asia pacific center for theoretical physics

Magnetic defects in conformal field theory

Matthew M. Roberts
Asia Pacific Center for Theoretical Physics & POSTECH

Based on arXiv:2405.06014 & 2408.11088, with Igal Arav, Jerome P. Gauntlett, Yusheng Jiao, Christopher Rosen

Inaugural INPP Demokritos-APCTP meeting, Athens, Oct 3, 2024

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(Look here for detailed references)

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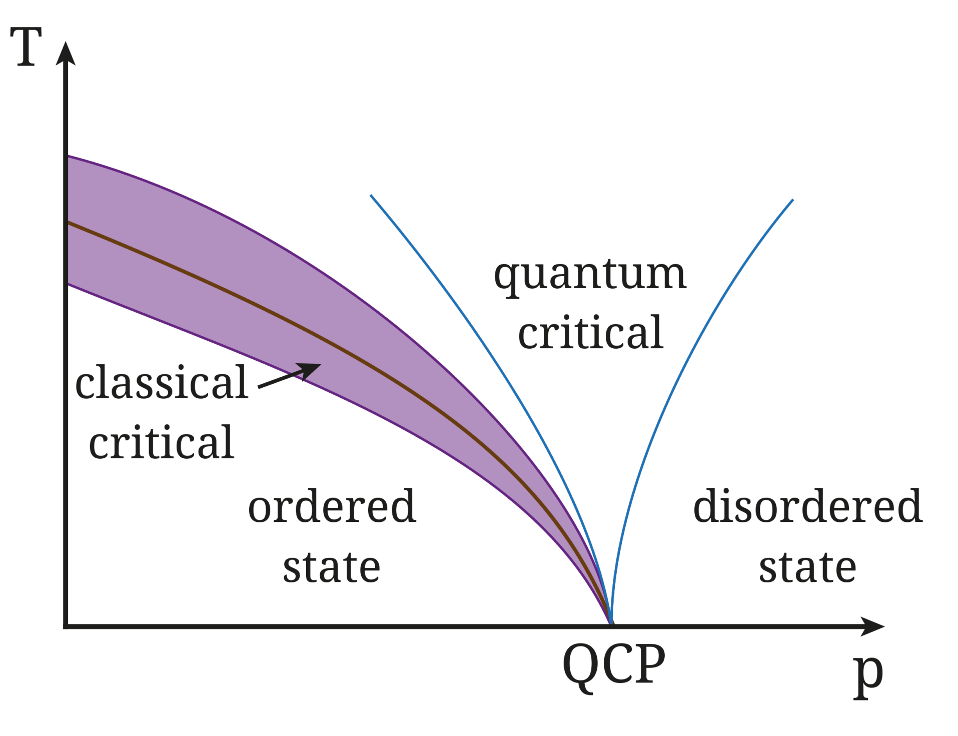
JeromeFest, Imperial College London, April 2024

Renormalization group flow

- Consider interacting d.o.f. with various scales (interaction energies, masses, etc)
- In deep IR limit, we are left with a scale invariant theory: Conformal field theory
- Usually very simple! But occasionally interesting
- Ex: 2nd order phase transition:
  - Ordered state IR: free goldstones (interactions irrelevant)
  - Disordered state IR: empty (everything massive)
  - Critical point: Massless theory, with novel scaling behavior

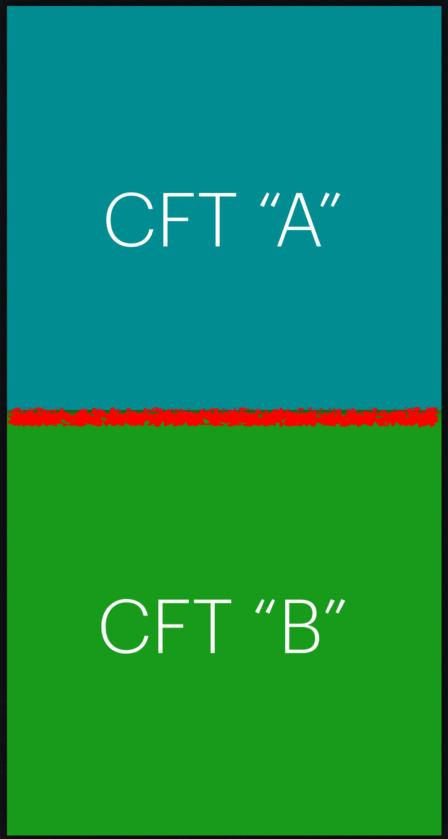


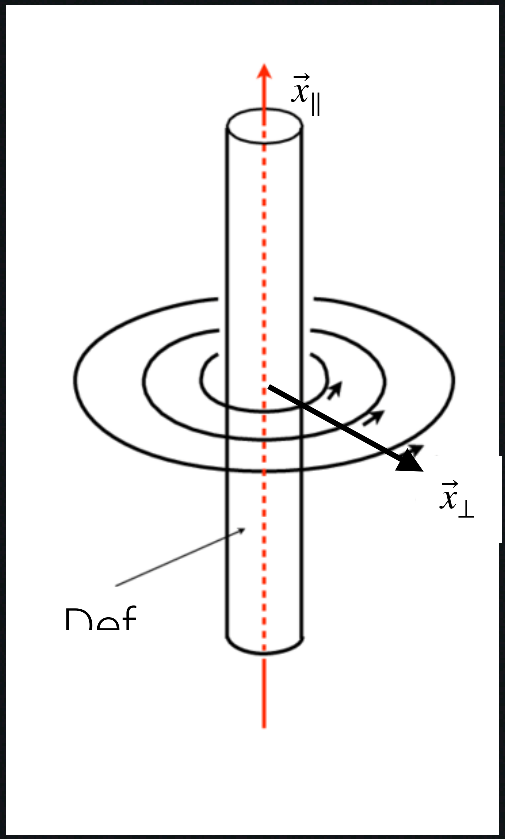
$$\langle \mathcal{O} \rangle \sim (p - p_c)^{\xi}$$

$$\langle \mathcal{O}(x)\mathcal{O}(y) \rangle \sim \frac{1}{|x-y|^{2\Delta}}$$

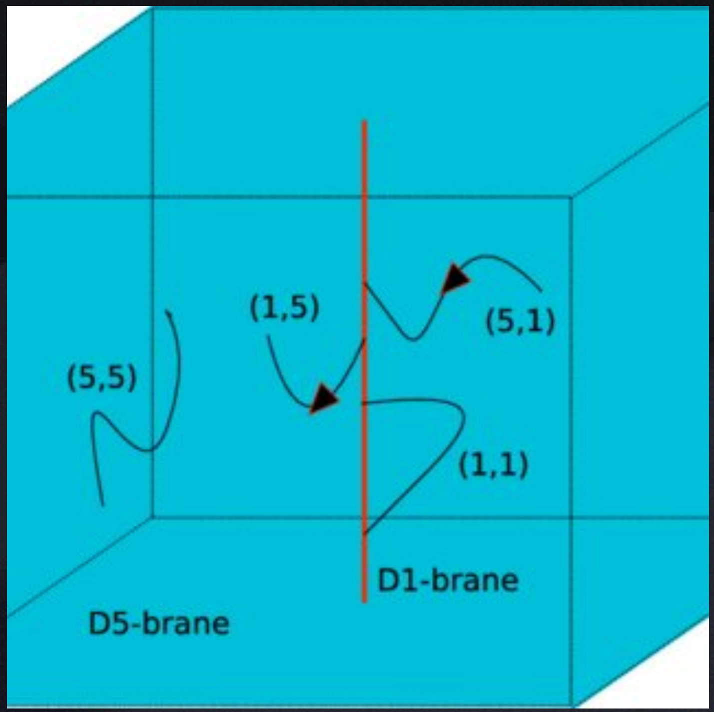
Renormalization group flow

- We know a lot about RG flows when things are Poincare invariant - what if they *aren't*?
    - Gluing two different systems together along a shared boundary
    - Lower dimensional system interacting with bulk ambient theory
    - Line defect inserted into larger theory



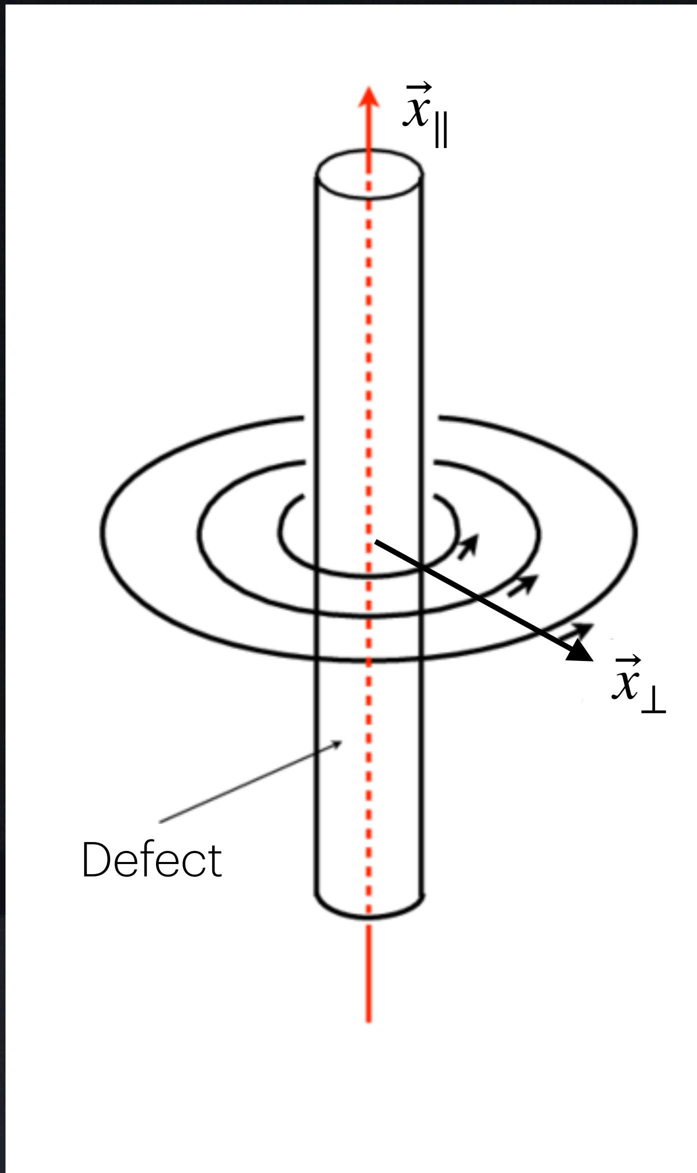


- How do RG flows behave here? What types of scale invariant fixed points?
- Can we generalize $c/F/a$ theorems to mixed dimensional systems? (monotonic measures of # d.o.f.)
- Extended operators are acted on by generalized symmetry operators



