Holographic mean field theory and Kondo Lattice

### Sang-Jin Sin (Hanyang U.) 2024.09@Athen



[Mean field theory and holographic Kondo lattice](https://inspirehep.net/literature/2803973), [2407.01978](https://arxiv.org/abs/2407.01978)

1 Mean field theory for strongly coupled systems: [Holographic approach](https://inspirehep.net/literature/2718941) *JHEP* 06 (2024) 100



# II. Holographic mean field theory

III. Kondo Physics

IV. Topology with Holography

### Frame of Physical thinking : reductionism

**Δημόκριτος | Democritus**



Matter =  $\sum$  atom

A frame of physical thinking:

 $Complex = \sum$  simple

### Simplicity is the key to the physics.

### Ways to the simplicity

- Physics=Seeking the simplicity. More than 5 parameter? Not much predictability.
- In condensed matter physics, there are  $10^{25}$  dof. How CM can be a physics? Ans= Periodic structure + 1 electron theory based on the weakness of int.)
- Even in particle physics SM, we need something for the simplicity i) Group structure (symmetry) ii) Hierarchy + Family structure. (repetition) iii) Weakness of coupling. (Independence of different sectors)
- In both PP & CM, the origin of the simplicity is the periodicity (repetition) & independence (weakness of int.)

### What happen if interaction is not weak? I

- 1. Particle nature is lost.
- 2. system is strongly entangled.

$$
H_{tot} = H(x_1) + H(x_2)
$$
  
\n
$$
\Rightarrow \psi_{tot} = \psi_i(x_1)\psi_j(x_2) \Rightarrow No \ entanglement
$$

$$
H_{tot} = H(x_1) + H(x_2) + H_{int}(x_1, x_2)
$$
  
\n
$$
\Rightarrow \psi_{tot} = \sum_{ij} c_{ij} \psi_i(x_1) \psi_j(x_2) \Rightarrow entanglement \iff \text{more even } c_{ij}
$$

### What if interaction is not weak? II

**•** Weak coupling: in  $\psi_{tot} = \sum_i c_{ij} \psi_i(x_1) \psi_j(x_2)$ , one term dominance. *ij*  $c_{ij}\psi_i(x_1)\psi_j(x_2)$  $\Rightarrow$   $\psi_{tot} = \psi_i(x_1)\psi_j(x_2)$  *separability* 

• For strong coupling, all the  $c_{ij}$  in  $\psi_{tot} = \sum c_{ij}\psi_i(x_1)\psi_j(x_2)$  are evenly distributed => No. of the important terms increases. *ij*

=> Entire system becomes one object.

- Inseparability is the characteristic of the strongly int. Sys.
- Simplicity restored! What one object? The black hole.

### view the whole as one body: QCP = BH



- Origin of simplification/universality in SIY = Information Loss  $=$  Democracy of scales  $=$  Emergence of physical law!
- Thermodynamic character: indeed, both have 0,1,2,3 law.
- Classification of OCP vs HSV:  $(z, \theta)$   $\omega = kz$ ,  $[s]=D-\theta \&$  sym.
- Equivalence is supported by exactly solvable models: AdS/SYM  $\rightarrow$

# Faq in AdS/CMT

• Postulate: gravity dual exits and Dictionary works.

GKP-Witten Relation

 $Z_{\text{gauge}} = Z_{\text{AdS}}$  $\langle \exp\left(i \int \phi^{(0)} O\right)\rangle = e^{i \underline{S}[\phi]_{u=0} = \phi^{(0)}]}.$ 

• Where is N of SU(N)? Large number of degen.



- Respect the bulk locality NOT the body locality.
- How to characterize a material?

## II. Holographic mean field theory

- Material = lattice\_structure +chem composition
- To characterize a CM, need to introduce a lattice.

Otherwise, we would not know what material we are dealing.

- 3 ways
	- 1. Explicit introduction. Brute force => PDE
	- 2. Explicit introduction. Tight Binding => ODE
	- 3. Implicit introduction by symmetry breaking

IR Probe scale = 1 meV =  $10^{-6}$ KeV (scale of lattice).

=> impossible to see the details of the lattice.

Proposal: Effect of the lattice = effect of the Symmetry breaking!

### Symmetry breaking and lattice

Proposal: in low E limit, Role of lattice  $= R$  or Tr symmetry breaking How to establish this? Calculate the effect of the order on the Fermion spectrum Mean field theory=Theory of symmetry breaking.

Conversely lattice can be identified as the spectrum generating symmetry breaking.

That is, material  $=$  spectrum ( $\sim$  band structure)

# Universal structure of MFT : Condensation and Order

 $\Delta \sim c_k c_{-k}$ , *BCS*  $\Delta \sim f_k^{\dagger} c_{-k}$ , *Kondo Condensation*  $M \sim c_k^{\dagger} \Gamma c_k$ , *Charge density or magnetic ordering* 

### Holographic MFT= Effect of order in fermion spectrum

 $Order$  :  $\langle \bar{c}\Gamma^A c\rangle \neq 0$ , Holographic dictionary: *Consider ψ dual to c*, and add  $\Phi_A \cdot \bar{\psi} \Gamma^A \psi$  to  $\mathscr{L}_0 = \bar{\psi} (\gamma^\mu i \partial_\mu - m) \psi$ .



Find the configuration of  $\Phi$  first, in the fixed BH gravity.  $\rightarrow$  Study  $\psi(z, x)$  in the fixed ( $g_{\mu\nu}$ ,  $\Phi$ )

to get spectrum of  $\chi$ .

### Structure of holographic MFT



p*<sup>g</sup>* ¯(*j*)

$$
JHEP 06 (2024) 100 \cdot e-Print: 2311.01897
$$

$$
S_{total} = S_{\psi} + S_{bdy} + S_{g,\overline{\Phi}} + \overline{S_{int}},
$$

2

*j*=1

⌘

 $d^d x \sum$ 

 $S_\psi = i$ 

Z

 $D_{\psi} = i \int d^{\pi}x \sum_{j=1}$ 



 $\overline{f}$ . Yuk Same  $\overline{f}$  and  $\overline{f}$ 

$$
S_{bdy} = \frac{i}{2} \int_{bdy} d^{d-1}x \sqrt{-h} \left( \bar{\psi}^{(1)} \psi^{(1)} \pm \bar{\psi}^{(2)} \psi^{(2)} \right),
$$
  
\n
$$
S_{g, \Phi} = \int d^{d}x \sqrt{-g} \left( R - 2\Lambda + |D_M \Phi_I|^2 - m_{\Phi}^2 |\Phi|^2 \right),
$$
  
\n
$$
S_{int} = \int d^{d}x \sqrt{-g} \left( \bar{\psi}^{(1)} \Phi \cdot \Gamma \psi^{(2)} + h.c \right)
$$

