The Gravitational Form Factor (GFF) of Hadrons from CFT in Momentum Space and the QCD Dilaton

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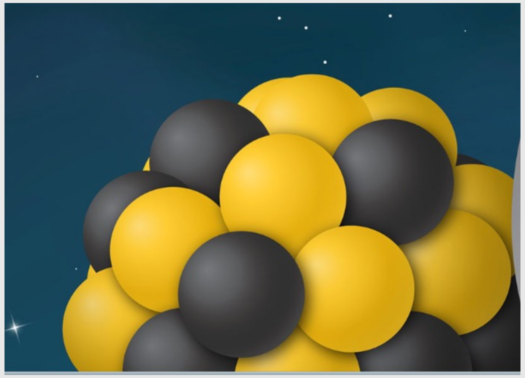
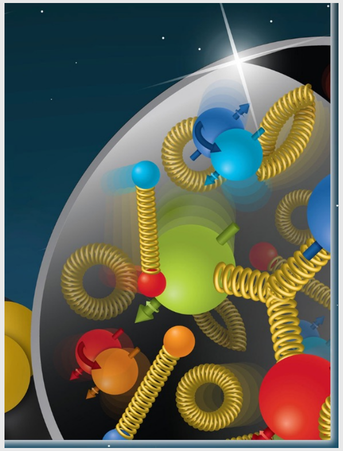


CFT_p for QCD



work with: **Stefano Lionetti, Dario Melle and Riccardo Tommasi**
graduate students at the University of Salento

Quantum Chromodynamics (QCD) in the gravitational form factor of the proton ad pion exhibits a dilaton pole, which is intrinsically linked to the trace anomaly as shown in QED (Giannotti and Mottola) (Armillis, Delle Rose, CC) and in QCD (Armillis, Delle Rose, CC)
This phenomenon is accompanied by a sum rule, which we verify in perturbation theory at the one-loop level.
The presence of such sum rules is a hallmark of chiral and conformal anomalies.

These contributions can be discussed quite clearly in the conformal limit, where a factorization approach to the proton form factor—commonly applied in exclusive processes—highlights their significance.

Looking ahead, future experiments in Deeply Virtual Compton Scattering (DVCS) at the Electron-Ion Collider (EIC) are expected to provide an opportunity to test this sum rule for the proton. Successful verification would serve as a strong indication of the exchange of a dilaton state, validating the theoretical predictions.

BRIEF OUTLINE

GFFs of hadrons (pion and poton) will be measured at **EIC** at BNL

The EIC is designed to explore the internal structure of protons, neutrons, and nuclei with unprecedented precision.
Improve our understanding of the 3D structure of protons and the distribution of their constituent quarks and gluons, as well as their correlations.

GPDs and proton tomography

DVCS is sensitive to **GPDs**, which encode information about the spatial distribution of quarks and gluons inside protons and nuclei. This is essential for a full understanding of the internal structure of hadrons.

Through DVCS, the EIC will contribute to producing a three-dimensional picture of the proton's internal structure, a major goal of modern nuclear physics.

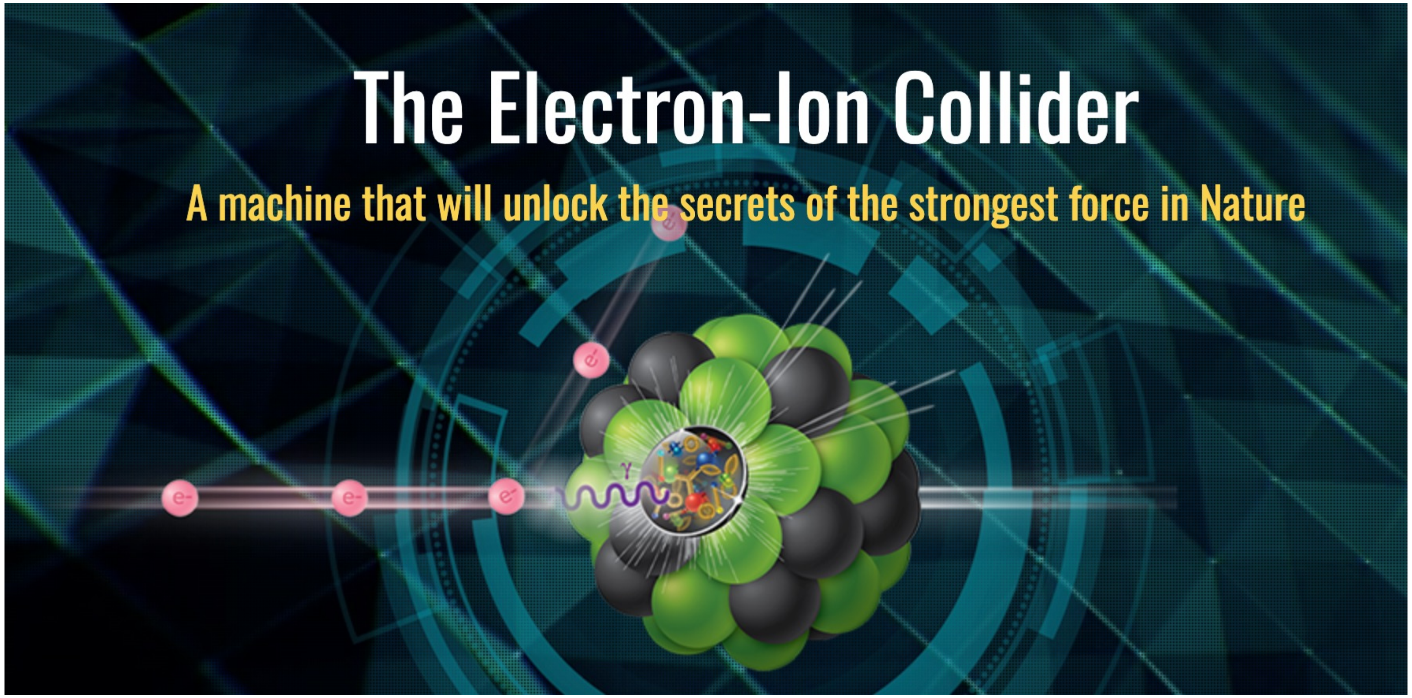
The Gravitational Form Factors of Hadrons from CFT, the Trace Anomaly, and the Perturbative Dilaton

Lionetti, Melle, Tommasi, CC, to appear

Brookhaven National Laboratory

Jefferson Lab

The Electron-Ion Collider (EIC) at Brookhaven National Laboratory is designed to have a highly flexible energy range, with the capability to collide electrons with protons and nuclei at center-of-mass energies ranging from approximately **20 GeV** to **140 GeV (e-p and e-Ions)**



Recent refs. for anomaly interactions, CFT_p,
relevant for axion and dilaton "poles" and sum rules

CFT_p

Gravitational chiral anomaly at finite temperature and density
Phys.Rev.D 110 (2024) 2, 025008 e-Print: 2404.06272 [hep-th]

parity-odd CFT_p

Axion-like Interactions and CFT in Topological Matter, Anomaly Sum Rules and the Faraday Effect
Adv.Phys.Res. (2024) e-Print: 2403.15641 [hep-ph]
Axionlike quasiparticles and topological states of matter: Finite density corrections of the chiral anomaly vertex
Phys.Rev.D 110 (2024) 2, 025014 2402.03151 [hep-ph]
with Mario Cretì, Stefano Lionetti, and Riccardo Tommasi

Parity-violating CFT and the gravitational chiral anomaly
Phys.Rev.D 109 (2024) 4, 045004 e-Print: 2309.05374 [hep-th]
CFT correlators and CP-violating trace anomalies
Eur.Phys.J.C 83 (2023) 9, 839 e-Print: 2307.03038 [hep-th]
Parity-odd 3-point functions from CFT in momentum space and the chiral anomaly
Eur.Phys.J.C 83 (2023) 6, 502 e-Print: 2303.10710 [hep-th]
with Stefano Lionetti and Matteo M. Maglio

CFT Constraints on Parity-odd Interactions with Axions and Dilatons
2408.02580 [hep-th]
S.Lionetti, C.C.

