Glueballs and exotics search strategies

Jean-Marie Frère, Physique Théorique , ULB

Avec le support de l'IISN, Belgique.

- Glueballs are the first prediction of QCD, and candidates have been around for a long time, but none has really been identified with certaintly
- Hybrids have been seen (most probably gluon + quark, but could of course be 4 quark states)

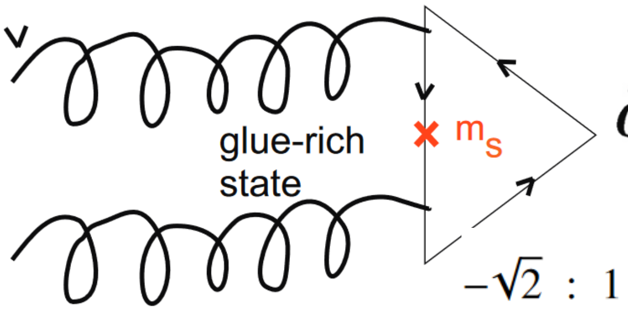
- Standard quark model approach : write a mixing matrix, then fit decays
  - Often unpractical with wide resonances, overlapping partial waves
- Search for sequences of glueballs decay paths
- Combine glue-rich decay products with glue-rich production;

Approach 1: find Glue-rich decay products

When no “safe” glueball has been identified, we must rely on theory

We know that the η (548 MeV) and η’ (958 MeV) have much too large masses to be pseudo-goldstone bosons like the pi and K respectively.
In fact, it was established that the chiral symmetry is broken by QCD (instantons). The connection between glue and the η system may then be established by comparison to the data.

In the naïve SU(3) flavour, chiral symmetry is only broken by quark masses, dominated by the strange quark



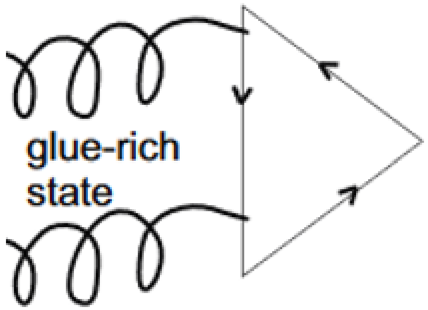
$$\partial_\mu j_5^\mu \Rightarrow \eta, \eta'$$

$$\partial_\mu A_8^\mu = \frac{2}{\sqrt{6}} \left(m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d - 2m_s \bar{s} i\gamma_5 s \right)$$

$$\partial_\mu A_0^\mu = \frac{2}{\sqrt{3}} \left(m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d + m_s \bar{s} i\gamma_5 s \right)$$

This seems to imply a ratio favoring the octet state $-\sqrt{2}\ :\ 1$

In fact, most of the analysis was (and sometimes still is) done in these terms.



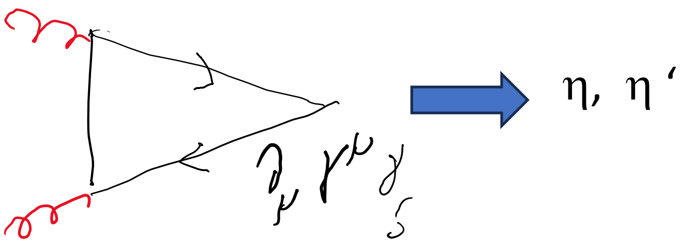
$$\partial_\mu j_5^\mu \Rightarrow \eta, \eta'$$

Of course, we are now familiar with the so-called anomaly.
Despite its name, this term is nothing weird; is is finite, and well-known to be responsible in a related context for $\pi^0 \to 2\gamma$
This term links in a fundamental way to the singlet (instead of the octet) current!

$$\partial_\mu A_8^\mu = \frac{2}{\sqrt{6}}\left(m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d - 2 m_s \bar{s} i\gamma_5 s\right)$$

$$\partial_\mu A_0^\mu = \frac{2}{\sqrt{3}}\left(m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d + m_s \bar{s} i\gamma_5 s\right)$$

$$+ \frac{1}{\sqrt{3}} \frac{3}{4} \frac{\alpha_s}{\pi} G^A_{\mu\nu} \widetilde{G}^{A\mu\nu},$$



This suppression is WRONG for the singlet channel !

Instead, the chiral anomaly gives a direct path to η η’

$$\partial^\mu A^8_\mu = \frac{2}{\sqrt{6}}(m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d - 2m_s \bar{s} i\gamma_5 s) \ ,$$

$$\partial^\mu A^0_\mu = \frac{2}{\sqrt{3}}(m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d + m_s \bar{s} i\gamma_5 s) + \boxed{\frac{1}{\sqrt{3}} \frac{3\alpha_s}{4\pi} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}}$$

$$\frac{< G^{\mu\nu} \widetilde{G}_{\mu\nu} \mid \eta' >}{< G^{\mu\nu} \widetilde{G}_{\mu\nu} \mid \eta >} \approx 3$$

As measured (at the suggestion of Gherstein) from the J/ψ radiative decays to η and η’

In fact, we have 2 competing effects, and the physical mixing falls in-between the "octet-singlet" and the strange-non-strange"

It is tempting to write a simple mixing equation

$$|\eta\rangle = |\eta_8\rangle \cos\theta - |\eta_0\rangle \sin\theta,$$

$$|\eta'\rangle = |\eta_8\rangle \sin\theta + |\eta_0\rangle \cos\theta.$$

This however lead to unending controversies as to the value of the mixing angle. In our initial review of the subject (which used the "anomalous" term to describe all the VVP process (including V into P gamma),a value of 15° appeared a reasonable compromise

An aside :

This is however certainly not the end of the story, since the view of a “rigid” mixing between 2 states which in reality live at very different masses (500 vs 1000MeV), both sides of the QCD transition, has proven not to be realistic (even if working in the “quark basis” seems to minimize the effect).

A more adapted “2-angle formalism” deals with this *(an alternate view would be to consider an energy-dependent angle).*

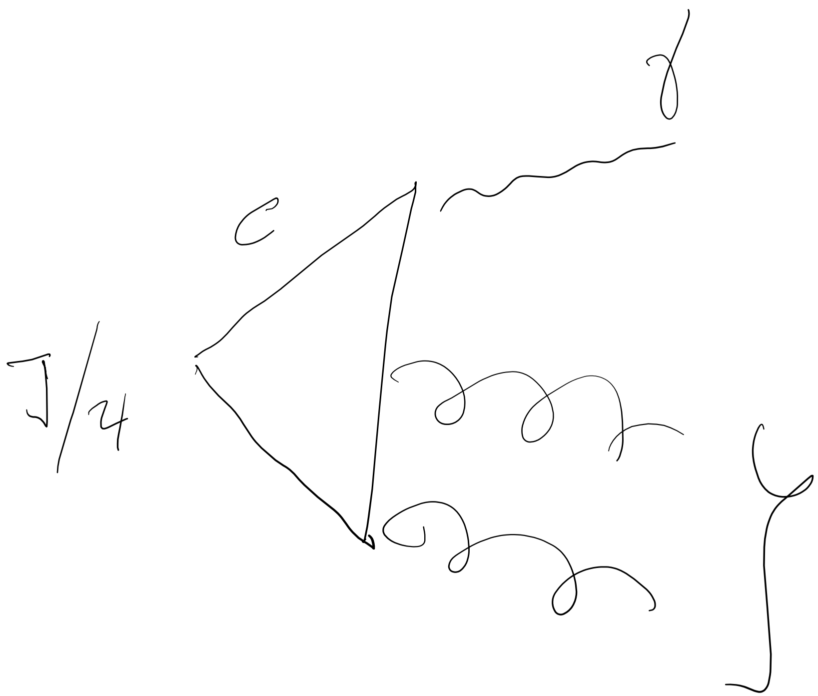
$$\langle 0|\frac{3\alpha_s}{4\pi}G\tilde{G}|\eta\rangle = \sqrt{\frac{3}{2}}m_\eta^2(f_8\cos\theta_8 - \sqrt{2}f_0\sin\theta_0) \ ,$$

$$\langle 0|\frac{3\alpha_s}{4\pi}G\tilde{G}|\eta'\rangle = \sqrt{\frac{3}{2}}m_{\eta'}^2(f_8\sin\theta_8 + \sqrt{2}f_0\cos\theta_0) \ .$$

$$\theta_8 = (-22.2 \pm 1.8)^\circ \ , \quad \theta_0 = (-8.7 \pm 2.1)^\circ \ , \quad f_0 = (1.18 \pm 0.04)f_\pi$$