Renormalization group flow

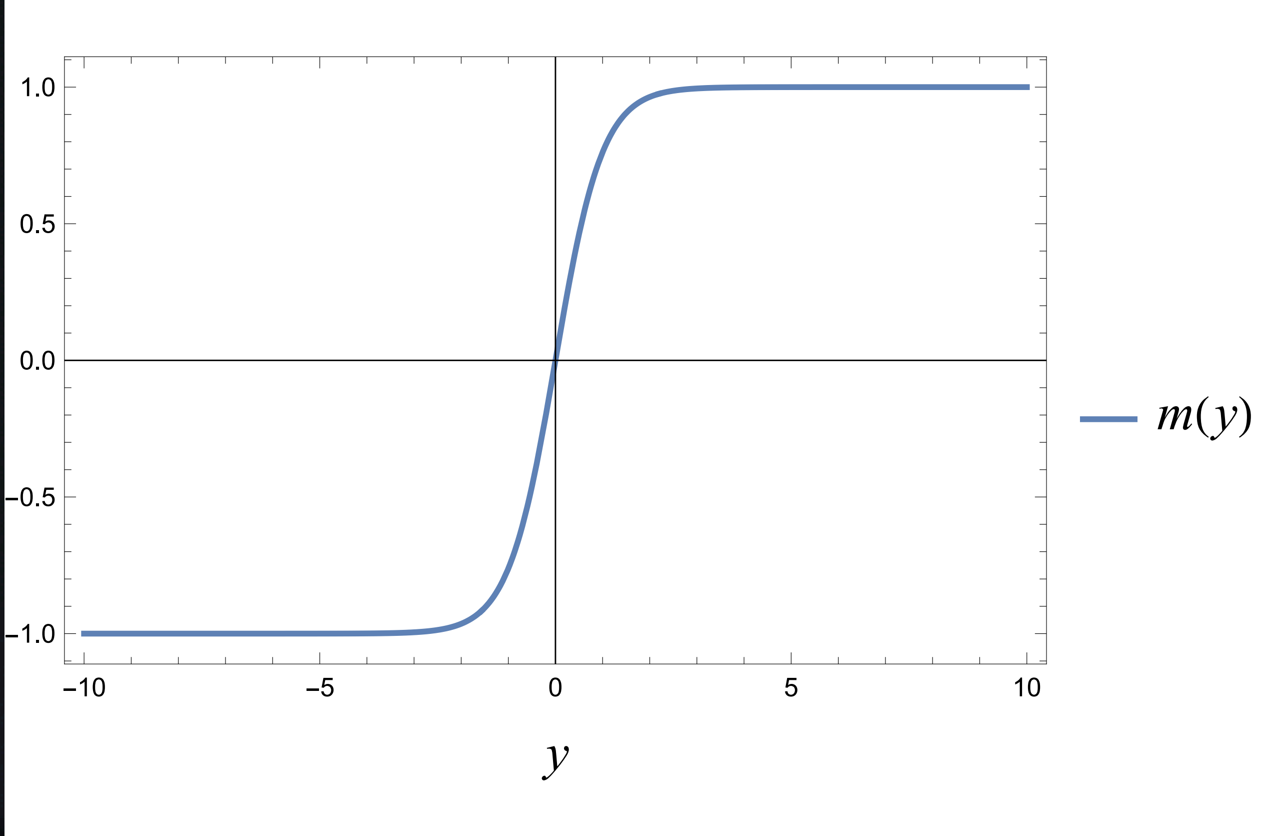
- Add defect (b.c., new d.o.f., ...) to $d_{||}$-dim'l subspace Σ in a CFT
- Take this whole system and flow to the infrared, resulting in a new mixed-dimensional scale invariant system
- If ambient space and defect are both flat, then
  - $SO(d,2) \to SO(d_{||},2) \times SO(d_{\perp})$
  - Lower dimensional system is not standard CFT, as there is no conserved stress tensor, $\nabla_{\mu} T^{\mu i} = \delta(\Sigma)\hat{D}^i \neq 0$
  - New "displacement" operator $\hat{D}^i$ on the defect
  - One-point functions can be nonzero, $\langle T \rangle \sim \dfrac{h_D}{|x_{\perp}|^d}$, $\langle J_{\theta} \rangle \sim \dfrac{C}{|x_{\perp}|^{d-2}}$,
  - Mixed system, with ambient (bulk) and defect operators



Simple example: Chern insulator

AKA Massive 2+1d fermion

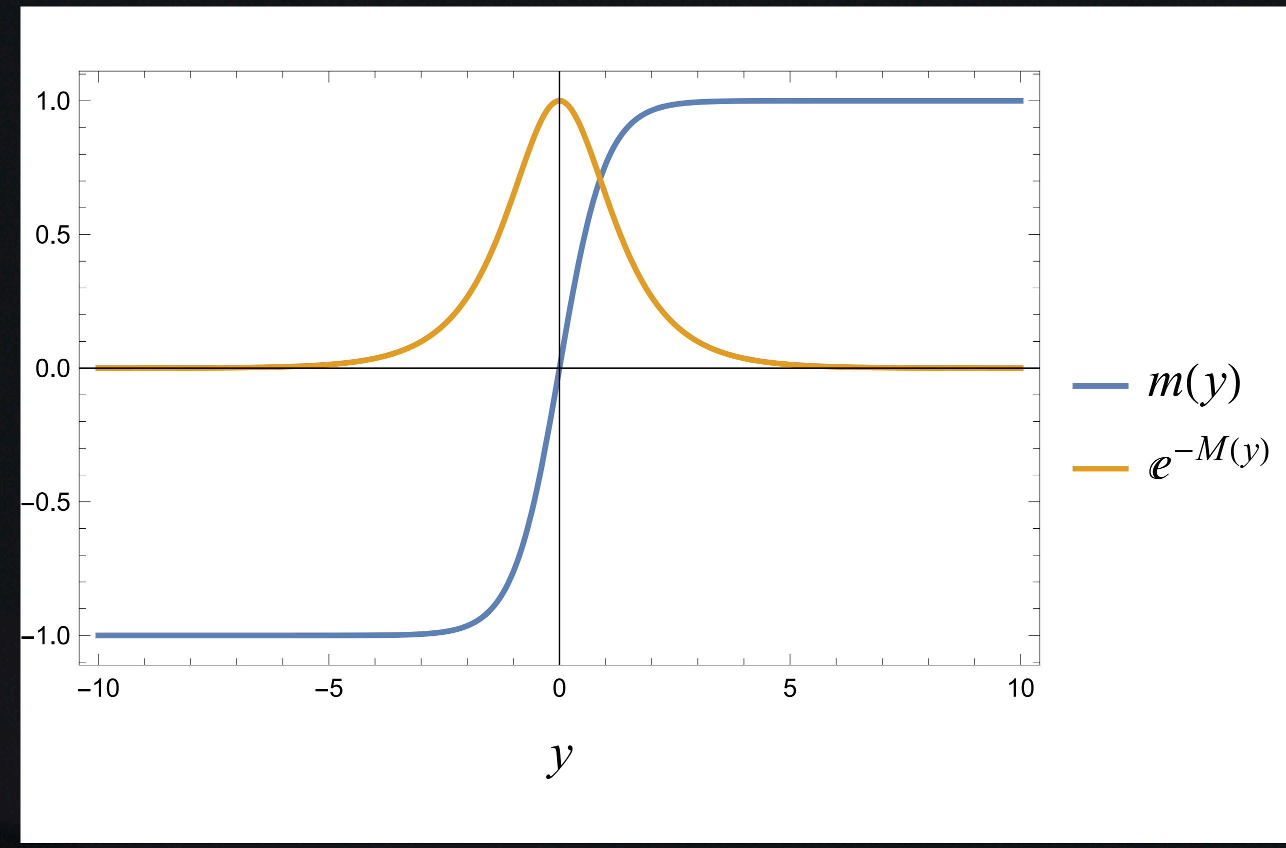
- Consider a Chern insulator, modeled by 2+1d fermion with finite mass:
  - $\left[i\gamma^\mu \partial_\mu - m(y)\right]\psi = 0$
  - Sign of mass breaks parity
- Build an interface between systems with opposite alignment (sign of $m(y)$)
- Away from interface, gapped (empty CFT)



Simple example: Chern insulator

AKA Massive 2+1d fermion

- Direct inspection shows that for
  - $\psi = \psi_0 e^{-i\omega t + ikx} e^{\mp M(y)}$, $k = \pm\,\omega$
  - $M'(y) = m(y)$, $\gamma^{01}\psi_0 = \pm\,\psi_0$
  - Crucially 2+1d, since then $\gamma^0\gamma^1 = -\,\gamma^2$
  - So for $e^{-M(y)}$ choice, we have a normalizable *chiral* edge mode

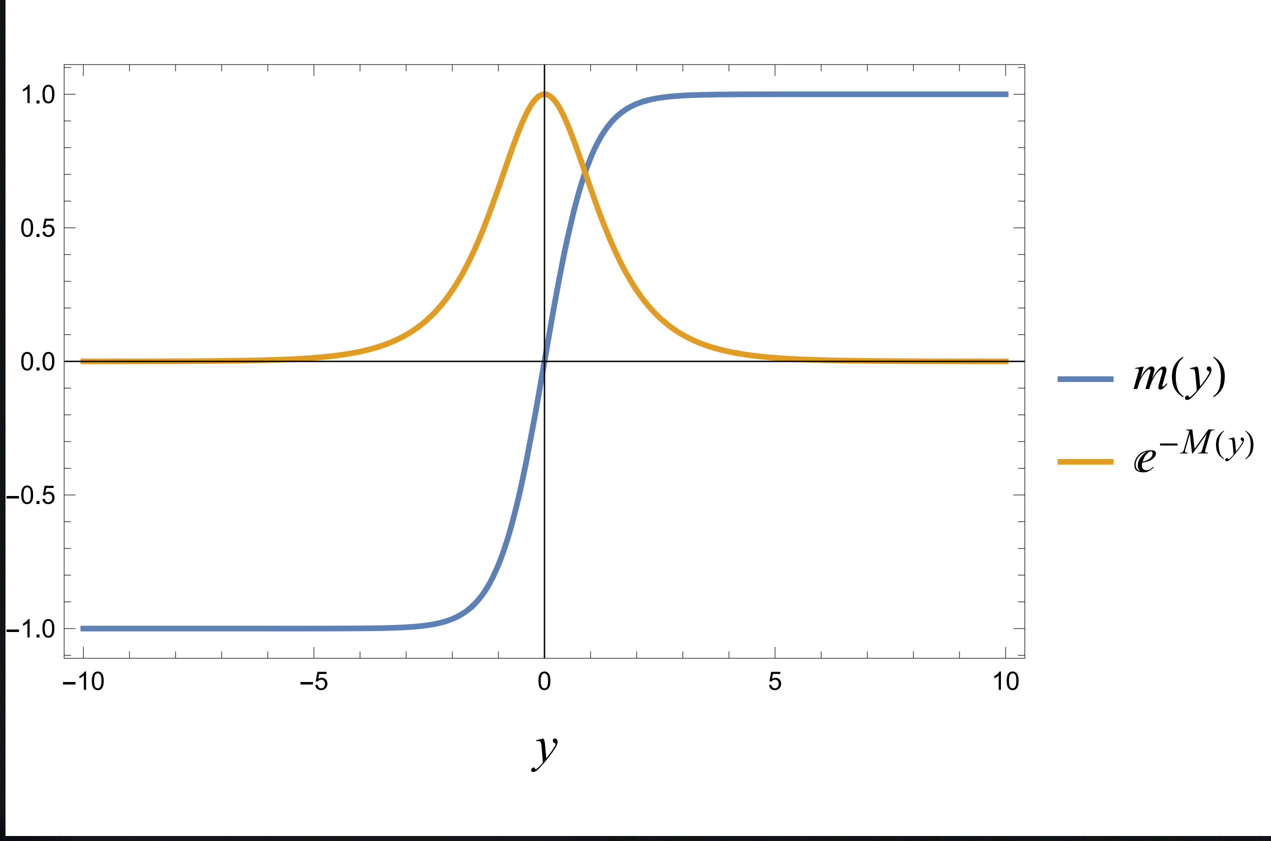


- Massless d.o.f., localized to edge, only moving right, not left, $\dfrac{\partial\omega}{\partial k} = +\,1$
  - Can generalize to system with interacting ambient space d.o.f. mixing with defect d.o.f.

