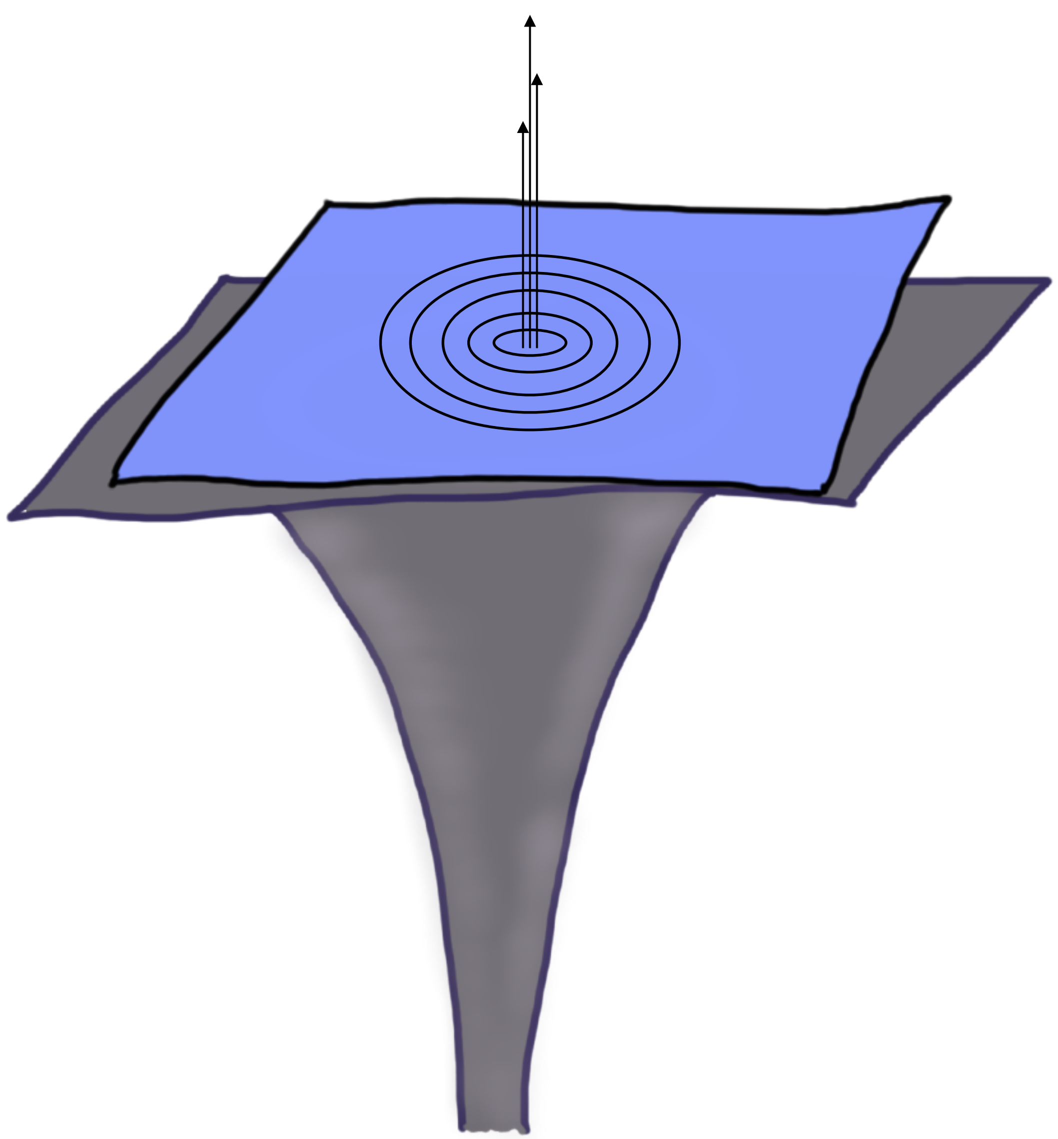
A (super)gravitational perspective on magnetic defects



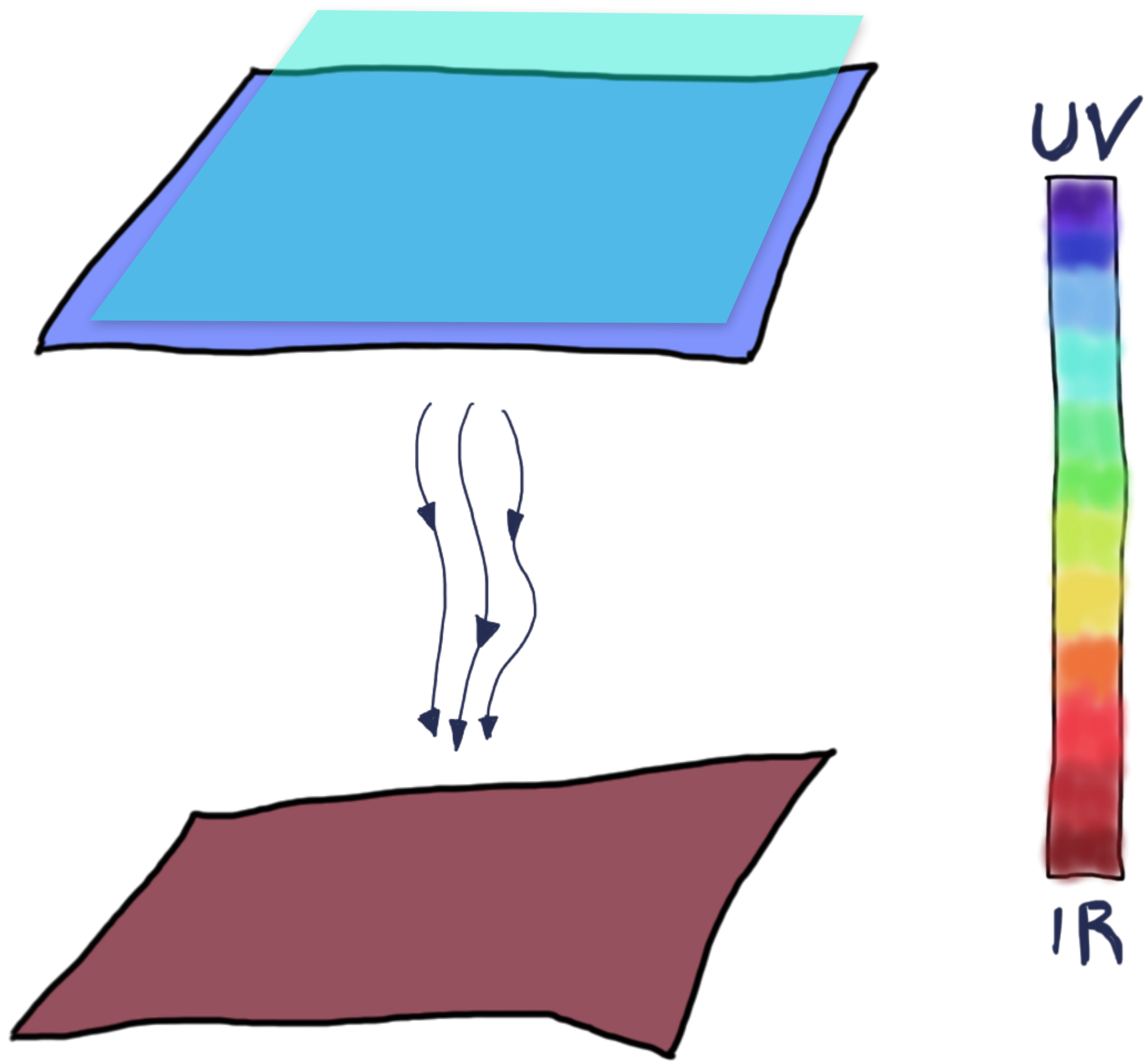
Chris Rosen
UoC

with

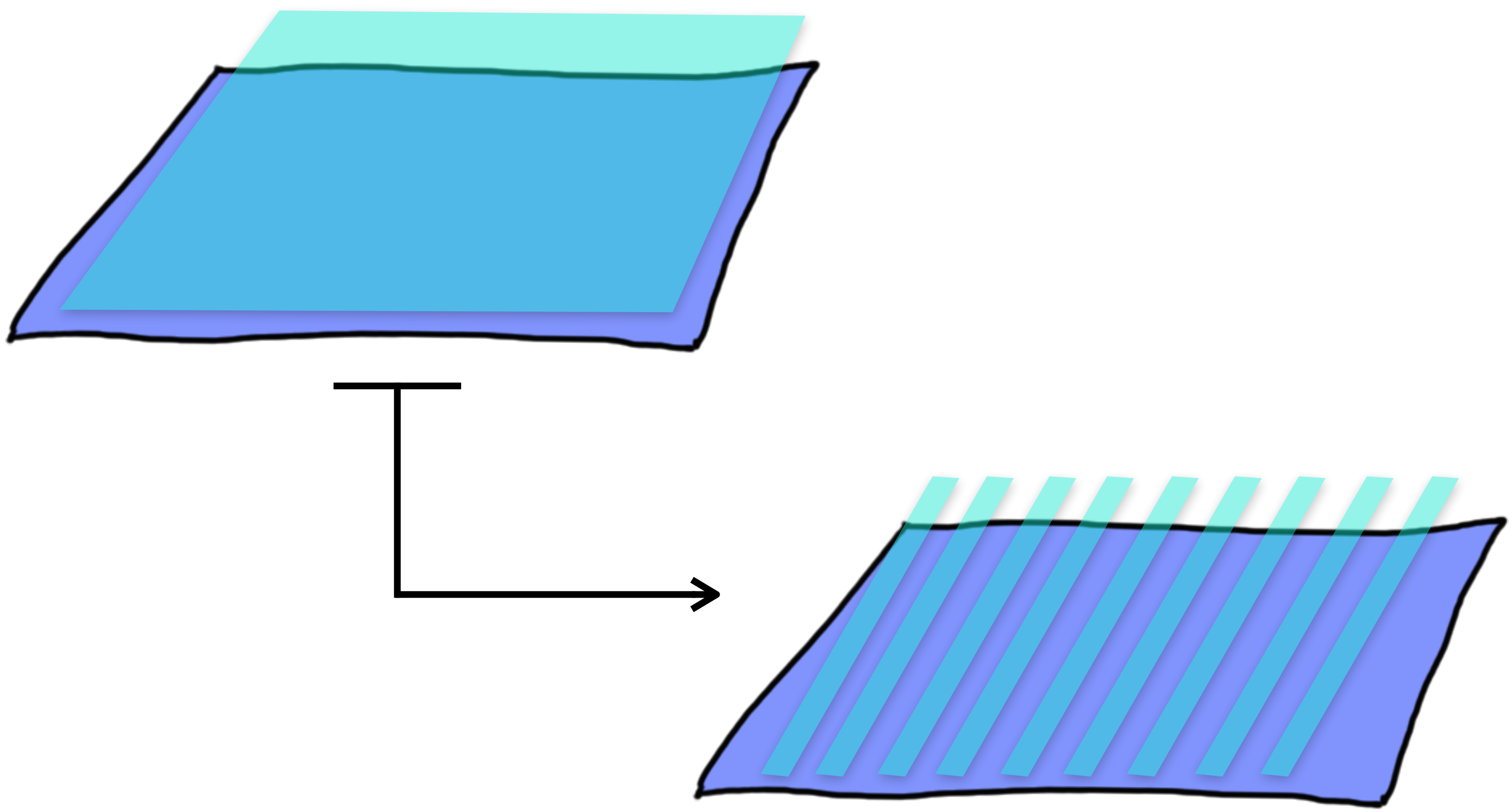
Arav, Gauntlett, Jiao & Roberts

$$S_{\mathrm{CFT}_d} \to S_{\mathrm{CFT}_d} + \int \mathrm{d}^d x \, g\mathcal{O}$$

