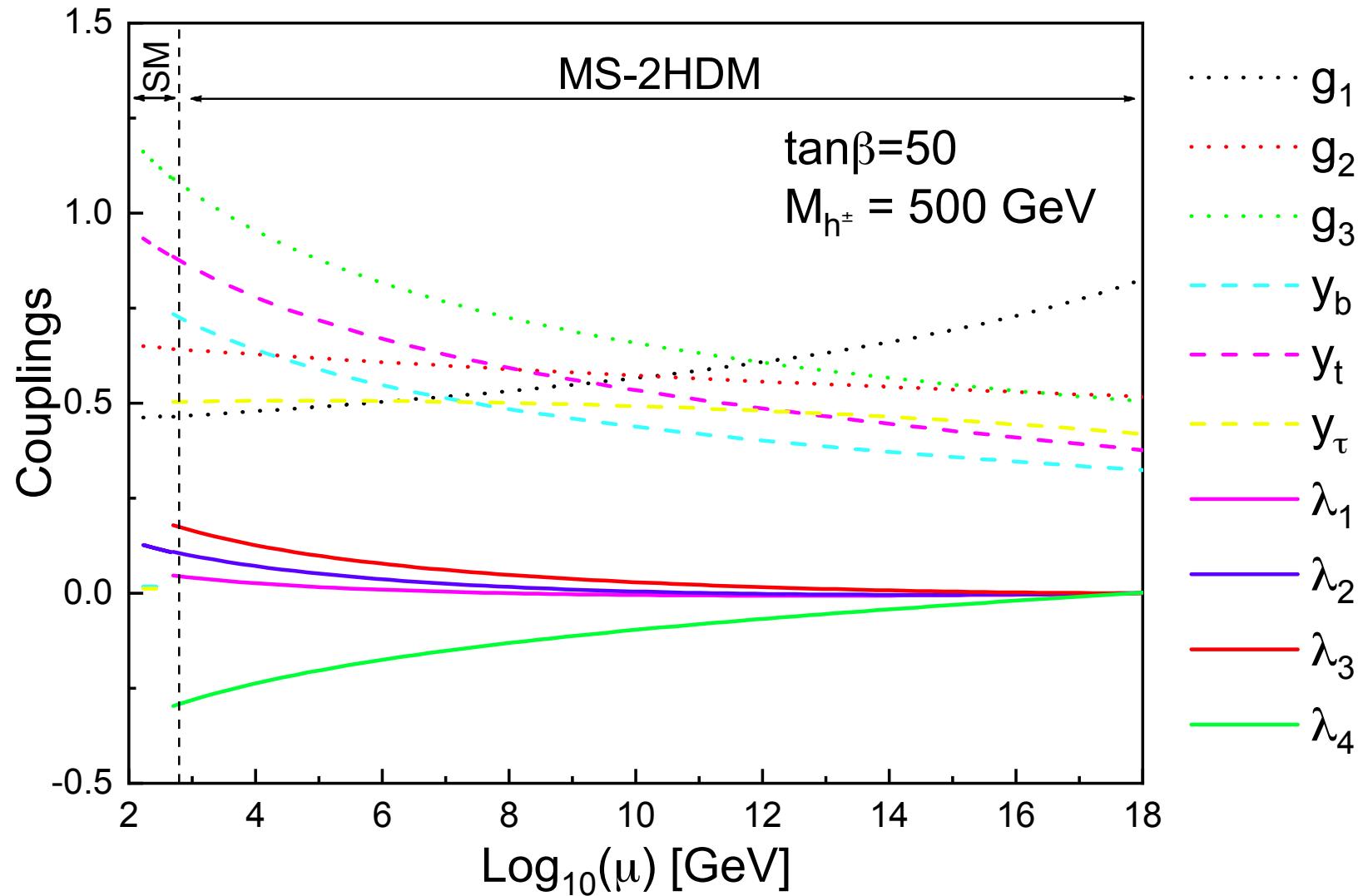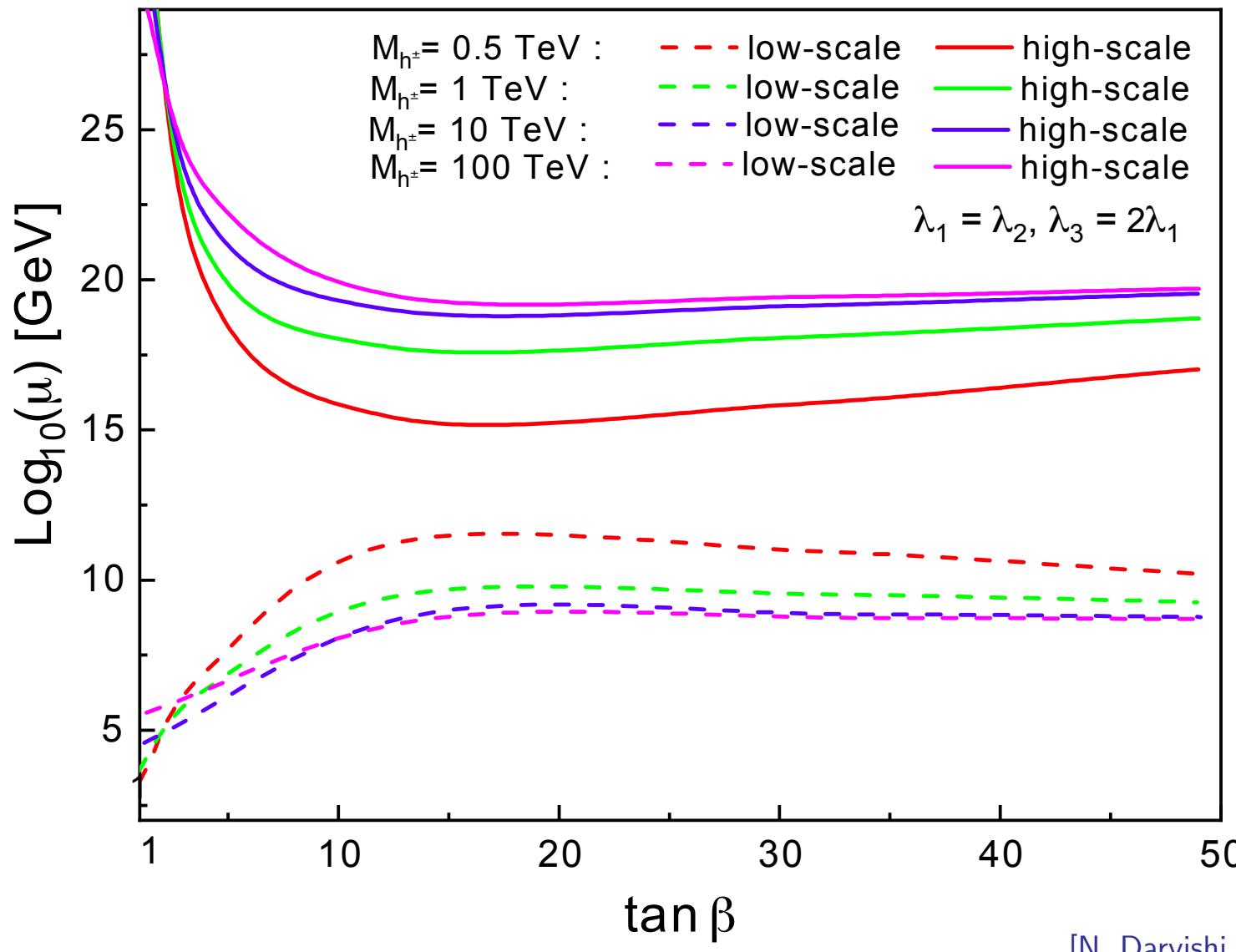
[N. Darvishi, AP '19]

