Model dependent and model independent enhancement to triple Higgs production

Gilberto Tetlalmatzi-Xolocotzi

A. Papaefstathiou, GTX: JHEP 06 (2024) 124 (2312.13562)

O. Karkout, A. Papaefstathiou, M. Postma, GTX, J. van de Vis, T du Pree (2404.12425)

A. Papaefstathiou, T. Robens, GTX: 2101.00037/ JHEP 05 (2021), 193

A. Papaefstathiou, GTX, M. Zaro: 1909.09166/ Eur.Phys.J.C 79 (2019) 11, 947

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Miniworkshop on HHH

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Center for Particle Physics Siegen

TRR 257 - Particle Physics Phenomenology after the Higgs Discovery

Universität Siegen

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Higgs Self-Interactions in the SM

$$V\left(\Phi^{\dagger}\Phi\right)=\mu^{2}\Phi^{\dagger}\Phi+\lambda_{SM}\left(\Phi^{\dagger}\Phi\right)^{2}$$

$$\Phi=\left(0,v_{0}+h\right)^{T}/\sqrt{2}$$

$$V\left(\Phi^{\dagger}\Phi\right)\supset\tfrac{1}{2}m_{h}^{2}h^{2}+\lambda_{SM}v_{0}h^{3}+\tfrac{\lambda_{SM}}{4}h^{4}$$

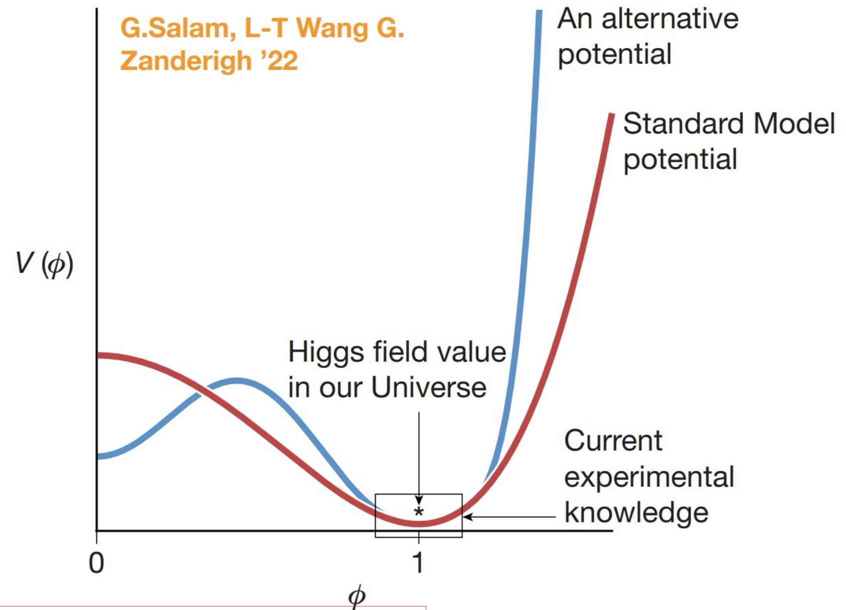
In the SM $\quad m_{h}^{2}=\lambda_{SM}v_{0}^{2}/2 \qquad v_{0}^{2}=-\mu^{2}/\lambda_{SM}$

Why to study triple Higgs production?

- The triple Higgs self coupling is sensitive to New Particles.

- It also gives the opportunity to test the Higgs quartic self couplings.

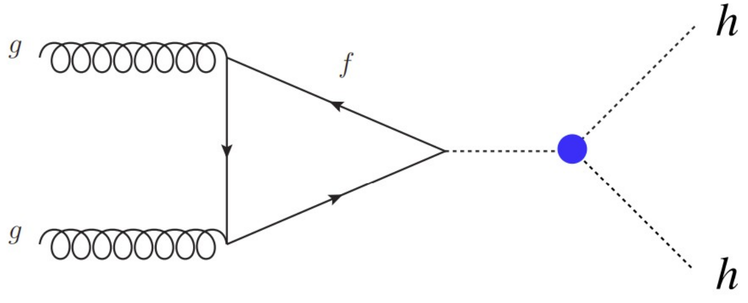
We do not know much about the shape of the Higgs potential.



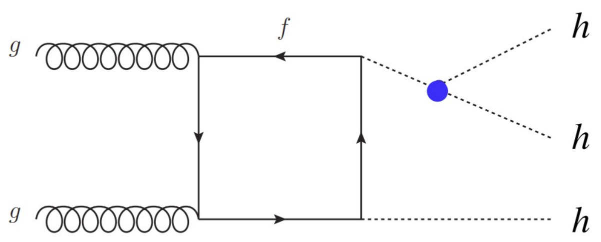
$$V\left(\Phi^{\dagger}\Phi\right) \supset \frac{1}{2} m_h^2 h^2 + a_3 h^3 + a_4 h^4$$

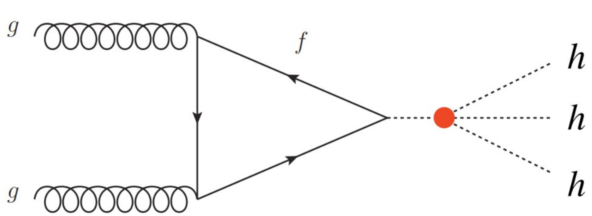
Why to study triple Higgs production?

Double Higgs production is the lowest multiplicity to probe for a_3 .



Triple Higgs production is the lowest multiplicity to probe for a_4.

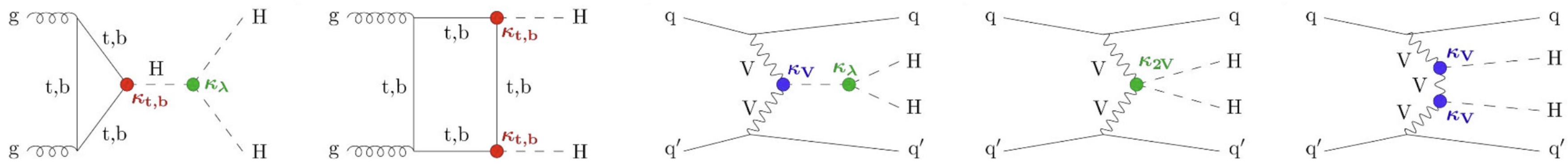




Status of double Higgs production

From the talk of Romain Bouquet (Moriond QCD 2024)

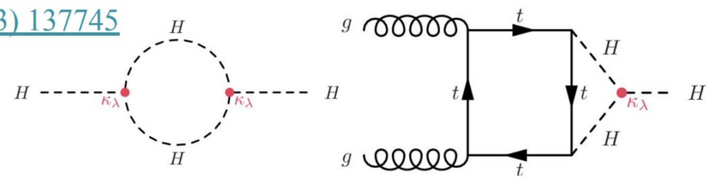
CMS, Nature **607**, 60–68 (2022)



Allows probing $\kappa_\lambda = \lambda / \lambda_{SM}$ & $\kappa_{2V} = \lambda_{2V} / \lambda_{2V,\,SM}$ ***modifiers***

Single Higgs analyses are indirectly sensitive to the Higgs self-coupling
(due to higher order corrections, 2 examples below)

ATLAS, Phys. Lett. B 843 (2023) 137745



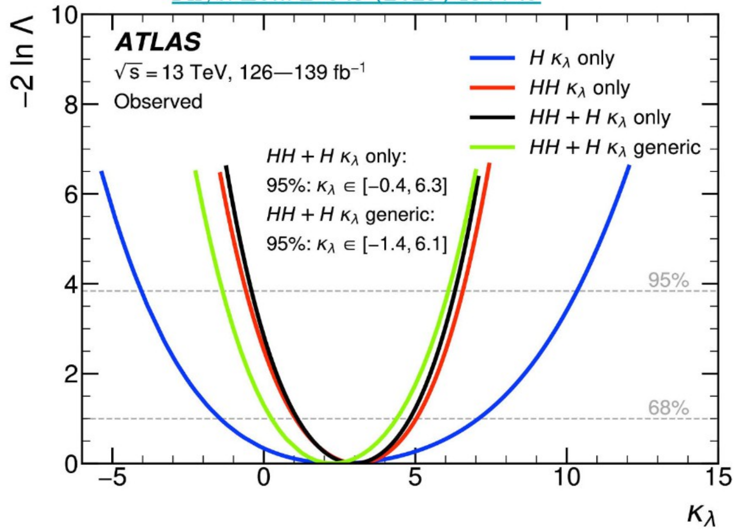
Challenging di-Higgs searches as $\boldsymbol{\sigma}_{\mathrm{HH}} / \boldsymbol{\sigma}_{\mathrm{H}} \sim 10^{-3}$

