Flavour deconstruction: from the electroweak scale to the GUT scale

CERN-TH BSM forum,
1st August 2024,
Geneva, Switzerland

Mario Fernández Navarro

Based on:

MFN, Stephen F. King, [2305.07690] hep-ph, JHEP 08 (2023) 020
MFN, Stephen F. King and Avelino Vicente, [2311.05683] hep-ph, JHEP 05 (2024) 130
MFN, Stephen F. King and Avelino Vicente, [2404.12442] hep-ph, JHEP 07 (2024) 147

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Outline

1. Introduction: Flavour in the SM (and beyond)

2. Flavour deconstruction: generics

3. Flavour deconstruction: from the electroweak scale

4. Flavour deconstruction: to the GUT scale

Family affair

who ordered *all* of that?

-Isidor Isaac Rabi after the discovery of the muon, a copy of the electron 100 times heavier, in 1936

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Personally I see no theoretical reason for a prejudice against an elementary spin-zero object. The real problem with Higgs - and this is one of those unresolved problems which I mentioned earlier and one which calls for greater depth in our theories - is the large number of parameters - 21 out of 26 in the standard 6-quark, (K-M) $SU(2) \times U(1) \times SU_C(3)$ model - attributable to the Higgs sector[4]. What is needed is an extension of the gauge (or a similar) principle to embrace the Higgs sector... almost certainly there is a more basic layer of structure underneath.

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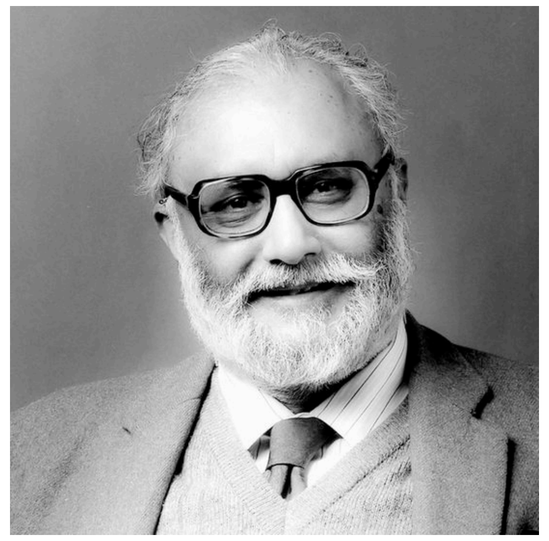

EPS HEP conference at CERN 1979

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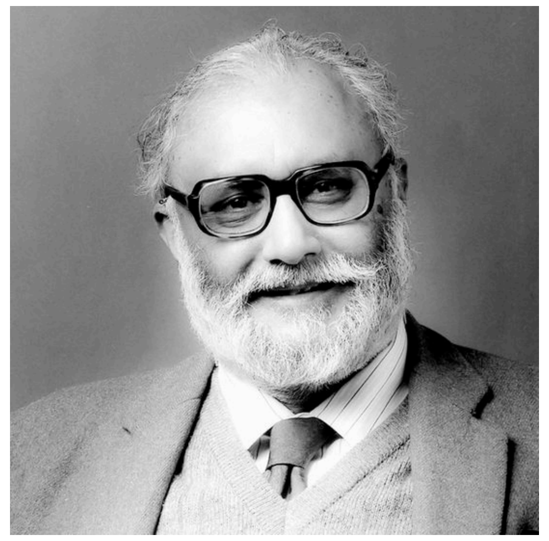

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In the Standard Model the masses of quarks and leptons take values proportional to the coupling constants in the interaction of these fermions with scalar fields, constants that in the context of this model are entirely arbitrary. But the peculiar hierarchical pattern of lepton and quark masses seems to call for a larger theory, in which in some leading approximation the only quarks and leptons with non-zero mass are those of the third generation, the tau, top, and bottom, with the other lepton and quark masses

Steven Weinberg, [2001.06582]

One of the longer-lasting puzzles in particle physics!

Flavour in the SM (and beyond)

- Flavour (generations, families) refers to copies of the gauge representation.
- The (chiral) fermions of the SM come in three identical (replicated) families under the gauge symmetry.
- Gauge anomalies cancel family by family.

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
q_{1L}	$\mathbf{3}$	$\mathbf{2}$	$1/6$
u_{1R}	$\mathbf{3}$	$\mathbf{1}$	$2/3$
d_{1R}	$\mathbf{3}$	$\mathbf{1}$	$-1/3$
ℓ_{1L}	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
e_{1R}	$\mathbf{1}$	$\mathbf{1}$	-1
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- At least three quark families needed to have *CP*-violating phase(s) in the CKM matrix, QCD asymptotic freedom requires less than nine (quark) families.

- Indeed all experimental data supports the existence of only three (chiral) families (see e.g. invisible decay width of Z boson)

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→ All these are *a posteriori* arguments

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Flavour in the SM (and beyond)

- Chiral fermions can couple to the Higgs doublet that breaks electroweak symmetry!

$$\mathcal{L}_{\text{Yukawa}} = y^u_{ij}\bar{q}_{iL}\widetilde{H}u_{jR} + y^d_{ij}\bar{q}_{iL}Hd_{jR} + y^e_{ij}\bar{\ell}_{iL}He_{jR} + \text{h.c.}$$

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- Chiral fermion masses appear as a consequence of spontaneous symmetry breaking

$$\langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_{\text{SM}} + h \end{pmatrix}$$

$$\mathcal{L}_{\text{Yukawa}} = \frac{1}{\sqrt{2}}(v_{\text{SM}} + h)\left[y^u_{ij}\bar{u}_{iL}u_{jR} + y^d_{ij}\bar{d}_{iL}d_{jR} + y^e_{ij}\bar{e}_{iL}e_{jR}\right] + \text{h.c.}$$

- $y^{u,d,e}_{ij}$ are generic 3×3 complex matrices

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- $y^{u,d,e}_{ij}$ are generic 3×3 complex matrices. In order to obtain mass eigenstates and (real and positive) eigenvalues, Yukawa matrices must be brought to diagonal form via bi-unitary transformations:

$$V_{u_L} y^u V^\dagger_{u_R} = \text{diag}(y_u, y_c, y_t) \qquad V_{d_L} y^d V^\dagger_{d_R} = \text{diag}(y_d, y_s, y_b) \qquad V_{e_L} y^e V^\dagger_{e_R} = \text{diag}(y_e, y_\mu, y_\tau)$$

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⟶ $m_\alpha = y_\alpha \dfrac{v_{\text{SM}}}{\sqrt{2}}$ Fermion masses are proportional to physical Yukawa couplings

⟶ Unitary matrices $V_{\psi_{L,R}}$ encode the mixing among fermions with the same gauge transformations

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Physical (weak interactions)	Unphysical (in the SM)
$V_{\text{CKM}} = V^\dagger_{u_L} V_{d_L}$	$V_{u_L}, V_{d_L}, V_{e_L}$
$V_{\text{PMNS}} = V^\dagger_{\nu_L} V_{e_L}$	$V_{u_R}, V_{d_R}, V_{e_R}$

But may become physical in BSM scenarios

Flavour in the SM (and beyond)

- Neutrino oscillations → (at least 2) neutrinos are massive and there exists mixing in the lepton sector
- In the SM(EFT) → dim-5 Weinberg operator $\frac{c_{ij}}{\Lambda}(\bar{\ell}^{C}_{Li}\widetilde{H})(\ell_{Lj}\widetilde{H})$ → neutrinos are Majorana

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- Simple origin via type I seesaw → add at least two gauge singlet (right-handed) neutrinos $N_{R1,2} \sim (\mathbf{1},\mathbf{1},0)$

$$m_D = \left(\begin{array}{c|cc} & N_{R1} & N_{R2} \\ \hline \bar{\ell}_1 & y^\nu_{11} & y^\nu_{12} \\ \bar{\ell}_2 & y^\nu_{21} & y^\nu_{22} \\ \bar{\ell}_3 & y^\nu_{31} & y^\nu_{32} \end{array}\right)\tilde{H} \qquad M_R = \left(\begin{array}{c|cc} & N_{R1} & N_{R2} \\ \hline \overline{N}^C_{R1} & M_{11} & M_{12} \\ \overline{N}^C_{R2} & M_{12} & M_{22} \end{array}\right)$$

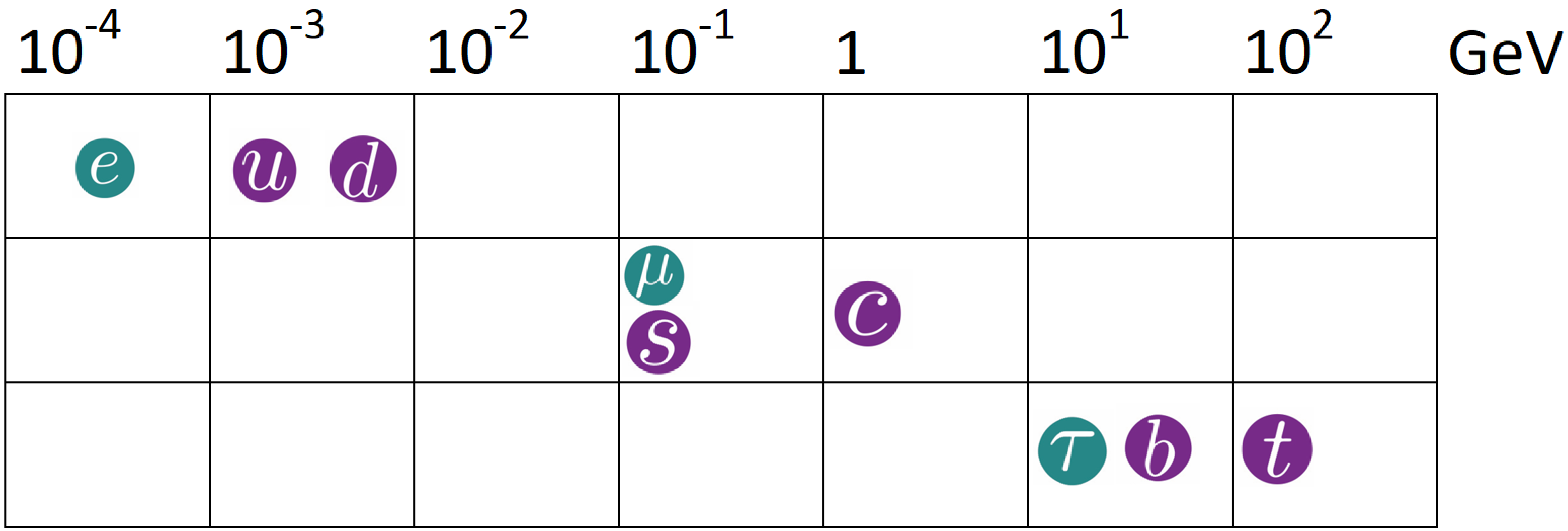
- Assume $m_D \ll M_R$: $\quad m_\nu \simeq m_D M_R^{-1} m_D^{\mathrm{T}}$ → $V_{\mathrm{PMNS}} \simeq V^\dagger_{\nu_L} V_{e_L}$

- $m_{\nu_1} = 0$ and normal ordering, then $m_{\nu_3} = \sqrt{\Delta m^2_{31}} \approx 0.05\,\mathrm{eV}$, if $y^\nu_{ij} \approx \mathcal{O}(1)$ then $M_{Rij} \approx \mathcal{O}(10^{15}\,\mathrm{GeV})$

Flavour in the SM (and beyond)

$$m_\alpha = y_\alpha \frac{v_{\rm SM}}{\sqrt{2}}$$

- Small y_α - natural a la t'Hooft
- But the three families are identical objects, all y_α enter the theory in the same way - Why hierarchies?



$$m_t \sim \frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_c \sim \lambda^{3.3}\frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_u \sim \lambda^{7.5}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$

$$m_b \sim \lambda^{2.5}\frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_s \sim \lambda^{5.0}\frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_d \sim \lambda^{7.0}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$

$$m_\tau \sim \lambda^{3.0}\frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_\mu \sim \lambda^{4.9}\frac{v_{\rm SM}}{\sqrt{2}}\,, \qquad m_e \sim \lambda^{8.4}\frac{v_{\rm SM}}{\sqrt{2}}\,,$$

where $v_{\rm SM} \simeq 246\,{\rm GeV}$ and $\lambda \simeq \sin\theta_C \simeq 0.224$

$$V_{\rm CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$V_{\rm PMNS} \sim \begin{pmatrix} 0.8 & 0.6 & 0.15 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

Flavour model building program

1. Understand why (how?) Nature has chosen three identical fermion families.

2. Understand the origin of flavour hierarchies, the origin of neutrino masses and the different mixing among quarks and leptons, in terms of a dynamical mechanism.

3. Ideally, all (flavour) parameters should be calculable from fundamental principles (i.e. theory with no free parameters) - or at least calculable in terms of $\mathcal{O}(1)$ parameters.

4. Ideally, testable!

Conventions

- 4-component (Dirac) spinor expressed in terms of 2-component (Weyl) spinor:

$$F = \begin{pmatrix} f_L \\ f_R \end{pmatrix} \equiv \begin{pmatrix} f \\ (f^c)^\dagger \end{pmatrix}, \quad \bar{F} = F^\dagger \gamma^0 = \begin{pmatrix} f^c & f^\dagger \end{pmatrix}, \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

see e.g. [Dreiner, Haber, Martin, 0812.1594], also Adam Falkowski GGI lectures

- In some BSM theories, LH fermions f unify in the same multiplet with conjugate RH fermions f^c (both are LH):

$$SU(5) \longrightarrow \begin{cases} \bar{\mathbf{5}}_i \sim \ell_i \oplus d_i^c \\ \mathbf{10}_i \sim q_i \oplus u_i^c \oplus e_i^c \end{cases}$$

$$SO(10) \longrightarrow \mathbf{16}_i \sim q_i \oplus \ell_i \oplus u_i^c \oplus d_i^c \oplus e_i^c \oplus \nu_i^c$$

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
q_{Li}	$\mathbf{3}$	$\mathbf{2}$	$1/6$
u_{Ri}	$\mathbf{3}$	$\mathbf{1}$	$2/3$
d_{Ri}	$\mathbf{3}$	$\mathbf{1}$	$-1/3$
ℓ_{Li}	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
e_{Ri}	$\mathbf{1}$	$\mathbf{1}$	-1

$\longrightarrow$

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
q_i	$\mathbf{3}$	$\mathbf{2}$	$1/6$
u_i^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$
d_i^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$
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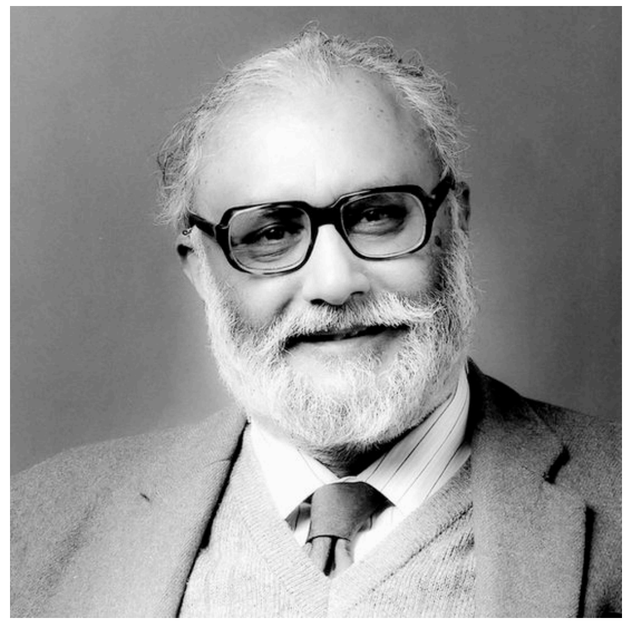
Flavour deconstruction

A GAUGE APPRECIATION OF DEVELOPMENTS IN PARTICLE PHYSICS - 1979

A. Salam,

ICTP, Trieste, Italy, and
Imperial College, London, England.



EPS HEP conference at CERN 1979

41) Consider one more example of the introduction of intermediate energy scales - and the plateau-breaking peaks - which may have their location almost anywhere, so far as the internal logic of the symmetry-breaking is concerned. The example is that of the tribal group $SU^{I}(5) \times SU^{II}(5) \times SU^{III}(5)$ corresponding to the Three Families. Assume each $SU^{i}(5)$ breaks to $[SU(2) \times U(1) \times SU_C(3)]^{i}$, i = I, II, III, with mass scales M^{i}. The final breaking stage corresponds to the emergence of the diagonal sum $[SU(2) \times U(1) \times SU_C(3)]^{I+II+III}$ ($\supseteq U(1) \times SU_C(3)$)

An oasis in the *SU*(5) desert vs *SO*(10) intermediate scales…

"Tribal groups"

↕

"Flavour deconstruction"

Flavour deconstruction

- Perhaps first concrete realisation is: [Li and Ma, 1981]

Gauge Model of Generation Nonuniversality

Xiao-yuan Li[(a)] and Ernest Ma
Department of Physics and Astronomy, University of Hawaii at Manoa, Honolulu, Hawaii 96822
(Received 13 October 1981)

An electroweak gauge model is discussed, where generations are associated with separate gauge groups with different couplings. The observed μ-e universality is the re-

We adopt the group $\mathrm{U}(1)\otimes[\mathrm{SU}(2)]^n$, and assign the ith generation of fermions to $\mathrm{U}(1)\otimes\mathrm{SU}(2)_i$, where $i=1,\ldots,n$, as in the standard model for that subgroup. The gauge couplings are g_0 for

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- More recent realisations:

[Craig, Green and Katz, 2011]

(De)Constructing a Natural and Flavorful Supersymmetric Standard Model

Nathaniel Craig,[1,2] Daniel Green,[1] and Andrey Katz[3]

SUSY $\quad \mathrm{SM}_{12}\times\mathrm{SM}_3$

[Bordone, Cornella, Fuentes-Martín, Isidori, 2017]

A three-site gauge model for flavor hierarchies and flavor anomalies

Marzia Bordone,[1,*] Claudia Cornella,[1,†] Javier Fuentes-Martín,[1,‡] and Gino Isidori[1,§]
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We present a three-site Pati-Salam gauge model able to explain the Standard Model flavor hierarchies while, at the same time, accommodating the recent experimental hints of lepton-flavor non-universality in B decays. The model is consistent with low- and high-energy bounds, and predicts a rich spectrum of new states at the TeV scale that could be probed in the near future by the high-p_T experiments at the LHC.

$\mathrm{PS}_1\times\mathrm{PS}_2\times\mathrm{PS}_3$

Flavour deconstruction

- Three fermion families are identical (replicated) objects under the SM symmetry, yet they interact so differently with the Higgs

→ Family (gauge) structure hidden at high energies?

- Flavour deconstruction: SM is embedded in a extended gauge symmetry that contains a separate factor for each fermion family:

$$G_{\text{universal}} \times G_1 \times G_2 \times G_3$$

[Salam 79', Rajpoot 81', Li and Ma 81', Georgi 82' … Bordone *et al* 17', Greljo and Stefanek 18', Fuentes-Martín *et al*, 20', Davighi and Isidori 23' …]

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  - Discriminates between the three families (no longer family replication).
  - The EW Higgs doublet is a third family particle (i.e. singlet of G_1 and G_2 but not of G_3).
  - Explains SM flavour structure with all fundamental (Yukawa) couplings being $\mathcal{O}(1)$.