– **Misalignment in the MS-2HDM**

CP-even mass matrix in Higgs basis:

$$\mathcal{M}_{\mathcal{S}}^2 = \begin{pmatrix} \widehat{A} & \widehat{C} \\ \widehat{C} & \widehat{B} \end{pmatrix} \xrightarrow[\textbf{approx.}]{\textbf{seesaw}} M_H^2 \simeq \widehat{A} - \frac{\widehat{C}^2}{\widehat{B}} \quad \& \quad M_h^2 \simeq \widehat{B} \gg \widehat{A}, \widehat{C}$$

Light-to-heavy scalar mixing:

$$\theta_{\mathcal{S}} \equiv \frac{\widehat{C}}{\widehat{B}} = \frac{v^2 s_\beta c_\beta \left[s_\beta^2 \left(2\lambda_2 - \lambda_{34} \right) - c_\beta^2 \left(2\lambda_1 - \lambda_{34} \right) \right]}{M_a^2 + 2v^2 s_\beta^2 c_\beta^2 \left(\lambda_1 + \lambda_2 - \lambda_{34} \right)} \ll 1$$

Higgs couplings to $V = W, Z$:

$$g_{HVV} \simeq 1 - \frac{1}{2}\,\theta_{\mathcal{S}}^2\,, \qquad g_{hVV} \simeq -\theta_{\mathcal{S}}$$

Higgs couplings to quarks:

$$g_{Huu} \simeq 1 + t_\beta^{-1}\,\theta_{\mathcal{S}}\,, \qquad\qquad g_{Hdd} \simeq 1 - t_\beta\,\theta_{\mathcal{S}}\,,$$
$$g_{huu} \simeq -\theta_{\mathcal{S}} + t_\beta^{-1}\,, \qquad\qquad g_{hdd} \simeq -\theta_{\mathcal{S}} - t_\beta\,.$$

Misalignment predictions in the MS-2HDM with low- and high-scale quartic coupling unification, assuming $M_{h\pm} = 500\,\text{GeV}$.

Couplings	ATLAS	CMS	$\tan\beta = 2$	$\tan\beta = 20$	$\tan\beta = 50$
$\|g_{HZZ}^{\text{low-scale}}\|$	[0.86, 1.00]	[0.90, 1.00]	0.9999	0.9999	0.9999
$\|g_{HZZ}^{\text{high-scale}}\|$			0.9981	0.9999	0.9999
$\|g_{Htt}^{\text{low-scale}}\|$	$1.31^{+0.35}_{-0.33}$	$1.45^{+0.42}_{-0.32}$	1.0049	1.0001	1.0000
$\|g_{Htt}^{\text{high-scale}}\|$			1.0987	1.0003	1.0001
$\|g_{Hbb}^{\text{low-scale}}\|$	$0.49^{+0.26}_{-0.19}$	$0.57^{+0.16}_{-0.16}$	0.9803	0.9560	0.9590
$\|g_{Hbb}^{\text{high-scale}}\|$			0.8810	0.9449	0.9427

$\rightarrow$ Misalignment predictions consistent with experiment

- **Maximally Symmetric Three Higgs Doublet Model (MS-3HDM)**

Breaking pattern:

$$\text{Sp}(6)/Z_2 \otimes \text{SU}(2)_L \xrightarrow{\ g' \neq 0\ } \text{SU}(3)_{\text{HF}} \otimes \text{U}(1)_Y \otimes \text{SU}(2)_L$$

$$\xrightarrow[\text{Type V}]{\mathbf{Y}^{u,d,e}} \text{U}(1)_{\text{PQ}} \otimes \text{U}(1)'_{\text{PQ}} \otimes \text{U}(1)_Y \otimes \text{SU}(2)_L$$

$$\xrightarrow[\text{soft } m_{ij}^2]{\langle \Phi_{1,2,3} \rangle} \text{U}(1)_{\text{em}}$$

– Quartic Coupling Unification in the MS-3HDM

Input parameters: $\tan\beta_1 = v_2/v_1$, $\tan\beta_2 = v_3/\sqrt{v_1^2 + v_2^2}$,

$M_{h_{1,2}^{\pm}}$ and $h_1^{\pm} h_2^{\mp}$-mixing angle: σ

[N. Darvishi, M. Masouminia, AP '21]

$M_{h_1^\pm}=500$ GeV

$\tan\beta_1=50$

$\tan\beta_2=0.018$

Legend:

High-Scale	Low-Scale
$1 - g_{HVV}^2$	$1 - g_{HVV}^2$
$1 - g_{Ht\bar{t}}^2$	$1 - g_{Ht\bar{t}}^2$
$1 - g_{Hb\bar{b}}^2$	$1 - g_{Hb\bar{b}}^2$
$1 - g_{H\tau\bar{\tau}}^2$	$1 - g_{H\tau\bar{\tau}}^2$

Misalignment vs $\text{Log}_{10}(\mu)$ [GeV]

Predictions:

Alignment of masses: $M_{h_1} \sim M_{a_1} \sim M_{h_1^\pm}$ $M_{h_2} \sim M_{a_2} \sim M_{h_2^\pm}$

Alignment of all heavy-sector mixing angles in the Higgs basis: $\alpha \simeq \rho \simeq \sigma$

● 2HDM potential

[TD Lee '73; AP, C Wagner '99;
Review: Branco, Ferreira, Lavoura, Rebelo, Sher, Silva '12.]

$$V = -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) - m_{12}^2(\phi_1^\dagger\phi_2) - m_{12}^{*2}(\phi_2^\dagger\phi_1)$$
$$+ \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)$$
$$+ \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1)$$
$$+ \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1) \,.$$

● Physical spectrum (CP-conserving limit):

CP-even Higgs bosons H and h;   CP-odd scalar a;   charged scalars $h^\pm$.

● Higgs coupling to gauge bosons $V = W, Z$:

$$g_{HVV} = \cos(\beta - \alpha)\,, \qquad g_{hVV} = \sin(\beta - \alpha)\,,$$

where $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle$ and α diagonalizes the CP-even mass matrix.