Status of double Higgs production

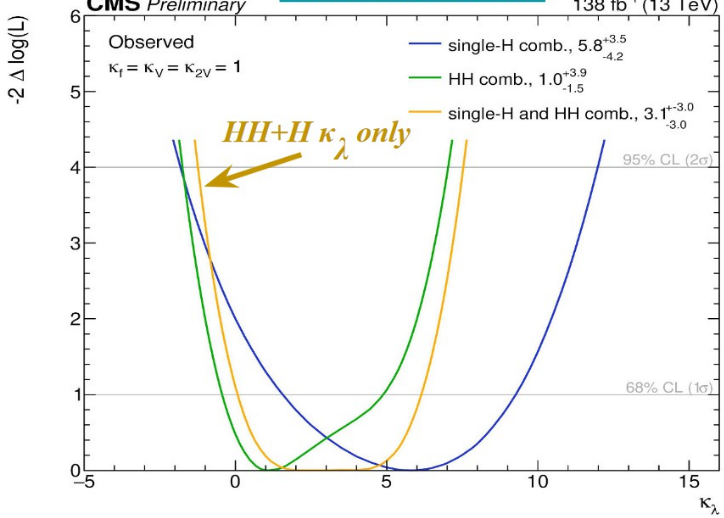
From the talk of Romain Bouquet (Moriond QCD 2024)

Single & double-Higgs combination

Phys. Lett. B 843 (2023) 137745



CMS-PAS-HIG-23-006



- *HH+H* κ_λ *only* = κ_λ *is the only floating parameter*
  - ATLAS *obs:* $-0.4 < \kappa_\lambda < 6.3$
  - CMS *obs:* $-1.2 < \kappa_\lambda < 7.5$

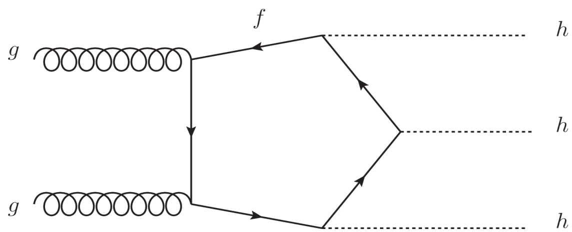
→ **ATLAS & CMS have very similar sensitivities to** κ_λ

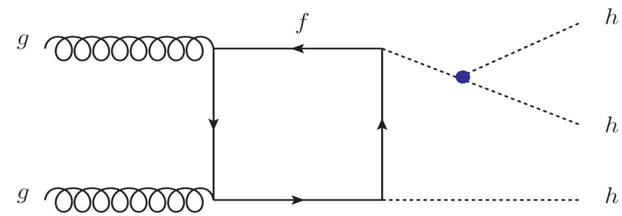
- *HH+H* κ_λ *generic* = κ_λ, κ_t, κ_b, κ_V & κ_τ **free parameters**
  - ATLAS *obs:* $-1.4 < \kappa_\lambda < 6.1$
  - CMS *obs:* $-1.4 < \kappa_\lambda < 7.8$

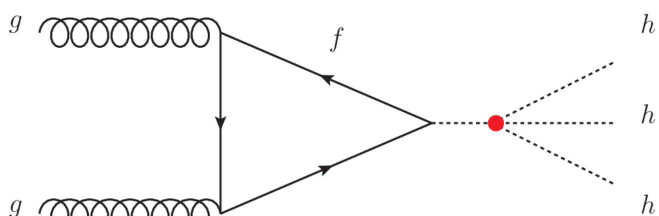
→ **Less assumptions required on the other couplings for *H+HH* combination**

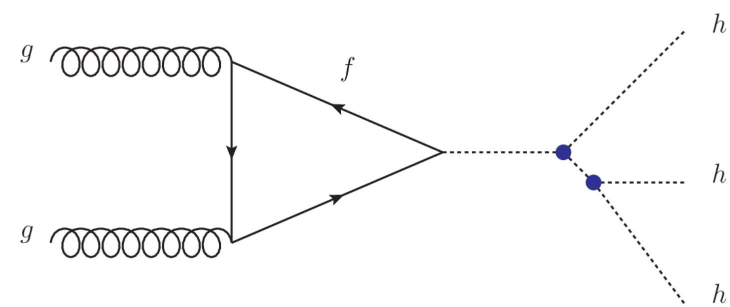
Triple Higgs production

$$g\,g \longrightarrow h\,h\,h$$









At the HL-LHC (pp @ 14 TeV)

$$\sigma = 0.1\ fb$$

~*O(100)* events

Hopeless!

At the FCC (pp @ 100 TeV)

$$\sigma \sim 5\ fb$$

Luminosity=*20 ab*$^{-1}$

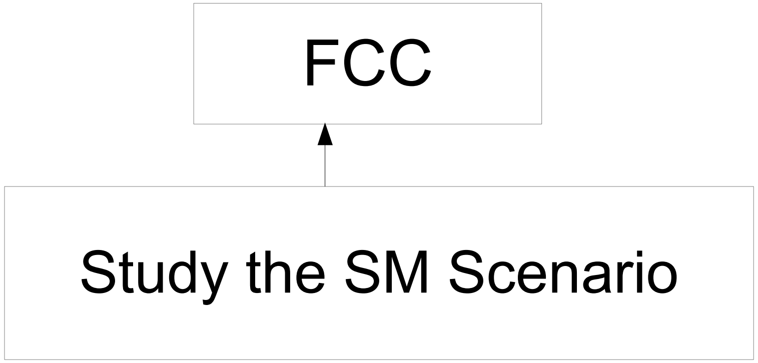
~100 000 events.

de Florian, Fabre, Mazzitelli: 1912.02760

Strategy

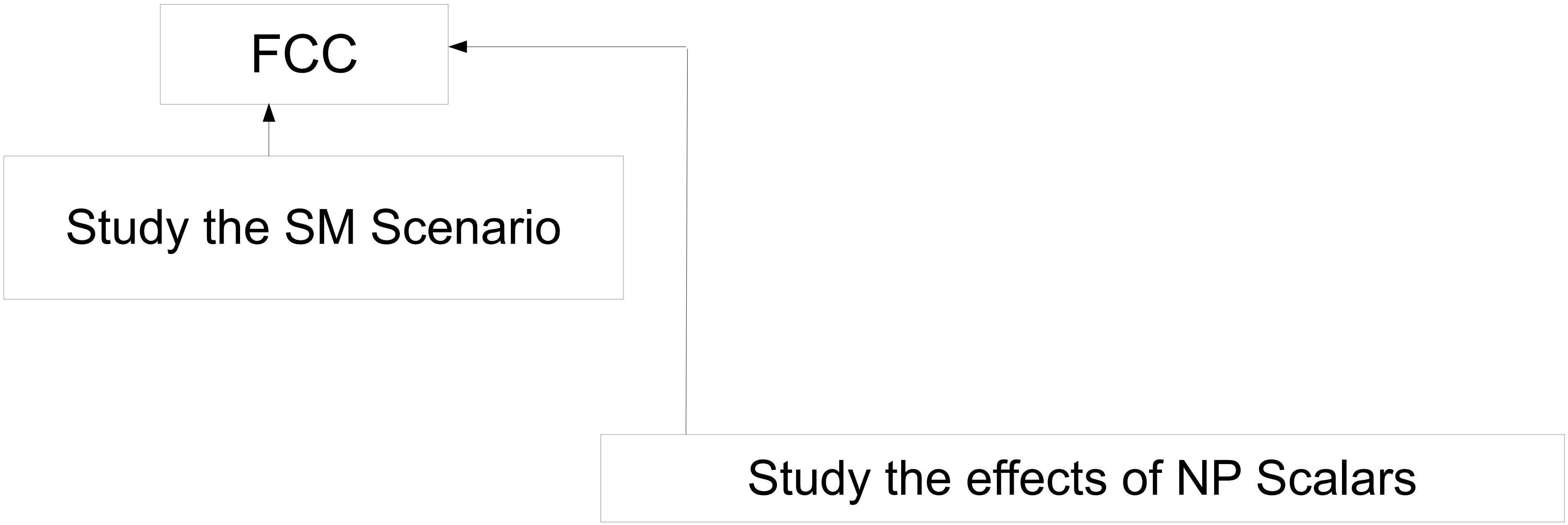
FCC

Strategy



Study the feasibility of measuring triple Higgs production as in the SM in the FCC

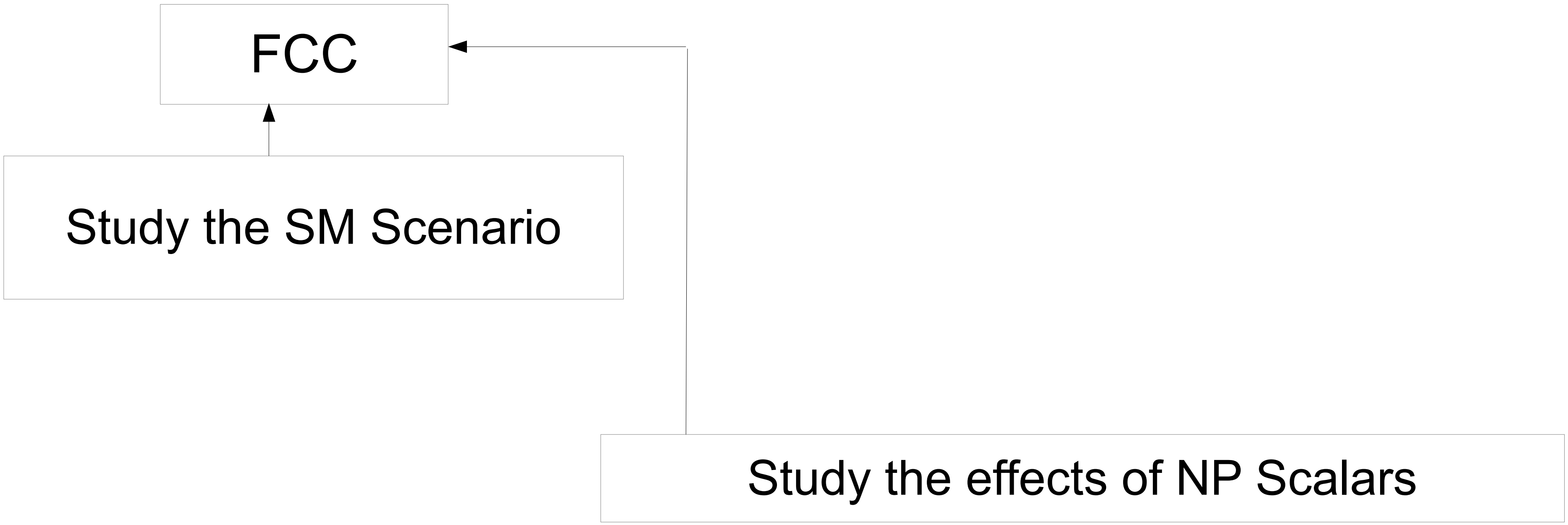
Strategy



Study the feasibility of measuring triple Higgs production as in the SM in the FCC

