The NNPDF4.0 aN^{3}LO PDFs

Giacomo Magni,
on behalf of NNPDF

Higgs WG1: aN^{3}LO PDF for Run3 & YR5
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email: gmagni@nikhef.nl

PDFs determination at aN^{3}LO

The NNPDF4.0 timeline

Sep 2021:
NNPDF4.0 [code & paper]

Jan 2024:
NNPDF4.0 MHOU
NNPDF4.0 QED

Feb 2024:
NNPDF4.0 aN3LO

Jun 2024:
NNPDF4.0 QED aN3LO

Several theoretical inputs are needed in a PDF fit:

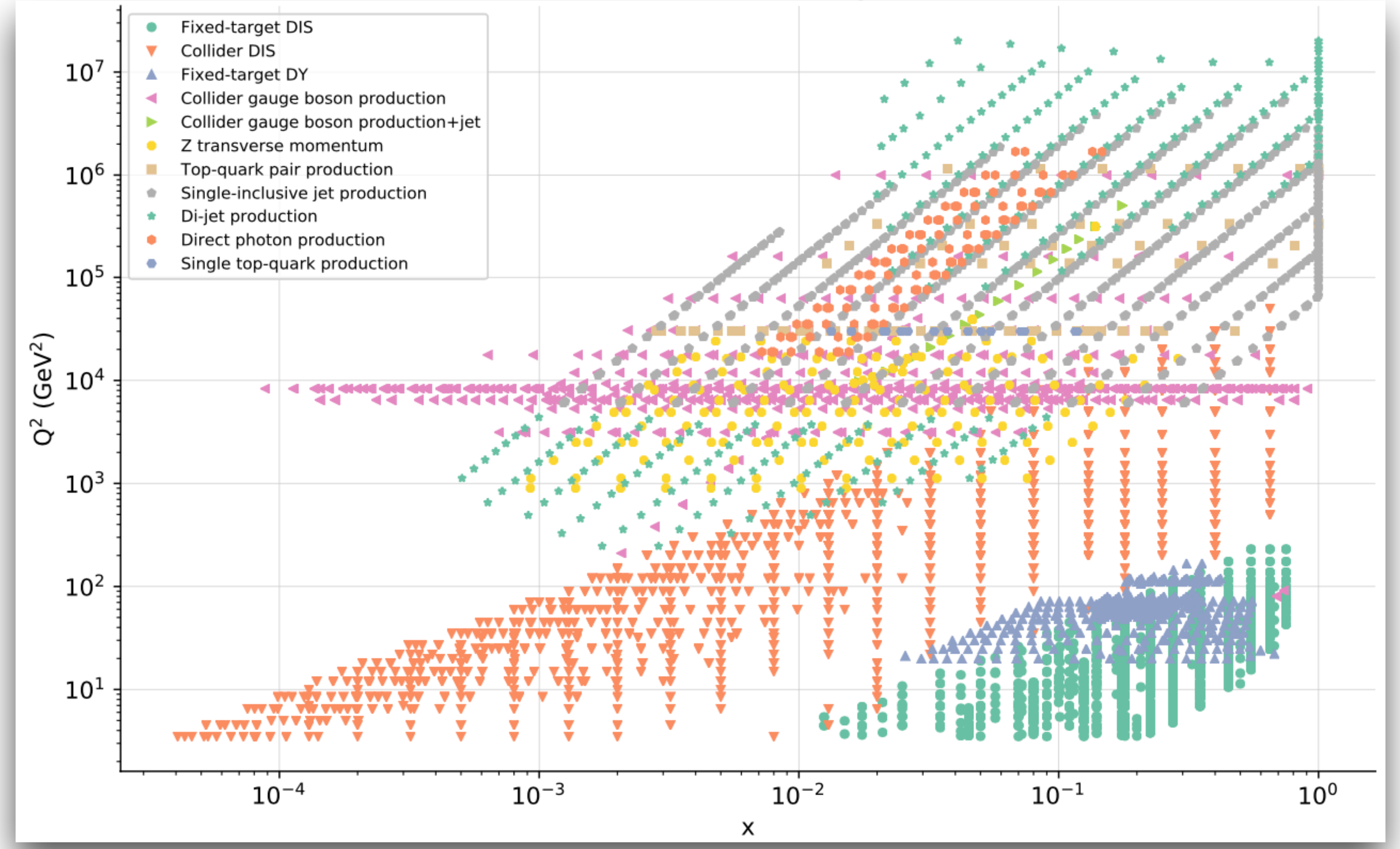
- QCD **splitting functions** which controls the DGLAP evolution.
- **VFNS matching conditions** for each running component.

$$f_i^{(n_f+1)}(x, Q^2) = A_{ij}(x, \alpha_s) f_j^{(n_f)}(x, Q^2)$$

- **Partonic coefficients** functions, accounting for massive corrections when possible.

$$\sigma(x, Q^2) = \sum_{i=0}^{n_f} C_{k,i}(x, \alpha_s) \otimes f_i(x, Q^2)$$

NNPDF4.0 Kinematic coverage



aN^3LO splitting functions

Analytical calculations of the complete N^3LO spitting functions are not available.
Large number of partial results available.

- **Large-n_f:** $\mathcal{O}(n_f^3)$, $P_{NS}^{(n_f^2)}$, Vogt et al. [arxiv:1610.07477]; $P_{qq,PS}^{(n_f^2)}$ Gehrmann et al. [arxiv:2308.07958]; $P_{gq}^{(n_f^2)}$ Falcioni et al. [arxiv:2310.01245];
- **NS small-x:** Davies et al. [arxiv:2202.10362] $P_{NS}^{(3)} \supset \sum_{k=0}^{6} \ln^k(x)$
- **Singlet small-x:** Bonvini, Marzani [arxiv:1805.06460] $P_{ij}^{(3)} \supset \sum_{k=0}^{3} \frac{\ln^k(x)}{x}$
- **Large-x:** Duhr et al. [arxiv:2205.04493]; Mistlberger et al. [arxiv:1911.10174]; Moch et al [arxiv:0912.0369].

$$P_{ii}^{(3)} \approx A_{4,i}\frac{1}{(1-x)_+} + B_{4,i}\delta(1-x) + C_{4,i}\ln(1-x) + D_{4,i}$$

$$P_{ij}^{(3)} \approx \sum_{k}^{6} \ln^k(1-x)$$

- 5 or 10 lowest **Mellin Moments:** Moch et al. [arxiv:1707.08315] [arxiv:2111.15561]; Falcioni et al. [arxiv:2302.07593], [arxiv:2307.04158] (more recent [arxiv:2404.09701], not included)

How do we combine the different limits?

- The approximation procedure is performed in **Mellin space** for each n_f part independently:

$$\gamma_{ij}^{(3)} = \gamma_{ij,n_f^3}^{(3)} + \gamma_{ij,N\to\infty}^{(3)} + \gamma_{ij,N\to 0}^{(3)} + \tilde{\gamma}_{ij}^{(3)}$$

- The parametrised part is constructed as:

$$\tilde{\gamma}_{ij} = \sum_{l} a_{ij}^{(l)} G_l(N)$$

- Vary the functions G_l to generate a set of approximation and determine **IHOU**

$$cov_{nm}^{(ij)} = \frac{1}{N_{ij}} \sum_{k=1}^{N_{ij}} \Delta_n^{(k)}\Delta_m^{(k)}, \quad \Delta_n^{(k)} = T_n^{(k)} - \bar{T}_n$$

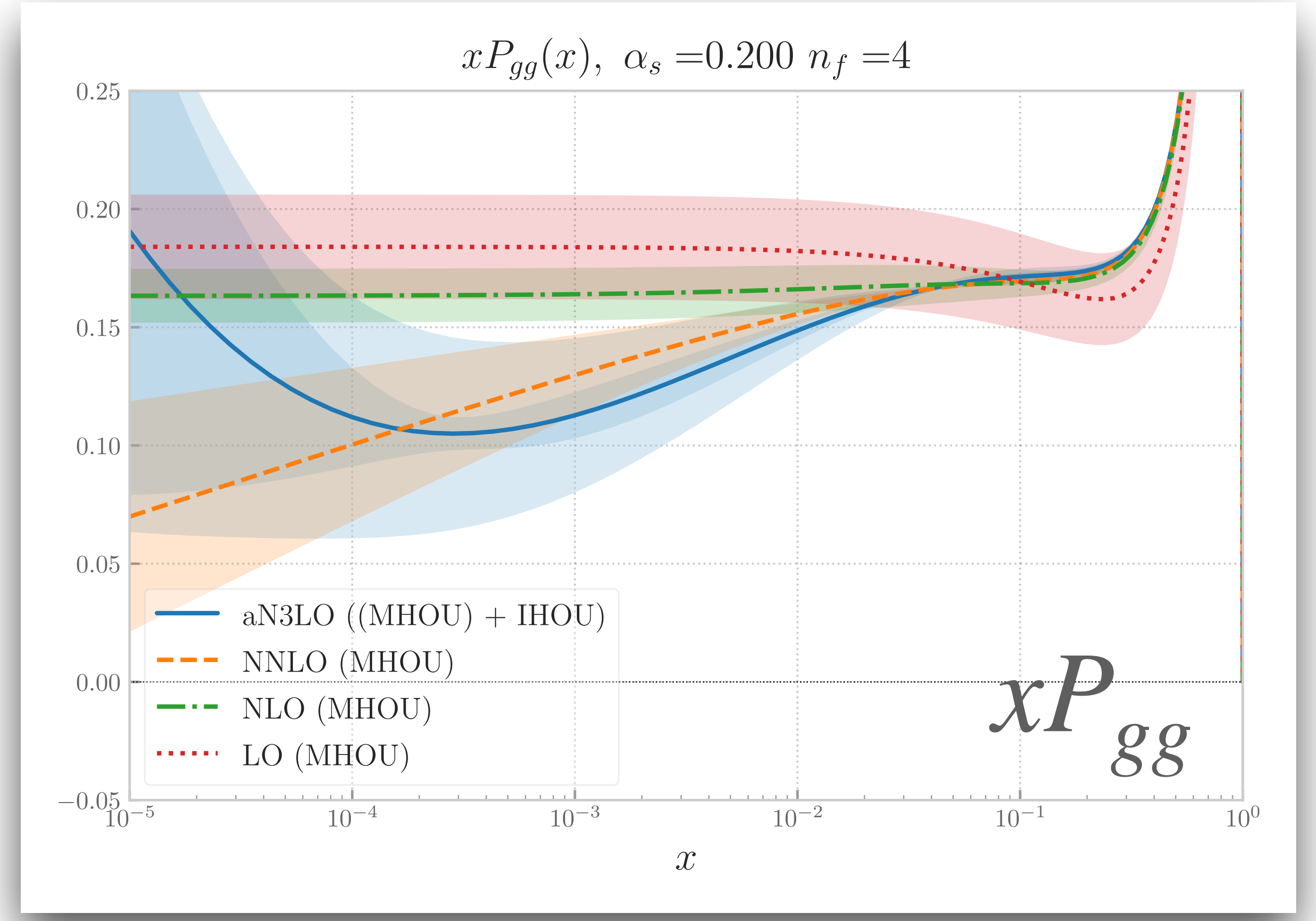
- Determine **MHOU from scale variation.**

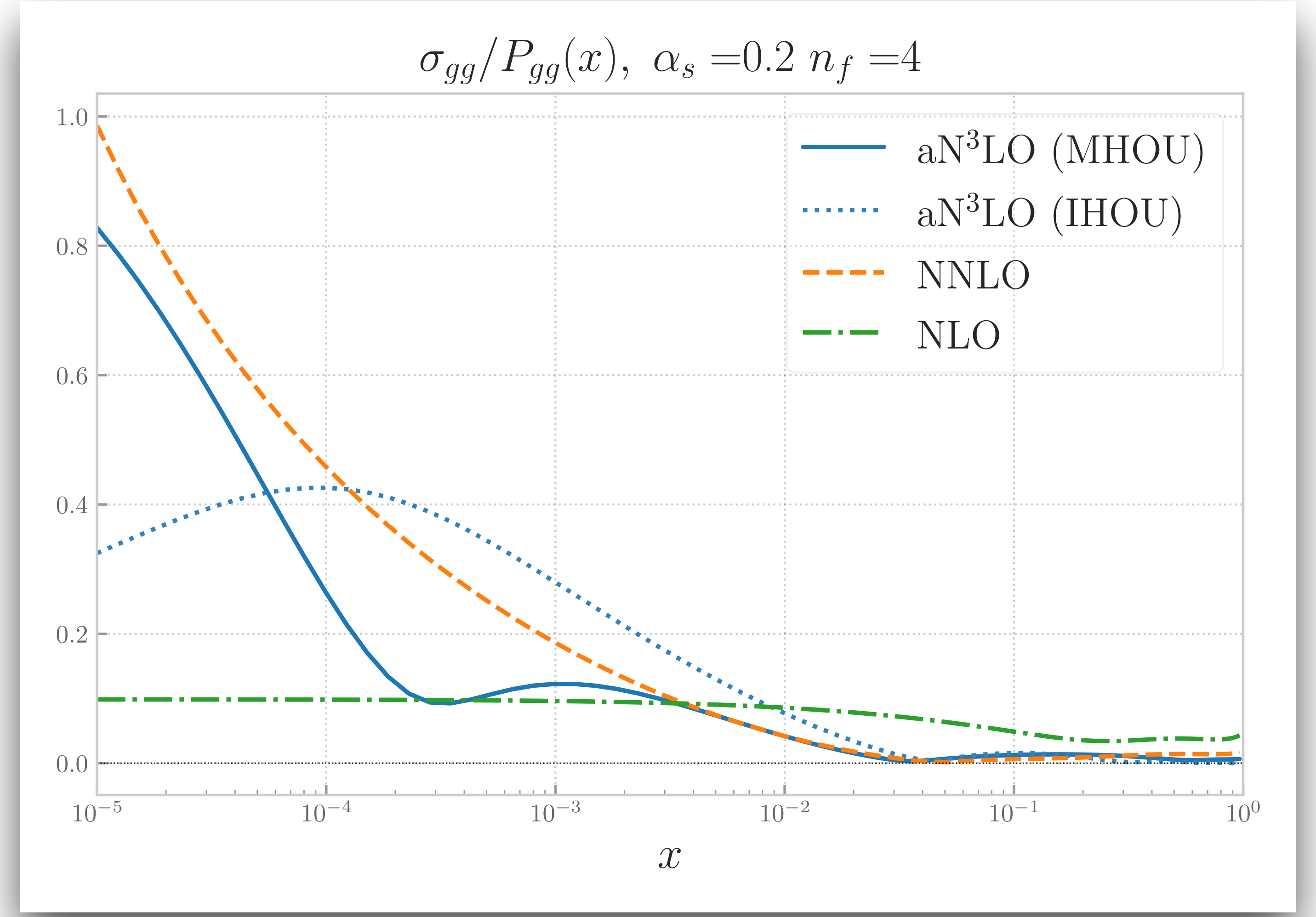
$$cov_{nm} = cov_{MHOU} + cov_{IHOU}$$

aN^{3}LO splitting functions

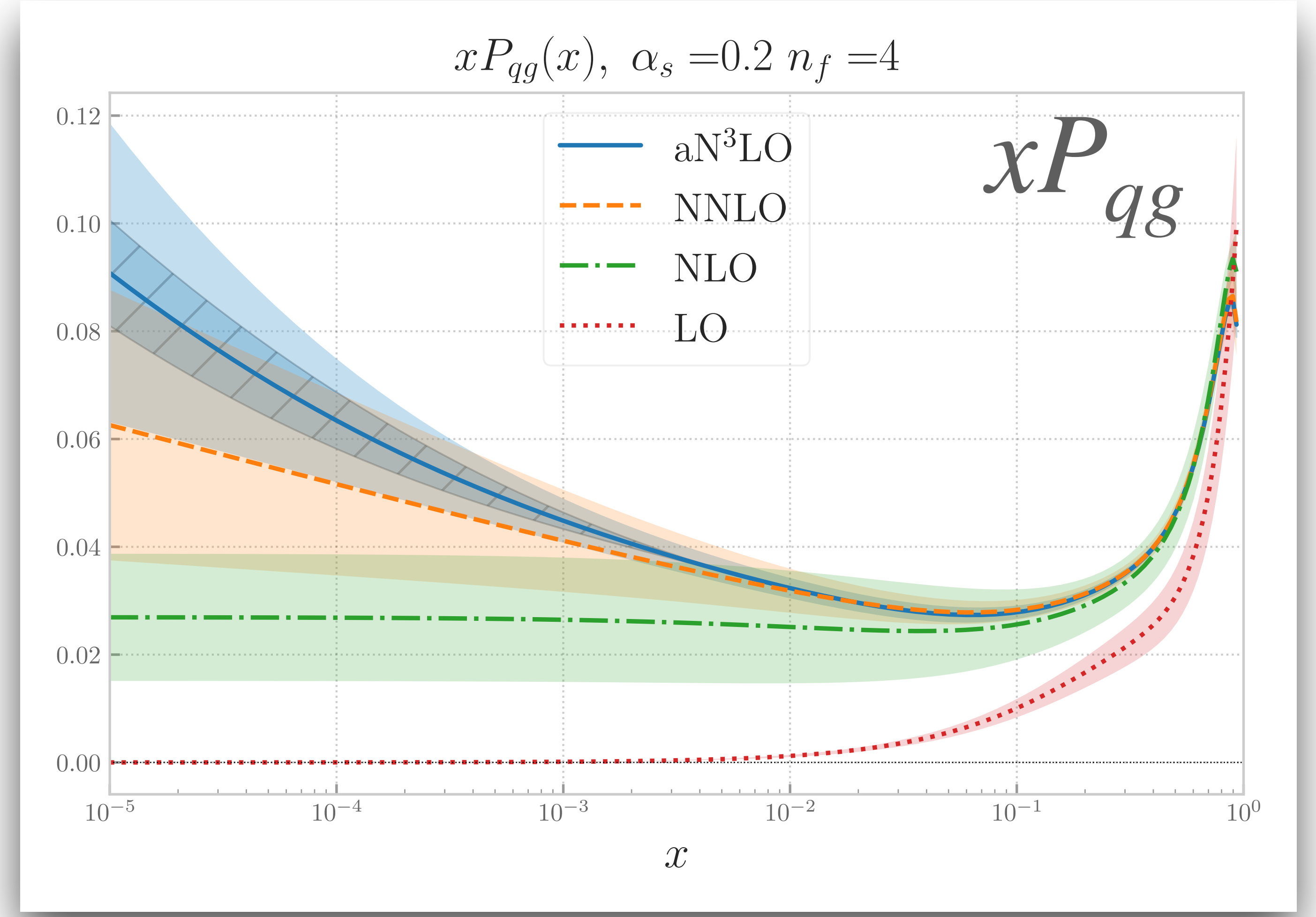
- For P_{qg}, P_{qq}, P_{gq} the N^{3}LO **approximation uncertainty is negligible** [IHOU < MHOU].
- In P_{gg} the **N^{3}LO approximation uncertainty is significant** [IHOU > MHOU for $x \geq 10^{-4}$].