Simple example: Chern insulator

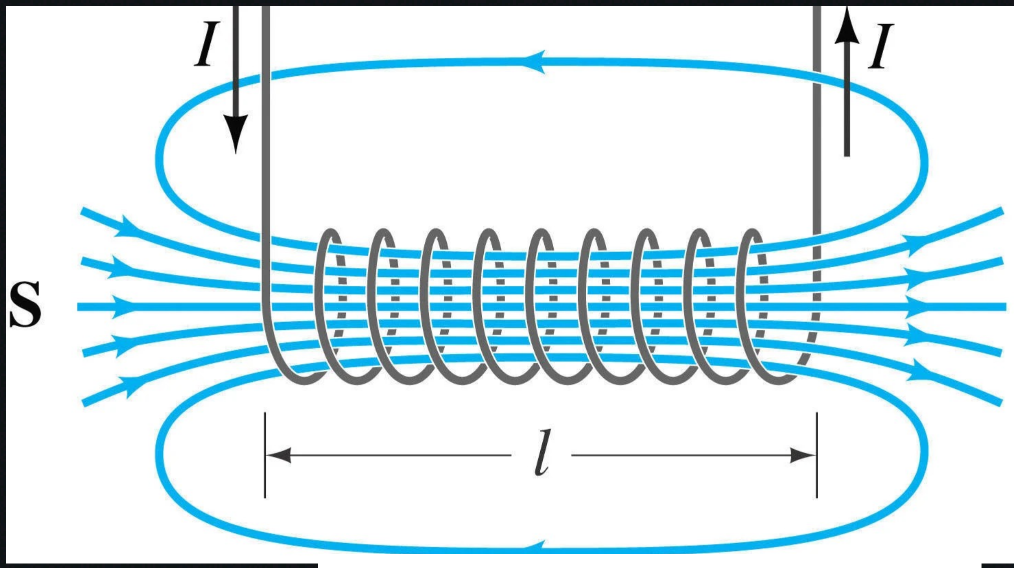
AKA Massive 2+1d fermion

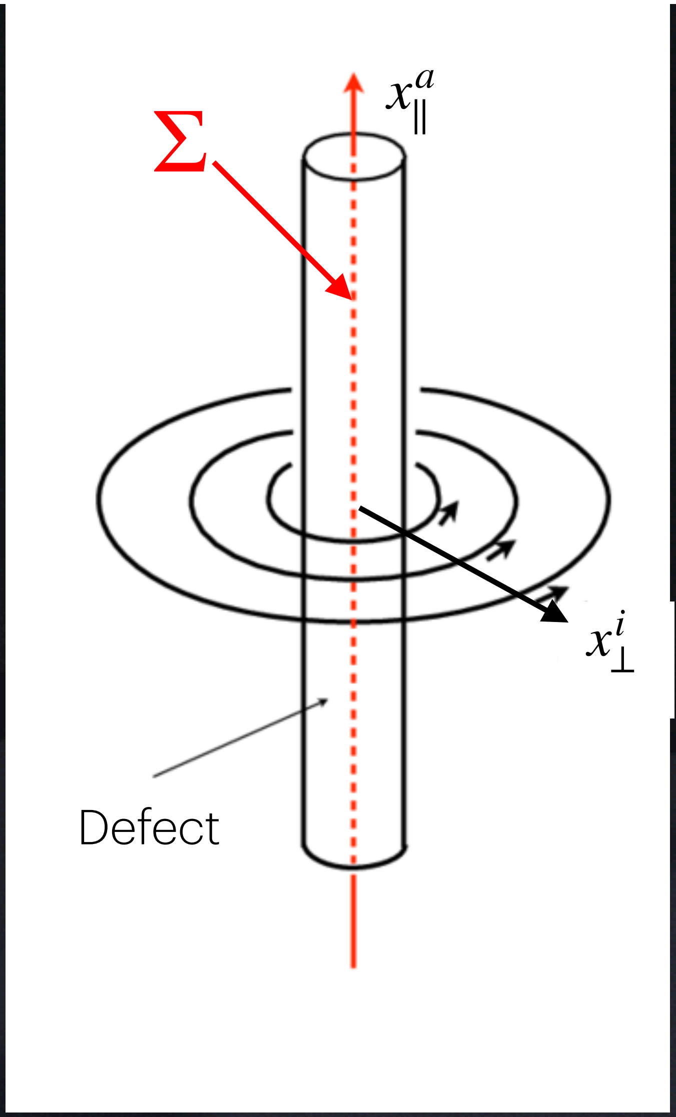
- Topologically protected edge state
- $\overline{\psi}\psi$ is parity odd
- Int. out massive Dirac fermion:
  - $S_{1PI}(A) \supset \dfrac{\text{sign}(m)}{2} \displaystyle\int \frac{1}{4\pi} AdA$
  - Away from defect, $S = \dfrac{1}{8\pi} \displaystyle\int_{y>0} AdA - \frac{1}{8\pi} \int_{y<0} AdA$
  - Needs $\displaystyle\int_{y=0} \overline{\chi}(D_t - D_x)\chi$ for gauge invariance (anomaly inflow)



Magnetic defects

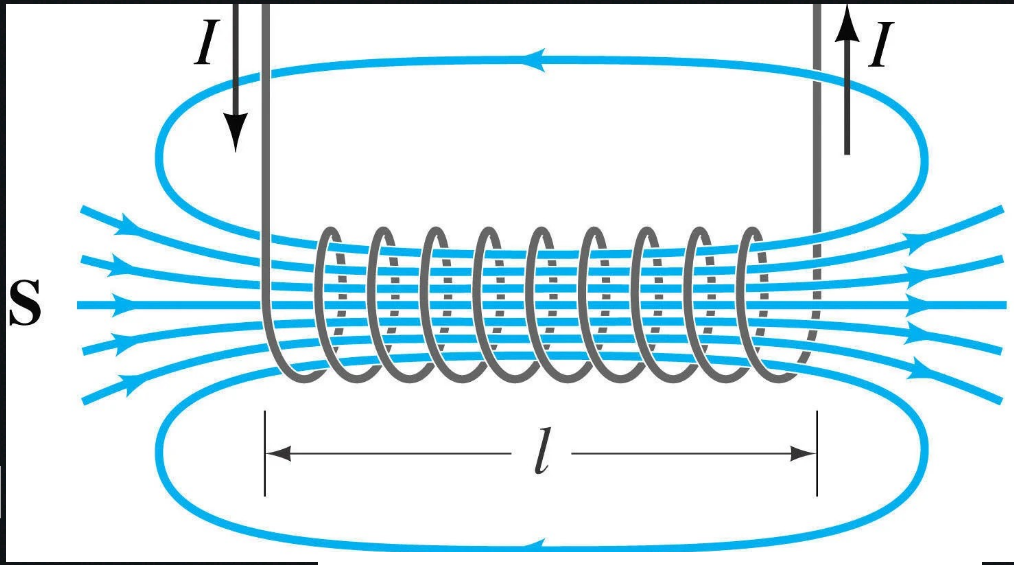
- Magnetic defects in 3+1d system: Consider magnetic flux for a background global symmetry
- In the limit of infinitely thin solenoid,
  - $A = \mu d\theta,\ F = 2\pi\ \mu\ \delta(\Sigma)$
  - $\frac{1}{2\pi}\int F = \frac{1}{2\pi}\oint A = \mu$
  - A.B. phase: $g = e^{2\pi i\mu}, \qquad \mu \sim \mu + 1$
- Scale invariant co-dimension two system:
  - $SO(4,2) \to SO(2,2) \times SO(2)$

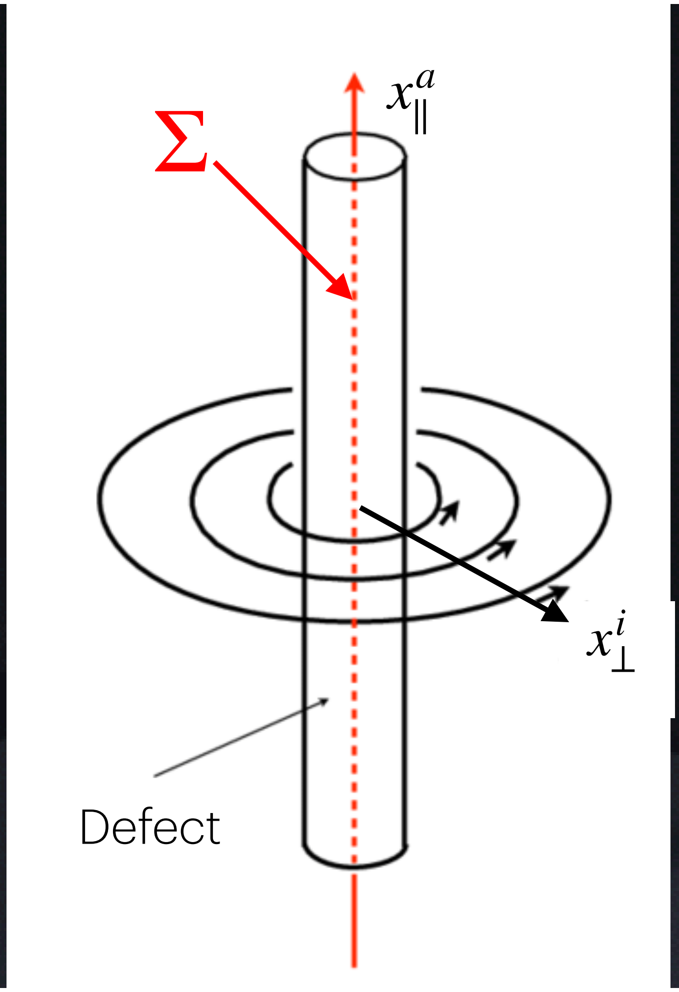


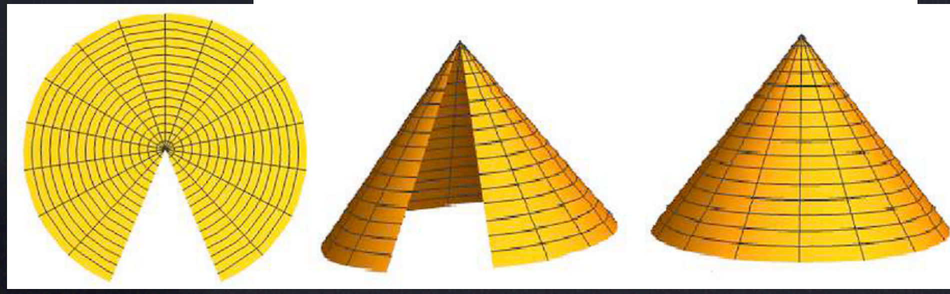


Magnetic defects

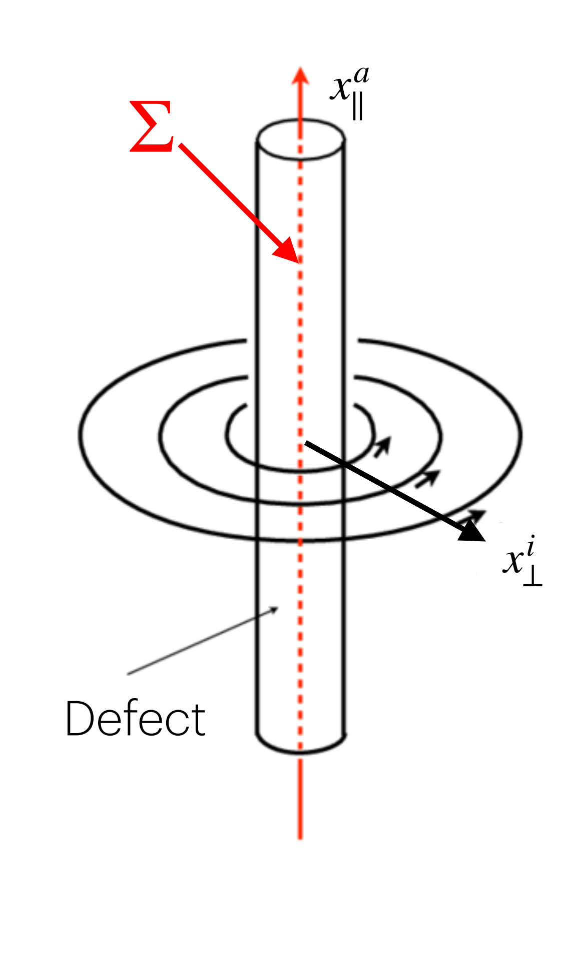
- $SO(4,2) \to SO(2,2) \times SO(2)$
- We can consider more general “flat” background with conical deficits:
  - $ds^2 = -dt^2 + dx^2 + d\rho^2 + n^2\rho^2 d\theta^2$
  - $\Delta\theta = 2\pi$
- This implies $R \sim (n-1)\delta(\Sigma)$, and may be necessary to cancel $U(1)_R$ monodromy
- Curvature version of solenoid
  - $0 < n < 1$: deficit, $n > 1$: excess







Why conical singularity?