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- Several different names in the literature: "tribal groups", "gauge non-universal theories of flavour (extensions of the SM)", "multi-scale theories of flavour", "flavour deconstruction"…

Outline

1. Introduction: Flavour in the SM (and beyond)
2. Flavour deconstruction: generics
3. Flavour deconstruction: from the electroweak scale
4. Flavour deconstruction: to the GUT scale

Simple example: Tri-hypercharge

A simple example: $SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$ → $Y_{\rm SM} \equiv Y = Y_1 + Y_2 + Y_3$

[MFN, King, 23'; MFN, King, Vicente, 24']

q_1	q_2	q_3
u_1^c	u_2^c	u_3^c
d_1^c	d_2^c	d_3^c
ℓ_1	ℓ_2	ℓ_3
e_1^c	e_2^c	e_3^c
		H_3

- "A separate (gauge) hypercharge for each fermion family" - no ad-hoc choices.

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Field	$SU(3)_c$	$SU(2)_L$	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$
q_1	$\mathbf{3}$	$\mathbf{2}$	$1/6$	0	0
u_1^c	$\mathbf{\bar{3}}$	$\mathbf{1}$	$-2/3$	0	0
d_1^c	$\mathbf{\bar{3}}$	$\mathbf{1}$	$1/3$	0	0
ℓ_1	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	0	0
e_1^c	$\mathbf{1}$	$\mathbf{1}$	1	0	0
q_2	$\mathbf{3}$	$\mathbf{2}$	0	$1/6$	0
u_2^c	$\mathbf{\bar{3}}$	$\mathbf{1}$	0	$-2/3$	0
d_2^c	$\mathbf{\bar{3}}$	$\mathbf{1}$	0	$1/3$	0
ℓ_2	$\mathbf{1}$	$\mathbf{2}$	0	$-1/2$	0
e_2^c	$\mathbf{1}$	$\mathbf{1}$	0	1	0
q_3	$\mathbf{3}$	$\mathbf{2}$	0	0	$1/6$
u_3^c	$\mathbf{\bar{3}}$	$\mathbf{1}$	0	0	$-2/3$
d_3^c	$\mathbf{\bar{3}}$	$\mathbf{1}$	0	0	$1/3$
ℓ_3	$\mathbf{1}$	$\mathbf{2}$	0	0	$-1/2$
e_3^c	$\mathbf{1}$	$\mathbf{1}$	0	0	1
H_3	$\mathbf{1}$	$\mathbf{2}$	0	0	$1/2$

q_1	q_2	q_3
u_1^c	u_2^c	u_3^c
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q_2	$\mathbf{3}$	$\mathbf{2}$	0	$1/6$	0
u_2^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	$-2/3$	0
d_2^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	$1/3$	0
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H_3	$\mathbf{1}$	$\mathbf{2}$	0	0	$1/2$

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- EW Higgs doublet only carries third family hypercharge - only third family Yukawas at renormalisable level.

- Light families are massless in first approximation:

$$\mathcal{L} = y_t q_3 H_3 u_3^c + y_b q_3 H_3 d_3^c + y_\tau \ell_3 H_3 e_3^c + \text{h.c.}$$

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ℓ_3	$\mathbf{1}$	$\mathbf{2}$	0	0	$-1/2$
e_3^c	$\mathbf{1}$	$\mathbf{1}$	0	0	1
$H_3^{u,d}$	$\mathbf{1}$	$\mathbf{2}$	0	0	$\pm 1/2$

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d_1^c	d_2^c	d_3^c
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- Type II 2HDM can take care of $m_{b,\tau}/m_t$ hierarchies via:

$$\tan\beta = v_u/v_d \approx \lambda^{-2} \simeq 20$$

Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

$\mathcal{O}(1)$ coupling

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$\mathcal{O}(1)$ coupling

- E.g. operator $q_2 H_3^d d_3^c \sim (0, \frac{1}{6}, -\frac{1}{6})$ forbidden by tri-hypercharge (gauge) symmetry.

Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Phi(0, -\frac{1}{6}, \frac{1}{6}) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

- E.g. operator $q_2 H_3^d d_3^c \sim (0, \frac{1}{6}, -\frac{1}{6})$ forbidden by tri-hypercharge (gauge) symmetry.
- Introduce a spurion $\Phi \sim (0, -\frac{1}{6}, \frac{1}{6})$, then we can write $\Phi q_2 H_3^d d_3^c$. Repeat for every entry in the matrix.

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- Promote spurion $\Phi \sim (0, -\frac{1}{6}, \frac{1}{6})$ to the physical scalar ("hyperon") , $\phi_{q23} \sim (0, -\frac{1}{6}, \frac{1}{6})$

$$\frac{\phi_{q23}}{\Lambda_2} q_2 H_3^d d_3^c$$

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- Promote spurion $\Phi \sim (0, -\frac{1}{6}, \frac{1}{6})$ to the physical scalar ("hyperon"), $\phi_{q23} \sim (0, -\frac{1}{6}, \frac{1}{6})$

$$\frac{\phi_{q23}}{\Lambda_2} q_2 H_3^d d_3^c$$

- We also promote the spurion in the (2,2) entry to hyperon $\Phi \sim (0, -\frac{1}{2}, \frac{1}{2}) \sim \phi_{\ell 23}$:

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Tri-hypercharge: spurions

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Flavour structure dynamically generated via tri-hypercharge SSB

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- So far we have two scalars (hyperons) $\phi_{\ell 23} \sim (0, -\frac{1}{2}, \frac{1}{2})$ and $\phi_{q23} \sim (0, -\frac{1}{6}, \frac{1}{6})$

$$Y_d = \begin{pmatrix} \approx 0 & \approx 0 & \approx 0 \\ \approx 0 & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q23}}{\Lambda_2} \\ \approx 0 & \frac{\phi_{\ell 23}}{\Lambda_2}\frac{\phi_{q23}}{\Lambda_2} & 1 \end{pmatrix}$$

Tri-hypercharge: spurions

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- Introduce $\phi_{\ell 12} \sim (-\frac{1}{2},\frac{1}{2},0)$ and $\phi_{q12} \sim (-\frac{1}{6},\frac{1}{6},0)$ to generate spurions in first row:

$$Y_d = \begin{pmatrix} \approx 0 & \approx 0 & \approx 0 \\ \approx 0 & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q23}}{\Lambda_2} \\ \approx 0 & \frac{\phi_{\ell 23}}{\Lambda_2}\frac{\phi_{q23}}{\Lambda_2} & 1 \end{pmatrix}$$

Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(-\frac{1}{6},-\frac{1}{3},\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(-\frac{1}{3},-\frac{1}{6},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(-\frac{1}{3},0,\frac{1}{3}) & \Phi(0,-\frac{1}{3},\frac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d + \mathrm{h.c.}$$

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$$Y_d = \begin{pmatrix} \frac{\phi_{\ell 12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_1} & \frac{\phi_{q12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q12}}{\Lambda_1}\frac{\phi_{q23}}{\Lambda_2} \\ \frac{\phi_{\ell 12}}{\Lambda_1}\frac{\tilde{\phi}_{q12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q23}}{\Lambda_2} \\ \frac{\phi_{q12}^2}{\Lambda_1^2}\frac{\phi_{q23}^2}{\Lambda_2^2} & \frac{\phi_{\ell 23}}{\Lambda_2}\frac{\phi_{q23}}{\Lambda_2} & 1 \end{pmatrix}$$

$$\phi_{\ell 12}\phi_{\ell 23} \sim \left(-\frac{1}{2},0,\frac{1}{2}\right)$$

$$\phi_{q12}\phi_{\ell 23} \sim \left(-\frac{1}{6},-\frac{1}{3},\frac{1}{2}\right)$$

$$\phi_{q12}\phi_{q23} \sim \left(-\frac{1}{6},0,\frac{1}{6}\right)$$

Tri-hypercharge: spurions

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(-\frac{1}{6},-\frac{1}{3},\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(-\frac{1}{3},-\frac{1}{6},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(-\frac{1}{3},0,\frac{1}{3}) & \Phi(0,-\frac{1}{3},\frac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d + \text{h.c.}$$

- So far we have two scalars (hyperons) $\phi_{\ell 23} \sim (0, -\frac{1}{2}, \frac{1}{2})$ and $\phi_{q23} \sim (0, -\frac{1}{6}, \frac{1}{6})$
- Introduce $\phi_{\ell 12} \sim (-\frac{1}{2}, \frac{1}{2}, 0)$ and $\phi_{q12} \sim (-\frac{1}{6}, \frac{1}{6}, 0)$ to generate spurions in first row:

$$Y_d = \begin{pmatrix} \frac{\phi_{\ell 12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_1} & \frac{\phi_{q12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q12}}{\Lambda_1}\frac{\phi_{q23}}{\Lambda_2} \\ \frac{\phi_{\ell 12}}{\Lambda_1}\frac{\tilde{\phi}_{q12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q23}}{\Lambda_2} \\ \frac{\phi_{q12}^2}{\Lambda_1^2}\frac{\phi_{q23}^2}{\Lambda_2^2} & \frac{\phi_{\ell 23}}{\Lambda_2}\frac{\phi_{q23}}{\Lambda_2} & 1 \end{pmatrix}$$

$$\phi_{\ell 12}\phi_{\ell 23} \sim \left(-\frac{1}{2}, 0, \frac{1}{2}\right)$$

$$\phi_{q12}\phi_{\ell 23} \sim \left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{2}\right)$$

$$\phi_{q12}\phi_{q23} \sim \left(-\frac{1}{6}, 0, \frac{1}{6}\right)$$

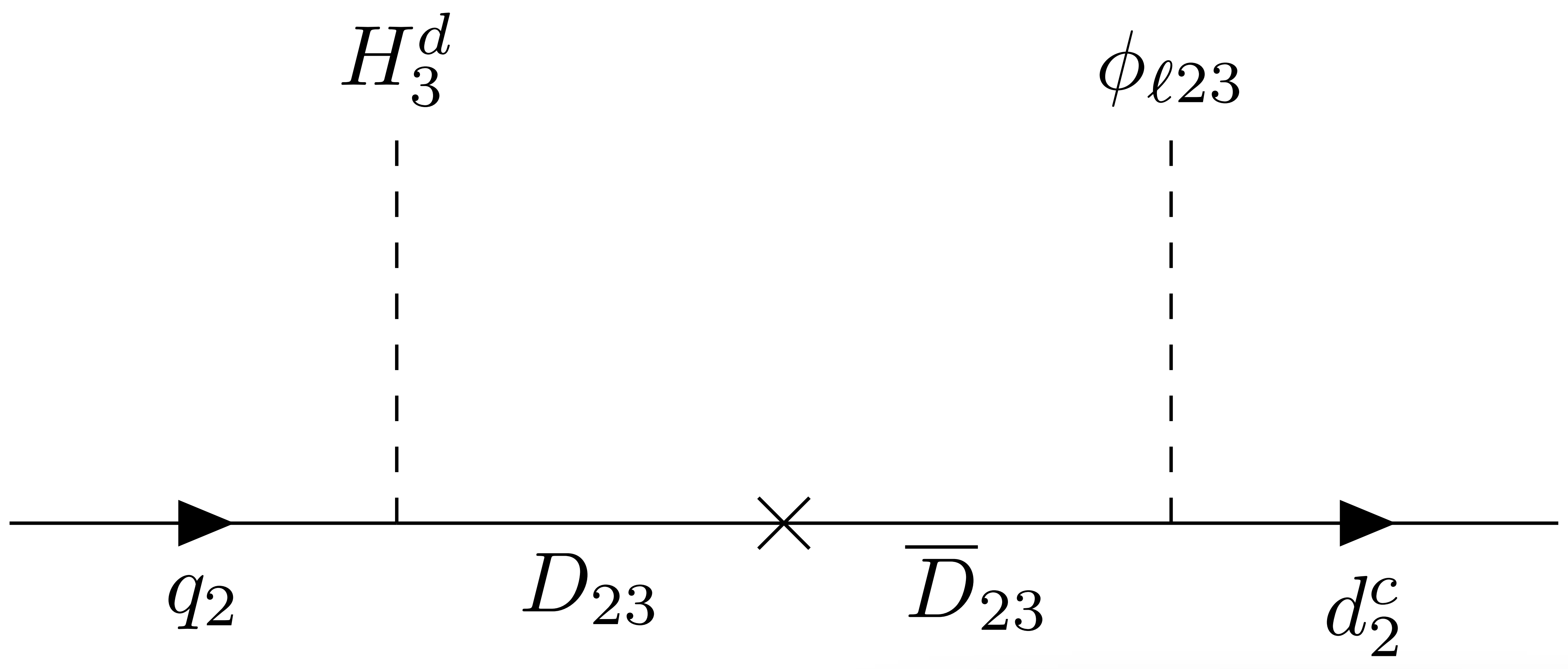
Heavy messengers needed for Λs!

Model 1

Field	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$	$SU(3)_c \times SU(2)_L$
$H_3^{u,d}$	0	0	$\pm 1/2$	$(\mathbf{1},\mathbf{2})$
ϕ_{q12}	-1/6	1/6	0	$(\mathbf{1},\mathbf{1})$
$\phi_{\ell 12}$	-1/2	1/2	0	$(\mathbf{1},\mathbf{1})$
ϕ_{q23}	0	-1/6	1/6	$(\mathbf{1},\mathbf{1})$
$\phi_{\ell 23}$	0	-1/2	-1/2	$(\mathbf{1},\mathbf{1})$
U_{12}	-1/6	-1/2	0	$(\overline{\mathbf{3}},\mathbf{1})$
U_{13}	-1/6	0	-1/2	$(\overline{\mathbf{3}},\mathbf{1})$
U_{23}	0	-1/6	-1/2	$(\overline{\mathbf{3}},\mathbf{1})$
D_{12}	-1/6	1/2	0	$(\overline{\mathbf{3}},\mathbf{1})$
D_{13}	-1/6	0	1/2	$(\overline{\mathbf{3}},\mathbf{1})$
D_{23}	0	-1/6	1/2	$(\overline{\mathbf{3}},\mathbf{1})$
E_{12}	1/2	1/2	0	$(\mathbf{1},\mathbf{1})$
E_{13}	1/2	0	1/2	$(\mathbf{1},\mathbf{1})$
E_{23}	0	1/2	1/2	$(\mathbf{1},\mathbf{1})$

$$Y_d = \begin{pmatrix} c_{11}^d \frac{\phi_{\ell 12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{12}}} & c_{12}^d \frac{\phi_{q12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{13}^d \frac{\phi_{q12}}{M_{D_{13}}} \frac{\phi_{q23}}{M_{D_{23}}} \\ c_{21}^d \frac{\phi_{\ell 12}}{M_{D_{12}}} \frac{\tilde{\phi}_{q12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{22}^d \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{23}^d \frac{\phi_{q23}}{M_{D_{23}}} \\ \approx 0 & \approx 0 & c_{33}^d \end{pmatrix}$$

$$Y_u = Y_d(d \to u, D \to U) \qquad Y_e = Y_d(d \to e, q \to \ell, D \to E)$$



- Completion via *SU*(2)-singlet VL fermions
- Simple scalar sector and potential

Model 1 spectrum

$$Y_d = \begin{pmatrix} c_{11}^d \frac{\phi_{\ell 12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{12}}} & c_{12}^d \frac{\phi_{q12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{13}^d \frac{\phi_{q12}}{M_{D_{13}}} \frac{\phi_{q23}}{M_{D_{23}}} \\ c_{21}^d \frac{\phi_{\ell 12}}{M_{D_{12}}} \frac{\tilde{\phi}_{q12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{22}^d \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{23}^d \frac{\phi_{q23}}{M_{D_{23}}} \\ \approx 0 & \approx 0 & c_{33}^d \end{pmatrix}$$

Model 1 spectrum

$$Y_d = \begin{pmatrix} c_{11}^d \frac{\phi_{\ell 12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{12}}} & c_{12}^d \frac{\phi_{q12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{13}^d \frac{\phi_{q12}}{M_{D_{13}}} \frac{\phi_{q23}}{M_{D_{23}}} \\ c_{21}^d \frac{\phi_{\ell 12}}{M_{D_{12}}} \frac{\tilde{\phi}_{q12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{22}^d \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{23}^d \frac{\phi_{q23}}{M_{D_{23}}} \\ \approx 0 & \approx 0 & c_{33}^d \end{pmatrix}$$

- Fit SM flavour structure via three naturally small parameters (includes also up-quarks and charged leptons):

$$\frac{m_2}{m_3} = \boxed{\frac{\langle \phi_{23} \rangle}{M_{23}} \sim \lambda^3\,,}$$

Model 1 spectrum

$$Y_d = \begin{pmatrix} c^d_{11}\frac{\phi_{\ell 12}}{M_{D_{13}}}\frac{\phi_{\ell 23}}{M_{D_{12}}} & c^d_{12}\frac{\phi_{q12}}{M_{D_{13}}}\frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{13}\frac{\phi_{q12}}{M_{D_{13}}}\frac{\phi_{q23}}{M_{D_{23}}} \\ c^d_{21}\frac{\phi_{\ell 12}}{M_{D_{13}}}\frac{\tilde{\phi}_{q12}}{M_{D_{13}}}\frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{22}\frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{23}\frac{\phi_{q23}}{M_{D_{23}}} \\ \approx 0 & \approx 0 & c^d_{33} \end{pmatrix}$$

- Fit SM flavour structure via three naturally small parameters (includes also up-quarks and charged leptons):

$$\frac{m_2}{m_3} = \boxed{\frac{\langle\phi_{23}\rangle}{M_{23}} \sim \lambda^3}\,, \qquad \sin\theta_c = \frac{V_{ub}}{V_{cb}} = \boxed{\frac{\langle\phi_{12}\rangle}{M_{12,13}} \sim \lambda}$$

Model 1 spectrum

$$Y_d = \begin{pmatrix} c^d_{11} \frac{\phi_{\ell 12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{12}}} & c^d_{12} \frac{\phi_{q12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{13} \frac{\phi_{q12}}{M_{D_{13}}} \frac{\phi_{q23}}{M_{D_{23}}} \\ c^d_{21} \frac{\phi_{\ell 12}}{M_{D_{13}}} \frac{\widetilde{\phi}_{q12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{22} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{23} \frac{\phi_{q23}}{M_{D_{23}}} \\ \approx 0 & \approx 0 & c^d_{33} \end{pmatrix}$$

- Fit SM flavour structure via three naturally small parameters (includes also up-quarks and charged leptons):

$$\frac{m_2}{m_3} = \boxed{\frac{\langle \phi_{23} \rangle}{M_{23}} \sim \lambda^3}\,, \qquad \sin\theta_c = \frac{V_{ub}}{V_{cb}} = \boxed{\frac{\langle \phi_{12} \rangle}{M_{12,13}} \sim \lambda} \qquad \frac{m_1}{m_3} = \frac{\langle \phi_{12} \rangle}{M_{12,13}} \frac{\langle \phi_{23} \rangle}{M_{12,13}} \sim \lambda^5 \Rightarrow \boxed{\frac{\langle \phi_{23} \rangle}{M_{12,13}} \sim \lambda^4}$$