See Escribano and JMF

A favoured Glue-Rich source : J/ψ radiative decays



Could also be done for ϒ; but BESIII has impressive results on J/ ψ

2 gluon state, tagged by photon,

For instance, applies to η and η’ explaining the large BR of η” despite phase space (Gherstein)

$$\frac{< G^{\mu\nu}\widetilde{G}_{\mu\nu} \mid \eta' >}{< G^{\mu\nu}\widetilde{G}_{\mu\nu} \mid \eta >} \approx 3$$

BR	$\gamma\,\eta'(958)$	$(\ 5.25\ \pm\ 0.07\ ) \times 10^{-3}$
	$\gamma\,\eta$	$(\ 1.085 \pm\ 0.018) \times 10^{-3}$

Approach 1 continued : identify “glue-rich” states

We have already done so with the η (548 MeV) and η’ (958 MeV) using mostly theory !

Another path to glue-rich states consists in looking for “exotics” : quark-gluon combinations with quantum numbers inaccessible to pure quark states;

Exotics -Hybrids

Maybe turning to exotics would make things easier?
Some hybrid quark-gluon states may be “exotic”

What are exotics?

We mainly mean meson states which cannot be realized with quarks alone.
In particular, their P and C properties may be incompatible with Spin/Isospin.

For instance, the π_1 I=1, J= 1^{-+} (previously called $\tilde{\rho}$)
could correspond to the interpolating current

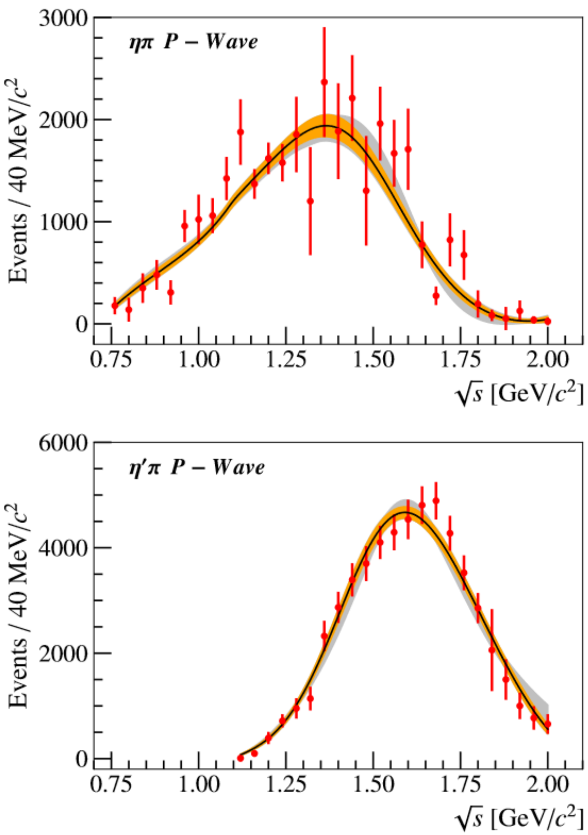
$$\phi^{\mu}_{\tilde{\rho}0} = g_s G^{\mu\nu}_a \, \tfrac{1}{2} (\bar{u}\gamma_\nu\gamma_5\lambda^a u - \bar{d}\gamma_\nu\gamma_5\lambda^a d) / f_{\tilde{\rho}} m^3_{\tilde{\rho}}$$

Unfortunately, at the quantum numbers level, the exotics with one gluon may also be realized trivially as an added quark-antiquark state (so-called 4-quark states).

The Parity of a q-antiqark system is given by $(-)^{l+1}$ with l the angular momentum, while the Charge conjugation is given by $(-)^{l+s}$, so the 1^{-+} state is forbidden in a strict 2-quark state.

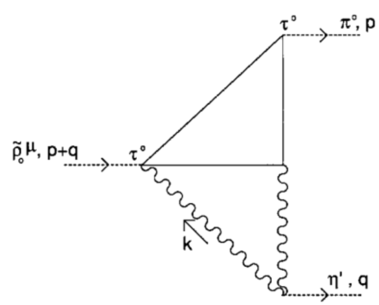
The π_1

$\pi_1(1400) + \pi_1(1600)$



Partial wave analysis of COMPASS data (Kopf et al.)

PDG now lists 2 states, but a recent coupled analysis (Kopf et al.) shows it could be one single state (around 1600)
Consistently seen in $\eta(')\,\pi$ modes, **while the traditional analysis favored $\rho\,\pi$ or b_1 p,…which are seen less sytematically .**



$$\frac{< G^{\mu\nu}\widetilde{G}_{\mu\nu} \mid \eta' >}{< G^{\mu\nu}\widetilde{G}_{\mu\nu} \mid \eta >} \approx 3$$

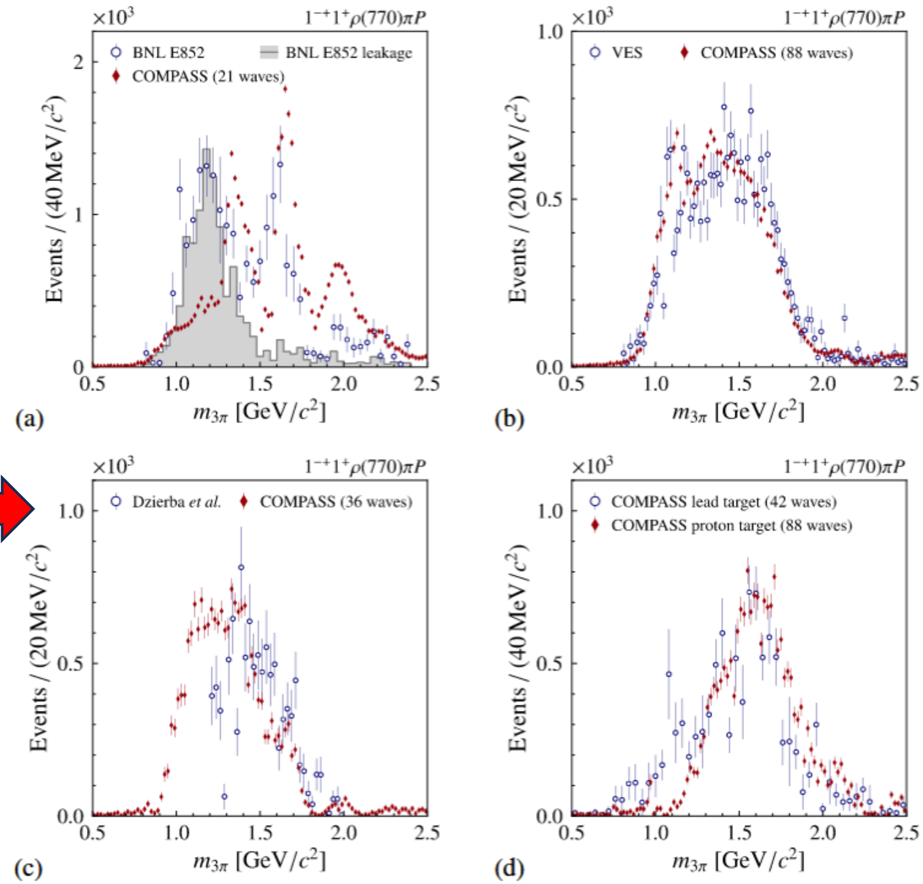
Jmf, S Titard 1988
…finally recognized
(see below)

Looking for the ρ π channel in COMPAS is no easy business!