• Symmetries of the 2HDM Potential

Introduce the $\mathsf{SU(2)}_L$-covariant 8D complex field multiplet

$$\mathbf{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix}, \quad \text{with } U_L \in \mathsf{SU(2)}_L : \mathbf{\Phi} \mapsto \mathbf{\Phi}' = U_L \mathbf{\Phi}.$$

$\mathbf{\Phi}$ satisfies the **Majorana constraint**

$$\mathbf{\Phi} = \mathsf{C}\,\mathbf{\Phi}^*,$$

where C is the **charge conjugation** 8D matrix

$$\mathsf{C} = \sigma^2 \otimes \sigma^0 \otimes \sigma^2 = \begin{pmatrix} \mathbf{0}_4 & \mathbf{1}_4 \\ -\mathbf{1}_4 & \mathbf{0}_4 \end{pmatrix} \otimes (-i\sigma_2).$$

• The SO(1,5) Bilinear Formalism

Introduce the *null* 6-Vector

$$
\mathsf{R}^A \;=\; \mathbf{\Phi}^\dagger\, \Sigma^A\, \mathbf{\Phi} \;=\;
\begin{pmatrix}
\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 \\[4pt]
\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1 \\[4pt]
-i\left[\phi_1^\dagger \phi_2 - \phi_2^\dagger \phi_1\right] \\[4pt]
\phi_1^\dagger \phi_1 - \phi_2^\dagger \phi_2 \\[4pt]
\phi_1^{\mathsf{T}} i\sigma^2 \phi_2 - \phi_2^\dagger i\sigma^2 \phi_1^* \\[4pt]
-i\left[\phi_1^{\mathsf{T}} i\sigma^2 \phi_2 + \phi_2^\dagger i\sigma^2 \phi_1^*\right]
\end{pmatrix},
$$

with $A = \mu,\ 4,\ 5$, and

$$
\Sigma^\mu \;=\; \frac{1}{2}\begin{pmatrix} \sigma^\mu & \mathbf{0}_2 \\ \mathbf{0}_2 & (\sigma^\mu)^{\mathsf{T}} \end{pmatrix} \otimes \sigma^0\,,
$$

$$
\Sigma^4 \;=\; \frac{1}{2}\begin{pmatrix} \mathbf{0}_2 & i\sigma^2 \\ -i\sigma^2 & \mathbf{0}_2 \end{pmatrix} \otimes \sigma^0\,, \qquad
\Sigma^5 \;=\; \frac{1}{2}\begin{pmatrix} \mathbf{0}_2 & -\sigma^2 \\ -\sigma^2 & \mathbf{0}_2 \end{pmatrix} \otimes \sigma^0\,.
$$

- **The 2HDM Potential in the SO(1,5) Formalism**

$$V_{2HDM} = -\frac{1}{2} M_A R^A + \frac{1}{4} L_{AB} R^A R^B,$$

with

$$M_A = \left(\mu_1^2 + \mu_2^2, \quad 2\text{Re}(m_{12}^2), \quad -2\text{Im}(m_{12}^2), \quad \mu_1^2 - \mu_2^2, \quad 0, \quad 0 \right),$$

$$L_{AB} = \begin{pmatrix} \lambda_1 + \lambda_2 + \lambda_3 & \text{Re}(\lambda_6 + \lambda_7) & -\text{Im}(\lambda_6 + \lambda_7) & \lambda_1 - \lambda_2 & 0 & 0 \\ \text{Re}(\lambda_6 + \lambda_7) & \lambda_4 + \text{Re}(\lambda_5) & -\text{Im}(\lambda_5) & \text{Re}(\lambda_6 - \lambda_7) & 0 & 0 \\ -\text{Im}(\lambda_6 + \lambda_7) & -\text{Im}(\lambda_5) & \lambda_4 - \text{Re}(\lambda_5) & -\text{Im}(\lambda_6 - \lambda_7) & 0 & 0 \\ \lambda_1 - \lambda_2 & \text{Re}(\lambda_6 - \lambda_7) & -\text{Im}(\lambda_6 - \lambda_7) & \lambda_1 + \lambda_2 - \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

- **Unitary Field Transformations:** [AP, Phys. Lett. B706 (2012) 465.]

$$\text{Sp}(4): \quad \mathbf{\Phi} \mapsto \mathbf{\Phi}' = U\,\mathbf{\Phi}, \quad \text{with} \quad U \in \mathsf{U}(4) \quad \underline{\text{and}} \quad UCU^{\mathsf{T}} = \mathsf{C}$$

$$\text{SO}(5): \quad \mathsf{R}^{\mathsf{I}} \mapsto \mathsf{R}'^{\mathsf{I}} = \mathsf{O}^{\mathsf{I}}{}_{\mathsf{J}}\,\mathsf{R}^{\mathsf{J}}, \quad \text{with} \quad \mathsf{O} \in \mathsf{SO}(5) \subset \mathsf{SO}(1,5)$$

$$\implies \quad \mathsf{SO}(5) \sim \mathsf{Sp}(4)/\mathbf{Z}_2$$

• **Symmetry Breaking** Scenarios and pseudo-Goldstone Bosons

No	Symmetry	Generators $T^a \leftrightarrow K^a$	Discrete Group Elements	Maximally Broken SO(5) Generators	Number of Pseudo-Goldstone Bosons	
1	$Z_2 \times O(2)$	T^0	D_{CP1}	–	0	
2	$(Z_2)^2 \times SO(2)$	T^0	D_{Z_2}	–	0	
3	$(Z_2)^3 \times O(2)$	T^0	D_{CP2}	–	0	
4	$O(2) \times O(2)$	$T^3,\, T^0$	–	T^3	1	(a)
✓ 5	$Z_2 \times [O(2)]^2$	$T^2,\, T^0$	D_{CP1}	T^2	1	(h)
✓ 6	$O(3) \times O(2)$	$T^{1,2,3},\, T^0$	–	$T^{1,2}$	2	$(h,\, a)$
7	$SO(3)$	$T^{0,4,6}$	–	$T^{4,6}$	2	$(h^\pm)$
8	$Z_2 \times O(3)$	$T^{0,4,6}$	$D_{Z_2} \cdot D_{CP2}$	$T^{4,6}$	2	$(h^\pm)$
9	$(Z_2)^2 \times SO(3)$	$T^{0,5,7}$	$D_{CP1} \cdot D_{CP2}$	$T^{5,7}$	2	$(h^\pm)$
10	$O(2) \times O(3)$	$T^3,\, T^{0,8,9}$	–	T^3	1	(a)
11	$SO(4)$	$T^{0,3,4,5,6,7}$	–	$T^{3,5,7}$	3	$(a,\, h^\pm)$
12	$Z_2 \times O(4)$	$T^{0,3,4,5,6,7}$	$D_{Z_2} \cdot D_{CP2}$	$T^{3,5,7}$	3	$(a,\, h^\pm)$
✓ 13	$SO(5)$	$T^{0,1,2,\dots,9}$	–	$T^{1,2,8,9}$	4	$(h,\, a,\, h^\pm)$

