Revisiting $D^+ \to \pi^+ \ell^+ \ell^-$ in SM using LCSR

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(Work in progress with Alexander Khodjamirian and Thomas Mannel)

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P H

TP1 Theoretical Particle Physics

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Introduction

- $c \to u\ell^+\ell^-$ transition : FCNC transition $\Longrightarrow$ short distance effects are strongly suppressed in SM due to GIM mechanism.
- FCNCs in charm sector are enhanced in various BSM scenarios $\Longrightarrow$ considered to be a good indicator of New Physics.
- $D \to \pi\ell^+\ell^-$: Simplest decay mode to study $c \to u\ell^+\ell^-$.
  - Dominated by weak singly Cabibbo suppressed (SCS) $D \to \pi$ transition combined with an electromagnetic emission of the lepton pair.
  - A simple mechanism: $D \to \pi\ell^+\ell^- \approx D \to \pi V(\to \ell^+\ell^-)$ (with $V = \rho, \omega, \phi, \ldots$).

V	$BR(D^+ \to \pi^+ V)$	$BR(V \to \mu^+\mu^-)$	$BR(D^+ \to \pi^+ V)_{V \to \mu^+\mu^-}$
$\rho^0(770)$	$(8.3 \pm 1.4) \times 10^{-4}$	$(4.55 \pm 0.28) \times 10^{-5}$	$(3.78 \pm 0.68) \times 10^{-8}$
$\omega(782)$	$(2.8 \pm 0.6) \times 10^{-4}$	$(7.4 \pm 1.8) \times 10^{-5}$	$(2.1 \pm 0.7) \times 10^{-8}$
$\phi(1020)$	$(5.7 \pm 0.14) \times 10^{-3}$	$(2.85 \pm 0.19) \times 10^{-4}$	$(1.62 \pm 0.12) \times 10^{-6}$

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- Other $D_{(s)} \to P\ell^+\ell^-$ channels ($P = \pi, K, \eta$)), Cabibbo favoured(CF) and doubly Cabibbo suppressed(DCS) are also interesting since they share long-distance dynamics (annihilation mechanism).

Effective Operators

- The effective Hamiltonian for $D \to \pi \ell^+ \ell^-$ (SCS)

suppressing factor

$V_{u\mathscr{D}} V^*_{c\mathscr{D}} \approx \lambda$

$V_{ub} V^*_{cb} \approx \lambda^5$

$$\mathscr{H}_{eff}^{\Delta S=0} = \frac{4G_F}{\sqrt{2}} \sum_{\mathscr{D}=d,s} \lambda_{\mathscr{D}} \left[C_1(\mu) O_1^{\mathscr{D}} + C_2(\mu) O_2^{\mathscr{D}} \right] - \lambda_b \sum_{i=3}^{10} C_i(\mu) O_i$$

$O_2^{\mathscr{D}} = (\bar{u}_L \gamma_\mu t^a \mathscr{D}_L)(\bar{\mathscr{D}}_L \gamma^\mu t^a c_L)$

$\ll C_{1,2} @ \mathcal{O}(m_c)$

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WCs @ $\mu = 1.3$ GeV at NNLO : $C_1 = 1.034, C_2 = -0.633$

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- The largest effect beyond GIM limit $\sim \lambda_b C_9$ ($C_9 = -0.488$)

Amplitude and Hadronic Matrix Element

- In the GIM limit ($\lambda_b = 0, \lambda_d = -\lambda_s$):,

$$\mathscr{A}(D^+ \to \pi^+ \ell^+ \ell^-) = \left(\frac{16\pi\alpha_{em} G_F}{\sqrt{2}} \right) \lambda_d \frac{\bar{u}_\ell \gamma^\mu \nu_\ell}{q^2} \mathscr{A}_\mu^{D^+ \to \pi^+ \gamma^*}(p, q)$$

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$$= \left[(p\,.\,q)q_\mu - q^2 p_\mu\right] \mathscr{A}_{\mathscr{D}}^{D^+\to\pi^+\gamma^*}(q^2)$$ (Due to conservation of EM current)

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The non-local invariant amplitude : dominated by long distance effects in the physical region of q^2.

$(4m_\ell^2 < q^2 < (m_D - m_\pi)^2)$

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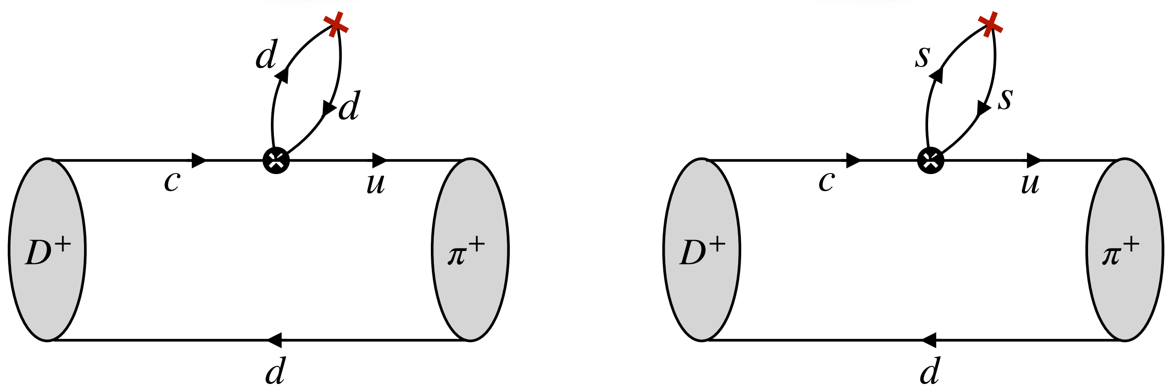
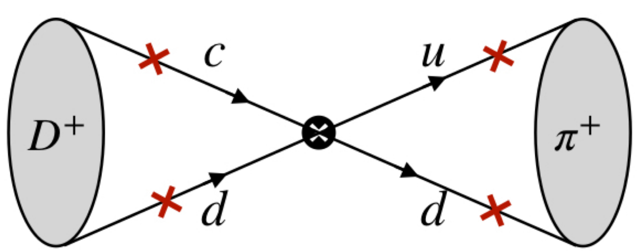
The object of our interest

Quark Topologies for $\mathscr{A}_\mu^{D^+\to\pi^+\gamma^*}(p,q)$

Loop Topology
(Only possible in SCS decays)



Annihilation Topology



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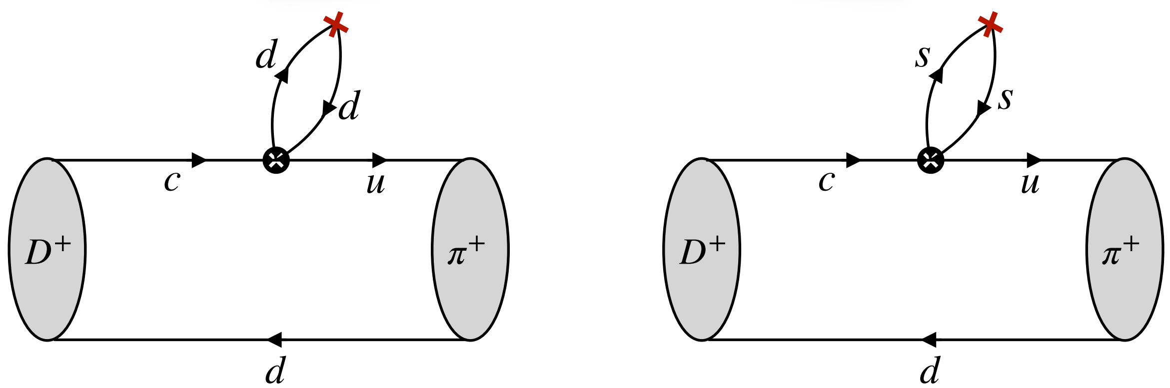

- In the $SU(3)_f$ (along with $\lambda_b \approx 0$), the two loops have complete GIM cancellation

L-topology with have non-zero contribution only due to $m_s \neq m_d$: Expected to be small

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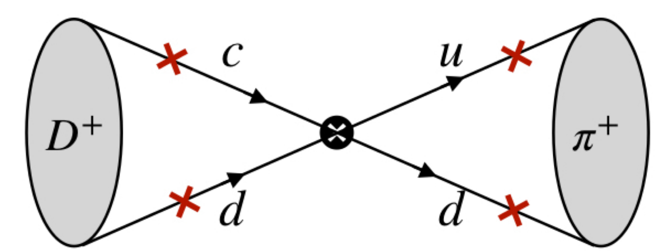