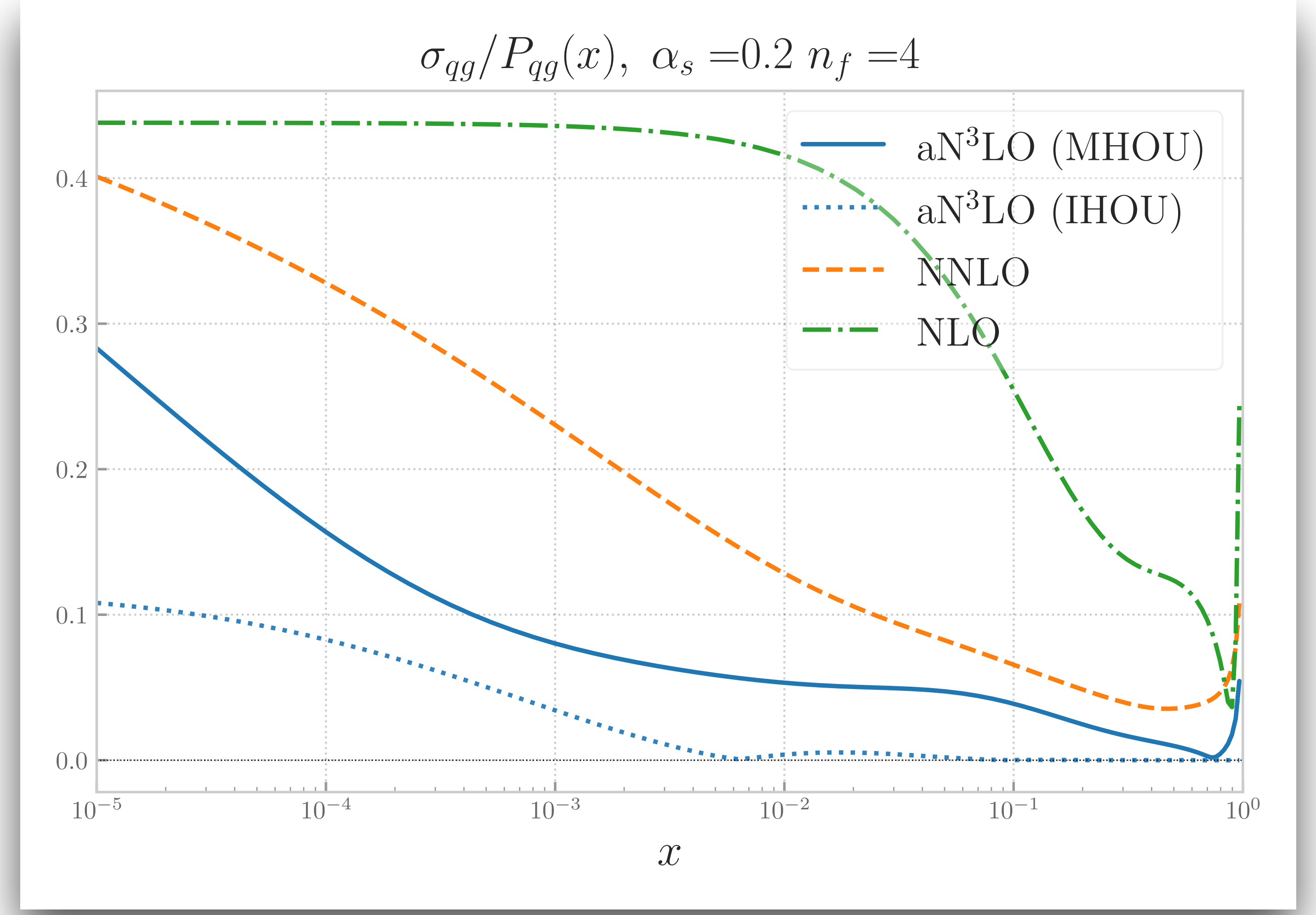
Gluon sector





Quark sector





- **IHOU** = incomplete higher order uncertainties [only for aN^{3}LO].
- **MHOU** = missing higher order uncertainties.

aN^{3}LO DIS coefficient functions

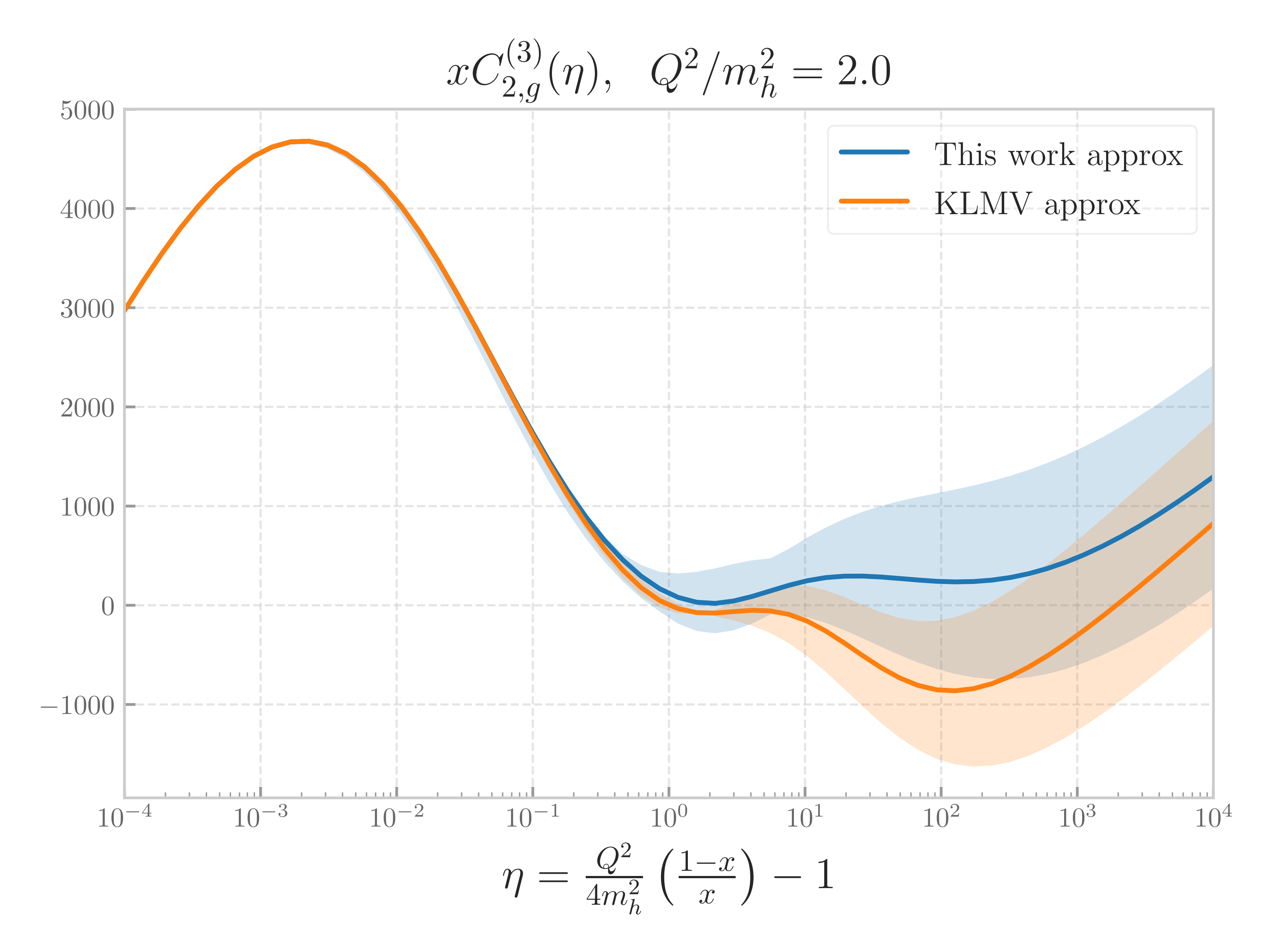
DIS structure functions are known at N^{3}LO in the **massless limit** for F_2, F_L, F_3:

- DIS NC: Larin, Nogueira, Van Ritbergen, Vermaseren [arxiv:9605317] Moch, Vermaseren, Vogt [arxiv:0411112], [arxiv:0504242] Blümlein et al. [arxiv:2208.14325].
- DIS CC: Davies, Moch, Vermaseren, Vogt [arxiv:0812.4168] [arxiv:1606.08907]

DIS **Heavy structure functions** can be parametrised joining the known limits ($Q \to m_h^2$ $Q \gg m_h^2$ and $x \to 0$) with proper damping functions f_1, f_2: N.Laurenti [arxiv:2401.12139]

$$C_{g,h}^{(3,0)} = C_{g,h}^{thr}(z, \frac{m_h}{Q})\, f_1(z) + C_{g,h}^{asy}(z, \frac{m_h}{Q})\, f_2(z)$$

Approximate N^{3}LO massive DIS



IHOU from massive coefficient are also taken into account.

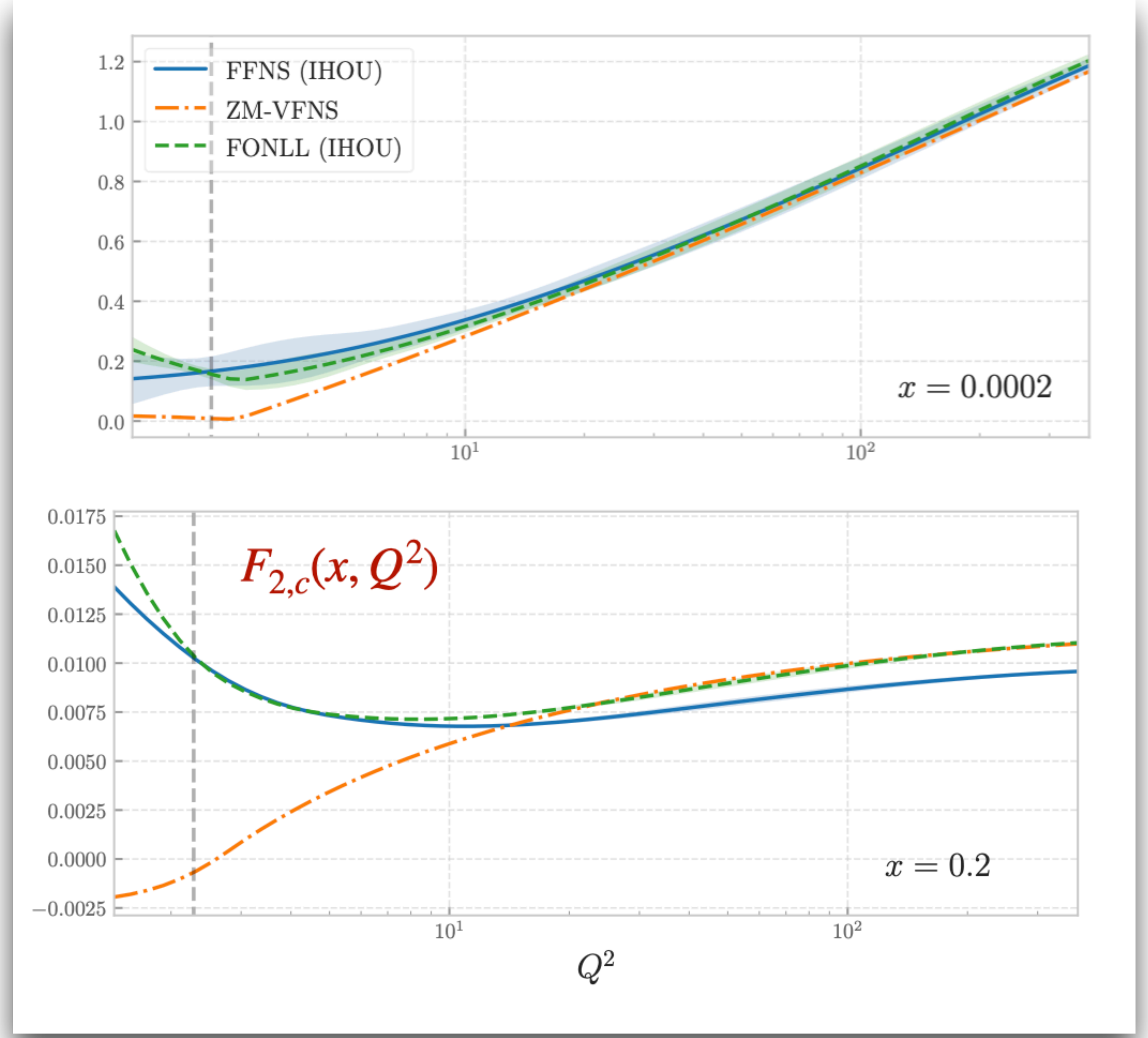
KLMV Kawamura, Lo Presti, Moch, Vogt [arxiv:1205.5727]

DIS VFNS at aN^{3}LO

To treat heavy quarks consistently during a PDF fit we must adopt a **Variable Flavor Number Scheme.**

PDFs matching conditions included at N^{3}LO almost completely: Blümlein et al. [arxiv:0904.3563] [arxiv:1008.3347] [arxiv:1402.0359] [arxiv:1409.1135] [arxiv:1406.4654] [arxiv:2211.0546] [arxiv:2311.00644] exception of $a_{H,g}^{(3)}$, computed in [arxiv:2403.00513]

$$\begin{pmatrix} g \\ \Sigma \\ h^+ \end{pmatrix}^{n_f+1} (\mu_h^2) = \mathbf{A}_{S,h^+}^{(n_f)}(\mu_h^2) \cdot \begin{pmatrix} g \\ \Sigma \\ h^+ \end{pmatrix}^{n_f} (\mu_h^2)$$



DIS structure functions are computed in the **FONLL** procedure: [arxiv:1001.2312]

- Extended up to N^{3}LO for the Heavy structure functions F_{heavy}
- Extended up to NNLO for light F_{light} + massless N^{3}LO contributions.

$$F_{h,FONLL} = F_{ZM}^{(n_f+1)} + F_{FFNS}^{(n_f)} - \lim_{m_h \to 0} F_{FFNS}^{(n_f)}$$

aN^{3}LO theory predictions

To produce our N^{3}LO PDF fit:

- We include **N^{3}LO corrections in DIS and DGLAP** with their respective IHOU.
- We adopt NNLO scale variation to estimate *unknown* N^{3}LO effects in DY, jets and top data.
- **MHOU and IHOU** are propagated to PDF fit with the **covariance formalism**:

$$\mathrm{Cov}_{tot} = \mathrm{Cov}_{exp} + \mathrm{Cov}_{DGLAP,IHOU} + \mathrm{Cov}_{DIS,IHOU} + \mathrm{Cov}_{HAD,MHOU}\ [+\mathrm{Cov}_{MHOU}]$$