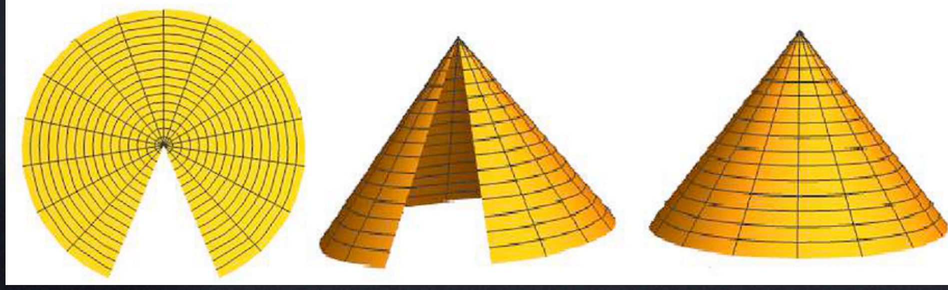
- Generalized background with conical singularities: Why? R-symmetry!

- The Killing spinor equation in the boundary theory is, roughly, $D_\mu \epsilon = \partial_\mu + \frac{1}{4}\gamma^{ab}\omega_{ab\mu} - iA_\mu^{(R)} = 0$

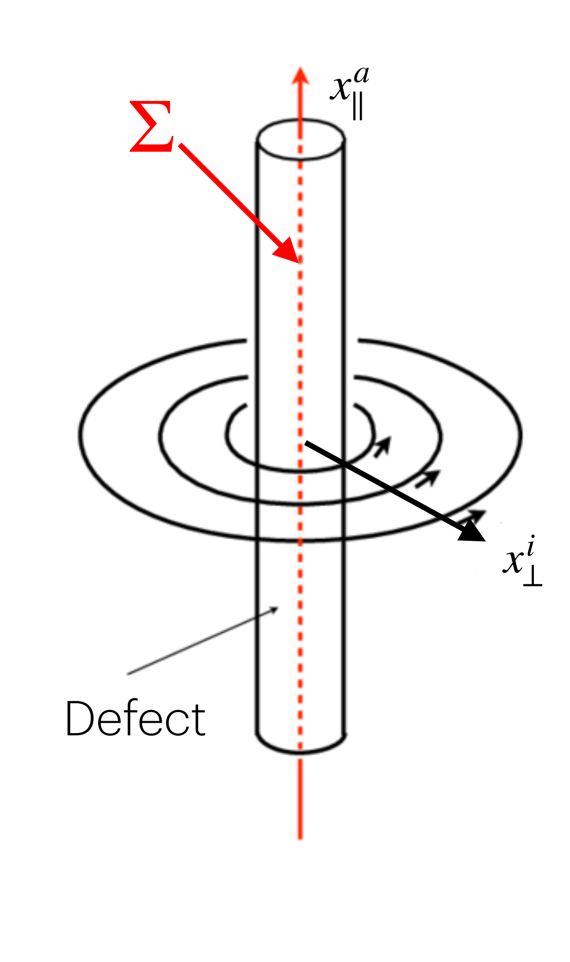
- Integrability: $[D_\mu, D_\nu]\epsilon = \left(\frac{1}{4}R_{ab}\gamma^{ab} + F^{(R)}\right)\epsilon = 0$

- If we have $F^{(R)} = \mu_R \delta(\Sigma)\ \rho d\rho \wedge d\theta$, we can fix the chirality of the spinor, $\gamma^{12}\epsilon = \pm\,\epsilon$ and satisfy with

- $R_{12} \sim \pm\,\mu_R \delta^{(2)}(\Sigma)\ \rho d\rho \wedge d\theta$:
  - conical singularity!

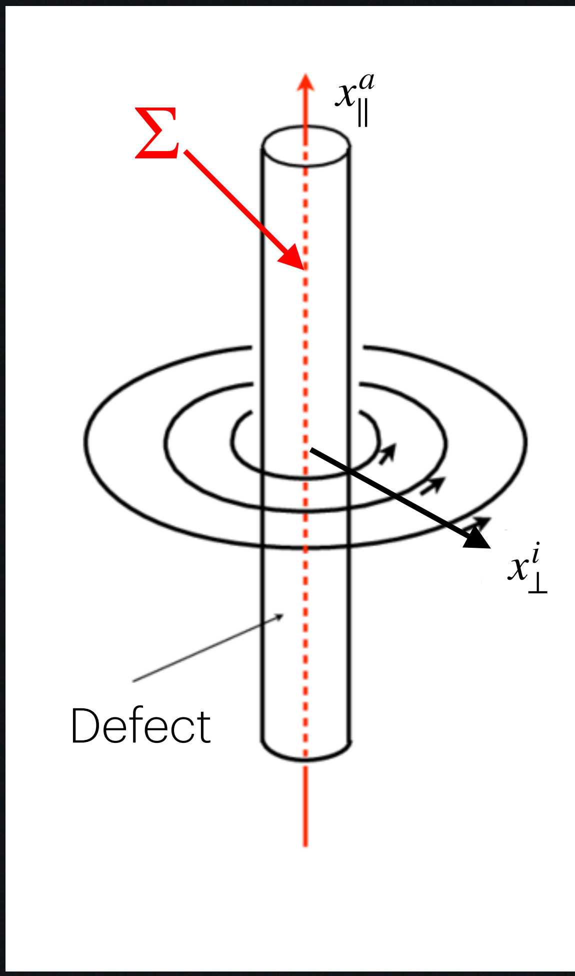


Monodromy defects



- How does the system respond to the solenoid?
  - By symmetries, we can have circulating currents
    - $\langle J_\theta \rangle = \frac{C(\mu)}{\rho^2}, \qquad \langle J_\rho \rangle = \langle J_{x_\parallel} \rangle = 0$
  - We also have nontrivial stress tensor
    - $\langle T^{ab} \rangle = -\frac{h_D(\mu)}{2\pi}\frac{\eta^{ab}}{\rho^4}, \langle T^{ij} \rangle = +\frac{h_D(\mu)}{2\pi}\frac{3\delta^{ij} - 4\frac{x_\perp^i x_\perp^j}{\rho^2}}{\rho^4}$
    - $\langle T \rangle = \frac{h_D(\mu)}{2\pi\rho^4}\left(dt^2 - dx^2 - d\rho^2 + 3\rho^2 d\theta^2\right)$
  - We can calculate $C(\mu), h_D(\mu)$. What other data?

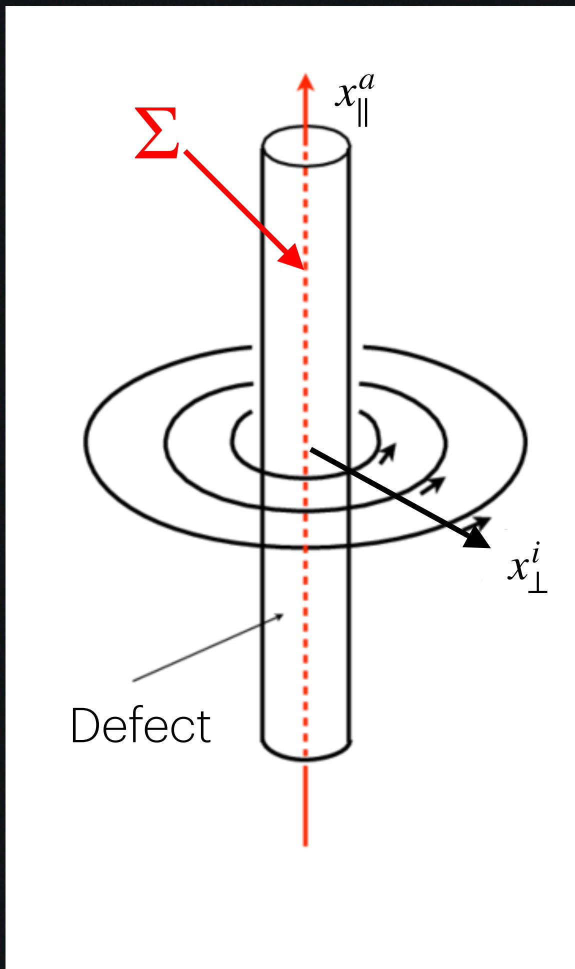
Monodromy defects



- Trace anomaly of 4d/2d system classified completely (ignoring parity odd possibilities ...)
- Standard ambient space terms, $\langle T^{\mu}{}_{\mu} \rangle = cW^2 - aE_4$
- A nontrivial defect gives three new central charges,
  - $\langle T^{\mu}{}_{\mu} \rangle = -\dfrac{1}{24\pi}\left(bR^{(\Sigma)} + d_1 \tilde{K}^{\mu}_{ab}\tilde{K}^{ab}_{\mu} - d_2 (\phi^* W)^{ab}{}_{ab} \right)$
  - b does not depend on defect marginal couplings (type A)
  - $d_{1,2}$ can (type B)
  - There is a b-theorem for flows driven by defect couplings, with $b_{IR} < b_{UV}$

Defect

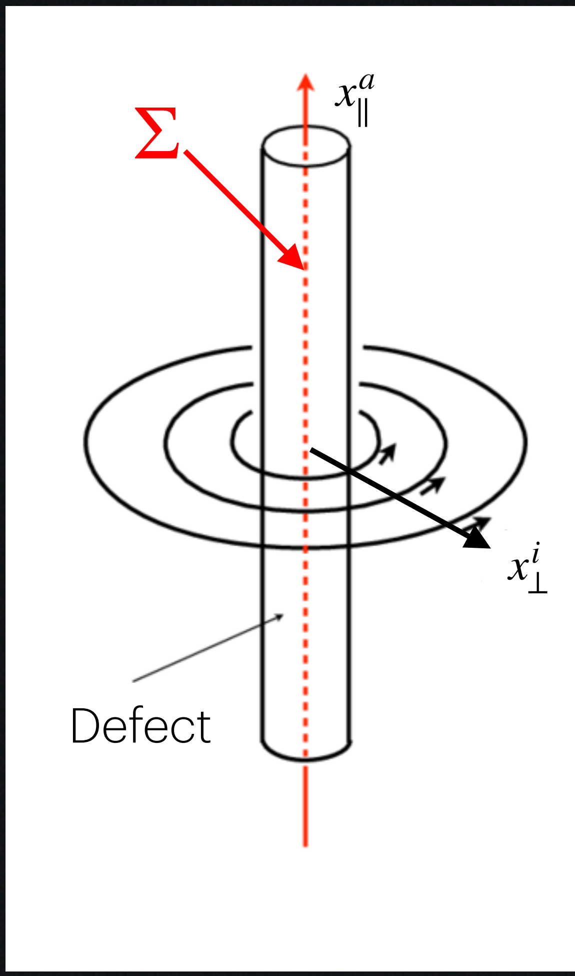
Monodromy defects



- $\langle T^{\mu}{}_{\mu}\rangle = -\frac{1}{24\pi}\left(bR^{(\Sigma)} + d_1 \tilde{K}^{\mu}_{ab}\tilde{K}^{ab}_{\mu} - d_2 (\phi^* W)^{ab}{}_{ab}\right)$
- For a flat defect, writing $x_{\perp}^{i} x_{\perp}^{i} = \rho^2$, recall
- $\langle T\rangle = \frac{h_D}{2\pi\rho^4}\left(dt^2 - dx^2 - d\rho^2 + 3\rho^2 d\theta^2\right),\ d_2 = 18\pi n h_D$
- With $(0,2)$ SUSY:
  - b indep. of some bulk marginal couplings, $d_1 = d_2$
  - $\langle (J_R)_\theta\rangle = \frac{n h_D}{2\pi\rho^2}, \qquad C_R = \frac{n h_D}{2\pi} = \frac{d_2}{(6\pi)^2}$