✓ : Natural SM Alignment $\longmapsto$

• Vacuum Topology of the 2HDM

$G_{HF/CP}$	$H_{HF/CP}$	$\mathcal{M}_\Phi^{HF/CP}$	Topological Defect
Z_2	$\mathbf{I}$	Z_2	Domain Wall
$U(1)_{PQ} \simeq S^1$	$\mathbf{I}$	S^1	Vortex
$SO(3)_{HF}$	$SO(2)_{HF}$	S^2	Global Monopole
$CP1 \simeq Z_2$	$\mathbf{I}$	Z_2	Domain Wall
$CP2 = Z_2 \otimes \Pi_2$	Π_2	Z_2	Domain Wall
$CP1 \otimes SO(2)$	$CP1$	S^1	Vortex

• **Energy density of the topological defect $\phi_{1,2}(\mathbf{r})$:**

$$\mathcal{E}(\phi_1, \phi_2) = (\nabla \phi_1^\dagger) \cdot (\nabla \phi_1) + (\nabla \phi_2^\dagger) \cdot (\nabla \phi_2) + V(\phi_1, \phi_2) + V_0 .$$

• **Gradient flow approach to numerically find $\phi_{1,2}(\mathbf{r})$**

$$-\frac{\delta E[\phi_{1,2}]}{\delta \phi_{1,2}(\mathbf{r}, \tau)} = \frac{\partial \phi_{1,2}(\mathbf{r}, \tau)}{\partial \tau} \to 0 , \quad \text{for } \tau \gg 1 .$$

• Z_2 Domain Walls

$$\begin{pmatrix} v_1^0 \\ v_2^0 \end{pmatrix} \quad \overset{U(1)_Y}{\longleftrightarrow} \quad \begin{pmatrix} -v_1^0 \\ -v_2^0 \end{pmatrix}$$

$$Z_2 \updownarrow \qquad\qquad\qquad \updownarrow Z_2$$

$$\begin{pmatrix} v_1^0 \\ -v_2^0 \end{pmatrix} \quad \overset{U(1)_Y}{\longleftrightarrow} \quad \begin{pmatrix} -v_1^0 \\ v_2^0 \end{pmatrix}$$
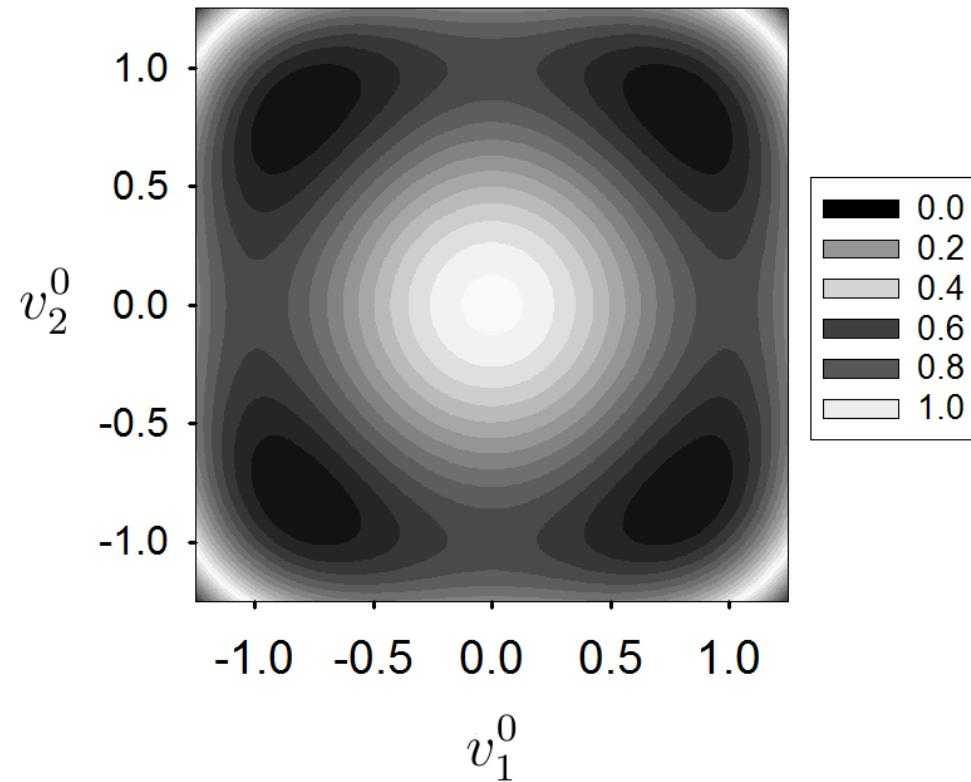
– Spatial profile of the $\mathbf{Z}_2$ domain wall

Introduce dimensionless quantities:

$$\hat{x} = \mu_2 x \,, \quad \hat{v}^0_{1,2}(\hat{x}) = \frac{v^0_{1,2}(\hat{x})}{\eta} \,, \quad \widehat{\mathrm{E}} = \frac{\lambda_2 \mathrm{E}}{\mu^3_2} \,, \quad \text{with } \eta = \frac{\mu_2}{\sqrt{\lambda_2}} \,.$$
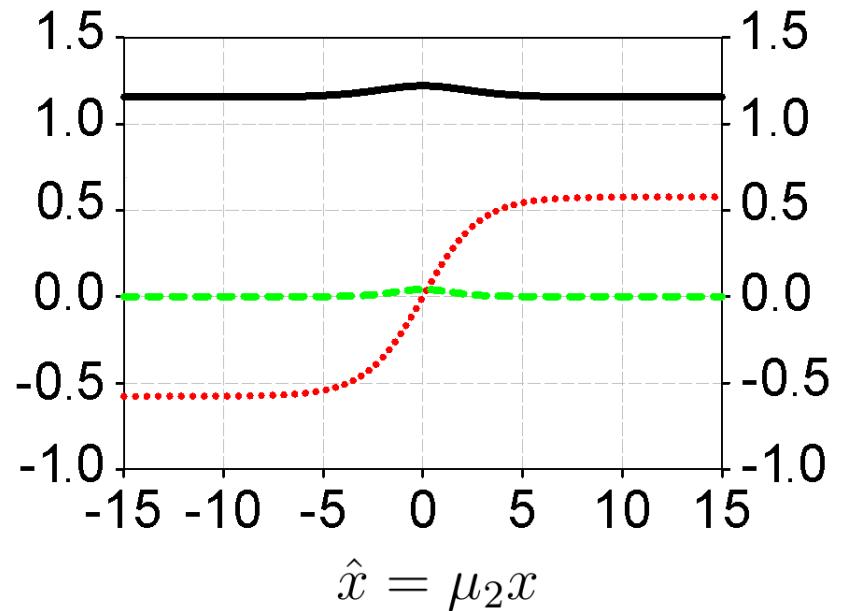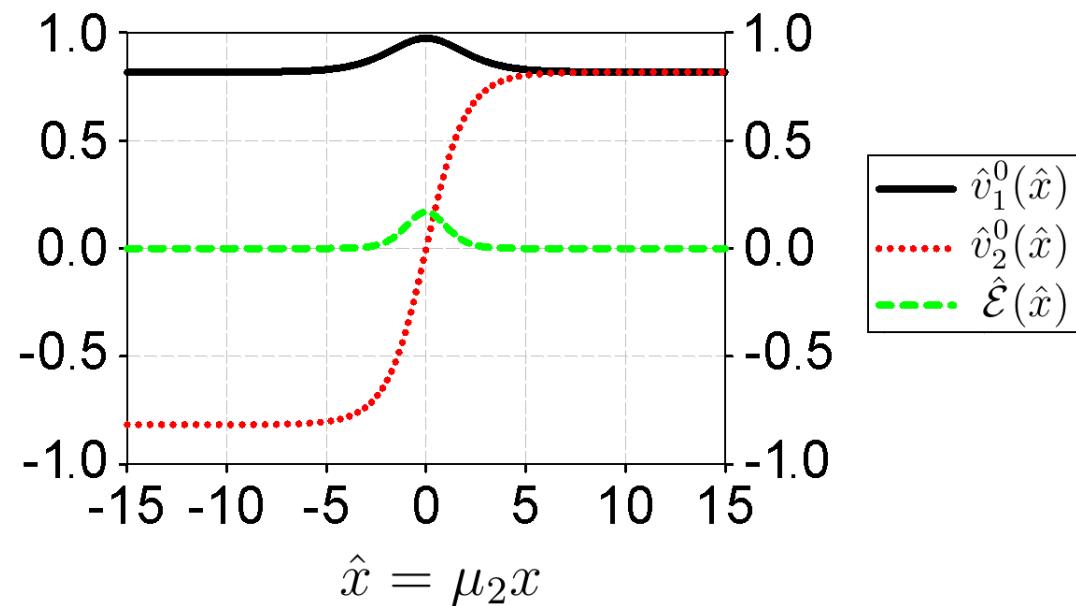
• Charge-**Violating** Domain Walls in the 2HDM

– Relatively **gauge-rotated** vacua at the boundaries:

$$\Phi_1(-\infty) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \Phi_2(-\infty) = \frac{1}{\sqrt{2}} \begin{pmatrix} v_+ \to 0 \\ -v_2\, e^{-i\xi} \end{pmatrix},$$

$$\Phi_1(+\infty) = U(+\infty)\, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \Phi_2(+\infty) = U(+\infty)\, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ +v_2\, e^{+i\xi} \end{pmatrix},$$

$$U(x) = e^{i\theta(x)}\, \exp\left(\frac{i\, G^i(x)\, \sigma^i}{v_{\text{SM}}}\, \frac{\sigma^i}{2} \right), \qquad \text{with} \quad U(-\infty) = \mathbf{1}_2\,.$$
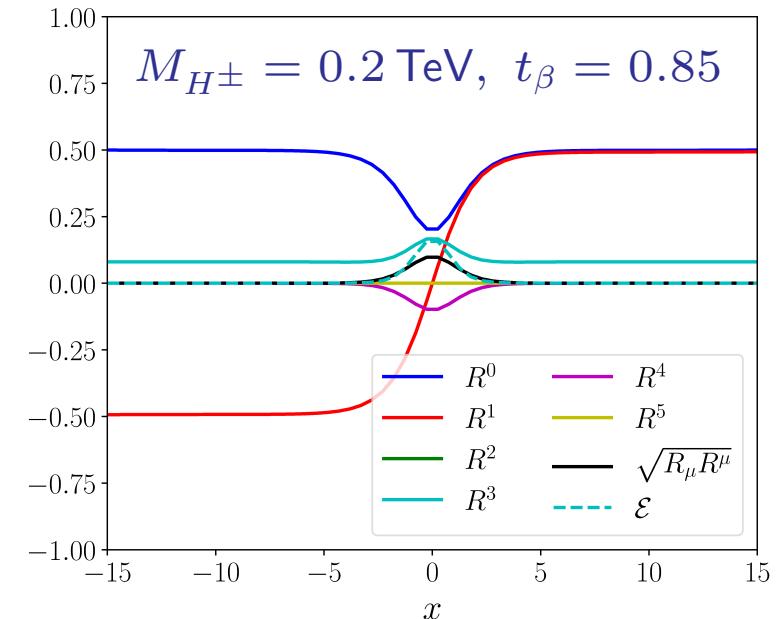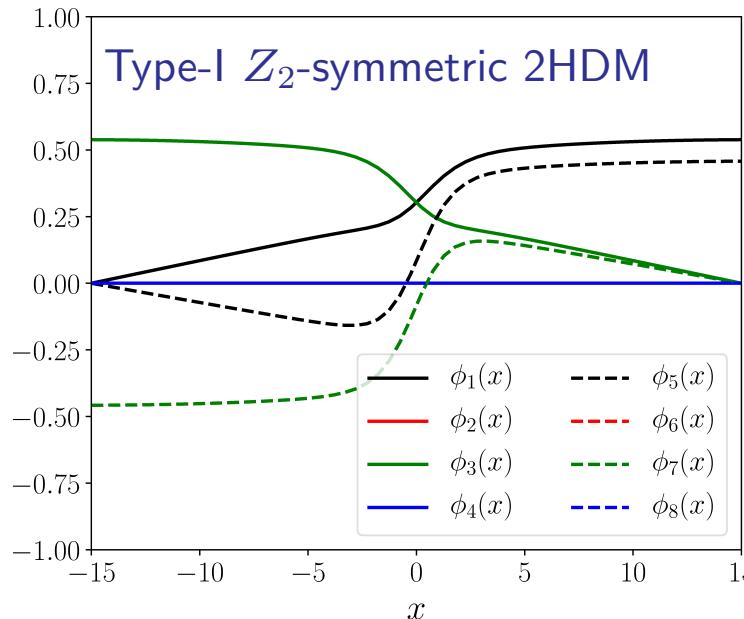
– **QCD** instantons in Type-II Z_2-symmetric 2HDM

[R Battye, AP, D Viatic, PRD102 (2020) 123536;