Model 1 spectrum

$$Y_d = \begin{pmatrix} c^d_{11}\frac{\phi_{\ell 12}}{M_{D_{13}}}\frac{\phi_{\ell 23}}{M_{D_{12}}} & c^d_{12}\frac{\phi_{q12}}{M_{D_{13}}}\frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{13}\frac{\phi_{q12}}{M_{D_{13}}}\frac{\phi_{q23}}{M_{D_{23}}} \\ c^d_{21}\frac{\phi_{\ell 12}}{M_{D_{13}}}\frac{\tilde{\phi}_{q12}}{M_{D_{13}}}\frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{22}\frac{\phi_{\ell 23}}{M_{D_{23}}} & c^d_{23}\frac{\phi_{q23}}{M_{D_{23}}} \\ \approx 0 & \approx 0 & c^d_{33} \end{pmatrix}$$

- Fit SM flavour structure via three naturally small parameters (includes also up-quarks and charged leptons):

$$\frac{m_2}{m_3} = \boxed{\frac{\langle\phi_{23}\rangle}{M_{23}} \sim \lambda^3}\,, \qquad \sin\theta_c = \frac{V_{ub}}{V_{cb}} = \boxed{\frac{\langle\phi_{12}\rangle}{M_{12,13}} \sim \lambda} \qquad \frac{m_1}{m_3} = \frac{\langle\phi_{12}\rangle}{M_{12,13}}\frac{\langle\phi_{23}\rangle}{M_{12,13}} \sim \lambda^5 \Rightarrow \boxed{\frac{\langle\phi_{23}\rangle}{M_{12,13}} \sim \lambda^4}$$

- Fixed relation between VEVs is predicted:

$$\boxed{\frac{\langle\phi_{23}\rangle}{\langle\phi_{12}\rangle} \sim \lambda^3 \approx 0.01}$$

→ Highly non-generic

Model 1 spectrum

- Fixed relation between VEVs is predicted:

$$\frac{\langle \phi_{23} \rangle}{\langle \phi_{12} \rangle} \sim \lambda^3 \approx 0.01$$

Spectrum up to $\mathcal{O}(1)$ variations

E

$\mathcal{O}(10^4\,\text{TeV})$ — $M_{U_{12,13},D_{12,13},E_{12,13}}$

$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$

$\mathcal{O}(1000\,\text{TeV})$ — $v_{12} \sim \langle \phi_{q12} \rangle,\ \langle \phi_{\ell 12} \rangle,\ M_{U_{23},D_{23},E_{23}}$

$\mathcal{O}(10\,\text{TeV})$ — $v_{23} \sim \langle \phi_{q23} \rangle,\ \langle \phi_{\ell 23} \rangle$

$174\,\text{GeV}$ — $v_{\text{SM}} \sim \langle H_3^{u,d} \rangle$

Model 1 spectrum

- Fixed relation between VEVs is predicted:

$$\frac{\langle \phi_{23} \rangle}{\langle \phi_{12} \rangle} \sim \lambda^3 \approx 0.01$$

Spectrum up to $\mathcal{O}(1)$ variations

E

$\mathcal{O}(10^4\ \text{TeV})$ — $M_{U_{12,13},D_{12,13},E_{12,13}}$

$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$

Z'_{12} ↓

$\mathcal{O}(1000\ \text{TeV})$ — $v_{12} \sim \langle \phi_{q12} \rangle,\ \langle \phi_{\ell 12} \rangle,\ M_{U_{23},D_{23},E_{23}}$

$SU(3)_c \times SU(2)_L \times U(1)_{Y_{12} \equiv Y_1 + Y_2} \times U(1)_{Y_3}$

$\mathcal{O}(10\ \text{TeV})$ — $v_{23} \sim \langle \phi_{q23} \rangle,\ \langle \phi_{\ell 23} \rangle$

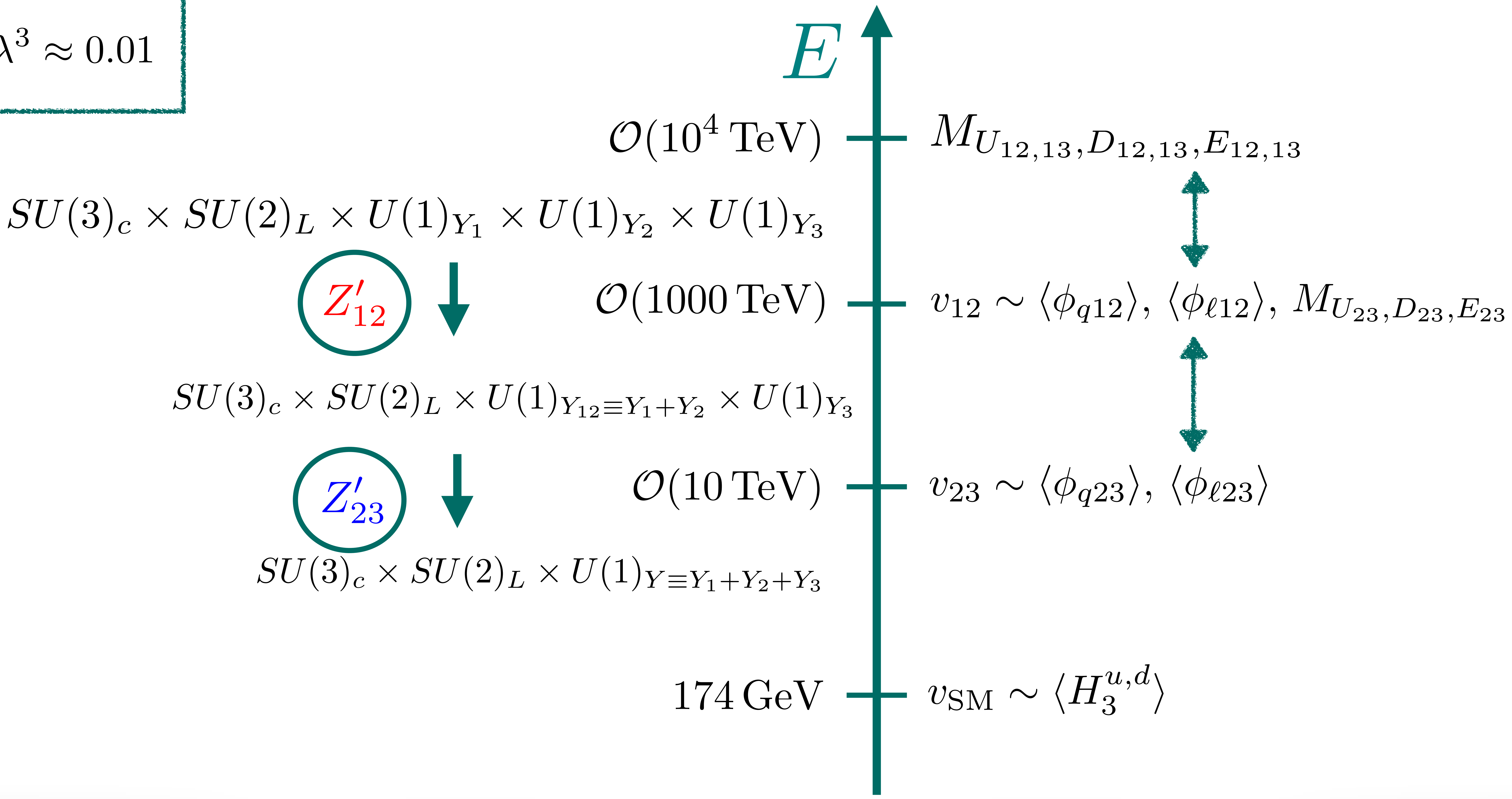
$174\ \text{GeV}$ — $v_{\text{SM}} \sim \langle H_3^{u,d} \rangle$

Model 1 spectrum

- Fixed relation between VEVs is predicted:

$$\frac{\langle \phi_{23} \rangle}{\langle \phi_{12} \rangle} \sim \lambda^3 \approx 0.01$$

Spectrum up to $\mathcal{O}(1)$ variations

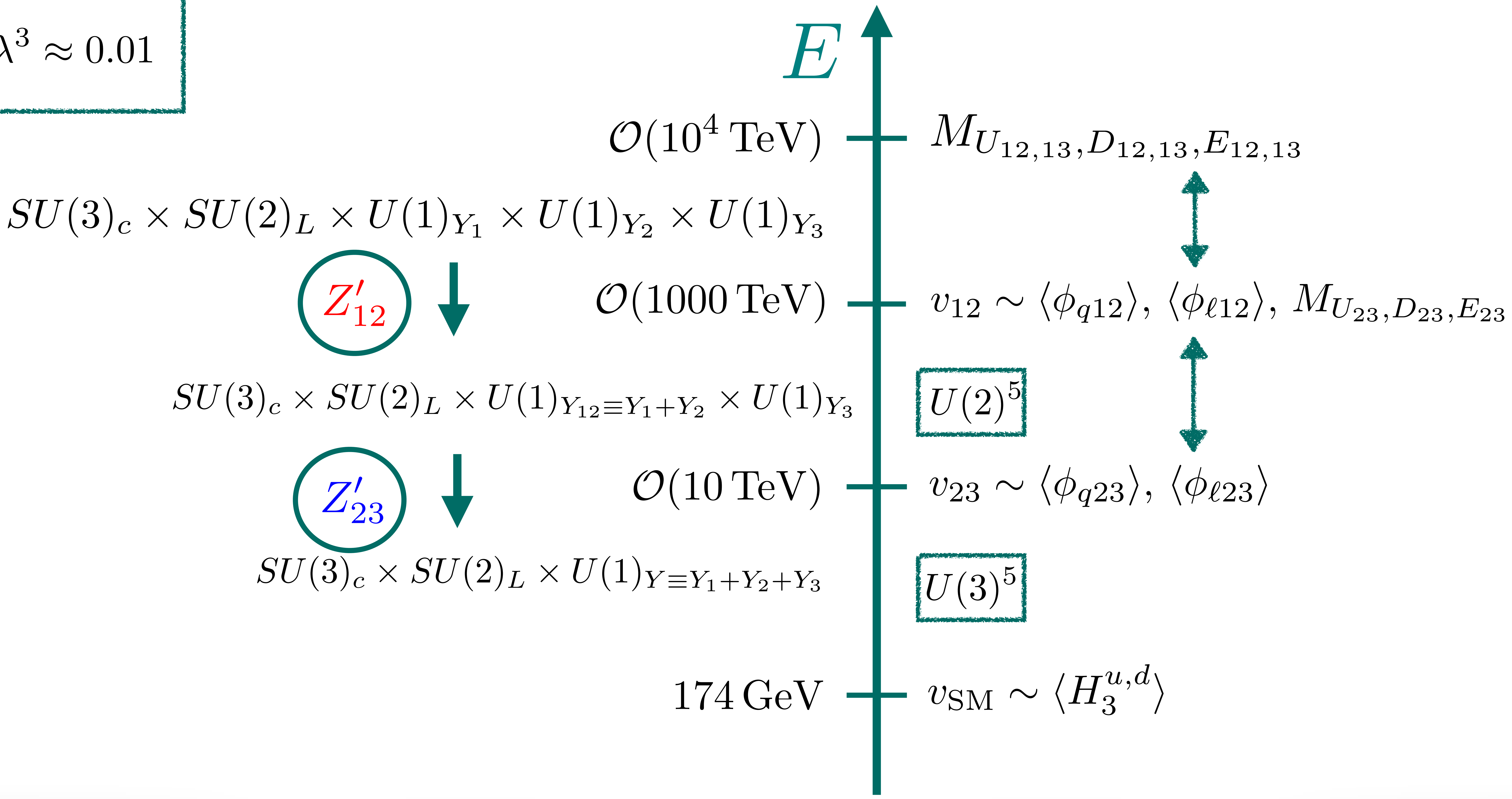


Model 1 spectrum

- Fixed relation between VEVs is predicted:

$$\frac{\langle \phi_{23} \rangle}{\langle \phi_{12} \rangle} \sim \lambda^3 \approx 0.01$$

Spectrum up to $\mathcal{O}(1)$ variations



Model 2

Field	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$	$SU(3)_c \times SU(2)_L$
$H_3^{u,d}$	0	0	$\pm 1/2$	$(\mathbf{1}, \mathbf{2})$
H_2^d	0	$-1/2$	0	$(\mathbf{1}, \mathbf{2})$
H_1^d	$-1/2$	0	0	$(\mathbf{1}, \mathbf{2})$
$\phi_{q_{12}}$	-1/6	1/6	0	$(\mathbf{1}, \mathbf{1})$
$\phi_{\ell_{12}}$	-1/2	1/2	0	$(\mathbf{1}, \mathbf{1})$
$\phi_{q_{23}}$	0	-1/6	1/6	$(\mathbf{1}, \mathbf{1})$
$\phi_{\ell_{23}}$	0	-1/2	1/2	$(\mathbf{1}, \mathbf{1})$
U_{12}	-1/6	-1/2	0	$(\overline{\mathbf{3}}, \mathbf{1})$
U_{13}	-1/6	0	-1/2	$(\overline{\mathbf{3}}, \mathbf{1})$
U_{23}	0	-1/6	-1/2	$(\overline{\mathbf{3}}, \mathbf{1})$

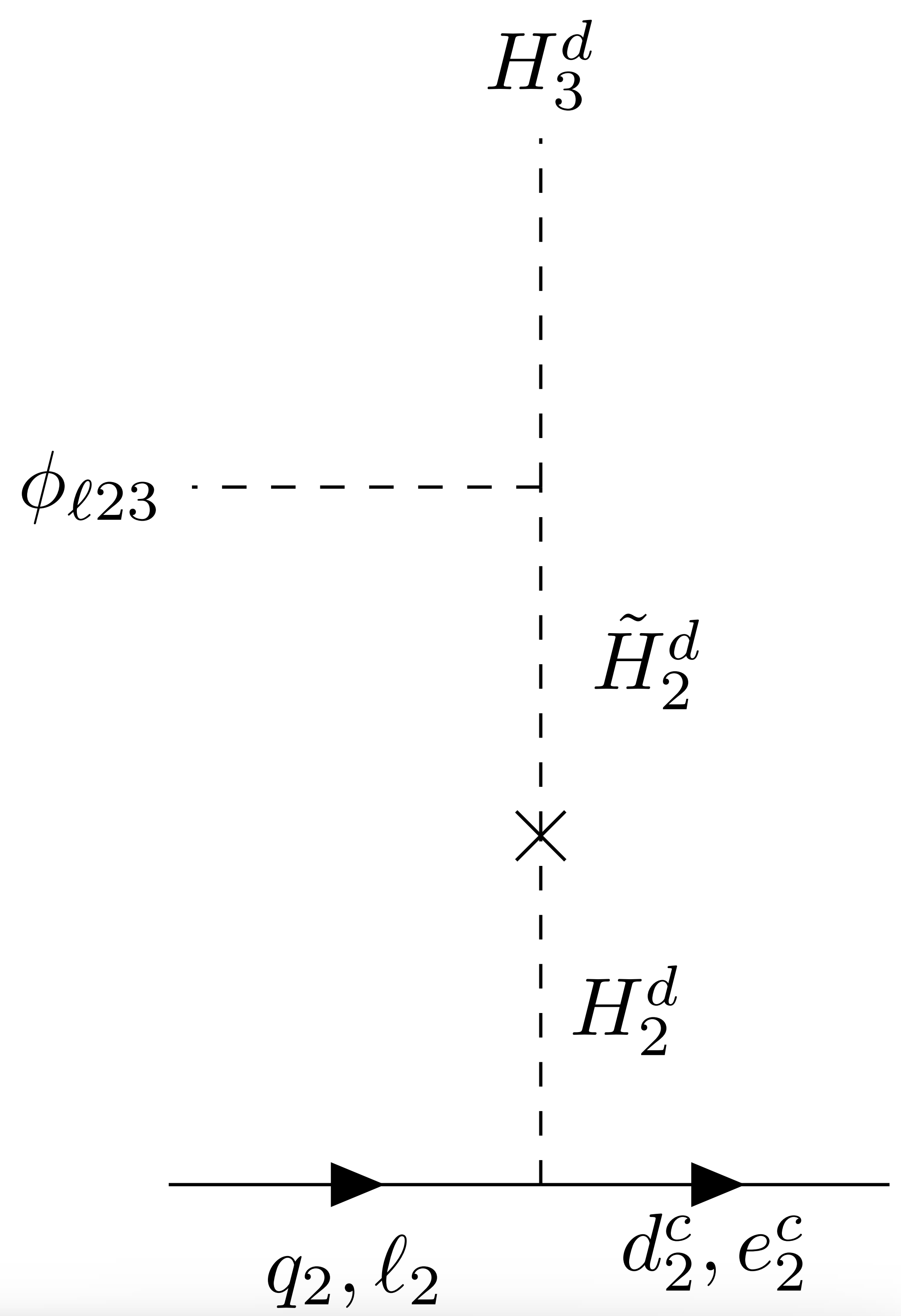
- Minimal number of degrees of freedom and representations
- Diagonal down-quark and charged lepton mass matrices
- CKM originates from up-sector only (down-aligned)

Model 2

Field	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$	$SU(3)_c \times SU(2)_L$
$H_3^{u,d}$	0	0	$\pm 1/2$	$(\mathbf{1},\mathbf{2})$
H_2^d	0	$-1/2$	0	$(\mathbf{1},\mathbf{2})$
H_1^d	$-1/2$	0	0	$(\mathbf{1},\mathbf{2})$
ϕ_{q12}	-1/6	1/6	0	$(\mathbf{1},\mathbf{1})$
$\phi_{\ell 12}$	-1/2	1/2	0	$(\mathbf{1},\mathbf{1})$
ϕ_{q23}	0	-1/6	1/6	$(\mathbf{1},\mathbf{1})$
$\phi_{\ell 23}$	0	-1/2	1/2	$(\mathbf{1},\mathbf{1})$
U_{12}	-1/6	-1/2	0	$(\overline{\mathbf{3}},\mathbf{1})$
U_{13}	-1/6	0	-1/2	$(\overline{\mathbf{3}},\mathbf{1})$
U_{23}	0	-1/6	-1/2	$(\overline{\mathbf{3}},\mathbf{1})$

$$Y_u = \begin{pmatrix} c_{11}^u \frac{\widetilde{\phi}_{\ell 12}}{M_{U_{13}}} \frac{\widetilde{\phi}_{\ell 23}}{M_{U_{12}}} & c_{12}^u \frac{\phi_{q12}}{M_{U_{13}}} \frac{\widetilde{\phi}_{\ell 23}}{M_{U_{23}}} & c_{13}^u \frac{\phi_{q12}}{M_{U_{13}}} \frac{\phi_{q23}}{M_{U_{23}}} \\ c_{21}^u \frac{\widetilde{\phi}_{\ell 12}}{M_{U_{12}}} \frac{\phi_{q12}}{M_{U_{13}}} \frac{\widetilde{\phi}_{\ell 23}}{M_{U_{23}}} & c_{22}^u \frac{\widetilde{\phi}_{\ell 23}}{M_{U_{23}}} & c_{23}^u \frac{\phi_{q23}}{M_{U_{23}}} \\ 0 & 0 & c_{33}^u \end{pmatrix}$$

$$Y_{d,e} = \begin{pmatrix} c_{11}^{d,e} \frac{\phi_{\ell 12}}{M_{H_1^d}} \frac{\phi_{\ell 23}}{M_{H_2^d}} & 0 & 0 \\ 0 & c_{22}^{d,e} \frac{\phi_{\ell 23}}{M_{H_2^d}} & 0 \\ 0 & 0 & c_{33}^{d,e} \end{pmatrix}$$