Defect

Monodromy defects



- For monodromy defects, for flavor currents
  - $\langle (J_I)_\theta \rangle = \dfrac{C_I(\mu)}{\rho^2}$,
- It was shown that $\dfrac{d}{d\mu_I} b(\mu) = \dfrac{1}{n} 12\pi^2 C_I(\mu)$
  - Calculation involves relating b to on-shell action in spherical Weyl frame
- To integrate b we need also $\partial_n b = -\dfrac{1}{n} d_2 + 12a$
  - Related to Weyl map which gives super-Renyi entropy
- And thus from calculating $\langle T \rangle$, $\langle J \rangle$ we can derive b

Monodromy defects

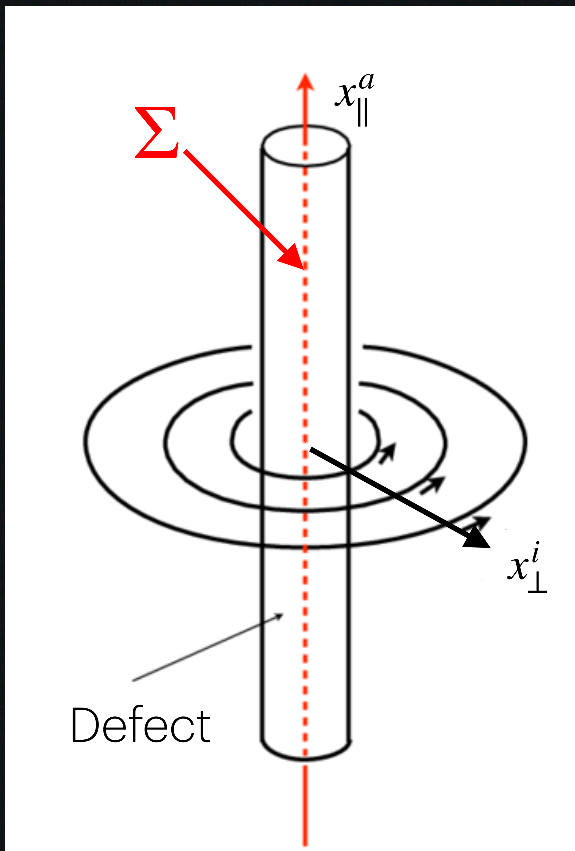
- We could also extract b from entanglement entropy: for $n = 1$

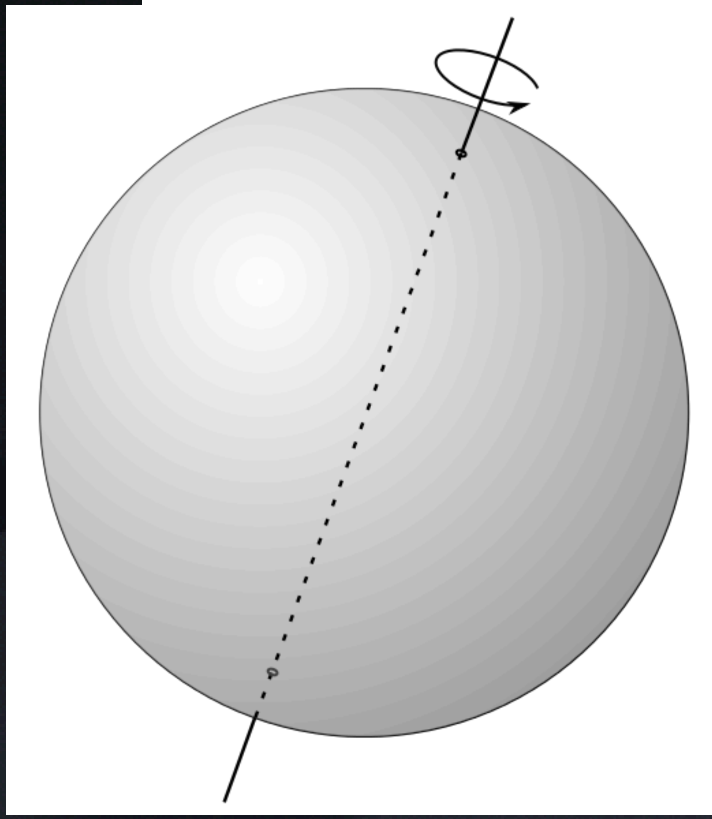
- $$S_{EE} = \gamma_1 \frac{\ell^2}{\epsilon^2} - 4a \ln(\ell/\epsilon) + \frac{1}{3}(b - d_2/3)\ln(\ell/\epsilon)$$

  - This is a more complicated calculation we leave to future work (hints how to proceed in literature...)

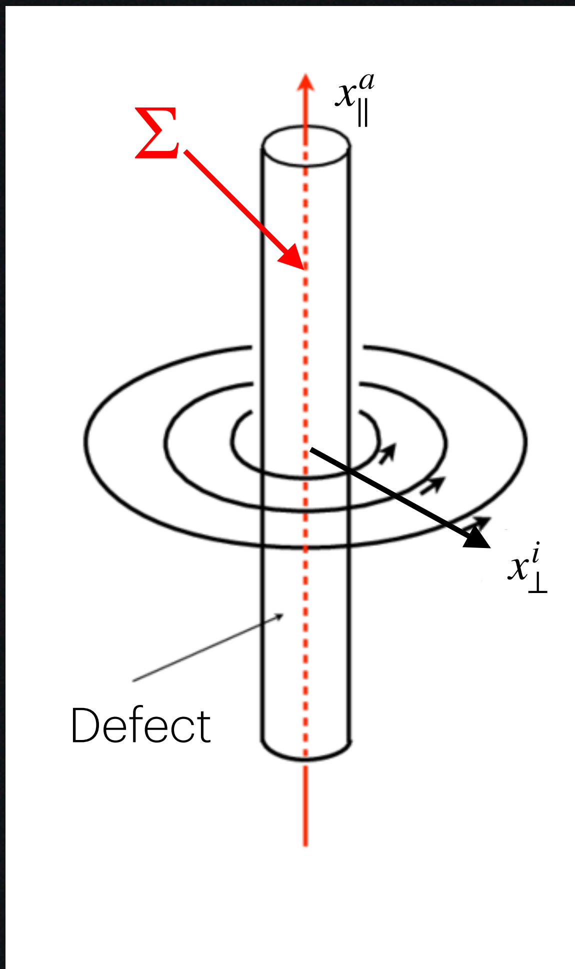
  - Note that the "universal" part of the entanglement is no longer monotonic

  - Unclear precisely how to generalize to general n



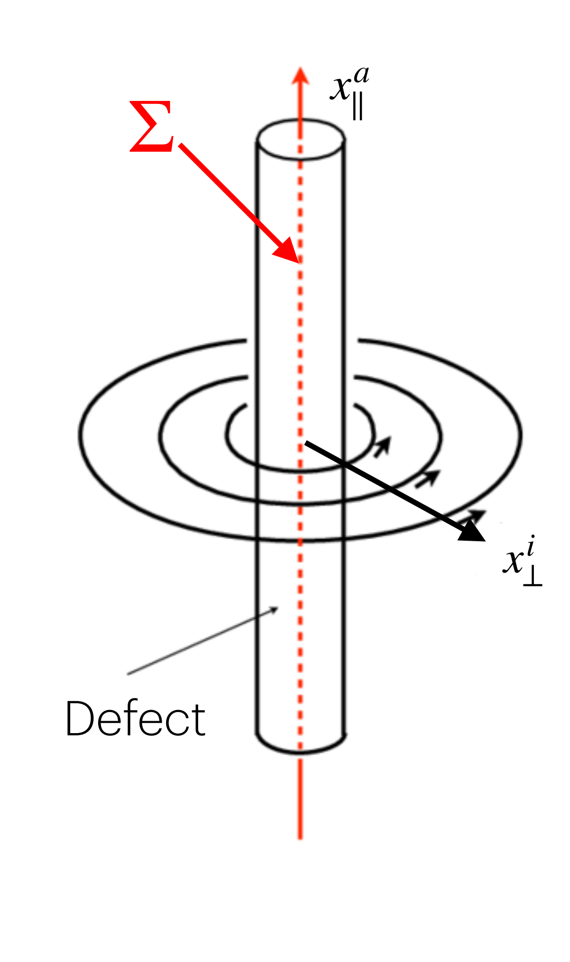


Holographic defects



- We wish to study holographic duals of conformal magnetic defects - natural starting point is $\mathcal{N} = 4$ SYM
  - Preserving only $(0,2)$ SUSY, we can turn on monodromies in $(U(1)^2)_F \times U(1)_R \subset SU(4)_R$
- We can also consider the $\mathcal{N} = 1$ LS theory
  - Turn on a mass for one of the three chiral multiplets in $\mathcal{N} = 4$ SYM, flows to strongly coupled $\mathcal{N} = 1$ theory
  - Monodromies in $U(1)_F \times U(1)_R \subset SU(2)_F \times U(1)_R$

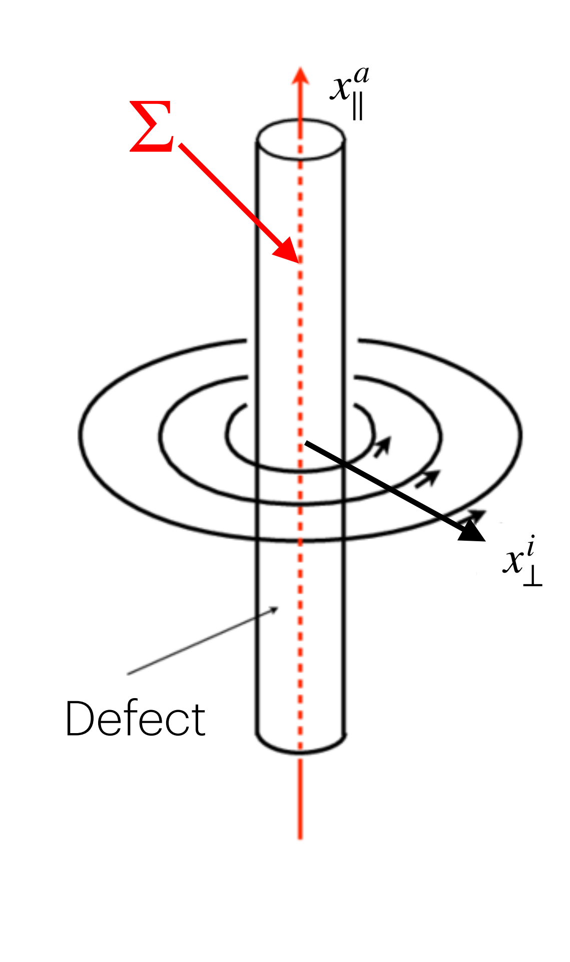
Holographic defects



- $\mathcal{N} = 4$: Monodromies in $(U(1)^2)_F \times U(1)_R$
- $\mathcal{N} = 1$: Monodromies in $U(1)_F \times U(1)_R$

- If we satisfy an algebraic constraint on the monodromies, we can also turn on the SUSY mass deformation to flow from $\mathcal{N} = 4$ to $\mathcal{N} = 1$
  - SUSY: $Dm_F = 0$, where $D = \partial + i(A^1 + A^2 - A^3)$
    - Holomorphic due to $(0,2)$ chirality
  - constant m_F then requires $\mu_1 + \mu_2 - \mu_3 = 0$
- In this restricted case, we can then compare $b,\ h_D$ at UV and IR