 $\sqrt{-g} \, \, \bar{\psi}^{(j)} \Big($ 

where  $\Phi$ <sub>*I*</sub> is order parameter field</sub> *d*5*x*  $\overline{d}$ ,  $\overline{d}$ ,  $\overline{d}$ ,  $\overline{d}$ ,  $\overline{d}$ ,  $\overline{d}$   $\overline{d}$  is constructed by cons where  $\Phi_I$  is order parameter field,  $\bar\psi^{(1)}\Phi\cdot\Gamma\psi^{(2)}$  is constructed by considering all possible Lorentz symmetry.  $4 \Box$   $\rightarrow$   $4 \Box$   $\rightarrow$   $4 \Box$   $\rightarrow$   $4 \Box$   $\rightarrow$   $\rightarrow$   $\Box$ 

$$
\Phi\cdot\Gamma=\Gamma^{\underline{\mu_1\mu_2\cdots\mu_I}}\Phi_{\underline{\mu_1\mu_2\cdots\mu_I}}.
$$

 $D\!\!\!\!/$  -  $m^{(j)}$ 

 $\setminus$ 

 $\psi^{(j)}$ 

## Classifying the MFT by the symmetry of the order

### 8 (half) of them have both simple pole and branch-cut types.

- $\bullet$   $\Phi, B_i, B_{ik}, B_{tu}$  (*AdS*<sub>5</sub>)
- $\bullet$   $\Phi$ ,  $\Phi$ <sub>5</sub>,  $B$ <sub>*i*</sub>,  $B$ <sub>5*i*</sub>,  $B$ <sub>*ik*</sub>,  $B$ <sub>*tu*</sub> (*AdS*<sub>4</sub>)



- 2-dimensional slice of the spectral density
- 3-dimensional spectral density

Figure: Simple pole and Branch-Cut types spectra

Appearing features: Gaps of s-,p-wave sym. 14 Flat bands of dim 1,2,3. Nodal rings of dim 1,2

# **Additional Supectral functions (pole types 1/4)** Analytic Green functions and their









## Summary of II

### Features in spectrum from Sym. Breaking Gaps of s-,p-wave sym. Flat bands of dim 1,2,3. Nodal rings of dim 1,2 Lattice <=> symmetry breaking

All many body theory assume:  $G \sim \frac{1}{\sqrt{2}}$ . *Z*  $\omega - \epsilon - \Sigma$ 

Some of Green fct has poles, indeed. but some of them are not. Branch cut singularity! => New class of Non-Fermi liquid 19

## III. Kondo Physics

- 1. Single Kondo
- 2. Multi-Kondo : Random impurities
- 3. Muti-Kondo : Kondo lattice

# What is Kondo physics



### **Scattering of [conduction electrons](https://en.wikipedia.org/wiki/Conduction_electrons) in a metal** by the [magnetic impurities](https://en.wikipedia.org/wiki/Magnetic_impurity).

1. Exp. Fact : Resistivity increase as T decrease after certain temp.



2. Theory: **Kondo**:  $\rho(T) = \rho_0 + aT^2 + bT^5 + c_m \ln \frac{\mu}{T}$ , Divergence as  $T \to 0$ . *T* 2. Theory: **Kondo**:  $\rho(T) = \rho_0 + aT^2 + bT^5 + c_m \ln \frac{\mu}{T}$ , Divergence as  $T \to 0$ 21 *formation of the Kondo Z. Theory.* **N** 

### $Saturation of  $\rho$  in  $T \rightarrow 0$$  $\mathsf{P}$  in the same state of  $\rho$  in  $\mathsf{I} \rightarrow \mathsf{U}$ the literature. degeneracy of such a magnetic ion is split, and provided there are an odd number of  $\alpha$

<sup>2</sup> ]) with an associated magnetic moment *M* = 2.64µ*B*. In a crystal, the 2 *j* + 1 fold

### In the transversal Kondo model defined by two independent parameters *g<sup>z</sup>* and *g*? = *g<sup>x</sup>* = *g<sup>y</sup>* nd. implimerant e couping goes strong in in. Complete screer degeneracy. (Fig. 2 and b.) **RG: imp-itinerant e coupling goes strong in IR: complete screening**