If $\partial g = 0$



Familiar ground: explore, quantify, catalogue…

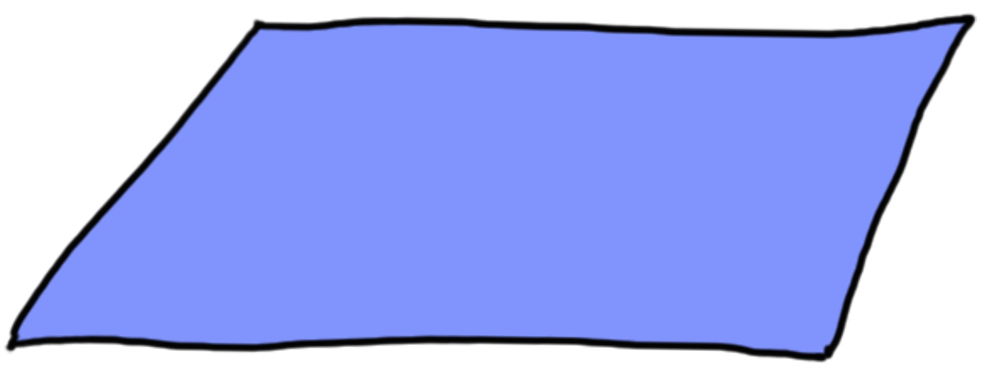


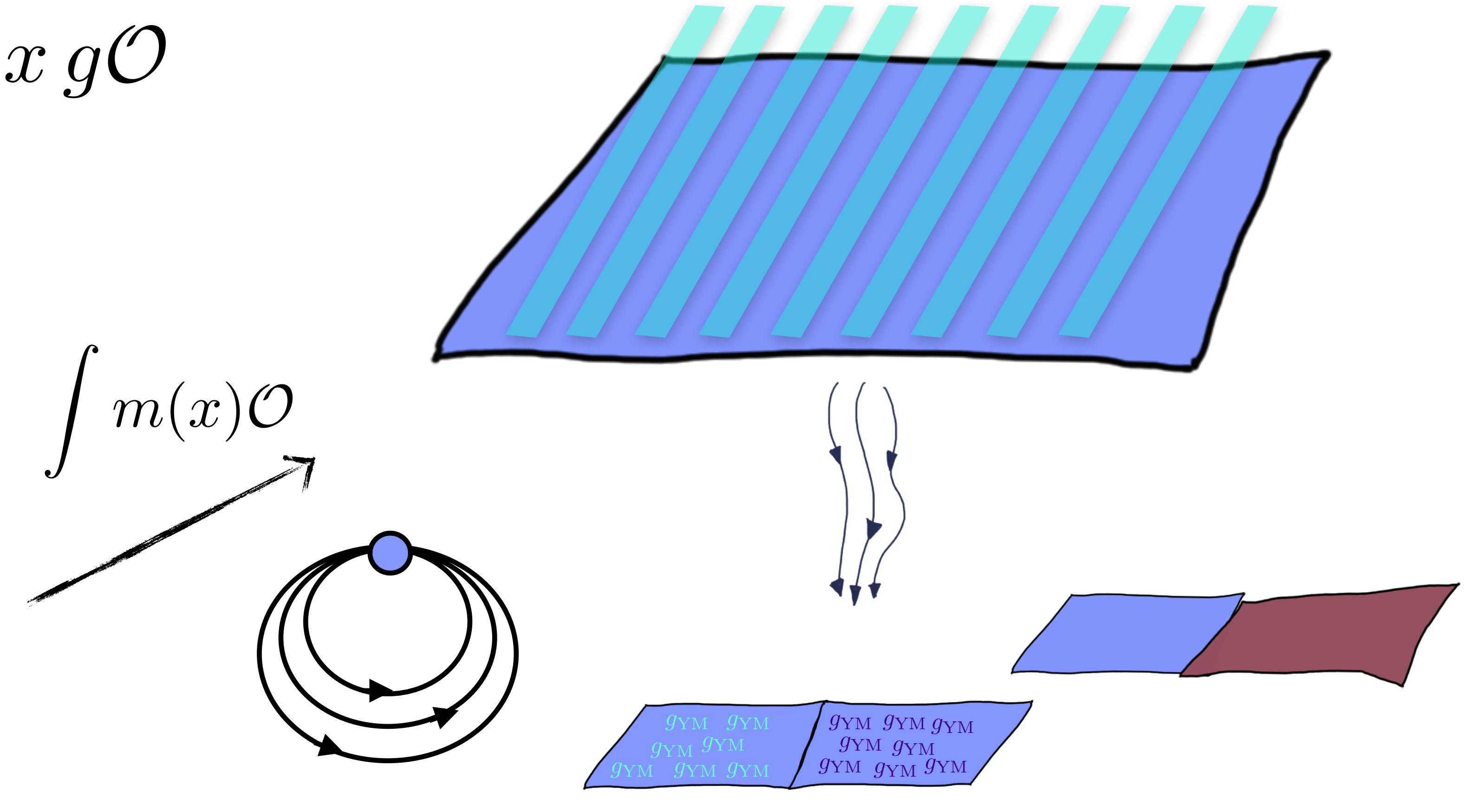
What if instead the deformation only preserves a subgroup of the CFT spacetime symmetries?

Much less is known, but these systems are of great interest theoretically and phenomenologically

$$S_{\mathrm{CFT}_d} \to S_{\mathrm{CFT}_d} + \int \mathrm{d}^d x \, g\mathcal{O}$$

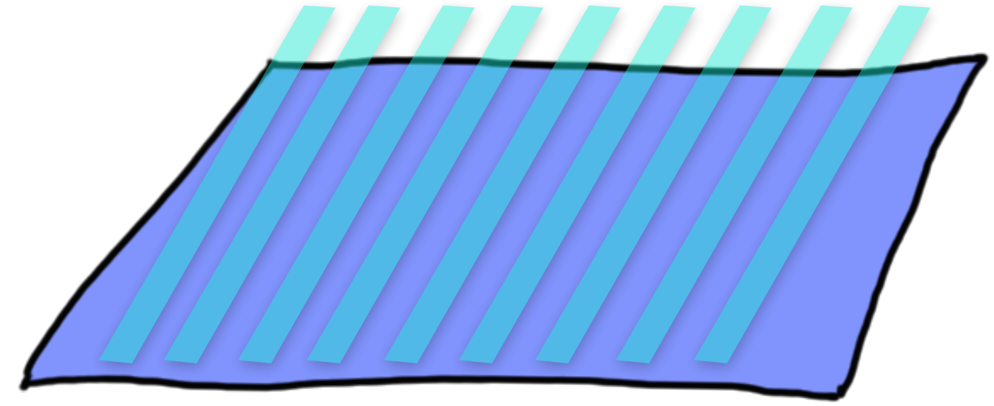
If $\partial g \neq 0$

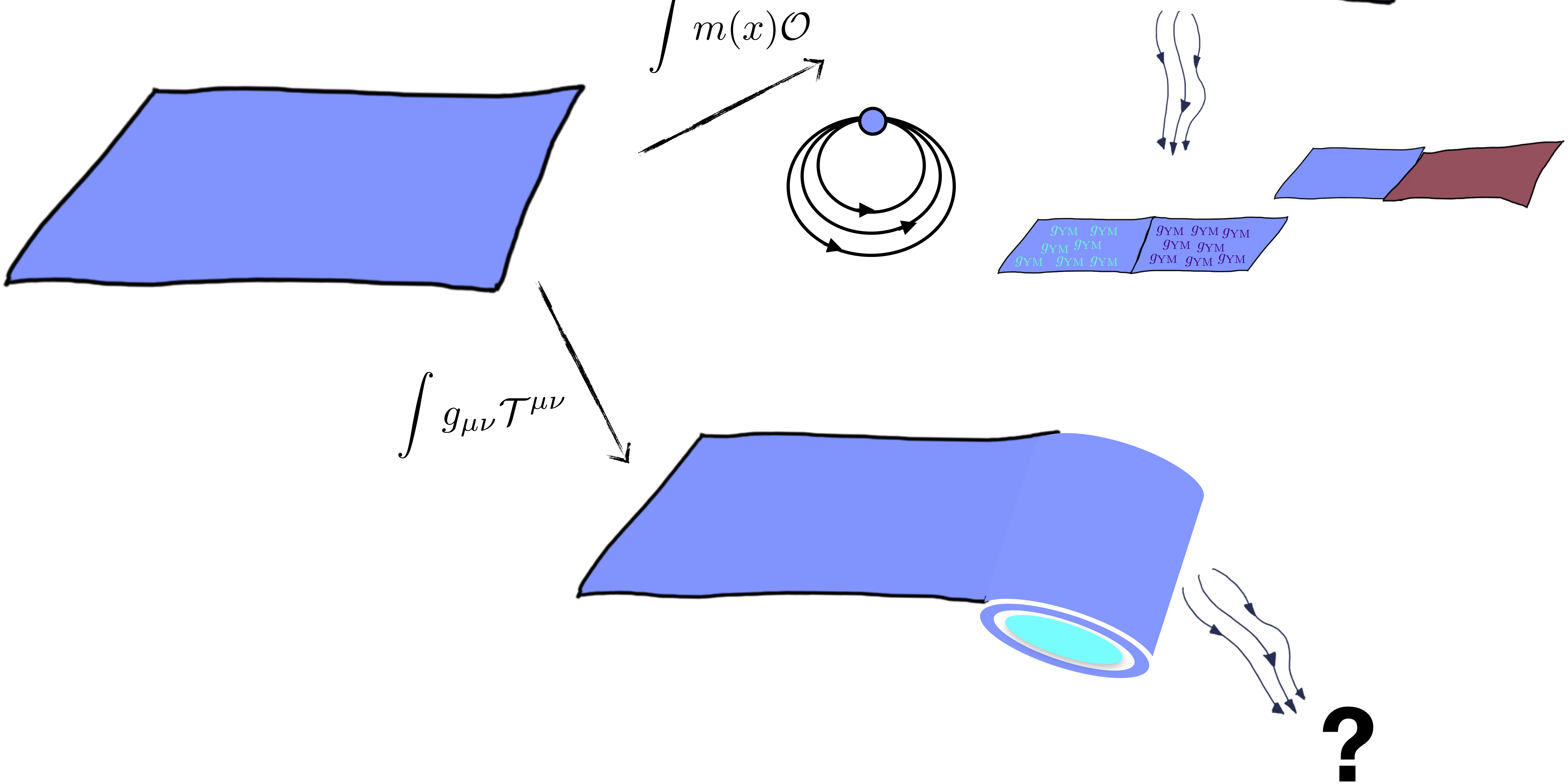


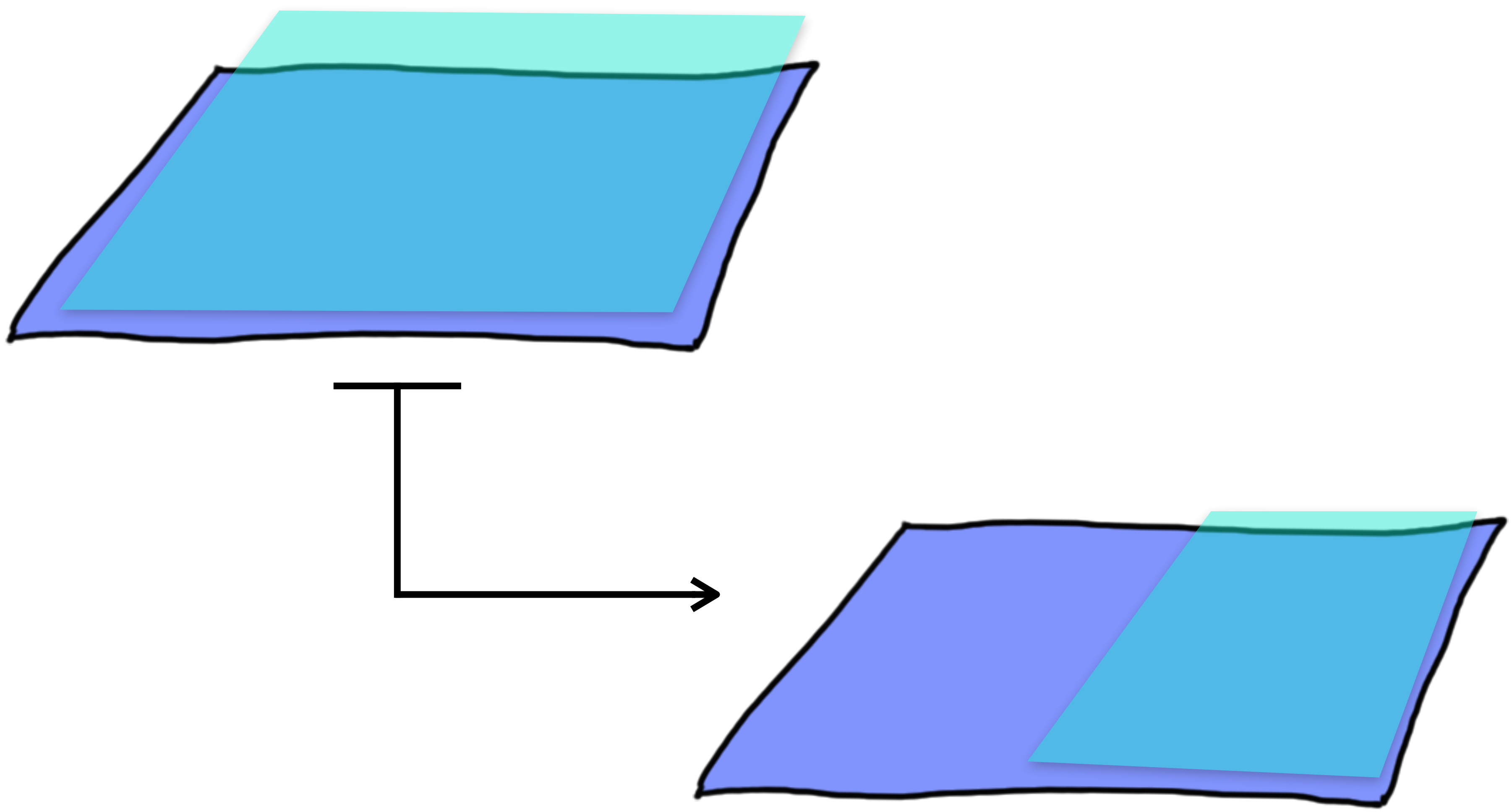


$$S_{\mathrm{CFT}_d} \to S_{\mathrm{CFT}_d} + \int \mathrm{d}^d x\, g\mathcal{O}$$