Various partial waves analysis (up to 88 waves!)

PHYSICAL REVIEW D **105,** 012005 (2022)

Exotic meson $\pi_1(1600)$ with $J^{PC} = 1^{-+}$ and its decay into $\rho(770)\pi$



points to remember

- Partial wave analysis may be extremely difficult and depends both on the **quality of the data and on the choice of waves to include**.
  (this is not a bijective transformation : many partial waves may give the same resulting curve → informed choices
- Ideally, a partial wave analysis should be verified by more than one production mode (in one production mode, one gets a coherent superposition of waves)

- The η' system , even if not the main decay , plays a key role, possibly for 2 reasons :
  - Theory predicts an affinity of η' and glue
  - It may be better protected from noise (by its mass, decay mode)

The η_1

Seen by GAMS,
and now by BESIII
decay modes : η η'

$\eta_1(1855)$ DECAY MODES

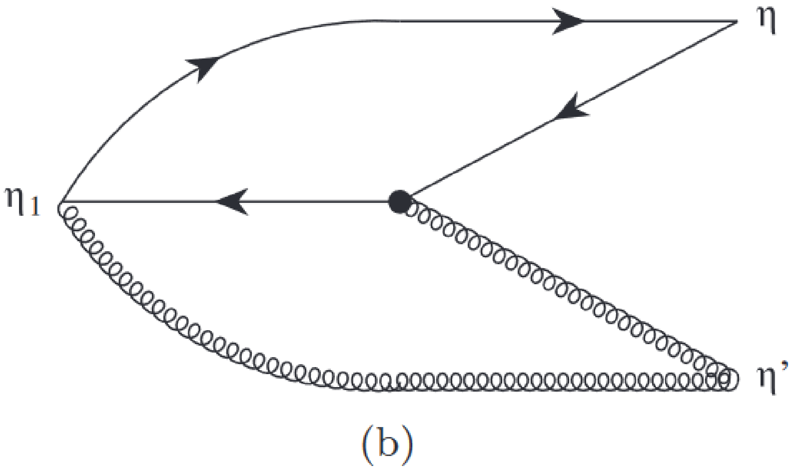
	Mode	Fraction (Γ_i/Γ)
Γ_1	$\eta\eta'$	seen

$\Gamma(\eta\eta')/\Gamma_{total}$ **Γ_1/Γ**

VALUE	*DOCUMENT ID*		*TECN*	*COMMENT*
seen	ABLIKIM	22AI	BES3	$J/\psi \to \gamma\eta\eta'$
seen	BARBERIS	00A		450 $pp \to p_f\eta\eta' p_s$
seen	ALDE	91B	GAM2	38 $\pi^- p \to \eta\eta' n$

“We find that the QCD axial anomaly enhances the decay width of the ηη′ channel
although this mode is strongly suppressed by the small P -wave phase space. Our results support
the interpretation of the η1(1855) recently observed by BESIII as the ¯ssg hybrid meson of IGJP C =0+1−+.

The QCD axial anomaly ensures the ηη′ decay mode to be a characteristic signal of the hybrid nature of the $\eta_1(1855)$ (Chen, Su and Zhu)



Once again, the ηη′ play a key role in the detection, even if not the main decay !

The difficult case : the "boring" states (read f0 , 0^{++} scalars with "vacuum" quantum nb)

There have been several attempts at the construction of mixing matrices usually with the 3 states f0 (1370) f0(1500) f0(1710), under various hypothesis.

While the role of the η and η' through anomalies was noted very early (with the GAMS discoveries) by Gherstein, the traditional mixing ignored them and insisted on the chiral suppression in the analysis of decays.

(hence the previous motivation based on exotics!)

The traditional approach (which worked for quark resonances) consists in writing “mixing matrices”, and trying to fit them to a variety of processes.

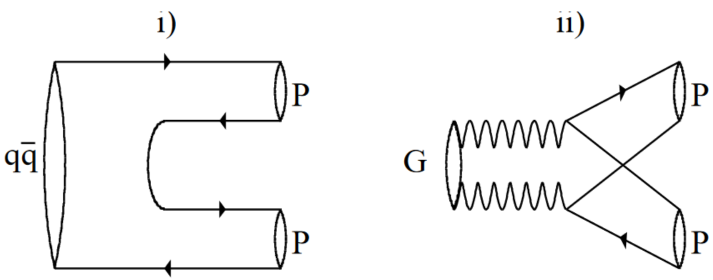
Various assumptions possible

- Flavour-blind (true at Lagrangian level)
- Mitigated by overlap of wave functions (favours heavy states)
- Some transitions chirally suppressed → strange quark favoured
- BUT : anomalies upset this (η η') then favoured, mostly singlet

Previous approaches (following Close et al, no anomaly contribution)

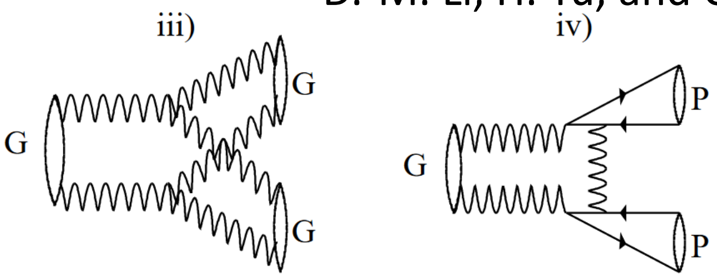
$$\begin{pmatrix} |f_0(1370)\rangle \\ |f_0(1500)\rangle \\ |f_0(1710)\rangle \end{pmatrix} = \begin{pmatrix} -0.91 & -0.07 & 0.40 \\ -0.41 & 0.35 & -0.84 \\ 0.09 & 0.93 & 0.36 \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix},$$

Later, more graphs included , notably gluon based



$$|f_1\rangle = -0.599|N\rangle + 0.326|S\rangle - 0.732|G\rangle,$$
$$|f_2\rangle = 0.795|N\rangle + 0.350|S\rangle - 0.495|G\rangle,$$
$$|f_3\rangle = 0.095|N\rangle - 0.878|S\rangle - 0.469|G\rangle.$$

D.-M. Li, H. Yu, and Q.-X. Shen, Eur.Phys.J. C19 (2001) 529–533,



No chiral suppression

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} \quad \begin{pmatrix} -0.85 & -0.38 & 0.37 \\ 0.53 & -0.63 & 0.57 \\ 0.02 & 0.68 & 0.73 \end{pmatrix} \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G \end{pmatrix}$$

Jmf, Julian Heeck Phys.Rev.D 92 (2015) 11, 114035

Chiral suppression

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.74 & 0.47 & 0.49 \\ -0.68 & 0.55 & 0.49 \\ -0.04 & -0.69 & 0.72 \end{pmatrix} \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G \end{pmatrix}$$

Another more recent analysis (no anomaly?)