Include extra scalars and asses the feasibility of the measurement at the FCC

Strategy

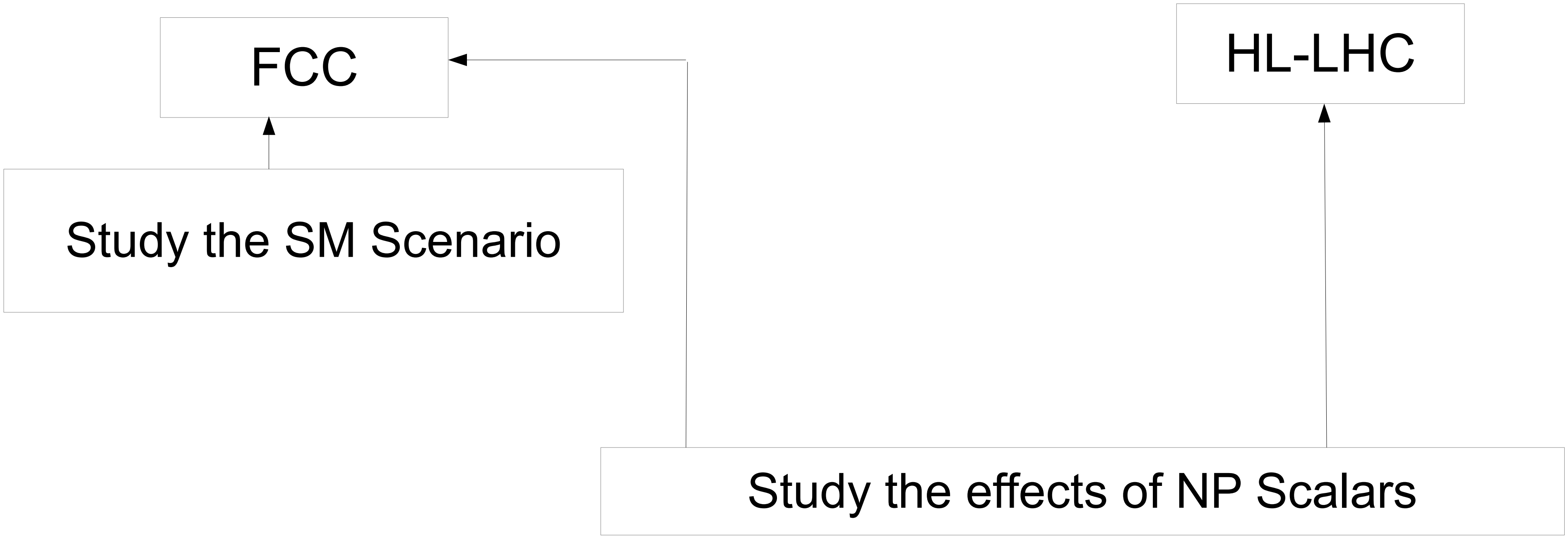


Study the feasibility of measuring triple Higgs production as in the SM in the FCC

Include extra scalars and asses the feasibility of the measurement at the FCC

NP scalars enhance the cross section!

Strategy



Study the feasibility of measuring triple Higgs production as in the SM in the FCC

Include extra scalars and asses the feasibility of the measurement at the FCC

NP scalars enhance the cross section!

Study triple Higgs production in the presence of NP scalar also at the LHC

FCC Study

$$h\,h\,h \longrightarrow X$$

Assuming a K-factor of 2

Maltoni, Vryonidou, Zaro: 1408.6542

X (Final State)	Br(%)	N(20 ab^{-1})
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	22207
$(b\bar{b})(b\bar{b})(WW_{1l})$	7.20	8328
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.31	7297
$(b\bar{b})(\tau\bar{\tau})(WW_{1l})$	1.58	1824
$(b\bar{b})(b\bar{b})(WW_{2l})$	0.98	1128
$(b\bar{b})(WW_{1l})(WW_{1l})$	0.90	1041
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.69	799
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.23	263

Papaefstathiou, GTX, Zaro: 1909.09166

Fuks, Kim, Lee: 1510.07697
1704.04298

Killian et al.: 1702.03554

Papaefstathiou, Sakurai.: 1508.06524
Chen et al.:1510.04013
Fuks, Kim, Lee: 1510.07697

6-b final state has the largest Branching Fraction

This is the channel we are focusing on in this talk

Backgrounds

In the FCC (pp @ 100 TeV)

FCC-hh Analysis (100 TeV)			
Process	$\sigma_{\rm NLO}(6\ b\text{-jet})$ [fb]	$\varepsilon_{\rm analysis}$	$N^{\rm cuts}_{20\ {\rm ab}^{-1}}$
hhh(SM)	1.14	0.0115	98.90
QCD $(b\bar{b})(b\bar{b})(b\bar{b})$	56.66×10^3	1.12×10^{-5}	4777.71
$pp \to Z(b\bar{b})(b\bar{b})$	1285.37	3.04×10^{-5}	294.63
$pp \to ZZ(b\bar{b})$	49.01	2.02×10^{-5}	7.48
$pp \to hZ(b\bar{b})$	9.87	3.04×10^{-5}	2.26
$pp \to hhZ$	0.601	5.95×10^{-4}	2.70
$pp \to hh(b\bar{b})$	0.096	8.095×10^{-5}	$\ll 1$
LI $gg \to hZZ$	8.28	1.62×10^{-4}	10.12
LI $gg \to ZZZ$	6.63	4.05×10^{-5}	2.03
LI $gg \to hhZ$	2.65	2.54×10^{-4}	5.07

$$\mathcal{L} = 20\ {\rm ab}^{-1}$$

In the HL-LHC (pp @ 14 TeV)

LHC Analysis (13.6 TeV)			
Process	$\sigma_{\rm NLO}(6\ b\text{-jet})$ [fb]	$\varepsilon_{\rm analysis}$	$N^{\rm cuts}_{3\times10^3\ {\rm fb}^{-1}}$
hhh(SM)	1.97×10^{-2}	0.12	2.77
QCD $(b\bar{b})(b\bar{b})(b\bar{b})$	6136.12	1.00×10^{-5}	69.67
$pp \to Z(b\bar{b})(b\bar{b})$	61.80	0.0045	318.17
$pp \to ZZ(b\bar{b})$	2.16	0.0059	14.3
$pp \to hZ(b\bar{b})$	0.45	0.0159	8.1
$pp \to hhZ$	0.0374	0.034	1.45
$pp \to hh(b\bar{b})$	0.0036	0.028	0.11
LI $gg \to hZZ$	0.143	0.022	3.62
LI $gg \to ZZZ$	0.124	0.013	1.76
LI $gg \to hhZ$	0.0458	0.047	2.42

$$\mathcal{L} = 3000\ {\rm fb}^{-1}$$

Details on the study of the 6b final state

- Parton level events (signal/background) generated with MadGraph5_aMC@NLO.

- The source of background with the highest XS is QCD-6b-Jets.

- The production of the 6b-final state is challenging, it was generated in the NIKHEF and Siegen computer clusters using the gridpack option available in MadGraph5_aMC@NLO.

- Parton shower and non-perturbative effects included with Herwig 7.