If $\partial g \neq 0$



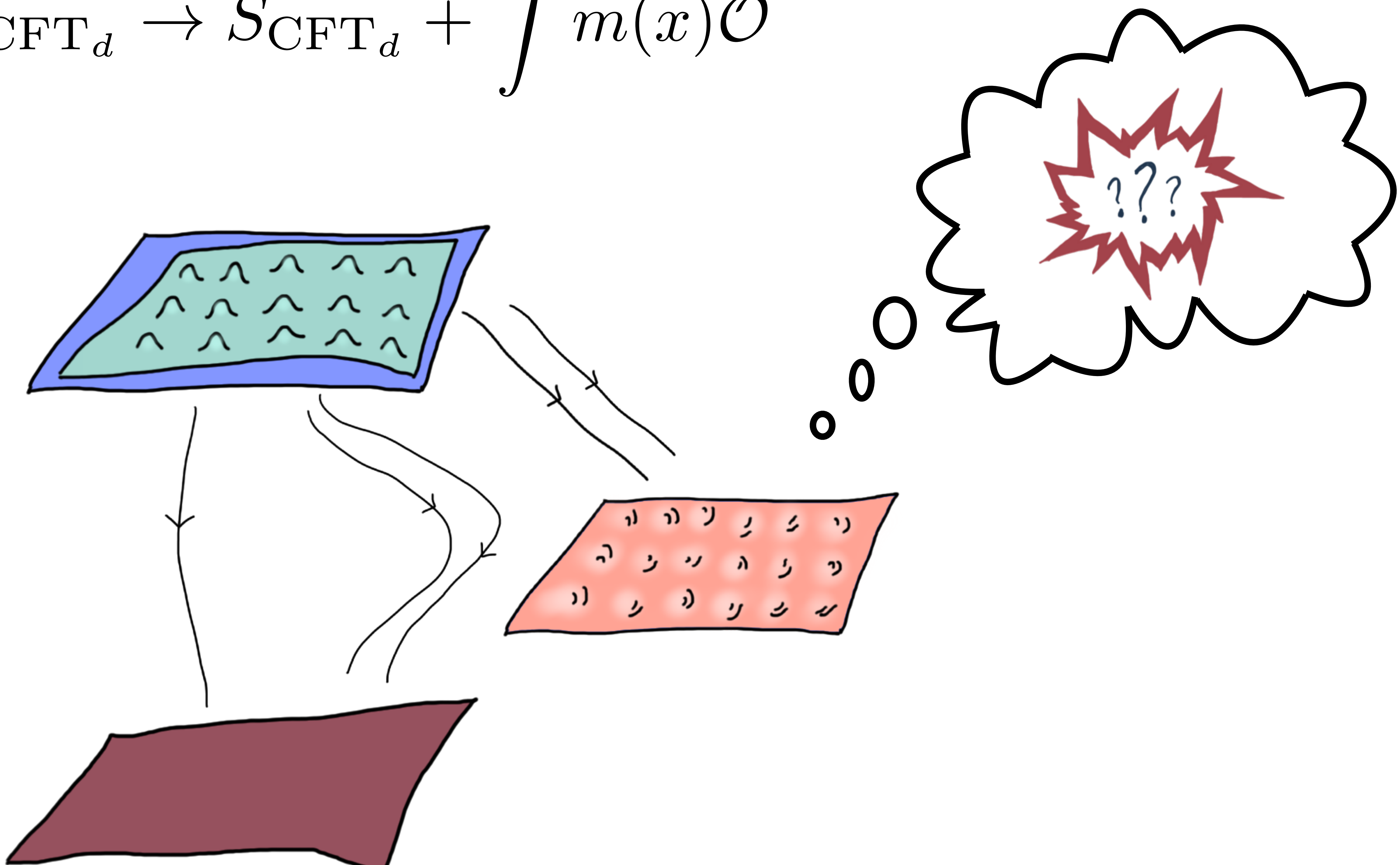




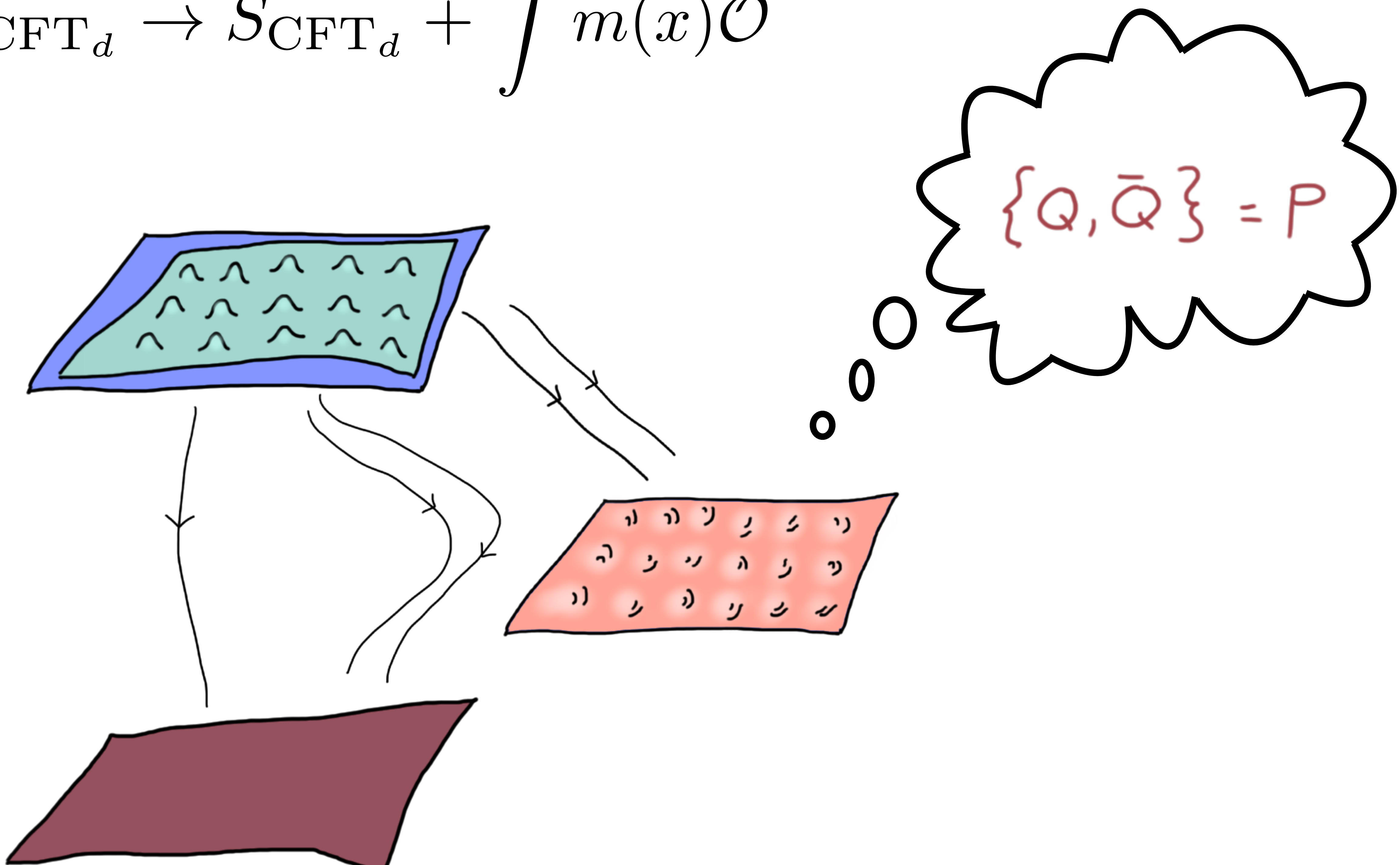
To get a handle on these systems, it is prudent to start slow, looking to deformations which preserve a large subgroup of the fixed point theory's symmetries

For example, start with a SCFT in d dimensions, and look for SUSY deformations which preserve SO(d-1,2)—the conformal group in one less dimension.

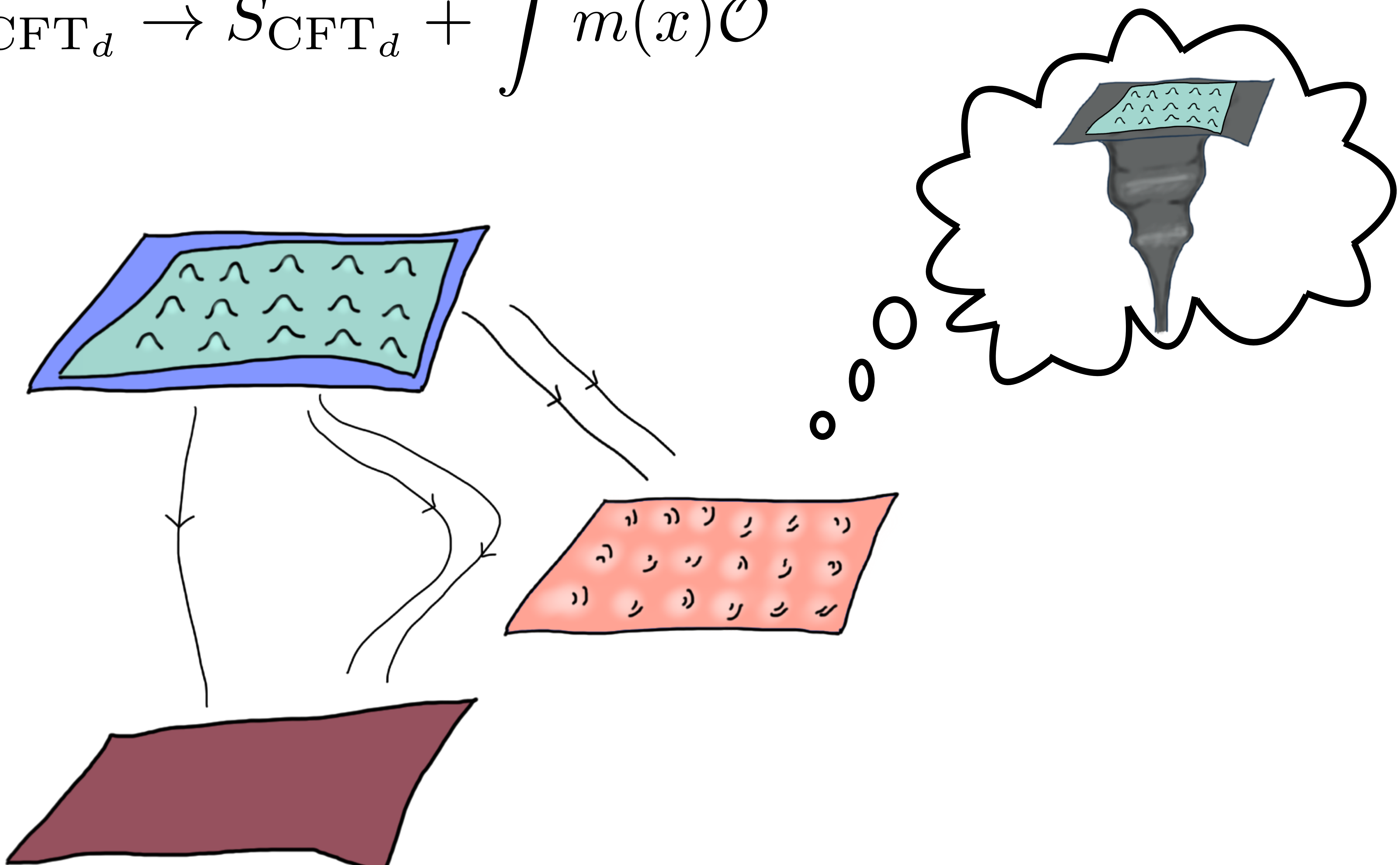
$$S_{\mathrm{CFT}_d} \to S_{\mathrm{CFT}_d} + \int m(x)\mathcal{O}$$