C.~Q.~Geng et al PhysRevD.110.014014 [arXiv:2403.07701 [hep-ph]].

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.981(019) & -0.121(242) & 0.039(118) \\ 0.117(241) & 0.981(019) & 0.078(076) \\ -0.048(119) & -0.072(078) & 0.994(006) \end{pmatrix} \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G_0 \end{pmatrix}.$$

Here the (current) preferred version is for the 1700 to be the glueball, and the other simple quark states.

Why so much discrepancy in the matrices ?

- We are not dealing with narrow resonances, the decays are highly momentum dependent (were it only by phase space !) , and partial waves interfere
- The matrix approach is by comparison merely an algebraic one, which discards all momentum dependence
- The branching ratios used for fitting the mixing matrix don't cover the full decay spectrum (for example, 4 pi or 2 σ)
- The PDG numbers vary in quality : often, recent analysis are not listed in main tables, some numbers are direct counting, others are partial wave analysis with specific choices
- The question of chiral suppression (and η η' role) is often treated in the "traditional" way

Looking in more detail at the f0 modes, we see clearly those distortions and effects

We start with a recent partial wave analysis (does not extend to BESiii) , and mainly pions and kaons

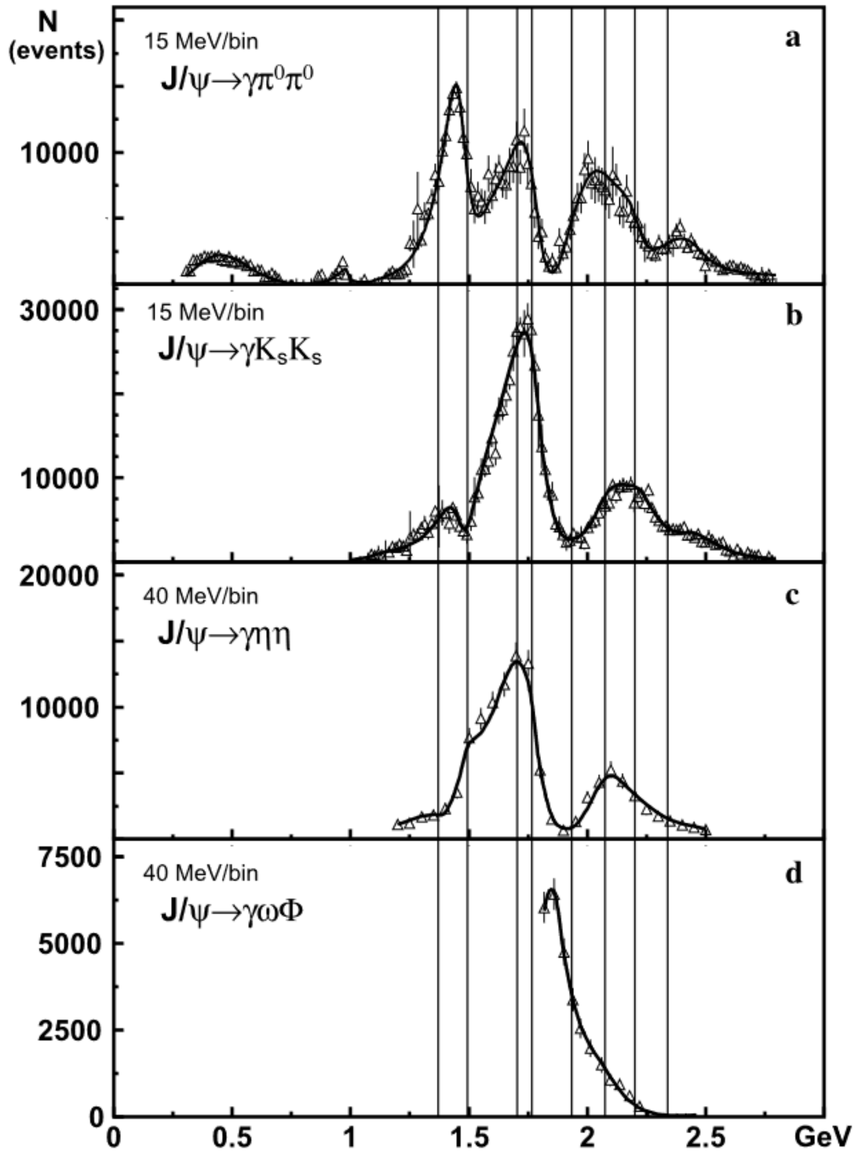
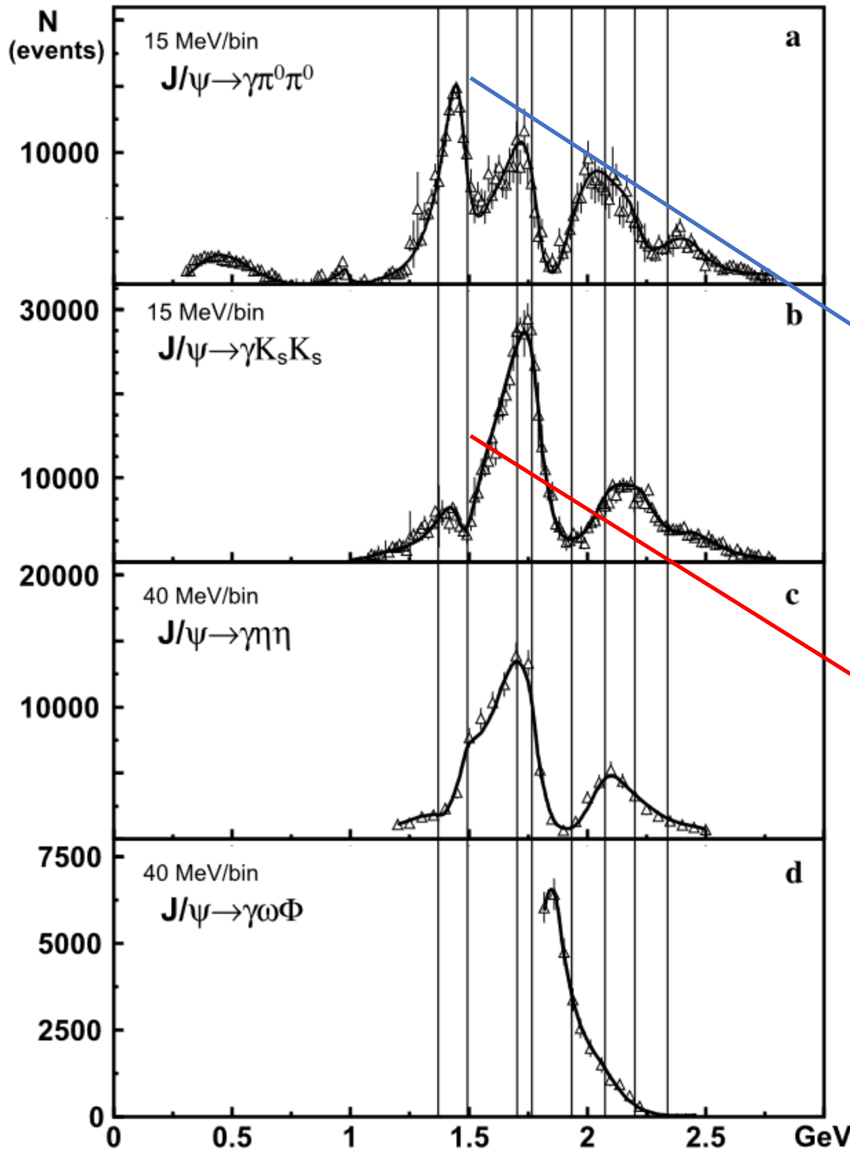


Fig. 1. Number of events in the S-wave as functions of the two-meson invariant mass from the reactions $J/\psi \to \gamma\ \pi^0\pi^0$ (a), $K_S K_S$ (b), $\eta\eta$ (c), $\phi\omega$ (d). (a) and (b) are based on the analysis of $1.3 \cdot 10^9$ J/ψ decays, (c) and (d) on $0.225 \cdot 10^9$ J/ψ decays.