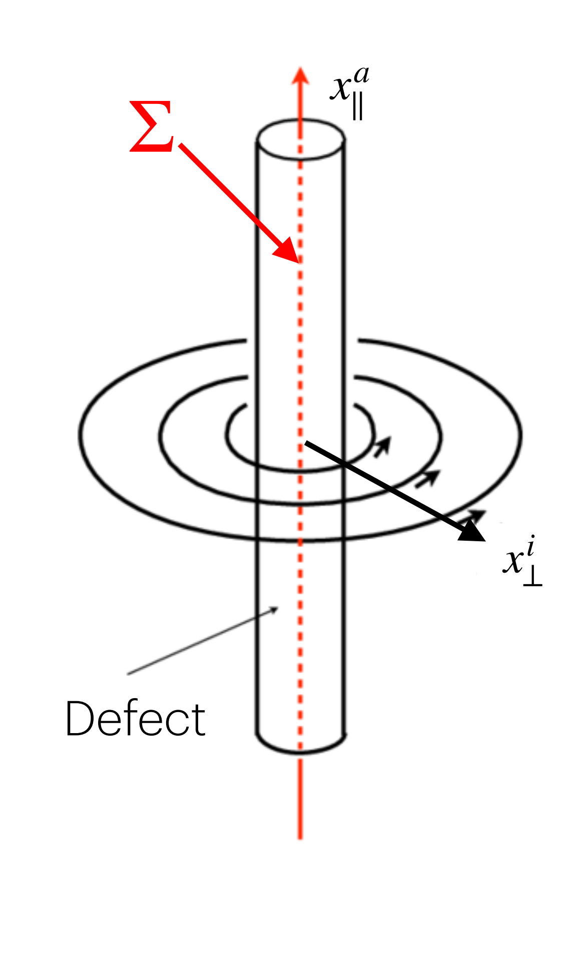
Why conical singularity?



- Any time $U(1)_R$ monodromy is nonzero, SUSY puts us on a conical background
  - From holographic perspective, no big change
    - Simply a modified coordinate periodicity
  - Must dial n to be able to integrate b from $\langle J \rangle$, $\langle T \rangle$
- $\frac{d}{d\mu_I} b(\mu, n) = \frac{1}{n} 12\pi^2 C_I(\mu, n),\ \partial_n b(\mu, n) = -\frac{1}{n} d_2(\mu, n) + 12a$

Some results

Lots of holography and SUSY magic under the hood



- As expected, our $\mathcal{N} = (0,2)$ grants us a great deal of analytic control
- We can use conserved quantities from the BPS equations to extract field asymptotics without having explicit solutions at hand, allowing us to extract VEVs
  - See the paper for the details (or Chris' talk!)
    - a lovely story for a SUGRA conference, but not here
- Instead, I will tell you some of our results

$\mathcal{N} = 4$ defects

- We have three monodromy parameters μ^i, and on the primary branch of solutions (adiabatically connected to the vacuum) we find
  - $\mu_R = \mu_1 + \mu_2 + \mu_3 = \kappa(1 - n)$
  - where $\kappa = \pm 1$ is the SUSY chirality
- We will parameterize solutions by μ_i remembering implicitly that we are always satisfying the above SUSY constraint.

$\mathcal{N} = 4$ defects

- Where do solutions exist? Not for all monodromies

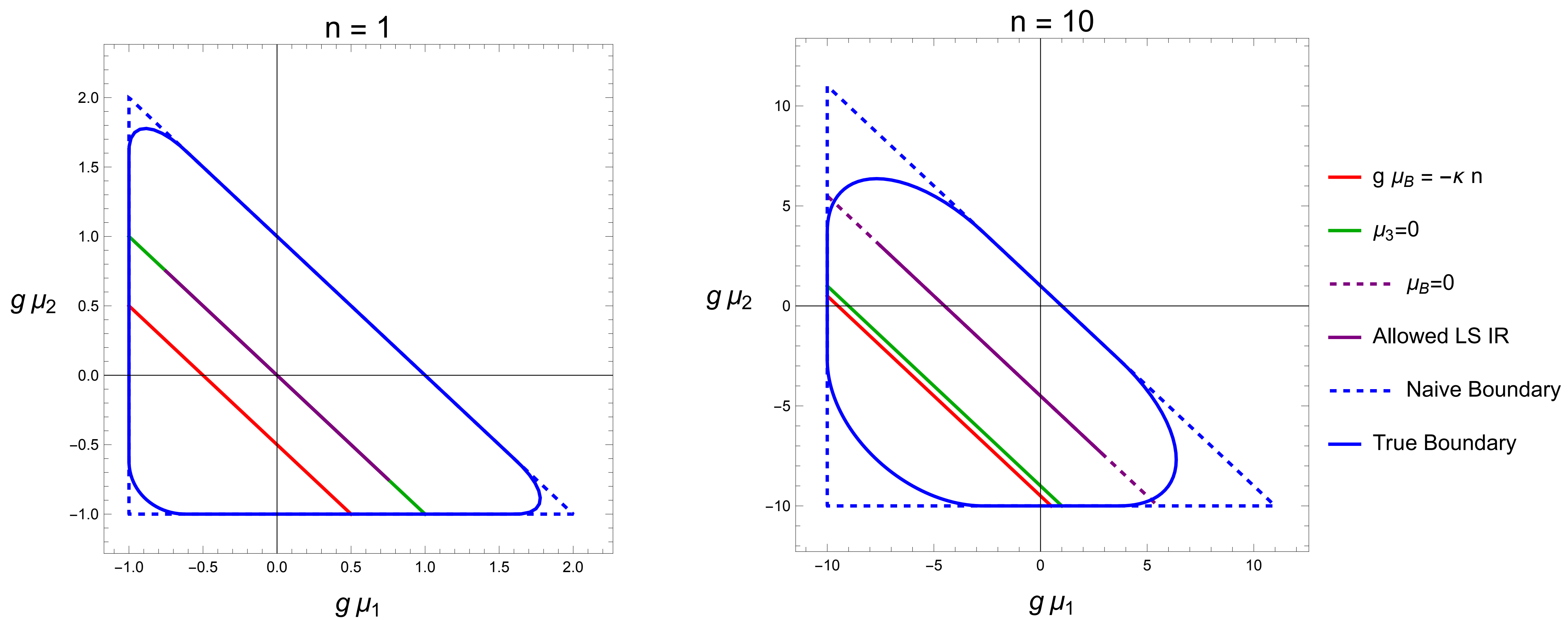


Figure 5: Solutions space of STU solutions as a function of two independent monodromy parameters $g\mu_1$, $g\mu_2$ for $n = 1$ (left plot) and $n = 10$ (right plot), for the main branch of solutions with $s = -\kappa/2$ (we have set $\kappa = +1$).

$\mathcal{N} = 4$ defects

- Currents:

$$\langle J_1 \rangle = \frac{N^2}{4\pi^2}\mu_1 \left(1 + \frac{\mu_2}{\kappa n}\right)\left(1 + \frac{\mu_3}{\kappa n}\right),$$
$$\langle J_2 \rangle = \frac{N^2}{4\pi^2}\mu_2 \left(1 + \frac{\mu_3}{\kappa n}\right)\left(1 + \frac{\mu_1}{\kappa n}\right),$$
$$\langle J_3 \rangle = \frac{N^2}{4\pi^2}\mu_3 \left(1 + \frac{\mu_1}{\kappa n}\right)\left(1 + \frac{\mu_2}{\kappa n}\right)$$

- Stress tensor:

$$h_D = -\frac{2\pi}{3\kappa n}\left(\langle J_1 \rangle + \langle J_2 \rangle + \langle J_3 \rangle\right) = -\frac{2\pi}{\kappa n}\langle J_R^{\mathcal{N}=4} \rangle$$

$\mathcal{N} = 4$ defects

- To calculate b recall

$$db = 12\pi^2 \left(\frac{1}{n} \sum_i \langle J_i \rangle d\mu_i - 3\frac{h_D}{2\pi} dn \right) + 12 a_{\mathcal{N}=4} dn$$

  - Where $a_{\mathcal{N}=4} = N^2/4$ at large N
  - It is a nontrivial check that the above is actually closed given our results, subject to the constraint $\mu_R = \mu_1 + \mu_2 + \mu_3 = \kappa(1-n)$

- It can be integrated to yield
  - $b_{\mathcal{N}=4} = -12 a_{\mathcal{N}=4} n (F^{\mathcal{N}=4} - 1) = -3N^2 n (F^{\mathcal{N}=4} - 1)$
  - Where $F^{\mathcal{N}=4} = \left(1 + \frac{\mu_1}{\kappa n}\right)\left(1 + \frac{\mu_2}{\kappa n}\right)\left(1 + \frac{\mu_3}{\kappa n}\right)$

$\mathcal{N} = 4$ defects

- $b_{\mathcal{N}=4} = -12a_{\mathcal{N}=4}n(F^{\mathcal{N}=4} - 1) = -3N^2n(F^{\mathcal{N}=4} - 1)$
- Where $F^{\mathcal{N}=4} = \left(1 + \frac{\mu_1}{\kappa n}\right)\left(1 + \frac{\mu_2}{\kappa n}\right)\left(1 + \frac{\mu_3}{\kappa n}\right)$ and $\mu_1 + \mu_2 + \mu_3 = \kappa(1 - n)$
- Remarkably, this agrees with free theory results (modulo assumptions about b.c. in free system)
  - When studying free field theory, there are marginal defect parameters related to b.c. at the core
  - We can identify these b.c. to match the above precisely!
  - Nominally continuous but matching fixes $\xi_i = 0$ or 1

$\mathcal{N} = 4$ defects

- We can also calculate the (appropriately renormalized) on-shell action, and we find
  - $S = \frac{N^2}{2\pi}\text{vol}(AdS_3)[nF^{\mathcal{N}=4}]$
  - We can equivalently write this as
  - $S = \text{vol}(AdS_3)\frac{1}{6\pi}\left(12a_{\mathcal{N}=4}n - b_{\mathcal{N}=4}\right)$

$\mathcal{N} = 1$ LS defects

- The mass deformation flowing to $\mathcal{N} = 1$ LS requires
  - $\mu_B = \mu_1 + \mu_2 - \mu_3 = 0$
- Which leaves us with one flavor monodromy
  - $\mu_F = \mu_1 - \mu_2$
- And the $U(1)_R$ monodromy related to the conical deficit,
- $\mu_R = \kappa(1 - n)$

$\mathcal{N} = 1$ LS defects

- Currents:
  - $\langle J_R^{LS} \rangle = \frac{N^2}{32\pi^2}\kappa\frac{n+1}{n^2}\left(1 - n^2 - \mu_F^2\right)$
  - $\langle J_F \rangle = \frac{N^2}{8\pi^2}\mu_F\left(1 + \frac{1}{2n}\right)$
- And as required by SUSY,
  - $h_D = -\frac{2\pi}{\kappa n}\langle J_R^{LS} \rangle$

$\mathcal{N} = 1$ LS defects

- To calculate b recall

$$db = 12\pi^2 \left(\frac{1}{n} \left(\langle J_R^{LS} \rangle d\mu_R + \langle J_F \rangle d\mu_F \right) - 3 \frac{h_D^{LS}}{2\pi} dn \right) + 12 a_{LS} dn$$

  - Where now $a_{LS} = \frac{27}{128} N^2$ at large N
  - Again a nontrivial check that the above is actually closed given our results, subject to the constraint $\mu_R = \kappa(1-n)$
- It can be integrated to yield
  - $b_{LS} = -12 a_{LS} n (F^{LS} - 1) = -\frac{81}{32} N^2 n (F^{LS} - 1)$
  - Where $F^{LS} = \left(1 + \frac{4\mu_1}{3\kappa n}\right)\left(1 + \frac{4\mu_2}{3\kappa n}\right)\left(1 + \frac{2\mu_3}{3\kappa n}\right)$