- Minimal number of degrees of freedom and representations
- Diagonal down-quark and charged lepton mass matrices
- CKM originates from up-sector only (down-aligned)

$$\longrightarrow \quad V_{\text{CKM}} \simeq V_{u_L}^\dagger$$

$$V_{d_L} \simeq \mathbb{I}$$

Model 2 spectrum

- Fixed relation between VEVs is predicted as in Model 1:

$$\frac{\langle\phi_{23}\rangle}{\langle\phi_{12}\rangle} \sim \lambda^3 \approx 0.01$$

Spectrum up to $\mathcal{O}(1)$ variations

E

$\mathcal{O}(10^4\,\text{TeV})$ — $M_{U_{12,13}}, M_{H_1^d}$

$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$

Z'_{12} ↓

$\mathcal{O}(1000\,\text{TeV})$ — $v_{12} \sim \langle\phi_{q12}\rangle, \langle\phi_{\ell 12}\rangle, M_{U_{23}}, M_{H_2^d}$

$SU(3)_c \times SU(2)_L \times U(1)_{Y_{12}\equiv Y_1+Y_2} \times U(1)_{Y_3}$ — $U(2)^5$

Z'_{23} ↓

$\mathcal{O}(10\,\text{TeV})$ — $v_{23} \sim \langle\phi_{q23}\rangle, \langle\phi_{\ell 23}\rangle$

$SU(3)_c \times SU(2)_L \times U(1)_{Y\equiv Y_1+Y_2+Y_3}$ — $U(3)^5$

$174\,\text{GeV}$ — $v_{\text{SM}} \sim \langle H_3^{u,d}\rangle$

Pheno: Z'_{23}

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

$$\xrightarrow{v_{12}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3} + Z'_{12}$$

$$\xrightarrow{v_{23}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3} + Z'_{23} + Z'_{12}$$

Pheno: Z'_{23}

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$
$$\overset{v_{12}}{\to} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3} + Z'_{12}$$
$$\overset{v_{23}}{\to} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3} + Z'_{23} + Z'_{12}$$

- Matching conditions connect gauge couplings with SM hypercharge coupling:

$$g_{12} = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}$$

$$g_Y = \frac{g_{12} g_3}{\sqrt{g_{12}^2 + g_3^2}} \simeq 0.36\,(M_Z)$$

$$g_i \geq g_Y \simeq 0.36\,(M_Z)$$

- Couplings cannot be small (unlike horizontal flavour symmetries), cannot be decoupled unless gauge bosons are very heavy

Pheno: Z'_{23}

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$
$$\xrightarrow{v_{12}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3} + Z'_{12}$$
$$\xrightarrow{v_{23}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3} + Z'_{23} + Z'_{12}$$

- For Z'_{23} :

$$\sin\theta_{23} = \frac{g_{12}}{\sqrt{g_{12}^2 + g_3^2}}$$

$$\mathcal{L}_{Z'_{23}} = Y_{f_{L,R}} \bar{f}_{L,R} \gamma^\mu \begin{pmatrix} -g_{12}\sin\theta_{23} & 0 & 0 \\ 0 & -g_{12}\sin\theta_{23} & 0 \\ 0 & 0 & g_3\cos\theta_{23} \end{pmatrix} f_{L,R} Z'_{23\mu}$$

- Matching conditions connect gauge couplings with SM hypercharge coupling:

$$g_{12} = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \qquad g_Y = \frac{g_{12} g_3}{\sqrt{g_{12}^2 + g_3^2}} \simeq 0.36\,(M_Z)$$

$$g_i \geq g_Y \simeq 0.36\,(M_Z)$$

- Couplings cannot be small (unlike horizontal flavour symmetries), cannot be decoupled unless gauge bosons are very heavy

- Coupled to all SM fermions
- Matching conditions: large g_3 then smaller g_{12} and viceversa
- Protected by accidental $U(2)^5$ symmetry (GIM on 12 families)

Pheno: Z'_{23}

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$
$$\overset{v_{12}}{\to} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3} + Z'_{12}$$
$$\overset{v_{23}}{\to} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3} + Z'_{23} + Z'_{12}$$

- Matching conditions connect gauge couplings with SM hypercharge coupling:

$$g_{12} = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \qquad g_Y = \frac{g_{12} g_3}{\sqrt{g_{12}^2 + g_3^2}} \simeq 0.36\,(M_Z)$$

- For Z'_{23}:

$$\sin\theta_{23} = \frac{g_{12}}{\sqrt{g_{12}^2 + g_3^2}}$$

$$g_i \geq g_Y \simeq 0.36\,(M_Z)$$

- Couplings cannot be small (unlike horizontal flavour symmetries), cannot be decoupled unless gauge bosons are very heavy

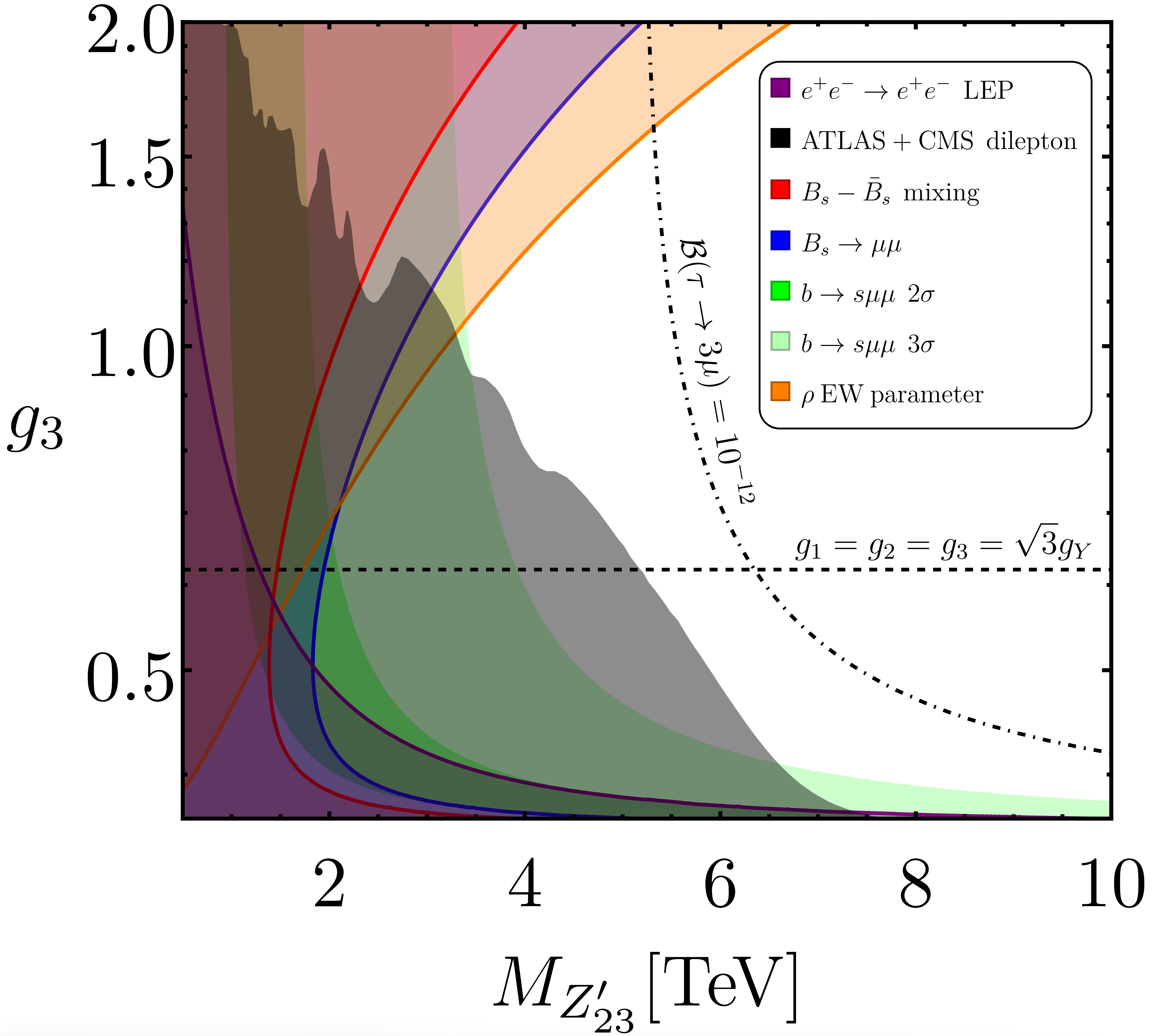
$$\mathcal{L}_{Z'_{23}} = Y_{f_{L,R}} \bar{f}_{L,R} \gamma^\mu \begin{pmatrix} -g_{12}\sin\theta_{23} & 0 & 0 \\ 0 & -g_{12}\sin\theta_{23} & 0 \\ 0 & 0 & g_3\cos\theta_{23} \end{pmatrix} f_{L,R} Z'_{23\mu}$$

- Coupled to all SM fermions
- Matching conditions: large g_3 then smaller g_{12} and viceversa
- Protected by accidental $U(2)^5$ symmetry (GIM on 12 families)

$$\sin\theta_{Z-Z'_{23}} = \frac{g_3 \cos\theta_{23}}{\sqrt{g_Y^2 + g_L^2}} \left(\frac{M_Z^0}{M_{Z'_{23}}^0}\right)^2 \Rightarrow \rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{1}{1 - g_3^2 \cos^2\theta_{23} \left(\frac{v_{SM}}{2M_{Z'_{23}}^0}\right)^2} > 1\,,$$

➡ Unavoidable mixing with SM Z boson

Pheno: Z'_{23}



[MFN, King, 2305.07690]

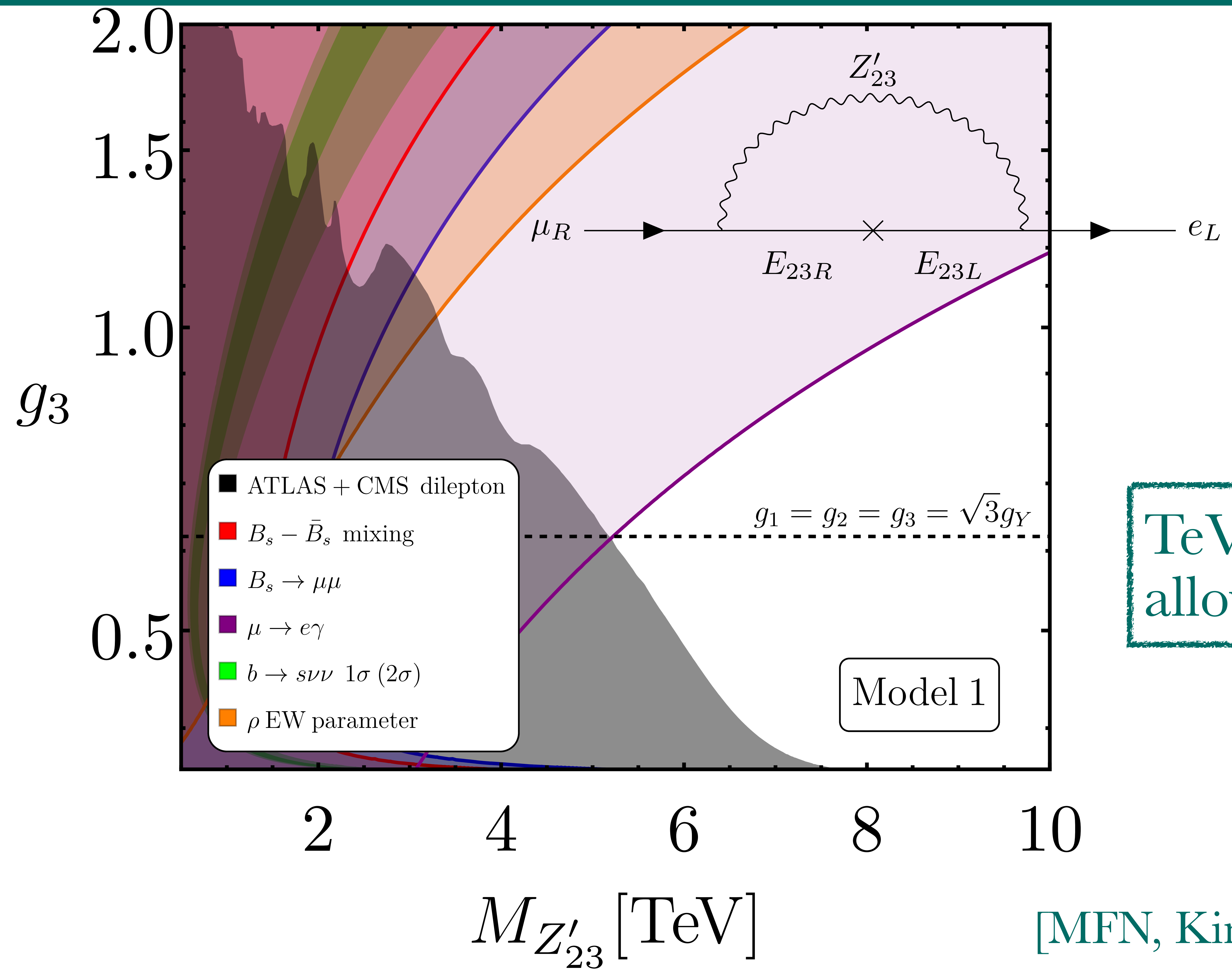
TeV masses allowed!

- EFT framework
- Assuming minimal breaking of $U(2)^5$, flavour observables non-competitive
- Model-independent:
  - EW precision
  - High-p_T dilepton searches

⟶ Potential for FCC program (ee+hh)

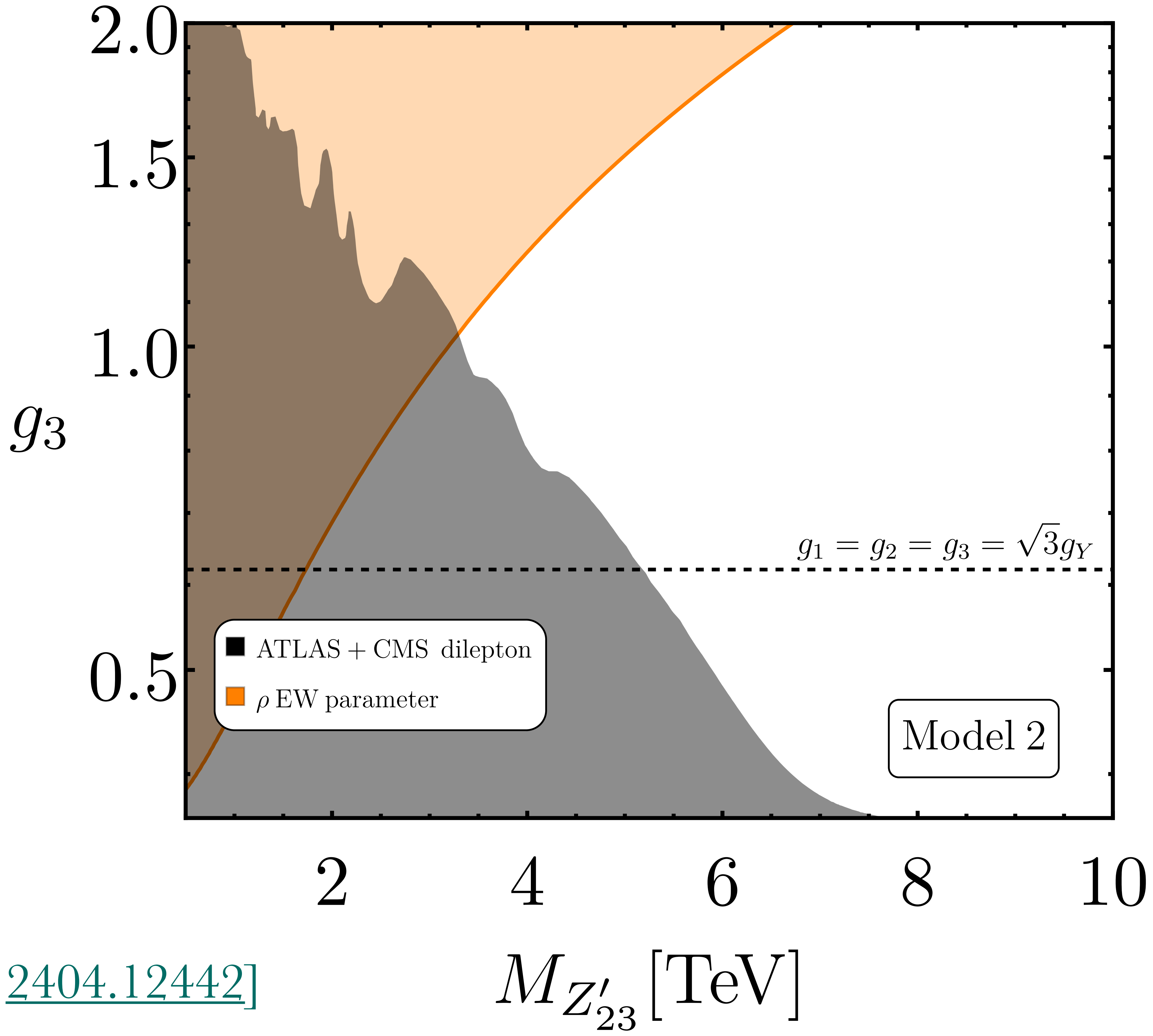
see also Davighi and Stefanek [2305.16280]

Pheno: Z'_{23}



TeV masses allowed!

[MFN, King, Vicente, 2404.12442]



- In Model 2, quark mixing originates from up-sector only ⟶ Top FCNCs only but poorly bounded (not shown)
- Non-minimal breaking of $U(2)^5$ ⟶ Potential for flavour precision program (including CLFV)

Pheno: Z'_{12} and VL fermions

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$
$$\xrightarrow{v_{12}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3} + Z'_{12}$$
$$\xrightarrow{v_{23}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3} + Z'_{23} + Z'_{12}$$

- Expect heavier Z'_{12}, e.g. :

$$M_{Z'_{23}} \sim \mathcal{O}(10\,\mathrm{TeV}) \longrightarrow M_{Z'_{12}} \sim \mathcal{O}(10^3\,\mathrm{TeV})$$

Since model predicts: $\langle \phi_{23} \rangle / \langle \phi_{12} \rangle \sim \lambda^3 \sim 0.01$

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$$\sin\theta_{12} = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$$

$$\mathcal{L}_{Z'_{12}} = Y_{f_{L,R}} \bar{f}_{L,R} \gamma^\mu \begin{pmatrix} -g_1 \sin\theta_{12} & 0 & 0 \\ 0 & g_2 \cos\theta_{12} & 0 \\ 0 & 0 & 0 \end{pmatrix} f_{L,R} Z'_{12\mu}$$

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- Coupled non-universally to first and second families

$\longrightarrow$ Despite expected to be very heavy, Z'_{12} can be tested via FCNCs!