In this interpretation, the authors argue for having the 2 lower states (1360) (1500) in a singlet and an octet , and for a complex (bump) at higher energy standing for the glueball .

A.V. Sarantsev , I. Denisenko , U. Thoma , E. Klempt Phys Lett B.2021.136227



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They interpret this as the interference of an octet and a singlet state (constructive vs destructive for the pion and K decays due to s contribution sign flip)
(quite different from the strange-non-strange usual approach)

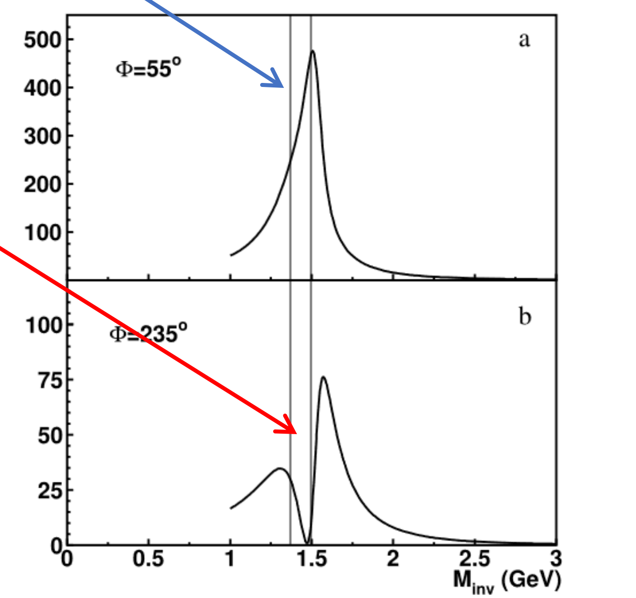


Fig. 2. Interference of two K-matrix poles for $f_0(1370)$ and $f_0(1500)$ with a relative phase of 55° (a) and 235° (b). Shown is the intensity in arbitrary units as a function of the two-meson invariant mass.

A recent partial wave analysis of the ηη' channel of radiative J/Psi decays by BES III does **not see the 1710 mode** and *concludes that it pleads for its being the only 0++ glueball .* (?)

But it is still interesting to look more in details at the first cancidate, the f0 (1500) f0 (1500) : seen by many expts : GAMS …crystal barrel, BES …

$f_0(1500)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor
Γ_1	$\pi\pi$	(34.5 ± 2.2) %	1.2
Γ_2	$\pi^+\pi^-$	seen	
Γ_3	$2\pi^0$	seen	
Γ_4	4π	(48.9 ± 3.3) %	1.2
Γ_5	$4\pi^0$	seen	
Γ_6	$2\pi^+2\pi^-$	seen	
Γ_7	$2(\pi\pi)_{S\text{-wave}}$	seen	
Γ_8	$\rho\rho$	seen	
Γ_9	$\pi(1300)\pi$	seen	
Γ_{10}	$a_1(1260)\pi$	seen	
Γ_{11}	$\eta\eta$	(6.0 ± 0.9) %	1.1
Γ_{12}	$\eta\eta'(958)$	(2.2 ± 0.8) %	1.4
Γ_{13}	$K\overline{K}$	(8.5 ± 1.0) %	1.1
Γ_{14}	$\gamma\gamma$	not seen	

A strange beast indeed (in fact large despite at threshold in η’ η

Although at the edge of phase space, the η η’ is enhanced ---- see work by Gherstein, Prokoshkin et al, following Novikov, Shifman Vaiinshtein,Zakharov. , and our later work on the nature of η’

Missing in most quark model analysis

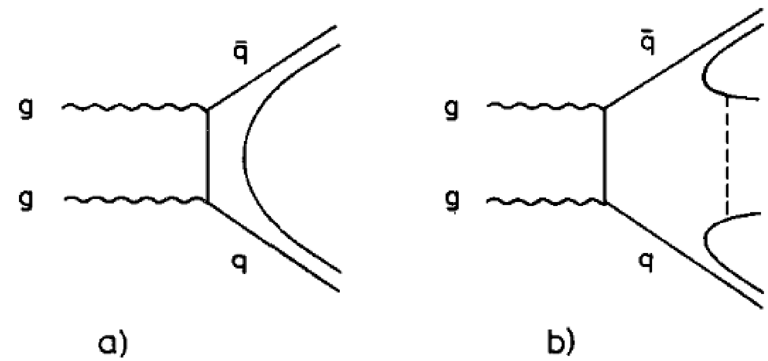


Fig. 1a and b. (See text)

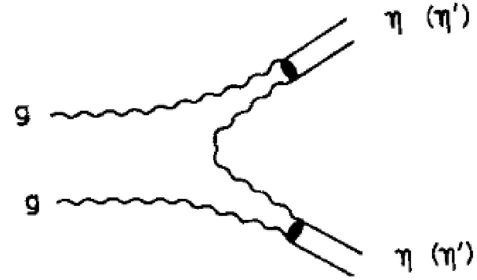
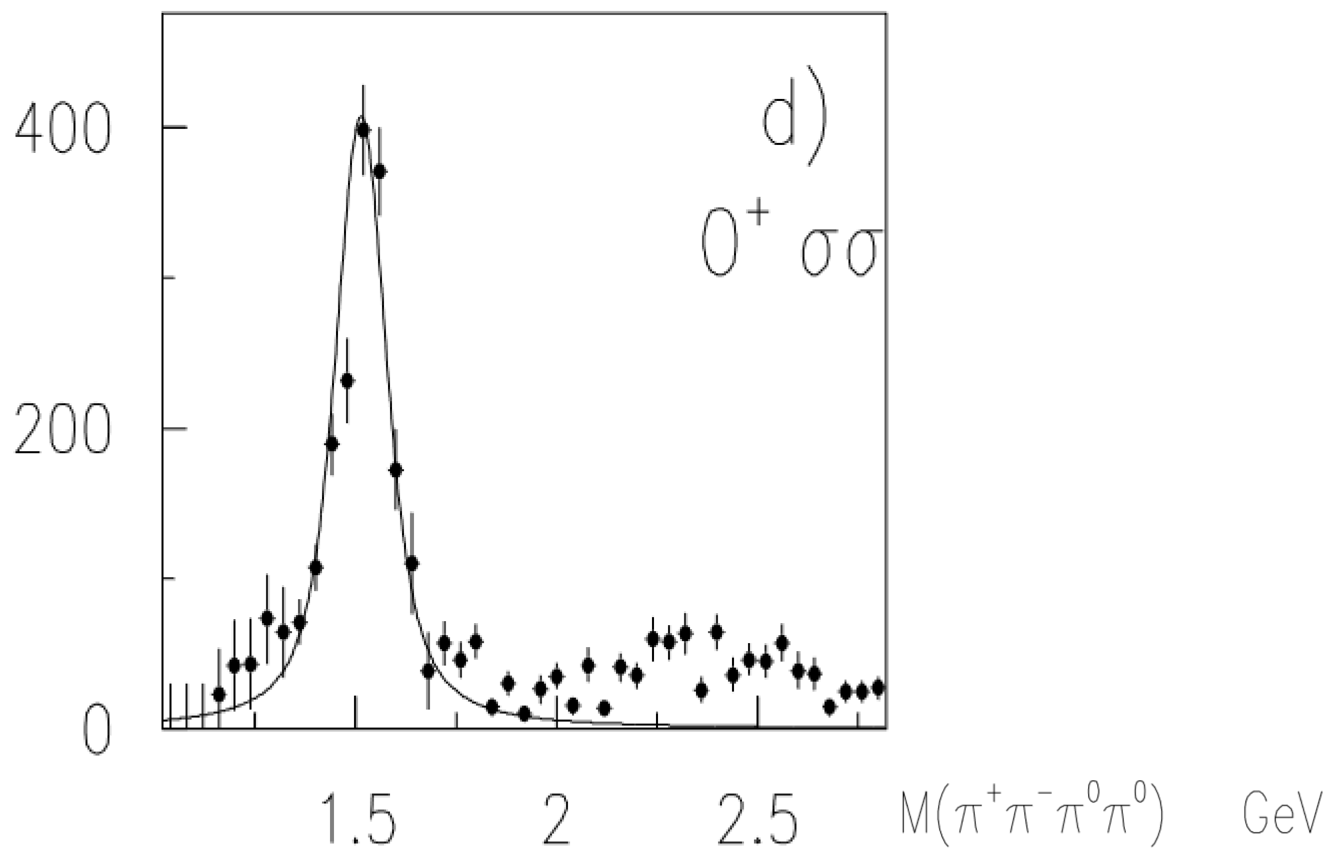