*<sup>d</sup>* ln *<sup>D</sup>* <sup>=</sup> 2*g*?*g<sup>z</sup>* ; *<sup>d</sup>* ln *<sup>D</sup>* <sup>=</sup> 2*g*<sup>2</sup> from which we obtain by integration [*g<sup>z</sup>*] ? = const*.* Therefore, the flow of the parameters *g<sup>z</sup>* and *g*? are located on a hyperbolic curve in the parameter space (*g<sup>z</sup>, g*?) which is depicted in figure 2. Since the RG-flow in Eq. (39) always when  $\mathcal{I}$  and  $\mathcal{I}$  and  $\mathcal{I}$  are  $\mathcal{I}$  and  $\mathcal{I}$  and  $\mathcal{I}$  are  $\mathcal{I}$  and  $\mathcal{I}$  are  $\mathcal{I}$  and  $\mathcal{I}$  are  $\mathcal{I}$  and  $\mathcal{I}$  are  $\mathcal{I}$  an  $\frac{1}{2}$  fundation  $\frac{1}{2}$ ? *<sup>&</sup>gt;* <sup>0</sup> and *<sup>g</sup><sup>z</sup> <sup>&</sup>lt;* <sup>0</sup>. If the transverse coupling is larger than the , *g<sup>z</sup> <* 0, the transversal coupling *g*? remains finite for *g<sup>z</sup>* = 0 and induces a sign change of  $g^2$ . The strong-coupling flow to the strong-coupling fixed-point ( $g^2$ (1*,* 1). These flow equations have one stable fixed point (*g<sup>z</sup>, g*?)=(1*,* 1) and one line of fixed points (*g<sup>z</sup> ,* 0). The latter are stable for a ferromagnetic *g<sup>z</sup> <* 0 and unstable for *g<sup>z</sup> >* 0. For a fully isotropic Kondo coupling, *g* = *g<sup>z</sup>* = *g*?, we only need to integrate the single differential  $\sqrt{2}$ *dg*  $\frac{dy}{d \ln D} = \frac{a}{e} \mathcal{G}(g) \frac{f}{t} = -\frac{2}{e} g^2 \frac{1}{t}$  **a p h**<br>**c c c h c n x** 11.12 Frithjof B. Anders The function of the function of the function in the second the literature and determines how the coupling of t constants flow while reducing the band width: a negative -function is a signature of weak signat interactions are the contractions and a growing interaction strength while  $dg$  is a growing the band width.  $\frac{u}{e} \frac{d}{d} \ln D$   $\frac{u}{r} \frac{d}{d} \frac{d}{d} \frac{d}{d}$  $g(D_0^{\mathbf{t}}) = \frac{\mathbf{k}}{i_1 + \mathbf{k_2}} \frac{q_0}{i_2 + \mathbf{k_3}}$  $1+2g_0 \ln(D_0^{\mathbf{h}}/D^{\prime})$ This solution obviously breaks down at a low energy scale *T<sup>K</sup>* = *D* at which the denominator 11.12 Frithjeffer B. Anders B The function is called the second the literature and determines how the coupling of the coupli constants flow which reducing the band with the band width: a negative reducing to the band with the band with<br>The band width: a negative of weak is a signature of weak is a signature of weak is a signature of weak is a s interactions are got high-energies and a growing interaction strength while  $\frac{t}{a}$  interaction strength width. With the initial values of the model *D*0*, g*0, we integrate this differential equation to  $\begin{array}{ccc} \n\frac{D}{c} & \frac{1}{c} & \frac{1}{c} \\ \n\frac{1}{c} & \frac{1}{c} & \frac{1}{c} \n\end{array}$  $\frac{1}{2}$ **g**  $\frac{1}{2}$ **g**  $\frac{1}{2}$ **g**  $\frac{1}{2}$ **g**  $\frac{1}{2}$  $\int \mathbf{S} \cdot \vec{S} \cdot \vec{I}$ diverges: **6. Origin** of excess to  $\theta$ The Hamiltonian of the Anderson model can be described by  $H_0 = \sum \varepsilon_k c$  $H' = \frac{1}{\sqrt{N}} \sum_{k,\sigma}$ where *n d* e that  $\mathbf{d}H$  m s  $\sigma$  $\mathcal{E}_{\pmb{k}} \mathcal{C}_{\pmb{k}\sigma}$ , 0 *k*  $k$ <sup> $\cup$ </sup> $k$  $H' = \frac{1}{\sqrt{N}} \sum_{k,\sigma} (V_{kd} c_{k\sigma}^{\dagger} d_{\sigma} + V_{dk} d_{\sigma}^{\dagger} c_{k\sigma})$  (perturbation)  $n_{d\sigma} = d_{\sigma}^{d} d_{\sigma}$  $\langle \sigma, d_{\sigma}^{\dagger} \rangle = 1$ , c<sub>g</sub><sub>r</sub>  $c_{k\sigma}$  +  $\varepsilon_d$   $\sum$  $\sigma$  $H_0 = \sum_{k,\sigma} \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + \varepsilon_d \sum_{\sigma} n_{d\sigma}^{\dagger} + U n_{d\uparrow} n_{d\downarrow}$  $(\sigma$  $\big\langle c_{k\sigma}$ **r**  $=\uparrow, \downarrow$  ).  $,c_{k' \sigma}^{\dagger}$   $\left\} = \delta_{k,k'}^{\phantom{\dagger}}$ Schrieffer-Wolff transformation —> Kondo Hamiltonian *M*<sup>2</sup>  $H_{\alpha}$   $+U_{n_{\alpha}+n_{\beta}}$ *<sup>B</sup> j*(*j* + 1), (2) angular momentum quantum number *j* and gyro-magnetic ratio ("g-factor") *g*. ✓ is the "Curie Where  $\sum_{n=1}^{\infty} \frac{1}{n}$  is the matrix of interactions between  $\sum_{n=1}^{\infty} \frac{1}{n}$  interactions between  $\sum_{n=1}^{\infty} \frac{1}{n}$  is the matrix of interactions between  $\sum_{n=1}^{\infty} \frac{1}{n}$  is the matrix of interactions b  $\begin{pmatrix} 1 & 1 \end{pmatrix}$  a metal presence of such local properties. The physics profoundly alters in the physics properties. The physics properties are properties. The physics properties are properties. The physics properties  $(\begin{array}{cccc} 0 & 0 \\ 0 & 0 \end{array})$  is defined by the *Ko*<sup>n</sup>  $\kappa$ <sub>0</sub>  $H =$  $\sum$  $k\sigma$  $\epsilon_k c^{\dagger}_{k\sigma} c_{k\sigma} +$  $\overline{d}H$  $\overrightarrow{r}$  $g \psi^\dagger(0) \vec{\sigma} \psi(0) \cdot \vec{S}$ *<sup>f</sup>* . (3) *g* al  $\mathbf{n}$ The inter Kasuya-Y **c**  $\mathbf{i}$ nature of My fi was written to the top of the top o electrical  $1$ temperatu impuritie dependen The **K** noble me s l  $\frac{1}{c}$  c n n r w Y r c a i n f spin glass. fie  $e$ t  $d\alpha$ l  $\sim$  9 f u<sub>71</sub>  $\sqrt{2}$ e $\mu$  111  $\mu$ n e **K** t  $K_{\Omega}$  ( )  $\bigcap$   $\mathfrak{b}$ e $\beta$  (  $\overline{\phantom{a}}$  o lized spins in c e n m w a r y a e of the intervals  $\mathbf n$  the antiferred antiferred and  $\mathbf n$ e K f<sub>a</sub>  $\overline{\phantom{a}}$   $\overline{\phantom{a}}$   $\phantom{a}$   $\phantom{a}$   $\phantom{a}$  $\cup$  or  $r$ e c **t and Jun K**  $\mathfrak{b} \setminus \ \_\_\_\mathfrak{u}$ o  $\frac{1}{1}$  $\mathbf{n}$   $\mathbf{A}$ e  $\wedge$  n m/ $u$ a  $\sqrt{u}$ ys an import  $e \t n$ t h romagnetic K c ald. He shows that the shows that the shows that  $\mathbf w$  $a/\sqrt{a}$  f e $\cup$  (*y* / t range. Even c h **K K** u i  $i \rightarrow s$ n i u g u h t s n e h a and ferroma c w w f f  $\qquad$   $\qquad$ t é at the present of  $e$ h in the set of  $\mathbf h$ i d  $s_{\Omega}$  1

diverges:

Anderson's poor man's scaling(1970) —>  $\displaystyle\leftarrow$  **F**  $\leftarrow$  : numerical RG (1975)  $\frac{22}{}$  $k$  creates a conduction  $\sum_{k=1}^{\infty} E_d + U$ critic and origin at the origin, where  $\frac{1}{\sqrt{2}}$  is the conduction in the conduction.

sea interacts with local moment via an antiferromagnetic contact interaction of strength *J*. The

 $\text{Hence } \mathbf{F} \setminus \{ \mathbf{F} \}$  is a small contribution on  $\text{Hence } \mathbf{F} \setminus \{ \mathbf{F} \}$ higher order processes will modify the -function. Nevertheless, we can use the new energy  $\overline{E_d}$  +  $\overline{E_d}$  +  $\overline{U}$  is only valid for small constants  $\overline{E_d}$  (4.075)  $h^{(1)}(n)$  -function. Nevertheless, we can use the new  $\frac{1}{2}$ *s <sup>k</sup> c*† *k*

scale to express the running coupling constant *g*(*D*<sup>0</sup>

 $T_K = D_0 e^{-1/2g_0} = D_0 e^{-1/\rho_0 J}$ 

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 $i^{\circ}$  d  $\sqrt{\nabla} h$ 

*<sup>d</sup>* ln *<sup>D</sup>* <sup>=</sup> 2*g<sup>x</sup>g<sup>y</sup>* (38c)

*.*  $\begin{array}{ccc}\n\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
a\n\end{array}$ 

### Classification of Multi Kondo : Random vs regular impurities

## single Kondo

**Figures and Captions**

 Kondo Lattice Both heavy fermion/ Kondo insulator



**RKKY** weak coupling

Random imp.

Kondo-Conden:

gap



### III.2. Discovery of a tiny gap in a dirty semiconductor



Physics revision 심사 중). (A) 측정 set-up 및 (B,C) 측정된 저항(Rd)–온도(<sup>T</sup> )/자기장(<sup>B</sup> ) 특성 곡선.