- The analysis was performed using HwSim. [*Papaefsathiou*, https://bitbucket.org/andreasp/hwsim]

Selection Analysis

- *Require 6 b-tagged jets*
- *Construct all the possible combinations of 3-pairs of b-jets:* I.
- *For each combination* I calculate the observable

$$\chi^{2,(6)} = \sum_{qr \in I} \left(M_{qr} - m_h\right)^2$$

- *Select the event based on the value of the combination which minimizes* $\chi^{2,(6)}$

- *The combination determining* $\chi^{2,(6)}_{min}$ *defines the best candidates for the set of 3-Higgs bosons in the event.*

Selection Analysis

Set of observables and optimized cuts applied during the selection analysis

Optimized cuts		
Observable	13.6 TeV	100 TeV
$p_{T,b}$ >	25.95 GeV	35.00 GeV
$\lvert\eta_b\rvert$ <	2.3	3.3
ΔR_{bb} >	0.3	0.3
p_{T,b_i} >	$[25.95, 25.95, 25.95]$ GeV $i = 1, 2, 3$	$[170.00, 135.00, 35.00]$ GeV
$\chi^{2,(6)}$ <	27.0 GeV	26.0 GeV
$\Delta m_{\text{min,med,max}}$ <	$[100, 200, 300]$ GeV	$[8, 8, 8]$ GeV
$\Delta R_{bb}(h^i)$ <	$[3.5, 3.5, 3.5]$	$[3.5, 3.5, 3.5]$
$\Delta R(h^i, h^j)$ <	$[3.5, 3.5, 3.5]$	$[3.5, 3.5, 3.5]$
$p_T(h^i)$ >	$[0.0, 0.0, 0.0]$ GeV	$[200.0, 190.0, 20.0]$ GeV
$p_{T\text{jet}}$ >	25 GeV	25 GeV
$\lvert\eta_{\text{jet}}\rvert$ <	4.0	4.0

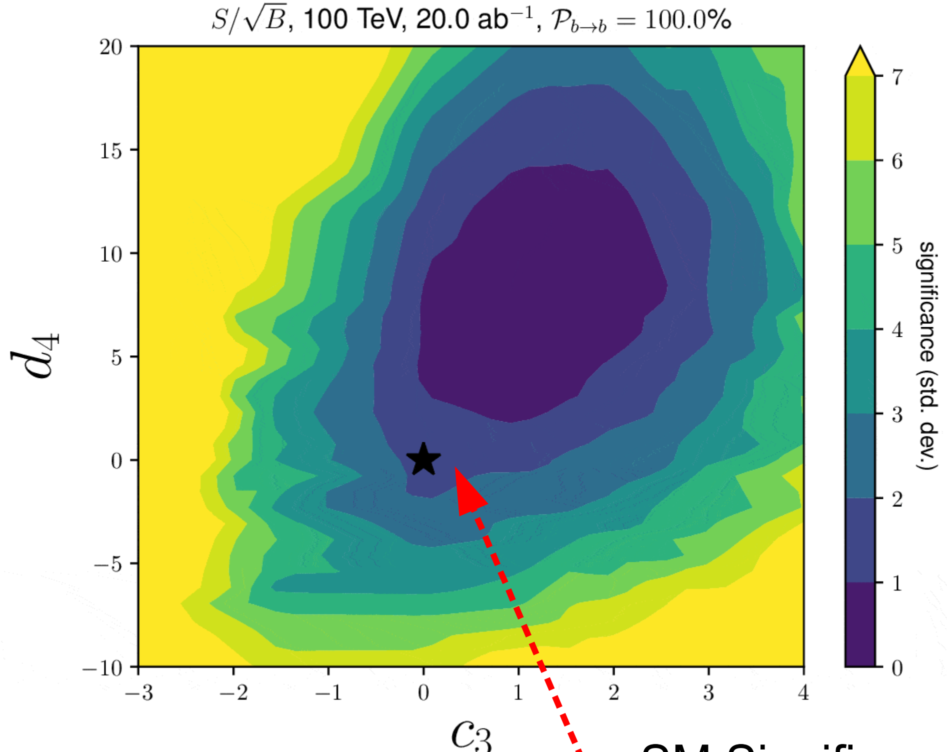
h^i : Higgs boson candidate

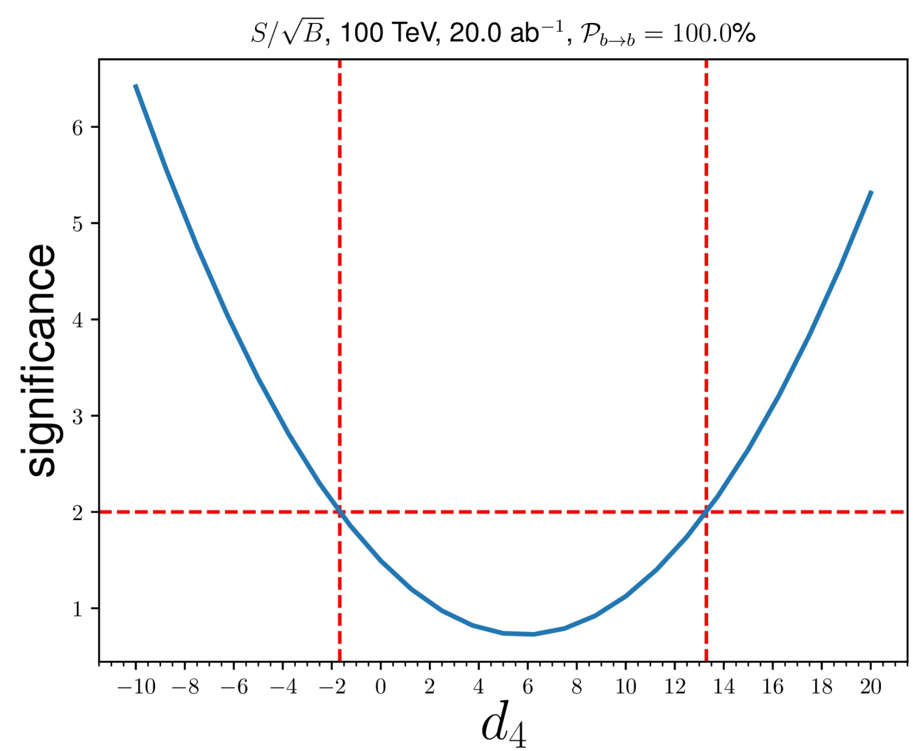
$i = 1,2,3$

Sensitivity to quartic-self couplings

Consider a generalized version of the SM scalar potential

$$V(h) = \frac{1}{2} m_h^2 h^2 + \lambda_{SM}(1+c_3) v_0 h^3 + \lambda_{SM} \frac{(1+d_4)}{4} h^4$$





SM Significance ~ 1.7 σ (Old)

SM Significance ~ 2.33 σ (New)

c_3=0

Anomalous couplings

Relevant phenomenological Lagriangian
to test anomalous couplings

$$
\begin{aligned}
\mathcal{L}_{\rm PhenoExp} = & -\lambda_{\rm SM} v\,(1+d_3)\,h^3 - \frac{\lambda_{\rm SM}}{4}\,(1+d_4)\,h^4 \\
& + \frac{\alpha_s}{12\pi}\left(c_{g1}\frac{h}{v} - c_{g2}\frac{h^2}{2v^2}\right) G^a_{\mu\nu} G^{\mu\nu}_a \\
& - \left[\frac{m_t}{v}\,(1+c_{t1})\,\bar{t}_L t_R h + \frac{m_b}{v}\,(1+c_{b1})\,\bar{b}_L b_R h + \text{h.c.}\right] \\
& - \left[\frac{m_t}{v^2} c_{t2} \bar{t}_L t_R h^2 + \frac{m_b}{v^2} c_{b2} \bar{b}_L b_R h^2 + \text{h.c.}\right] \\
& - \left[\frac{m_t}{v^3}\left(\frac{c_{t3}}{2}\right)\bar{t}_L t_R h^3 + \frac{m_b}{v^3}\left(\frac{c_{b3}}{2}\right)\bar{b}_L b_R h^3 + \text{h.c.}\right],
\end{aligned}
$$

Obtained by considering D=6 EFT operators (SILH, 0703164) and breaking correlations (ATLAS and CMS)

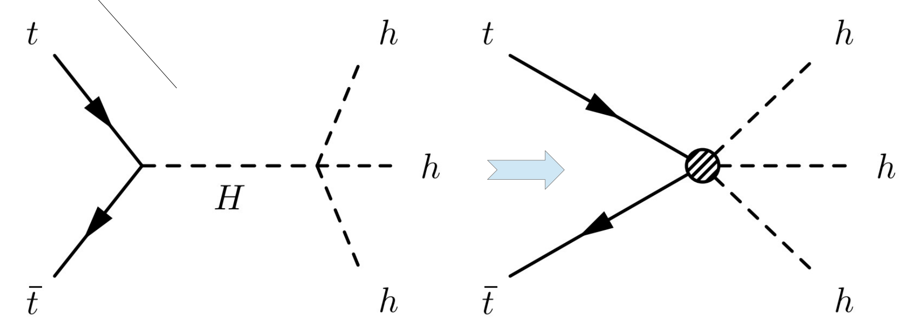
Can also be obtained from the Electroweak chiral Lagrangian

Anomalous couplings

Relevant phenomenological Lagriangian
to test anomalous couplings

$$
\begin{aligned}
\mathcal{L}_{\text{PhenoExp}} =& -\lambda_{\text{SM}} v\,(1+d_3)\,h^3 - \frac{\lambda_{\text{SM}}}{4}\,(1+d_4)\,h^4 \\
&+ \frac{\alpha_s}{12\pi}\left(c_{g1}\frac{h}{v} - c_{g2}\frac{h^2}{2v^2}\right) G^a_{\mu\nu} G_a^{\mu\nu} \\
&- \left[\frac{m_t}{v}\,(1+c_{t1})\,\bar{t}_L t_R h + \frac{m_b}{v}\,(1+c_{b1})\,\bar{b}_L b_R h + \text{h.c.}\right] \\
&- \left[\frac{m_t}{v^2} c_{t2} \bar{t}_L t_R h^2 + \frac{m_b}{v^2} c_{b2} \bar{b}_L b_R h^2 + \text{h.c.}\right] \\
&- \left[\frac{m_t}{v^3}\left(\frac{c_{t3}}{2}\right)\bar{t}_L t_R h^3 + \frac{m_b}{v^3}\left(\frac{c_{b3}}{2}\right)\bar{b}_L b_R h^3 + \text{h.c.}\right],
\end{aligned}
$$