For parity odd correlators

a pole structure + CWIs
are sufficient to completely determine a parity odd interaction

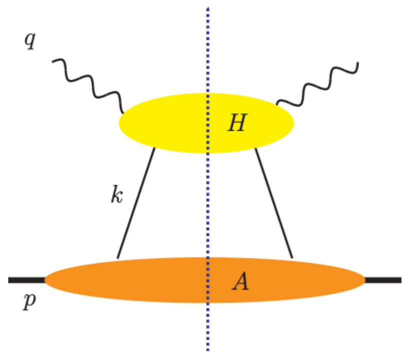
$$W^{\mu\nu} = \frac{1}{4\pi}\int \mathrm{d}^4 y e^{iq\cdot y}\sum_X \langle A|j^\mu(y)|X\rangle\langle X|j^\nu(0)|A\rangle$$

$$F_1(x,Q^2)\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)$$

$$+F_2(x,Q^2)\frac{(p^\mu - q^\mu p\cdot q/q^2)\,(p^\nu - q^\nu p\cdot q/q^2)}{p\cdot q},$$

FROM DIS
TO
DVCS

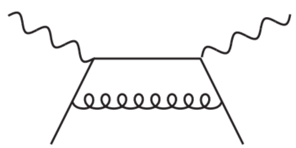
inclusive, factorizable

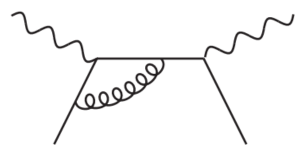


$$W^{\mu\nu}(q^\mu,p^\mu) = \sum_a \int_x^1 \frac{\mathrm{d}\xi}{\xi}\, f_{a/A}(\xi,\mu)\; H_a^{\mu\nu}(q^\mu,\xi p^\mu,\mu,\alpha_s(\mu)) + \text{remainder}.$$

$$F_1(x,Q^2) = \sum_a \int_x^1 \frac{\mathrm{d}\xi}{\xi}\, f_{a/A}(\xi,\mu)\; H_{1a}\left(\frac{x}{\xi},\frac{Q}{\mu},\alpha_s(\mu)\right) + \text{remainder},$$

$$\frac{1}{x}F_2(x,Q^2) = \sum_a \int_x^1 \frac{\mathrm{d}\xi}{\xi}\, f_{a/A}(\xi,\mu)\; \frac{\xi}{x}H_{2a}\left(\frac{x}{\xi},\frac{Q}{\mu},\alpha_s(\mu)\right) + \text{remainder},$$





Factorization of Hard Processes in QCD*

J. Collins, D. Soper, G. Sterman Adv.Ser.Direct.High Energy Phys.5:1-91,1988
hep-ph/0409313,

see also **M. Diehl**, Lectures at Ecole Joliot Curie 2018

for exclusive processes

A. Khodiamirian "Hadron form factors"

light cone, gauge invariant matrix elements (forward)

matrix elements of quark/gluon operators

collinear factorization

$$f_q(x) = \int \frac{dz^-}{2\pi}\, e^{ixp^+z^-} \, \langle p \,|\, \bar{q}(0)\, \tfrac{1}{2}\gamma^+ W[0,z]\, q(z)\,|\, p\rangle \Big|_{z^+=0,\, z_T=0}$$

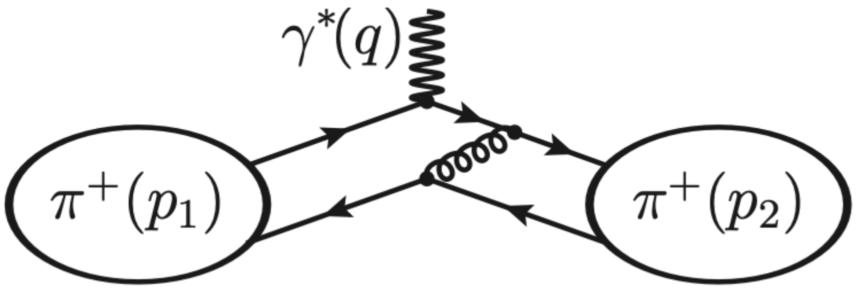
Form factors

DA's

$$\varphi_\pi(u,\mu) = 6u\bar{u}\Big(1 + a_2(\mu)C_2^{3/2}(u-\bar{u}) + a_4(\mu)C_4^{3/2}(u-\bar{u})\Big)\,,$$

Evolving with ERBL (Efremov-Radyushkin-Brodsky-Lepage) evolution equations

at intermediate momentum transfers



interpolate with the Fock vacuum factorization

$$|\pi^+(p_1)\rangle = \int_0^1 d\alpha_1\, \tilde{\phi}_\pi(\alpha_1) |\{u(\alpha_1 p_1)\bar{d}(\bar{\alpha}_1 p_1)\}_\pi\rangle,$$

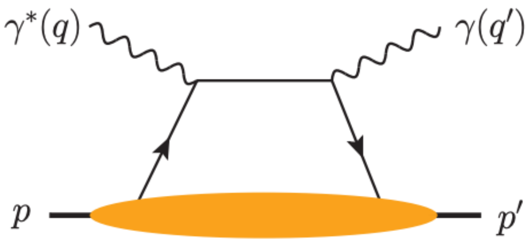
$$\langle \pi^+(p_2)| = \int_0^1 d\alpha_2\, \tilde{\phi}_\pi(\alpha_2)\, \langle\{u(\alpha_2 p_2)\bar{d}(\bar{\alpha}_2 p_2)\}_\pi|.$$

$$F_\pi(Q^2) = f_\pi^2 \int_0^1 d\alpha_2 \int_0^1 d\alpha_1 \tilde{\phi}_\pi(\alpha_1,\mu) T(Q^2,\alpha_1,\alpha_2,\mu)\tilde{\phi}_\pi(\alpha_2,\mu)\,,$$

Sudakov suppression
(Li and Sterman)

- *Nucl.Phys.B* 381 (1992)

QCD in exclusive processes



low momentum transfers
but how low?

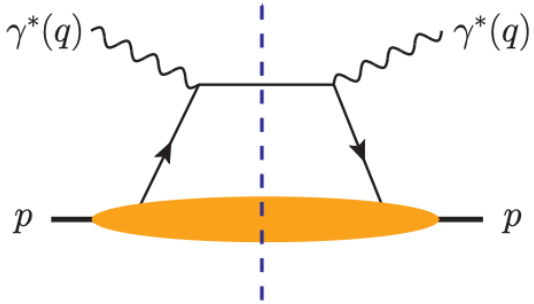
X. Ji, A Radyushkin GPD's

exclusive cross section

$$\propto \left|\mathcal{A}(\gamma^* p \to \gamma p)\right|^2$$

square of amplitude

DIS



measure in $ep \to eX$

Bjorken limit: $Q^2 = -q^2 \to \infty$ at fixed $x_B = Q^2/(2p \cdot q)$

$$\sigma_{\text{tot}}(\gamma^* p \to X) \xrightarrow{\text{opt. theorem}} \text{Im}\,\mathcal{A}(\gamma^* p \to \gamma^* p)$$

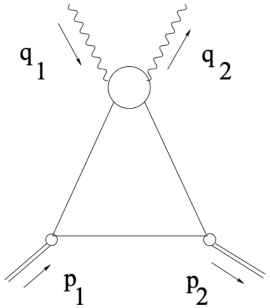
forward amplitude

A. Radyushkin

X. Ji 1998

Non forward parton distributions

(**PRE-DVCS**) Compton scattering at intermediate energy, non factorizable in QCD



Exclusive Processes at Intermediate Energy, Quark–Hadron Duality and the Transition to Perturbative QCD

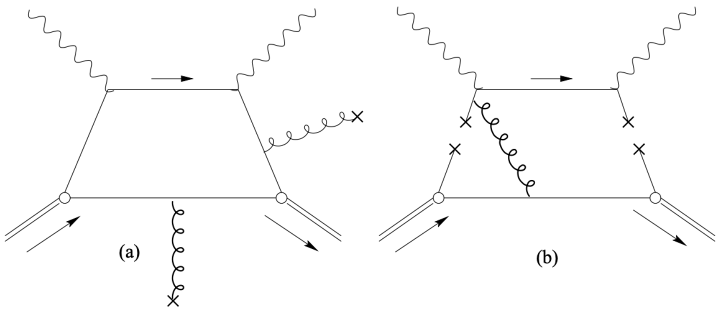
H.N. Li, CC *JHEP* 07 (1998) 008 e-Print: hep-ph/9805406 [hep-ph]

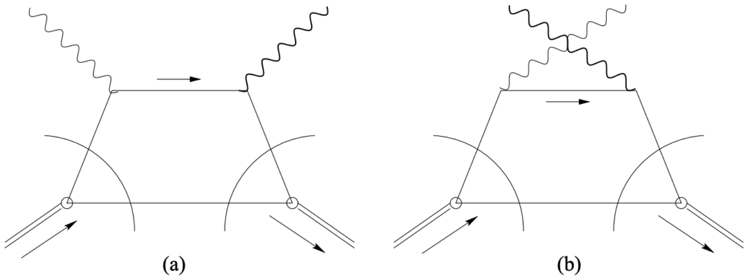
The Transition to Perturbative QCD in Compton Scattering

H.N. Li, CC

Nucl.Phys.B 434 (1995) 535-564

e-Print: hep-ph/9405295 [hep-ph]





QCD Sum Rules and Compton Scattering

A. Radyushkin, G. Sterman, CC (1994)

dispersive approach in the analysis of Compton scattering

Generalized Bjorken region: additional scaling variables

several analysis
D. Muller, Geyer, Robashik, Blumlein
The standard DVCS format due to Ji and Radyushkin

A general formalism, more cumbersome, applicable to a wide range of processe.

Example:
Leading Twist Amplitudes for Exclusive Neutrino Interactions in the Deeply Virtual Limit
Phys. Rev. D
M. Guzzi, CC arXiv:hep-ph/0411253

Neutrino scattering on nucleons in the regime of deeply virtual kinematics studied both in the charged and the neutral eletroweak sectors using a formalism developed by Blümlein, Robashik, Geyer and Collaborators.