$\mathcal{N} = 1$ LS defects

- We can also calculate the (appropriately renormalized) on-shell action, and we find

  - $S = \frac{N^2}{2\pi}\mathrm{vol}(AdS_3)[nF^{LS}]$

  - We can equivalently write this as

  - $S = \mathrm{vol}(AdS_3)\frac{1}{6\pi}\left(12a_{LS}n - b_{LS}\right)$

$\mathcal{N} = 4$/LS bulk flows

- We have calculated properties of defects for both $\mathcal{N} = 4$ and LS theories - let's compare them (when mass def. allowed)

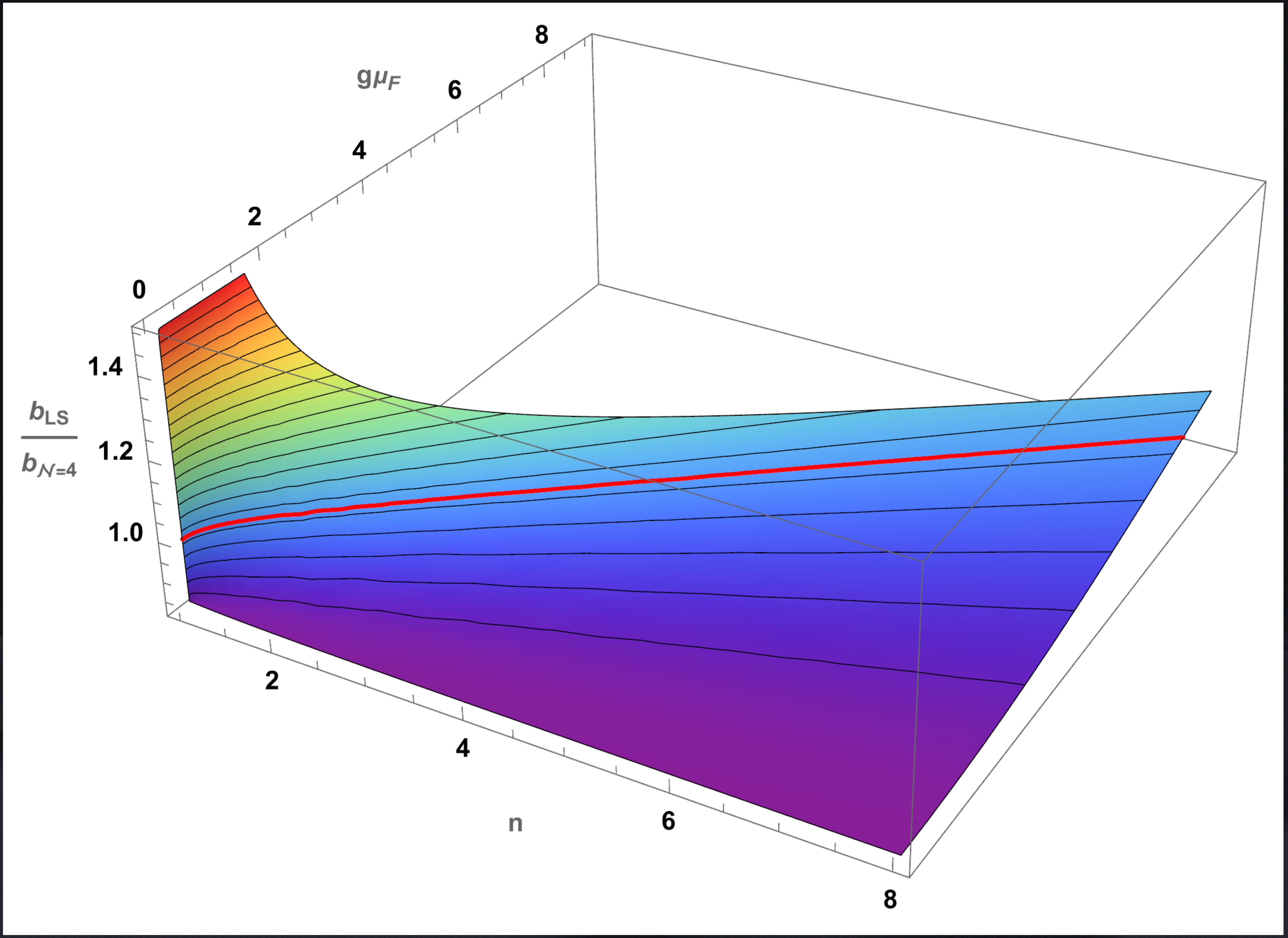
- $b_{\mathcal{N}=4}|_{\mu_B=0} = \dfrac{3N^2}{32n^2}\left((n-1)(1+8n+23n^2)+4(1+n)\mu_F^2\right)$

- $b_{LS} = \dfrac{3N^2}{32n^2}\left((n-1)(1+7n+19n^2)+4(1+2n)\mu_F^2\right)$

- First, note that for $n = 1$, $b_{LS} = \dfrac{3}{2}b_{\mathcal{N}=4}$, it increases!

- For $n \neq 1$, it can either increase or decrease!

$\mathcal{N} = 4$/LS bulk flows - b_{IR}/b_{UV}



$\mathcal{N} = 4$/LS bulk flows

- We can similarly compare the on-shell actions, which depend only on $a,\ b$ central charges.

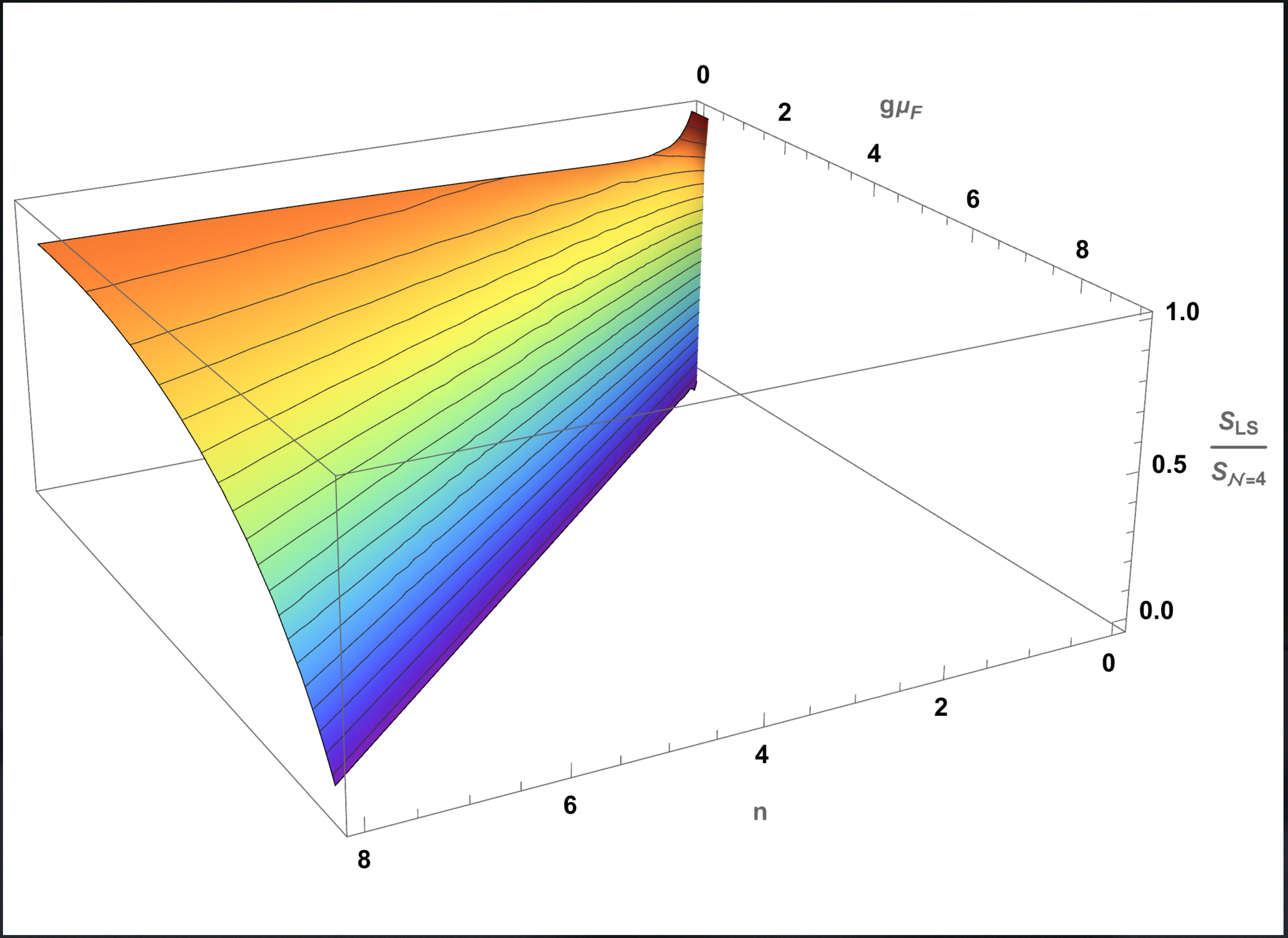
- $$S = \text{vol}(AdS_3)\frac{1}{6\pi}\left(12an - b\right)$$

- One can use our previous results to explicitly demonstrate that $S_{LS}/S_{\mathcal{N}=4} < 1$

  - (Explicit expressions ugly, in the paper)

- Strongly indicative that $12an - b$ is a good RG monotone

$\mathcal{N} = 4$/LS bulk flows - S_{IR}/S_{UV}

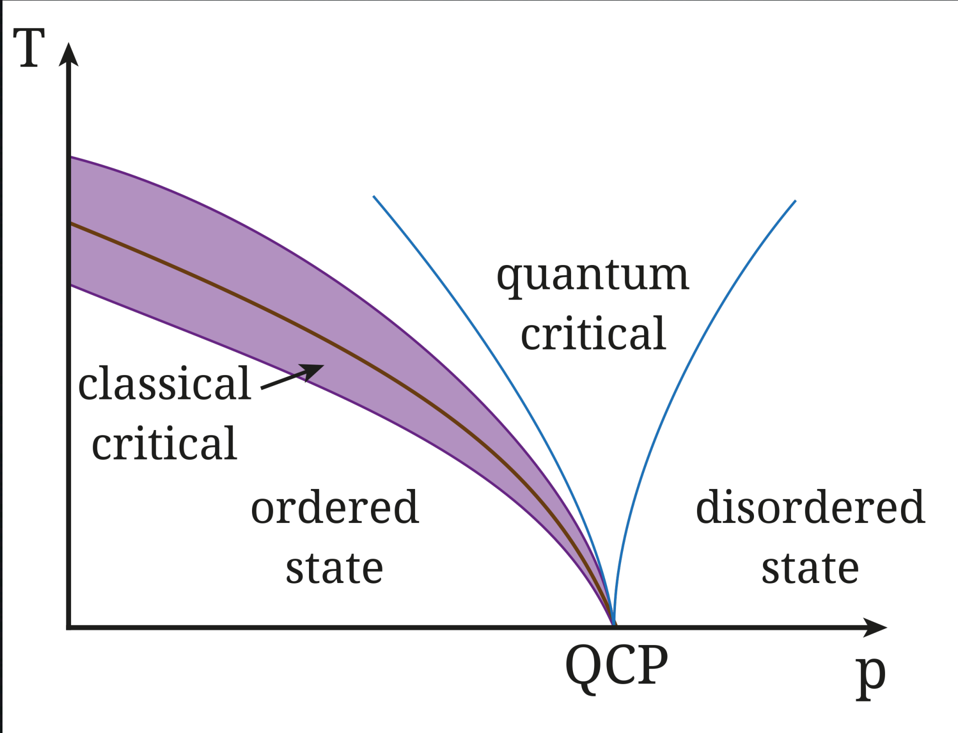


Topics I don't have time for

- Matching to free results
- Supersymmetric Renyi entropy (via the conical singularity)
- "Baroque" conformal defects with $m_F = \dfrac{\lambda}{\rho}$
- Details of how we extracted all these VEVs without analytic solutions
- Periodicity of monodromies / boundary conditions for regular solutions
- The 3d/1d analog (VEVs similar, but no anomalies in 3d/1d)