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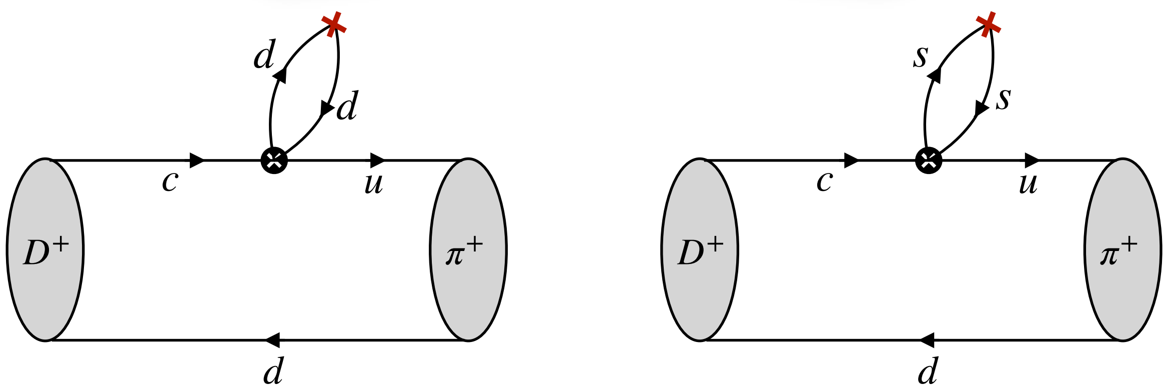

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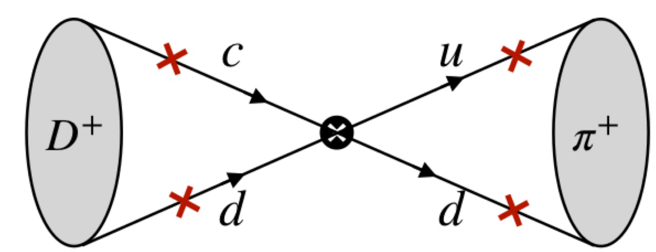

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A-topology is the main contribution.

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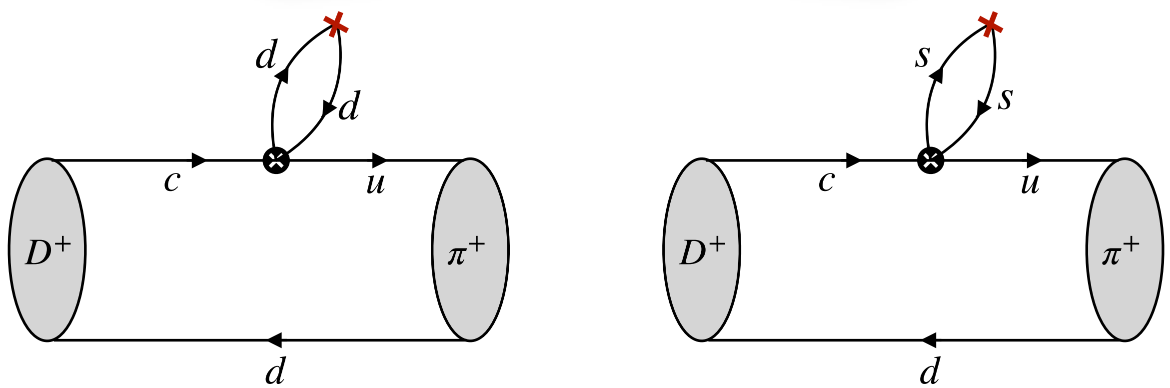

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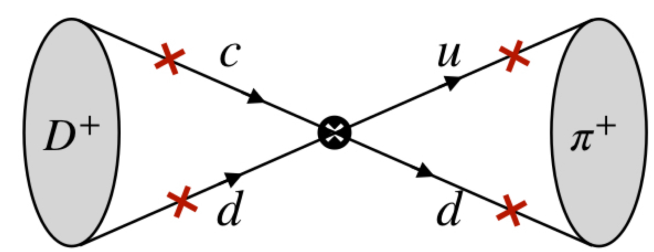

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A-topology is the main contribution.

- At NLO, there will be multiple diagrams with the exchange of virtual gluons : Out of the scope of the present study.

The use of U-spin

- Combining Two approximations: GIM limit, $\lambda_b = 0, \lambda_d = -\lambda_s$ and $SU(3)_{fl}$ limit, $m_s = m_{u,d}$
- The Hamiltonians of CF, SCS, and DSC modes form a U-triplet: **(Only annihilation topology)**

$$O_1^{(U=1)} \equiv \begin{pmatrix} (\bar{u}_L \gamma_\mu s_L)(\bar{d}_L \gamma^\mu c_L) \\ \frac{1}{\sqrt{2}}\left[(\bar{u}_L \gamma_\mu d_L)(\bar{d}_L \gamma^\mu c_L) - (\bar{u}_L \gamma_\mu s_L)(\bar{s}_L \gamma^\mu c_L)\right] \\ (\bar{u}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu c_L) \end{pmatrix} = \begin{pmatrix} |1, +1\rangle \\ -|1,0\rangle \\ |1, -1\rangle \end{pmatrix}$$

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- As j_μ^{em} is a U-singlet, the matrix element of interest:

$$\langle P^+|j_\mu^{em}(x)O_1^{(U=1)}|D^+\rangle \longrightarrow$$

Two ways to make a U-spin singlet

$$\langle P^+_{(U=1/2)}|j_\mu^{em}(x)O_1^{(U=1)}|D^+_{(U=1/2)}\rangle$$

$$\begin{pmatrix} |K^+\rangle = |u\bar{s}\rangle \\ |\pi^+\rangle = |u\bar{d}\rangle \end{pmatrix} = \begin{pmatrix} |1/2,+1/2\rangle \\ -|1/2,-1/2\rangle \end{pmatrix}$$

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$$\begin{pmatrix} |K^0\rangle = |d\bar{s}\rangle \\ \frac{\sqrt{3}}{2}|\eta_8\rangle - \frac{1}{2}|\pi^0\rangle = \frac{1}{\sqrt{2}}|d\bar{d} - s\bar{s}\rangle \\ |\bar{K}^0\rangle = |s\bar{d}\rangle \end{pmatrix} = \begin{pmatrix} |1,+1\rangle \\ -|1,0\rangle \\ -|1,-1\rangle \end{pmatrix}$$

U-spin relations

$$\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = -\mathscr{A}^{(D_s^+\to K^+\gamma^*)}(q^2) = \mathscr{A}^{(D_s^+\to\pi^+\gamma^*)}(q^2) = \mathscr{A}^{(D^+\to K^+\gamma^*)}(q^2)$$

$$\mathscr{A}^{(D^0\to\bar{K}^0\gamma^*)}(q^2) = \mathscr{A}^{(D^0\to K^0\gamma^*)}(q^2) = -\frac{1}{2}\mathscr{A}^{(D^0\to\pi^0\gamma^*)}(q^2) + \frac{\sqrt{3}}{2}\mathscr{A}^{(D^0\to\eta^0\gamma^*)}(q^2)$$

$$\mathscr{A}^{(D^0\to\eta_8\gamma^*)}(q^2) = -\sqrt{3}\mathscr{A}^{(D^0\to\pi^0\gamma^*)}(q^2)$$

$$\mathscr{A}^{(D^0\to\eta'\gamma^*)}(q^2) = 0$$

D^0, η': U-spin singlets.

- Measuring the CF modes, e.g. $D_s \to \pi^+\ell^+\ell^-$ will allow to disentangle this topology.

What do we know from Experiments?

- Upper bounds from PDG:

Decay mode	Cabibbo hierarchy	BR, exp. upper limit
$D^+ \to \pi^+\ell^+\ell^-$	SCS	$1.1 \times 10^{-6}(\ell = e)$ $6.7 \times 10^{-8}(\ell = \mu)$
$D^+ \to K^+\ell^+\ell^-$	DCS	$8.5 \times 10^{-7}(\ell = e)$ $5.4 \times 10^{-8}(\ell = \mu)$
$D^0 \to \bar{K}^0\ell^+\ell^-$	CF	$2.4 \times 10^{-5}(\ell = e)$ $2.6 \times 10^{-4}(\ell = \mu)$
$D^0 \to \pi^0\ell^+\ell^-$	SCS	$4 \times 10^{-6}(\ell = e)$ $1.8 \times 10^{-4}(\ell = \mu)$
$D^0 \to \eta\ell^+\ell^-$	SCS	$3 \times 10^{-6}(\ell = e)$ $5.3 \times 10^{-4}(\ell = \mu)$
$D^0 \to \eta'\ell^+\ell^-$	SCS	-
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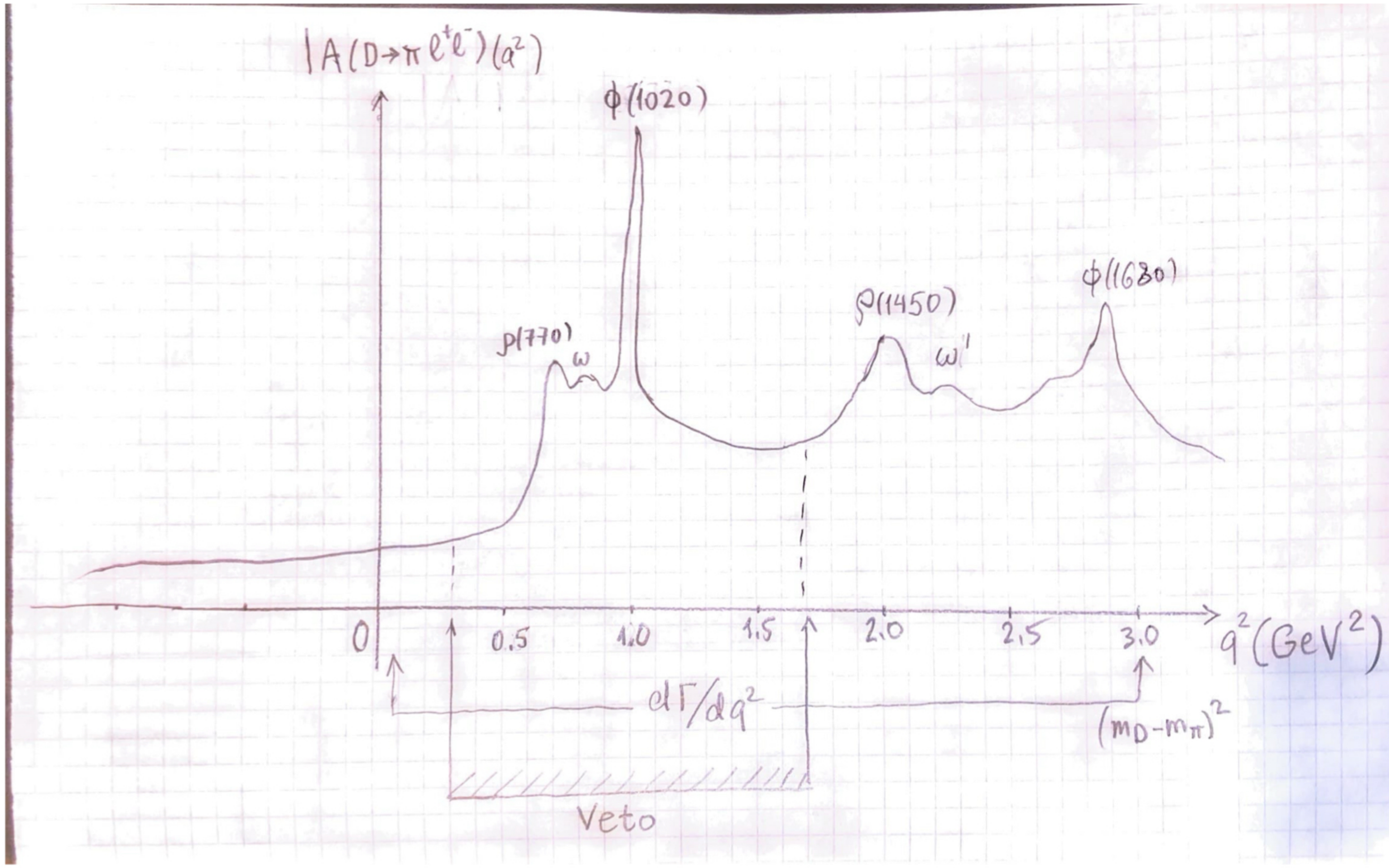
$D^0 \to \eta\ell^+\ell^-$	SCS	$3 \times 10^{-6}(\ell = e)$ $5.3 \times 10^{-4}(\ell = \mu)$
$D^0 \to \eta'\ell^+\ell^-$	SCS	-
$D^0 \to K^0\ell^+\ell^-$	DCS	-
$D_s^+ \to \pi^+\ell^+\ell^-$	CF	$5.5 \times 10^{-6}(\ell = e)$ $1.8 \times 10^{-7}(\ell = \mu)$
$D_s^+ \to K^+\ell^+\ell^-$	SCS	$3.7 \times 10^{-6}(\ell = e)$ $1.4 \times 10^{-7}(\ell = \mu)$

[PDG]

- Most recent upper bound on $(D^+ \to \pi^+\mu^+\mu^-)$: vetoing the resonance region. [LHCb, (JHEP06 (2021) 044)]

Can we really isolate resonances?



- The full amplitude represented via hadronic dispersion relation :

Decay constant

Amplitude for $D \to \pi V$

Continuum and higher resonances

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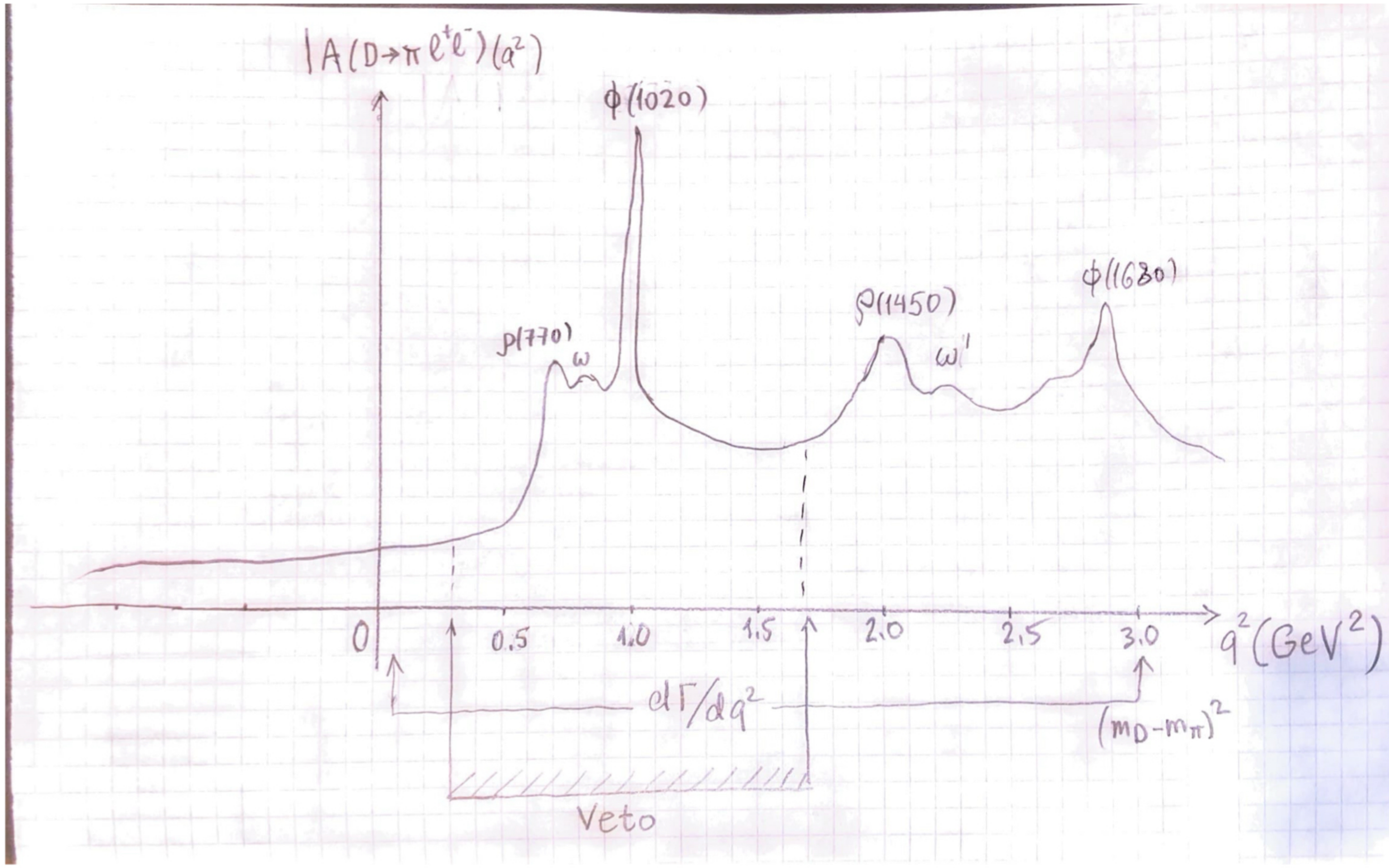