Deeply virtual neutrino scattering (DVNS)

Amore, M. Guzzi. CC (JHEP 2005)

arXiv:hep-ph/0404121

$$P_{1,2} = \bar{P} \pm \frac{\Delta}{2} \qquad q_{1,2} = \bar{q} \mp \frac{\Delta}{2}$$

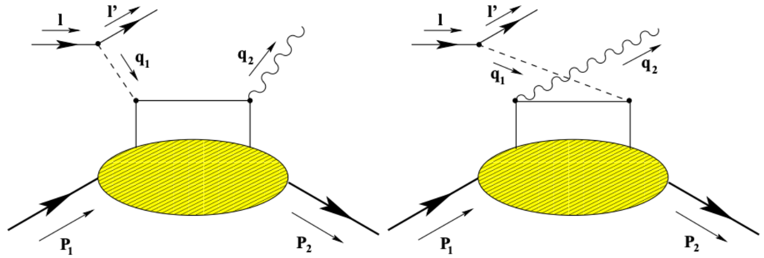
with $-\Delta = P_2 - P_1$ being the momentum transfer.

$$\bar{P} \cdot \Delta = 0, \quad t = \Delta^2 \quad \bar{P}^2 = M^2 - \frac{\upsilon}{4}$$

$$\xi = -\frac{\bar{q}^2}{2\bar{P} \cdot \bar{q}} \qquad \eta = \frac{\Delta \cdot \bar{q}}{2\bar{P} \cdot \bar{q}}$$

X Ji's formulation for the kinematics

Example with the Z neutral current



two scaling variables, related to the average momentum of the struck quark and the longitudinal momentum exchange

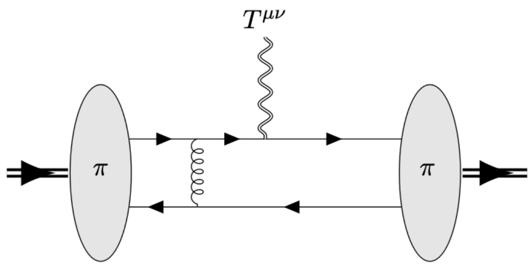
$$\int \frac{d\lambda}{(2\pi)} e^{i\lambda z} \langle P' | \overline{\psi}\left(-\frac{\lambda n}{2}\right) \gamma^\mu \psi\left(\frac{\lambda n}{2}\right) | P \rangle =$$

$$H(z,\xi,\Delta^2)\overline{U}(P')\gamma^\mu U(P) + E(z,\xi,\Delta^2)\overline{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P) + ...$$

$$\int \frac{d\lambda}{(2\pi)} e^{i\lambda z} \langle P' | \overline{\psi}\left(-\frac{\lambda n}{2}\right) \gamma^\mu \gamma^5 \psi\left(\frac{\lambda n}{2}\right) | P \rangle =$$

$$\tilde{H}(z,\xi,\Delta^2)\overline{U}(P')\gamma^\mu\gamma^5 U(P) + \tilde{E}(z,\xi,\Delta^2)\overline{U}(P')\frac{\gamma^5\Delta^\mu}{2M}U(P) + ...$$

The link betweed DVCS and GFFs



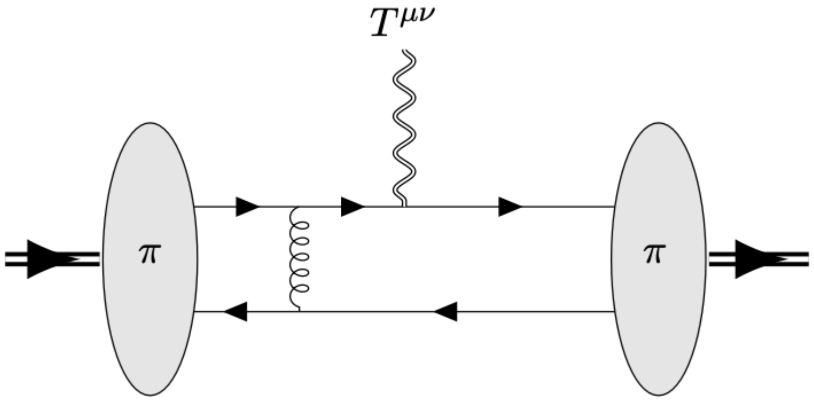
$$\langle p', s'|T_{\mu\nu}(0)|p, s\rangle = \bar{u}' \left[A(t)\, \frac{\gamma_{\{\mu}P_{\nu\}}}{2} + B(t)\, \frac{i\, P_{\{\mu}\sigma_{\nu\}\rho}\Delta^{\rho}}{4M} + D(t)\, \frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M} + M \sum_{\hat{a}} \bar{c}^{\hat{a}}(t)\, g_{\mu\nu} \right] u$$

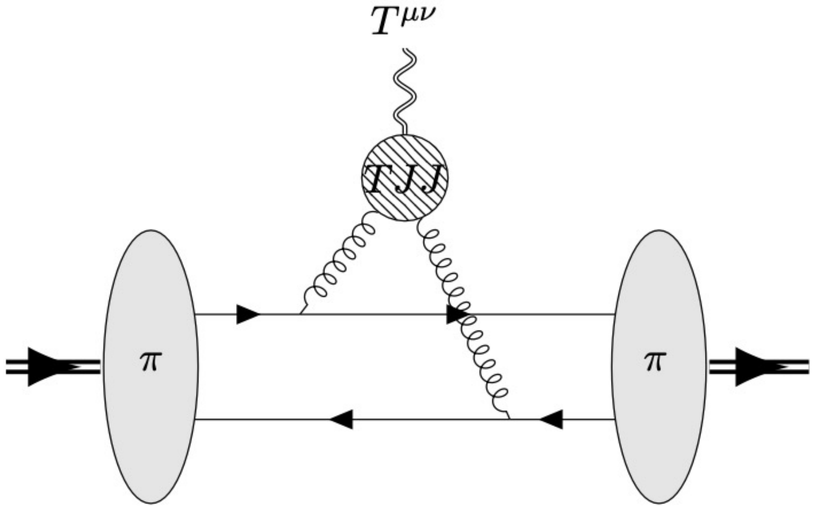
where $u(p)$ and $\overline{u}(p')$ are the proton spinors, $P = (p + p')/2$ is the average momentum, $\Delta = p' - p$ is the momentum transfer, $t = \Delta^2$, and M is the mass of the proton. $\gamma^{(\mu}P^{\nu)}$denotes the symmetric combination $\gamma^\mu P^\nu + \gamma^\nu P^\mu$.

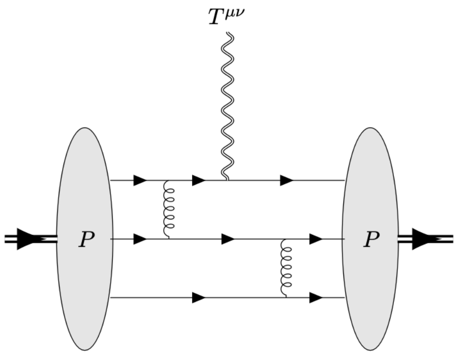
$$\langle p'\,|\hat{T}_{\mu\nu}(0)|p\rangle = \left[2P_\mu P_\nu\, A(t) + \frac{1}{2}(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)\, D(t) + 2\; m^2\, \bar{c}(t)\, g_{\mu\nu} \right].$$

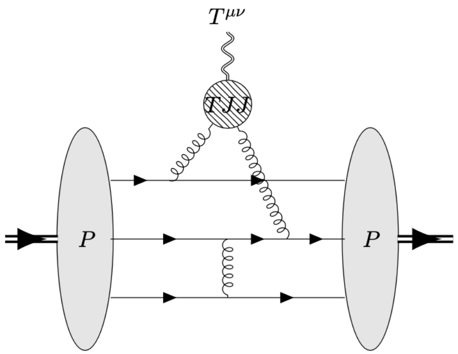
$$\hat{T}^{\;\mu}_{\mu} \equiv \beta(g)\; F^{a,\mu\nu} F^{a,}{}_{\mu\nu} + (1 + \gamma_m) \sum_q m_q \bar{\psi}_q \psi_q \;,$$

In the factorization picture: the TJJ correlator









The TJJ is the crucial brings in
the manifestation of the anomaly
(dilaton) pole

In CFT_p
Maglio, CC

in QED studied by Giannotti and Mottola, Armillis, Delle Rose CC,
in QCD by Armillis, Delle Rose, CC

$$J_q + J_g = \frac{1}{2}[A_q(0) + B_q(0)] + \frac{1}{2}[A_g(0) + B_g(0)] = \frac{1}{2}.$$

angular momentum sum rule at t=0

A and B can be determined as Mellin moments of the DVCS invariant amplitudes

$$\int_{-1}^{1} \mathrm{d}x \, x \, H^a(x,\xi,t) = A^a(t) + \xi^2 D^a(t)\,, \qquad \int_{-1}^{1} \mathrm{d}x \, x \, E^a(x,\xi,t) = B^a(t) - \xi^2 D^a(t)\,.$$