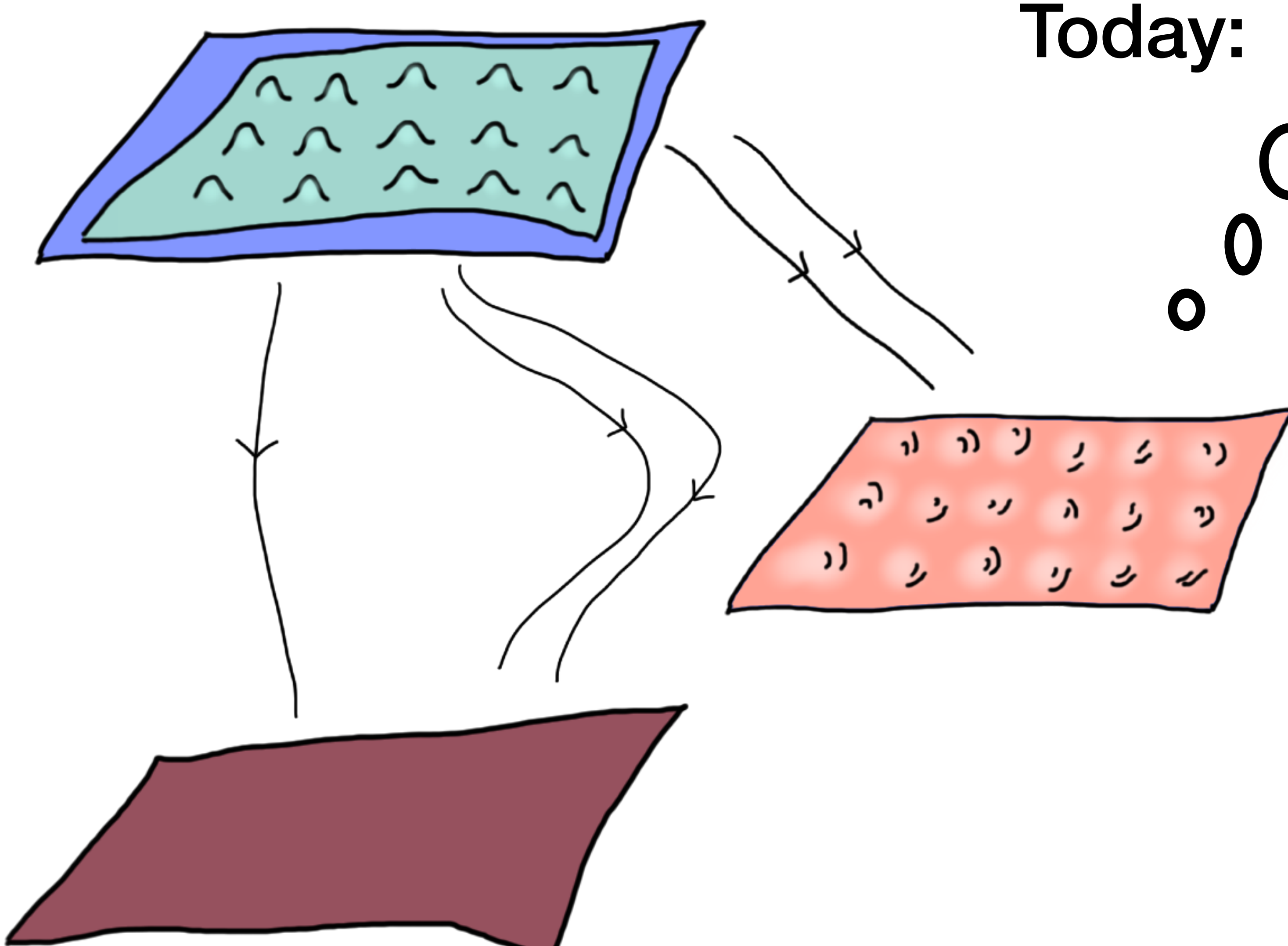
$$S_{\mathrm{CFT}_d} \to S_{\mathrm{CFT}_d} + \int m(x)\mathcal{O}$$



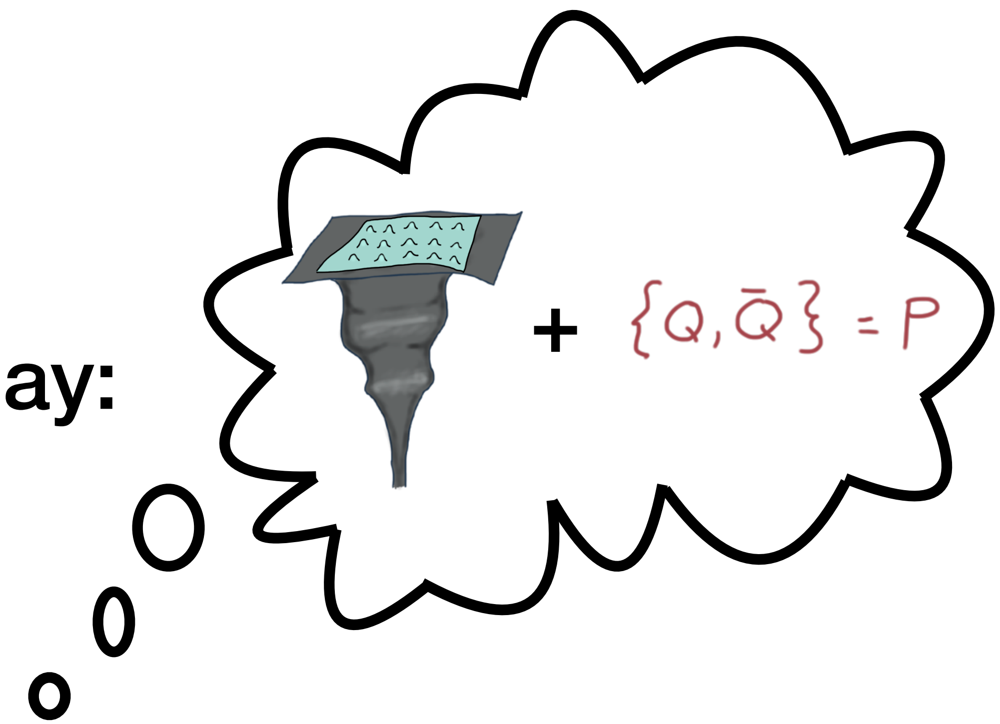
$$S_{\mathrm{CFT}_d} \to S_{\mathrm{CFT}_d} + \int m(x)\mathcal{O}$$

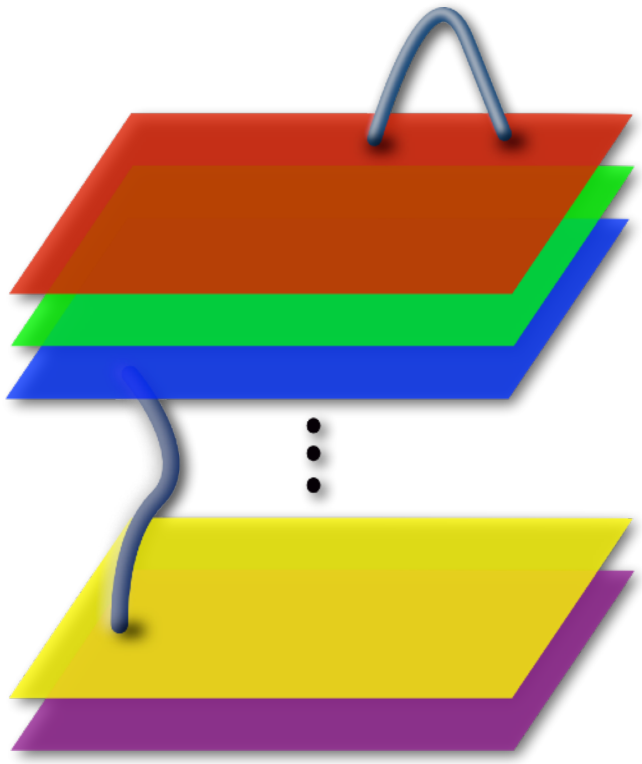


$$S_{\mathrm{CFT}_d} \to S_{\mathrm{CFT}_d} + \int m(x)\mathcal{O}$$

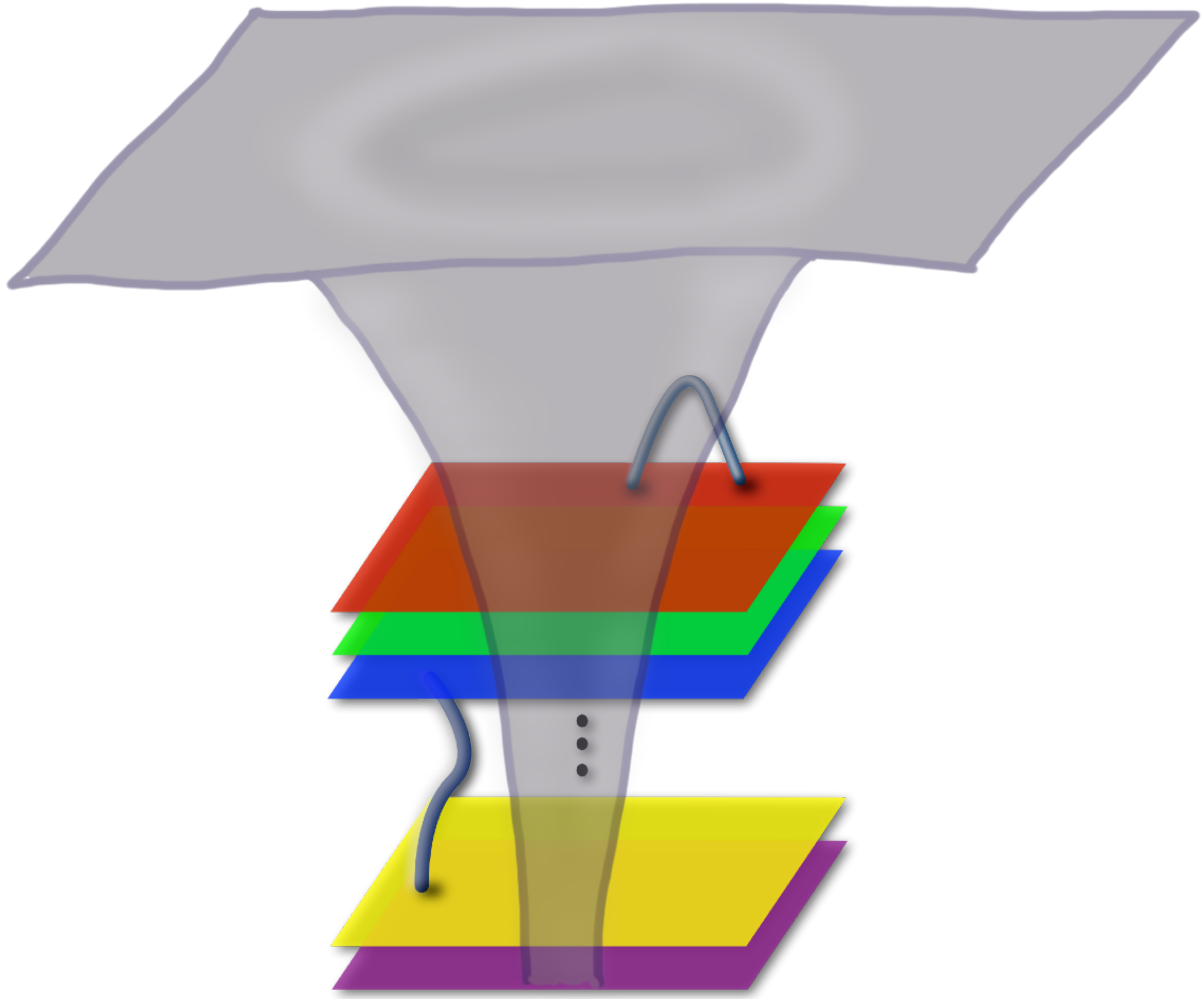


Today:

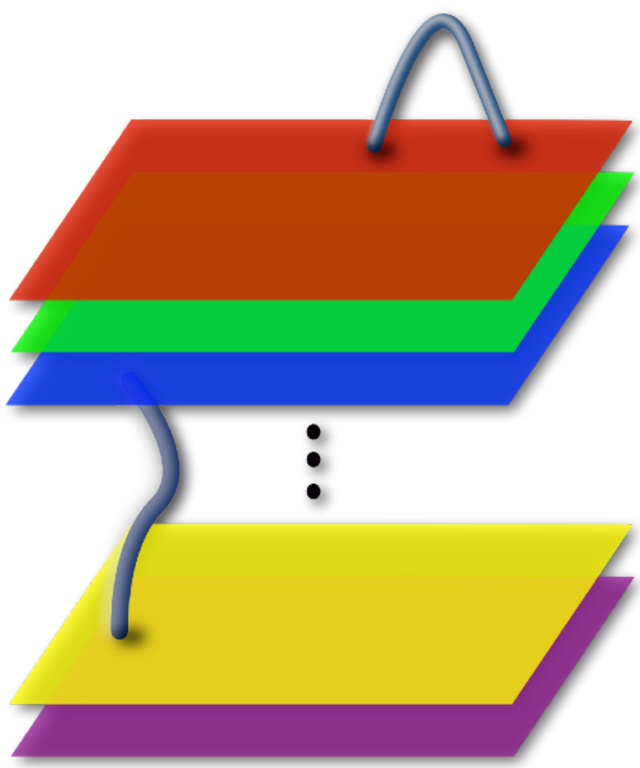




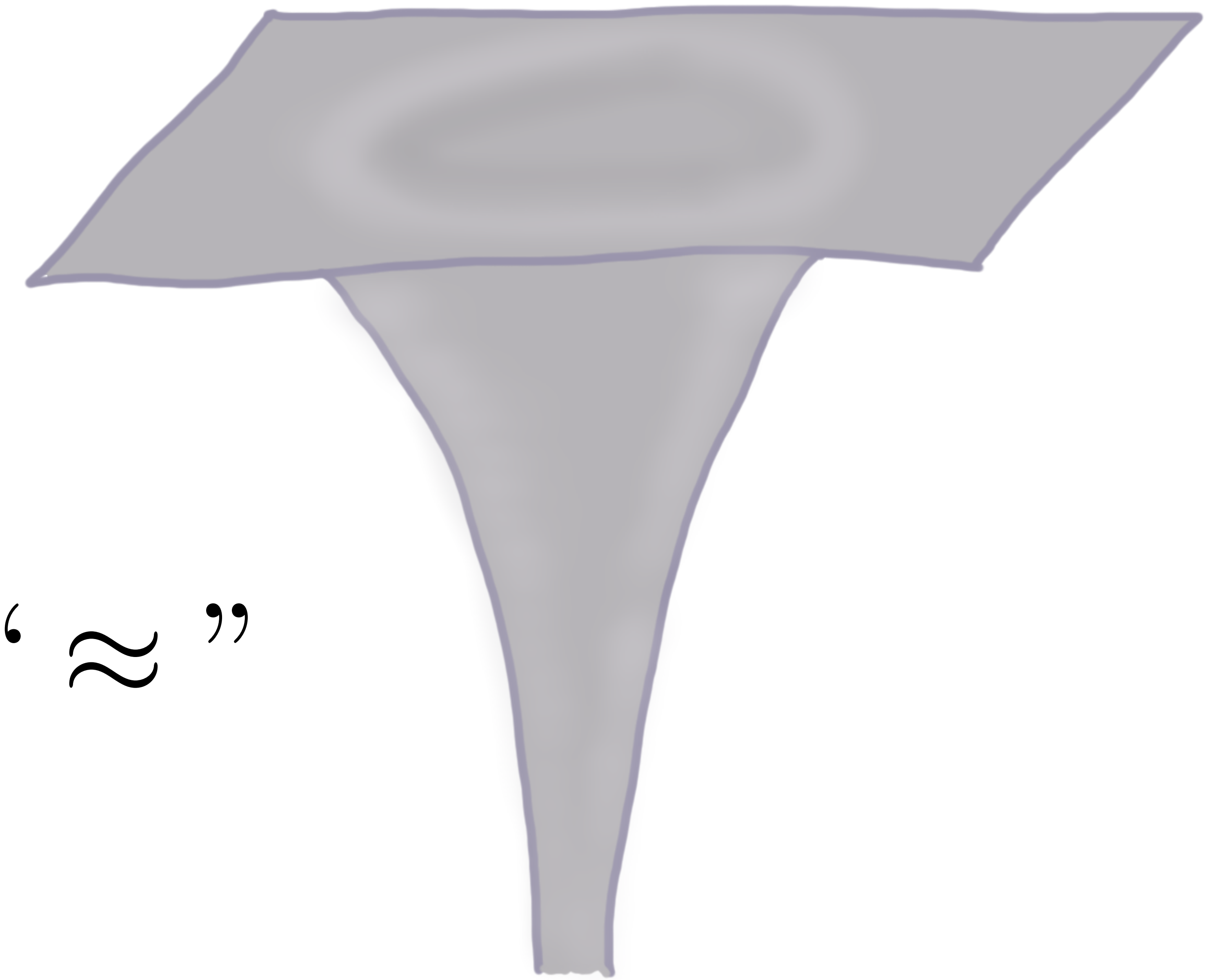
Dual Descriptions: **A gauge theory**



Dual Descriptions: A (super) gravitational theory

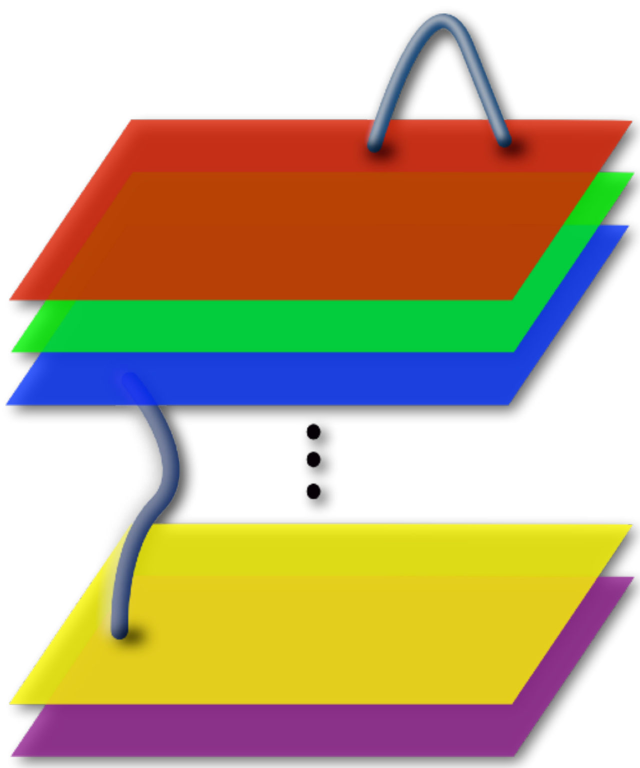


“ ≈ ”

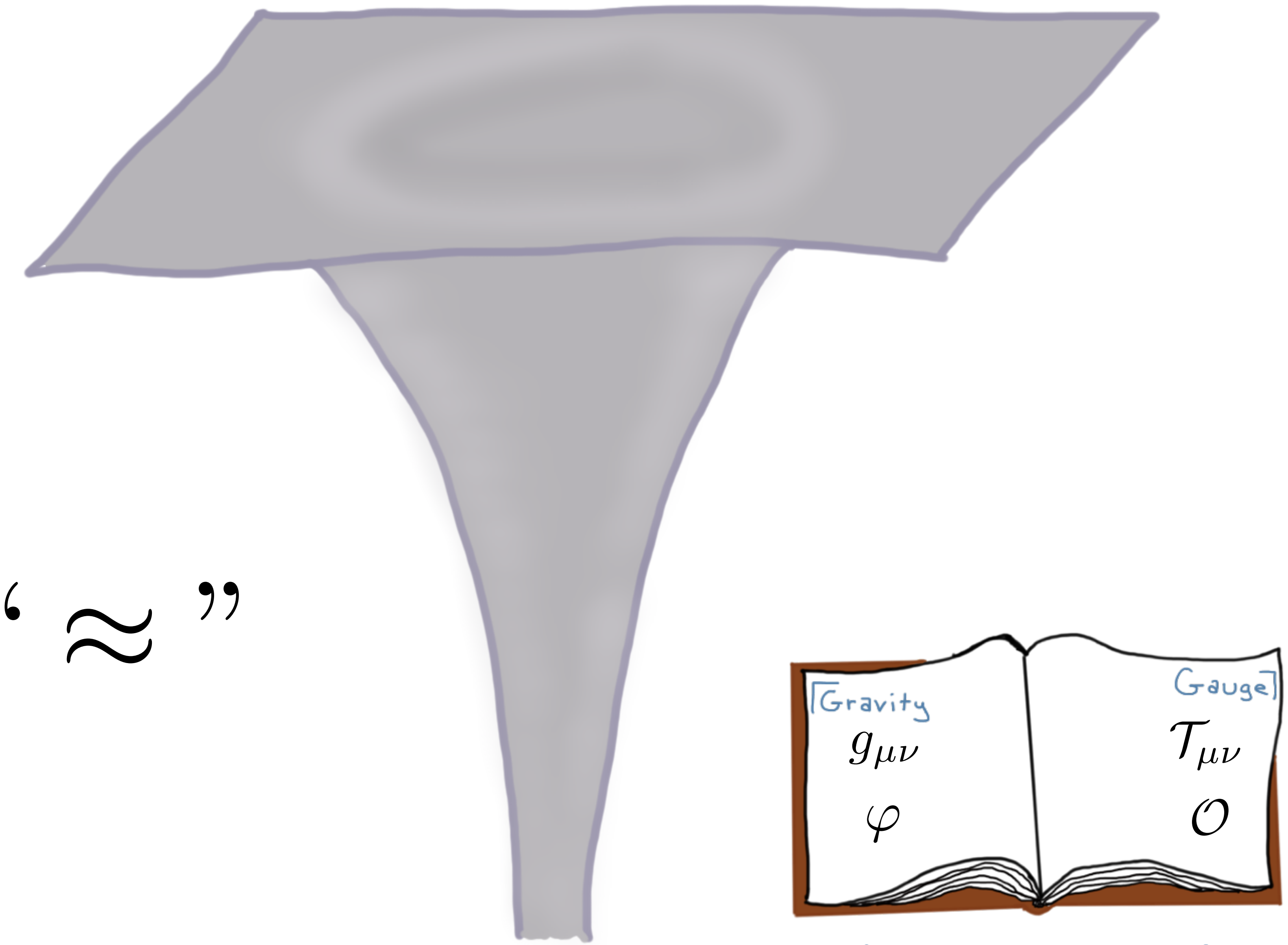


Gauge / **Gravity** **Duality**

(Confromal Field Theory—CFT) (Anti-de Sitter—AdS)



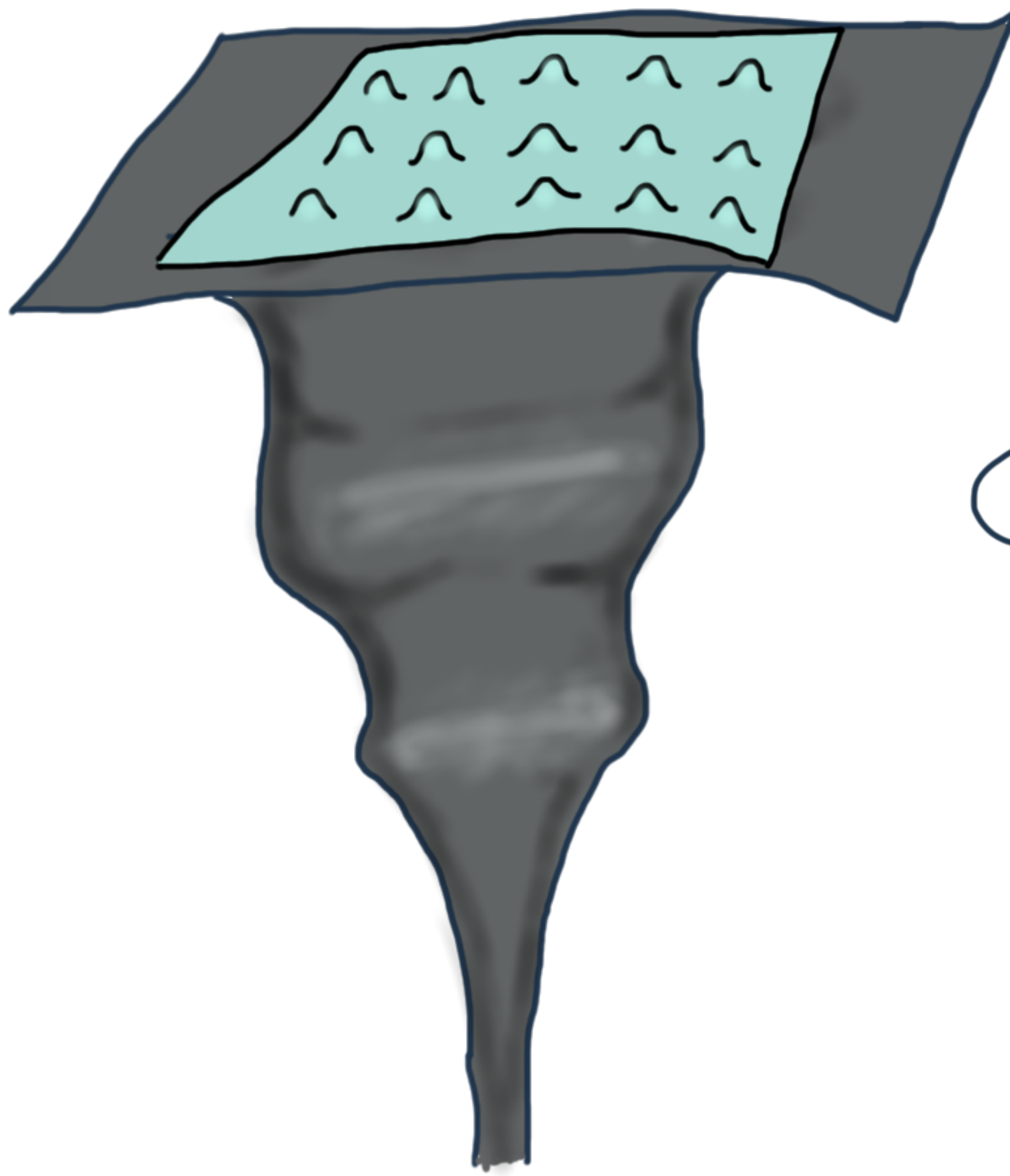
“ $\approx$ ”