## Difficulty of our system as Kondo lattice

• If no periodicity—> No momentum ! No band.

The whole picture of Kondo-lattice break down.

- No calculational scheme.
- In fact, random singlet picture
- $\rightarrow$  No gap!
- However, ……

A gap is found in random impurity similar to Indirect gap of Kondo lattice

### lattice point group symmetry either to a quartet and doublet in cubic crystal, or three Kramers doublets in a tetragonal environment. Taking into a single Kramers doublet only a single Kramers doublet on the Overlapping Kondo cloud => Kondo condensation : Our proposal: dense Random multi-Kondo

$$
H = \sum_{i\sigma}\varepsilon_i^f f_{i\sigma}^\dagger f_{i\sigma} + U n_{i\uparrow} n_{i\downarrow} + \sum_{\vec{k}\sigma}\varepsilon_{\vec{k}\sigma} c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma} + \sum_{i,\vec{k},\sigma} V_k \left( g_{\rm I}^{i\vec{k}\vec{R}_i} f_{\rm II}^\dagger c_{\rm II} + g_{\rm II}^{i\vec{k}\vec{R}_i} c_{\rm III}^\dagger f_{\rm III} + g_{\rm III}^{i\vec{k}\vec{R}_i} c_{\rm III}^{\dagger} c_{\rm III}^{\dagger} c_{\rm III} + g_{\rm III}^{i\vec{k}\vec{R}_i} c_{\rm III}^{\dagger} c_{\rm III}^
$$

- Cooper pair=cc  $\cdot \cdot \langle c \rangle \times (0) \rightarrow 0$  superconductivity explain some basic properties of HF materials [24], a more realistic description realistic description require<br>The more requires the more realistic description requires the more realistic description requires the more rea
- $\blacksquare$ • Kondo pair =  $f^{\dagger}c$  :  $\langle f^{\dagger}c \rangle \neq 0 \rightarrow$  Kondo condensation effects are clearly seen in the specific heat or transport measurements [24].



Famamoto et. al.<br>"Observation of the Kondo screening cloud" Yamamoto et. al. Nature 2020

extension of the Anderson model (PAM) and the Anderson model (PAM) and the Anderson model (PAM) and the Anders<br>The Anderson model (PAM) and the Anderson model (PAM) and the Anderson model (PAM) and the Anderson model (PAM

#### Kondo condensation model and its result The action for the fermion for the fermion and the real spacetime is given by S155 dimensional spacetime is gi<br>S155 dimensional spacetime is given by S155 dimensional spacetime is given by S155 dimensional spacetime is gi Kondo condensation model and its result similar approach is true when we fix *ψ*−. The former defines the standard quantisation, and le condensation model and it  $\blacksquare$

! + 3R!) (C. 7)

**S8.2. Spectral function and density of state in holographic theory**

where  $T_{\rm B}$  is the BH temperature chosen to be  $T_{\rm B}$  and  $\mu$   $\sim$   $2$   $\mu$   $\sim$   $2$   $\mu$   $\sim$   $2$   $\mu$   $\sim$   $2$   $\mu$ 

E-@E0AB/1 = 3-0 . The subscript *D* denotes the Dirac fermion and the covariant

4 (2NOS) + 24N<br>O5 (2NO5 + 24N)<br>O5 (2NO5 + 24N)

 $\frac{1}{2}$ 

A±

 $\mathbf{F}$ 

%3 =

lm

,

$$
S_D = \int d^{d+1}x \sqrt{-g} \overline{\psi} (\Gamma^M D_M - m - \Phi) \psi + \int d^{d+1}x \sqrt{-g} \left( |\partial_\mu \Phi|^2 - m^2 \Phi^2 \right)
$$
  

$$
D_M = \partial_M + \frac{1}{4} \omega_{abM} \Gamma^{ab} - iqA_M,
$$

$$
\Phi = \frac{\Phi^{(0)}}{r} + \frac{\Phi^{(1)}}{r^2} + \cdots
$$
\n
$$
\Phi^{(0)} = 0, \quad \Phi^{(1)} = M_0 \sqrt{1 - T/T^*}
$$
\n
$$
f(r) = 1 - \frac{r_0^3}{r^3} - \frac{r_0 \mu^2}{r^3} + \frac{r_0^2 \mu^2}{r^4}.
$$



### **nature physics**





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**Article Article and the actual of the state of the state of the state of the https://doi.org/10.1038/s41567-022-01930-3** 

**, Jonathan R. Prance <sup>6</sup>**

### **Observation of Kondo condensation in a degenerately doped silicon metal Observation of Kondo in a degenerately doped silicon metal**

**Yuri A. Pashkin <sup>6</sup>**

**Hyungsang Kim<sup>1</sup>, Eun Ky⊌Kim®<sup>3</sup>⊠, and the same of the songs, and the same of the same o Yonuk Chong <sup>4</sup> , Woon Song5 Taewon Yuk3 , Sang-Jin Sin <sup>3</sup> , Soonjae Moon3 , Jonathan R. Prance <sup>6</sup> , , Sang-Jin Sin <sup>3</sup> , Soonjae Moon3** Yuri A. Pashkin <sup>®</sup> & Jaw-Shen Tsai<sup>2,7</sup> **Hyunsik Im 1,2 , Dong Uk Lee <sup>3</sup> , Yongcheol Jo1 , Jongmin Kim1 ,** 

 Check for updates  $\Omega$  $\sim$ 

 $\bullet$   $\boxtimes$ 

### **Remark: random vs lattice** Remark: random vs lattice K-condensation vs K-insulator  $\mathsf{R}$  and  $\mathsf{R}$  set-up  $\mathsf{R}$  set-up  $\mathsf{R}$ of Kondo hybridization between the Sm local moments and the conduction electrons and is consistent with  $\sim$



tibility deviates from high-temperature independent-spin

gaplike features in the low-temperature point-contact spectroscopy, but definitive explanations of the shape and the temperature evolution are lacking. In our study, the temperature dependence of the differential point-contact conductance data, dI=dV, measured using a Ag-SmB6 junction, is summarized in Fig. 1(c). Below 100 K, the local term in Fig. 1(c). Below 100 K, the local term in

#### K-lattice : asymmetric gap K-cond: symmetric gap  $V$  lattice, is corresponded con  $\ldots$  iaconco  $\ldots$  ac $\ldots$ 1.  $\ldots$  and  $\ldots$   $\ldots$ **f** 1  $\blacksquare$  $\boldsymbol{\mathsf{\Omega}}$  $\blacktriangledown$ Ceco(In0.9985Hg). The Ceco(In0.9985Hg) is the Ceco(In0.9985Hg).  $\blacksquare$  $\sim$   $\sim$ the corresponding surface  $\mathcal{L}$  of  $\mathcal{L}$  at 20 K (dashed line). B, Averaged line). B, Averaged line  $t_{\rm m}$ metric son  $K_{\rm m}$  $\sum_{i=1}^n$



 $SLP$ Si:P larger than the amplitude of the hybridization with the in-plane spd

150 mk 이다. (C) 양자 상전이 phase diagram 및 각 phase 에서 측정된 DOS 스펙트럼.  $\blacksquare$  directly probe the energy of  $\blacksquare$ **dual in the 115 material symmetric** stap troscopic map  $\mathcal{L}$  mapping with the STM that enables us to visualize  $\mu$ momentum  $\sigma$  $\gamma$ illiliclic gap discrete Fourier transforms ( $\frac{1}{\sqrt{2}}$ corresponding surface B of CeRhIn5 at 20 K (dashed line). c, d, Tunnelling

### **III.3 Physics of Kondo lattice**

![](_page_29_Figure_1.jpeg)

Essence of the Kondo Lattice physics:

Electron trapped and propagate rarely from site to site.