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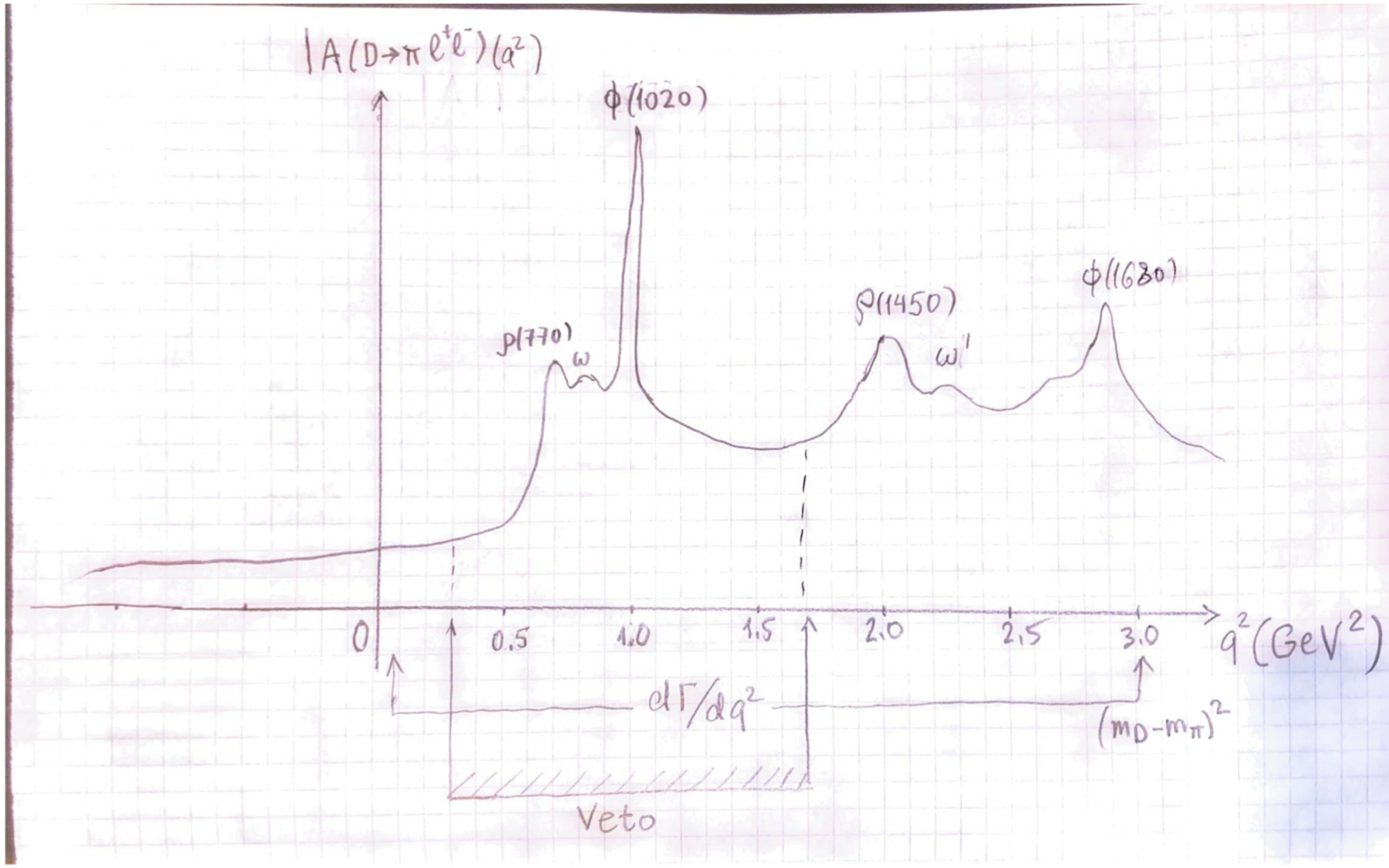

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As, the experimental bounds are approaching theory predictions, it is important to revisit it within the Standard Model.

Our methodology: LCSR-supported dispersion relation

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(Resembling partly the analysis of nonlocal effects in $B \to K^*\ell^+\ell^-$)

[A. Khodjamirian, T. Mannel, A. Pivovarov, Y. Wang, 1211.0234]

[A. Khodjamirian, A. V. Rusov, 1703.04765] , N. Gubernari, M. Rebound, D. van Dyk, J. Virto, 2011.09813

A brief overview of LCSR method

TOOLS TO DERIVE LCSR

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(Computing correlation function as a product of perturbatively calculated Hard scattering kernel and pion DAs)

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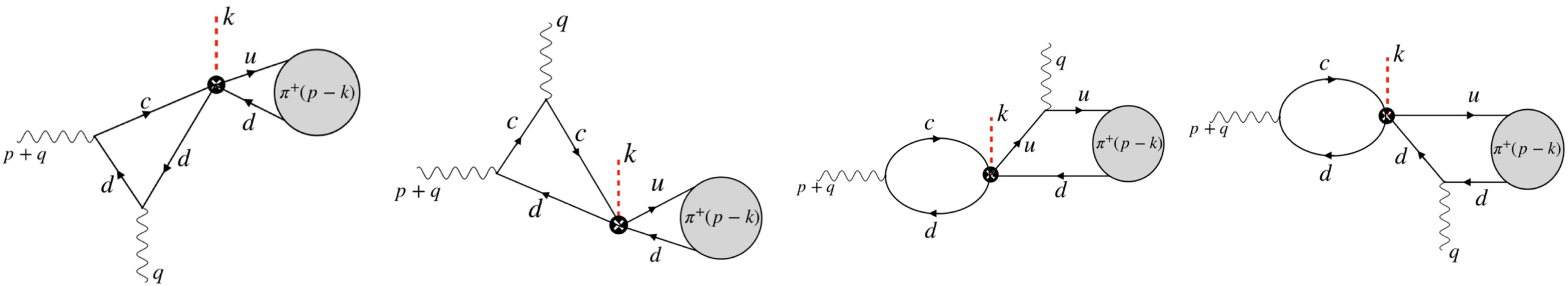
Only O_1^d contributes. The O_2 contribution vanishes after Fierz transformation.



(a) (b) (c) (d)

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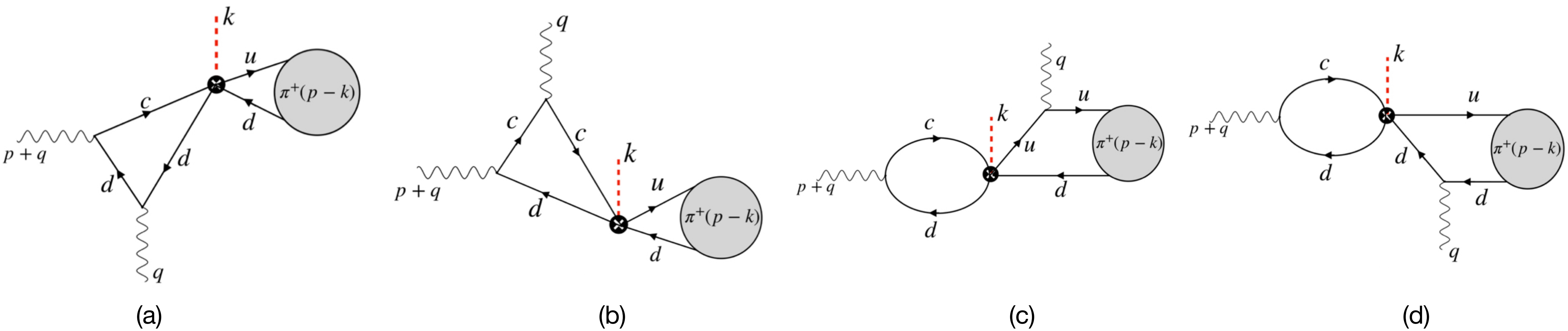

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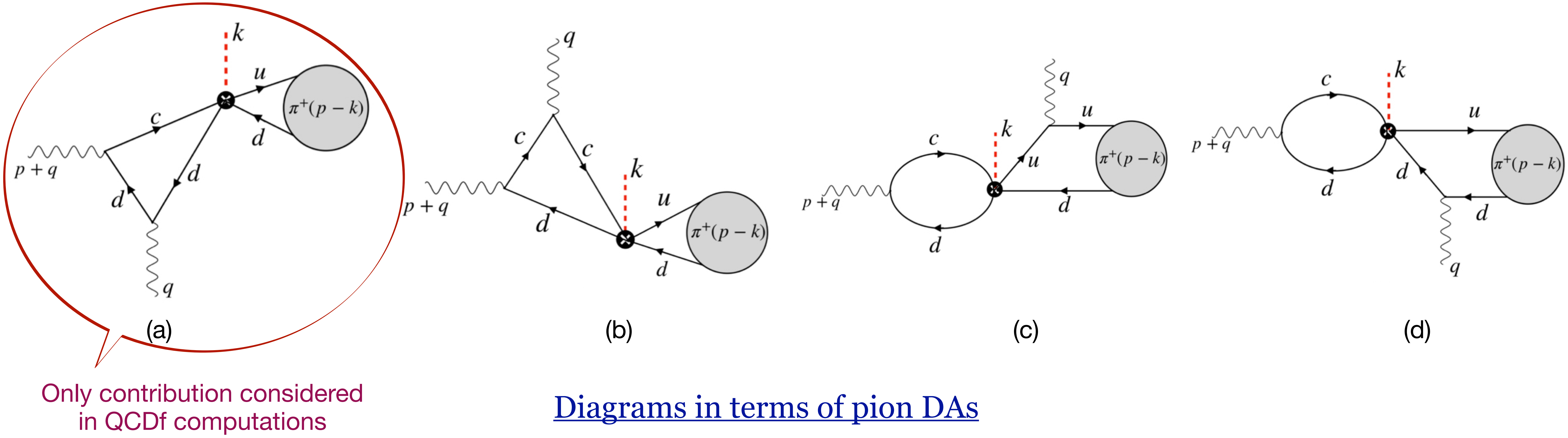