$\left(\frac{c_{t3}}{2}\right)\bar{t}_L t_R h^3$



Anomalous couplings

Current and expected bounds on the anomalous couplings

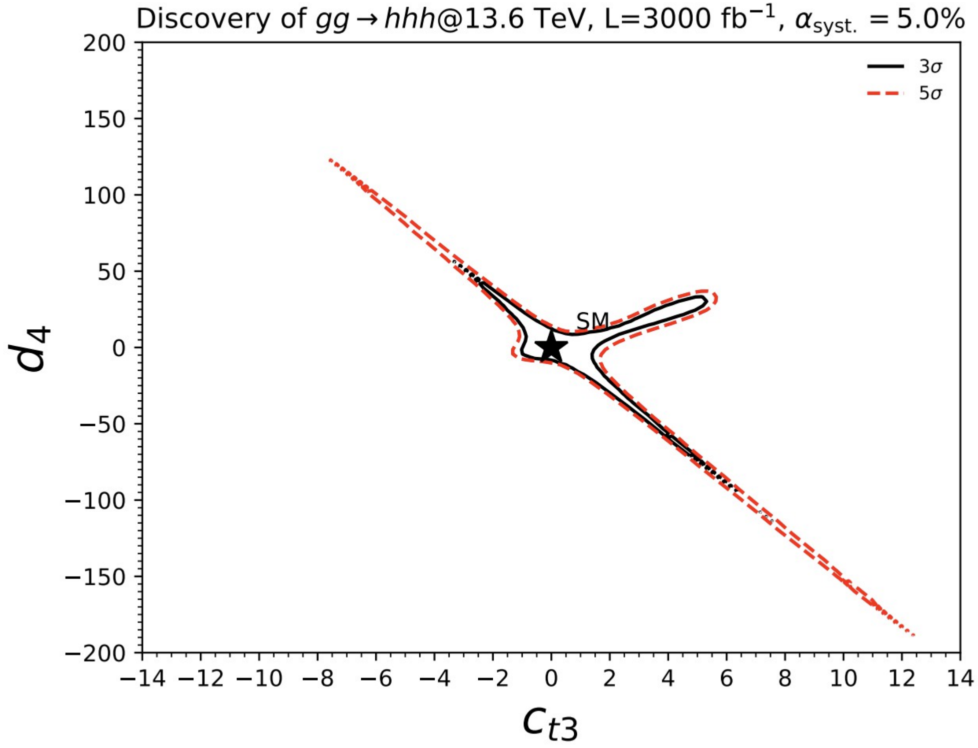
Percentage uncertainties			
	HL-LHC	FCC-hh	Ref.
$\delta(d_3)$	50	5	[1905.03764]
$\delta(c_{g1})$	2.3	0.49	[1905.03764]
$\delta(c_{g2})$	5	1	[1502.00539]
$\delta(c_{t1})$	3.3	1.0	[1905.03764]
$\delta(c_{t2})$	30	10	[1502.00539]
$\delta(c_{b1})$	3.6	0.43	[1905.03764]
$\delta(c_{b2})$	30	10	[1502.00539]

The couplings d_4 and c_{t3} can be bounded by triple Higgs production

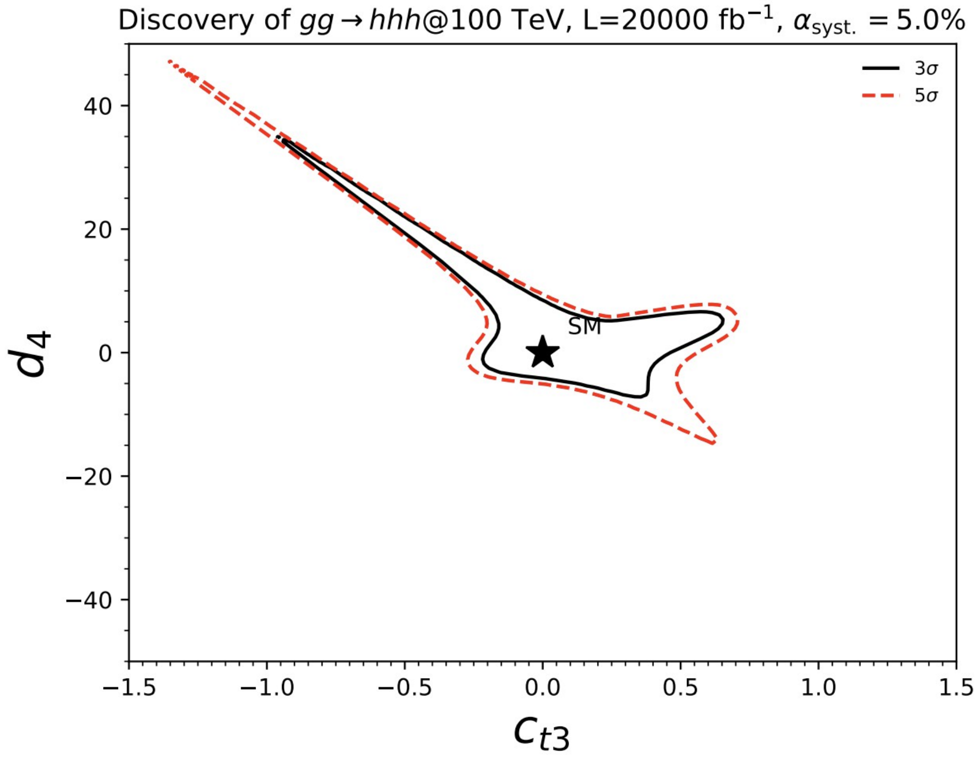
Anomalous couplings

Evidence and discovery regions for triple Higgs production at proton-proton colliders