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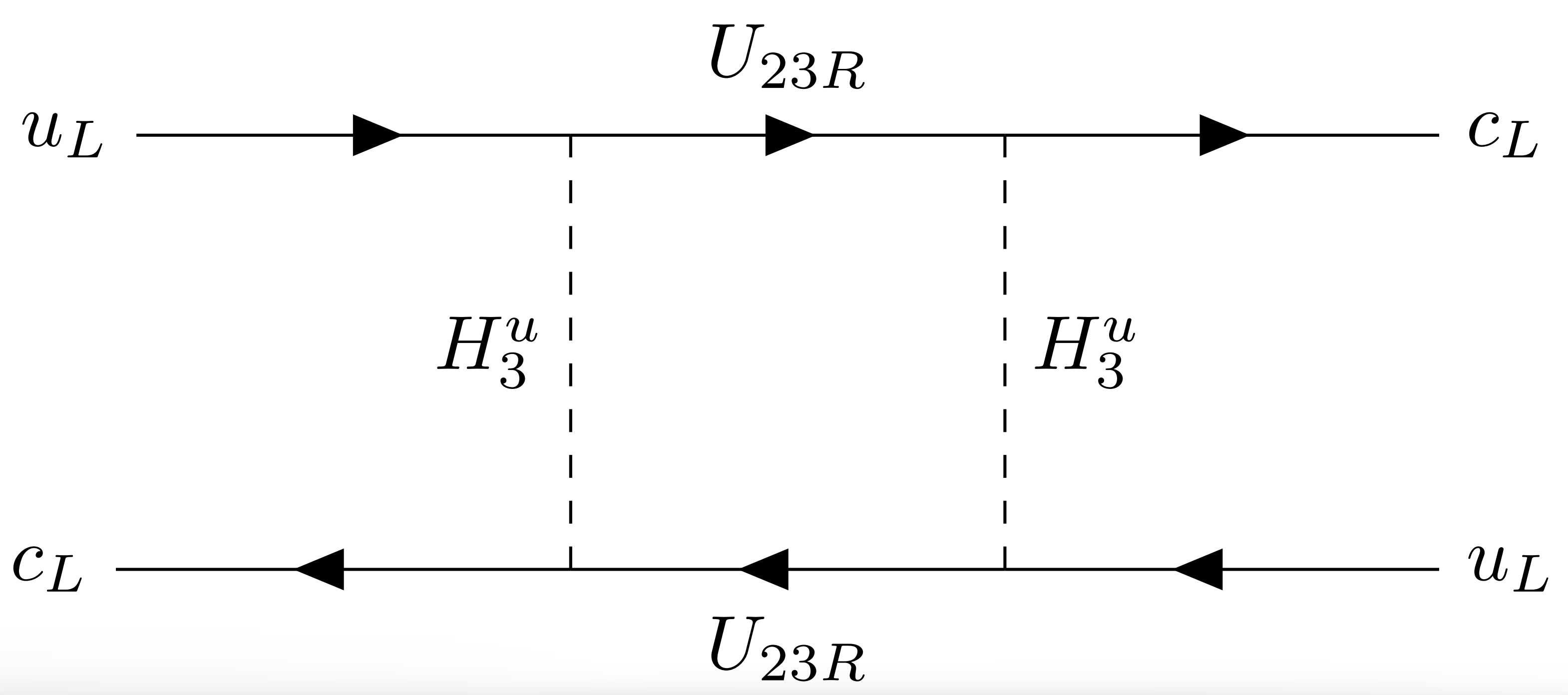
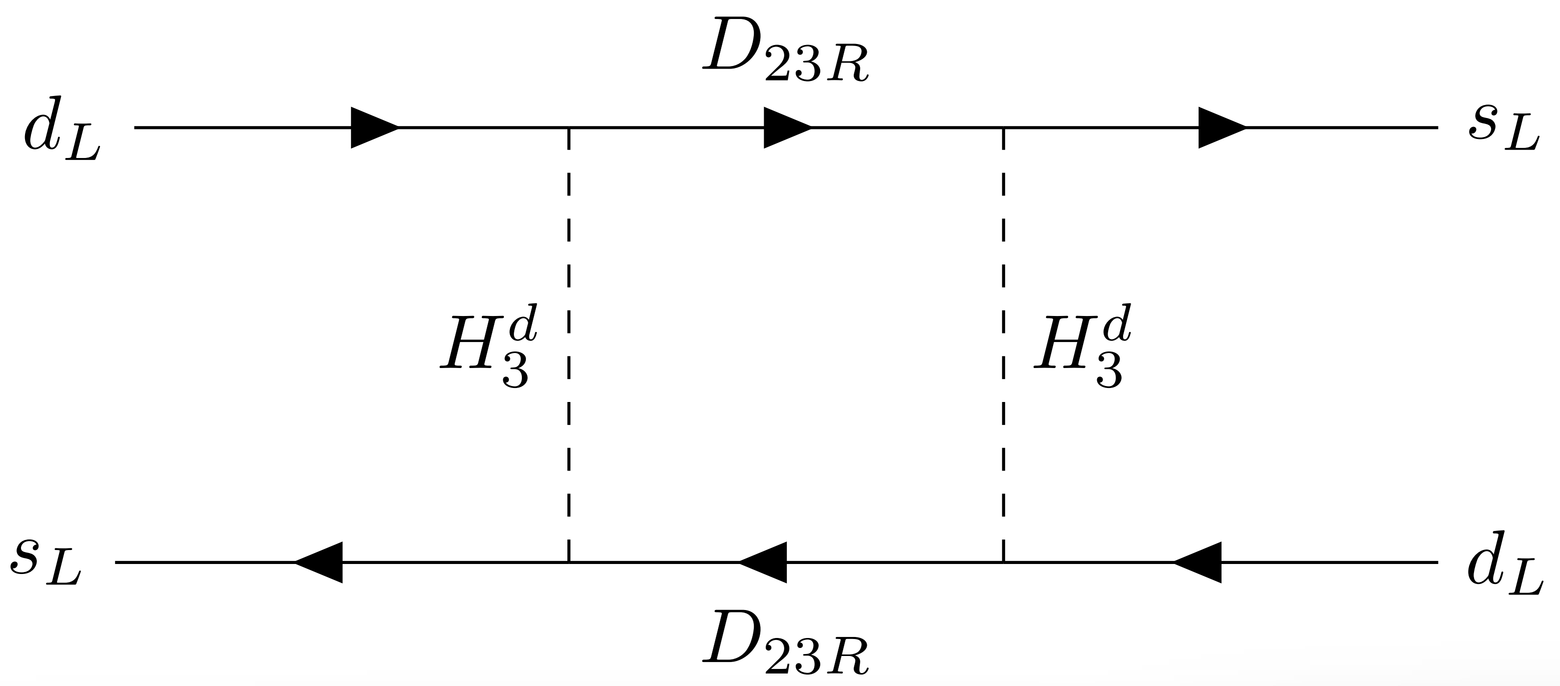
→ Despite expected to be very heavy, Z'_{12} can be tested via FCNCs!

- VL fermions can also mediate significant FCNCs:

$D - \overline{D}$





$K - \overline{K}$

Pheno: Z'_{12} and VL fermions

Model	Observable	Mediator	Bound (TeV)
1	$K-\bar{K}$ (Re)	Z'_{12}	$M_{Z'_{12}}/g_1 > 340 \times \lvert\mathrm{Re}\left[\frac{c^d_{12}}{c^d_{22}}\frac{c^d_{21}}{c^d_{22}}\right]\rvert$
	$K-\bar{K}$ (Im)	Z'_{12}	$M_{Z'_{12}}/g_1 > 3\cdot 10^3 \times \lvert\mathrm{Im}\left[\frac{c^d_{12}}{c^d_{22}}\frac{c^d_{21}}{c^d_{22}}\right]\rvert$
	$\mu \to e\gamma$	Z'_{12}	$M_{Z'_{12}}/g_1 > 30 \times \lvert c^e_{12}/c^e_{22}\rvert$
		Z'_{23}	$M_{Z'_{23}}/g_3 > 8 \times \lvert y^e_{62}(y^e_{65}y^e_{15})^*\rvert$
	$\mu \to 3\,e$	Z'_{12}	$M_{Z'_{12}}/g_1 > 30 \times \lvert c^e_{12}/c^e_{22}\rvert$
2	$D-\bar{D}$ (Re)	Z'_{12}	$M_{Z'_{12}}/g_1 > 150 \times \lvert\mathrm{Re}\left[\frac{c^u_{12}}{c^u_{22}}\frac{c^u_{21}}{c^u_{22}}\right]\rvert$
	$D-\bar{D}$ (Im)	Z'_{12}	$M_{Z'_{12}}/g_1 > 500 \times \lvert\mathrm{Im}\left[\frac{c^u_{12}}{c^u_{22}}\frac{c^u_{21}}{c^u_{22}}\right]\rvert$

- FCNCs via Z'_{12} are sensitive to fermion mixing predicted by each model → can discriminate among models
- FCNCs involving 1st and 2nd family quarks most competitive, e.g. $K-\bar{K}$ and $D-\bar{D}$ mixing, also CLFV

→ Potential for flavour precision program (including CLFV)

- Very sensitive to RH quark mixing and complex phases $\quad Q_5^{sd} = (\bar{s}^\alpha_L d^\beta_R)(\bar{s}^\beta_R d^\alpha_L)$

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1	$K-\bar{K}$ (Re)	Z'_{12}	$M_{Z'_{12}}/g_1 > 340 \times \left\|\mathrm{Re}\left[\frac{c^d_{12}}{c^d_{22}}\frac{c^d_{21}}{c^d_{22}}\right]\right\|$
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Order	Observable	Bound (TeV)
Tree-level (Z-mediated)	$D-\bar{D}$ (Re)	$M_{U_{23}} > 2.5 \times \|\mathrm{Re}\,[y^u_{26}(y^u_{65}y^u_{15})^*]\|$
	$D-\bar{D}$ (Im)	$M_{U_{23}} > 4 \times \|\mathrm{Im}\,[y^u_{26}(y^u_{65}y^u_{15})^*]\|$
	$K-\bar{K}$ (Re)	$M_{D_{23}} > 0.1 \times \|\mathrm{Re}\,[y^d_{26}(y^d_{65}y^d_{15})^*]\|$
	$K-\bar{K}$ (Im)	$M_{D_{23}} > 0.3 \times \|\mathrm{Im}\,[y^d_{26}(y^d_{65}y^d_{15})^*]\|$
	$\mu\to 3e$	$M_{E_{23}} > 3 \times \|y^e_{26}(y^e_{65}y^e_{15})^*\|$
1-loop (box with $H_3^{u,d}$)	$D-\bar{D}$ (Re)	$M_{U_{23}} > 15 \times \|\mathrm{Re}\,[y^u_{26}(y^u_{65}y^u_{15})^*]\|$
	$D-\bar{D}$ (Im)	$M_{U_{23}} > 130 \times \|\mathrm{Im}\,[y^u_{26}(y^u_{65}y^u_{15})^*]\|$
	$K-\bar{K}$ (Re)	$M_{D_{23}} > 14 \times \|\mathrm{Re}\,[y^d_{26}(y^d_{65}y^d_{15})^*]\|$
	$K-\bar{K}$ (Im)	$M_{D_{23}} > 170 \times \|\mathrm{Im}\,[y^d_{26}(y^d_{65}y^d_{15})^*]\|$

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- FCNCs involving 1st and 2nd family quarks most competitive, e.g. $K-\bar{K}$ and $D-\bar{D}$ mixing, also CLFV

  ⟶ Potential for flavour precision program (including CLFV)

- Very sensitive to RH quark mixing and complex phases $\quad Q_5^{sd} = (\bar{s}^\alpha_L d^\beta_R)(\bar{s}^\beta_R d^\alpha_L)$
- FCNCs also sensitive to VL fermion masses directly! Instead of ratios of the form $\langle\phi\rangle/M$…

Neutrino sector in flavour deconstruction

- Proceeding as in seesaw type I, add (at least 2) RH neutrino singlets:

$$\mathcal{L} = c_{3i}\ell_3 H_3^u \nu_i^c + c_{2i}\overbrace{\frac{\phi_{\ell 23}}{\Lambda}}^{\epsilon}\ell_2 H_3^u \nu_i^c + \ldots + M_{ij}\nu_i^c \nu_j^c \quad m_D = \begin{pmatrix} & \nu_1^c & \nu_2^c \\ \hline \ell_1 & \epsilon^2 & \epsilon^2 \\ \ell_2 & \epsilon & \epsilon \\ \ell_3 & 1 & 1 \end{pmatrix} H_3^u \quad M_R = \begin{pmatrix} & \nu_1^c & \nu_2^c \\ \hline \nu_1^c & M_{11} & M_{12} \\ \nu_2^c & M_{12} & M_{22} \end{pmatrix}$$

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- Problematic to explain large (anarchic?) neutrino mixing. Two ways forward:

1. $\epsilon \sim \mathcal{O}(1)$ ⟶ Go beyond EFT ⟶ Work in complete model [MFN, King, 23'; MFN, King, Vicente, 24'] and more…

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2. Introduce hierarchies in M_R to cancel those in m_D. [Greljo and Isidori, 2406.01696]

$$m_D = \begin{pmatrix} \epsilon^2 & \lesssim \epsilon & \lesssim 1 \\ \lesssim \epsilon^2 & \epsilon & \lesssim 1 \\ \lesssim \epsilon^2 & \lesssim \epsilon & 1 \end{pmatrix} \qquad M_R = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}$$

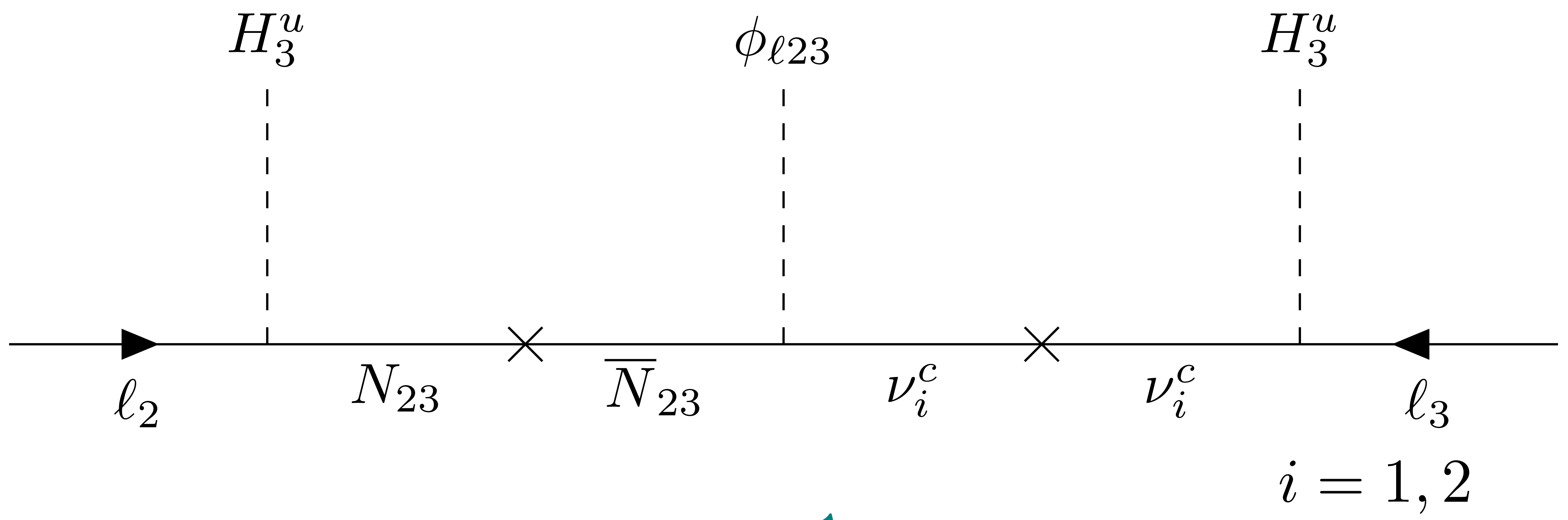
- $SU(3)^2 \times SU(2)_L \times U(1)_R^3 \times U(1)_{B-L}$
- $SU(3) \times SU(2)_L^3 \times U(1)_R \times U(1)_{B-L}^3$
- $SU(3) \times SU(2)_L^3 \times U(1)_R^3 \times U(1)_{B-L}^3$
- Need to impose conditions over various ϵ's, but can be achieved in complete models
- All scales very high or consider inverse seesaw

$$\epsilon_L = \epsilon_R = \epsilon_\ell \equiv \epsilon$$

[see also Greljo and Stefanek 1802.04274, Fuentes-Martín *et al* 2012.10492]

Neutrino sector in tri-hypercharge

- Generate $\epsilon \sim \mathcal{O}(1)$ in complete model: e.g. add messenger neutrinos, similar in spirit to VL fermions of charged sectors:



	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$	$SU(3)_c \times SU(2)_L$
ν_1^c	0	0	0	$(\mathbf{1}, \mathbf{1})$
ν_2^c	0	0	0	$(\mathbf{1}, \mathbf{1})$
N_{12}	1/2	-1/2	0	$(\mathbf{1}, \mathbf{1})$
N_{13}	1/2	0	-1/2	$(\mathbf{1}, \mathbf{1})$
N_{23}	0	1/2	-1/2	$(\mathbf{1}, \mathbf{1})$

$$\mathcal{L} = c_{3i} \ell_3 H_3^u \nu_i^c + c_{2i} \underbrace{\frac{\phi_{\ell 23}}{M_{N_{23}}}}_{\approx 1} \ell_2 H_3^u \nu_i^c + \ldots + M_{ij} \nu_i^c \nu_j^c + \text{h.c.}$$

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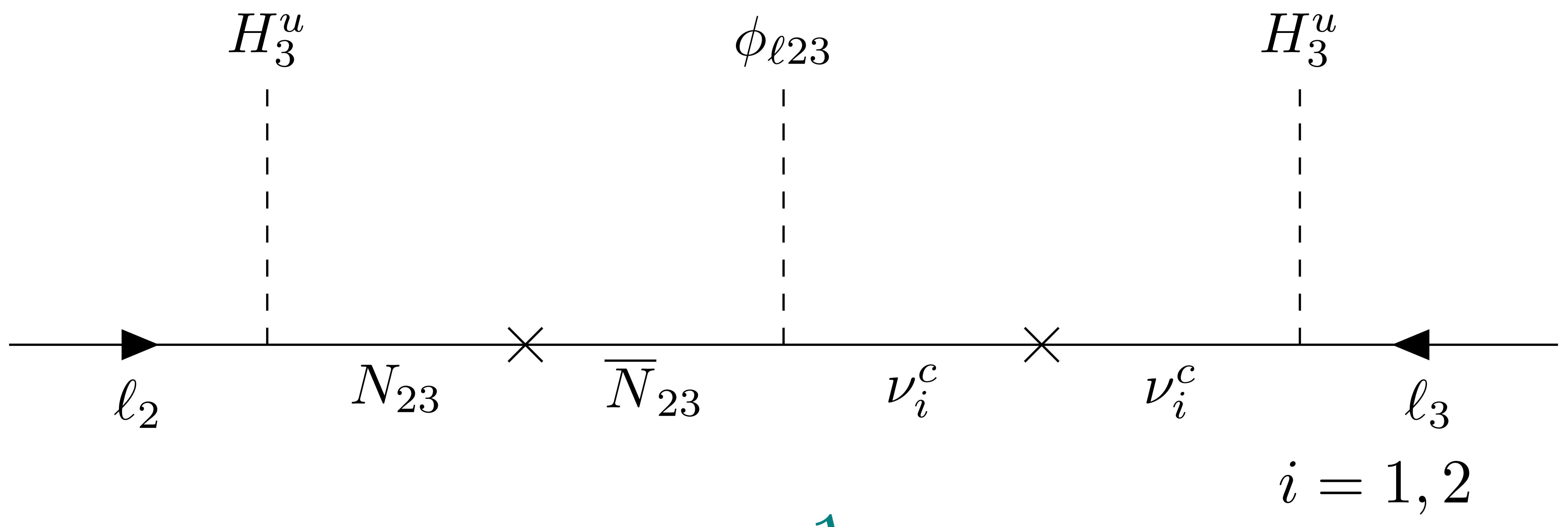

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$$m_D = \left(\begin{array}{c|cc} & \nu_1^c & \nu_2^c \\ \hline \ell_1 & c_{11} & c_{12} \\ \ell_2 & c_{21} & c_{22} \\ \ell_3 & c_{31} & c_{32} \end{array} \right) H_3^u \,, \quad M_R = \left(\begin{array}{c|cc} & \nu_1^c & \nu_2^c \\ \hline \nu_1^c & M_{11} & M_{12} \\ \nu_2^c & M_{21} & M_{22} \end{array} \right)$$

$$m_\nu \simeq m_D M_R^{-1} m_D^{\mathrm{T}}$$

Seesaw mechanism!