(X. Ji)

by measuring DVCS -> Determination of the gravitational form factor

The CFT_p analysis allows us to uncover some important features of the anomaly interactions, by the analysis of the Conformal Ward Identities.
This require a more general formalism, that has been developed in the last 10 years

Delle Rose, Mottola Serino, CC
Bzowski, McFadden, Skenderis

CFT_p **CFT in momentum space** (free field theory realization in QCD at one-loop)

introduced in

stress energy tensor in QCD

$$
\begin{aligned}
T_{\mu\nu} &= -g_{\mu\nu}\mathcal{L}_{QCD} - F^a_{\mu\rho}F^{a\rho}_{\nu} - \frac{1}{\xi}g_{\mu\nu}\partial^\rho(A^a_\rho\partial^\sigma A^a_\sigma) + \frac{1}{\xi}(A^a_\nu\partial_\mu(\partial^\sigma A^a_\sigma) + A^a_\mu\partial_\nu(\partial^\sigma A^a_\sigma)) \\
&+ \frac{i}{4}\Big[\overline{\psi}\gamma_\mu(\overrightarrow{\partial}_\nu - igT^aA^a_\nu)\psi - \overline{\psi}(\overleftarrow{\partial}_\nu + igT^aA^a_\nu)\gamma_\mu\psi + \overline{\psi}\gamma_\nu(\overrightarrow{\partial}_\mu - igT^aA^a_\mu)\psi \\
&- \overline{\psi}(\overleftarrow{\partial}_\mu + igT^aA^a_\mu)\gamma_\nu\psi\Big] + \partial_\mu\overline{c}^a(\partial_\nu c^a - gf^{abc}A^c_\nu c^b) + \partial_\nu\overline{c}^a(\partial_\mu c^a - gf^{abc}A^c_\mu c^b).
\end{aligned}
$$

$$
\begin{aligned}
T^{g.f.}_{\mu\nu} &= \frac{1}{\xi}\left[A^a_\nu\partial_\mu(\partial\cdot A^a) + A^a_\mu\partial_\nu(\partial\cdot A^a)\right] - \frac{1}{\xi}g_{\mu\nu}\left[-\frac{1}{2}(\partial\cdot A)^2 + \partial^\rho(A^a_\rho\partial\cdot A^a)\right], \\
T^{gh}_{\mu\nu} &= \partial_\mu\overline{c}^aD^{ab}_\nu c^b + \partial_\nu\overline{c}^aD^{ab}_\mu c^b - g_{\mu\nu}\partial^\rho\overline{c}^aD^{ab}_\rho c^b.
\end{aligned}
$$

$$
T_{\mu\nu} = T^{f.s.}_{\mu\nu} + T^{ferm.}_{\mu\nu} + T^{g.fix.}_{\mu\nu} + T^{ghost}_{\mu\nu}.
$$

$$
T^{f.s.}_{\mu\nu} = \eta_{\mu\nu}\frac{1}{4}F^a_{\rho\sigma}F^{a\,\rho\sigma} - F^a_{\mu\rho}F^{a\,\rho}_{\nu}
$$

QCD in background gravity

(Lionetti, Melle, Tommasi, CC)

$$Z[J,\eta,\bar\eta,\chi,\bar\chi,g] = \mathcal{N}\int \mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\bar\psi\,\mathcal{D}c\,\mathcal{D}\bar c\,\exp\Big\{i\int\sqrt{-g}d^4x\,(\mathcal{L}+J^a_\mu A^{\mu a}$$
$$+\bar\eta\psi+\bar\psi\eta+\bar\chi c+\bar c\chi)\Big\},$$

conformal symmetry broken by the gauge fixing/ghost sector

$$\begin{aligned} F^a_{\mu\nu} &= \nabla_\mu A^a_\nu - \nabla_\nu A^a_\mu + g f^{abc}A^b_\mu A^c_\nu \\ &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc}A^b_\mu A^c_\nu, \\ \nabla_\mu A^{\nu a} &= \partial_\mu A^{\nu a} + \Gamma^\lambda_{\mu\nu}A^{\lambda a} \end{aligned}$$

insert QCD on a gravitational background

$$D_\mu\psi = \left(\partial_\mu\psi + A^a_\mu T^a + \frac{1}{4}\omega^{\underline{ab}}_\mu\sigma_{\underline{ab}}\right)\psi$$

$$D_\mu V^{\underline{a}} = \partial_\mu V^{\underline{a}} + \omega^{\underline{a}}_{\mu\underline{b}}V^{\underline{b}}$$

$$\Gamma^\rho_{\mu\nu} = e_{\underline{a}}^{\ \rho}\left(\partial_\mu e^{\underline{a}}_{\ \nu} + \omega^{\underline{a}}_{\mu\underline{b}}e^{\underline{b}}_{\ \nu}\right).$$

$$\nabla_\mu V^\rho = e_{\underline{a}}^{\ \rho}D_\mu V^{\underline{a}}$$

$$\omega^{\underline{ab}}_\mu(x) = e_{\underline{a}}^{\ \nu}(x)e_{\underline{b}\nu;\mu}(x),$$

$$\mathcal{L}_f = \sqrt{-g}\left\{\frac{i}{2}\left[\bar\psi\gamma^\mu(\mathcal{D}_\mu\psi) - (\mathcal{D}_\mu\bar\psi)\gamma^\mu\psi\right] - m\bar\psi\psi\right\}$$

perform a Legendere transformation to define the effective action

We recall that a special conformal transformation in flat space is characterised by

$$x'^{\mu} = \frac{(x^{\mu} - b^{\mu}x^2)}{\Omega(x)} \qquad \text{with} \qquad \Omega(x) = 1 - 2b\cdot x + b^2x^2 \qquad \text{and} \qquad J_c \equiv \left|\frac{\partial x'}{\partial x}\right| = \Omega^{-d}$$

$$g'_{\mu\nu}(x') = \Omega^2 g_{\mu\nu}(x) \qquad \phi'(x') = J_c^{-\Delta/d}\phi(x) = \Omega^{\Delta}\phi(x).$$

and for a spin-1 field

$$J'^{\mu}(x') = \Omega^{\Delta_J}\frac{\partial x'^{\mu}}{\partial x^{\nu}}J^{\nu}(x).$$

Differential equations can be derived for the special conformal transformation. Expanding these relations for $b \ll 1$ and taking the finite part one obtains

$$\mathcal{K}^k\phi(x) = \left(-x^2\frac{\partial}{\partial x^{\kappa}} + 2x^{\kappa}x^{\alpha}\frac{\partial}{\partial x^{\alpha}} + 2\Delta_{\phi}x^{\kappa}\right)\phi(x)$$

$$\mathcal{K}^k J^{\mu}(x) = \left(-x^2\frac{\partial}{\partial x^{\kappa}} + 2x^{\kappa}x^{\alpha}\frac{\partial}{\partial x^{\alpha}} + 2\Delta_J x^{\kappa}\right)J^{\mu}(x) + 2\left(\delta^{\mu\kappa}x_{\rho} - \delta^{\kappa}_{\rho}x^{\mu}\right)J^{\rho}(x).$$

Conformal symmetry is broken in QCD by the gauge-fixing and ghost contributions (see also Braun, Manashov, Moch, Strohmaier *Phys.Lett.B* 793 (2019) 78-84 e-Print: 1810.04993 [hep-th] for possible use)

The quark and gluon sectors, however, at one-loop, can be treated separately

$$g_{\mu\nu}\langle T^{\mu\nu}\rangle = b\,C^2 + b'\left(E - \frac{2}{3}\,\Box\,R\right) + b''\,\Box\,R + c\,F^{a\,\mu\nu}F^a_{\mu\nu},$$

Einstein Gauss-Bonent
and Weyl tensor

$$g_{\mu\nu}\langle T^{\mu\nu}\rangle = \beta F^{a\,\mu\nu}F^a_{\mu\nu}.$$

$$\mathcal{S}_{eff}(g, A_c) \equiv \sum_{n=1}^{\infty}\frac{1}{2^n n!}\int d^dx_1\ldots d^dx_n\sqrt{g_1}\ldots\sqrt{g_n}\,\langle T^{\mu_1\nu_1}\ldots T^{\mu_n\nu_n}\rangle_{\bar g}\delta g_{\mu_1\nu_1}(x_1)\ldots\delta g_{\mu_n\nu_n}(x_n),$$
$$\times\delta g_{\mu_1\nu_1}(x_1)\ldots\delta g_{\mu_n\nu_n}(x_n)\delta A^{a_{n+1}}_{c\ \mu_{n+1}}(x_{n+1})\ldots\delta A^{a_{n+k}}_{c\ \mu_{n+k}}(x_{n+k}) + GT\ldots$$

effective action
easy to control in flat space

$$S_{anom}[g,A] =$$
$$\frac{1}{8}\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}\left(E - \frac{2}{3}\Box R\right)_x \Delta_4^{-1}(x,x')\left[2b\,F + b'\left(E - \frac{2}{3}\Box R\right) + 2\,c\,F_{\mu\nu}F^{\mu\nu}\right]_{x'}.$$

$$\Delta_4 \equiv \nabla_\mu\left(\nabla^\mu\nabla^\nu + 2R^{\mu\nu} - \frac{2}{3}Rg^{\mu\nu}\right)\nabla_\nu = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu + \frac{1}{3}(\nabla^\mu R)\nabla_\mu - \frac{2}{3}R\Box$$

nonlocal
conformal anomaly action
for generic backgrounds

$$S_{anom}[g,A] \to -\frac{c}{6}\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}\,R^{(1)}_x\,\Box^{-1}_{x,x'}\,[F^a_{\alpha\beta}F^{a\,\alpha\beta}]_{x'}\,,$$

at linearized level the TJJ
(abelian at least)
can be expressed by Riegert's action

$$S_{anom}[g,A] = \frac{1}{8}\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}\left(E-\frac{2}{3}\Box R\right)_x \Delta_4^{-1}(x,x')\left[2b\,F + b'\left(E-\frac{2}{3}\Box R\right)+2\,c\,F_{\mu\nu}F^{\mu\nu}\right]_{x'} .$$