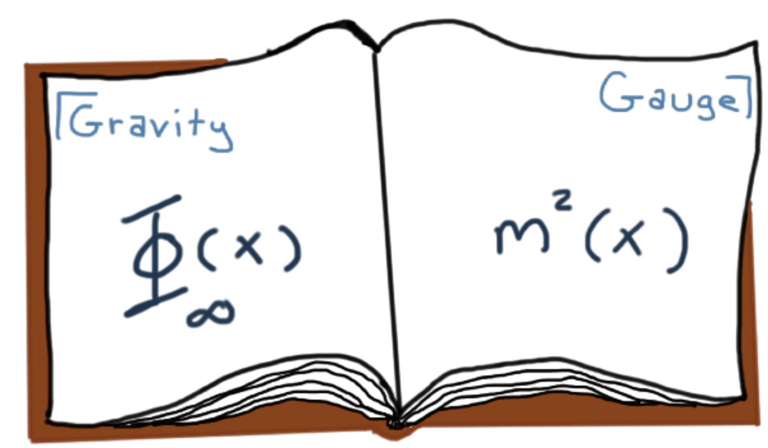
Gauge / **Gravity**

Duality

Holographic RG



$$G_{\mu\nu} = T_{\mu\nu} \quad \vdots \quad \Phi^{i}_{\infty}(x)$$



RG “geometrized”

sDefects: an N=4 example

$$\int \mathcal{L}_{\mathcal{N}=4} + \varphi_{ij}\left(D^{ij}{}_{kl} + E^2\right)\varphi^{kl} - \frac{1}{2}\bar{\psi}^i E_{ij}\,\psi^j + V_\mu{}^i{}_j\,\mathcal{J}^{\mu\, j}{}_i + \dots$$

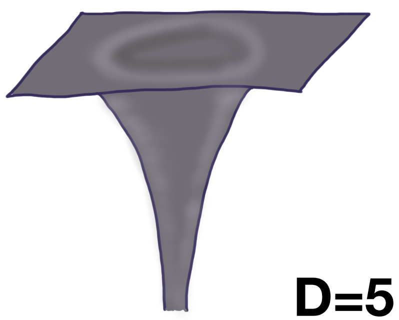
$(\mathbf{6})$ $\quad(\mathbf{20'})$ $\quad(\bar{\mathbf{4}})$ $\quad(\mathbf{10})$ $\quad(\mathbf{15})$

$(\partial\tau = 0,\ M_p \to \infty)$

An efficient and powerful way to study supersymmetric deformations of CFTs involves coupling the theory to (non-dynamical) background fields of off-shell conformal supergravity, e.g. Festuccia-Seiberg.

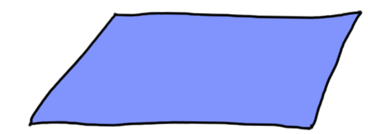
The coupling of N=4 SYM to N=8 conformal supergravity was accomplished in the 80's by de Roo et al.

sDefects: an N=4 example



$$e^{S_{\rm os}(\Phi)}$$

$$\left\langle e^{\int \Phi_b \mathcal{O}} \right\rangle_{\rm CFT}$$



D=5 N=8 SO(6) gauged SUGRA ⟺ **D=4 N=4 SYM in CSG background**

$$\mathcal{U}\ :\ \mathbf{42} = \mathbf{1} + \mathbf{1} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{20'} \qquad E_{ij}\ ,\ D^{ij}{}_{kl}$$

$$\mathcal{A}_\mu\ :\ \mathbf{15} = \mathbf{15} \qquad V_\mu{}^i{}_j$$

Ex

$$\left(E_{33}\,,\, D^{12}{}_{34}\,,\, D^{12}{}_{12} \right)$$

$$\phi \qquad \mathcal{O}_\phi\ :\ \operatorname{tr}\chi_3\chi_3 + \ldots + \text{h.c.}$$

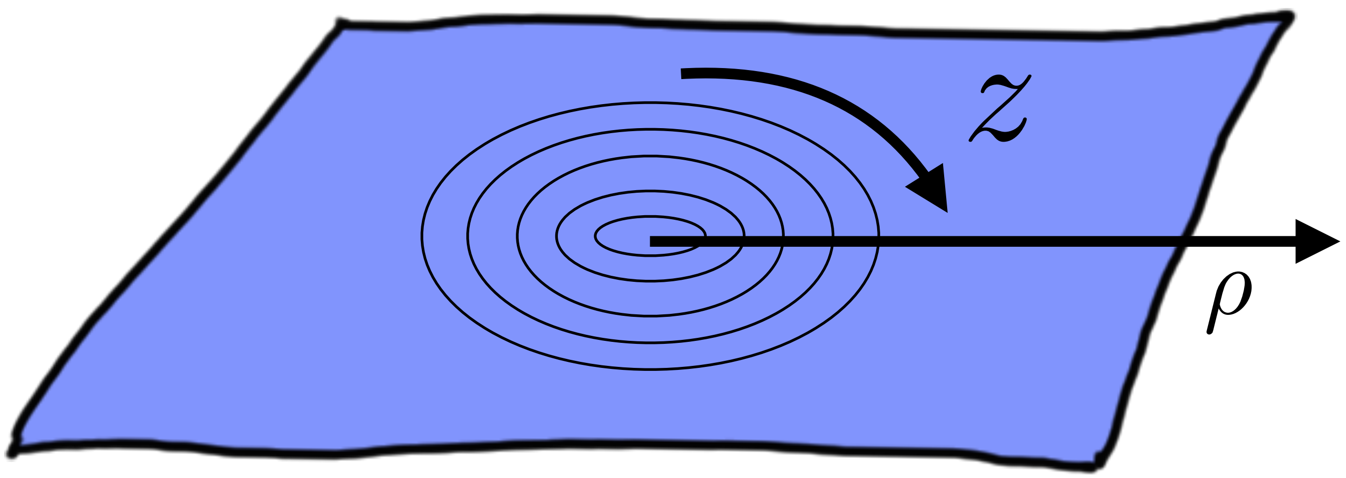
$$\alpha \qquad \mathcal{O}_\alpha\ :\ \operatorname{tr} Z_3^2 + \text{h.c.}$$

$$\beta \qquad \mathcal{O}_\beta\ :\ \operatorname{tr}\left(|Z_1|^2 + |Z_2|^2 - 2|Z_3|^2\right) + \text{h.c.}$$

AdS/CFT provides an explicit realisation of the deformation of N=4 SYM via coupling to background conformal SUGRA. With some care, we can identify the precise correspondence between our field theory deformations and their gravitational dual…

Monodromy defects

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 + \mathrm{d}\rho^2 + \rho^2 \mathrm{d}z^2$$



Monodromy defects

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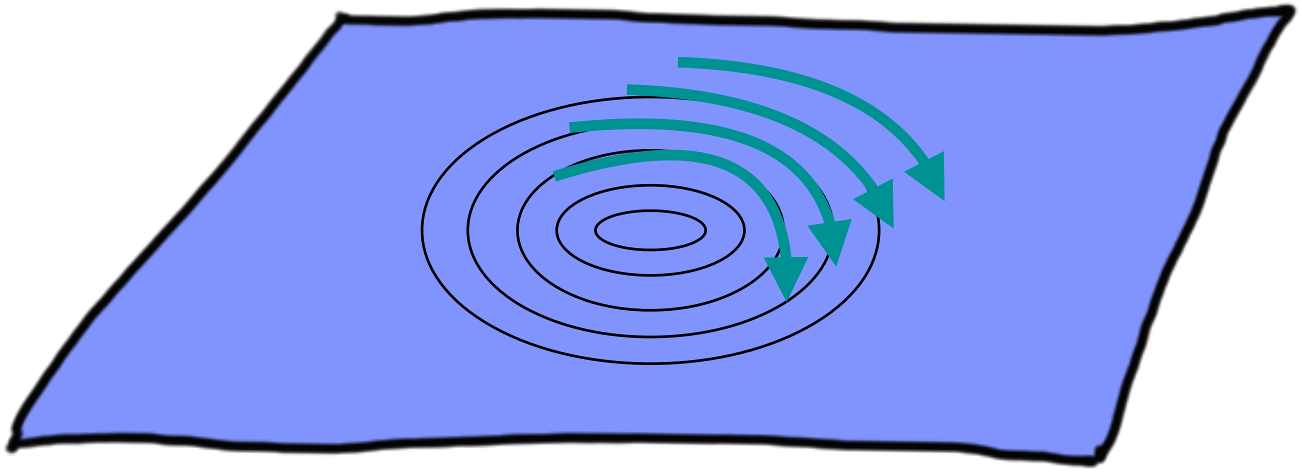
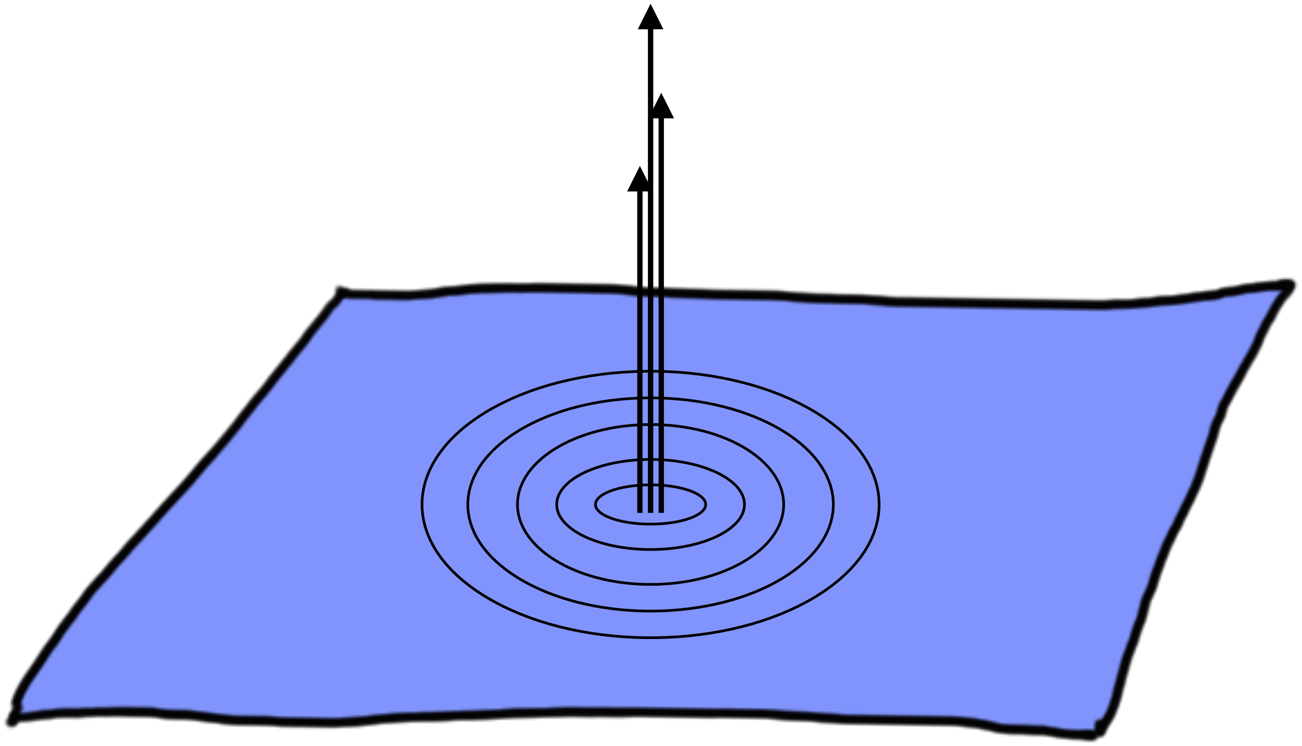


$$V = \alpha\,\mathrm{d}z$$

Monodromy defects

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 + \mathrm{d}\rho^2 + \rho^2 \mathrm{d}z^2$$



$$V = \alpha\, \mathrm{d}z$$

$$\int F = \int_\partial V$$

$$F = \alpha\delta(\rho)\, \mathrm{d}\rho \wedge \mathrm{d}z$$

Monodromy defects

$$\mathrm{d}s^2 = \underbrace{-\mathrm{d}t^2 + \mathrm{d}\vec{x}^2}_{x^a} + \overbrace{\mathrm{d}\rho^2 + \rho^2 \mathrm{d}z^2}^{x^i}$$

$$\langle T^{ab} \rangle = -\frac{h_D}{2\pi} \frac{\eta^{ab}}{\rho^4} \qquad \langle T^{ij} \rangle = \frac{h_D}{2\pi} \frac{3\delta^{ij} - 4\frac{x^i x^j}{\rho^2}}{\rho^4}$$

Co-dimension two conformal defects in e.g. four dimensions imply one point functions constrained by the preserved spacetime symmetries

$$\langle J^R{}_z \rangle = \frac{C}{\rho^2}$$

Monodromy defects

$$ds^2 = \underbrace{-dt^2 + d\vec{x}^2}_{x^a} + \overbrace{d\rho^2 + \rho^2 dz^2}^{x^i}$$

$$\langle T^{ab} \rangle = -\frac{h_D}{2\pi} \frac{\eta^{ab}}{\rho^4} \qquad \langle T^{ij} \rangle = \frac{h_D}{2\pi} \frac{3\delta^{ij} - 4\frac{x^i x^j}{\rho^2}}{\rho^4}$$

Co-dimension two conformal defects in e.g. four dimensions imply one point functions constrained by the preserved spacetime symmetries

$$\langle J_z \rangle = \frac{C}{\rho^2} \quad \xrightarrow{\mathcal{N} = (0,2)} \quad \langle {J^R}_z \rangle = \frac{h_D}{2\pi\rho^2}$$

Appealing to the enhanced symmetry algebra for superconformal defects yields relationships between outwardly distinct one-point functions.