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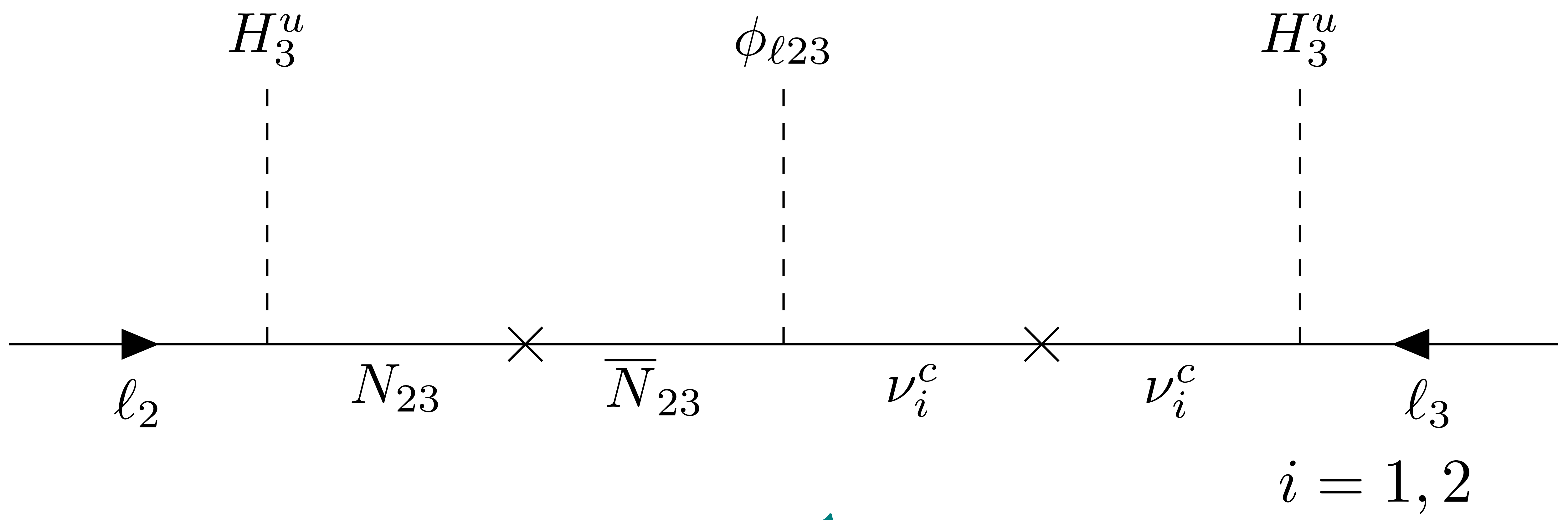

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Seesaw mechanism!

✓ $M \approx 10^{15}$ GeV

✓ No need of small couplings nor v_{12}, v_{23} being very heavy

✓ No need of adding extra scalars

✓ $M_{N_{23}} \approx v_{23} \gtrsim \mathcal{O}(10\,\text{TeV})$, potential pheno

Outline

1. Introduction: Flavour in the SM (and beyond)
2. Flavour deconstruction: generics
3. Flavour deconstruction: from the electroweak scale
4. Flavour deconstruction: to the GUT scale

Going up

- Simple models/Bottom-up:

$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$ [This talk]

$SU(3)_c \times SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \times U(1)_Y$ [Li and Ma, PRL 81'; Muller and Nandi, hep-ph/9602390; Davighi *et al* 2312.13346; Capdevila *et al*, 2401.00848 …..]

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  - Naturalness: Identify parameter space of 23-layers where corrections to Higgs mass are minimal:

    [Allwicher *et al* 2011.01946, Davighi and Stefanek 2305.16280, Davighi *et al* 2312.13346]
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    [Allwicher *et al* 2011.01946, Davighi and Stefanek 2305.16280, Davighi *et al* 2312.13346]
  - Naturalness: SUSY/Composite Higgs [Craig *et al* 1103.3708, Fuentes-Martín and Stangl 2004.11376, Fuentes-Martín *et al*, 2203.01952, Davighi *et al* 2407.10950]

- Electroweak-flavour unification: $Sp(6) \to SU(2)_{L1} \times SU(2)_{L2} \times SU(2)_{L3} \to SU(2)_L$

  [Davighi and Tooby-Smith 2206.04482, Davighi *et al* 2212.06163]

Reunification at high energies!

"Deconstructed" GUT? Tri-unification

- Gauge sector of flavour deconstructed models may contain up to 9 gauge couplings:

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

[This talk]

$$SU(3)_c \times SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \times U(1)_Y$$

[Li and Ma, PRL 81'; Muller and Nandi, hep-ph/9602390 … Chiang *et al*, 0911.1480; Allwicher *et al*, 2011.01946; Davighi *et al* 2312.13346; Capdevila *et al*, 2401.00848 …]

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- "Deconstructed" theories seem to preserve an approximate $\mathbb{Z}_3$ (cyclic permutation symmetry) relating the three sites (i.e. approx. same matter content under the three sites):
  - E.g. $\{\phi_{\ell 12}^{(\frac{1}{2},-\frac{1}{2},0)}, \phi_{\ell 13}^{(\frac{1}{2},0,-\frac{1}{2})}, \phi_{\ell 23}^{(0,\frac{1}{2},-\frac{1}{2})}\}$, $\{H_1^{(\frac{1}{2},0,0)}, H_2^{(0,\frac{1}{2},0)}, H_3^{(0,0,\frac{1}{2})}\}$, $\{D_{12}^{(-\frac{1}{6},\frac{1}{2},0)}, D_{13}^{(-\frac{1}{6},0,\frac{1}{2})}, D_{23}^{(0,-\frac{1}{6},\frac{1}{2})}\}$

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  - If $\mathbb{Z}_3$ is exact at very high energies, then:

$$\boxed{SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times \mathbb{Z}_3}$$

[Salam 79', Rajpoot 81', Georgi 82', de Rújula, Georgi, Glashow 84', $SU(3)_c \times SU(3)_L \times SU(3)_R \times \mathbb{Z}_3$ …]

with $\mathbb{Z}_3$ permuting the three $SU(5)$, contains a single gauge coupling in the UV.

✓ Tri-unification may be the origin of low energy flavour deconstructed models.

$SU(5)^3$: an explicit example

- Note that the model must have the same matter under each $SU(5)$ (to preserve $\mathbb{Z}_3$).

Field	$\boldsymbol{SU(5)_1}$	$\boldsymbol{SU(5)_2}$	$\boldsymbol{SU(5)_3}$
F_1	$\mathbf{\bar{5}}$	$\mathbf{1}$	$\mathbf{1}$
F_2	$\mathbf{1}$	$\mathbf{\bar{5}}$	$\mathbf{1}$
F_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{\bar{5}}$
T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
T_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
T_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
Ω	$\mathbf{24}$	$\mathbf{24}$	$\mathbf{24}$
H_1	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
H_2	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$
H_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$

$$SU(5)^3 \to \mathrm{SM}_1 \times \mathrm{SM}_2 \times \mathrm{SM}_3$$

$$SU(5)^3 \to SU(3)_c \times SU(2)_L \times U(1)_Y^3$$

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$$F_i \to d_i^c \oplus \ell_i \qquad T_i \to q_i \oplus u_i^c \oplus e_i^c$$

$SU(5)^3$: an explicit example

- Note that the model must have the same matter under each $SU(5)$ (to preserve $\mathbb{Z}_3$).
- General embedding of bottom-up deconstruction: broken via large adjoint or bi-adjoints

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\bar{5}$	1	1
F_2	1	$\bar{5}$	1
F_3	1	1	$\bar{5}$
T_1	10	1	1
T_2	1	10	1
T_3	1	1	10
Ω	24	24	24
H_1	5	1	1
H_2	1	5	1
H_3	1	1	5

$F_i \to d_i^c \oplus \ell_i \qquad T_i \to q_i \oplus u_i^c \oplus e_i^c$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
Ω_1	24	1	1
Ω_2	1	24	1
Ω_3	1	1	24

$$SU(5)^3 \to \mathrm{SM}_1 \times \mathrm{SM}_2 \times \mathrm{SM}_3$$

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Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
Ω_{12}	24	24	1
Ω_{13}	24	1	24
Ω_{23}	1	24	24

$SU(5)^3$: an explicit example

- For tri-hypercharge, need of deconstructing $SU(2)$ and $SU(3)$ as well for embedding in $SU(5)^3$ This suggests a possible SM^3 intermediate scale, i.e. two options:

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
F_2	$\mathbf{1}$	$\overline{\mathbf{5}}$	$\mathbf{1}$
F_3	$\mathbf{1}$	$\mathbf{1}$	$\overline{\mathbf{5}}$
T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
T_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
T_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
Ω_{12}	$\mathbf{24}$	$\mathbf{24}$	$\mathbf{1}$
Ω_{13}	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{24}$
Ω_{23}	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{24}$
H_1	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
H_2	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$
H_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$

$F_i \to d_i^c \oplus \ell_i \qquad T_i \to q_i \oplus u_i^c \oplus e_i^c$

Minimal breaking chain

$$
\begin{aligned}
&SU(5)^3 \\
&\xrightarrow{v_{24}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_1 \times U(1)_2 \times U(1)_3 \\
&\xrightarrow{v_{12}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2} \times U(1)_3 \\
&\xrightarrow{v_{23}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2+3}\,.
\end{aligned}
$$

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Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
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T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
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Ω_{12}	$\mathbf{24}$	$\mathbf{24}$	$\mathbf{1}$
Ω_{13}	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{24}$
Ω_{23}	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{24}$
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Minimal breaking chain

$$SU(5)^3 \xrightarrow{v_{\mathbf{24}}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_1 \times U(1)_2 \times U(1)_3 \xrightarrow{v_{12}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2} \times U(1)_3 \xrightarrow{v_{23}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2+3}\,.$$

Intermediate scale

$$SU(5)^3 \xrightarrow{v_{\mathbf{24}}} \mathrm{SM}_1 \times \mathrm{SM}_2 \times \mathrm{SM}_3 \xrightarrow{v_{\mathrm{SM}3}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_1 \times U(1)_2 \times U(1)_3 \xrightarrow{v_{12}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2} \times U(1)_3 \xrightarrow{v_{23}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2+3}\,.$$

Tri-hypercharge layer

	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$	$SU(3)_c \times SU(2)_L$
$H_3^{u,d}$	0	0	$\pm 1/2$	$(\mathbf{1},\mathbf{2})$
$H_2^{u,d}$	0	$\pm 1/2$	0	$(\mathbf{1},\mathbf{2})$
$H_1^{u,d}$	$\pm 1/2$	0	0	$(\mathbf{1},\mathbf{2})$
$\phi_{q_{12}}$	-1/6	1/6	0	$(\mathbf{1},\mathbf{1})$
$\phi_{q_{13}}$	-1/6	0	1/6	$(\mathbf{1},\mathbf{1})$
$\phi_{q_{23}}$	0	-1/6	1/6	$(\mathbf{1},\mathbf{1})$
$\phi_{\ell 12}$	-1/2	1/2	0	$(\mathbf{1},\mathbf{1})$
$\phi_{\ell 13}$	-1/2	0	1/2	$(\mathbf{1},\mathbf{1})$
$\phi_{\ell 23}$	0	-1/2	1/2	$(\mathbf{1},\mathbf{1})$
Q_1	1/6	0	0	$(\mathbf{3},\mathbf{2})$
Q_2	0	1/6	0	$(\mathbf{3},\mathbf{2})$
Q_3	0	0	1/6	$(\mathbf{3},\mathbf{2})$

$$Y_d = \begin{pmatrix} c_{11}^d \frac{\phi_{\ell 13}}{M_{H_1^d}} & c_{12}^d \frac{\phi_{q12}}{M_{Q_2}} \frac{\phi_{\ell 23}}{M_{H_2^d}} & c_{13}^d \frac{\phi_{q12}}{M_{Q_2}} \frac{\phi_{q23}}{M_{Q_3}} \\ c_{21}^d \frac{\phi_{\ell 13}}{M_{H_1^d}} \frac{\phi_{q12}}{M_{Q_1}} & c_{22}^d \frac{\phi_{\ell 23}}{M_{H_2^d}} & c_{23}^d \frac{\phi_{q23}}{M_{Q_3}} \\ c_{31}^d \frac{\phi_{\ell 13}}{M_{H_1^d}} \frac{\phi_{q12}}{M_{Q_2}} \frac{\phi_{q23}}{M_{Q_3}} & c_{32}^d \frac{\phi_{\ell 23}}{M_{H_2^d}} \frac{\phi_{q23}}{M_{Q_2}} & y_b \end{pmatrix}$$

$$Y_u = Y_d(d \to u)$$

$$Y_e = \begin{pmatrix} c_{11}^e \frac{\phi_{\ell 13}}{M_{H_1^d}} & 0 & 0 \\ 0 & c_{22}^e \frac{\phi_{\ell 23}}{M_{H_2^d}} & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

- $SU(2)_L$-doublets VL quarks and heavy Higgs generate flavour structure AND help with $SU(2)_L$ RGE for gauge coupling unification
- Same matter under each hypercharge! ⟶ $\mathbb{Z}_3$ ⟶ Less minimal, more symmetric
- Minimal breaking of $U(2)^5$ at 23-scales ⟶ Safe for pheno

Example of gauge coupling unification

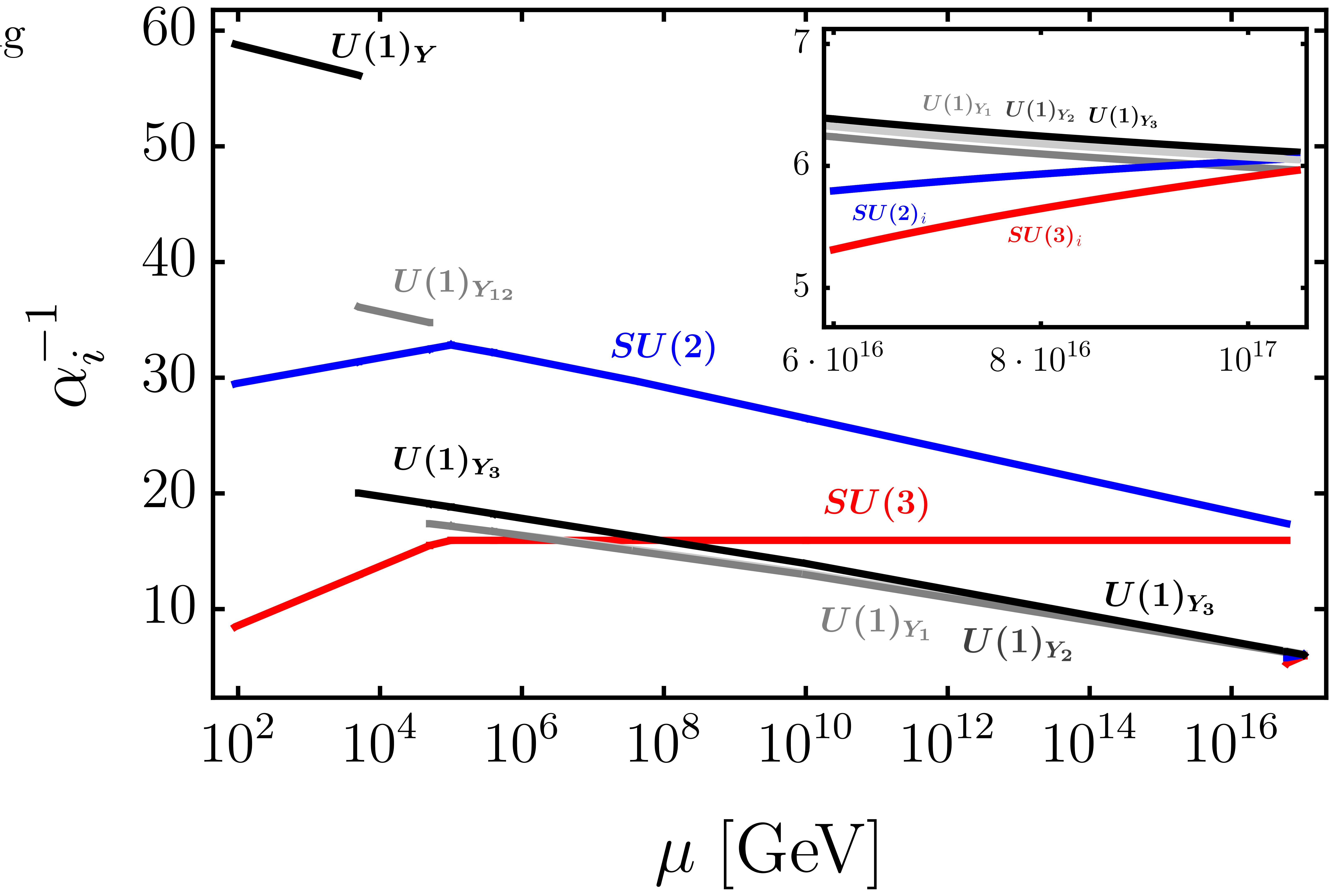
- Discontinuities due to gauge coupling matching conditions:

$$\alpha_{Y_{12}}^{-1} + \alpha_{Y_3}^{-1} = \alpha_Y^{-1}(v_{23})$$

$$\alpha_{Y_1}^{-1} + \alpha_{Y_2}^{-1} = \alpha_{Y_{12}}^{-1}(v_{12})$$

$$\alpha_i = \frac{g_i^2}{4\pi}$$

$$\alpha_{s,L,1}^{-1} + \alpha_{s,L,2}^{-1} + \alpha_{s,L,3}^{-1} = \alpha_{s,L}^{-1}(v_{\mathrm{SM}^3})$$