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Loop diagram from LCSR

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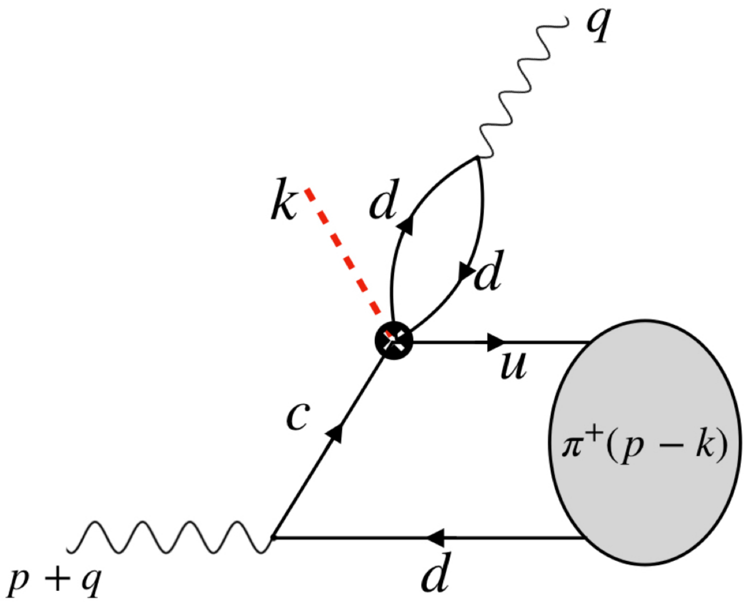

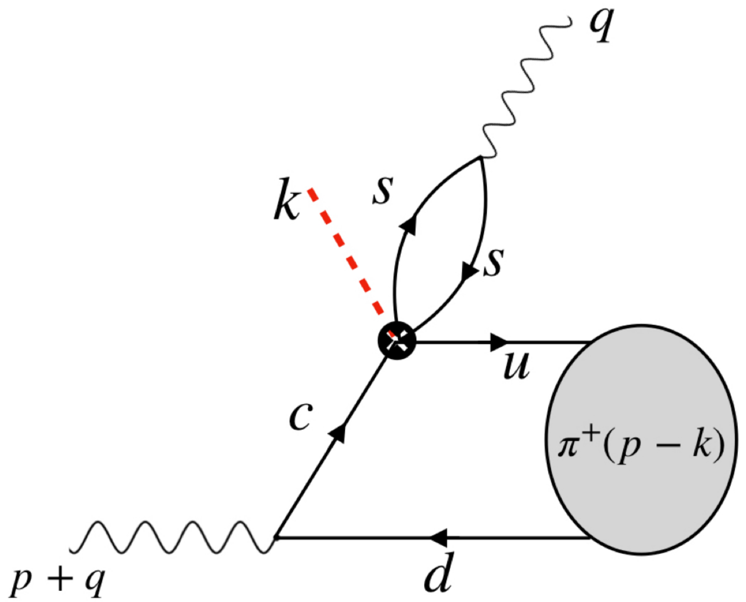

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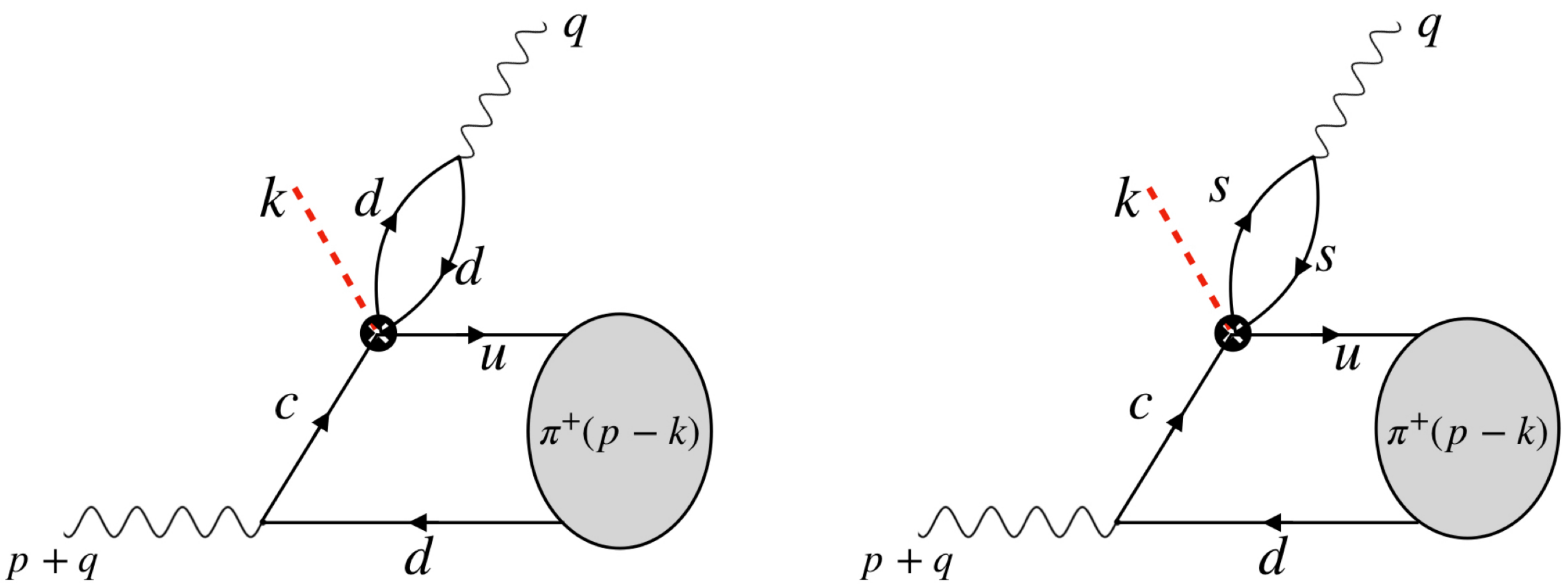

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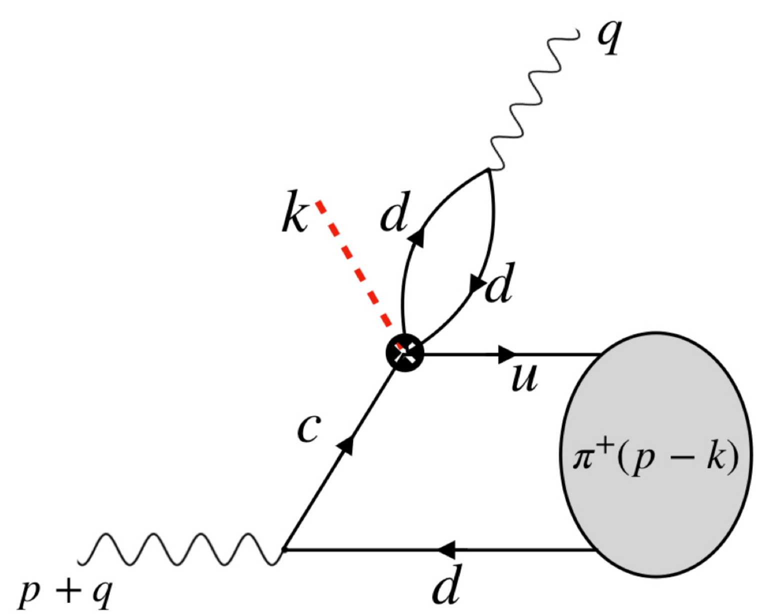