On a larger length scale, a very slow coherent motion

a quasi-particle with a large effective mass.

#### **Assumed on refs. [44], MET for the Kondo lattice in refs. [44], we consider the Kondo lattice in relationships a non-relationships a non-relationships a non-relationships a non-relationships a non-relationships a non-rela** ~ ) *·* (*†* MFT for the Kondo lattic <sup>2</sup> ( *†*  $\mathbf{F} = \mathbf{F} \mathbf{$ matrices. *g<sup>l</sup>* is the light-light coupling constant. *g<sup>s</sup>* and *g<sup>v</sup>* are the scalar- and vector-type **heavy-light coupling coupling coupling coupling coupling coupling coupling the Fierz identity, we can write the Fierz iden**  $\zeta$  and  $\alpha$  +  $\zeta$ 2*m* <sup>+</sup> *<sup>g</sup><sup>l</sup>* <sup>2</sup> *<sup>g</sup>s*( *†*

*i*

@*<sup>t</sup>*

i.

<sup>2</sup> ( *†*

@*<sup>t</sup>* <sup>+</sup>

2*m*

@*<sup>t</sup>*

) *gv*( *†*

~ ) *·* (*†*

is the mass of the light fermion. *µ* is the chemical potential for the light fermion. is

) *gv*( *†*

$$
\mathcal{L} = \psi^{\dagger} \left( i \frac{\partial}{\partial t} + \frac{\nabla^2}{2m} + \mu \right) \psi + \chi^{\dagger} \left( i \frac{\partial}{\partial t} - \lambda \right) \chi + \frac{g_l}{2} (\psi^{\dagger} \psi)^2 - g_s(\psi^{\dagger} \psi)(\chi^{\dagger} \chi) - g_v(\psi^{\dagger} \vec{\sigma} \psi) \cdot (\chi^{\dagger} \vec{\sigma} \chi).
$$

+ *†*

Using the Fierz identity, *g* Fierz identity.

2.1 Setup

continuum limit as follows:

✓

@

*L* = *†*

) *gv*( *†*

◆

*i*

@*<sup>t</sup>* <sup>+</sup>

2*m*

the energy level of the heavy fermion without hybridization. ~ = (1*,* 2*,* 3) are the Pauli

+ *µ*

+ *†*

)(*†*

$$
\mathcal{L} = \psi^{\dagger} \left( i \frac{\partial}{\partial t} + \frac{\nabla^2}{2m} + \mu \right) \psi + \chi^{\dagger} \left( i \frac{\partial}{\partial t} - \lambda \right) \chi \n+ \frac{g_l}{2} (\psi^{\dagger} \psi)^2 + g_s' (\psi^{\dagger} \chi)(\chi^{\dagger} \psi) + g_v' (\psi^{\dagger} \vec{\sigma} \chi) \cdot (\chi^{\dagger} \vec{\sigma} \psi), \qquad g_s' \coloneqq \frac{g_s + 3g_v}{2}, \quad g_v' \coloneqq \frac{g_s - g_v}{2}.
$$
\n
$$
\mathcal{L}_{\text{MF}} = \Psi^{\dagger} D \Psi - U,
$$

$$
\Psi^{\dagger} := (\psi^{\dagger} \chi^{\dagger}), \quad \Psi := \begin{pmatrix} \psi \\ \chi \end{pmatrix}, \qquad \langle \psi^{\dagger} \psi \rangle \equiv -\frac{M}{g_l}, \quad \langle \psi^{\dagger} \chi \rangle \equiv \frac{\Delta_s}{g_s'}, \quad \langle \psi^{\dagger} \vec{\sigma} \chi \rangle \equiv \frac{\vec{\Delta}_v}{g'_v},
$$
\n
$$
D := \begin{pmatrix} i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} + \mu - M \Delta_s^* + \vec{\sigma} \cdot \vec{\Delta}_v^* \\ \Delta_s + \vec{\sigma} \cdot \vec{\Delta}_v & i\frac{\partial}{\partial t} - \lambda \end{pmatrix},
$$
\n
$$
U := \frac{M^2}{2g_l} + \frac{|\Delta_s|^2}{g'_s} + \frac{|\vec{\Delta}_v|^2}{g'_v}.
$$
\n31

### MFT for the Kondo lattice (continued)

![](_page_31_Figure_1.jpeg)

 $T_K \sim V^2/D$  : 1 − *Kondo Temp*.

FS in gap- $>$  K insulator, otherwise (a)  $\omega(p)$  without condensation. (b)  $\omega(p)$  with condensation. **Heavy Fermion w/ larger FS** 

#### MFT for the Kondo lattice 2 *|v|* = 0. Black and blue lines show the direct and indirect hybridization gaps, respectively. **The direct gap is approximately linear in**  $\mathbb{M}$  **is a for the Kondo lattice 2.** *G*1(!*, p*~) := ! *<sup>p</sup>*<sup>2</sup> <sup>2</sup>*<sup>m</sup>* + *µ M* ⇤ *<sup>s</sup>* <sup>+</sup> ~ *·* <sup>~</sup> ⇤ *v* e Kondo lat *,* (2.10) *<sup>s</sup>* <sup>+</sup> ~ *·* <sup>~</sup> *<sup>v</sup>* !

!

*<sup>s</sup>* <sup>+</sup> ~ *·* <sup>~</sup> ⇤

[! !*i*(*p*~)]*.* (2.11)

<sup>2</sup>

*|s|* = 0, *|v|* = 0, ✓ = 0. (b) *M* = 0*.*1, *|s|* = 0*.*1, *|v|* = 0*.*05, ✓ = 1. (c) *M* = 0,

<sup>2</sup>*<sup>m</sup>* + *µ M* ⇤

$$
\Omega = U + \frac{1}{V} \sum_{|\vec{p}| < \Lambda} \sum_{i=1}^{4} \left\{ -\frac{1}{2} |\omega_i(\vec{p})| - \frac{1}{\beta} \ln \left[ 1 + e^{-\beta |\omega_i(\vec{p})|} \right] \right\}
$$
  