Fig. 2. (See Text)

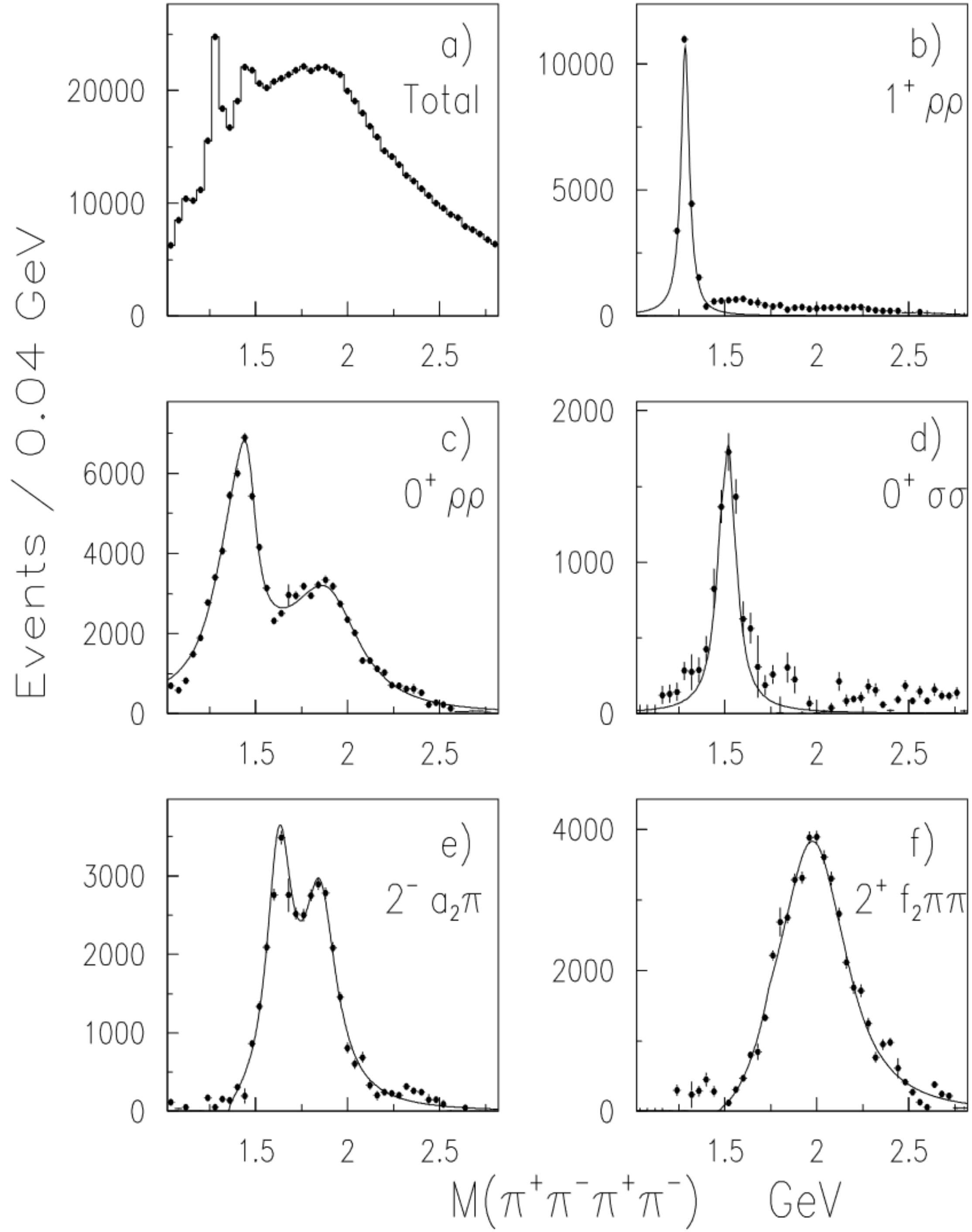
Approach 2 : try to look for “cascades” of glue-linked states.

Another example of such cascade of states f0(1500) decays notably into 4 pions, grouped in 2 f0(500) (also called σ)



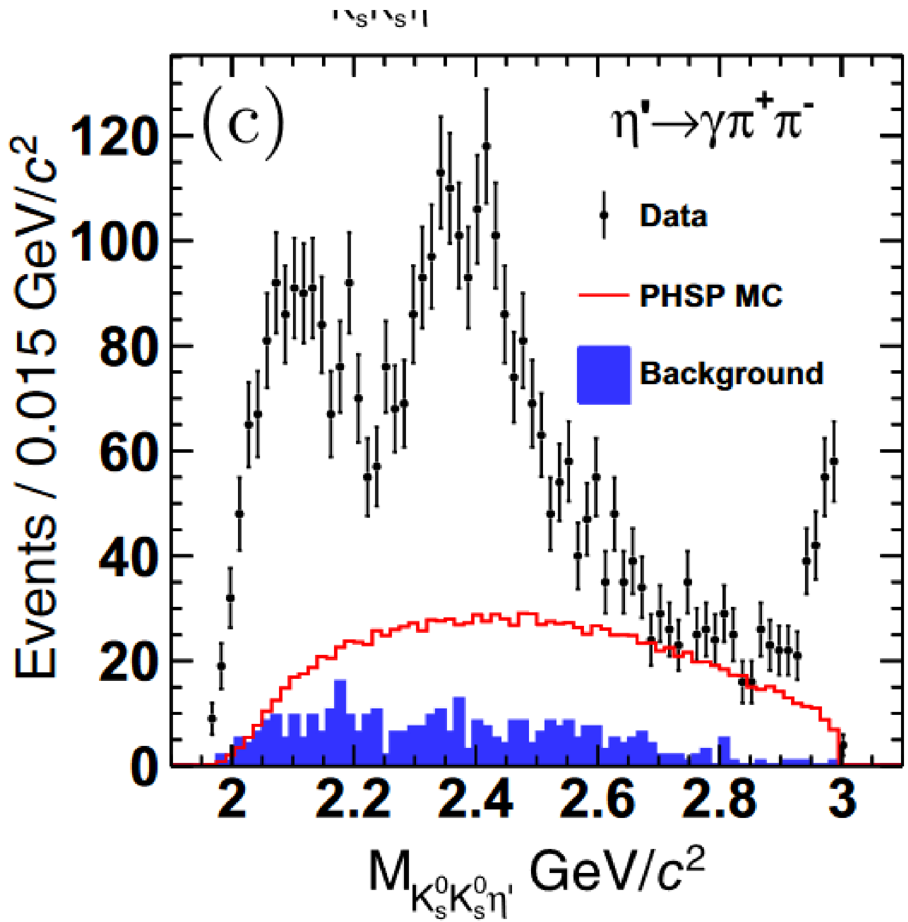
Phys.Lett.B471:440-448,2000 WA102 collaboration