Experimental Covariance matrix

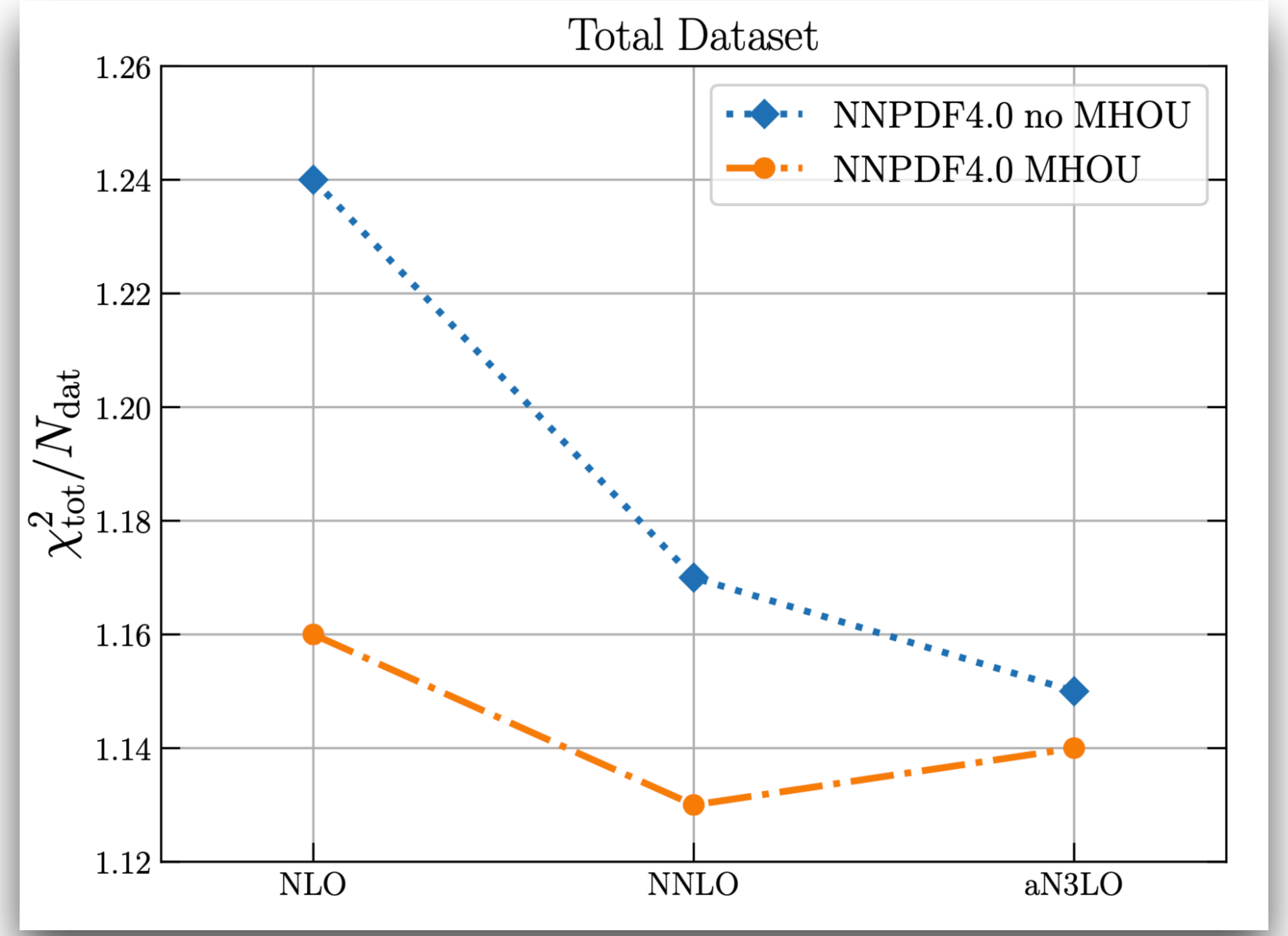
N^{3}LO DGLAP

Massive corrections to DIS

Missing N^{3}LO for Hadronic processes

MHOU Covariance matrix

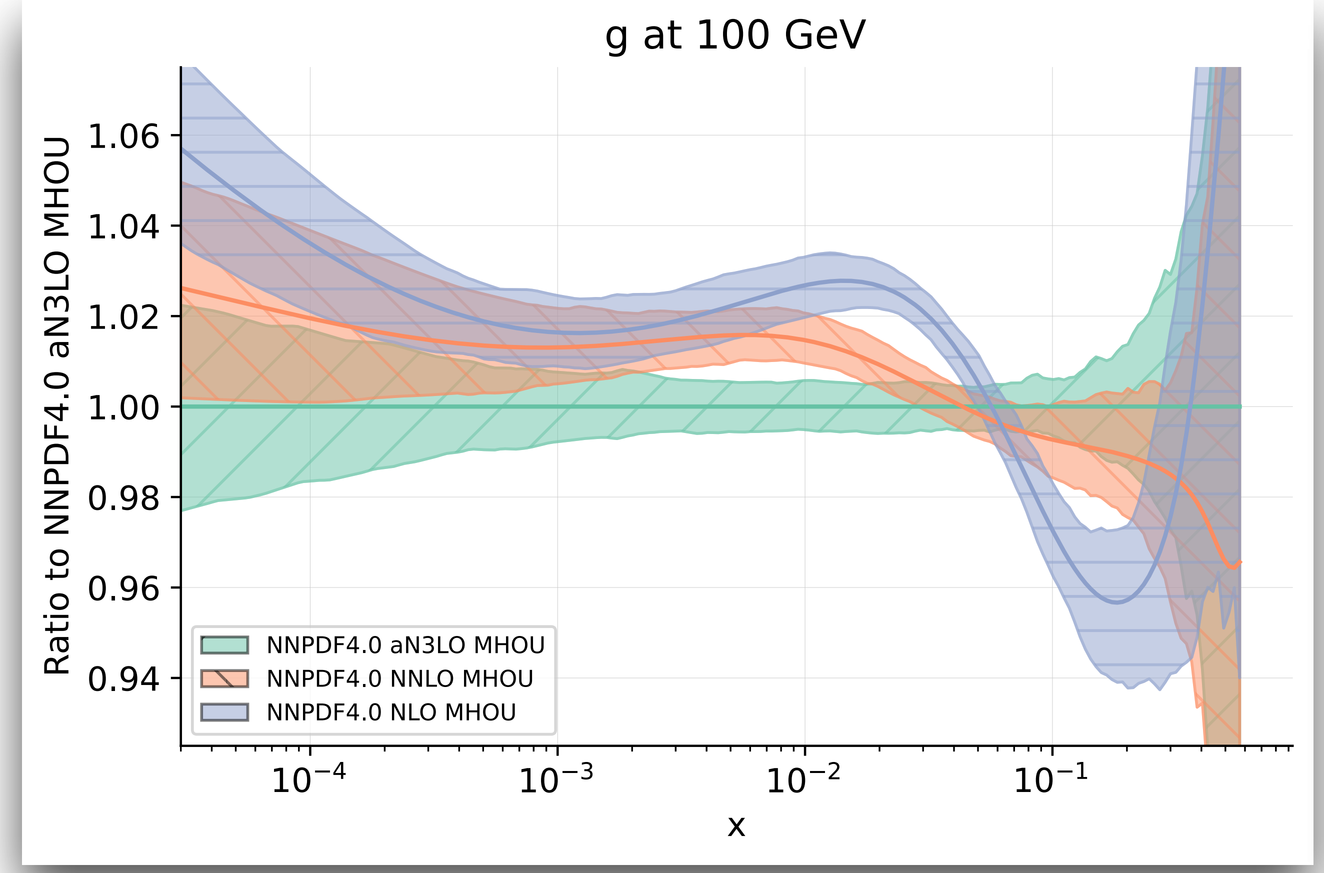
Total χ^2 at different QCD orders

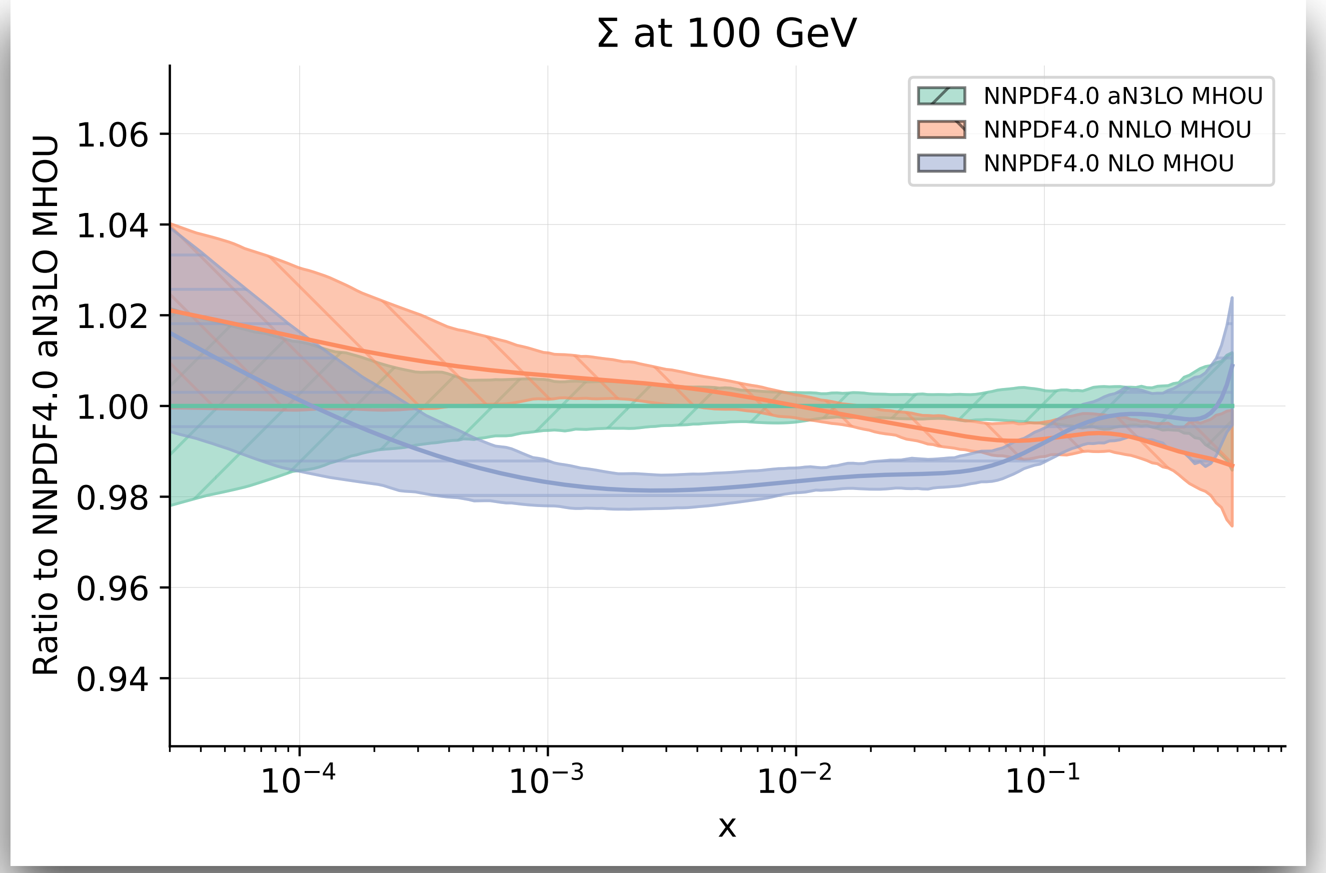


- **MHOU stabilise the fit**: χ^2 is less dependent on QCD order.
- MHOU can shift central value, resolve tensions among datasets. Mainly de-weight jets datasets.

The NNPDF4.0 aN3LO PDF set

Perturbative convergence

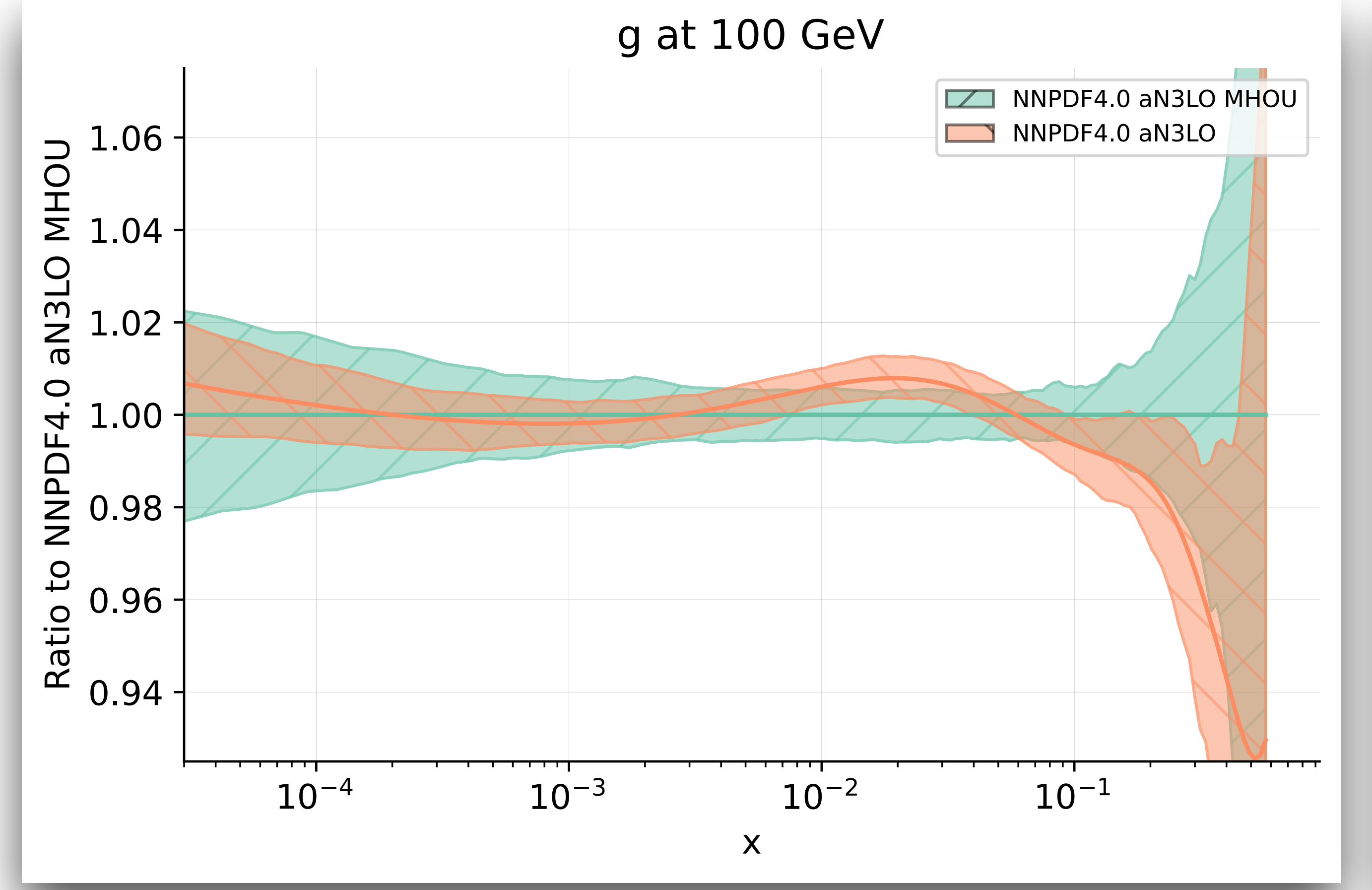


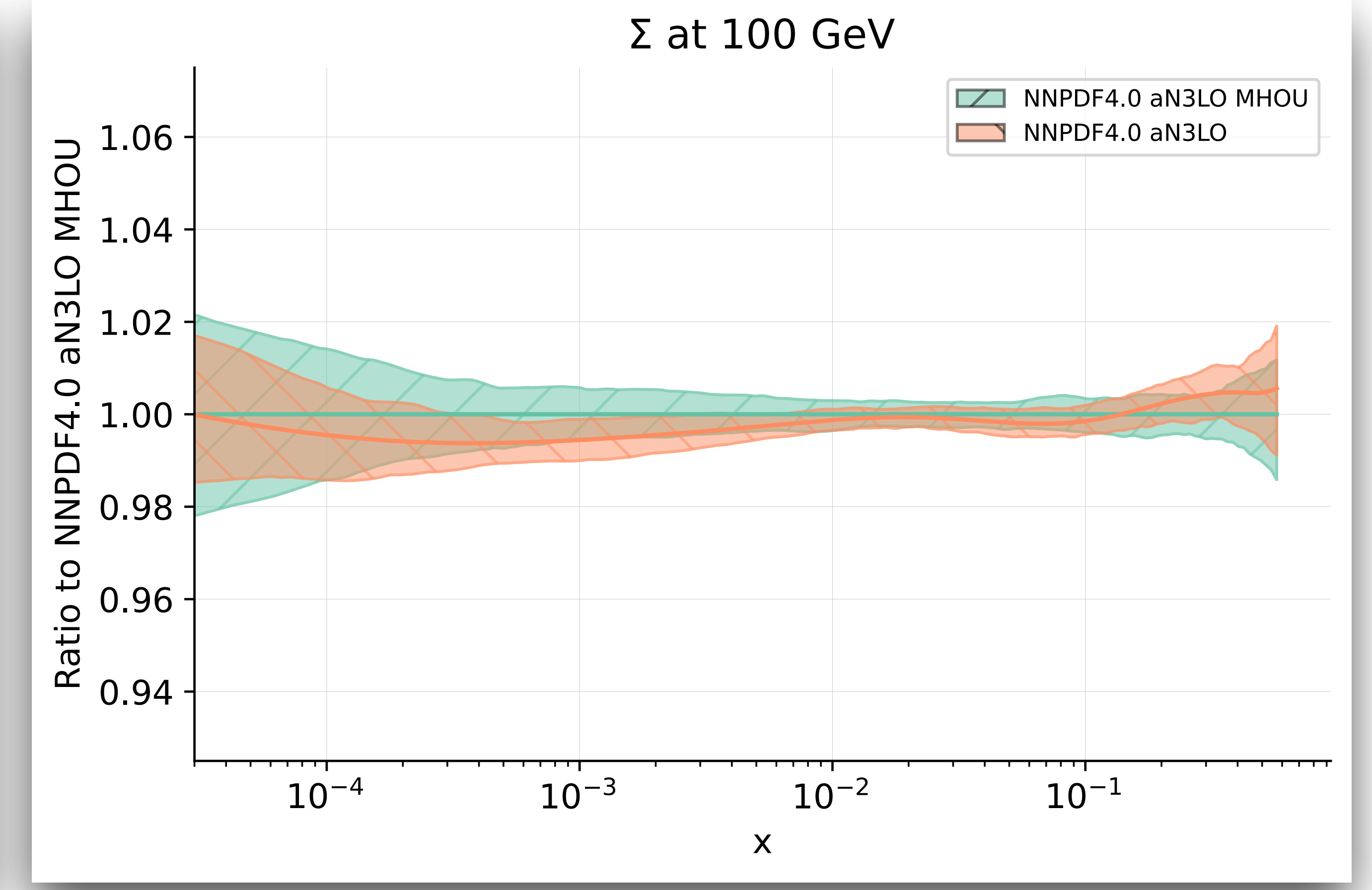


- Good perturbative convergence in the data region.
- Impact of **aN^{3}LO** corrections is **mild on quarks PDFs.**
- $\sim 2\,\%$ effect on the **gluon around $x \approx 10^{-2}$.**

Impact of MHOU

- aN^{3}LO PDFs with/without MHOU are compatible.
- aN^{3}LO corrections have a larger effect on the small-x, low-Q DIS data.