HL-LHC



FCC

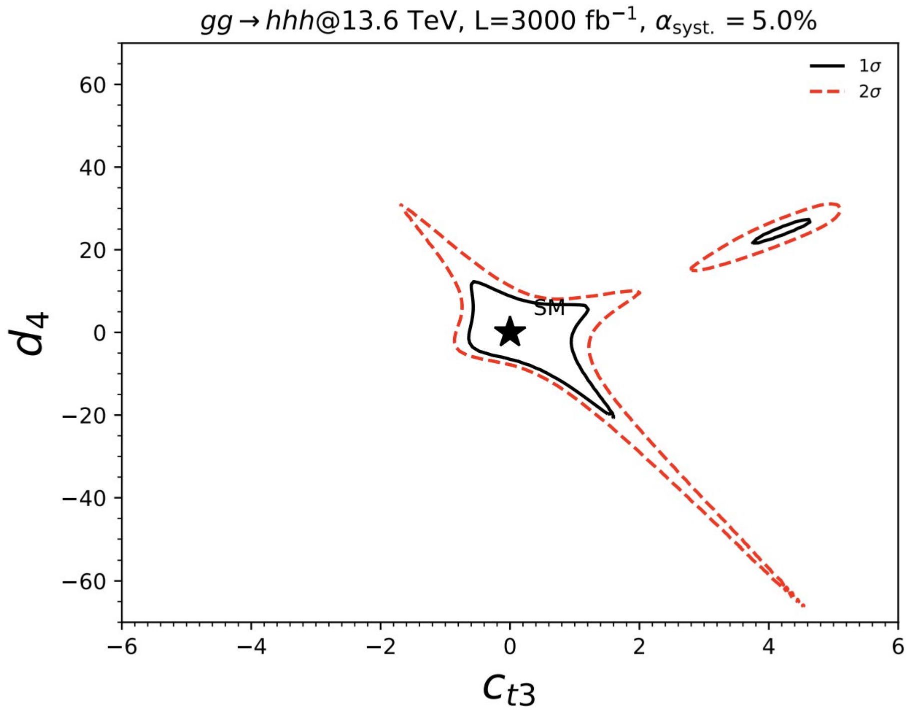


Evidence and discovery contours at proton colliders

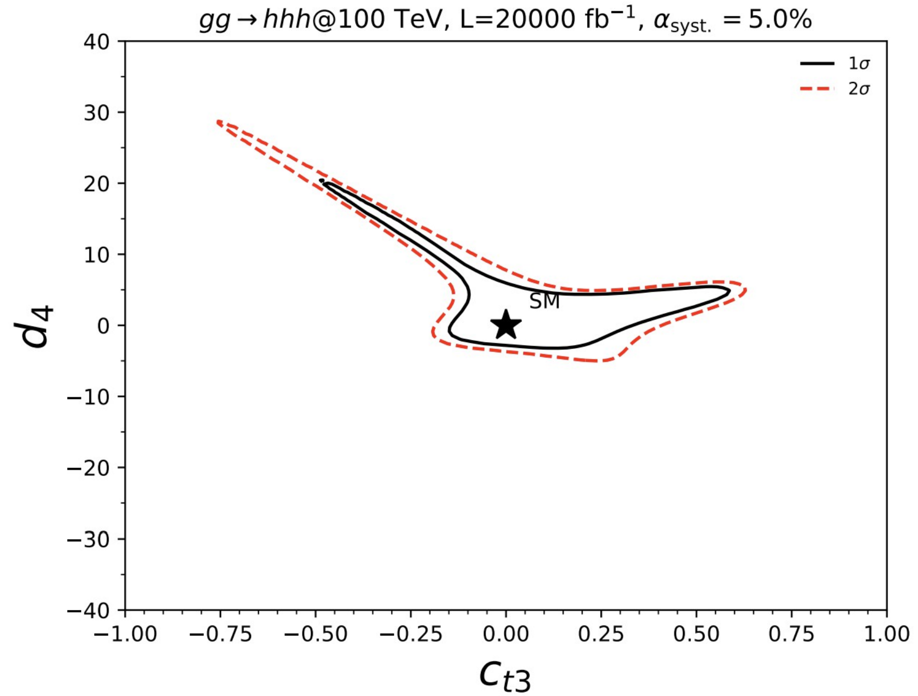
Anomalous couplings

Confidence regions on the anomalous couplings at proton-proton colliders

HL-LHC



FCC



In this plot it is assumed that the SM is the underlying theory

Adding an Extra-Scalar Singlet

The x-SM potential

$$V(\Phi, S) = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \left(\frac{a_1}{2}\right)(\Phi^\dagger \Phi) S$$

$$+ \left(\frac{a_2}{2}\right)(\Phi^\dagger \Phi) S^2 + \left(\frac{b_2}{2}\right) S^2 + \left(\frac{b_3}{3}\right) S^3 + \left(\frac{b_4}{4}\right) S^4$$

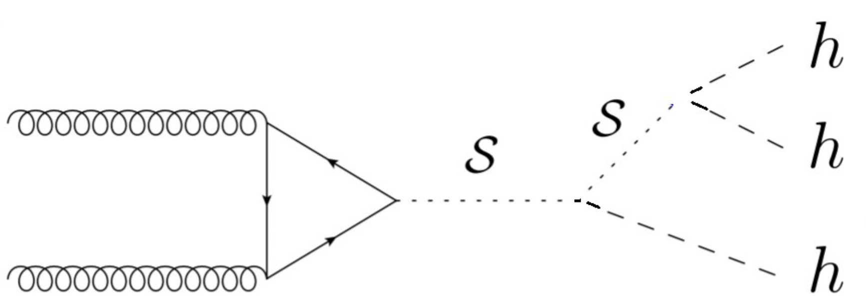
Kotwal et al. 1605.06123

Mass Eigenstates

$$h_1 = h\cos\theta + \phi_s \sin\theta$$

$$h_2 = -h\sin\theta + \phi_s \cos\theta$$

$$S = (\phi_S + v_S)/\sqrt{2}$$



Triple Higgs production in the presence of an extra-scalar

Analysis results

Benchmark points which lead to a Strong-First Order EW Phase Transition

Benchmark	$\cos\theta$	$\sin\theta$	m_2 (GeV)	Γ_{h_2} (GeV)	x_0 (GeV)	λ	a_1 (GeV)	a_2	b_3 (GeV)	b_4	$\frac{\sigma(h_1h_1)}{\sigma(hh)_{SM}}$	$\frac{\sigma(h_1h_1h_1)}{\sigma(hhh)_{SM}}$
B1max	0.976	0.220	341	2.42	257	0.92	-377	0.392	-403	0.77	22.44	60.55
B2max	0.982	0.188	353	2.17	265	0.99	-400	0.446	-378	0.69	22.43	56.69
B3max	0.983	0.181	415	1.59	54.6	0.17	-642	3.80	-214	0.16	6.43	3.01
B4max	0.984	0.176	455	2.08	47.4	0.18	-707	4.63	-607	0.85	5.19	3.37
B5max	0.986	0.164	511	2.44	40.7	0.18	-744	5.17	-618	0.82	3.49	2.94
B6max	0.988	0.153	563	2.92	40.5	0.19	-844	5.85	-151	0.083	2.79	3.60
B7max	0.992	0.129	604	2.82	36.4	0.18	-898	7.36	-424	0.28	2.51	4.70
B8max	0.994	0.113	662	2.97	32.9	0.17	-976	8.98	-542	0.53	2.28	4.91
B9max	0.993	0.115	714	3.27	29.2	0.18	-941	8.28	497	0.38	1.98	2.68
B10max	0.996	0.094	767	2.83	24.5	0.17	-920	9.87	575	0.41	1.95	2.35
B11max	0.994	0.105	840	4.03	21.7	0.19	-988	9.22	356	0.83	1.76	1.03

Identification of the
Extra-scalar at *100 TeV*

Benchmark	Significance
B1max	46.6
B2max	42.9
B3max	2.9
B4max	3.7
B5max	3.0
B6max	3.8
B7max	5.3
B8max	7.8
B9max	5.9
B10max	4.9
B11max	2.3

Two Real Singlet Extension of the SM
TRSM

$$V(\Phi,\phi_i)=V_{SM}(\Phi)+V(\Phi,S,X)$$

Reduce the number of parameters by imposing

$$\mathbb{Z}_2^S: S\to -S\,,\ X\to X$$

$$\mathbb{Z}_2^X: S\to S\,,\ X\to -X$$

$$V(\Phi,X,S)=\mu_\Phi^2\Phi^\dagger\Phi+\lambda_\Phi(\Phi^\dagger\Phi)^2+\mu_S^2 S^2+\lambda_S S^4 +\mu_X^2 X^2+\lambda_X X^4+\lambda_{\Phi S}\Phi^\dagger\Phi X^2+\lambda_{SX} S^2 X^2$$

$$S=(\phi_S+v_S)/\sqrt{2}$$

$$X=(\phi_X+v_X)/\sqrt{2}$$

Change to the physical basis

$$\begin{pmatrix}h_1\\h_2\\h_3\end{pmatrix}=R(\theta_X,\theta_S)\begin{pmatrix}\phi_h\\\phi_S\\\phi_X\end{pmatrix}$$

$h_1=h$ *is the SM Higgs boson*

$$M_1=125\,GeV$$

Free independent parameters

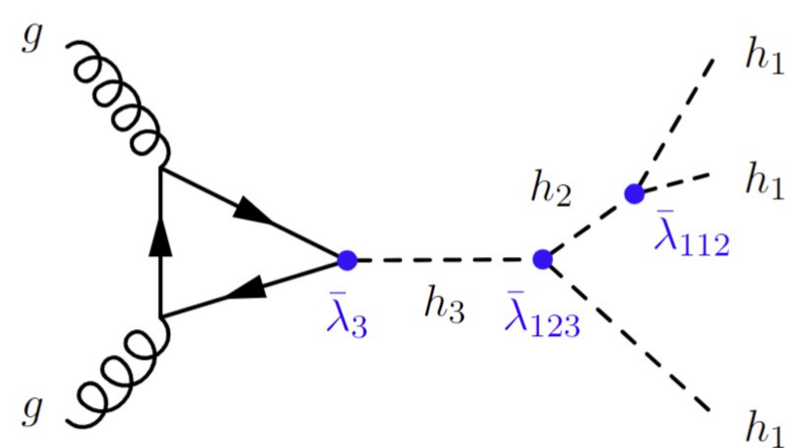
$$M_2, M_3, \theta_{hS}, \theta_{hX}, \theta_{SX}, v_S, v_X$$