RD Peccei, HR Quinn '77]

$$V_{\text{inst}} \sim \Lambda_{\text{QCD}}^4 \left[\left(\frac{\Phi_1^\dagger \Phi_2}{v_{\text{SM}}^2} \right)^{n_{\text{G}}} - \left(\frac{\Phi_1^\dagger \Phi_2 \, e^{i\theta_{\text{QCD}}}}{v_{\text{SM}}^2} \right)^{n_{\text{G}}} \right] + \text{H.c.}$$

$$\lesssim \frac{\Lambda_{\text{QCD}}^4}{v_{\text{SM}}^2} s_\beta^2 c_\beta^2 \left(1 - \cos\left(n_{\text{G}} \theta_{\text{QCD}} \right) \right) \Phi_1^\dagger \Phi_2 + \text{H.c.},$$

$$\implies \quad \theta_{\text{QCD}} \gtrsim \frac{10^{-11}}{\sin\beta \cos\beta}$$

From neutron EDM limit: $\theta_{\text{QCD}} \lesssim 10^{-11} - 10^{-10}$

Loose constraint: $\boxed{\mathbf{0.3 \lesssim \tan\beta \lesssim 3}}$

– Biased initial conditions in Z_2-symmetric 2HDMs

$$N_{\mathrm{dw}} = A t^{-n} \exp(-\alpha t)$$

Legend (left panel):
- $\varepsilon = 0.03$
- $\alpha = 0.00016$, $n = 0.88512$
- $\varepsilon = 0.04$
- $\alpha = 0.00056$, $n = 0.76261$
- $\varepsilon = 0.05$
- $\alpha = 0.00106$, $n = 0.66388$
- $\varepsilon = 0.06$
- $\alpha = 0.00176$, $n = 0.50527$

Right panel: $y = 0.0533x - 0.0015$
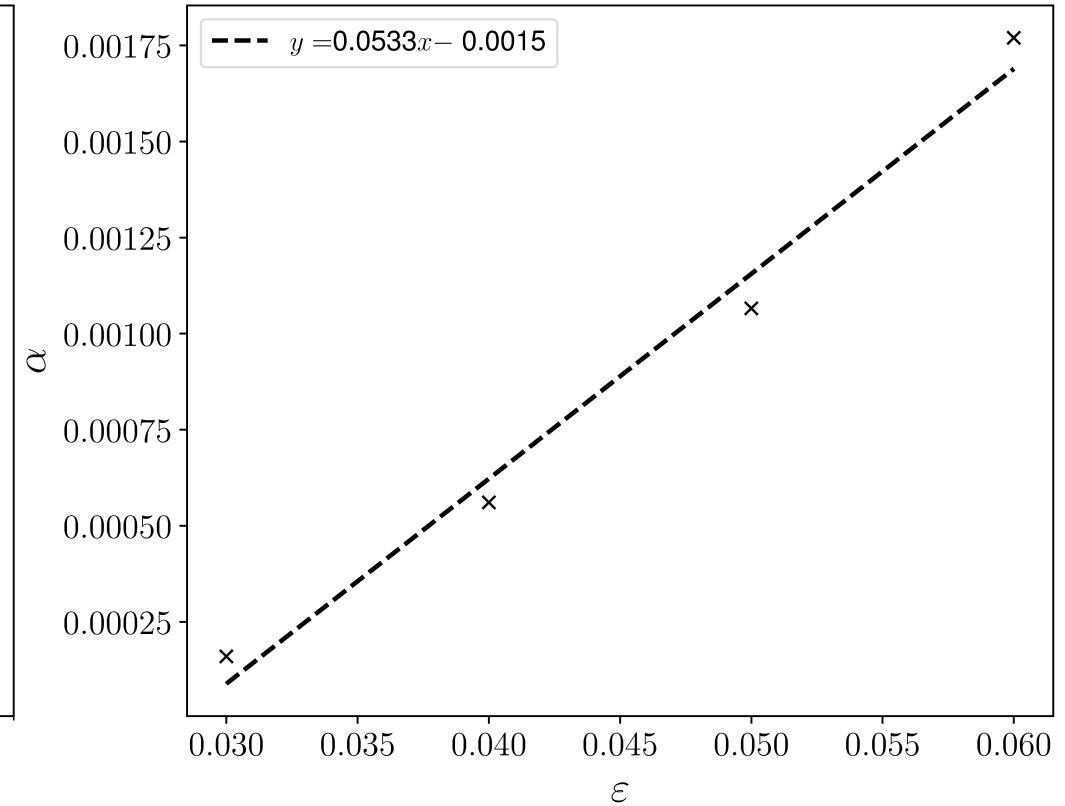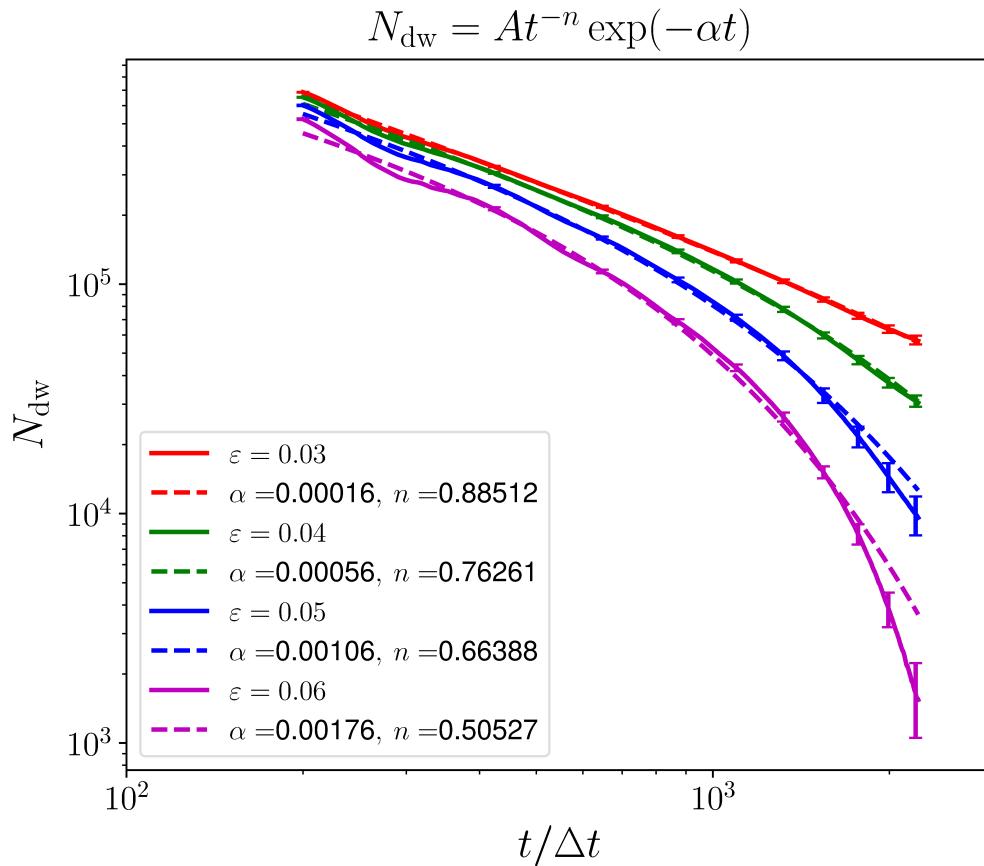
Avoidance of DW domination in the Universe:

$$\varepsilon \; > \; \frac{640\pi}{3}\frac{A\widehat{E}}{e}\left(\frac{v_{\mathrm{SM}}^{3/2}}{M_{\mathrm{Pl}}}\right)^2 \; \simeq \; 2.5 \times 10^{-29} A\widehat{E} \ \text{GeV}, \quad \text{with } A, \widehat{E} \sim 1.$$