LHC phenomenology: Higgs production

- aN^3LO PDFs effects are visible in gluon fusion, leading to a $2.1\,\%$ **suppression w.r.t NNLO PDFs**.

$$\text{NNLO PDF:}\quad \sigma(gg \to H) = 44.73 \pm 0.26\ (\text{pdf}) \pm 2.1\ (\text{scale})\ [\text{pb}]$$

$$\text{aN3LO PDF:}\quad \sigma(gg \to H) = 43.78 \pm 0.24\ (\text{pdf}) \pm 2.0\ (\text{scale})\ [\text{pb}]$$

- Higgs VBF is more stable at different perturbative orders, although the PDF dependency is not negligible.

- More faithful estimation of NNLO TH uncertainties due to HO, YR4 estimate is too optimistic ($\sim 1\,\%$):

Effect of aN^3LO PDF

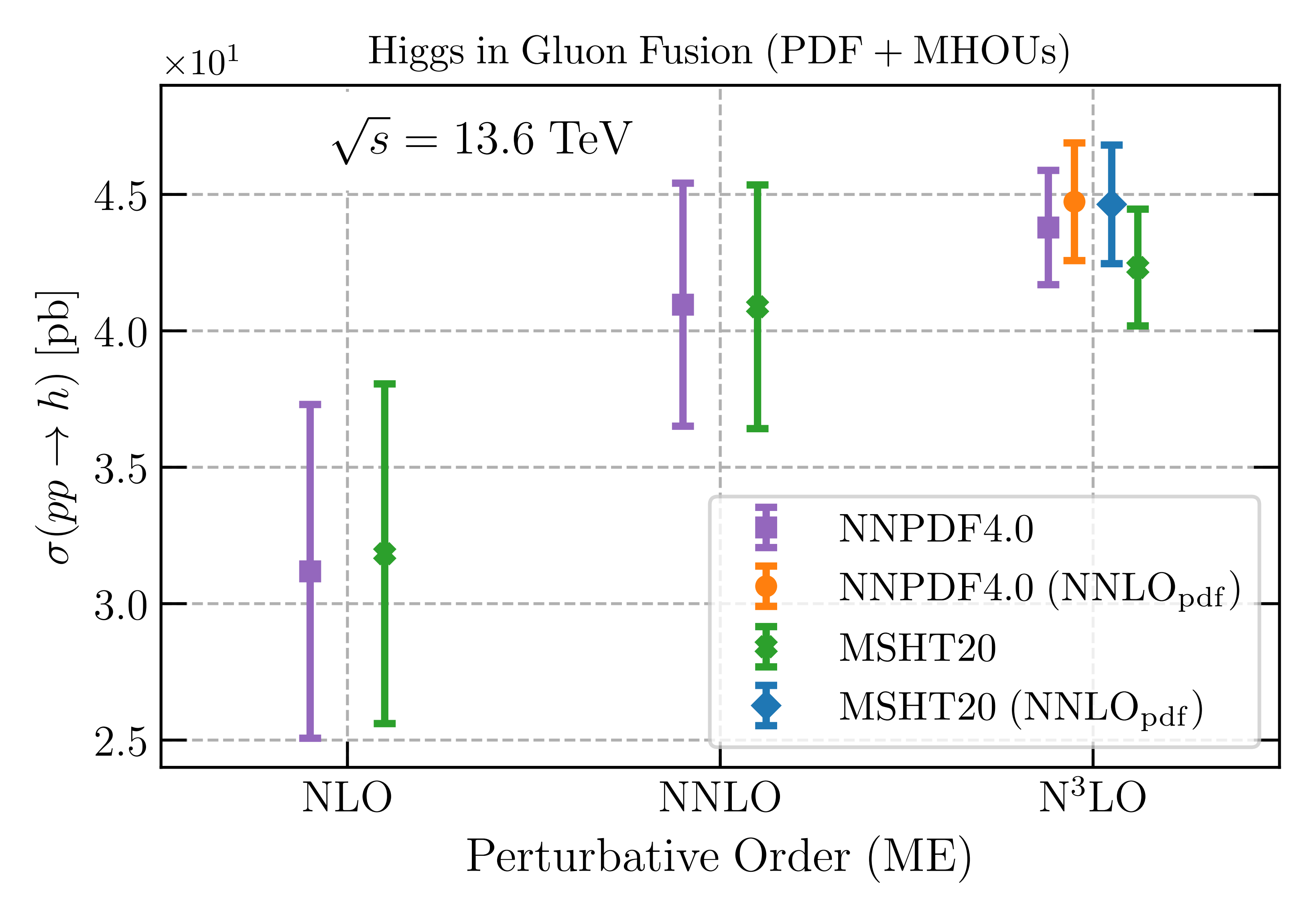
$$\Delta^{\text{exact}}_{\text{NNLO}} \equiv \left| \frac{\sigma^{\text{N}^3\text{LO}}_{\text{N}^3\text{LO-PDF}} - \sigma^{\text{N}^3\text{LO}}_{\text{NNLO-PDF}}}{\sigma^{\text{N}^3\text{LO}}_{\text{N}^3\text{LO-PDF}}} \right|$$

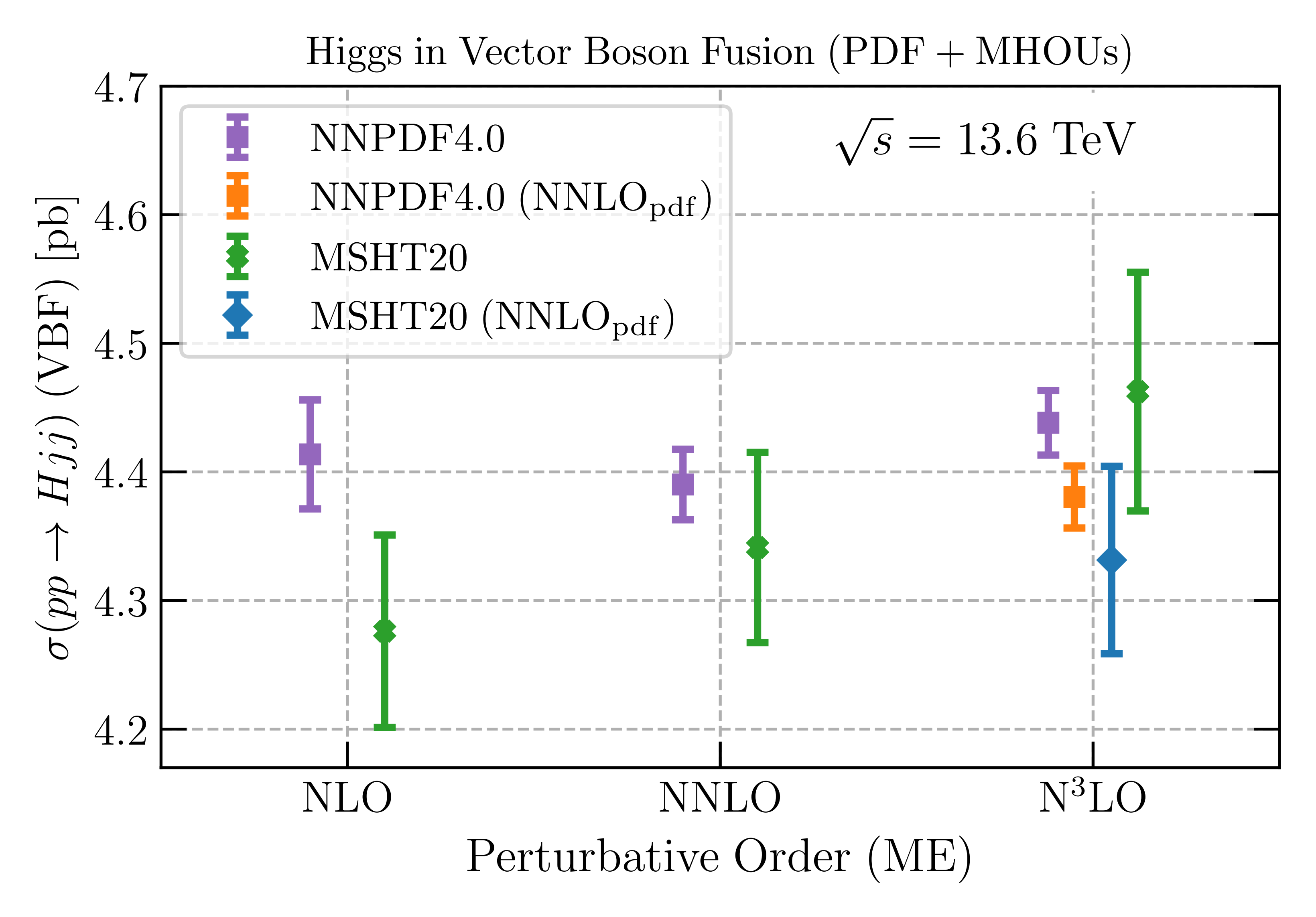
YR4 estimate

$$\Delta^{\text{app}}_{\text{NNLO}} \equiv \frac{1}{2} \left| \frac{\sigma^{\text{NNLO}}_{\text{NNLO-PDF}} - \sigma^{\text{NNLO}}_{\text{NLO-PDF}}}{\sigma^{\text{NNLO}}_{\text{NNLO-PDF}}} \right|$$

Relative uncertainty (%)

Process	NNPDF4.0					
	σ (pb)	δ_{th}	$\delta^{\text{noMHOU}}_{\text{PDF}}$	$\delta^{\text{MHOU}}_{\text{PDF}}$	$\Delta^{\text{app}}_{\text{NNLO}}$	$\Delta^{\text{exact}}_{\text{NNLO}}$
$gg \to h$	43.8	4.8	0.6	0.7	0.2	2.2
h VBF	4.44	0.6	0.5	0.6	0.2	1.3
hW^+	0.97	0.6	0.5	0.6	0.2	0.5
hW^-	0.61	0.6	0.6	0.6	0.2	0.3
hZ	0.87	0.5	0.4	0.5	0.1	0.3





aN^{3}LO PDF with QED corrections

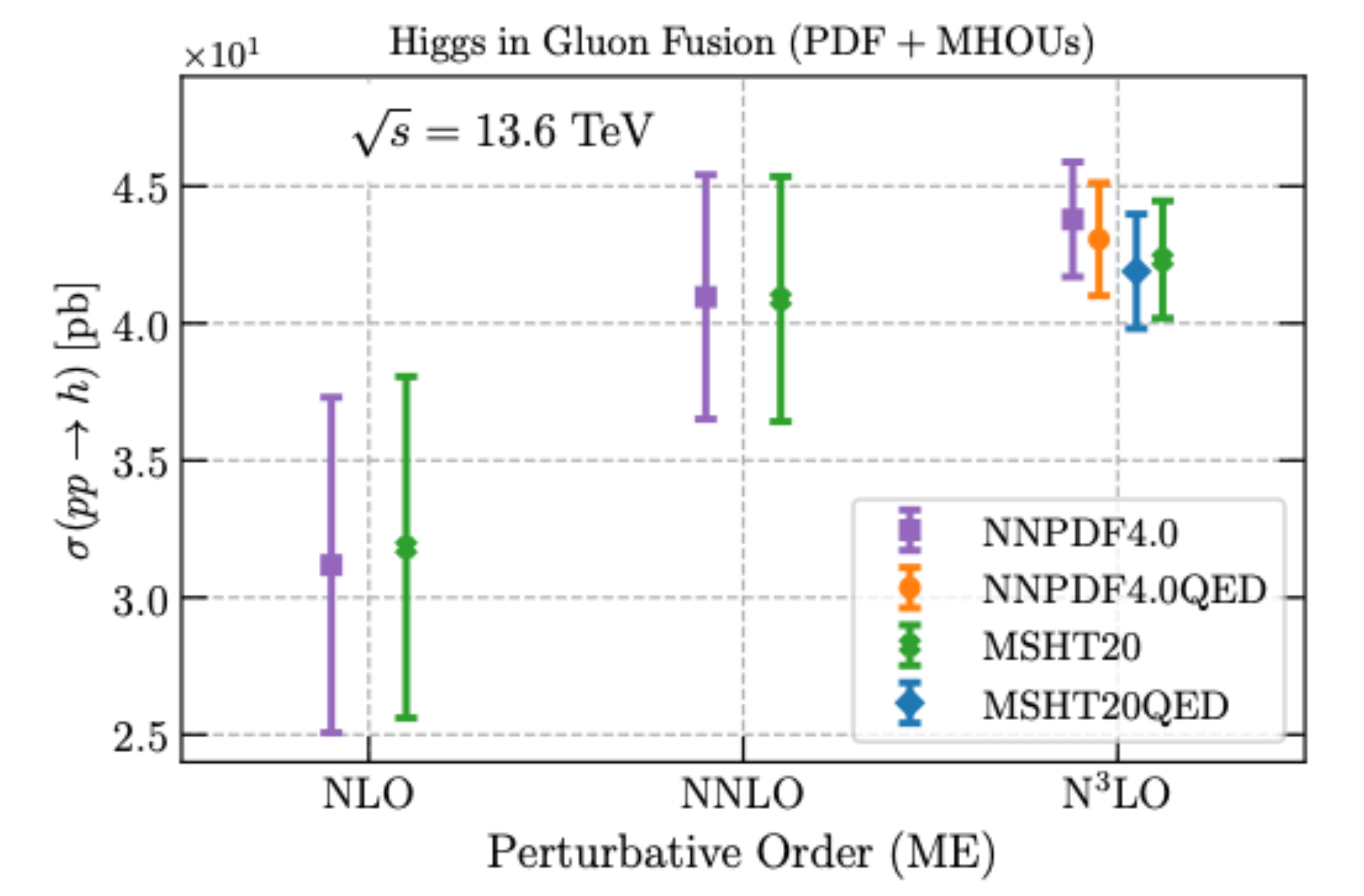
Barontini, Laurenti, Rojo [arxiv:2406.01779]

Recently we have also provided an additional global fits:

✓ **NNPDF40 QED aN3LO**

The photon **PDF is computed from DIS** structure functions at a given **high Q^2** scale.
[LuxQED Manohar et al. [arxiv:1607.04266] [arxiv:1708.01256]]

DGLAP with mixed $QED \otimes QCD$: $\mathcal{O}(\alpha_s \alpha_{em}), \mathcal{O}(\alpha_{em}^2)$

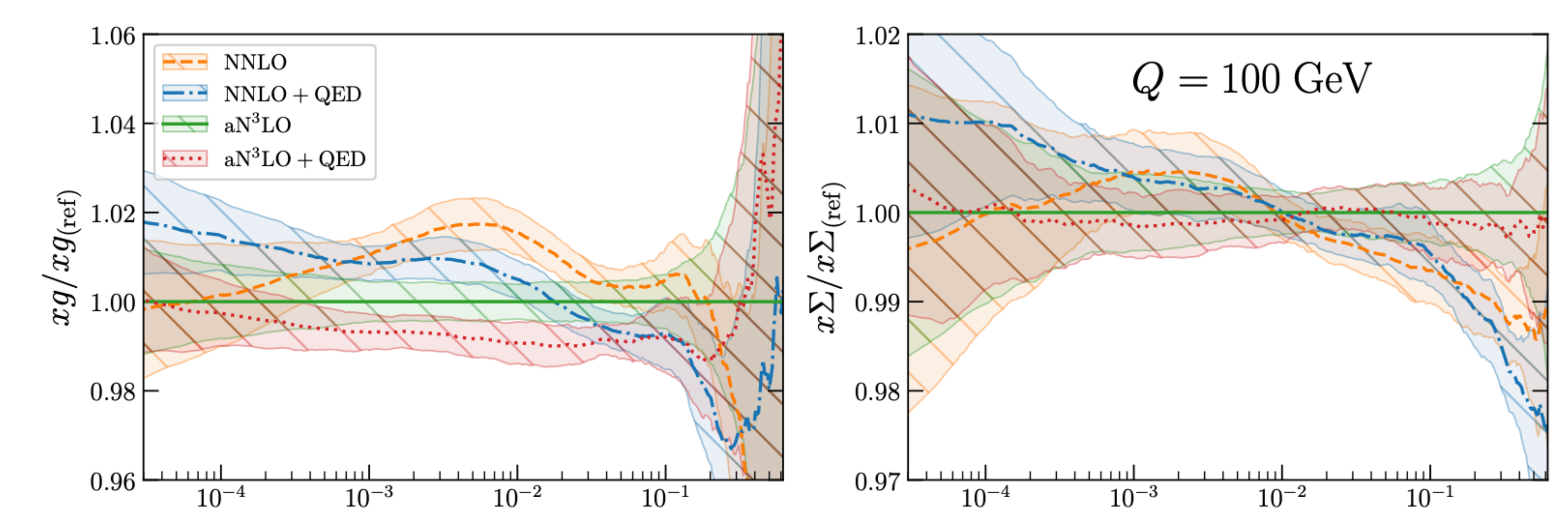


- The photon PDF subtracts some momentum from other partons (especially gluon):

$$\int_0^1 x dx \left(g(x) + \sum_i q_i^+(x) + \gamma(x) \right) = 1$$

- **QED** effects on the PDFs **are comparable to QCD aN^{3}LO** corrections, **both must be taken into account** to achieve best accuracy.