Monodromy defects from SUGRA

$$
\begin{aligned}
\mathrm{d}s^2 &= -\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 + \mathrm{d}\rho^2 + \rho^2 \mathrm{d}z^2 \\
&= \rho^2 \left(\frac{-\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 + \mathrm{d}\rho^2}{\rho^2} + \mathrm{d}z^2 \right) \\
&= \rho^2 \mathrm{d}s^2 \left(\mathrm{AdS}_3 \times S^1 \right)
\end{aligned}
$$

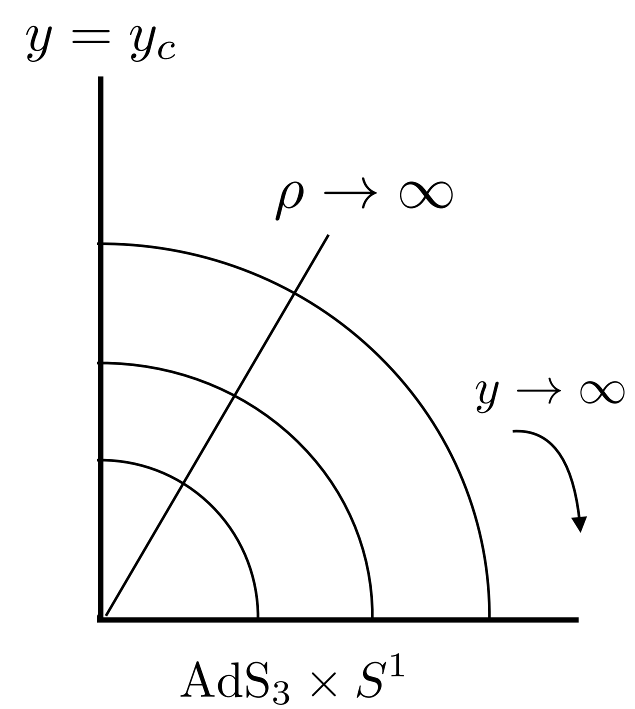
This Weyl frame is well positioned for a holographic description

Monodromy defects from SUGRA

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 + \mathrm{d}\rho^2 + \rho^2 \mathrm{d}z^2$$

$$= \rho^2 \left(\frac{-\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 + \mathrm{d}\rho^2}{\rho^2} + \mathrm{d}z^2 \right)$$

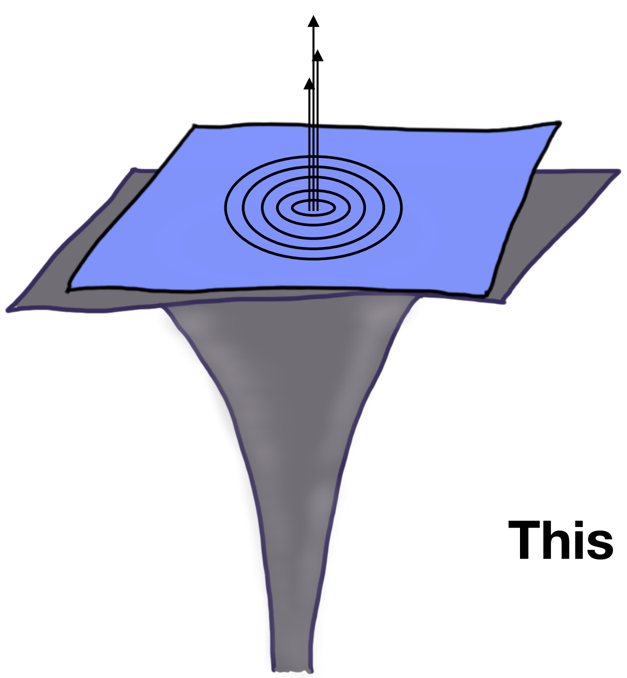
$$= \rho^2 \mathrm{d}s^2 \left(\mathrm{AdS}_3 \times S^1\right)$$



$$\mathrm{d}s_5^2 = e^{2B}\mathrm{d}s^2\left(\mathrm{AdS}_3\right) + h^2 \mathrm{d}z^2 + f^2 \mathrm{d}y^2$$

$$A^I = a^I\,\mathrm{d}z$$

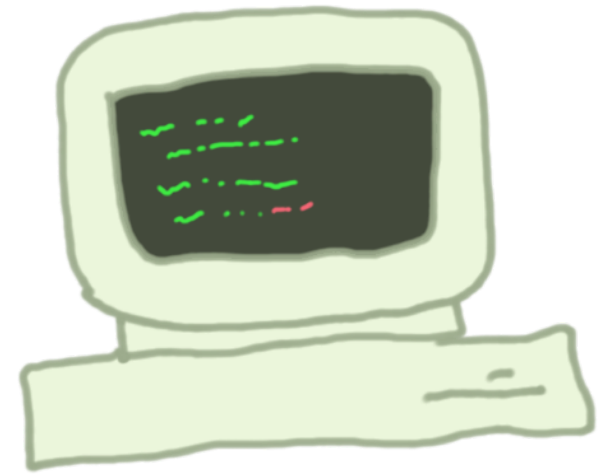
$$(\alpha, \beta, \ldots)$$

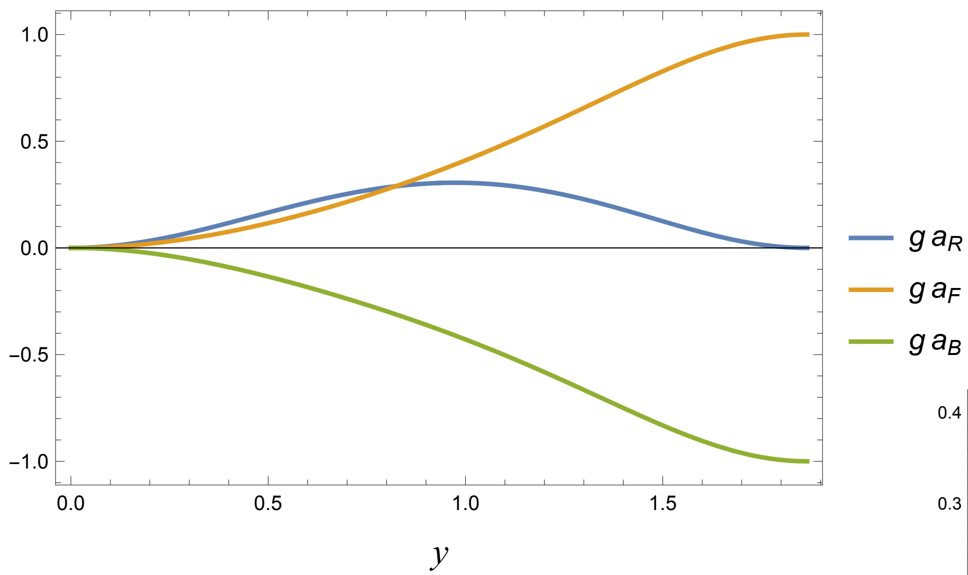


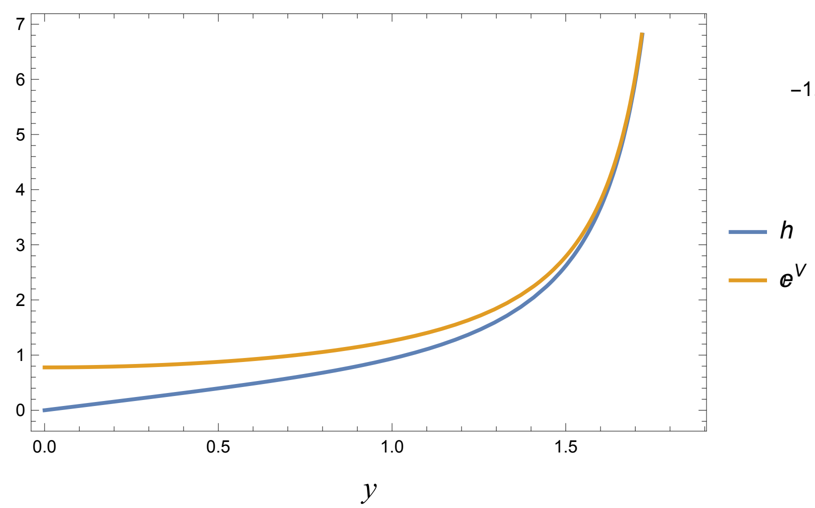
This Weyl frame is well positioned for a holographic description

Monodromy defects from SUGRA

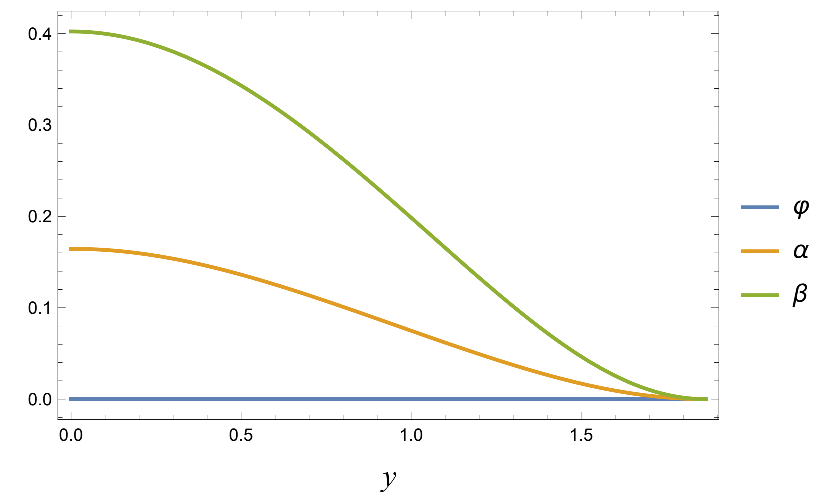
$$\delta\Psi_\mu^i = \delta\chi^{ijk} = 0$$







“Messy”



Monodromy defects from SUGRA

$$\delta\Psi^i_\mu = \delta\chi^{ijk} = 0$$

$$\partial_y \mathcal{E}^I = 0$$

$$\mathcal{E}^I \propto \langle J^I_z \rangle$$

$$\langle J^I_z \rangle = C^I(\alpha^J)$$

$$\langle T_{mn} \rangle \mathrm{d}x^m \otimes \mathrm{d}x^n = -\frac{h_D}{2\pi}\left(\mathrm{d}s^2(\mathrm{AdS}_3) - 3\mathrm{d}z^2\right)$$

$$h_D = h_D\left(\alpha^I\right)$$

“Powerful”

There exists an attractor-like mechanism at play…

Monodromy defects from SUGRA

$$\delta\Psi_\mu^i = \delta\chi^{ijk} = 0$$

$$\partial_y \mathcal{E}^I = 0$$

$$\mathcal{E}^I \propto \langle J_z^I \rangle$$

$$S_{\text{CFT}}^{\text{os}}\left(\alpha^I\right)$$

⋮

$$b\left(\alpha^I\right), d_1\left(\alpha^I\right), d_2\left(\alpha^I\right)$$

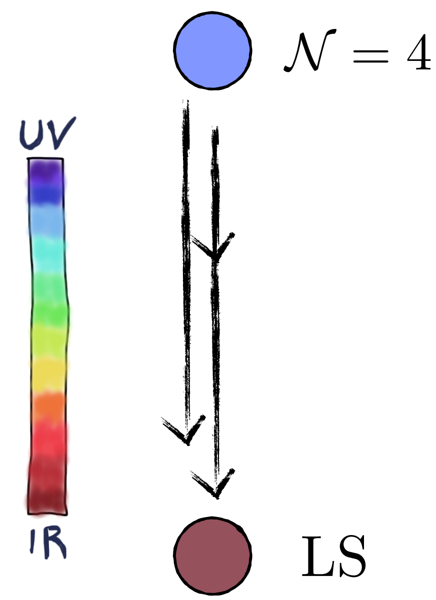
“Powerful”