the branching ratio of f0(1500) to ρρ/σσ = 2.6 ± 0.4

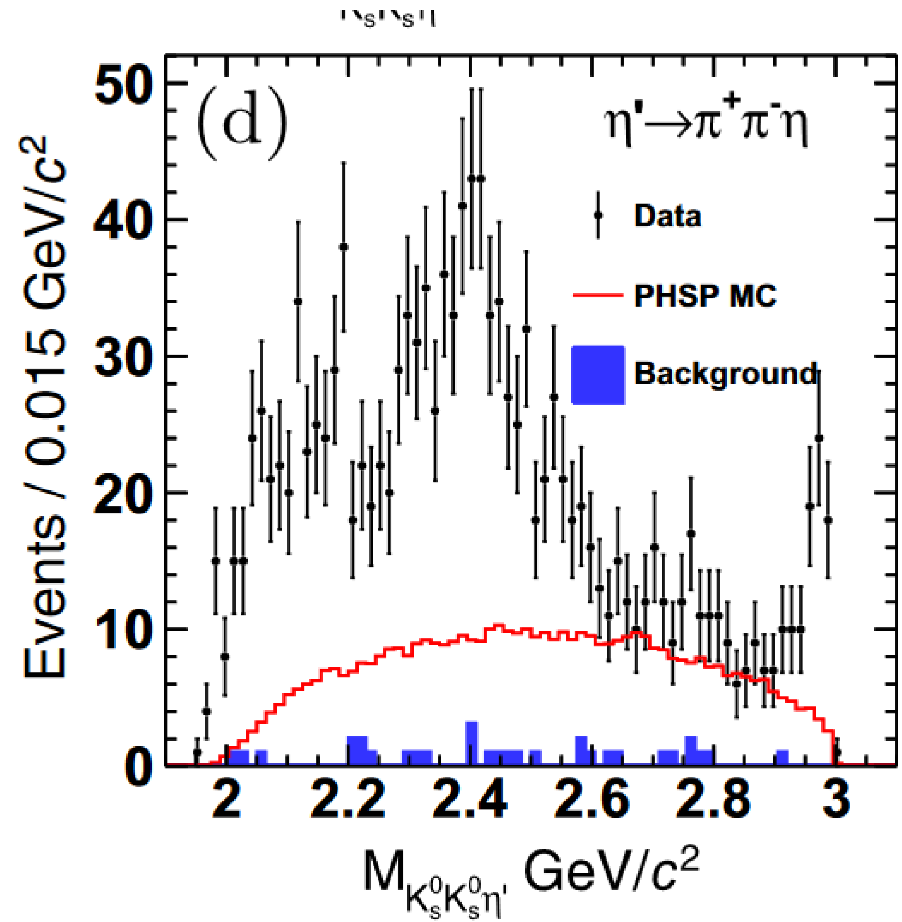

a)
Total
b)
1^+ ρρ
c)
0^+ ρρ
d)
0^+ σσ
e)
2^- $a_2\pi$
f)
2^+ $f_2\pi\pi$
Events / 0.04 GeV
$M(\pi^+\pi^-\pi^+\pi^-)$ GeV

BESIII discovery of the X(2370) a candidate for 0^{-+} glueball

It decays via a η’ and the K° K° channel also shows an affinity for f0 (980)



Eur. Phys. J. C (2020) 80:746
https://doi.org/10.1140/epjc/s10052-020-8078-4



Phys. Rev. Lett. 132 (2024) 181901

Now, the f0(1500) appears in a sequential glue decay)

FROM .~X.~Chen, et al, Rept. Prog. Phys. {86} (2023)

Recently, the BESIII collaboration studied the $J/\psi \to \gamma\pi^+\pi^-\eta'$ process and observed a new resonance $X(2600)$ in the $\pi^+\pi^-\eta'$ invariant mass spectrum with a statistical significance larger than 20σ [906], as shown in Fig. 68(c). Its mass and width were measured to be

$$X(2600) : M = 2617.8 \pm 2.1^{+18.2}_{-\ 1.9} \text{ MeV}\,, \qquad (134)$$
$$\Gamma = 200 \pm 8^{+20}_{-17} \text{ MeV}\,,$$

and its spin-parity quantum number was determined to be $J^{PC} = 0^{-+}$ or 2^{-+}. Interestingly, BESIII found its strong correlation to the scalar meson $f_0(1500)$:

$$\mathcal{B}(J/\psi \to \gamma X(2600)) \cdot \mathcal{B}(X(2600) \to f_0(1500)\eta') \cdot \mathcal{B}(f_0(1500) \to \pi^+\pi^-)$$
$$= (3.39 \pm 0.18^{+0.91}_{-0.66}) \times 10^{-5}\,, \qquad (135)$$
$$\mathcal{B}(J/\psi \to \gamma X(2600)) \cdot \mathcal{B}(X(2600) \to f_2'(1525)\eta') \cdot \mathcal{B}(f_2'(1525) \to \pi^+\pi^-)$$
$$= (2.43 \pm 0.13^{+0.31}_{-1.11}) \times 10^{-5}\,. \qquad (136)$$

Note the lattice QCD simulations strongly indicate the pseudoscalar glueball lies around 2.6 GeV. The $X(2600)$ is closely related to the $f_0(1500)$ and $f_2'(1525)$. If the $f_0(1500)$ were the scalar glueball, the $X(2600)$ could be the pseudoscalar glueball. Otherwise, if either the $f_0(1500)$ or $f_2'(1525)$ were dominated by the $\bar{q}q$ component, the $X(2600)$ could be an excited $\bar{q}q$ state. More investigations are necessary to understand their nature.