• U(1)$_{\mathrm{PQ}}$ Vortices   [R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

$$\phi_1(r) \;=\; \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1^0(r) \end{pmatrix}, \qquad \phi_2(r,\chi) \;=\; \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2^0(r)e^{in\chi} \end{pmatrix}.$$
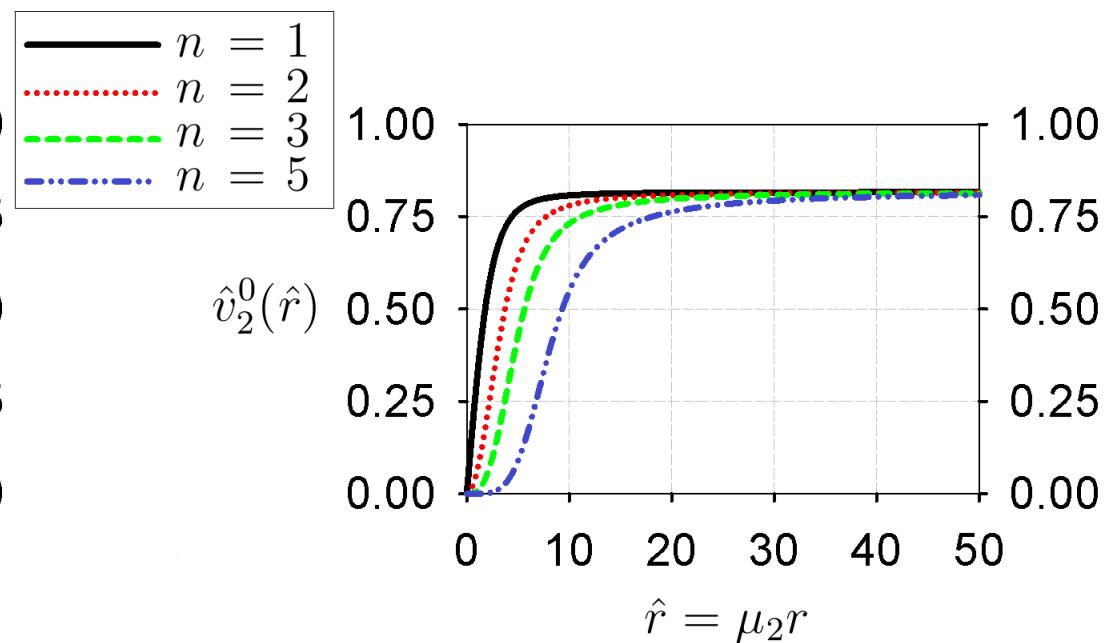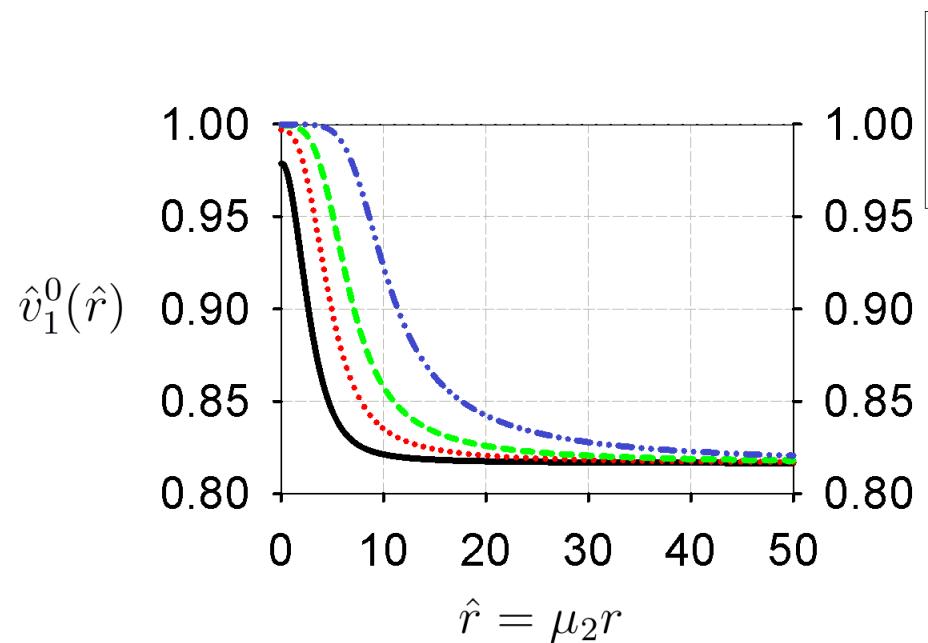
Energy dependence of the **U(1)**$_\mathrm{PQ}$ **Vortex**

Energy per unit length:

$$\mathrm{E} \;=\; 2\pi \int_0^\infty r dr \;\; \mathcal{E}(\phi_1, \phi_2) \;,$$

with

$$\mu^2 \;=\; \frac{\mu_1^2}{\mu_2^2} \;.$$

$$\phi_1(r,\chi) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(r)\cos(n\chi) \end{pmatrix}, \qquad \phi_2(r,\chi) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -v(r)\sin(n\chi) \end{pmatrix}.$$

- **SO(3)$_{\mathrm{HF}}$ Global Monopole**

$$\phi_1(r,\chi) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v(r)\sin\chi \end{pmatrix}, \qquad \phi_2(r,\chi,\psi) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v(r)e^{i\psi}\cos\chi \end{pmatrix}.$$