- At large-x, similar effect on the $\gamma(x, Q^2)$ PDF as in MSTH20 aN3LO QED Cridge et al. [arxiv:2312.07665]



Summary & outlook

Newest NNPDF4.0 releases:

- ✓ NNLO theory uncertainties through scale variations.
- ✓ aN^{3}LO QCD: state of the art **DGLAP** and **DIS**, along with theory uncertainties.
- ✓ Determination of Photon PDF.

- aN^{3}LO PDFs can be used both **with N^{3}LO partonic matrix elements.**
- aN^{3}LO PDFs can be used to **evaluate** more precisely **missing higher order** effects.
- Reduction of the N^{3}LO $gg \to H$ cross section: $-2.1\,\%$ `(aN3LO PDF)`, $-3.7\,\%$ `(aN3LO + QED PDF)` w.r.t. NNLO PDFs (and fixed matrix element)

Possible combination of `NNPDF40 aN3LO` and `MSTH20 aN3LO` is technically feasible (PDF4LHC like combination)

Jan 2024:
NNPDF4.0 MHOU
NNPDF4.0 QED

Feb 2024:
NNPDF4.0 aN3LO

Jun 2024:
NNPDF4.0 QED aN3LO

...

WIP:
Towards NNPDF4.1

Summary & outlook

Newest NNPDF4.0 releases:

✓ Theory uncertainties **[NNPDF4.0 MHOU]**

✓ aN^{3}LO effects in DGLAP and DIS **[NNPDF4.0 aN3LO]**

✓ Photon PDF **[NNPDF4.0 QED, NNPDF4.0 QED aN3LO]**.

Possible combination of NNPDF40 aN3LO and MSTH20 aN3LO is technically feasible (PDF4LHC like combination)

Towards NNPDF4.1 :

- Full NNLO: removal of NNLO k-factors (Matrix-Pineappl interface)
- EWK corrections through k-factors
- Improved methodology: for ex. extended Hyperoptimization
- Extension of fitted data (LHC 13 TeV): DY, Top, Jets; DIS + Jet

Jun 2024:
NNPDF4.0 QED aN3LO

Jun 2024:
NNPDF4.0 MC

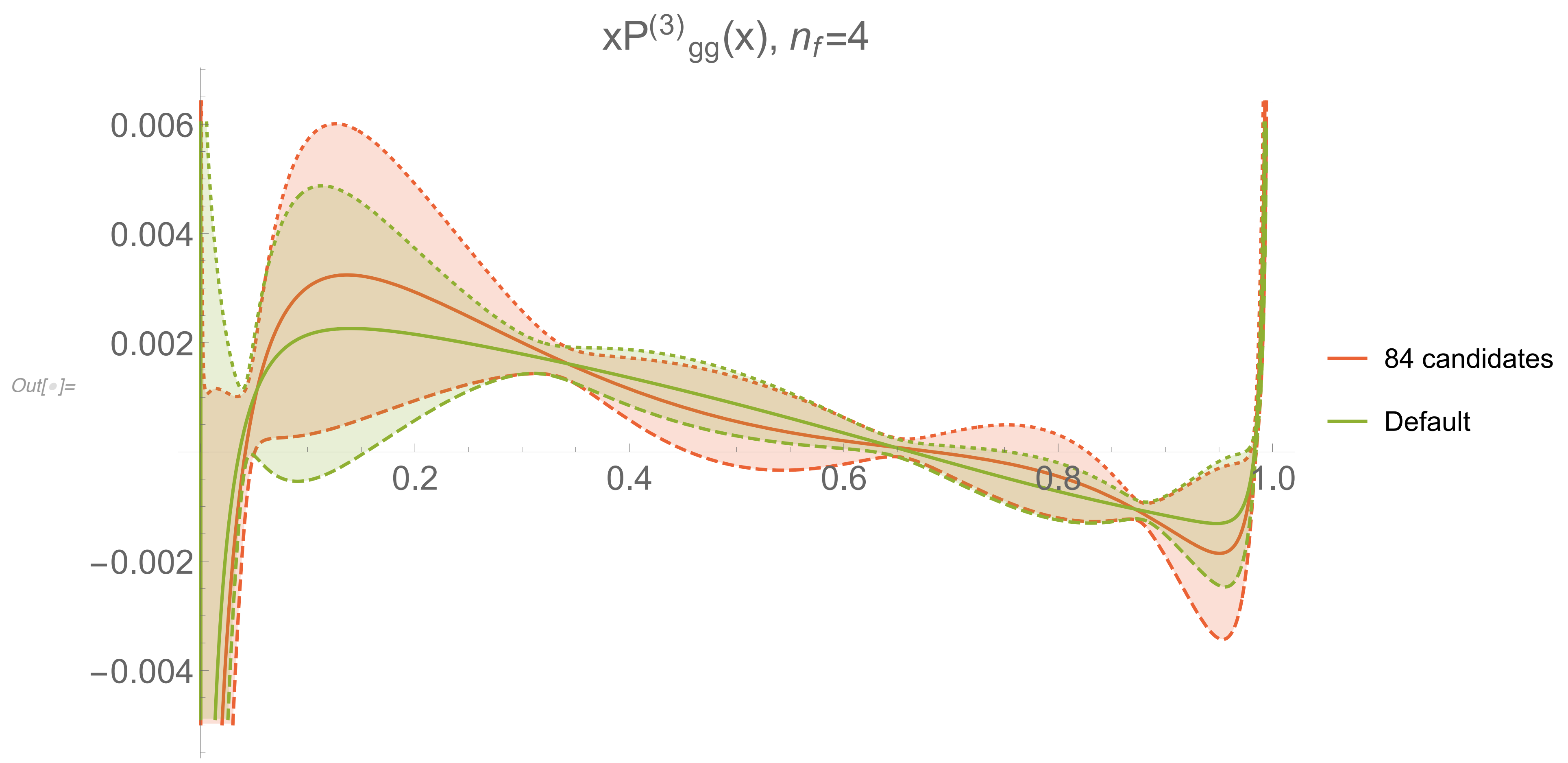
WIP:
Determination of α_s

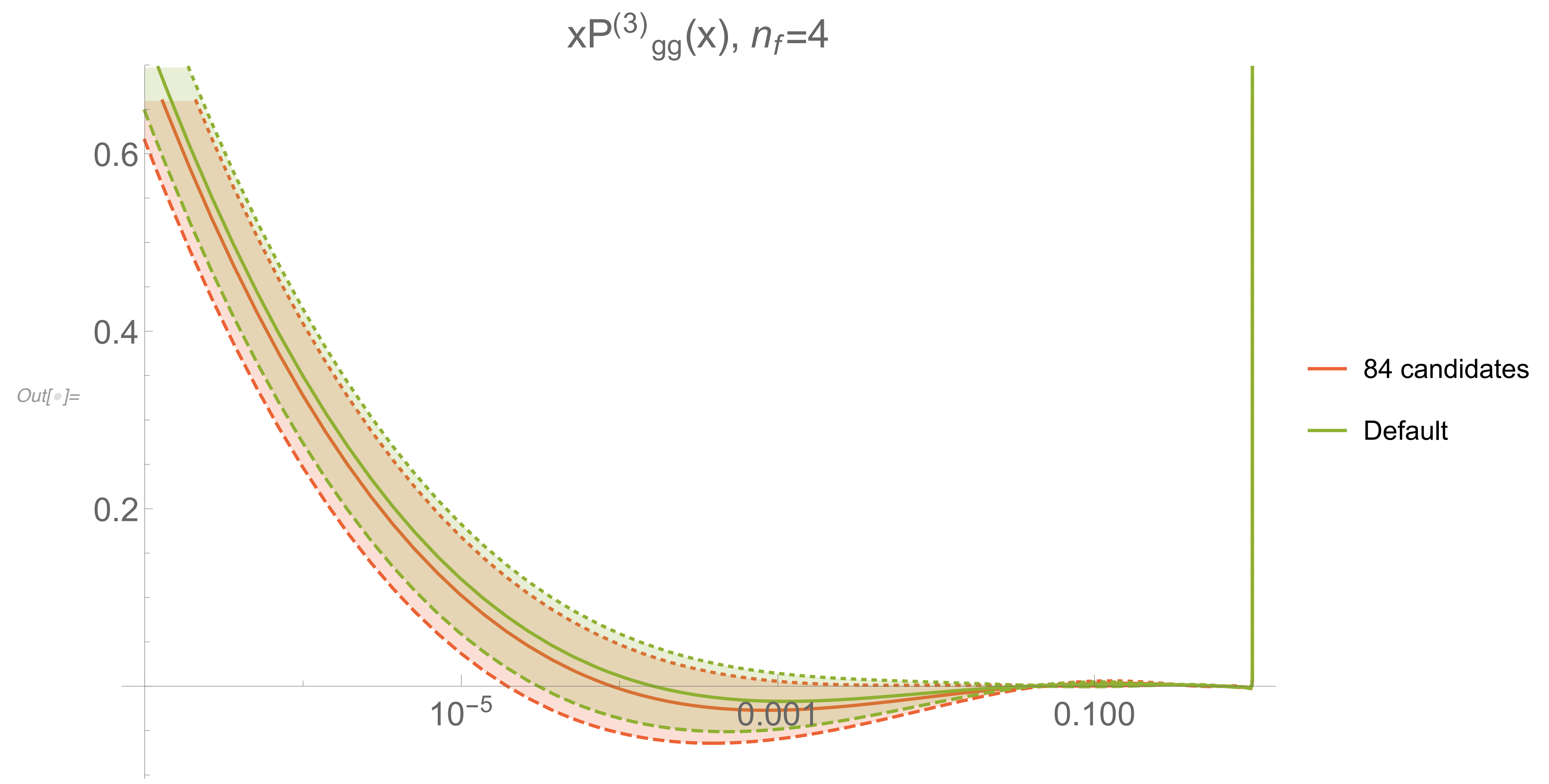
...

WIP:
Towards NNPDF4.1

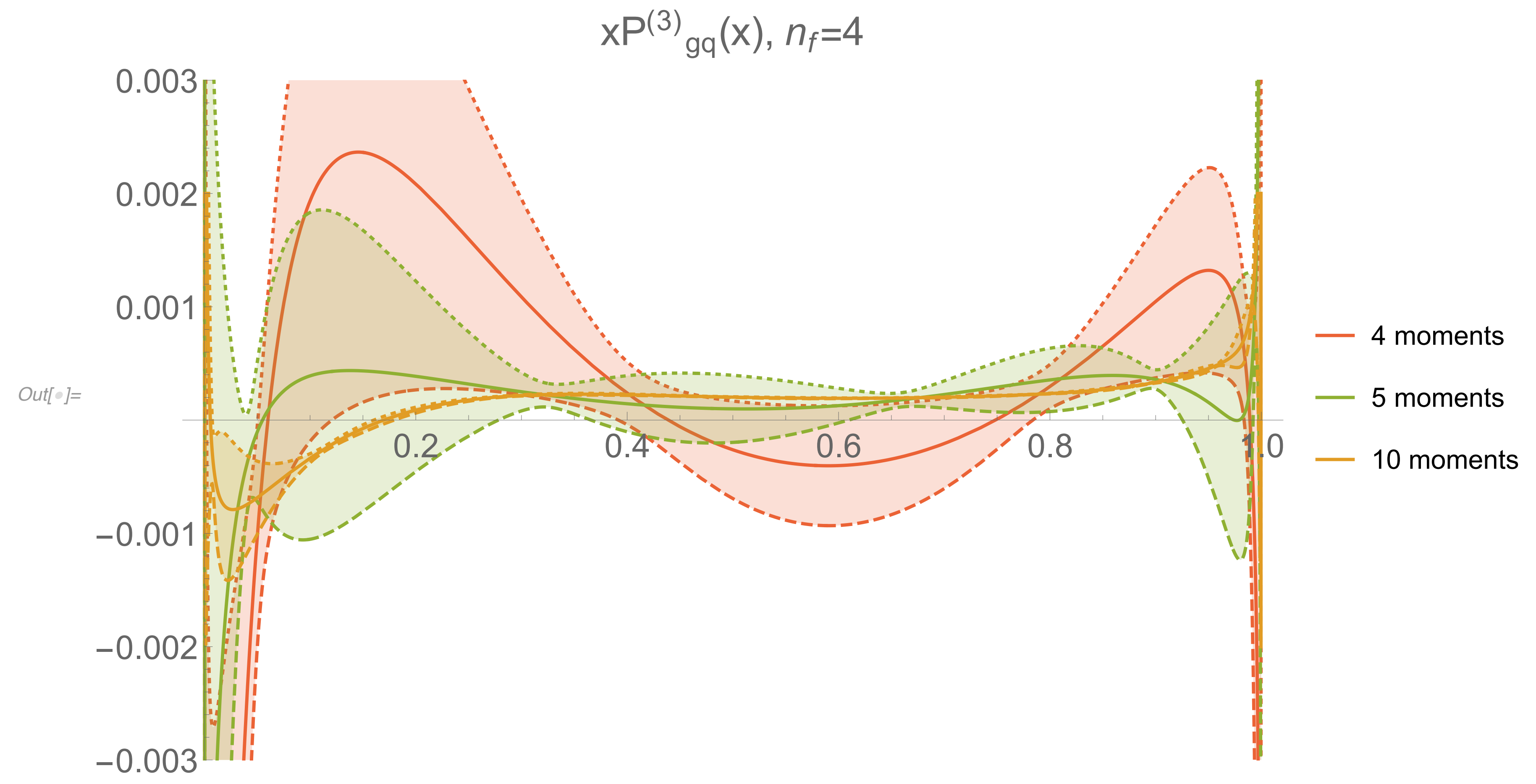
aN^{3}LO splitting functions approxmation

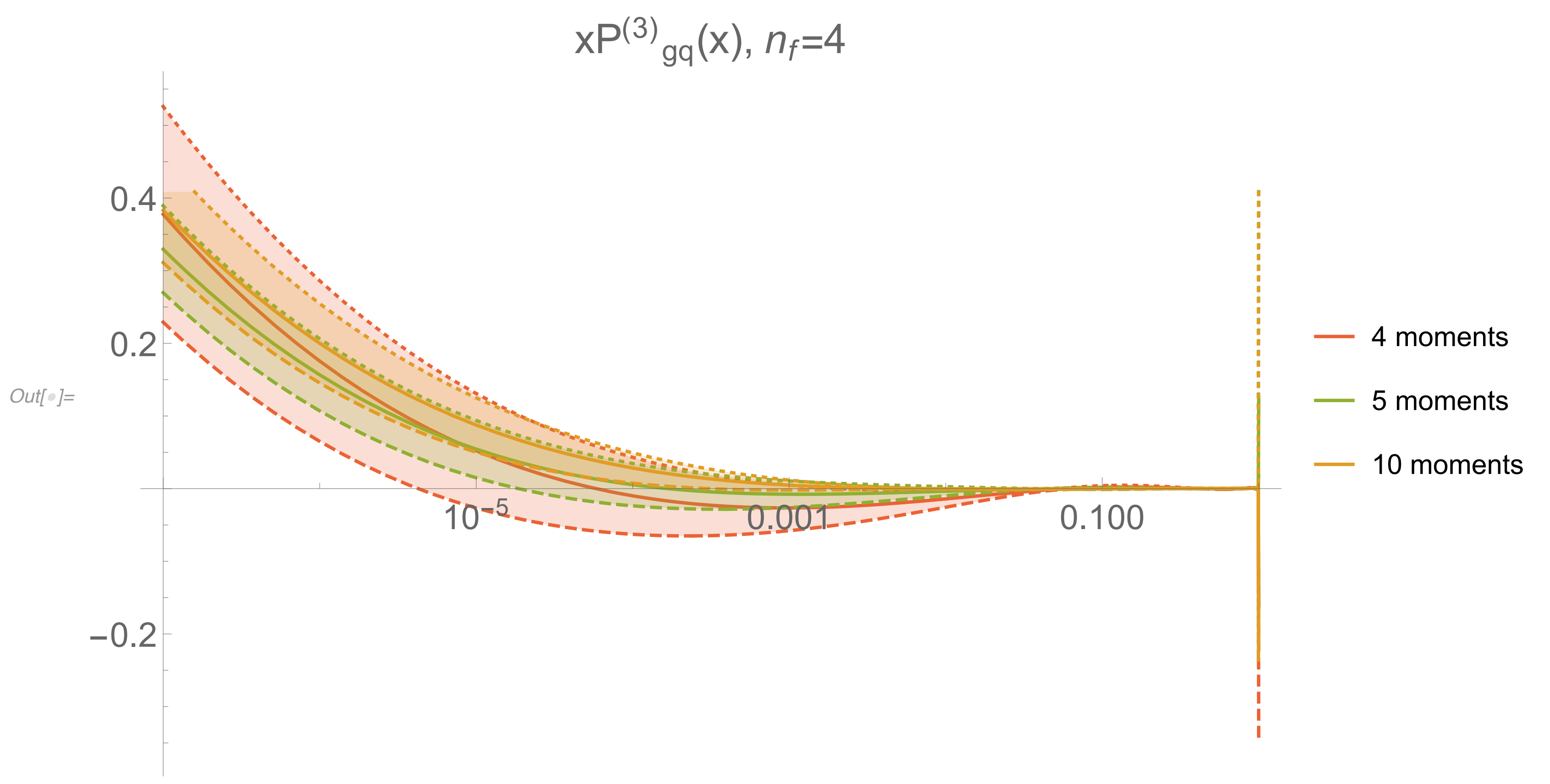
How does the approximation change if we add more test functions?