This effective action is derived from a variational solution of the conformal anomaly

$$\Delta_4 \equiv \nabla_\mu\left(\nabla^\mu\nabla^\nu + 2R^{\mu\nu} - \frac{2}{3}Rg^{\mu\nu}\right)\nabla_\nu = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu + \frac{1}{3}(\nabla^\mu R)\nabla_\mu - \frac{2}{3}R\Box$$

Paneitz operator (conformal geometry) conformally covariant under Weyl rescalings

A review on the origin and use of CFT_p methods

Phys.Rept. 952 (2022) 1-95 e-Print: 2005.06873 [hep-th]
M. Maglio, C.C.

Problems of this action beyond 3 point functions

Four-point functions of gravitons and conserved currents of CFT in momentum space: testing the nonlocal action with the TTJJ

•*Eur.Phys.J.C* 83 (2023) 5, 427 Print: 2212.12779 [hep-th] Maglio, Tommasi, CC

Nonlocal Gravity, Dark Energy and Conformal Symmetry: Testing the Hierarchies of Anomaly-Induced Actions

•*PoS* CORFU2023 (2024) 165

$$\Delta_4 \equiv \nabla_\mu \left(\nabla^\mu \nabla^\nu + 2R^{\mu\nu} - \frac{2}{3} R g^{\mu\nu} \right) \nabla_\nu = \Box^2 + 2R^{\mu\nu} \nabla_\mu \nabla_\nu + \frac{1}{3} (\nabla^\mu R) \nabla_\mu - \frac{2}{3} R \Box \,.$$

Paneitz operator

$$\sqrt{-g}\, \Delta_4 \chi_0 = \sqrt{-\bar{g}}\, \bar{\Delta}_4 \chi_0,$$

Weyl invariant if acting on conformal scalars (ie fields of vanishing scaling dimensions)

TTT in agreement with the free field theory realization and the general CFT derivation.
The general solution depends in 3 constants and can be matched by free feld theory (3 sectors)

In the parity-even case, due to renormalization,
one can write CWIs directly at d=4, where the same pole structure appears

M. Maglio,E. Mottola, CC

TTT in CFT: Trace Identities and the Conformal Anomaly Effective Action

•*Nucl.Phys.B* 942 (2019) 303-328 **1703.08860** [hep-th]

CFT_p allows to treat the anomaly contributions, which are essentially ignored by CFT in coordinate space

Anomalies come from regions of the correlator where all the external points are in coincidence.

In momentum space they are associated with an interesting light cone behaviour of the 3-point functions (this is valid both for conformal and chiral anomalies)

The case of the TJJ in QCD: How to parameterize the Proton Form factor effectively and uncover the dilaton pole

We proceed from the AVV

the **longitudinal/transverse (LT) decomposition**

$$W^{\lambda\mu\nu} = \frac{1}{8\pi^2}\left[W^{L\,\lambda\mu\nu} - W^{T\,\lambda\mu\nu}\right],$$

$$W^{L\,\lambda\mu\nu} = w_L\, k^\lambda \varepsilon[\mu,\nu,k_1,k_2]$$

De Rafael et al

developed in the study of g-2 of the muon

It corrects an erro r in the book by Kerson Huang on particle theory

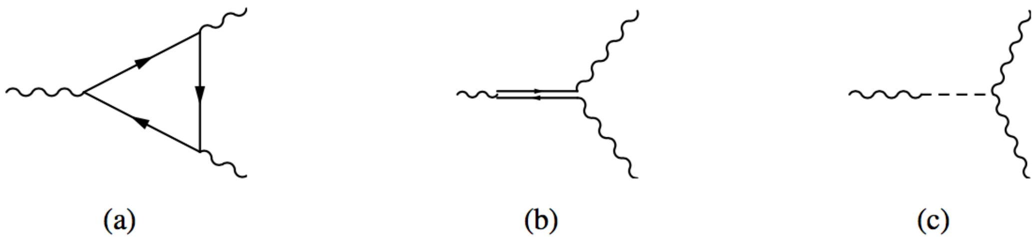
Only the L part contributes to the Ward Identity

$$\begin{aligned} W^{T}{}_{\lambda\mu\nu}(k_1,k_2) \;&=\; w_T^{(+)}\left(k^2,k_1^2,k_2^2\right)\, t^{(+)}_{\lambda\mu\nu}(k_1,k_2) + w_T^{(-)}\left(k^2,k_1^2,k_2^2\right)\, t^{(-)}_{\lambda\mu\nu}(k_1,k_2) \\ &+ \widetilde{w}_T^{(-)}\left(k^2,k_1^2,k_2^2\right)\, \widetilde{t}^{(-)}_{\lambda\mu\nu}(k_1,k_2), \end{aligned}$$

$$\begin{aligned} t^{(+)}_{\lambda\mu\nu}(k_1,k_2) \;&=\; k_{1\nu}\,\varepsilon[\mu,\lambda,k_1,k_2] - k_{2\mu}\,\varepsilon[\nu,\lambda,k_1,k_2] - (k_1\cdot k_2)\,\varepsilon[\mu,\nu,\lambda,(k_1-k_2)] \\ &\quad + \frac{k_1^2+k_2^2-k^2}{k^2}\, k_\lambda\; \varepsilon[\mu,\nu,k_1,k_2]\;, \\ t^{(-)}_{\lambda\mu\nu}(k_1,k_2) \;&=\; \left[(k_1-k_2)_\lambda - \frac{k_1^2-k_2^2}{k^2}\, k_\lambda\right]\varepsilon[\mu,\nu,k_1,k_2] \\ \widetilde{t}^{(-)}_{\lambda\mu\nu}(k_1,k_2) \;&=\; k_{1\nu}\,\varepsilon[\mu,\lambda,k_1,k_2] + k_{2\mu}\,\varepsilon[\nu,\lambda,k_1,k_2] - (k_1\cdot k_2)\,\varepsilon[\mu,\nu,\lambda,k]. \end{aligned}$$

Tensor structures involved
In the LT parameterization

$$
\begin{aligned}
w_L(s_1, s_2, s) &= -\frac{4i}{s} \\
w_T^{(+)}(s_1, s_2, s) &= i\frac{s}{\sigma} + \frac{i}{2\sigma^2}\Big[(s_{12}+s_2)(3s_1^2 + s_1(6s_{12}+s_2) + 2s_{12}^2)\log\frac{s_1}{s} \\
&+ (s_{12}+s_1)(3s_2^2 + s_2(6s_{12}+s_1) + 2s_{12}^2)\log\frac{s_2}{s} \\
&+ s(2s_{12}(s_1+s_2) + s_1 s_2(s_1+s_2+6s_{12}))\Phi(s_1, s_2)\Big]
\end{aligned}
$$

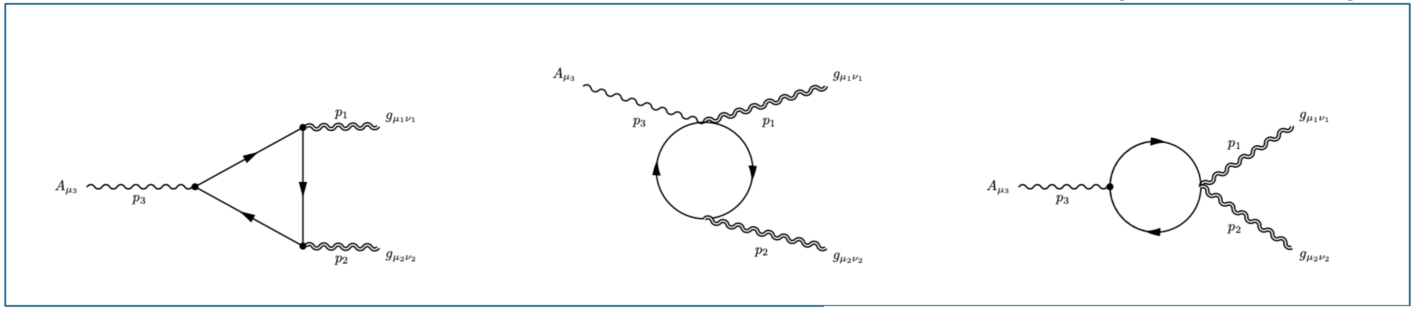


The triangle diagram in the fermion case (a), the collinear fermion configuration responsible for the anomaly (b) and a diagrammatic representation of the exchange via an intermediate state (dashed line) (c).