Robens, Stefaniak, Wittbrodt: 1908.08554

Old Benchmark Scenario of Study BP3

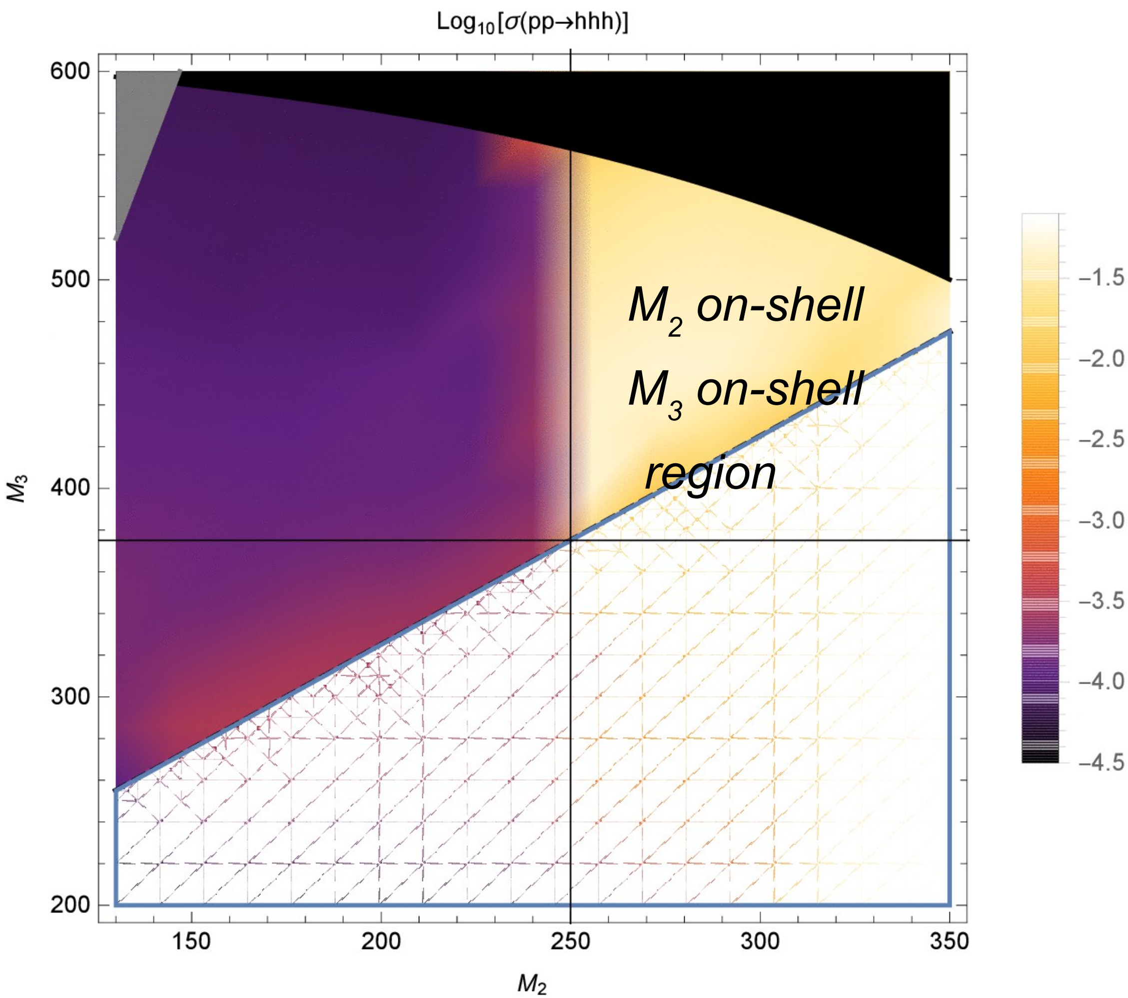
The BP3 Scenario introduced in 1908.08554 which allows for a large $h_1h_1h_1$ production while obeying current theoretical and experimental constraints.

Parameter	Value
M_1	125.09 GeV
M_2	[125, 500] GeV
M_3	[255, 650] GeV
θ_{hS}	−0.129
θ_{hX}	0.226
θ_{SX}	−0.899
v_S	140 GeV
v_X	100 GeV



We consider the mass hierarchy $M_1 < M_2 < M_3$

Production cross section



The X-Section can reach up to 50 fb for M_2~(263,280) GeV and M_3~450 GeV

Old benchmark points

Label	(M_2, M_3) [GeV]	$\varepsilon_{\text{Sig.}}$	$S\vert_{300\text{fb}^{-1}}$	$\varepsilon_{\text{Bkg.}}$	$B\vert_{300\text{fb}^{-1}}$	$\text{sig}\vert_{300\text{fb}^{-1}}$	$\text{sig}\vert_{3000\text{fb}^{-1}}$
A	$(255, 504)$	0.025	14.12	8.50×10^{-4}	19.16	2.92	9.23
B	$(263, 455)$	0.019	17.03	3.60×10^{-5}	8.11	4.78	15.11
C	$(287, 502)$	0.030	20.71	9.13×10^{-5}	20.60	4.01	12.68
D	$(290, 454)$	0.044	37.32	1.96×10^{-4}	44.19	5.02	15.86
E	$(320, 503)$	0.051	32.54	2.73×10^{-4}	61.55	3.76	11.88
F	$(264, 504)$	0.028	18.18	9.13×10^{-5}	20.60	3.56	11.27
G	$(280, 455)$	0.044	38.70	1.96×10^{-4}	44.19	5.18	16.39
H	$(300, 475)$	0.054	41.27	2.95×10^{-4}	66.46	4.64	14.68
I	$(310, 500)$	0.063	41.42	3.97×10^{-4}	89.59	4.09	12.94
J	$(280, 500)$	0.029	20.67	9.14×10^{-5}	20.60	4.00	12.65

These points are associated with large couplings which can break perturbativity at the energy scale MZ

New Benchmark points (LHC)

Determine phase space that enhances triple Higgs production in the TRSM based on

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Perturbative conditions

$$\lambda_{11} < \frac{\pi^2}{3} \approx 3.3, \quad \lambda_{22}, \lambda_{33} < \frac{4\pi^2}{9} \approx 4.4, \quad \lambda_{12}, \lambda_{13}, \lambda_{23} < 2\pi^2 \approx 20$$

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Relevant HiggsBounds Experimental Analyses

Processes	Experiment	Int. Luminosity	arXiv ref.
$gg \to S \to W^+W^-, ZZ$	ATLAS	139 fb^{-1}	2004.14636 [57]
$gg \to S \to ZZ$	ATLAS	139 fb^{-1}	2009.14791 [58]
$gg \to S \to h_1 h_1 \to (b\bar{b})(\tau^+\tau^-)$	CMS	137 fb^{-1}	2106.10361 [59]
$(b\bar{b}, \tau^+\tau^-, W^+W^-, ZZ, \gamma\gamma)(b\bar{b})$		35.9 fb^{-1}	1811.09689 [60]
$gg \to S \to h_1 h_1 \to (b\bar{b}, \tau^+\tau^-, W^+W^-, \gamma\gamma)^2$	ATLAS	36.1 fb^{-1}	1906.02025 [61]
$gg \to S \to h_1 h_1 \to (b\bar{b})(\gamma\gamma)$	ATLAS	36.1 fb^{-1}	1807.04873 [62]
$gg \to S \to W^+W^-, ZZ$	ATLAS	36.1 fb^{-1}	1808.02380 [63]
$pp \to S \to ZZ$ (incl. VBF)	CMS	35.9 fb^{-1}	1804.01939 [64]
$gg \to S \to h_1 h_1 \to (b\bar{b})(b\bar{b})$	CMS	35.9 fb^{-1}	1806.03548 [65]
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Boundedness from below

Experimental constraints from HiggsTools (HiggsSignals and HiggsBounds)

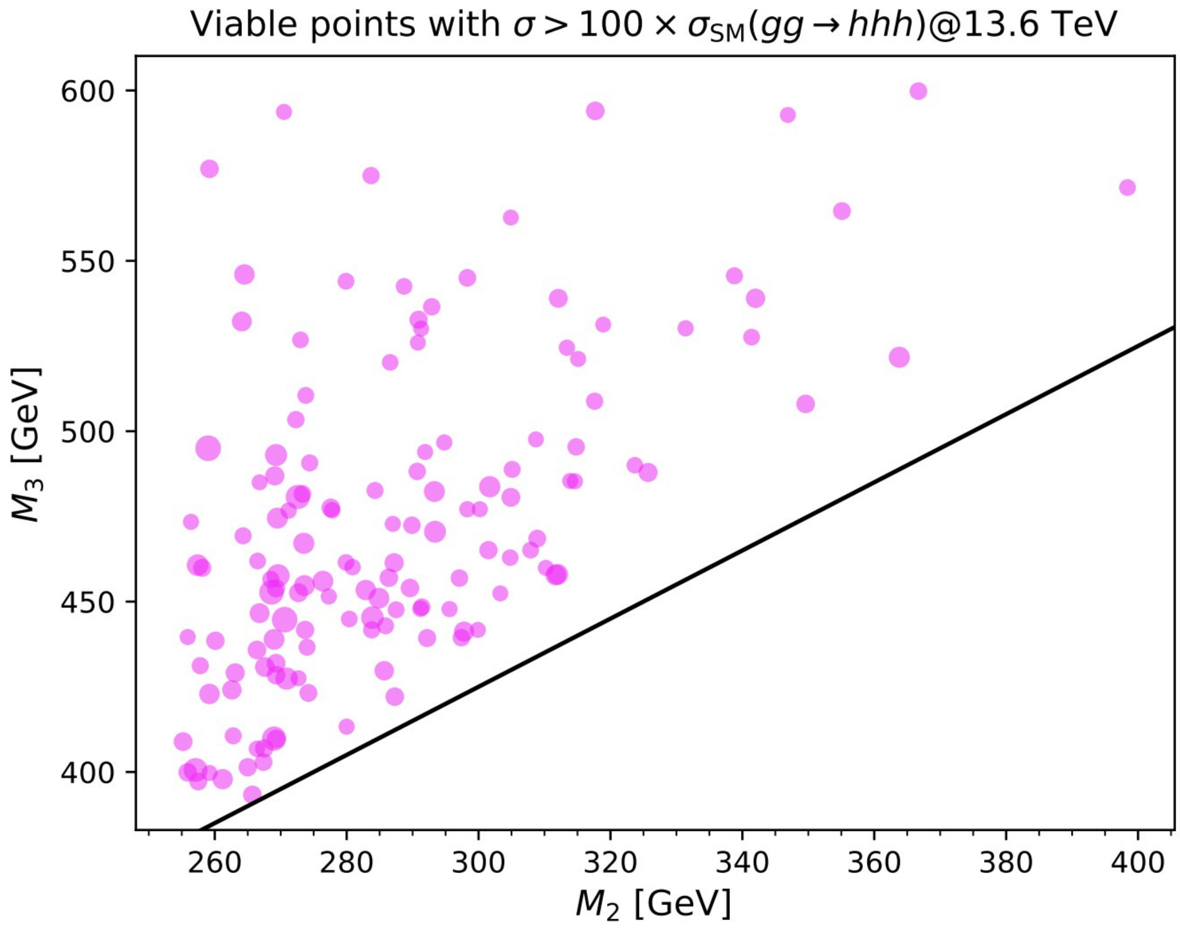
We consider the threshold

$$\sigma_{3h_1} > 100\, \sigma_{3h_1}^{\mathrm{SM}},$$

Our analysis entailed 530,000 phase space points

Only 130 points fulfilled all the conditions

New Benchmark points (LHC)