PHYSICAL REVIEW LETTERS **129,** 042001 (2022)

BESIII
X(2600)

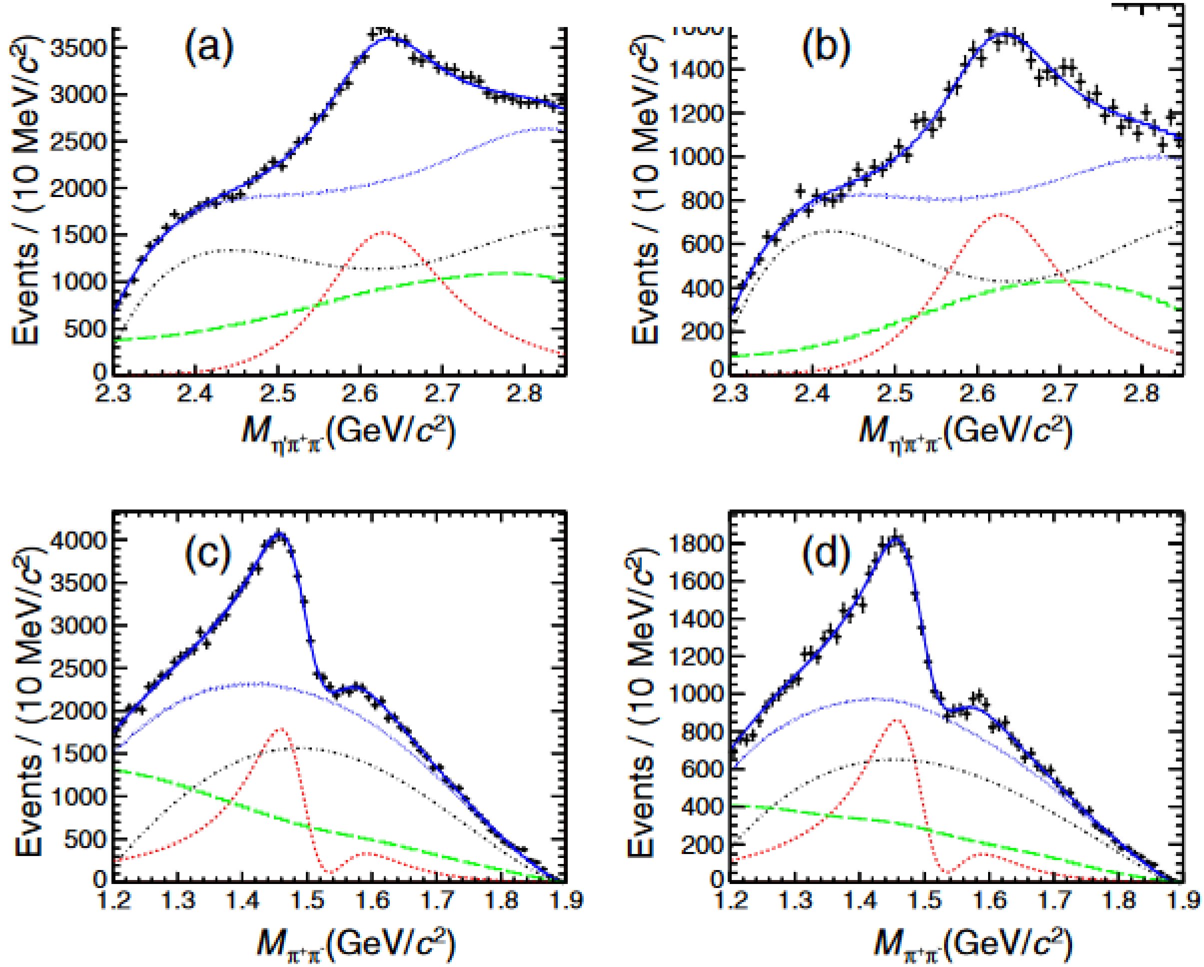


FIG. 2. The $\pi^+\pi^-\eta'$ and $\pi^+\pi^-$ mass spectra distributions with the two decay channels of η', $\eta' \to \gamma\pi^+\pi^-$ and $\eta' \to \pi^+\pi^-\eta$, with

Approach 3 : find new production modes

Ambiguities of partial wave analysis
-→look for other production mechanisms, with different combination of modes produced

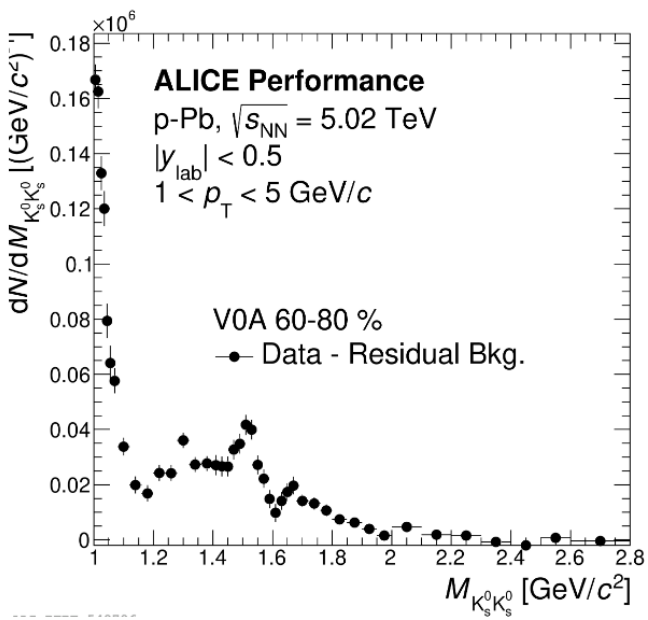
Already explored

- Central production (GAMS, WA102, COMPASS..) in fixed target
- J/ψ radiative decay
- B decays (LHCb) useful to (test the “s quark content”
- B and Bs decay to J/ψ test the “quark” content ,
  little activity seen above 1500 MeV → support glueball interpretation (Sarantsev, Klempt)

Under study

- ALICE

ALICE : heavy ion and p collisions (p-Pb; Pb-Pb) **concentrates on K° K°** , analysis based on PDG masses
and accent placed on production mode (central , ion vs p)



Alice poster in Quark matter 2023 (Satoshi Yano)

Approach 0

Another approach is provided Implicitly by experimenters !

Combine measure of « glue production » and glue-like decay

$J/\Psi \to \gamma X \to \gamma \eta \eta'$

Another approach is provided directly by experimenters !

Combine measure of « glue production » and glue-like decay

$J/\Psi \to \gamma X \to \gamma \eta \eta'$

	Resonance	M (MeV/c^2)	Γ (MeV)	$M_{\rm PDG}$ (MeV/c^2)	$\Gamma_{\rm PDG}$ (MeV)	B.F. ($\times 10^{-5}$)	Sig.
	$f_0(1500)$	1506	112	1506	112	3.05 ± 0.07	$\gg 30\sigma$
	$f_0(1810)$	1795	95	1795	95	0.07 ± 0.01	7.6σ
	$f_0(2020)$	1935 ± 5	266 ± 9	1992	442	1.67 ± 0.07	11.0σ
	$f_0(2100)$	2109 ± 11	253 ± 21	2086	284	0.33 ± 0.03	5.2σ
$\gamma\eta\eta'$	$f_0(2330)$	2327 ± 4	44 ± 5	2314	144	0.07 ± 0.01	8.5σ

Seems the good old f0(1500) remains a favorite (but is not alone)
(that it was previously named G(1590) results from the phase space distortion near threshold in eta etaprime mode)

Most of the time, the search proceeds through a partial wave analysis of the decay products.

This is practically limited to few bodies final states, typically 2 ordinary mesons (π, ρ, K…) , although some candidates have been seen in more complex finals states for instance in 4 π .

Main points

- The algebraic approach (including ours) is seductive but has limitations
  In fact, we want to extract amplitudes, but phase space distorts and interferences complicates the extraction (case of f0 into eta etaprime is a good illustration)
- The fit is made using only a fraction of the decays (mosty 2-body) , but f0(1500) main decay is 4pi (actually 2 f0(500)?)
- Meanwhile, the role of eta(prime) , long talked down, has become prominent, notably in the exotics decays
- New data can be expected from ALICE (heavy ion collisions) , but for the time being, they are not examined in detail for understanding glueballs,

4 π is larger than 2 π , (could be 2 σ)

KK is less than 2 π, $\eta\eta'$ with respect to $\eta\eta$ is significant , despite being « at threshold »

Are we missing modes?
For instance, we could be missing 3 or 4 σ as they are not searched for (or searchable for) in BESIII

Probably not possible to determine from inclusive, due to clutter of states!

$J/\Psi \rightarrow \gamma X$

… the ones that got away!