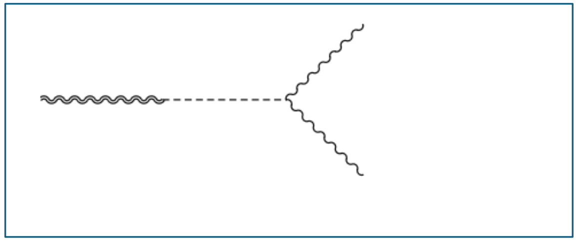
The signature of the chiral anomaly is in the the generation of 1 pole in the axial vector channel

$$\langle t^{\mu_1\nu_1} t^{\mu_2\nu_2} j_{5\,loc}^{\mu_3}\rangle = p_3^{\mu_3}\, \Pi^{\mu_1\nu_1}_{\alpha_1\beta_1}(p_1)\, \Pi^{\mu_2\nu_2}_{\alpha_2\beta_2}(p_2)\, \varepsilon^{\alpha_1\alpha_2 p_1 p_2} \left(F_1\, g^{\beta_1\beta_2} + F_2\, p_1^{\beta_2} p_2^{\beta_1}\right)$$

Longitudinal terms coming from the anomaly deetermine
By the anomaly



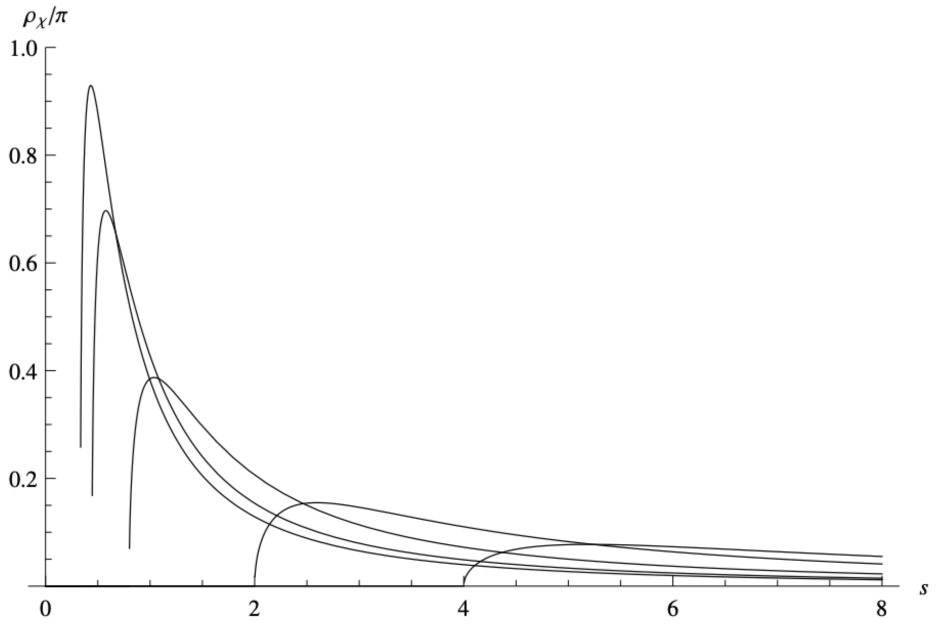
$$F_1 = \frac{16ia_2(p_1\cdot p_2)}{p_3^2}, \qquad F_2 = -\frac{16ia_2}{p_3^2}.$$



The pole, from the perturbative perspective, is fue to the exchange of a
Pseudoscalar mode (a collinear fermion antifermion pair)

$$\langle t^{\mu_1\nu_1} t^{\mu_2\nu_2} j_{5\,loc}^{\mu_3}\rangle = 4ia_2\frac{p_3^{\mu_3}}{p_3^2}(p_1\cdot p_2)\left\{\left[\varepsilon^{\nu_1\nu_2 p_1 p_2}\left(g^{\mu_1\mu_2} - \frac{p_1^{\mu_2}p_2^{\mu_1}}{p_1\cdot p_2}\right) + (\mu_1\leftrightarrow\nu_1)\right] + (\mu_2\leftrightarrow\nu_2)\right\}.$$

In the conformal limit we exchange the pole. Away from the conformal limit we have a sum rule fixed by the anomaly



Implications for the cosmology of the very early universe.

I will consider only the case of the Chern Simons current for the J5TT (orrelator chiral gravitational anomaly)

Chiral anomaly interactions determined by an anomaly pole+ conformal symmetry. the same is true for a Chern Simons current . Our analysis does not depend on the current

$$J^{\lambda}_{5f} = \bar{\psi}\gamma_5\gamma^{\lambda}\psi$$

$$J^{\lambda}_{CS} = \epsilon^{\lambda\mu\nu\rho} A_{\mu}\partial_{\nu}A_{\rho},$$

Gravitational anomalies induced **by Chern-Simons currents**. Do they have anomaly poles? **Yes, they do**.One can easily show that the perturbative analysis are associated with sum rules

$$\begin{aligned}\int_{4m^2}^{\infty} ds\, \Delta_{AVV}(s,m) &= 2\, d_{AVV} \\ \int_{4m^2}^{\infty} ds\, \Delta_{J_fTT}(s,m) &= \frac{2}{3}\, d_{J_fTT} \\ \int_{4m^2}^{\infty} ds\, \Delta_{J_{CS}TT}(s,m) &= \frac{14}{45}\, d_{J_{CS}TT},\end{aligned}$$

Sum rule
(Lionetti, Maglio, CC)

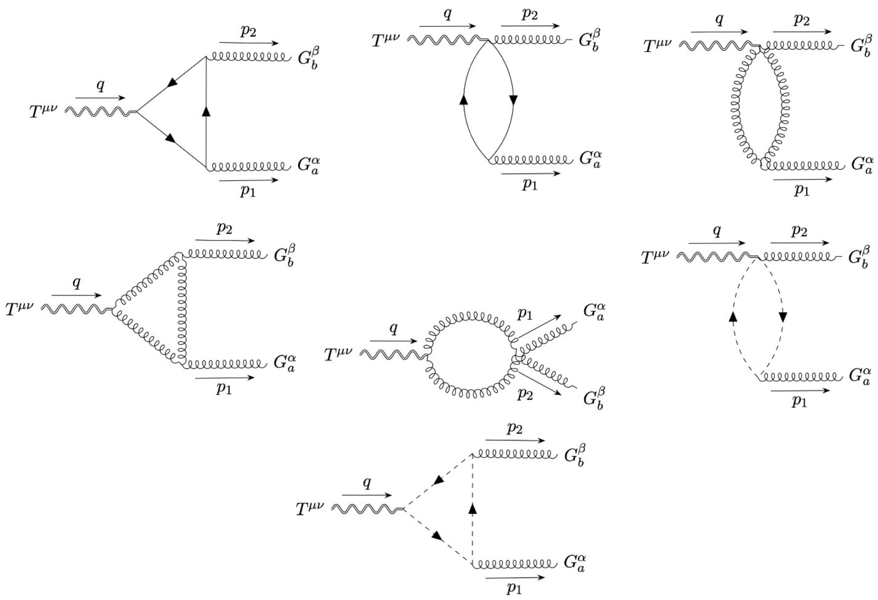
$$\begin{aligned}\langle 0|J^{\mu}_{f}|\gamma\gamma\rangle &= f_1(q^2)\frac{q^{\mu}}{q^2}F_{\kappa\lambda}\tilde{F}^{\kappa\lambda} \\ \langle 0|J^{\mu}_{f}||gg\rangle &= f_2(q^2)\frac{q^{\mu}}{q^2}R_{\kappa\lambda\rho\sigma}\tilde{R}^{\kappa\lambda\rho\sigma} \\ \langle 0|J^{\mu}_{CS}|gg\rangle &= f_3(q^2)\frac{q^{\mu}}{q^2}R_{\kappa\lambda\rho\sigma}\tilde{R}^{\kappa\lambda\rho\sigma},\end{aligned}$$

$$\lim_{m\to 0}\Delta(q^2,m) \propto \delta(q^2)$$

$$\begin{aligned}\Delta_{AVV}(q^2,m) \equiv \mathrm{Im} f_1(q^2) &= \frac{a_{AVV}}{q^2}(1-v^2)\log\frac{1+v}{1-v} \\ \Delta_{J_fTT}(q^2,m) \equiv \mathrm{Im} f_2(q^2) &= \frac{d_{J_fTT}}{q^2}(1-v^2)^2\log\frac{1+v}{1-v} \\ \Delta_{J_{CS}TT}(q^2,m) \equiv \mathrm{Im} f_3(q^2) &= \frac{d_{J_{CS}TT}}{q^2}v^2(1-v^2)^2\log\frac{1+v}{1-v}\end{aligned}$$

photons and gravitons are on-shell

with $v = \sqrt{1-4m^2/q^2}$ and d_{AVV}, d_{J_fTT} and $d_{J_{CS}TT}$ being the corresponding anomaly coefficients. Notice the different forms of Δ_{J_fTT} and $\Delta_{J_{CS}TT}(q^2,m)$ away from the conformal limit.