$$\hat{r} = \mu_1 r$$
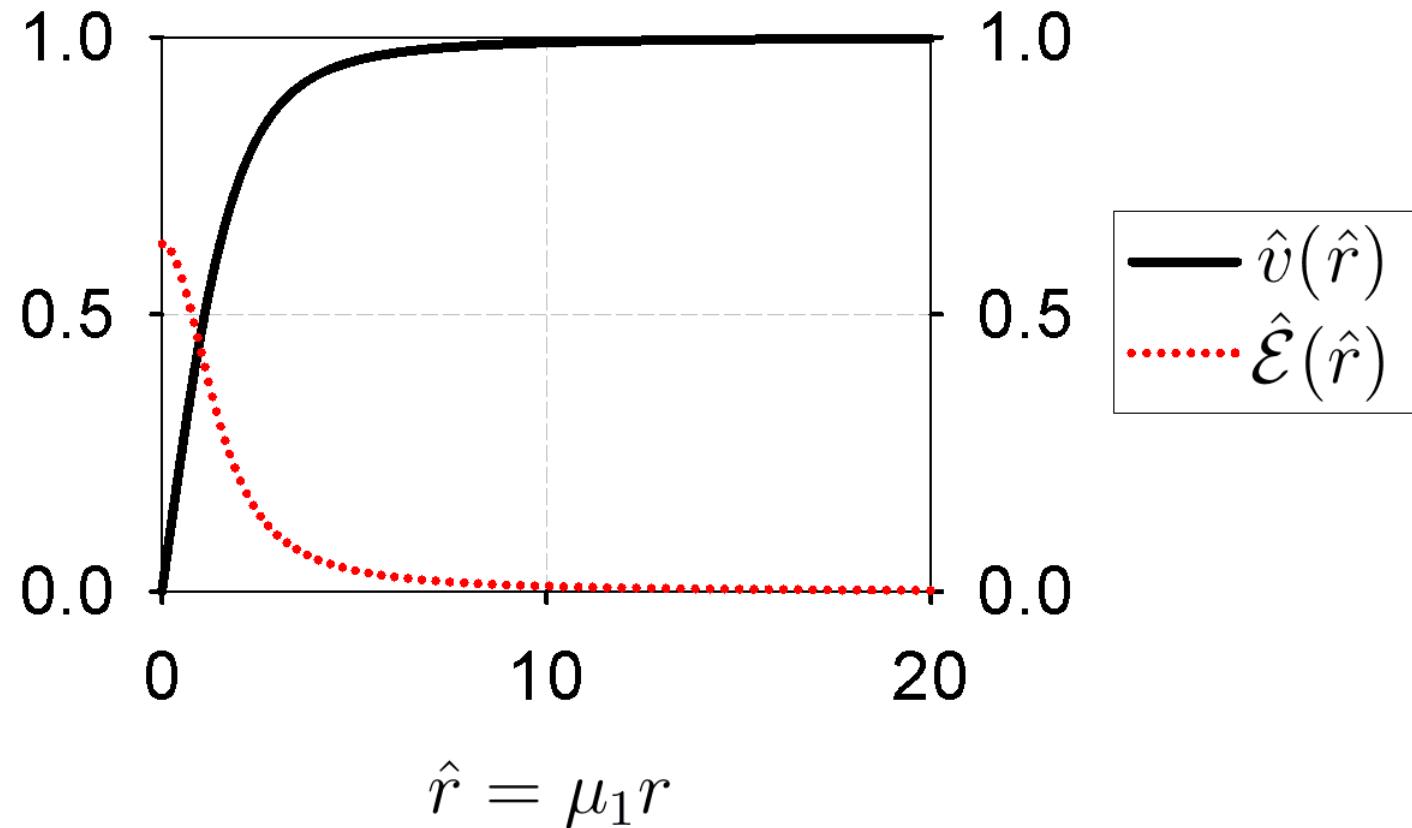
– n**HDM potential** with m **inert scalar doublets:**

$$V_{n\text{HDM}} = V_{\text{sym}} + V_{\text{inert}} + \Delta V_{\text{soft}} ,$$

– **3 continuous alignment symmetries** in the **field space of the active EWSB sector** $(N_H = n - m)$:

(i) $\text{Sp}(2N_H) \times \mathcal{D}$ (ii) $\text{SU}(N_H) \times \mathcal{D}$ (iii) $\text{SO}(N_H) \times \mathcal{CP} \times \mathcal{D}$,

where $\mathcal{D}$ acts on the inert sector *only*.

– **Symmetry invariants:**

(i) $S = \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \ldots = \frac{1}{2}\boldsymbol{\Phi}^\dagger \boldsymbol{\Phi}$

(ii) $D^a = \Phi_1^\dagger \sigma^a \Phi_1 + \Phi_2^\dagger \sigma^a \Phi_2 + \ldots$

(iii) $T = \Phi_1 \Phi_1^\mathsf{T} + \Phi_2 \Phi_2^\mathsf{T} + \ldots$

– **Symmetric part of the scalar potential:**

$$V_{\text{sym}} = -\mu^2\, S + \lambda_S\, S^2 + \lambda_D\, D^a D^a + \lambda_T\, \text{Tr}\,(T\, T^*) .$$

– **Inert part of the scalar potential:**

$$V_{\text{inert}} \;=\; \widehat{m}^2_{\hat{a}\hat{b}}\, \widehat{\Phi}^\dagger_{\hat{a}}\widehat{\Phi}_{\hat{b}} \;+\; \lambda_{\hat{a}\hat{b}\hat{c}\hat{d}}\,(\widehat{\Phi}^\dagger_{\hat{a}}\widehat{\Phi}_{\hat{b}})(\widehat{\Phi}^\dagger_{\hat{c}}\widehat{\Phi}_{\hat{d}}) \;+\; \lambda_{\hat{a}\hat{b}cd}\,(\widehat{\Phi}^\dagger_{\hat{a}}\widehat{\Phi}_{\hat{b}})(\Phi^\dagger_c\Phi_d)$$

$$+\; \lambda_{a\hat{b}\hat{c}d}\,(\Phi^\dagger_a\widehat{\Phi}_{\hat{b}})(\widehat{\Phi}^\dagger_{\hat{c}}\Phi_d) \;+\; \left[\,\lambda_{a\hat{b}c\hat{d}}\,(\Phi^\dagger_a\widehat{\Phi}_{\hat{b}})(\Phi^\dagger_c\widehat{\Phi}_{\hat{d}}) \;+\; \text{H.c.}\right]$$

$$\mathsf{Z}^{\mathsf{I}}_2:\quad \Phi_a \to \Phi_a \quad (a = 1,\,2,\,\ldots,\,N_H)\,,\qquad \widehat{\Phi}_{\hat{b}} \to -\,\widehat{\Phi}_{\hat{b}} \quad (\hat{b} = \hat{1},\,\hat{2},\,\ldots,\,\widehat{m})$$

– **Soft-symmetry Breaking:**

$$\Delta V_{\text{soft}} \;=\; m^2_{ab}\,\Phi^\dagger_a\Phi_b$$

– **Minimal Symmetry of Alignment in the Higgs basis:**

$$\mathsf{Z}^{\mathsf{EW}}_2:\quad \Phi'_1 \to \Phi'_1\,,\qquad \Phi'_{a'} \to -\,\Phi'_{a'}\qquad (a' = 2,\,3,\,\ldots,\,N_H)$$

where m^2_{ab} becomes diagonal.

$\Longrightarrow$ $\boxed{\textbf{Minimal Alignment Symmetry:}\quad \mathsf{Z}^{\mathsf{EW}}_2 \times \mathsf{Z}^{\mathsf{I}}_2}$ [AP '16]