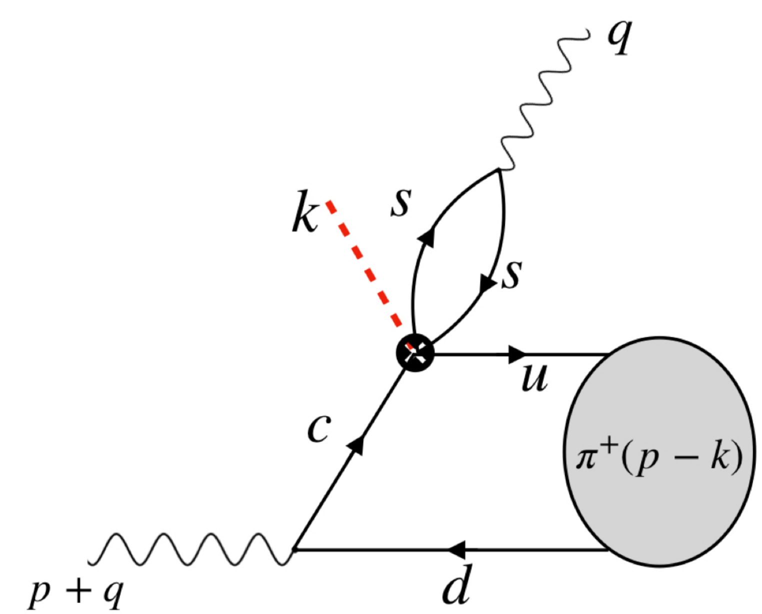

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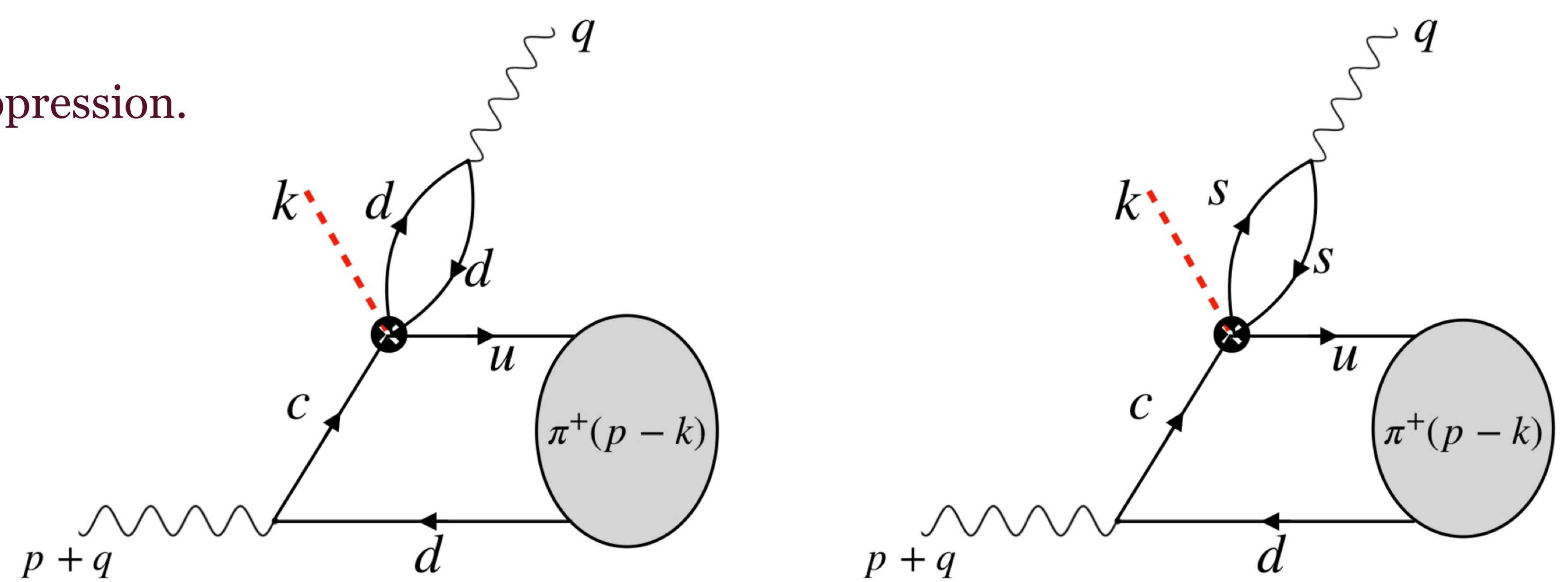
- Both WCs (C_1 and C_2) contribute in this case.
- The contribution is small due to GIM suppression.



LCSR Results

- The final sum rule read as (for $q^2 < 0$):

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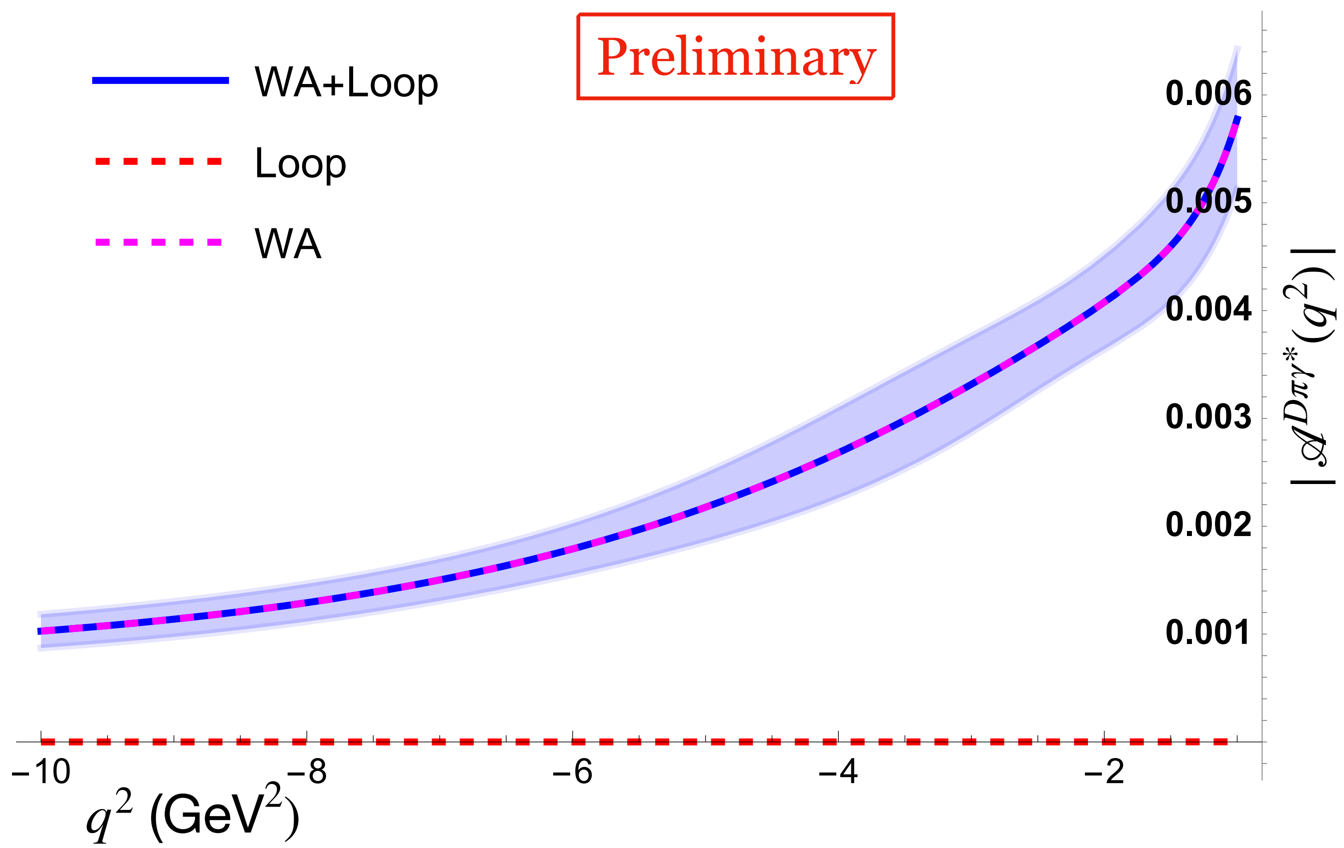

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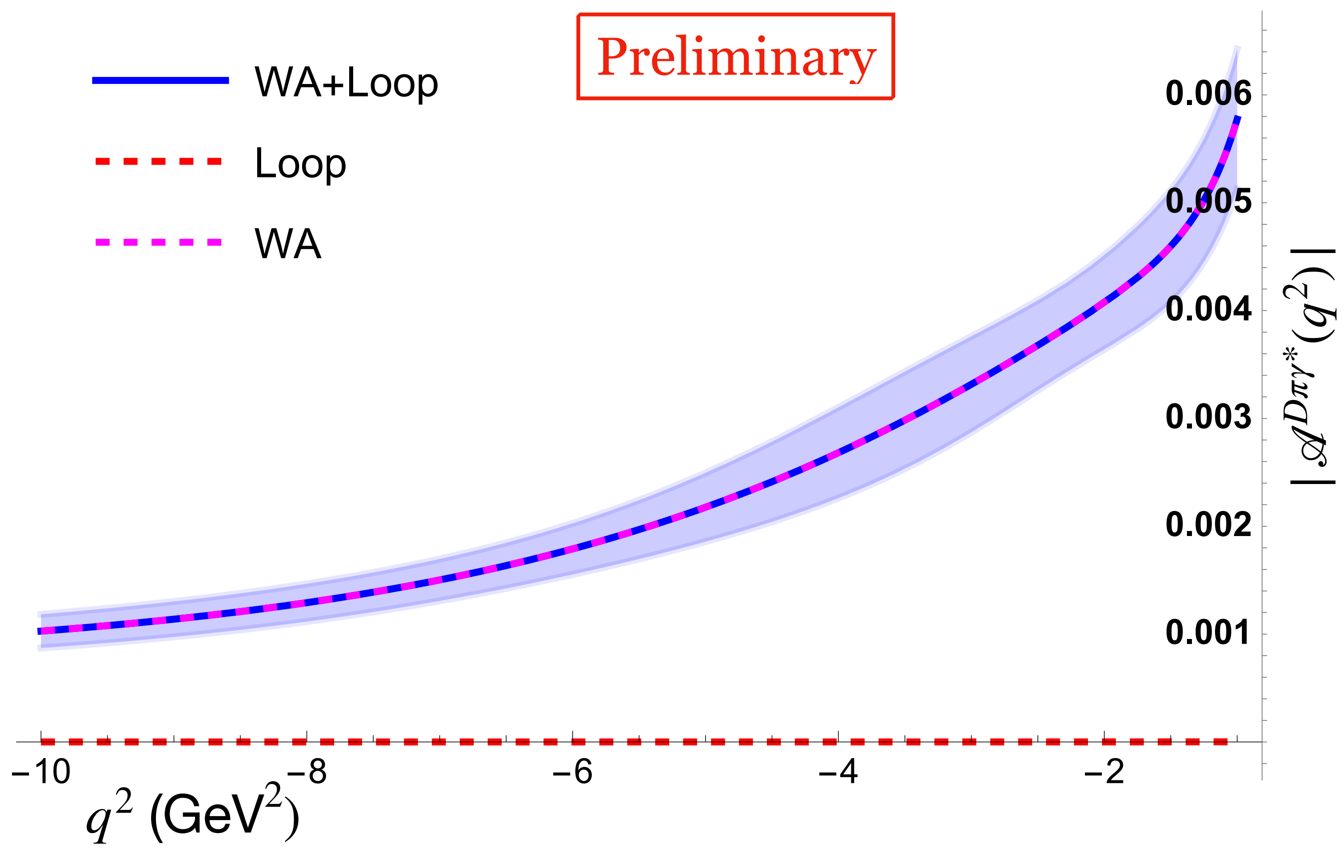
- F^{OPE} include contribution from twist-2 distribution amplitude (DA) of pion (using 2 Gegenbauer moments).