The dilaton pole in QCD

quark sector

transverse traceless sector

$$\langle t^{\mu\nu}(p_1) j^{a\,\alpha}(p_2) j^{b\,\beta}(p_3)\rangle_q = \Pi_1{}^{\mu\nu}_{\mu_1\nu_1}\pi_2{}^{\alpha}_{\alpha_1}\pi_3{}^{\beta}_{\beta_1}\Big(A_1^{(q)ab}\; p_2^{\mu_1}p_2^{\nu_1}p_3^{\alpha_1}p_1^{\beta_1} + A_2^{(q)ab}\;\delta^{\alpha_1\beta_1}p_2^{\mu_1}p_2^{\nu_1} + A_3^{(q)ab}\;\delta^{\mu_1\alpha_1}p_2^{\nu_1}p_1^{\beta_1}$$
$$+A_3^{(q)ab}(p_2\leftrightarrow p_3)\delta^{\mu_1\beta_1}p_2^{\nu_1}p_3^{\alpha_1} + A_4^{(q)ab}\;\delta^{\mu_1\beta_1}\delta^{\alpha_1\nu_1}\Big)\,. \quad (8$$

$$\langle T^{\mu\nu}\,J^{a\,\alpha}\,J^{b\,\beta}\rangle = \langle t^{\mu\nu}\,j^{a\,\alpha}\,j^{b\,\beta}\rangle + \langle T^{\mu\nu}\,J^{a\,\alpha}\,j^{b\,\beta}_{loc}\rangle + \langle T^{\mu\nu}\,j^{a\,\alpha}_{loc}\,J^{b\,\beta}\rangle + \langle t^{\mu\nu}_{loc}\,J^{a\,\alpha}\,J^{b\,\beta}\rangle$$
$$- \langle T^{\mu\nu}\,j^{a\,\alpha}_{loc}\,j^{b\,\beta}_{loc}\rangle - \langle t^{\mu\nu}_{loc}\,j^{a\,\alpha}_{loc}\,J^{b\,\beta}\rangle - \langle t^{\mu\nu}_{loc}\,J^{a\,\alpha}\,j^{b\,\beta}_{loc}\rangle + \langle t^{\mu\nu}_{loc}\,j^{a\,\alpha}_{loc}\,j^{b\,\beta}_{loc}\rangle\,.$$

$$\langle T_{\mu_1\nu_1}(\boldsymbol{p}_1)J^{\mu_2 a_2}(\boldsymbol{p}_2)J^{\mu_3 a_3}(\boldsymbol{p}_3)\rangle_q = \langle t_{\mu_1\nu_1}(\boldsymbol{p}_1)j^{\mu_2 a_2}(\boldsymbol{p}_2)j^{\mu_3 a_3}(\boldsymbol{p}_3)\rangle_q$$
$$+\,2\mathscr{T}_{\mu_1\nu_1}{}^{\alpha}(\boldsymbol{p}_1)\Big[\delta^{\mu_3}_{[\alpha}p_{3\beta]}\langle J^{\mu_2 a_2}(\boldsymbol{p}_2)J^{\beta a_3}(-\boldsymbol{p}_2)\rangle_q + \delta^{\mu_2}_{[\alpha}p_{2\beta]}\langle J^{\mu_3 a_3}(\boldsymbol{p}_3)J^{\beta a_2}(-\boldsymbol{p}_3)\rangle_q\Big]$$
$$+\,\frac{1}{d-1}\,\pi_{\mu_1\nu_1}(\boldsymbol{p}_1)\mathcal{A}_q^{\mu_2\mu_3 a_2 a_3},$$

$$\bar{A}_1^{(q)ab} = \frac{\delta^{ab}}{48\left(p_1^2 p_2^2-(p_1\cdot p_2)^2\right)^4}\Big[A_{10}+A_{11}B_0(p_1^2)+A_{12}B_0(p_2^2)+A_{13}B_0(q^2)+A_{14}C_0(p_1^2,p_2^2,q^2)\Big]$$
$$\bar{A}_2^{(q)ab} = -\frac{\delta^{ab}}{144\left(p_1^2 p_2^2-(p_1\cdot p_2)^2\right)^3}\Big[A_{20}^{(q)}+A_{21}^{(q)}B_0(p_1^2)+A_{22}^{(q)}B_0(p_2^2)+A_{23}^{(q)}B_0(q^2)+A_{24}^{(q)}C_0(p_1^2,p_2^2,q^2)\Big]$$
$$\bar{A}_3^{(q)ab} = \frac{\delta^{ab}}{72\left(p_1^2 p_2^2-(p_1\cdot p_2)^2\right)^3}\Big[A_{31}^{(q)}B_0(p_1^2)+A_{32}^{(q)}B_0(p_2^2)+A_{33}^{(q)}B_0(q^2)+A_{34}^{(q)}C_0(p_1^2,p_2^2,q^2)\Big]$$
$$\bar{A}_4^{(q)ab} = -\frac{\delta^{ab}}{72\left(p_1^2 p_2^2-(p_1\cdot p_2)^2\right)^2}\Big[A_{40}^{(q)}+A_{41}^{(q)}B_0(p_1^2)+A_{42}^{(q)}B_0(p_2^2)+A_{43}^{(q)}B_0(q^2)+A_{44}^{(q)}C_0(p_1^2,p_2^2,q^2)\Big].$$

computable in pQCD.
They coincide with the conformal solution (nonn Lagrangian),

Gluon sector. This sector has been investigated by us without resporting to the CWIs, which are broken by this sector. But the sector decomposiiton still holds and shows the presence of an dilaton pole

$$\langle T^{\mu\nu}(q)J^{a\alpha}(p_1)J^{b\beta}(p_2)\rangle_g = \langle t^{\mu\nu}(q)j^{a\alpha}(p_1)j^{b\beta}(p_2)\rangle_g + \langle t^{\mu\nu}(q)j^{a\alpha}_{loc}(p_1)j^{b\beta}(p_2)\rangle_g + \langle t^{\mu\nu}(q)j^{a\alpha}(p_1)j^{b\beta}_{loc}(p_2)\rangle_g$$
$$+\,2\mathcal{I}^{\mu\nu\rho}(q)\left[\delta^{\beta}_{[\rho}p_{2\sigma]}\langle J^{a\alpha}(p_1)J^{b\sigma}(-p_1)\rangle_g+\delta^{\alpha}_{[\rho}p_{1\sigma]}\langle J^{b\beta}(p_2)J^{a\sigma}(-p_2)\rangle\right]_g+\frac{1}{d-1}\pi^{\mu\nu}(q)\left[\mathcal{A}_g^{\alpha\beta ab}+\mathcal{B}_g^{\alpha\beta ab}\right] \quad (0.1)$$

The final reusult for the qcd dilaton pole

$$\langle T^{\mu\nu}(q)J^{a\alpha}(p_1)J^{b\beta}(p_2)\rangle = \langle t^{\mu\nu}(q)j^{a\alpha}(p_1)j^{b\beta}(p_2)\rangle + \langle t^{\mu\nu}(q)j^{a\alpha}_{loc}(p_1)j^{b\beta}(p_2)\rangle_g + \langle t^{\mu\nu}(q)j^{a\alpha}(p_1)j^{b\beta}_{loc}(p_2)\rangle_g$$
$$+ 2\mathcal{I}^{\mu\nu\rho}(q)\left[\delta^{\beta}_{[\rho}p_{2\sigma]}\langle J^{a\alpha}(p_1)J^{b\sigma}(-p_1)\rangle + \delta^{\alpha}_{[\rho}p_{1\sigma]}\langle J^{b\beta}(p_2)J^{a\sigma}(-p_2)\rangle\right] + \frac{1}{3\,q^2}\hat{\pi}^{\mu\nu}(q)\left[\mathcal{A}^{\alpha\beta ab} + \mathcal{B}^{\alpha\beta ab}_g\right]$$

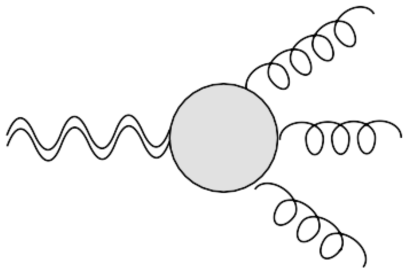
where

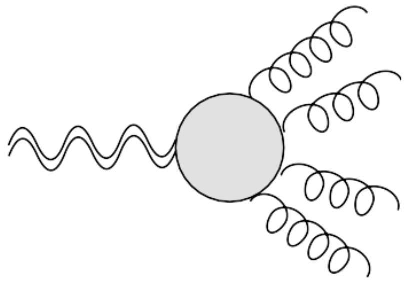
$$\mathcal{A}^{\alpha\beta ab}_g + \mathcal{B}^{\alpha\beta ab}_g = g_{\mu\nu}\langle T^{\mu\nu}(q)\, J^{a\alpha}(p_1)\, J^{b\beta}(p_2)\,\rangle_g\Big|$$

$$\mathcal{B}^{\alpha\beta ab}_g = \left[C^{ab}_1\, p_1^{\alpha}\, p_1^{\beta} + C^{ab}_2\, p_1^{\alpha}\, p_2^{\beta} + C^{ab}_3\, p_1^{\beta}\, p_2^{\alpha} + C^{ab}_4\, p_2^{\alpha}\, p_2^{\beta} + C^{ab}_5\, \delta^{\alpha\beta}\right]$$