How does the approximation change if we add more Mellin moments?





Hadronic processes: DY, Jets, Top

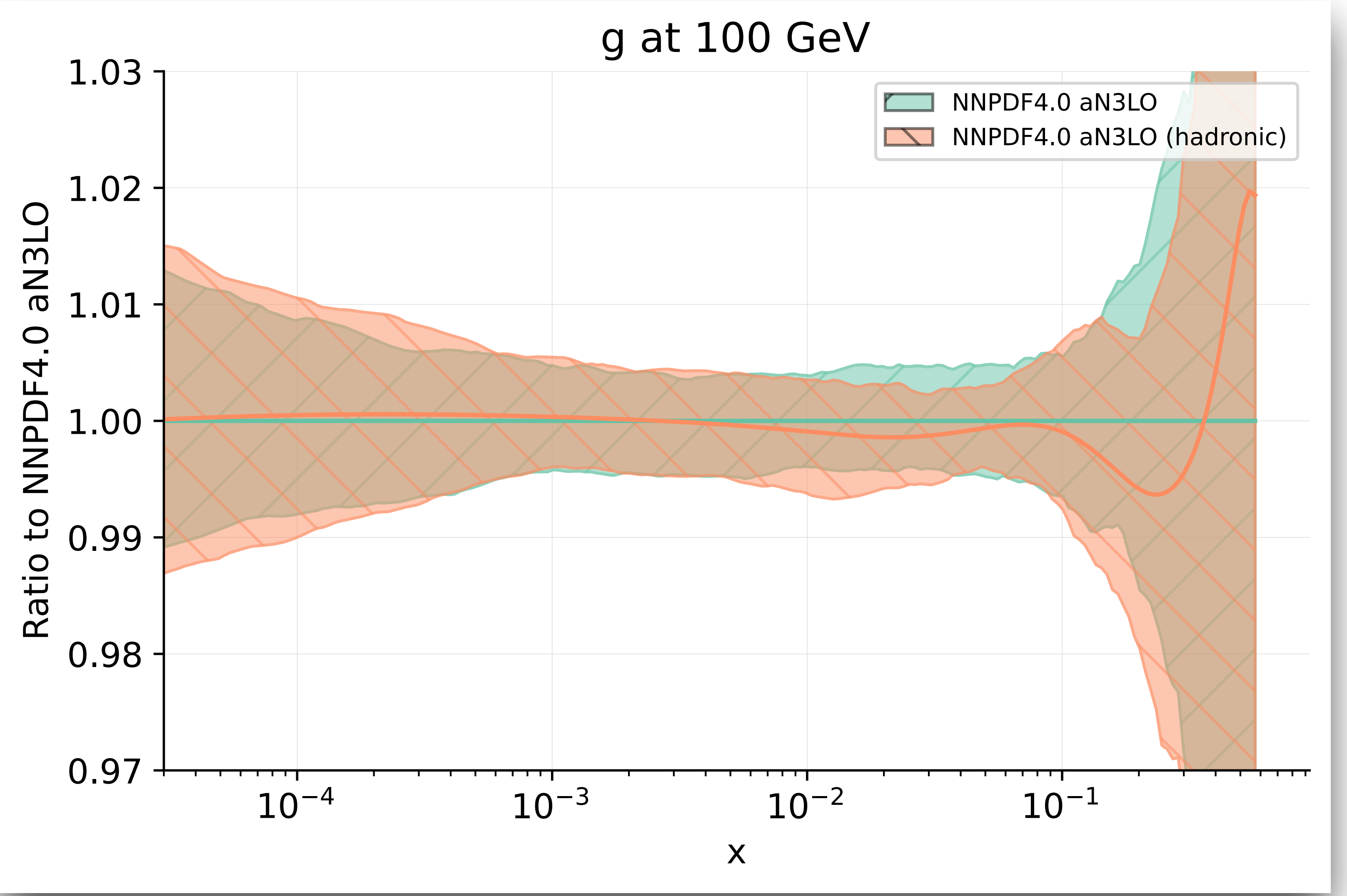
Single boson production (DY):

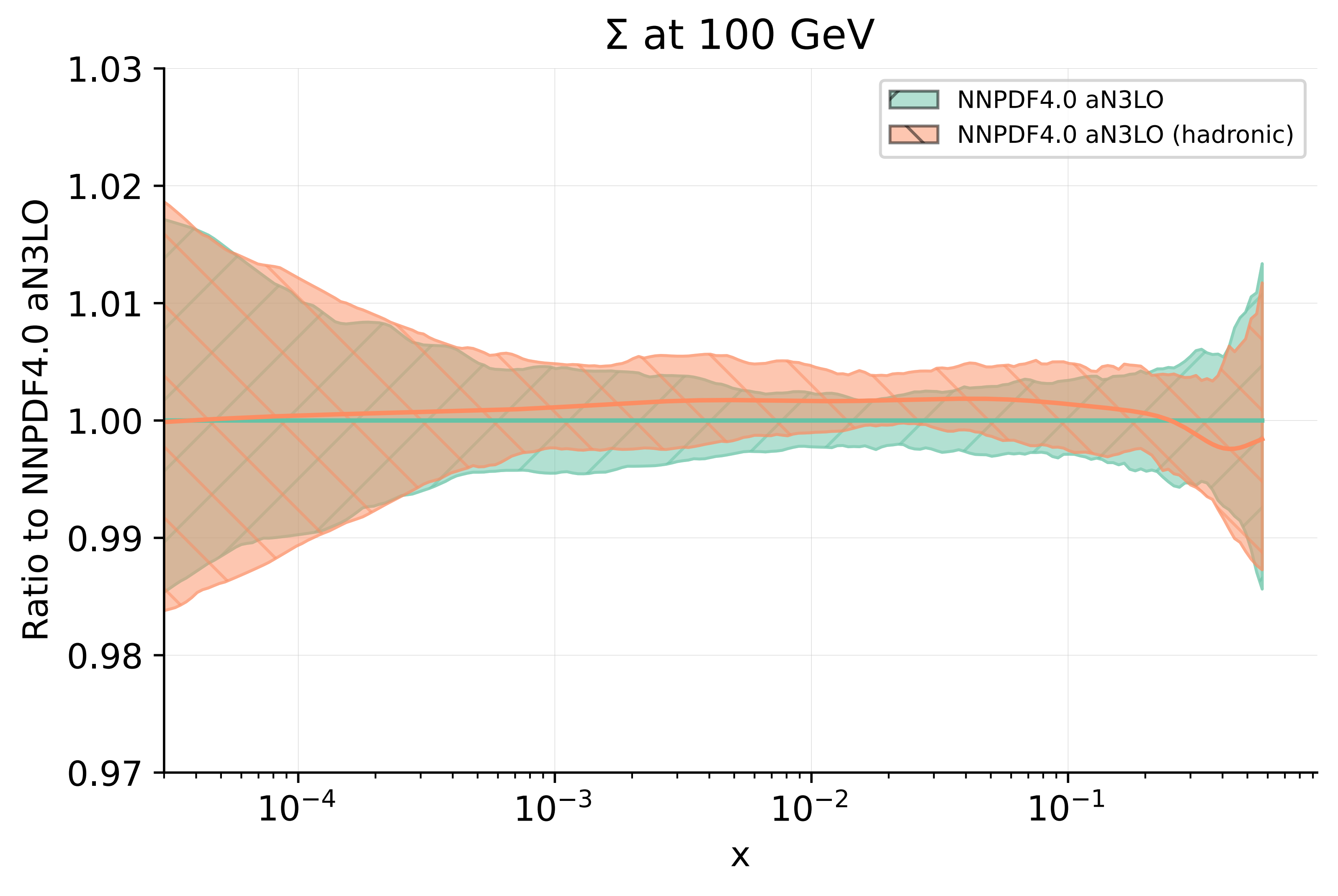
- N^{3}LO corrections to Z and $W^{\pm}$ differential in $m_{\ell\ell}$ or y_Z, can be included through k-factors. Effects are around 1-2% of the total cross sections, and quite flat in the boson rapidity.
- Effect at PDF level is negligible (limited number of data). **N^{3}LO DY k-factors not included in the default fit**.
- Differential distributions in p_t are available only up to NNLO.

Jets, Dijets, Top:

- N^{3}LO corrections are not known or public available.

We use NNLO MHOU from 3pt renormalisation scale variation to estimate unknown N^{3}LO effects.





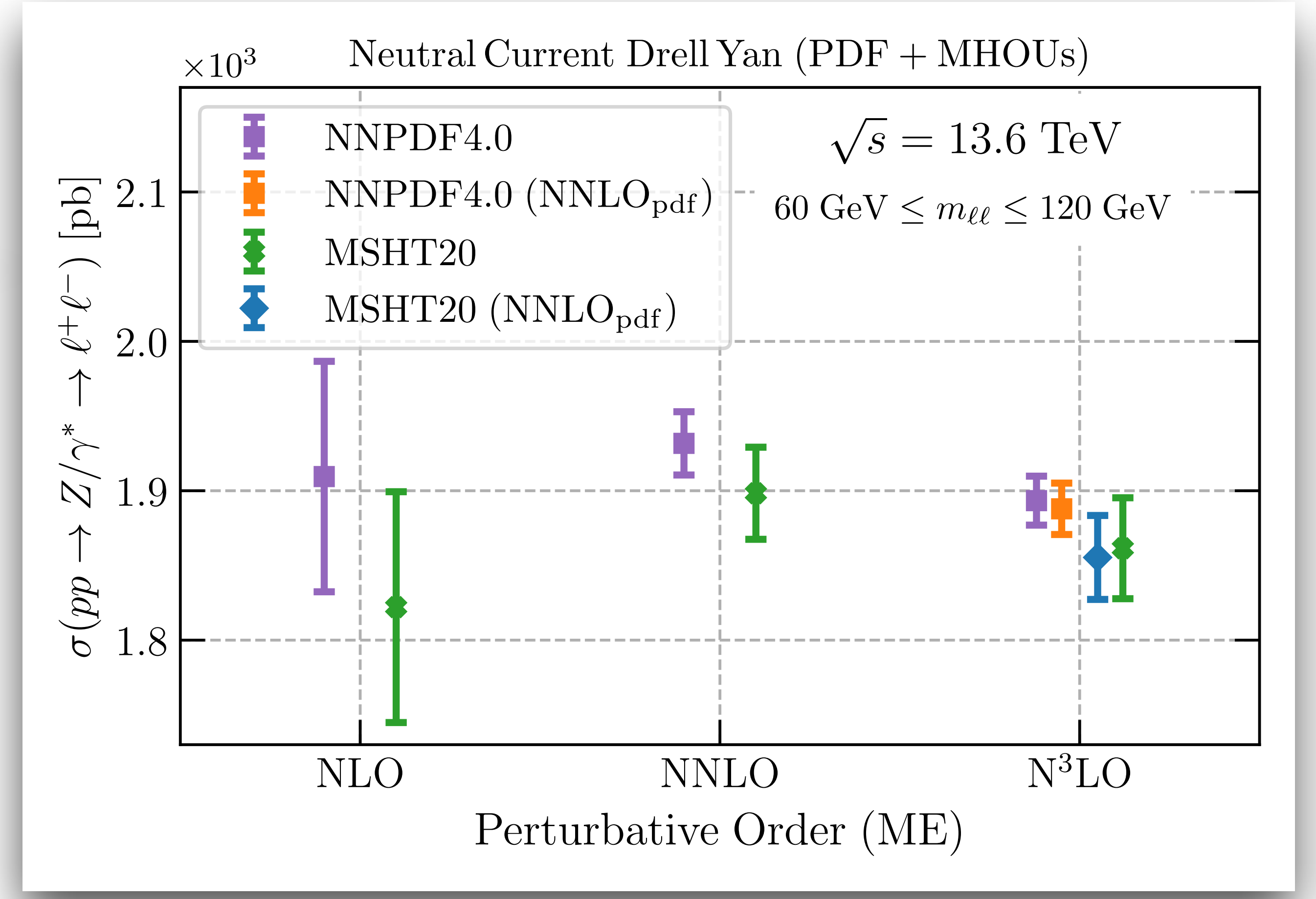
LHC phenomenology: Drell-Yan

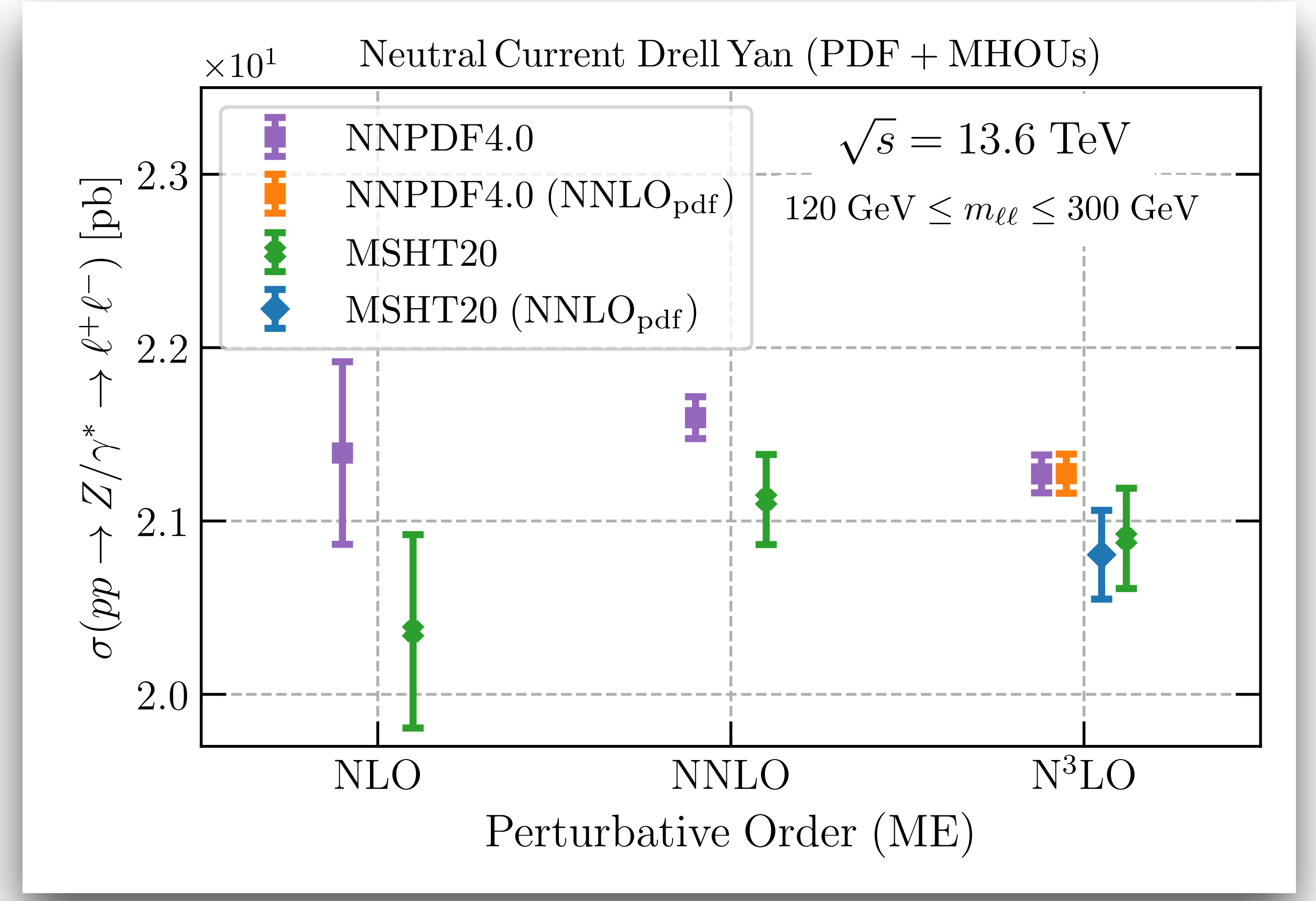
- Also for gauge boson production (depending on quark luminosities), the usage of aN^{3}LO PDFs **improve the perturbative convergence**.