= 
$$
U - \frac{1}{4\pi^2} \int_0^{\Lambda} dp p^2 \sum_{i=1}^{4} |\omega_i(p)| - \frac{1}{2\pi^2 \beta} \int_0^{\Lambda} dp p^2 \sum_{i=1}^{4} \ln \left[ 1 + e^{-\beta |\omega_i(p)|} \right],
$$

det *<sup>G</sup>*1(!*, <sup>p</sup>*~) ⌘ <sup>Y</sup>

a<br>Ma

*i*=1

! *<sup>p</sup>*<sup>2</sup>

$$
\omega_{i=1,\dots,4} = \mathcal{E}_+ \pm \sqrt{\mathcal{E}_-^2 + |\Delta_s|^2 + |\vec{\Delta}_v|^2 \pm \sqrt{(|\Delta_s|^2 + |\vec{\Delta}_v|^2)^2 - |\Delta_s^2 - \vec{\Delta}_v \cdot \vec{\Delta}_v|^2}},
$$
  

$$
\mathcal{E}_\pm \coloneqq \frac{1}{2} \left[ \left( \frac{p^2}{2m} - \mu + M \right) \pm \lambda \right].
$$

![](_page_32_Figure_3.jpeg)

33 (a)  $\Omega$  versus  $|\Delta|$ . (b)  $\Omega$  with strong  $g'_s > g'_v > g_c$ . (c)  $\Omega$  with strong  $g'_v > g'_s >$  3

### Holographic Kondo Lattice probe spinor fields (1*,*2) in AdS4:

Consider a metric field *g*, a *U*(1) gauge field *A*, two neutral real scalar fields s*,*ps, and two

$$
S_{\text{tot}} = S_{\text{bg}} + S_{\text{spin}},
$$
  
\n
$$
S_{\text{bg}} = S_{\text{bg},\text{bdy}} + \int d^4x \sqrt{-g} \left( R + \frac{6}{L^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)
$$
  
\n
$$
+ \int d^4x \sqrt{-g} \left[ -(\partial_{\mu} \Phi_8)(\partial^{\mu} \Phi_8) - m_8^2 \Phi_8^2 - (\partial_{\mu} \Phi_{\text{ps}})(\partial^{\mu} \Phi_{\text{ps}}) - m_{\text{ps}}^2 \Phi_{\text{ps}}^2 \right],
$$
  
\n
$$
S_{\text{spin}} = S_{\text{spin},\text{bdy}} + \sum_{j=1}^2 \int d^4x \sqrt{-g} i \bar{\psi}^{(j)} \left[ \frac{1}{2} \left( \vec{B}^{(j)} - \vec{B}^{(j)} \right) - m_j \right] \psi^{(j)}
$$
  
\n
$$
+ \int d^4x \sqrt{-g} \left( \frac{\bar{\psi}^{(1)}}{\bar{\psi}^{(2)}} \right)^T \left( g_1 \Phi_{\text{ps}} \cdot \mathbf{r}^5 \ \mathbf{V} \Phi_8 \cdot i \mathbb{I}_4 \right) \left( \psi^{(1)} \right),
$$
  
\n
$$
S_{\text{spin},\text{bdy}} = \frac{1}{2} \int d^3x \sqrt{-h} [\bar{\psi}^{(1)}(i\mathbb{I}_4) \psi^{(1)} + \bar{\psi}^{(2)} \Gamma^{\text{XX}} \psi^{(2)}],
$$
  
\n
$$
\psi^{(j)} = \Gamma^a e_a^B \left( \partial_B + \frac{1}{4} \omega_{\text{bed}} \Gamma^{cd} - i q_j A_B \right),
$$
  
\n
$$
\mu = g g^{ua},
$$
  
\n
$$
L = 1,
$$
  
\n
$$
\bar{\psi}^{(j)} = \psi^{(j) \dagger} \Gamma^{\text{t}}_{\text{sp}}
$$

 $i\sigma_2$  0

 $\setminus$ 

 $\Gamma^{\underline{u}} = \sigma_3 \otimes \sigma_0 =$ 

2

 $[\Gamma^a,\Gamma^b]$ 

 $\Gamma^{ab} = \frac{1}{2}$ 

 $0\,\,\,\sigma_3$ 

 $\sigma_3$  0

 $\sqrt{ }$ 

 $\Gamma^\mathrm{Y} = \sigma_1 \otimes \sigma_3 =$ 

 $\Gamma^5 = i\Gamma^{\underline{t}}\Gamma^{\underline{X}}\Gamma^{\underline{Y}}\Gamma^{\underline{u}}.$ 

![](_page_33_Picture_2.jpeg)

] . 24

 $\setminus$ 

*,* (3.7)

 $\sigma_1$  0

 $\sigma_0$  0

 $0 - \sigma_0$ 

 $\sqrt{ }$ 

#### <sup>y</sup> <sup>=</sup> <sup>1</sup> ⌦ <sup>3</sup> <sup>=</sup>  $\overline{\mathbf{S}}$  $=$  3  $\pm$  3  $\pm$ = *i*txyu *, ab* **1 Molographic Kondo Latti** [*<sup>a</sup> , <sup>b</sup>* nor<br>2 iphic Kon do Lattice 2 *D/* (*j*) ⌘ Holographic Kondo Lattice 2

*,* u

<sup>0</sup> 0

] *.* (3.9)

(*j*)

*,* (3.8)

0 <sup>3</sup>

*j*=1

5

$$
+ \int \mathrm{d}^4x \sqrt{-g} \Bigg( \frac{\bar{\psi}^{(1)}}{\bar{\psi}^{(2)}} \Bigg)^{\prime} \, \Bigg( \frac{g_1 \Phi_{\mathrm{ps}} \cdot \Gamma^5}{V \Phi_{\mathrm{s}} \cdot i \mathbb{I}_4} \, \frac{V \Phi_{\mathrm{s}} \cdot i \mathbb{I}_4}{g_2 \Phi_{\mathrm{s}} \cdot i \mathbb{I}_4} \Bigg) \Bigg( \frac{\psi^{(1)}}{\psi^{(2)}} \Bigg),
$$

•  $g_1$  is the coupling strength of  $\psi^{(1)}(\Phi_{ps} \cdot \Gamma^5)\psi^{(1)}$  that makes a hyperbolic spectrum of the light fermion dual to  $\psi^{(1)}$  (to see why we have not chosen the scalar-type interaction, see appendix D). 1 Z  $\frac{1}{\sqrt{2}}$ *x*upli ng strength of  $\psi^{(1)}(\Phi_{\text{ps}} \cdot \Gamma^5)\psi^{(1)}$  that makes a hyperbolic spectrum

 $\overline{z}$ 

- We consider the standard-mixed quantization to flatten the spectrum of the heavy fermion dual to  $\psi^{(2)}$  (see eq. (3.4) and refs. [70, 71, 83]). The flat spectrum comes from the cancellation of the spinor components making the compact localized states (CLS) [71, 84].  $\frac{1}{2}$
- $g_2$  is the coupling strength of  $\bar{\psi}^{(2)}(\Phi_s \cdot i\mathbb{I}_4)\psi^{(2)}$  that isolates the flat spectrum from others (see appendix  $D$  and ref. [71]).
- *V* is the coupling constant of the inter-flavor interaction  $\bar{\psi}^{(1)}(\Phi_s \cdot i\mathbb{I}_4)\psi^{(2)}$  hybridizing the light and heavy fermions.