- The major source of calculated LCSR uncertainties are the uncertainties in s_0^D and the DA parameters.



LCSR Results

- The final sum rule read as (for $q^2 < 0$):

$$m_D^2 f_D \mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) e^{-m_D^2/M^2} = \frac{1}{\pi}\int_{m_c^2}^{s_0^D} ds\, e^{-s/M^2} \mathrm{Im}(F^{(OPE)}(s, q^2, m_D^2))$$

- M^2 (Borel parameter) and s_0^D (effective threshold) are the sum rule parameters taken to be:

$$M^2 = (4.5 \pm 1.0)\ \mathrm{GeV}^2$$

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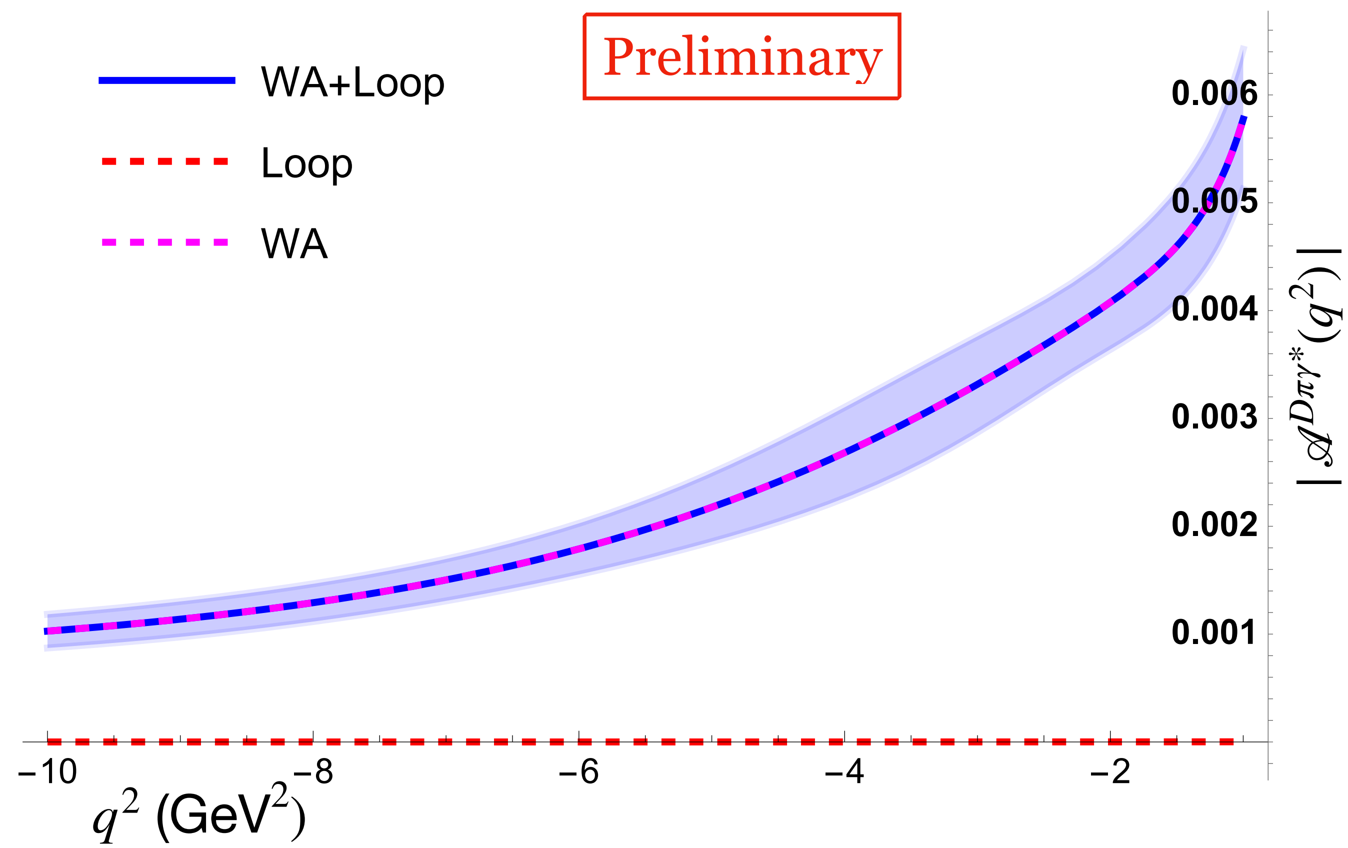
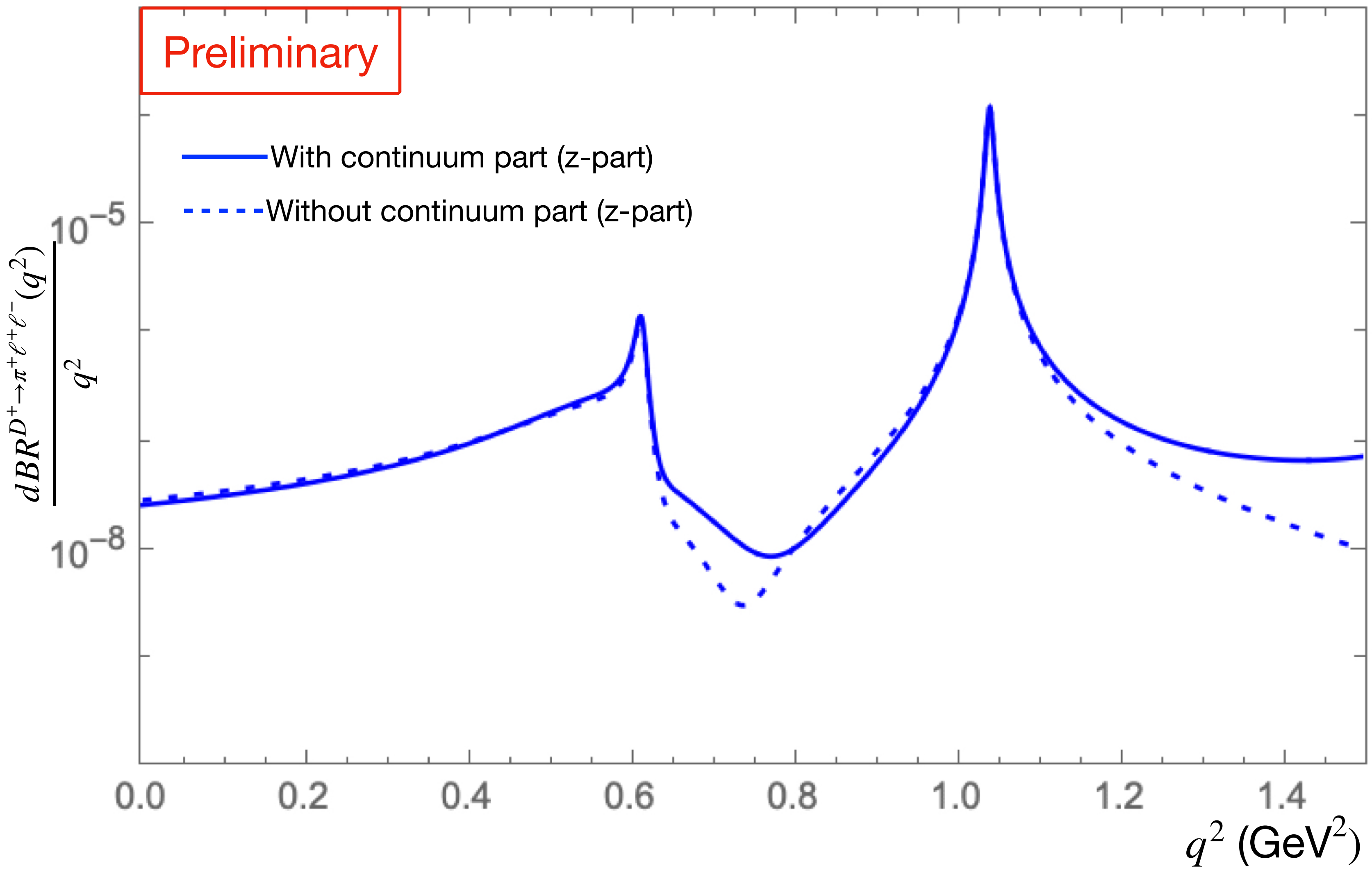


- The contribution to the decay amplitude from O_9 varies from $\sim 1.5 \times 10^{-6}$ to $\sim 7.5 \times 10^{-6}$ at $0 < q^2 < (m_D - m_\pi)^2$: at least three order of magnitudes smaller than the main amplitude