New Benchmark points (LHC)

Benchmark points for enhanced triple Higgs production										
M_2	M_3	v_2	v_3	θ_{12}	θ_{13}	θ_{23}	$\frac{\sigma}{\sigma_{SM}}$	Res. Frac.	μ_{pert}	$\frac{\mu_{\text{pert}}}{\mu_{\text{pole}}}$
259.0	495.0	215.8	180.8	6.191	0.163	5.691	306.025	0.955	2.7×10^2	7.3
270.6	444.7	122.4	847.2	0.268	0.030	0.522	302.361	0.929	1.8×10^2	7.4
268.6	452.7	137.8	784.8	0.263	0.023	0.645	275.616	0.954	2.4×10^2	7.3
272.6	480.7	928.3	143.7	3.098	2.9	2.375	267.245	0.948	1.4×10^2	7.3
269.0	409.8	138.0	599.4	0.244	0.004	0.773	266.439	0.976	2.4×10^2	7.4
269.1	486.9	227.5	307.9	0.074	6.149	2.631	157.583	0.956	4.3×10^2	7.3
259.2	577.0	289.0	275.6	0.137	6.148	2.324	145.470	0.781	1.2×10^4	7.3
283.7	575.0	259.4	330.4	0.137	6.152	2.299	122.546	0.779	3.0×10^3	7.3
264.3	469.3	207.3	359.5	0.285	6.277	0.692	119.121	0.999	5.4×10^3	7.3
266.5	461.9	653.1	229.0	2.889	3.046	1.015	112.794	0.863	5.3×10^4	7.4
259.2	399.7	444.5	217.0	2.917	3.046	1.047	103.717	0.973	1.2×10^5	7.4

Update of
A. Papaefstathiou, T. Robens, GTX: 2101.00037/ JHEP 05 (2021), 193

New Benchmark points (LHC)

Benchmark points for enhanced triple Higgs production

						23	$\frac{\sigma}{\sigma_{SM}}$	Res. Frac.	μ_{pert}	$\frac{\mu_{\text{pert}}}{\mu_{\text{pole}}}$
						691	306.025	0.955	2.7×10^2	7.3
						522	302.361	0.929	1.8×10^2	7.4
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						375	267.245	0.948	1.4×10^2	7.3
						773	266.439	0.976	2.4×10^2	7.4
						631	157.583	0.956	4.3×10^2	7.3
						324	145.470	0.781	1.2×10^4	7.3
						299	122.546	0.779	3.0×10^3	7.3
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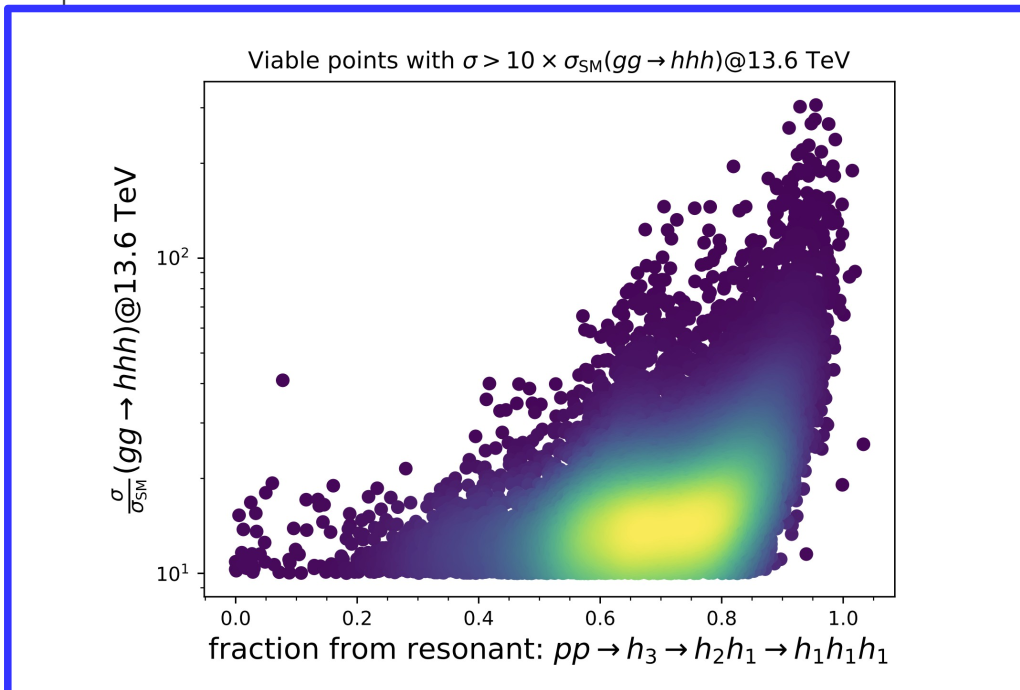

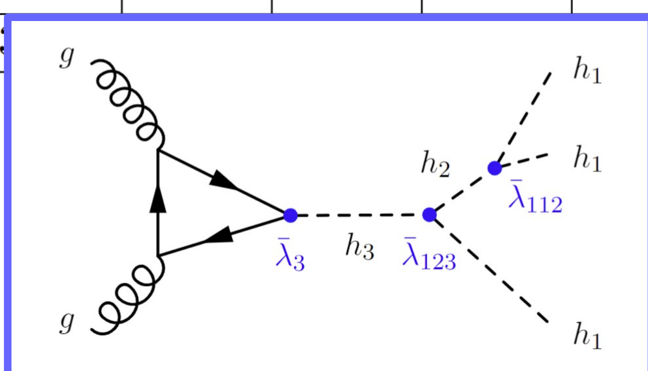

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In practice our points fulfil the following theoretical relationship

$$\ln(\mu_{pole}/\mu_{pert}) = 2$$

$$\mu_{pole} \approx 7.4\mu_{pert}$$

Closing Remarks

- Triple Higgs production $h_1h_1h_1$ as in the SM cannot be probed at the LHC due to its tiny cross section.
- The improved luminosity and center of mass energy of a 100 TeV collider can make the identification of the SM $h_1h_1h_1$ possible.
- The 6-b jets final state is a good candidate to search for $h_1h_1h_1$ *within and beyond the SM*
- *Extended scalar sectors can be probed through $h_1h_1h_1$ even in the HL-LHC (consider for instance the TRSM).*

Acknowledgements

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 945422

$$
\begin{aligned}
\mathcal{L}_{h^n} =& -\mu^2|H|^2 - \lambda|H|^4 - \left(y_t\bar{Q}_L H^c t_R + y_b \bar{Q}_L H b_R + \text{h.c.}\right) \\
&+ \frac{c_H}{2\Lambda^2}(\partial^\mu|H|^2)^2 - \frac{c_6}{\Lambda^2}\lambda_{\text{SM}}|H|^6 + \frac{\alpha_s c_g}{4\pi\Lambda^2}|H|^2 G^a_{\mu\nu}G^{\mu\nu}_a \\
&- \left(\frac{c_t}{\Lambda^2}y_t|H|^2\bar{Q}_L H^c t_R + \frac{c_b}{\Lambda^2}y_b|H|^2\bar{Q}_L H b_R + \text{h.c.}\right),
\end{aligned}
$$

Model independent statistical analyses

$$\delta_{\mathrm{B}} = \sqrt{B + (\alpha B)^2}$$

$$P(\{c_i\}) = \frac{1}{\sqrt{2\pi}\delta_{\mathrm{B}}} \exp\left[-\frac{S(\{c_i\})^2}{2\delta_{\mathrm{B}}^2}\right]$$

$$P_C(c_i) = \frac{1}{\sqrt{2\pi}\delta(c_i)} \exp\left[-\frac{c_i^2}{2\delta(c_i)^2}\right]$$

$$P_M(c_{t3}, d_4) = \sum_{(d_3, c_{t2})} \Delta d_3 \Delta c_{t2} P_C(d_3) P_C(c_{t2}) P(\{c_i\})$$

Model independent statistical analyses

$$\delta_{\mathrm{SM+B}} = \sqrt{S_{\mathrm{SM}} + B + (\alpha B)^2} \qquad \bar{P}(\{c_i\}) = \frac{1}{\sqrt{2\pi}\delta_{\mathrm{SM+B}}} \exp\left[-\frac{(S_{\mathrm{SM}} - S(\{c_i\}))^2}{2\delta^2_{\mathrm{SM+B}}}\right]$$

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