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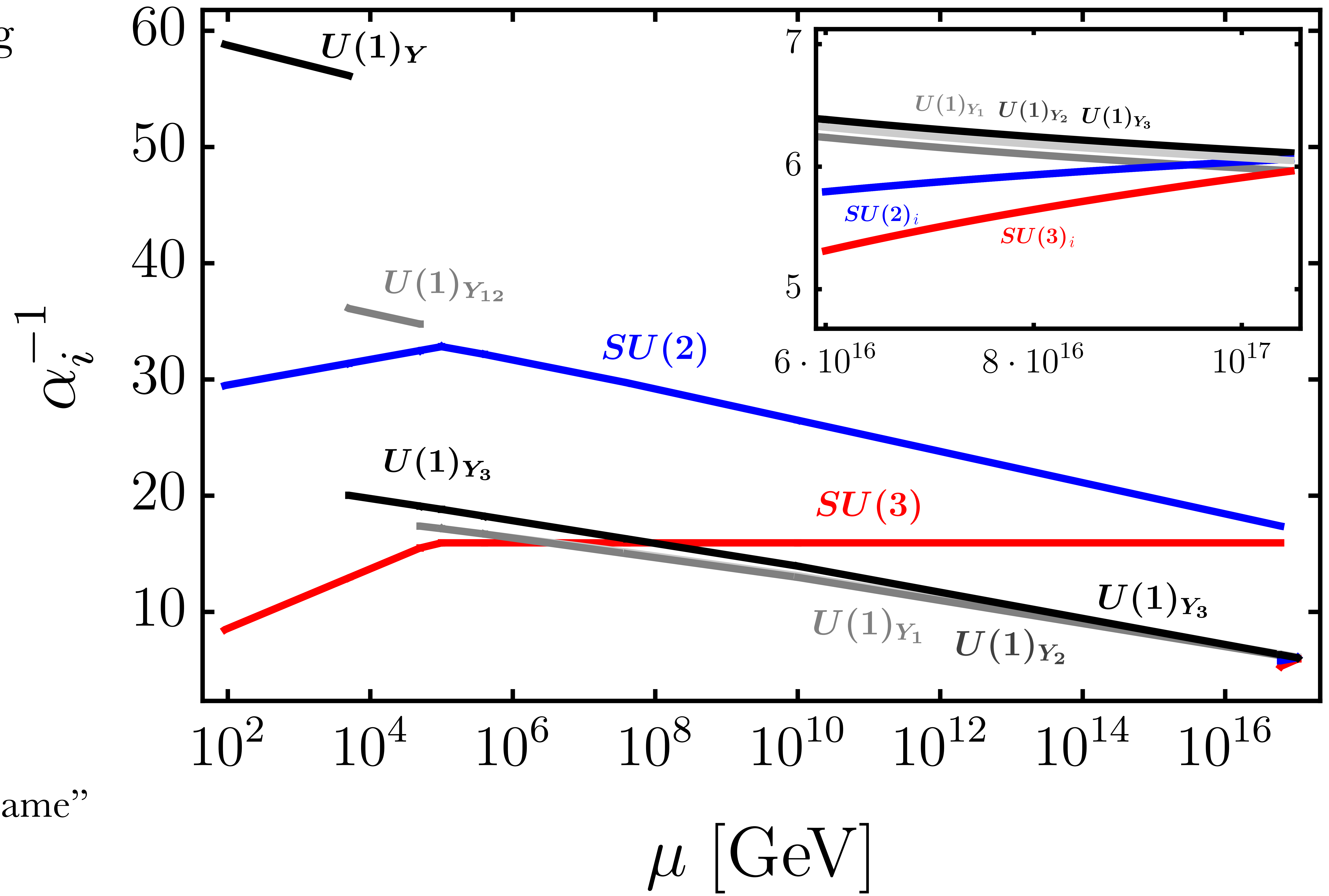
- VL quarks Q_i help bend SU(2).
- Colour octet $\Theta_i \sim (\mathbf{8},\mathbf{1},0)_i$ from cyclic **24** at v_{12} scale to bend SU(3) (non-SUSY).
- Gauge couplings approximately "run the same" thanks to approximate $\mathbb{Z}_3$ at low energies, which becomes exact at high energies.



Example of gauge coupling unification

- Discontinuities due to gauge coupling matching conditions:

$$\alpha^{-1}_{Y_{12}} + \alpha^{-1}_{Y_3} = \alpha^{-1}_{Y}(v_{23})$$

$$\alpha^{-1}_{Y_1} + \alpha^{-1}_{Y_2} = \alpha^{-1}_{Y_{12}}(v_{12})$$

$$\alpha_i = \frac{g_i^2}{4\pi}$$

$$\alpha^{-1}_{s,L,1} + \alpha^{-1}_{s,L,2} + \alpha^{-1}_{s,L,3} = \alpha^{-1}_{s,L}(v_{\mathrm{SM}^3})$$

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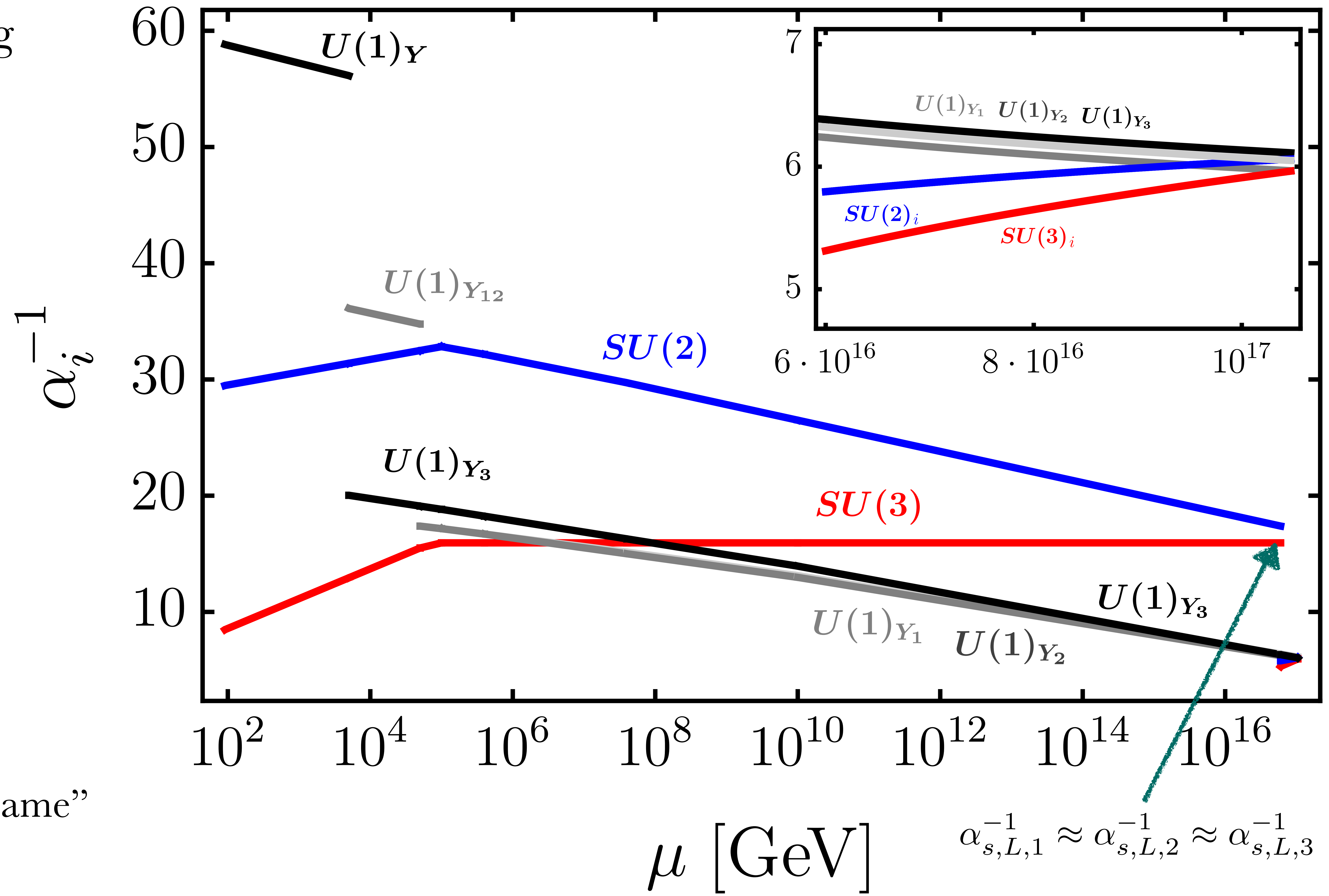
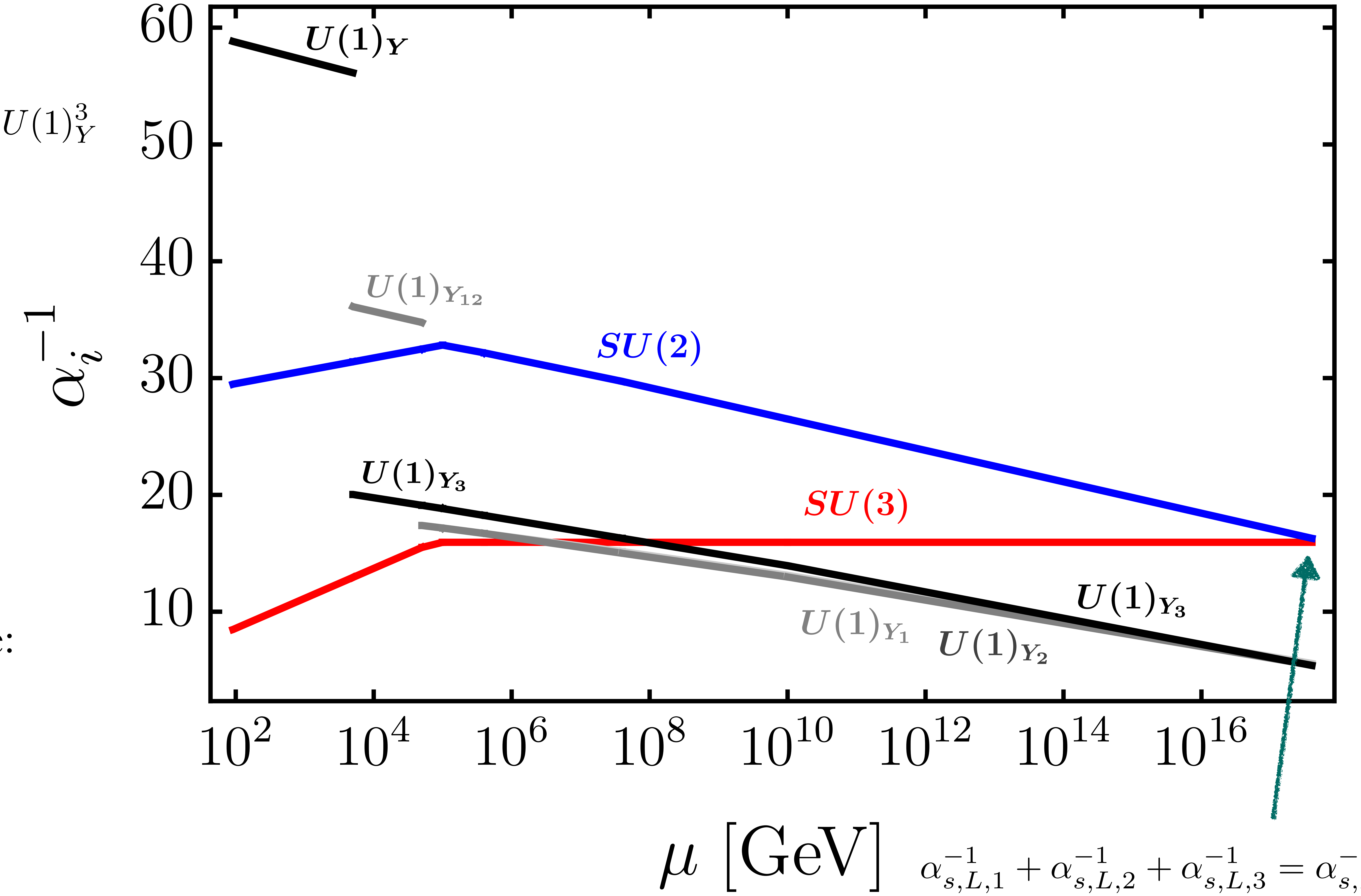


$v_{\mathrm{SM}^3} = 6 \times 10^{16}$ GeV → $M_{\mathrm{GUT}} = 10^{17}$ GeV

Gauge coupling unification: Example 2

No intermediate SM^3 scale:

$SU(5)^3 \xrightarrow{M_{\rm GUT}} SU(3)_c \times SU(2)_L \times U(1)_Y^3$



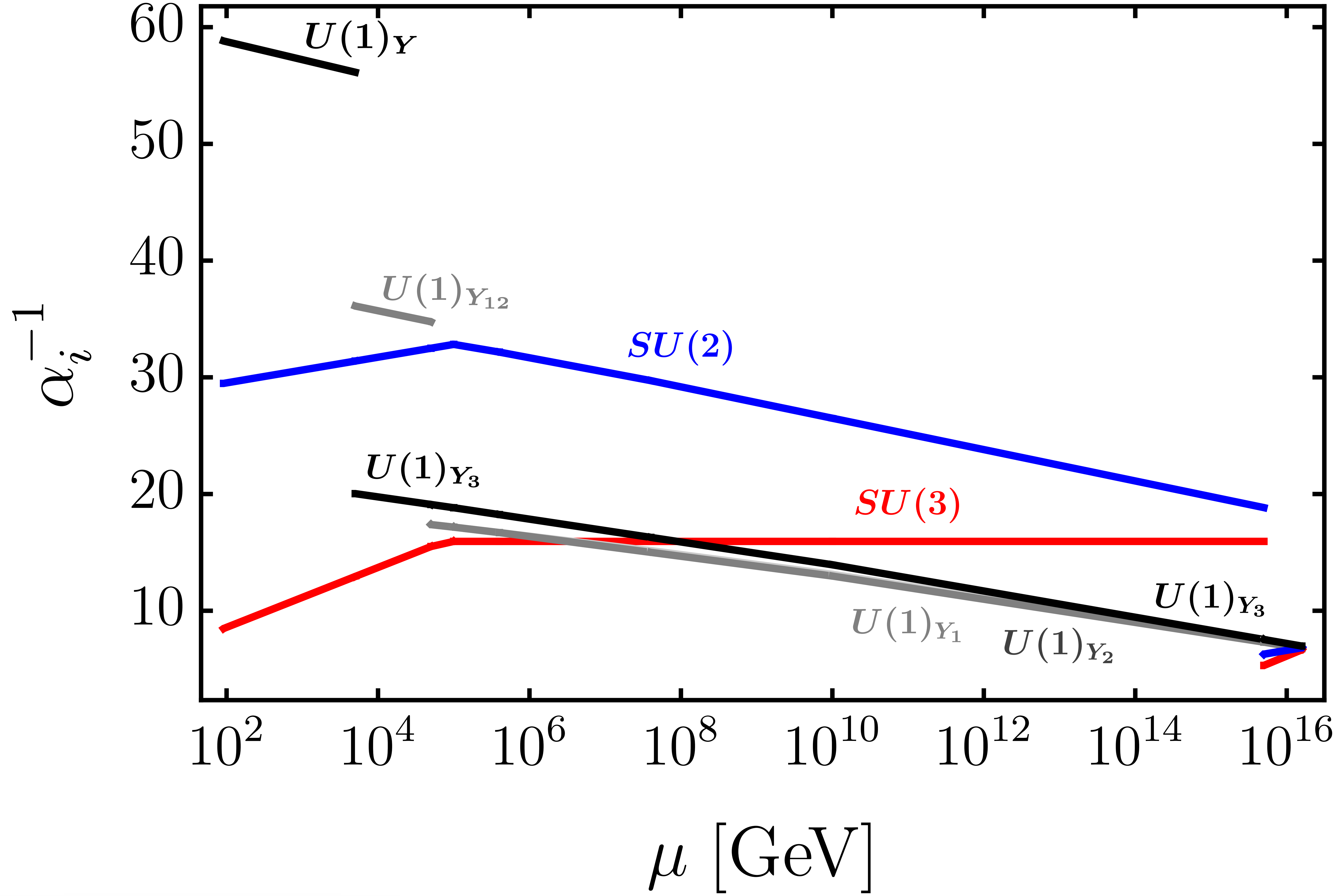
$\alpha_{s,L,1}^{-1} + \alpha_{s,L,2}^{-1} + \alpha_{s,L,3}^{-1} = \alpha_{s,L}^{-1}(M_{\rm GUT})$

GUT scale in minimal example:

$M_{\rm GUT} \simeq 5 \times 10^{17}$ GeV

Gauge coupling unification: Example 3

How low can can we deconstruct $SU(3)_c$ and $SU(2)_L$?



Intermediate scale

$$SU(5)^3 \xrightarrow{M_{\rm GUT}} \mathrm{SM}_1 \times \mathrm{SM}_2 \times \mathrm{SM}_3$$
$$\xrightarrow{v_{\rm SM^3}} SU(3)_c \times SU(2)_L \times U(1)^3_{\rm Y}$$

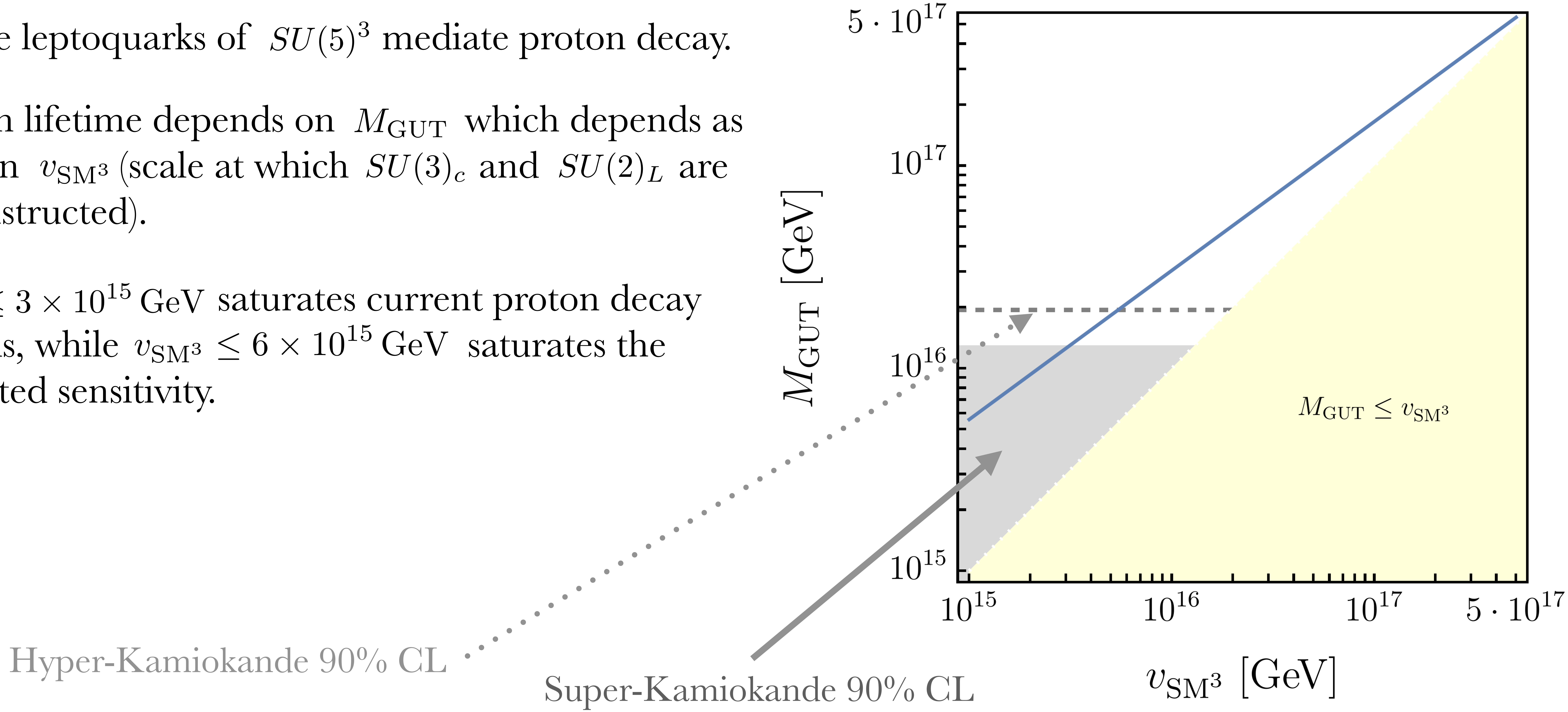
$$v_{\rm SM^3} = 5 \times 10^{15}\,\mathrm{GeV}$$

→ $M_{\rm GUT} \simeq 1.8 \times 10^{16}\,\mathrm{GeV}$

proton decay!