Final Results



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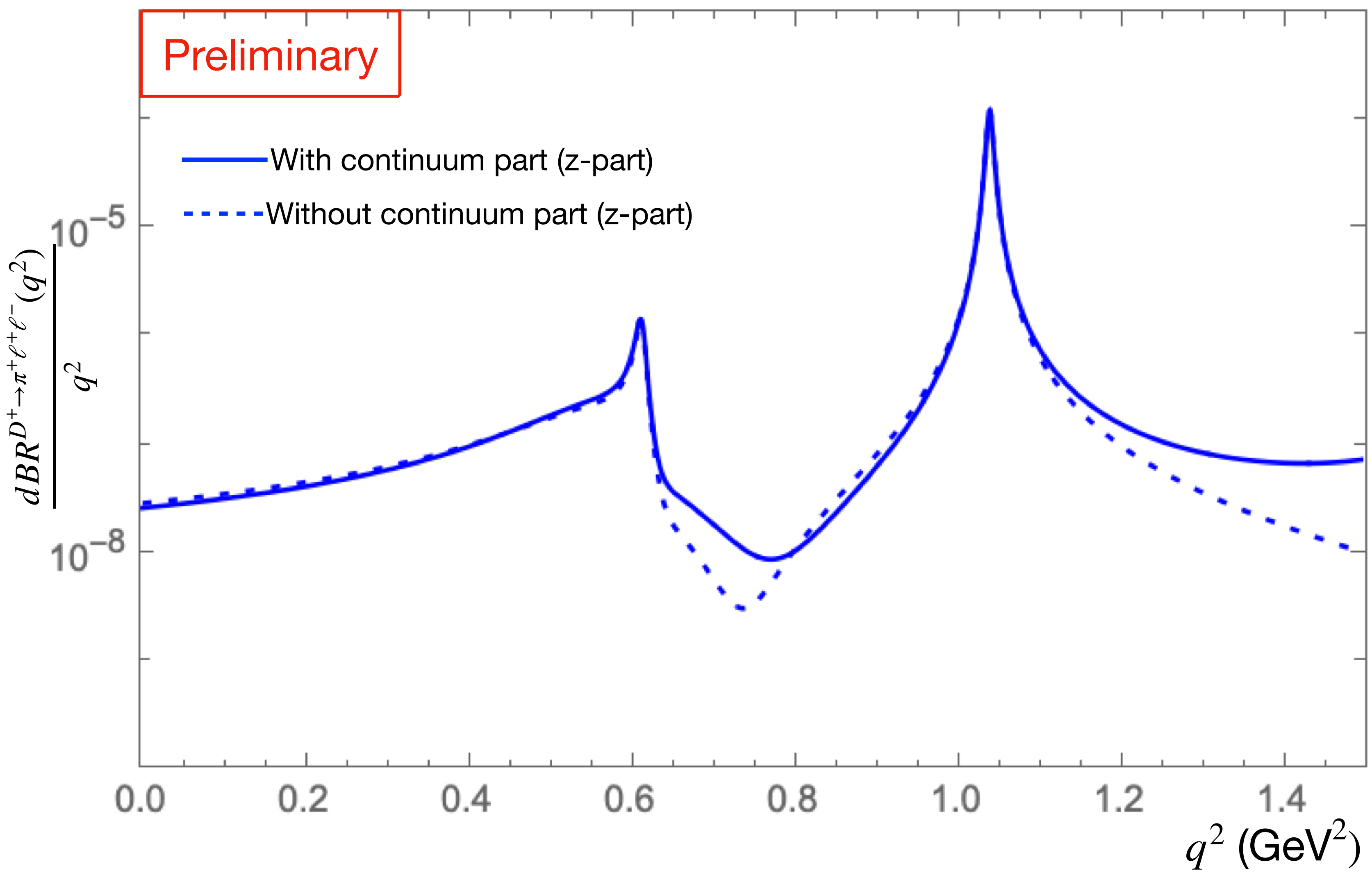

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Figure: The results for the differential branching fraction using the dispersion relation with the fitted parameters

- The low q^2 region is generated by the “tail” of the resonances, the intermediate and high q^2 region is influenced by excited states.
- The low q^2 region ($(0.250)^2 \leq q^2 \leq (0.525)^2$), integrated branching fraction $\sim 5.5 \times 10^{-9}$ ($\sim$ 2 times the QCDf predictions).

[A. Bharucha, D. Boito, C. Méaux, JHEP 04 (2021) 158]

Summary and Outlook

- In this work, we study the long distance effects in $D^+ \to \pi^+ \ell^+ \ell^-$ decays using LCSR supported dispersion relation.
- We found that the amplitude is mainly dominated by the weak annihilation topologies (loop and short distance contributions are tiny).
- We present the preliminary results for the differential decay width for $D^+ \to \pi^+ \ell^+ \ell^-$ decays.
- In low q^2 region ($(0.250)^2 \leq q^2 \leq (0.525)^2$) (in GeV^2), integrated branching fraction is $\sim 5.5 \times 10^{-9}$ (preliminary).

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- Important message for experimental analysis:

There is no way to isolate long distance effects in $D_{(s)} \to P\ell^+\ell^-$ decays by simply vetoing resonances, one need measurements of the differential decay rates in the whole q^2 region.

Thank you for your attention !!

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Back up!

What do we already know from theory: QCD factorization?

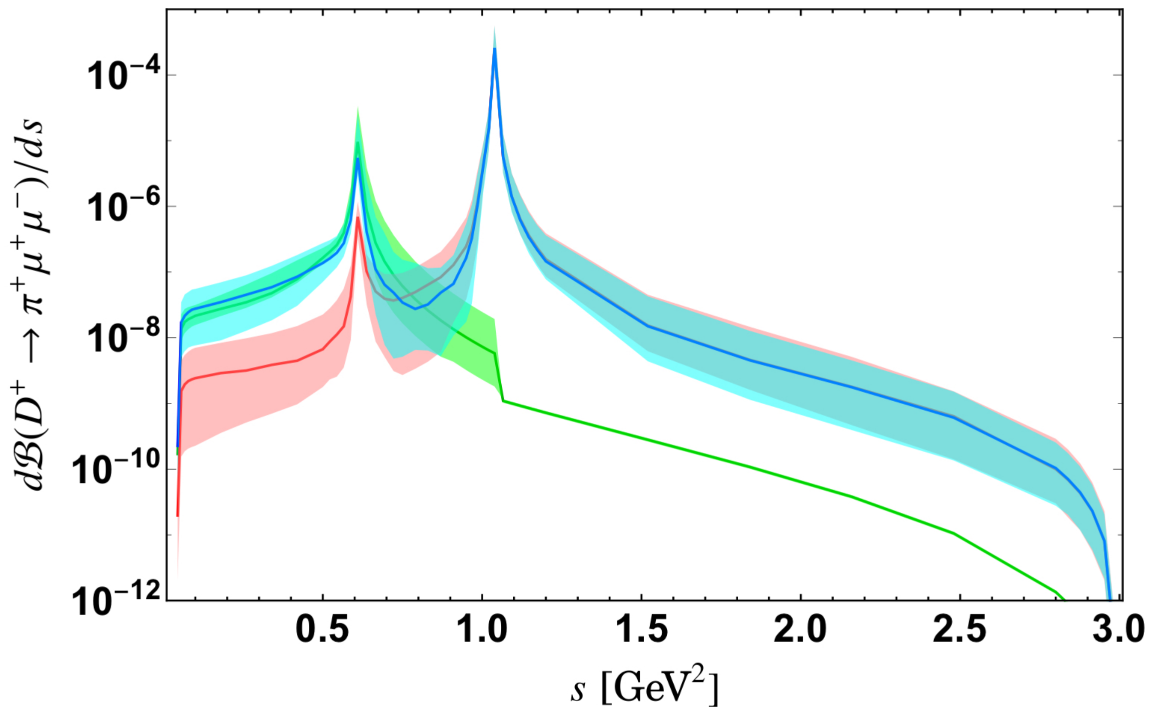
- The method was originally suggested for $B \to K^* \ell^+ \ell^-$.

- First use for charm decays in $D \to \rho \ell^+ \ell^-$:

  The loop topology diagram modified to include resonances. : Shifman model of loop-resonance duality

- Later, a similar method applied to $D \to \pi \ell^+ \ell^-$ (with the main focus on new physics).

$$\mathcal{B}(D^+ \to \pi^+ \mu^+ \mu^-)\Big|_{\text{low } q^2}^{\text{SM}} = (8.1^{+5.9}_{-6.1}) \times 10^{-9},$$
$$\mathcal{B}(D^+ \to \pi^+ \mu^+ \mu^-)\Big|_{\text{high } q^2}^{\text{SM}} = (2.7^{+4.0}_{-2.6}) \times 10^{-9},$$



- Major missing:
  - Includes only one of the four annihilation diagrams (emission from the initial d-quark) :
    - ✴ Other three diagrams turns out to be important.
  - $\frac{1}{m_c^2}$ corrections eg. from the use of D-meson distribution amplitudes:
    - ✴ Expected to be large (atleast compared to the B-meson case).

Therefore, with the experimental bounds approaching theory predictions, it is important to revisit it within the Standard Model.