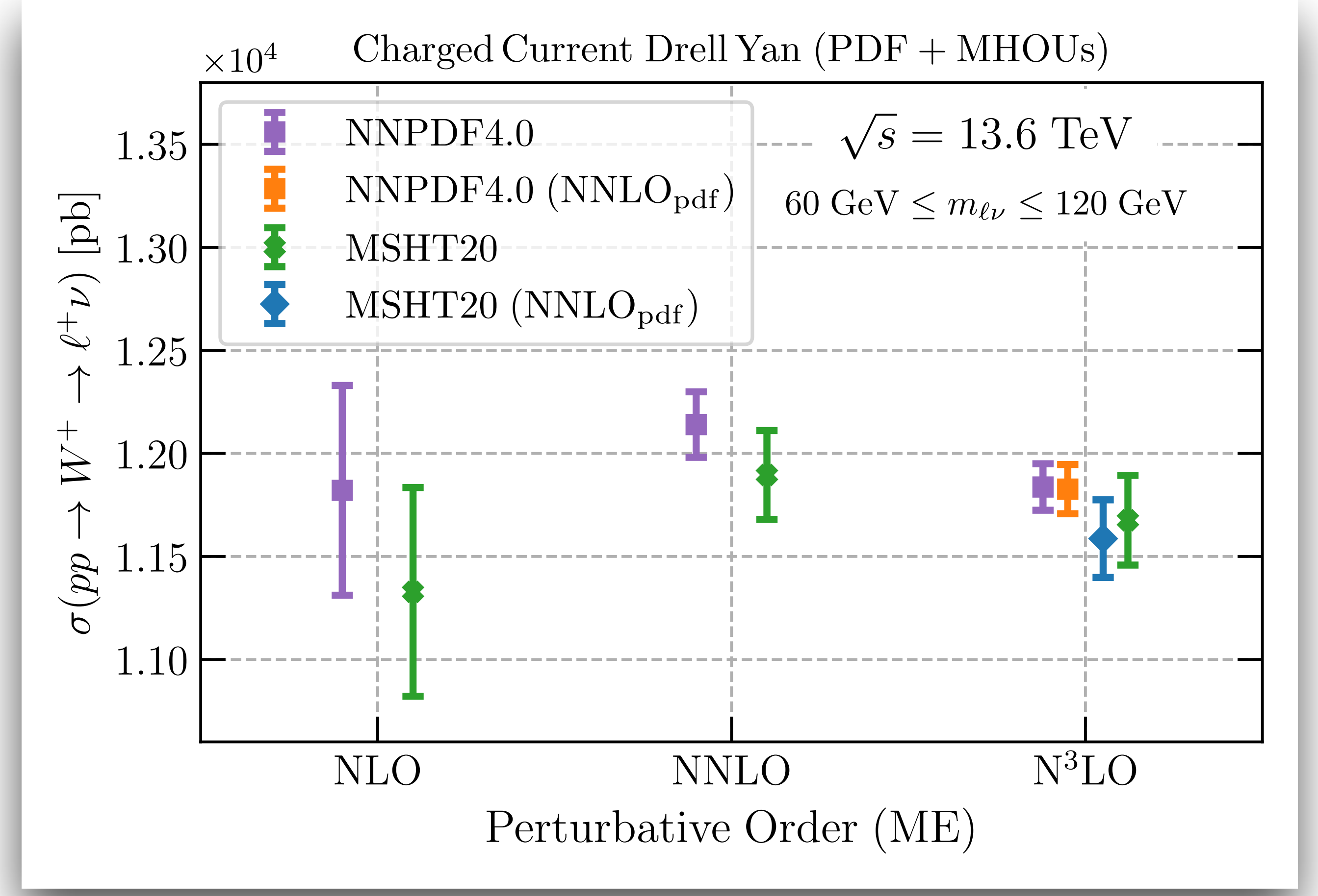
- **Similar N^{3}LO/NNLO ratio** to `MSHT20 aN3LO`.

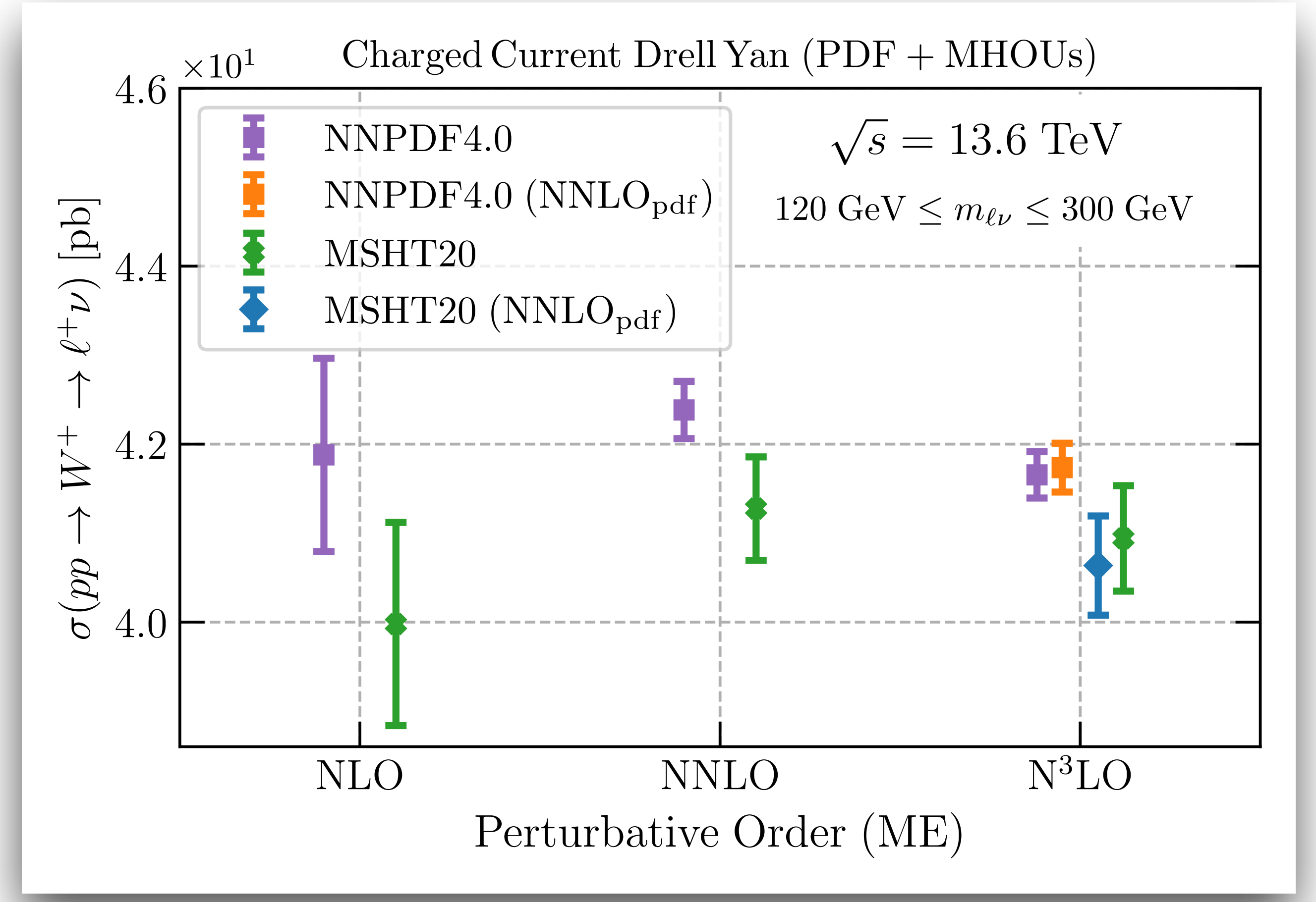
- For DY processes we find: $\Delta^{exact}_{NNLO} \leq \Delta^{app}_{NNLO}$

Process	NNPDF4.0					
	σ (pb)	$\delta_{\rm th}$	$\delta^{\rm noMHOU}_{\rm PDF}$	$\delta^{\rm MHOU}_{\rm PDF}$	$\Delta^{\rm app}_{\rm NNLO}$	$\Delta^{\rm exact}_{\rm NNLO}$
W^+ (p)	1.2×10^4	1.0	0.5	0.5	1.1	0.1
W^- (p)	8.8×10^3	1.0	0.5	0.5	1.1	0.1
Z (p)	1.9×10^3	0.9	0.4	0.5	1.1	0.3
W^+ (hm)	4.7×10^{-4}	2.8	2.8	3.3	3.2	1.1
W^- (hm)	1.4×10^{-4}	2.9	2.9	3.3	3.3	0.1
Z (hm)	2.1×10^{-4}	2.3	2.3	2.5	3.4	0.3





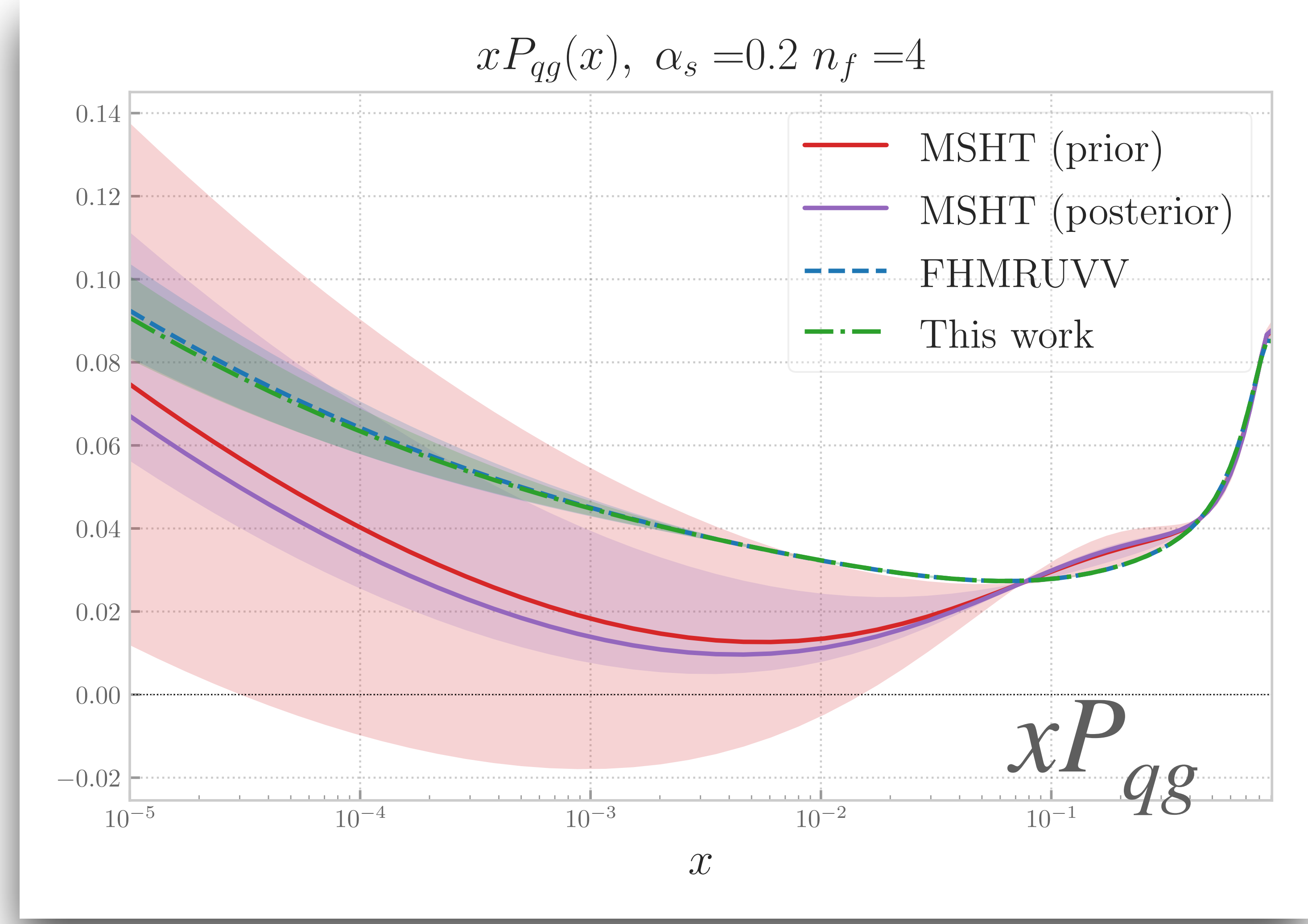


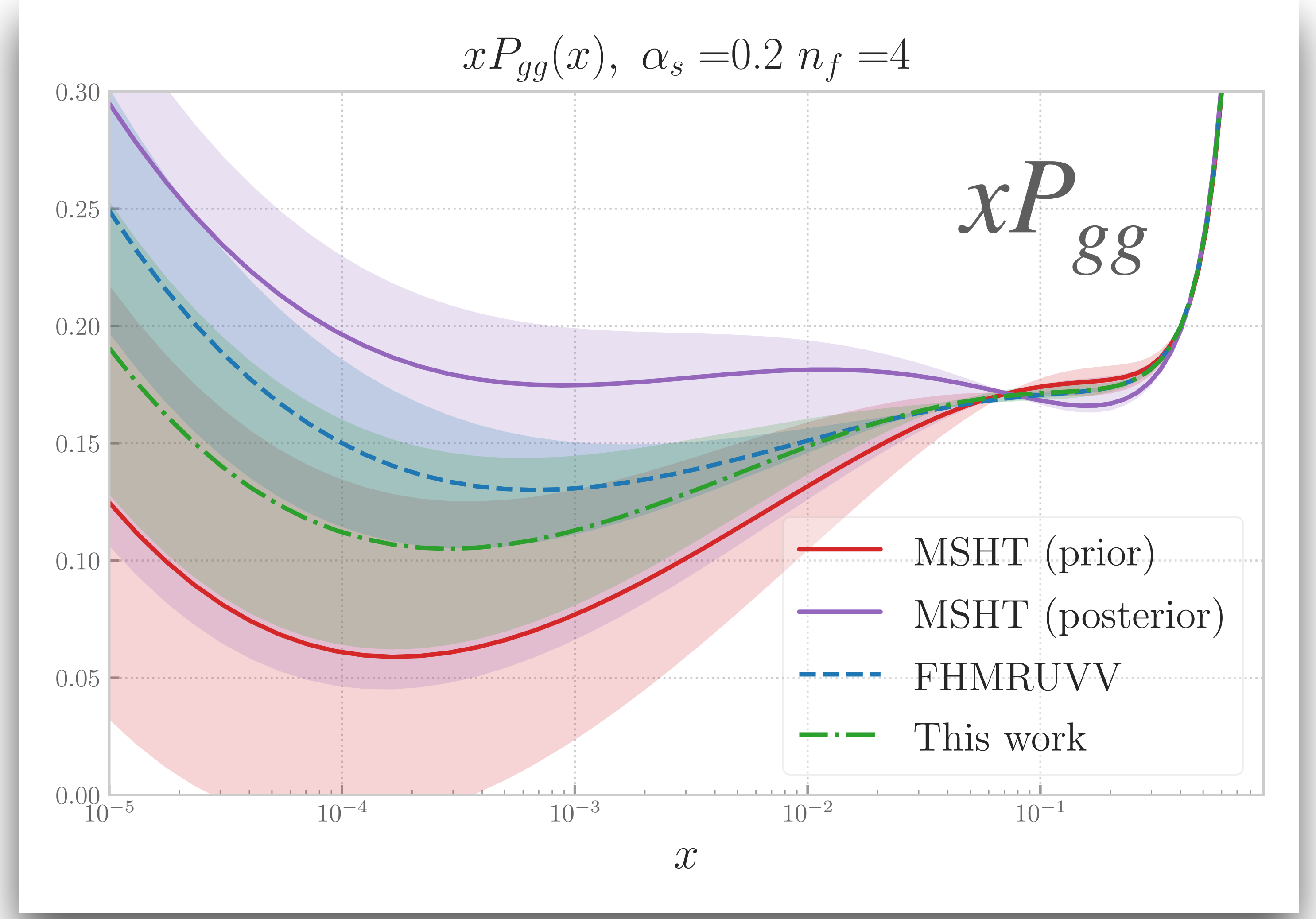


Comparison to MSHT20 aN3LO

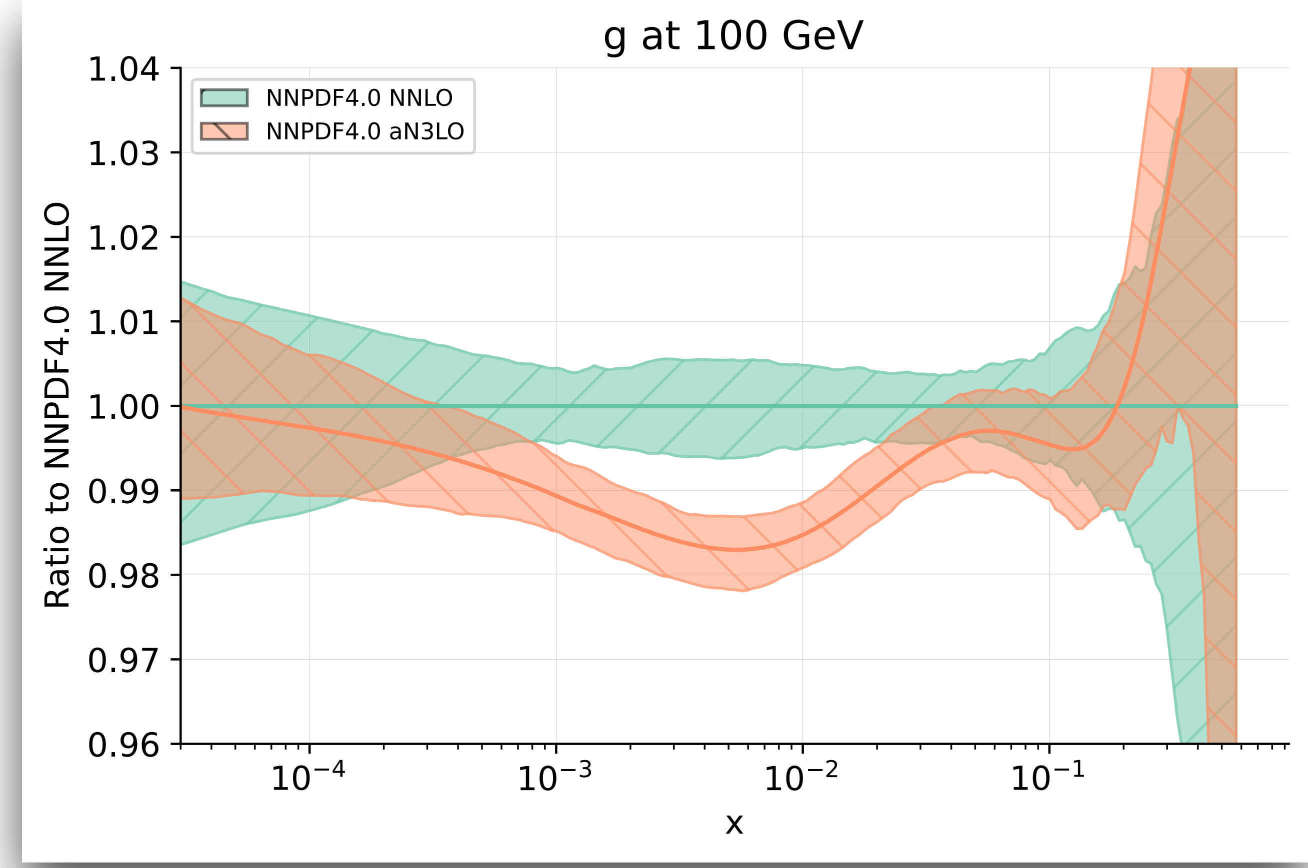
McGowan, Cridge, Harland-Lang, Thorne [arxiv:2207.04739]