### 10 Holographic Kondo Lattice 3 )((*j*) *<sup>i</sup>*u) (*j*)

*S*spin = (equations of motion term)

*j*=1

$$
\left[ \begin{pmatrix} \overrightarrow{D} - m_1 & 0 \\ 0 & \overrightarrow{D} - m_2 \end{pmatrix} - i \begin{pmatrix} g_1 \Phi_{\text{ps}} \cdot \Gamma^5 & V \Phi_{\text{s}} \cdot i \mathbb{I}_4 \\ V \Phi_{\text{s}} \cdot i \mathbb{I}_4 & g_2 \Phi_{\text{s}} \cdot i \mathbb{I}_4 \end{pmatrix} \right] \begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \end{pmatrix} = 0.
$$

![](_page_35_Figure_2.jpeg)

# IV. Topology in interacting system

• Topological Hamiltonian Method and Eigenvectors  $(\omega = 0)$ 

$$
\mathcal{H}_t(\boldsymbol{k}) = -\mathbb{G}^{-1}(0,\boldsymbol{k})
$$

where eigenvector of  $H_t$  and  $H$  share the same eigenvector,  $|n\rangle$ .

$$
\mathcal{F}_c = \nabla \times \langle n | \partial_{\mathbf{k}} | n \rangle \tag{2}
$$

Alternative method: "Cubic of Green's function"

$$
\mathcal{F}_c = \frac{1}{3!} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \epsilon_{\mu\nu\rho c} \text{Tr} \left[ \mathbb{G} (\partial_{\mu} \mathbb{G}^{-1}) \mathbb{G} (\partial_{\nu} \mathbb{G}^{-1}) \mathbb{G} (\partial_{\rho} \mathbb{G}^{-1}) \right]
$$
(3)

![](_page_36_Figure_7.jpeg)

Monopole Number:

$$
C_n = \oint \mathcal{F}_c \cdot dS = i \oint \nabla \times \langle n | \partial_{\mathbf{k}} | n \rangle \cdot dS
$$

#### Critical case  $(\Phi = 0)$ For zero temperature, we use the Green's function derived in our previous work, which in our previous work, which Figure 1:  $\text{Critical case (} \mathbb{Q} = 0 \text{)}$ <sup>2</sup>*k*3*/*<sup>2</sup> **<sup>1</sup>** (5.4) *Fkz,k<sup>x</sup>* = *Fkx,k<sup>z</sup>* = *ky* <sup>2</sup>*k*3*/*<sup>2</sup> **<sup>1</sup>** (5.5)

$$
\mathcal{A}^{11} = \mathcal{A}^{22} = \frac{|\mathbf{k}| - k_x}{2|\mathbf{k}|(k_y^2 + k_z^2)} (0, -k_z, k_y)^{\mathrm{T}}
$$
(5.1)

$$
\mathcal{A}^{33} = \mathcal{A}^{44} = \frac{|\mathbf{k}| + k_x}{2|\mathbf{k}| (k_y^2 + k_z^2)} (0, -k_z, k_y)^{\mathrm{T}}
$$
(5.2)

$$
\mathcal{A}^{13} = \mathcal{A}^{24} = \mathcal{A}^{31*} = \mathcal{A}^{42*} = \frac{\sqrt{k^2 - k_x^2}}{2k^2(k_y^2 + k_z^2)} (-i(k_y^2 + k_z^2), ik_x k_y + |\mathbf{k}|k_z, ik_x k_z - |\mathbf{k}|k_y)^{\mathrm{T}}
$$
\n(5.3)

#### F=dA+A^A =>for Abelian case, denote F=  $F = dA + A^A A =$ curvature in the will consider the seeding of the *1-a2*  $\alpha$  $\Omega$ = *Fky,k<sup>x</sup>* = *kz*  $L-d\Lambda$   $\Lambda\Lambda$   $\rightarrow$  for  $\Lambda$  holian case, denote  $L$

(a) Berrey Curvature with closed integral surface (b) Phase Diagram

temperature with closed surface. The curvature shows the same feature with the dirac monopole.

$$
\Omega = \frac{1}{k^{3/2}} (k_x, k_y, k_z)^{\text{T}} \qquad \text{flux} = \int_{\mathcal{S}} \Omega \cdot dS = 2\pi
$$

### Topological Liquid : scalar order without gap

$$
S_{\psi} = \int d^5 x \sum_{j=1}^2 \sqrt{-g} \ \overline{\psi}^{(j)} \Big( \frac{\overrightarrow{\not{D}} - \overleftarrow{\not{D}}}{2} - m^{(j)} \Big) \psi^{(j)}, \tag{5}
$$

$$
S_{g,\Phi} = \int d^5x \sqrt{-g} \Big( R - 2\Lambda - \nabla_M \Phi^2 - m_{\Phi}^2 |\Phi|^2 \Big) \tag{6}
$$

$$
S_{int} = \int d^5 x \sqrt{-g} \Big( i \Phi \bar{\psi}^{(1)} \psi^{(2)} + h.c \Big). \tag{7}
$$

where  $D\!\!\!\!/\,\,=\Gamma^M D_M$ ,  $D_M=(\partial_M-i q A_M + \frac{1}{4} \omega_{M\alpha\beta} \Gamma^{\alpha\beta})$ 

![](_page_38_Picture_5.jpeg)

# Spectrum is pole type, differ from critical case. *<sup>k</sup>*<sup>2</sup> !<sup>2</sup> (1)

![](_page_39_Figure_2.jpeg)

 $\mathsf{S}$  Berrey Curvature with  $\mathsf{S}$ Fermion

!<br>!!<br>!!

 $A_{\rm eff}$  the fermions propagator shows as  $\Gamma$ 

for both cases

coupled Fermion (b) Berry curvature density in momentum space where *k*<sup>2</sup> = *k*<sup>2</sup>

 $\overline{A_0}$ However, Berry Curvature is Identical to critical case. Before trace over occupied bands, we get following curvature The same Dirac monopole

### Scalar Interaction case (SA quantization) future work.

- Gapped spectrum
- •Trivial topology The main feature of this interaction is the gap generation, as it was noticed in [24, 38,

$$
\text{Tr}\,\mathbb{G}_{M_0}^{(SA)}=\frac{4\omega}{\sqrt{\bm{k}^2-\omega^2+M_0^2}},
$$

that the chirality cannot be defined in odd dimensions. We postpone this problem to the

![](_page_40_Figure_4.jpeg)

#### Vector Interaction : Separated Dirac monopole BB@ 0 *<sup>k</sup><sup>y</sup>* 2 (*bxkx*)2+*k*<sup>2</sup> *<sup>y</sup>*+*k*<sup>2</sup> *z* <sup>3</sup>*/*<sup>2</sup> C<br>CCA<br>CCA 1990 - Paris Maria M