Conclusions

- New extensive study of $(0,2)$ 2d/4d magnetic defects
  - Explicit expressions for new central charges
- **New conjectured monotonic quantity**!
- To do:
  - Scattering (two-point functions), entanglement entropy, ...
  - Detailed analysis of defect core holographic renormalization


T
quantum
critical
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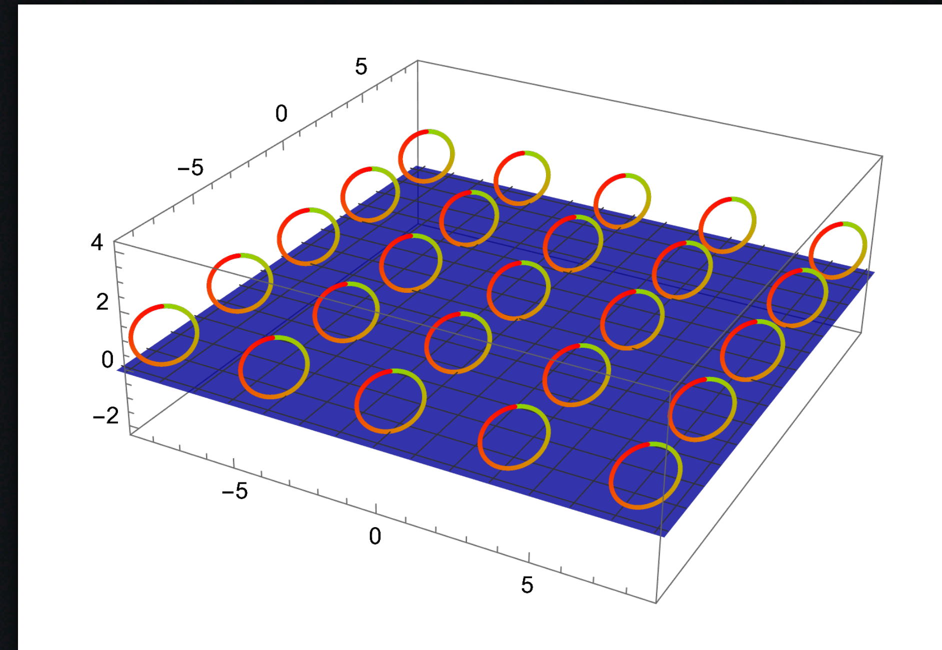
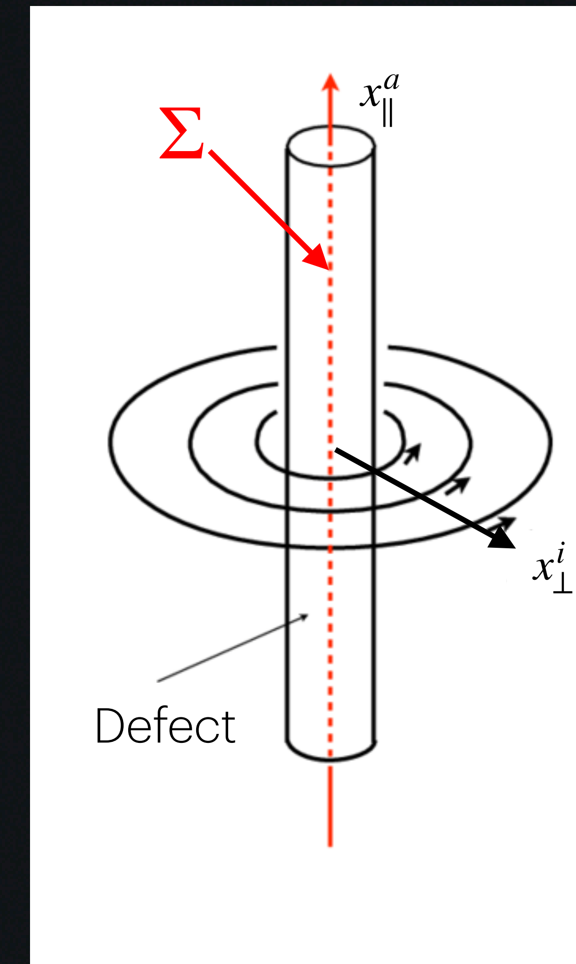
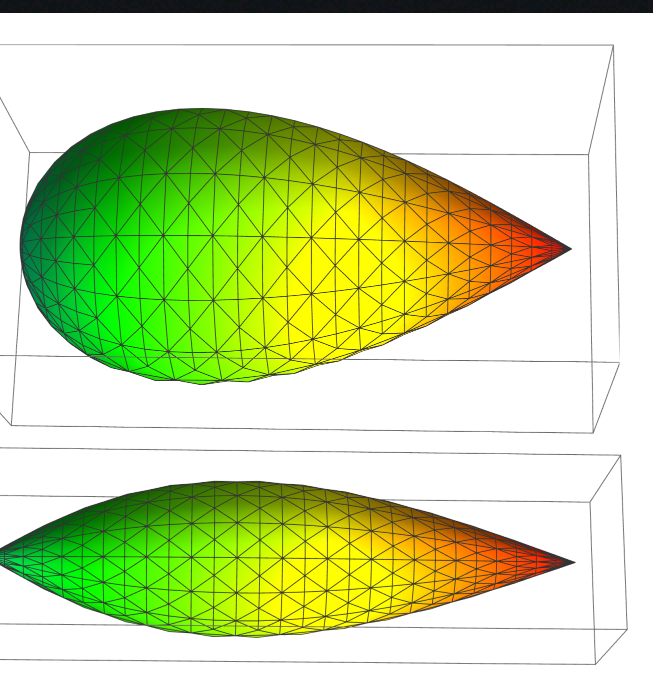
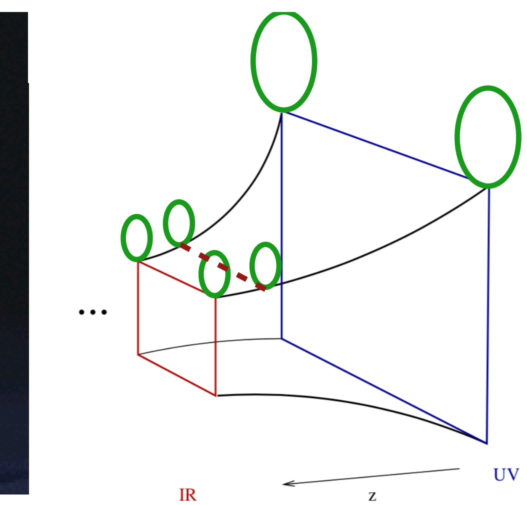
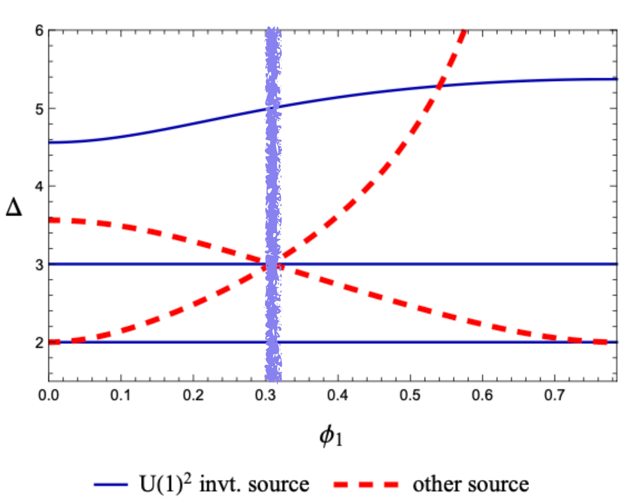


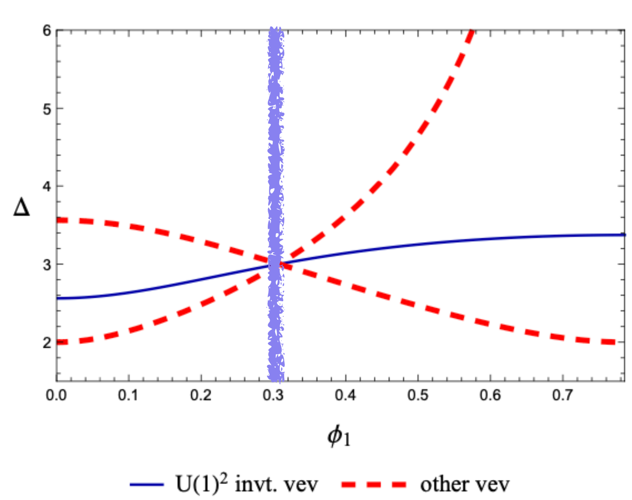

Σ
Defect

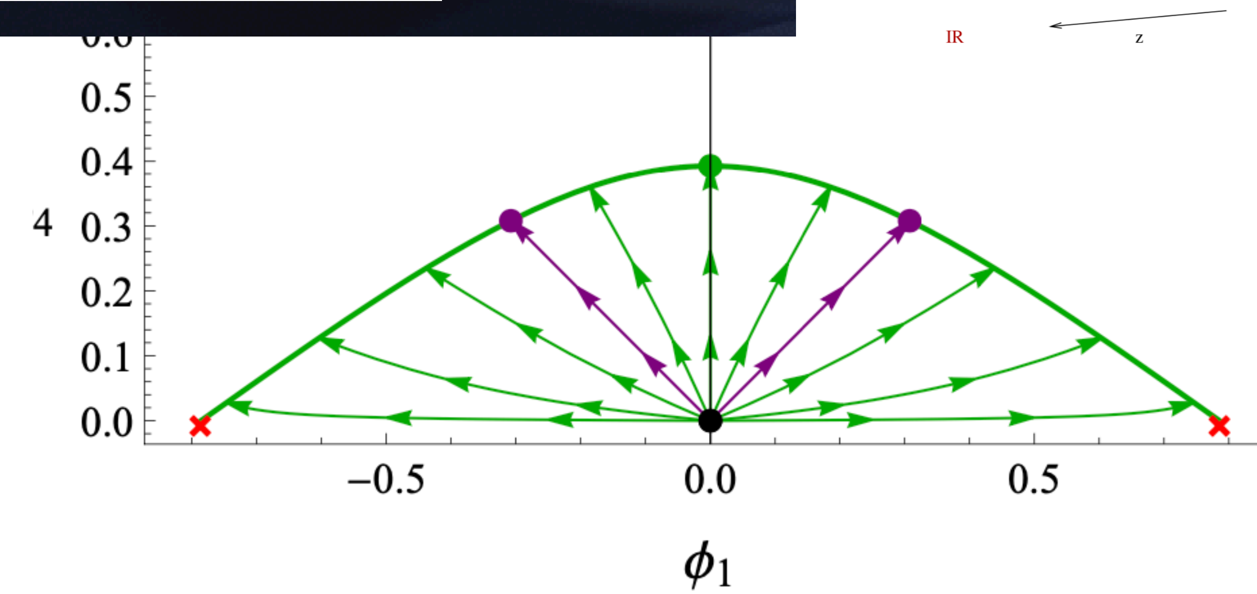


Thank you!


IR
z
UV


Δ
ϕ_1
U(1)² invt. source
other source


Δ
ϕ_1
U(1)² invt. vev
other vev


ϕ_1