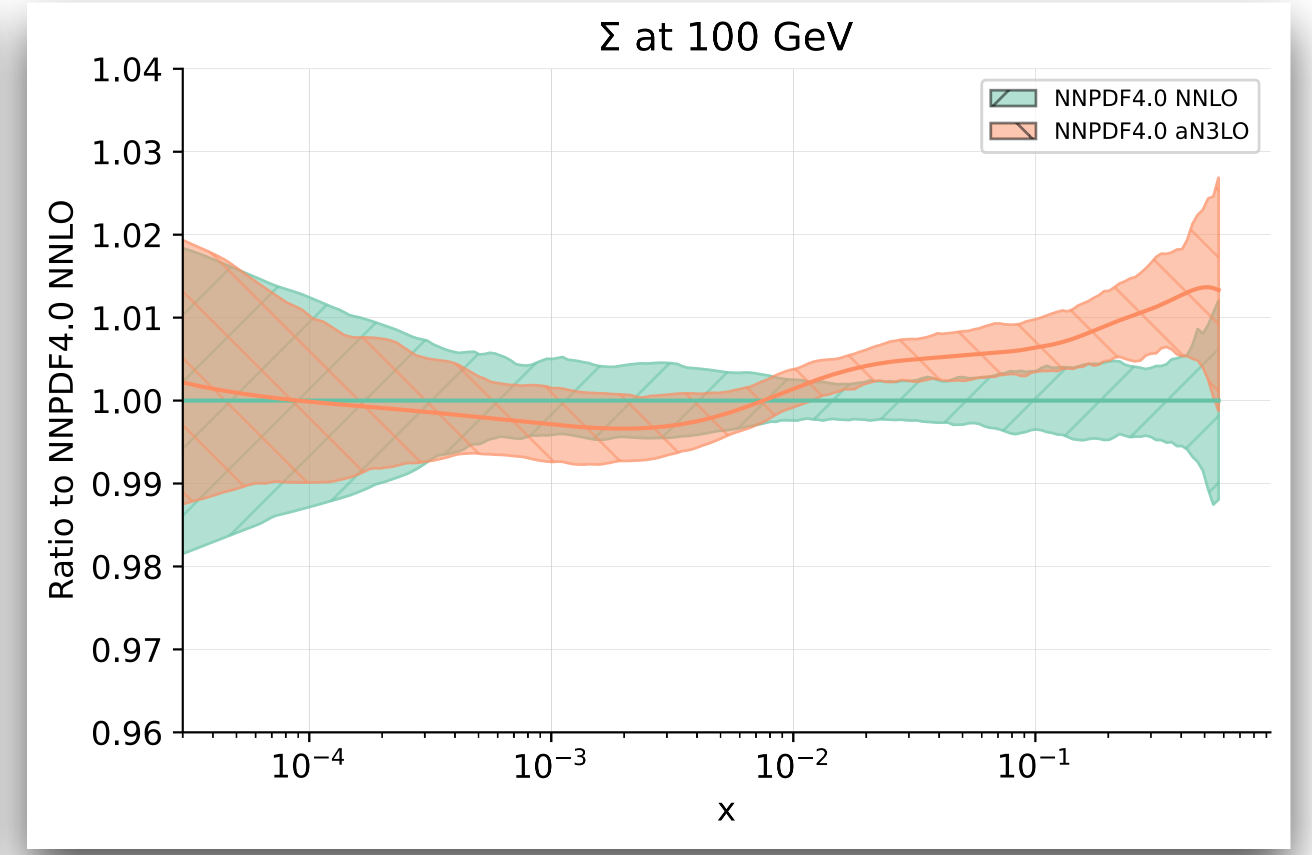
N^3LO Splitting functions



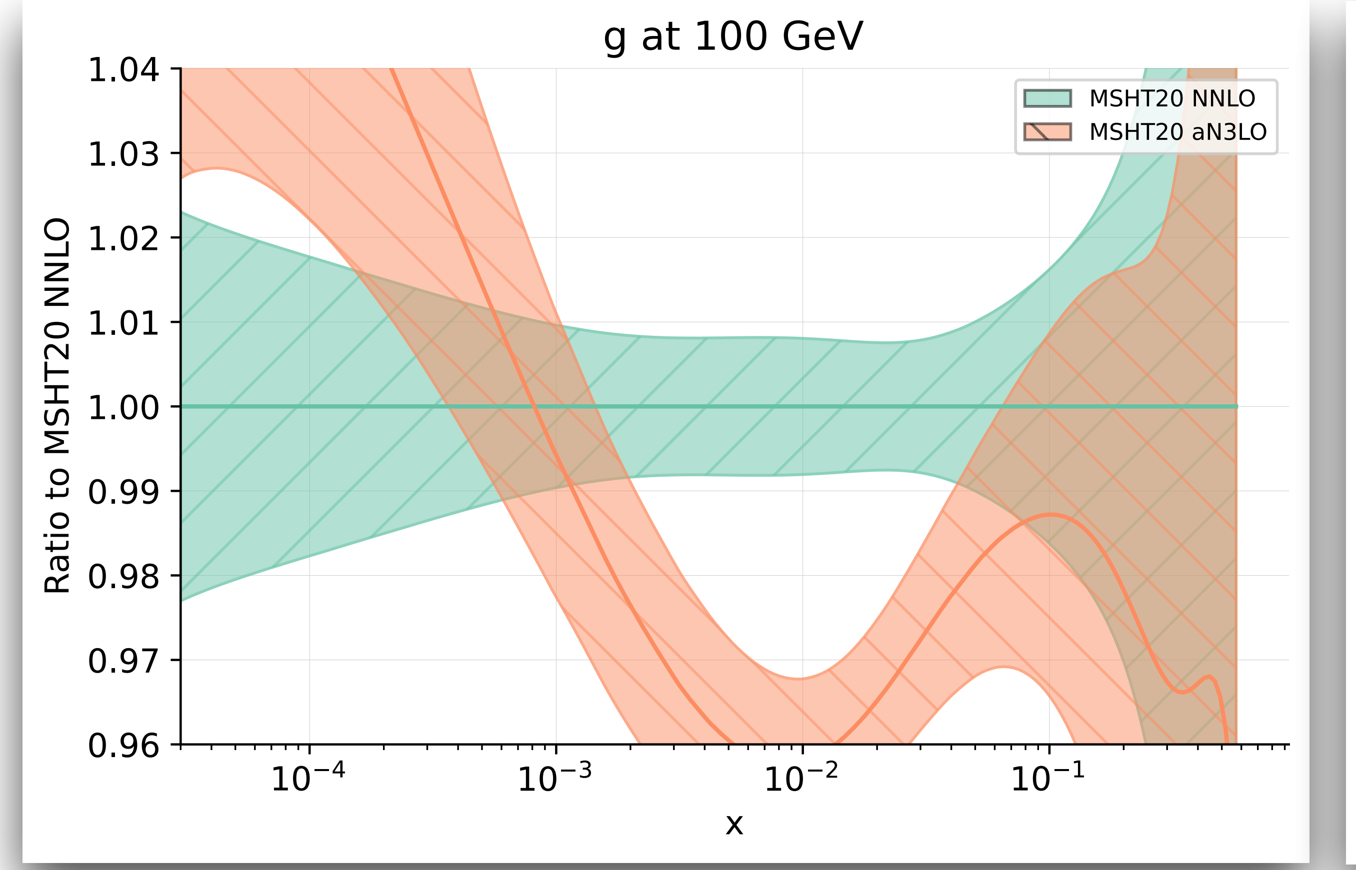


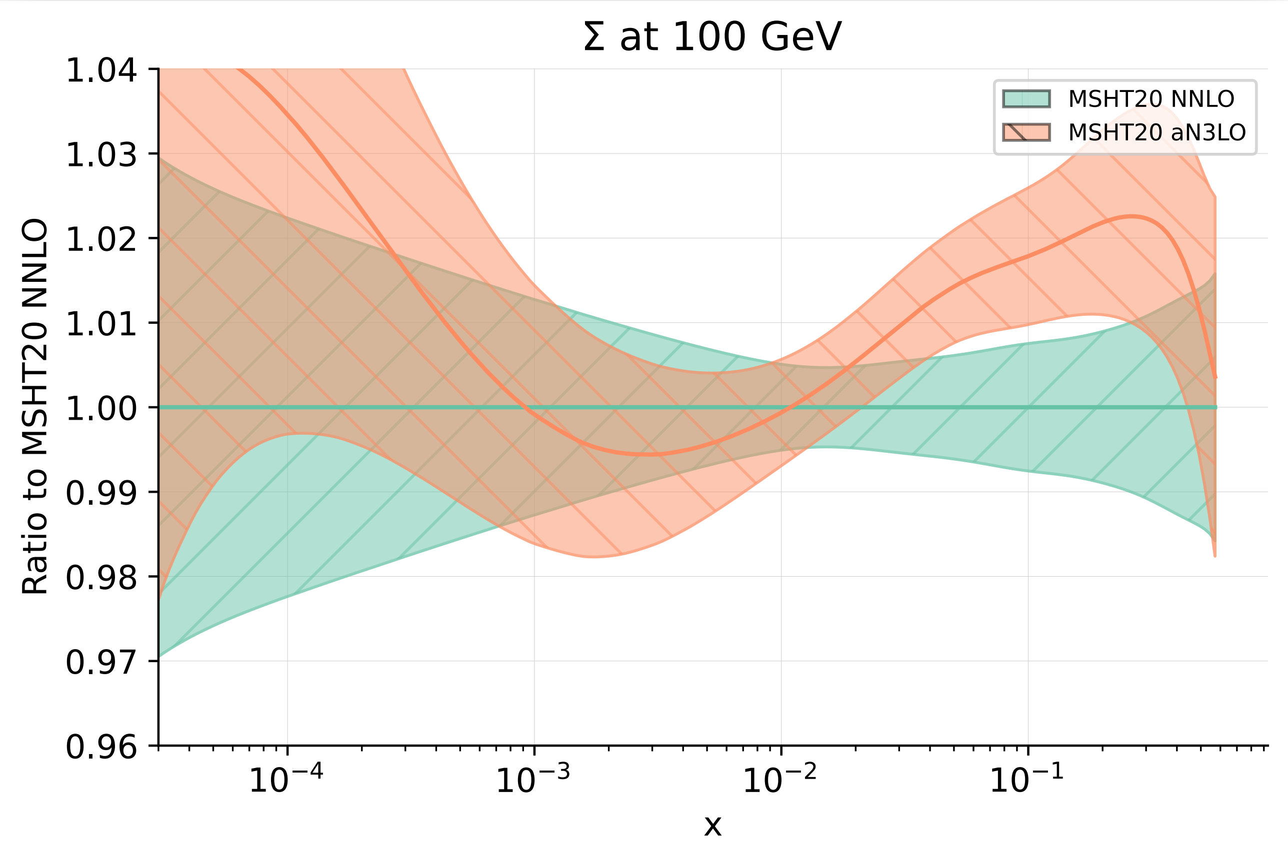
NNPDF4.0 aN^3LO / NNLO





MSHT20 aN^3LO / NNLO





Comparison to MSHT20 aN3LO [pheno]

McGowan, Cridge, Harland-Lang, Thorne [arxiv:2207.04739]

$$\Delta^{\text{exact}}_{\text{NNLO}} \equiv \left| \frac{\sigma^{\text{N}^3\text{LO}}_{\text{N}^3\text{LO}-\text{PDF}} - \sigma^{\text{N}^3\text{LO}}_{\text{NNLO}-\text{PDF}}}{\sigma^{\text{N}^3\text{LO}}_{\text{N}^3\text{LO}-\text{PDF}}} \right|$$

$$\Delta^{\text{app}}_{\text{NNLO}} \equiv \frac{1}{2} \left| \frac{\sigma^{\text{NNLO}}_{\text{NNLO}-\text{PDF}} - \sigma^{\text{NNLO}}_{\text{NLO}-\text{PDF}}}{\sigma^{\text{NNLO}}_{\text{NNLO}-\text{PDF}}} \right|$$

Relative uncertainty (%)

Process	NNPDF4.0						MSHT20				
	σ (pb)	δ_{th}	$\delta^{\text{noMHOU}}_{\text{PDF}}$	$\delta^{\text{MHOU}}_{\text{PDF}}$	$\Delta^{\text{app}}_{\text{NNLO}}$	$\Delta^{\text{exact}}_{\text{NNLO}}$	σ (pb)	$\delta_{\text{th}\sigma}$	δ_{PDF}	$\Delta^{\text{app}}_{\text{NNLO}}$	$\Delta^{\text{exact}}_{\text{NNLO}}$
$gg \to h$	43.8	4.8	0.6	0.7	0.2	2.2	42.3	5.1	1.7	1.4	5.3
h VBF	4.44	0.6	0.5	0.6	0.2	1.3	4.46	2.1	2.0	1.3	2.9
hW^+	0.97	0.6	0.5	0.6	0.2	0.5	0.95	1.5	1.4	0.8	0.9
hW^-	0.61	0.6	0.6	0.6	0.2	0.3	0.60	1.6	1.5	0.9	1.0
hZ	0.87	0.5	0.4	0.5	0.1	0.3	0.85	1.4	1.4	1.1	0.8

Process	NNPDF4.0						MSHT20				
	σ (pb)	δ_{th}	$\delta^{\text{noMHOU}}_{\text{PDF}}$	$\delta^{\text{MHOU}}_{\text{PDF}}$	$\Delta^{\text{app}}_{\text{NNLO}}$	$\Delta^{\text{exact}}_{\text{NNLO}}$	σ (pb)	$\delta_{\text{th}\sigma}$	δ_{PDF}	$\Delta^{\text{app}}_{\text{NNLO}}$	$\Delta^{\text{exact}}_{\text{NNLO}}$
W^+ (p)	1.2×10^4	1.0	0.5	0.5	1.1	0.1	1.2×10^4	1.9	1.7	2.3	0.8
W^- (p)	8.8×10^3	1.0	0.5	0.5	1.1	0.1	8.7×10^3	1.9	1.6	2.1	0.0
Z (p)	1.9×10^3	0.9	0.4	0.5	1.1	0.3	1.9×10^3	1.8	1.6	2.6	0.3
W^+ (hm)	4.7×10^{-4}	2.8	2.8	3.3	3.2	1.1	4.6×10^{-4}	4.0	3.9	2.0	1.3
W^- (hm)	1.4×10^{-4}	2.9	2.9	3.3	3.3	0.1	1.5×10^{-4}	4.2	4.2	2.0	0.6
Z (hm)	2.1×10^{-4}	2.3	2.3	2.5	3.4	0.3	2.2×10^{-4}	3.6	3.6	2.7	0.2

QED corrections in Higgs gluon fusion

- The **photon induced effects** (**Blue** vs **Orange**) are essentially negligible.
- The **QED corrections** to quarks and gluons (**Red** vs **Orange**) are at most $\mathcal{O}(2\%)$.