2  $\overline{a}$ 

i<br>L

(*bx*+*kx*)2+*k*<sup>2</sup>

(*bx*+*kx*)2+*k*<sup>2</sup>

*<sup>y</sup>*+*k*<sup>2</sup>

Berry curvature *<sup>y</sup>*+*k*<sup>2</sup> Berry curvatu

<sup>3</sup>*/*<sup>2</sup> <sup>0</sup>

<sup>3</sup>*/*<sup>2</sup> <sup>0</sup>

$$
\Omega = \frac{1}{2((b_x + k_x)^2 + k_y^2 + k_z^2)^{3/2}} (k_x + b_x, k_y, k_z)^{\mathrm{T}} + \frac{1}{2((b_x - k_x)^2 + k_y^2 + k_z^2)^{3/2}} (k_x - b_x, k_y, k_z)^{\mathrm{T}}
$$

Spectrum

![](_page_41_Figure_4.jpeg)

![](_page_41_Figure_5.jpeg)

(a)  $B_x^{(0)(55)}, \omega$ - $k_x$  (b) Berry curvature on  $k_z$ - $k_x$  plane

![](_page_42_Picture_0.jpeg)

$$
S_{\psi} = \int d^5 x \sum_{j=1}^2 \sqrt{-g} \, \bar{\psi}^{(j)} \Big( \frac{\vec{D} - \vec{D}}{2} - m^{(j)} \Big) \psi^{(j)}, \tag{8}
$$

$$
S_{g,B_{\mu\nu}} = \int d^5 x \sqrt{-g} \Big( R - 2\Lambda - |D_M \Phi_I|^2 - m_\Phi^2 |\Phi|^2 \Big), \tag{9}
$$

$$
S_{int} = \int d^5x \sqrt{-g} \Big( B_{\mu\nu} \bar{\psi}^{(1)} \Gamma^{\mu\nu} \psi^{(2)} + h.c \Big). \tag{10}
$$

where  $D\!\!\!\!/\,\,\,=\Gamma^M D_M$ ,  $D_M=(\partial_M+\frac{1}{4}\omega_{M\alpha\beta}\Gamma^{\alpha\beta})$ , and  $B=B_{xy}(u)\;dx\wedge dy$ 

![](_page_42_Picture_5.jpeg)

### Topology of Flat band

![](_page_43_Figure_1.jpeg)

# Summary ( $AdS_5$  or 3d topology)

![](_page_44_Figure_1.jpeg)

### Single monopole

### Separated monopole

![](_page_44_Figure_4.jpeg)

#### $AdS_4$ : scalar vs pseudo-scalar In AdS4, 2-flavors always gives zero curvature so that the topology is trivial. However, the topology is trivial. However,  $\mathcal{A} = \mathcal{I} \mathcal{L} \mathcal{L}$ One can see that there is gap-gappless phase transition where sign of *M*<sup>0</sup> see figure 7 .  $\Delta dS$  regions of  $\Delta s$ In contrast, the 1-flavor with psudo scalar *·* <sup>=</sup> 5*M*<sup>5</sup> can give a gap phase also.

regradless of spectral functions.

!

<sup>5</sup> )3*/*<sup>2</sup> (7.5)

the scalar  $\Gamma \cdot \Phi = iM_0$  Green's function is given by

$$
\mathbb{G} = \begin{pmatrix} \frac{k_x + \omega}{-M_0 + \sqrt{k_x^2 + k_y^2 + M_0^2 - \omega^2}} & \frac{k_y}{M_0 - \sqrt{k_x^2 + k_y^2 + M_0^2 - \omega^2}}\\ \frac{k_y}{M_0 - \sqrt{k_x^2 + k_y^2 + M_0^2 - \omega^2}} & \frac{k_x + \omega}{M_0 - \sqrt{k_x^2 + k_y^2 + M_0^2 - \omega^2}} \end{pmatrix}
$$
  
Tr  $\mathbb{G} = \frac{2\omega}{-M_0 + \sqrt{k_x^2 + k_y^2 + M_0^2 - \omega^2}}$ 

Therefore, even in the case that the spectral function look similar the spectral function look similar the top

regrades of spectral functions.<br>The spectral functions of spectral functions. Iopological Liquid (8<sup>-</sup>0) Topological Liquid (g<0)

1-flavor cases are non-trivial topology.

7.1 scalar vs pseudo-scalar

**But in b** But in both case  $Q = 0$  *<sup>k</sup><sup>x</sup>* <sup>+</sup> ! *k<sup>y</sup>* <sup>+</sup> *iM*<sup>5</sup>  $\frac{y}{2}$  $\Omega_{\mathbf{x}\mathbf{y}}=0$ 

1 by the 1-flavor with psudo scalar  $\Gamma \cdot \Phi = \Gamma^5 M_5$  can give a gap for with psudo scalar  $\Gamma \cdot \Phi = \Gamma^5 M_5$  can give a gap if

In this case, the Green function gives  $\sigma$  the Green function gives  $\sigma$  that the topology is trivial,  $\sigma$ 

In this case, this case, this case, the Green function gives  $\sigma$  that the topology is trivial, the topology is trivial,  $\sigma$ 

$$
\int \mathbb{G} = \frac{1}{\sqrt{k_x^2 + k_y^2 + M_5^2 - \omega^2}} \begin{pmatrix} k_x + \omega & -k_y + iM_5 \ -k_y - iM_5 & -k_x + \omega \end{pmatrix},
$$
  
Tr  $\mathbb{G} = \frac{2\omega}{\sqrt{k_x^2 + k_y^2 + M_5^2 - \omega^2}}$ 

Spectrum->	gap	(g>0)
Topological	Liquid	(g<0)
But in both case	$\Omega = \frac{M_5}{2(k_x^2 + k_y^2 + M_5^2)^{3/2}}$	
$c_1 = \frac{1}{2\pi} \int F = 1$		

By using topological Hamiltonian method we get the curvature:

# Topology in finite temperature

1. Non-interacting (single particle) theory: Finite temperature is ensemble average. Each band has its own topological number  $c_n$ . Therefore the topological number  $=$  average of  $c_n$ : Actually Uhlmann defined a T-dependent c.  $c(T) = \sum p_n(T) c_n$ 

Q: But does it make sense for a topology to be dependent on T, a continuous deformation?

Q: What holography says about it?

## Monopole number at Finite T in holography

### Findite Temperature Monopole Temperature Monopole Number of Temperature Monopole International Method 1: A & F are T-independent, though G depends on T. Method 2:  $GdG^{-1}$  depends on T.

![](_page_47_Figure_2.jpeg)

![](_page_47_Figure_3.jpeg)

Figure: monopole numbers over the evolution of temperature by various integration sphere radius.

Figure: Monopole charge with increasing of temperature, with a fixed sphere surface

Flux over Large enough Surface => temperature independent result.

### **Observation**

```
1. In holography, c_1(T) = c_1(0).
```
2. Why this happen? In AdS/CFT dictionary, finite temperature  $\sim$  black hole  $\sim$  (a pure) state!

![](_page_48_Figure_3.jpeg)

### Conclusion

- 1. Lattice = symmetry breaking mechanism =>spectrum generation CLS=Atom, essence of both=localization of electron identify  $f$  orbital = flat band by CLS.
- 2. Topology of strongly interaction can be handled and holography gives a T-independent Topology.
- 3. Kondo lattice = flat band hybridized with s-band.

Thank you