Proton decay

- Gauge leptoquarks of $SU(5)^3$ mediate proton decay.
- Proton lifetime depends on $M_{\rm GUT}$ which depends as well on $v_{\rm SM^3}$ (scale at which $SU(3)_c$ and $SU(2)_L$ are deconstructed).
- $v_{\rm SM^3} \leq 3 \times 10^{15}\,\rm GeV$ saturates current proton decay bounds, while $v_{\rm SM^3} \leq 6 \times 10^{15}\,\rm GeV$ saturates the projected sensitivity.



Take home messages

- Why three families? This now becomes a question about gauge symmetry. 😑

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Take home messages

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- SM flavour parameters calculable (at least in terms of $\mathcal{O}(1)$ parameters)?
Yes but models contain many of them (and NP scales). 😑 May arise from GUT frameworks. 😃

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- Testable? Translates SM flavour structure into simple (perhaps correlated) NP scales that carry meaningful pheno. The lowest scale may be close to TeV. 😃

➡ Collider (LHC, FCC) physics, EWPOs, flavour physics and perhaps more. Couplings are large, but no clear reason for the scales to be low (perhaps hierarchy problem, anomalies?)

Take home messages

Thank you!

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Backup: CKM alignment

$$V_{u_L} y^u V_{u_R}^\dagger = \mathrm{diag}(y_u, y_c, y_t) \qquad V_{d_L} y^d V_{d_R}^\dagger = \mathrm{diag}(y_d, y_s, y_b) \qquad V_{e_L} y^e V_{e_R}^\dagger = \mathrm{diag}(y_e, y_\mu, y_\tau)$$

- Notice that all mixing matrices may become physical in extensions of the SM, including EFTs and SMEFT scenarios, consider the example:

$$\mathcal{L} \supset C_u (\overline{u}_{L3} \gamma_\mu u_{L3})(\overline{u}_{L3} \gamma^\mu u_{L3})$$

$$u_{L3} = (V_{u_L}^\dagger)_{13} \hat{u}_{L1} + (V_{u_L}^\dagger)_{23} \hat{u}_{L2} + (V_{u_L}^\dagger)_{33} \hat{u}_{L3}$$

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$$u_{L3} = (V_{u_L}^\dagger)_{13} \hat{u}_{L1} + (V_{u_L}^\dagger)_{23} \hat{u}_{L2} + (V_{u_L}^\dagger)_{33} \hat{u}_{L3}$$ → Off-diagonal operators

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$$V_{u_L} y^u V_{u_R}^\dagger = \mathrm{diag}(y_u, y_c, y_t) \qquad V_{d_L} y^d V_{d_R}^\dagger = \mathrm{diag}(y_d, y_s, y_b) \qquad V_{e_L} y^e V_{e_R}^\dagger = \mathrm{diag}(y_e, y_\mu, y_\tau)$$

- Notice that all mixing matrices may become physical in extensions of the SM, including EFTs and SMEFT scenarios, consider the example:

$$\mathcal{L} \supset C_u (\overline{u}_{L3}\gamma_\mu u_{L3})(\overline{u}_{L3}\gamma^\mu u_{L3}) \longrightarrow \text{Off-diagonal operators} \longrightarrow \text{FCNCs!}$$

$$u_{L3} = (V_{u_L}^\dagger)_{13}\hat{u}_{L1} + (V_{u_L}^\dagger)_{23}\hat{u}_{L2} + (V_{u_L}^\dagger)_{33}\hat{u}_{L3}$$

- It is common to make assumptions about the mixing matrices for pheno studies: $V_{\mathrm{CKM}} = V_{u_L}^\dagger V_{d_L}$

$V_{\mathrm{CKM}} \approx V_{d_L}\,,$	$V_{u_L}^\dagger \approx \mathbb{I}$	▸ Up-aligned (CKM originates from down sector)
$V_{\mathrm{CKM}} \approx V_{u_L}^\dagger\,,$	$V_{d_L} \approx \mathbb{I}$	▸ Down-aligned (CKM originates from up sector)

- All mixing matrices may be predicted in theories of flavour.

Backup: $U(2)^5$

- Due to having three replicated families, the SM enjoys an accidental flavour symmetry:

$$U(3)^5 = U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$$

- Minimally broken by SM Yukawa couplings. Since third family is the heaviest, the leading breaking comes from third family Yukawas:

$$U(3)^5 \to U(2)^5 = U(2)_q \times U(2)_u \times U(2)_d \times U(2)_\ell \times U(2)_e$$

[Barbieri *et al* 96', Agashe *et al* 05', Barbieri *et al* 11' …]

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- First and second family Yukawa couplings provide small breaking effects - in the spurion formalism:

$$\mathcal{L}_{\text{Yukawa}} = \begin{pmatrix} \Delta Y_u & x_u V_q \\ 0 & y_t \end{pmatrix} \bar{q}_L \tilde{H} u_R + \begin{pmatrix} \Delta Y_d & x_d V_q \\ 0 & y_b \end{pmatrix} \bar{q}_L H d_R + \begin{pmatrix} \Delta Y_e & x_e V_\ell \\ 0 & y_\tau \end{pmatrix} \bar{\ell}_L H e_R + \text{h.c.}$$

$$\Delta Y_u \sim \mathbf{2}_q \times \mathbf{2}_u\,, \quad \Delta Y_d \sim \mathbf{2}_q \times \mathbf{2}_d\,, \quad \Delta Y_e \sim \mathbf{2}_\ell \times \mathbf{2}_d\,, \quad V_q \sim \mathbf{2}_q\,, \quad V_\ell \sim \mathbf{2}_\ell$$

Minimal (SM-like) breaking of $U(2)^5$

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$$\Delta Y_u \sim \mathbf{2}_q \times \mathbf{2}_u\,, \quad \Delta Y_d \sim \mathbf{2}_q \times \mathbf{2}_d\,, \quad \Delta Y_e \sim \mathbf{2}_\ell \times \mathbf{2}_d\,, \quad V_q \sim \mathbf{2}_q\,, \quad V_\ell \sim \mathbf{2}_\ell$$

Minimal (SM-like) breaking of $U(2)^5$

Different spurions in your BSM theory? → GIM-like protection from 1-2 FCNCs is lost, e.g. $\lambda_u \sim \mathbf{2}_u\,, \quad \lambda_d \sim \mathbf{2}_d$

Backup: breaking $SU(5)^3$

- $SU(5)^3$ can be broken to deconstructed $SU(2)_L^3$ and $SU(3)_c^3$ via bi-fundamentals:

$$\mathbf{5}_i \times \overline{\mathbf{5}}_j = [(\mathbf{3}, \mathbf{1}, -1/3) \oplus (\mathbf{1}, \mathbf{2}, 1/2)]_i \times [(\overline{\mathbf{3}}, \mathbf{1}, 1/3) \oplus (\mathbf{1}, \mathbf{2}, -1/2)]_j$$

- $SU(5)^3$ can be broken to tri-hypercharge $U(1)^3$ via bi-adjoints:

$$\mathbf{24}_1 \times \mathbf{24}_2 \supset (\mathbf{8}, \mathbf{1}, 0)_1 \times (\mathbf{8}, \mathbf{1}, 0)_2 = (\mathbf{8}, \mathbf{1}, 0)_{1+2} \times (\mathbf{8}, \mathbf{1}, 0)_{1+2} = (\mathbf{1}, \mathbf{1}, 0)_{1+2} \oplus \ldots$$

$$\mathbf{24}_1 \times \mathbf{24}_2 \supset (\mathbf{1}, \mathbf{3}, 0)_1 \times (\mathbf{1}, \mathbf{3}, 0)_2 = (\mathbf{1}, \mathbf{3}, 0)_{1+2} \times (\mathbf{1}, \mathbf{3}, 0)_{1+2} = (\mathbf{1}, \mathbf{1}, 0)_{1+2} \oplus \ldots$$

Backup: pseudo-Goldstone bosons

$$\mathcal{L}_d = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2},0,\frac{1}{2}) & \Phi(-\frac{1}{6},-\frac{1}{3},\frac{1}{2}) & \Phi(-\frac{1}{6},0,\frac{1}{6}) \\ \Phi(-\frac{1}{3},-\frac{1}{6},\frac{1}{2}) & \Phi(0,-\frac{1}{2},\frac{1}{2}) & \Phi(0,-\frac{1}{6},\frac{1}{6}) \\ \Phi(-\frac{1}{3},0,\frac{1}{3}) & \Phi(0,-\frac{1}{3},\frac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d + \text{h.c.}$$

- So far we have two scalars (hyperons) $\phi_{\ell 23} \sim (0,-\frac{1}{2},\frac{1}{2})$ and $\phi_{q23} \sim (0,-\frac{1}{6},\frac{1}{6})$
- Introduce $\phi_{\ell 12} \sim (-\frac{1}{2},\frac{1}{2},0)$ and $\phi_{q12} \sim (-\frac{1}{6},\frac{1}{6},0)$ to generate spurions in first row:

$$Y_d = \begin{pmatrix} \frac{\phi_{\ell 12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_1} & \frac{\phi_{q12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q12}}{\Lambda_1}\frac{\phi_{q23}}{\Lambda_2} \\ \frac{\phi_{\ell 12}}{\Lambda_1}\frac{\tilde{\phi}_{q12}}{\Lambda_1}\frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q23}}{\Lambda_2} \\ \frac{\phi_{q12}^2}{\Lambda_1^2}\frac{\phi_{q23}^2}{\Lambda_2^2} & \frac{\phi_{\ell 23}}{\Lambda_2}\frac{\phi_{q23}}{\Lambda_2} & 1 \end{pmatrix}$$

$$V \supset \lambda_{12}\phi_{q12}^3\phi_{\ell 12} + \lambda_{23}\phi_{q23}^3\phi_{\ell 23} + \text{h.c.}$$

Heavy messengers needed for Λs!

Backup: Simple deconstructions

- Comparison of minimal complete models with the same number of scalars and VL fermions d.o.f.s

$$\epsilon_{23} = \frac{\langle \phi_{23} \rangle}{M_{23}} \qquad \epsilon_{12} = \frac{\langle \phi_{12} \rangle}{M_{12,13}} \qquad \epsilon_{23/12} = \frac{\langle \phi_{23} \rangle}{M_{12,13}}$$

Model 1 in [MFN, King, Vicente, 2404.12442]

$$SU(3)_c \times SU(2)_L \times U(1)_Y^3 \qquad Y_d \sim \begin{pmatrix} \epsilon_{12}\epsilon_{23/12} & \epsilon_{12}\epsilon_{23} & \epsilon_{12}\epsilon_{23} \\ \epsilon_{12}^2\epsilon_{23} & \epsilon_{23} & \epsilon_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

- Three small parameters 😑
- Successful flavour structure
- Correlation between 12 and 23 VEVs

[Davighi, Gosnay, Miller, Renner, 2312.13346]

$$SU(3)_c \times SU(2)_L^3 \times U(1)_Y \qquad Y_d \sim \begin{pmatrix} \epsilon_{12}\epsilon_{23} & \epsilon_{12}\epsilon_{23} & \epsilon_{12}\epsilon_{23} \\ \epsilon_{23} & \epsilon_{23} & \epsilon_{23} \\ 1 & 1 & 1 \end{pmatrix}$$

- Two small parameters
- Too heavy 1st family since $\epsilon_{12} \approx \sin\theta_C \approx \lambda$
- No correlation between 12 and 23 VEVs

Model A in [Barbieri and Isidori, 2312.14004]

$$Y_d \sim \begin{pmatrix} \epsilon_{12}\epsilon_{23} & \epsilon_{23} & \epsilon_{23} \\ \epsilon_{12}\epsilon_{23} & \epsilon_{23} & \epsilon_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

$$SU(3)_c \times SU(2)_L \times U(1)_{R_1} \times U(1)_{R_2} \times U(1)_{(B-L)_{12}} \times U(1)_{Y_3}$$

- Two small parameters
- Need $\mathcal{O}(\lambda)$ coefficients for Cabibbo angle and V_{ub}/V_{cb}, which is acceptable
- No correlation between 12 and 23 VEVs
- Larger gauge symmetry

Backup: full theory

$$\mathcal{L}_Y^d = \left(\begin{array}{c|cccccc} & d_1^c & d_2^c & d_3^c & D_{12} & D_{13} & D_{23} \\ \hline q_1 & 0 & 0 & 0 & 0 & y_{15}^d H_3^d & 0 \\ q_2 & 0 & 0 & 0 & 0 & 0 & y_{26}^d H_3^d \\ q_3 & 0 & 0 & y_3^d H_3^d & 0 & 0 & 0 \\ \overline{D}_{12} & y_{41}^d \tilde{\phi}_{\ell 12} & y_{42}^d \phi_{q12} & 0 & M_{D_{12}} & y_{45}^d \phi_{\ell 23} & 0 \\ \overline{D}_{13} & 0 & 0 & 0 & y_{54}^d \tilde{\phi}_{\ell 23} & M_{D_{13}} & y_{56}^d \phi_{q12} \\ \overline{D}_{23} & 0 & y_{62}^d \tilde{\phi}_{\ell 23} & y_{63}^d \phi_{q23} & 0 & y_{65}^u \tilde{\phi}_{q12} & M_{D_{23}} \end{array} \right) + \text{h.c.} \tag{1}$$

Backup: Model 3 spectrum

Spectrum up to $\mathcal{O}(1)$ variations:

E

$\mathcal{O}(10^5\,\text{TeV})$ — $M_{H_1^{u,d}},\ M_{Q_{1,2}}$

$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$

$\mathcal{O}(1000\,\text{TeV})$ — $v_{12} \sim \langle \phi_{q12} \rangle,\ M_{Q_3},\ M_{H_2^{u,d}}$

$\mathcal{O}(10\,\text{TeV})$ — $v_{23} \sim \langle \phi_{q23} \rangle,\ \langle \phi_{\ell 23} \rangle,\ \langle \phi_{\ell 13} \rangle$

174 GeV — $v_{\text{SM}} \sim \langle H_3^{u,d} \rangle$

$$\frac{\langle \phi_{\ell 23} \rangle}{M_{H_2^{u,d}}} \sim \lambda^3 \qquad \frac{\langle \phi_{q23} \rangle}{M_{Q_3}} \sim \lambda^2$$

$$\frac{\langle \phi_{\ell 13} \rangle}{M_{H_1^{u,d}}} \sim \lambda^6 \qquad \frac{\langle \phi_{q12} \rangle}{M_{Q_{1,2}}} \sim \lambda$$

No fixed connection among VEVs is predicted

Backup: Model 3 spectrum

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$\mathcal{O}(10^5\,\text{TeV})$ — $M_{H_1^{u,d}},\ M_{Q_{1,2}}$

$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$

↓

$\mathcal{O}(1000\,\text{TeV})$ — $v_{12} \sim \langle \phi_{q12} \rangle,\ M_{Q_3},\ M_{H_2^{u,d}}$

$SU(3)_c \times SU(2)_L \times U(1)_{Y_{12} \equiv Y_1 + Y_2} \times U(1)_{Y_3}$

$\mathcal{O}(10\,\text{TeV})$ — $v_{23} \sim \langle \phi_{q23} \rangle,\ \langle \phi_{\ell 23} \rangle,\ \langle \phi_{\ell 13} \rangle$

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$SU(3)_c \times SU(2)_L \times U(1)_{Y \equiv Y_1 + Y_2 + Y_3}$

$174\ \text{GeV}$ — $v_{\text{SM}} \sim \langle H_3^{u,d} \rangle$

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No fixed connection among VEVs is predicted

Backup: SU(5) cube model table

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\overline{5}$	1	1
F_2	1	$\overline{5}$	1
F_3	1	1	$\overline{5}$
T_1	10	1	1
T_2	1	10	1
T_3	1	1	10
χ_1	10	1	1
χ_2	1	10	1
χ_3	1	1	10
N_{12}	5	$\overline{5}$	1
N_{13}	5	1	$\overline{5}$
N_{23}	1	5	$\overline{5}$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
Ω_1	24	1	1
Ω_2	1	24	1
Ω_2	1	1	24
$H_1^{u,d}$	5, $\overline{5}$	1	1
$H_2^{u,d}$	1	5, $\overline{5}$	1
$H_3^{u,d}$	1	1	5, $\overline{5}$
Φ_{12}^F	5	$\overline{5}$	1
Φ_{13}^F	$\overline{5}$	1	5
Φ_{23}^F	1	5	$\overline{5}$
Φ_{12}^T	$\overline{10}$	10	1
Φ_{13}^T	10	1	$\overline{10}$
Φ_{23}^T	1	$\overline{10}$	10

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
H_1^{45}	45	1	1
H_2^{45}	1	45	1
H_3^{45}	1	1	45

Backup: SU(5) cube model table

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
F_2	$\mathbf{1}$	$\overline{\mathbf{5}}$	$\mathbf{1}$
F_3	$\mathbf{1}$	$\mathbf{1}$	$\overline{\mathbf{5}}$
T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
T_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
T_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
χ_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
χ_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
χ_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
N_{12}	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$
N_{13}	$\mathbf{5}$	$\mathbf{1}$	$\overline{\mathbf{5}}$
N_{23}	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
Ω_1	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
Ω_2	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$
Ω_2	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$
$H_1^{u,d}$	$\mathbf{5},\ \overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
$H_2^{u,d}$	$\mathbf{1}$	$\mathbf{5},\ \overline{\mathbf{5}}$	$\mathbf{1}$
$H_3^{u,d}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5},\ \overline{\mathbf{5}}$
Φ_{12}^F	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$
Φ_{13}^F	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$
Φ_{23}^F	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$
Φ_{12}^T	$\overline{\mathbf{10}}$	$\mathbf{10}$	$\mathbf{1}$
Φ_{13}^T	$\mathbf{10}$	$\mathbf{1}$	$\overline{\mathbf{10}}$
Φ_{23}^T	$\mathbf{1}$	$\overline{\mathbf{10}}$	$\mathbf{10}$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
$H_1^{\mathbf{45}}$	$\mathbf{45}$	$\mathbf{1}$	$\mathbf{1}$
$H_2^{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{45}$	$\mathbf{1}$
$H_3^{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{45}$

cyclic **45** to split down/charged lepton masses as in conventional $SU(5)$