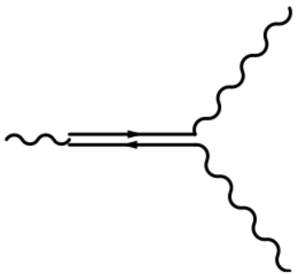
$$u^{\alpha\beta}(p_1, p_2) \equiv (p_1\cdot p_2)g^{\alpha\beta} - p_2^{\alpha}p_1^{\beta}$$

This is FF in momentum space differentiated twice wrt the gluons





we refer to our paper for a discussion of the other non abelian contributions



the dilaon pole in the trace anomaly

For on shell gluons

Delle Rose, Armillis, CC

$$\Gamma^{\mu\nu\alpha\beta\,ab}(p,q) = \Gamma_g^{\mu\nu\alpha\beta\,ab}(p_1,p_2) + \Gamma_q^{\mu\nu\alpha\beta\,ab}(p_1,p_2) = \sum_{i=1}^{3} \Phi_i(s,0,0)\,\delta^{ab}\,\phi_i^{\mu\nu\alpha\beta}(p_1,p_2)\,,$$

$$\begin{aligned}
\phi_1^{\mu\nu\alpha\beta}(p_1,p_2) &= (s\,g^{\mu\nu} - q^\mu q^\nu)\,u^{\alpha\beta}(p_1,p_2),\\
\phi_2^{\mu\nu\alpha\beta}(p_1,p_2) &= -2\,u^{\alpha\beta}(p_1,p_2)\left[s\,g^{\mu\nu} + 2(p_1^\mu\,p_1^\nu + p_2^\mu\,p_2^\nu) - 4\,(p_1^\mu\,p_2^\nu + p_2^\mu\,p_1^\nu)\right],\\
\phi_3^{\mu\nu\alpha\beta}(p_1,p_2) &= (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu) g^{\alpha\beta} + \frac{s}{2}\left(g^{\alpha\nu}g^{\beta\mu} + g^{\alpha\mu}g^{\beta\nu}\right)\\
&\quad - g^{\mu\nu}\left(\frac{s}{2}g^{\alpha\beta} - p_2^\alpha p_1^\beta\right) - \left(g^{\beta\nu}p^\mu + g^{\beta\mu}p_1^\nu\right)p_2^\alpha - \left(g^{\alpha\nu}p_2^\mu + g^{\alpha\mu}p_2^\nu\right)p_1^\beta,
\end{aligned}$$

$$\begin{aligned}
\Phi_1(s,0,0) &= -\frac{g^2}{72\pi^2\,s}\,(2n_f - 11C_A) + \frac{g^2}{6\pi^2}\sum_{i=1}^{n_f} m_i^2\left\{\frac{1}{s^2} - \frac{1}{2s}\mathcal{C}_0(s,0,0,m_i^2)\left[1 - \frac{4m_i^2}{s}\right]\right\},\\
\Phi_2(s,0,0) &= -\frac{g^2}{288\pi^2\,s}\,(n_f - C_A)\\
&\quad - \frac{g^2}{24\pi^2}\sum_{i=1}^{n_f} m_i^2\left\{\frac{1}{s^2} + \frac{3}{s^2}\mathcal{D}(s,0,0,m_i^2) + \frac{1}{s}\mathcal{C}_0(s,0,0,m_i^2)\,\left[1 + \frac{2m_i^2}{s}\right]\right\},\\
\Phi_3(s,0,0) &= \frac{g^2}{288\pi^2}\,(11n_f - 65C_A) - \frac{g^2\,C_A}{8\pi^2}\left[\frac{11}{6}\mathcal{B}_0^{\overline{MS}}(s,0) - \mathcal{B}_0^{\overline{MS}}(0,0) + s\,\mathcal{C}_0(s,0,0,0)\right]\\
&\quad + \frac{g^2}{8\pi^2}\sum_{i=1}^{n_f}\left\{\frac{1}{3}\mathcal{B}_0^{\overline{MS}}(s,m_i^2) + m_i^2\left[\frac{1}{s} + \frac{5}{3s}\mathcal{D}(s,0,0,m_i^2) + \mathcal{C}_0(s,0,0,m_i^2)\,\left[1 + \frac{2m_i^2}{s}\right]\right]\right\},
\end{aligned}$$

$$\Phi_{1,2q}(k^2, m^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\rho_{1,2q}(s, m^2)}{s - k^2},$$

$$\frac{1}{\pi} \int_0^\infty ds\, \rho_{1q}(s, m^2) = \frac{g^2}{36\pi^2}, \qquad \frac{1}{\pi} \int_0^\infty ds\, \rho_{2q}(s, m^2) = \frac{g^2}{288\pi^2},$$

$$\bar{\rho}_{1q}(s, m^2) \equiv \frac{36\pi^2}{g^2} \rho_{1q}(s, m^2) \qquad \bar{\rho}_{2q}(s, m^2) \equiv \frac{288\pi^2}{g^2} \rho_{2q}(s, m^2)$$

the pole is exchanged
in the conformal limit

$$\lim_{m \to 0} \bar{\rho}_{1q} = \lim_{m \to 0} \bar{\rho}_{2q} = \delta(s).$$

Only one pole contributes to the
treace anomaly

A similar pattern is found in the gluon sector, which obviously is not affected by the mass term. In this case the on-shell and transverse condition on the external gluons brings to three very simple form factors whose expressions are

$$\Phi_{1\,g}(k^2) = \frac{11\, g^2}{72\pi^2\, k^2}\, C_A\,, \qquad \Phi_{2\,g}(k^2) = \frac{g^2}{288\pi^2\, k^2}\, C_A\,,$$

$$\Phi_{3\,g}(k^2) = -\frac{g^2}{8\pi^2} C_A \left[\frac{65}{36} + \frac{11}{6} \mathcal{B}_0^{\overline{MS}}(k^2, 0) - \mathcal{B}_0^{\overline{MS}}(0, 0) + k^2\, \mathcal{C}_0(k^2, 0) \right].$$

A similar pattern is found in the gluon sector, which obviously is not affected by the mass term. In this case the on-shell and transverse condition on the external gluons brings to three very simple form factors whose expressions are

$$\begin{aligned}\Phi_{1\,g}(k^2) &= \frac{11\,g^2}{72\pi^2\,k^2}\,C_A\,, \qquad \Phi_{2\,g}(k^2) = \frac{g^2}{288\pi^2\,k^2}\,C_A\,,\\ \Phi_{3\,g}(k^2) &= -\frac{g^2}{8\pi^2}C_A\left[\frac{65}{36}+\frac{11}{6}\mathcal{B}_0^{\overline{MS}}(k^2,0)-\mathcal{B}_0^{\overline{MS}}(0,0)+k^2\,\mathcal{C}_0(k^2,0)\right].\end{aligned}$$

Also in this case, it is clear that the simple poles in $\Phi_{1\,g}$ and $\Phi_{2\,g}$, the two form factors which are not affected by the renormalization, are accounted for by two spectral densities which are proportional to $\delta(s)$. The anomaly pole in $\Phi_{1\,g}$ is accompanied by a second pole in the non anomalous form factor $\Phi_{2\,g}$. Notice that Φ_{3g} is affected by renormalization, and as such it is not considered relevant in the spectral analysis.

1. **We have analyzed the hard scattering amplitude of gravitational form factors (GFFs) of hadrons at one-loop level, considering their connection to conformal field theory (CFT) within the QCD factorization framework for hard exclusive processes at large momentum transfers.**

These form factors are crucial for studying quark and gluon angular momentum within hadrons, as they relate to the Mellin moments of Deeply Virtual Compton Scattering (DVCS) invariant amplitudes.

The analysis employs a diffeomorphism invariant approach, utilizing the gravitational effective action formalism and conformal symmetry in momentum space to discuss quark and gluon contributions.

The interpolating correlator in the hard scattering of any GFF is identified as the non-Abelian TJJ (stress-energy/gluon/gluon) 3-point function at order O(\alpha_s^2) .

2. This correlator reveals an effective dilaton interaction in the t-channel, manifested as a massless anomaly pole due to the trace anomaly, constrained by a sum rule on its spectral density.

We haveinvestigated the role of quarks, gauge-fixing, and ghost contributions in reconstructing the hard scattering amplitude, which is expressed in terms of its transverse traceless, longitudinal, and trace components as identified from CFT in momentum space.

A convenient parameterization of the hard scattering amplitude is presented, which is relevant for future experimental investigations of DVCS/GFF amplitudes, particularly at the Electron-Ion Collider at BNL.

There is a perturbative sum rule for the TJJ the proton that can be experimentally tested

Thank
you