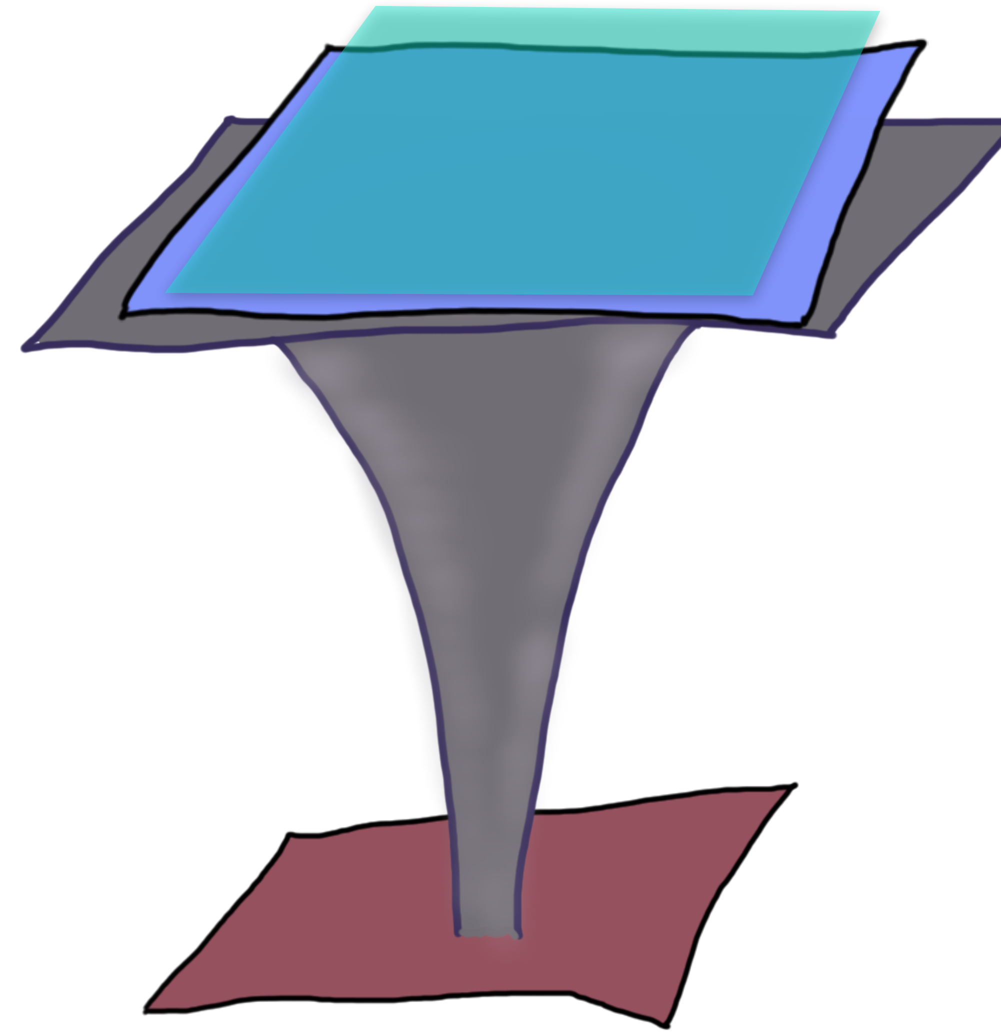
There exists an attractor-like mechanism at play…

Monodromy defects and RG

$$\Delta\mathcal{L} \sim \varphi\Big(\partial m + m^2\Big)\bar{\varphi} \, - \, m\,\bar{\psi}\psi$$

$$\partial m = 0$$



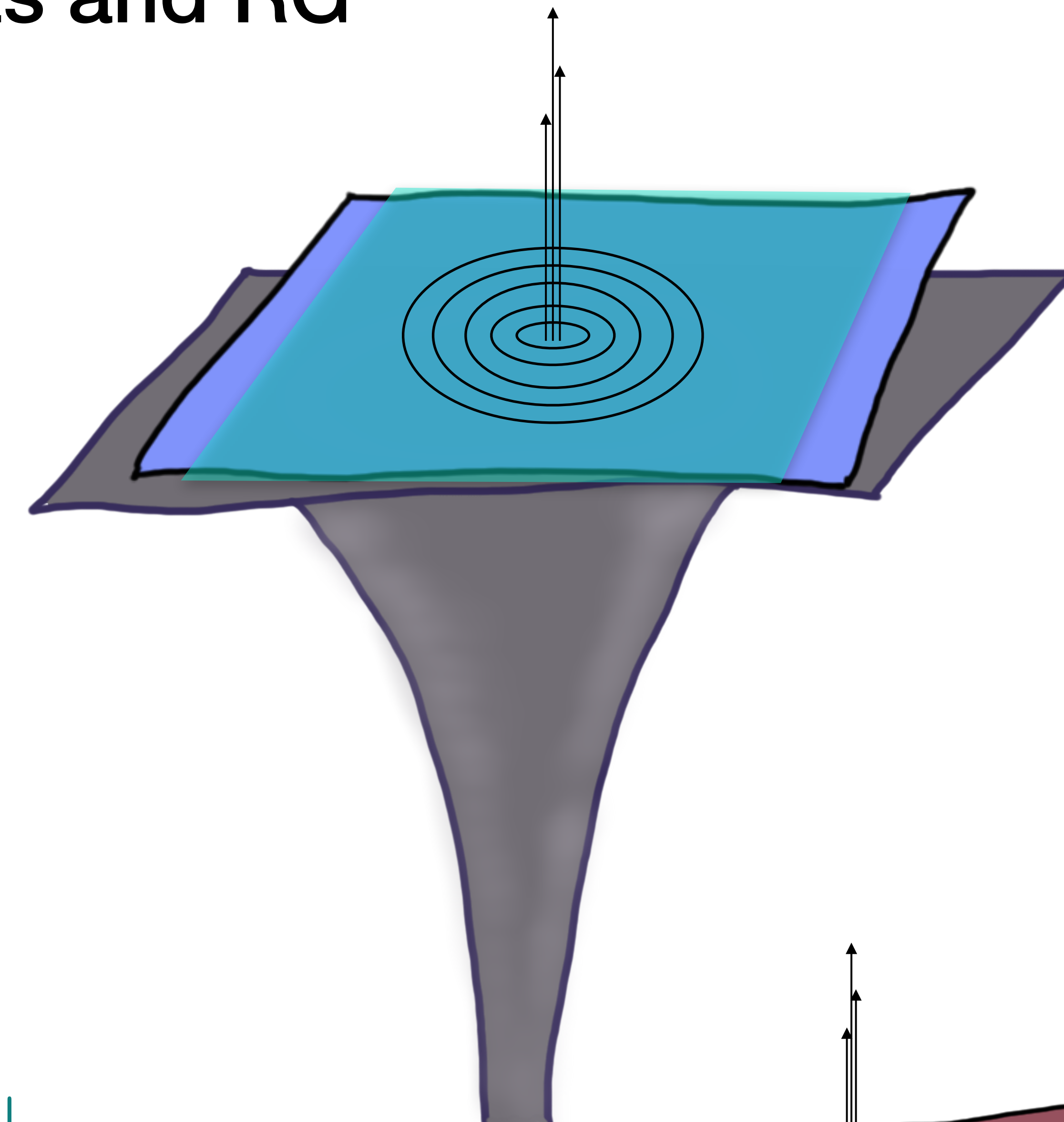


$$\langle T^{\mu}{}_{\mu}\rangle \sim aE_4 + \ldots$$

then

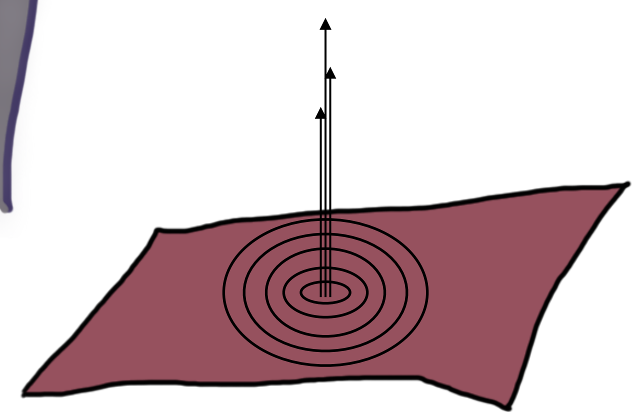
$$a_{\rm UV} > a_{\rm IR}$$

Monodromy defects and RG



$$\left.\frac{S^{\text{os}}_{\text{LS}}\left(\alpha^I\right)}{S^{\text{os}}_{\mathcal{N}=4}\left(\alpha^I\right)}\right|_{\alpha} < 1$$

??

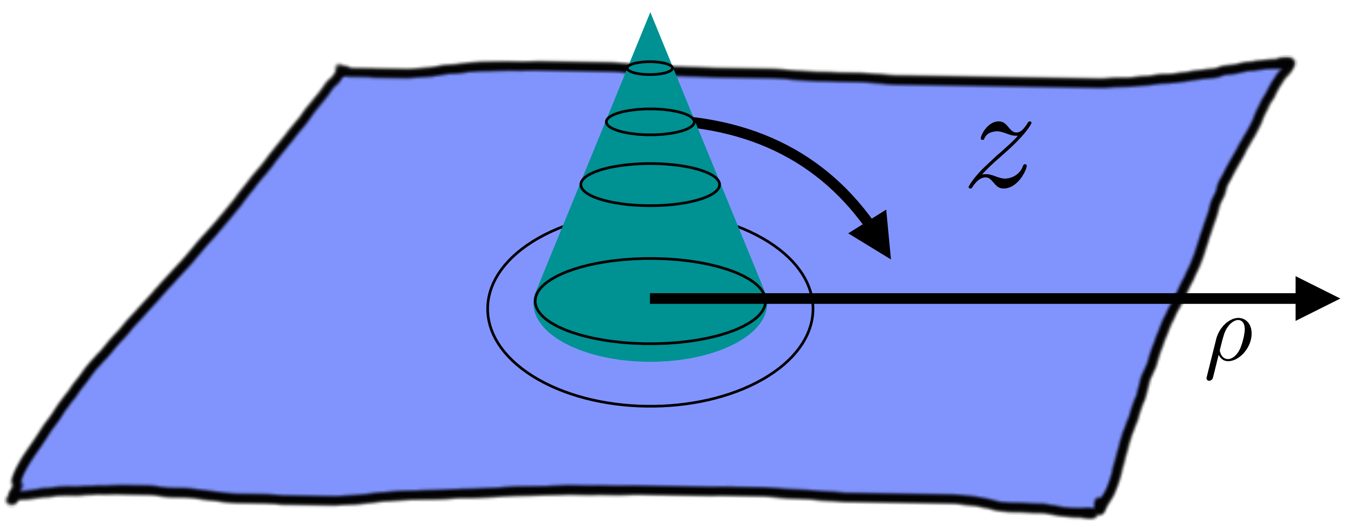


then

$$(12a_{\text{UV}} - b_{\text{UV}}) > (12a_{\text{IR}} - b_{\text{IR}})$$

Extensions

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 + \mathrm{d}\rho^2 + n^2\rho^2\mathrm{d}z^2$$



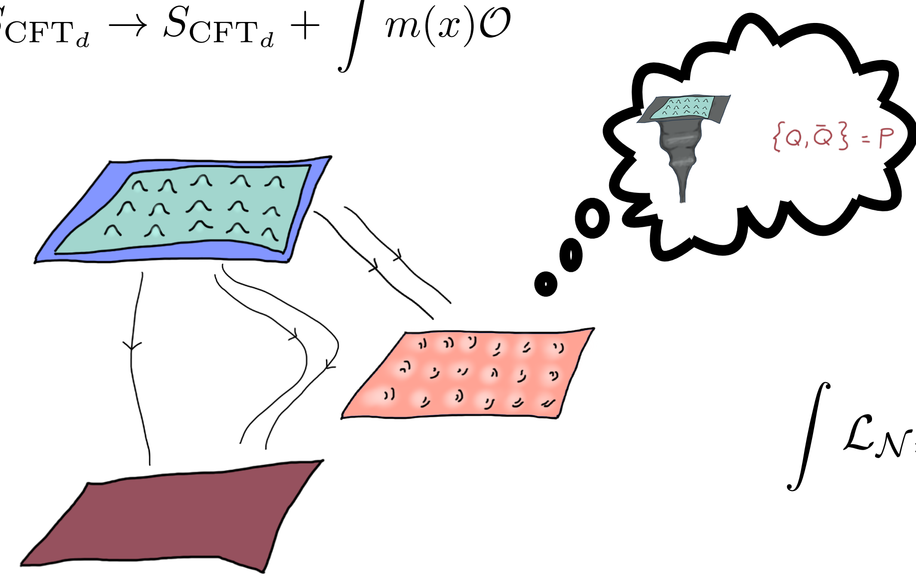
$$h_D = h_D\left(\alpha^I, n\right), \quad S^{\text{os}}_{\text{CFT}}\left(\alpha^I, n\right), \quad b\left(\alpha^I, n\right), \ \ldots$$

Allowing for conical singularities refines results and allows for new extensions (e.g. charged and SUSY Renyi entropies).

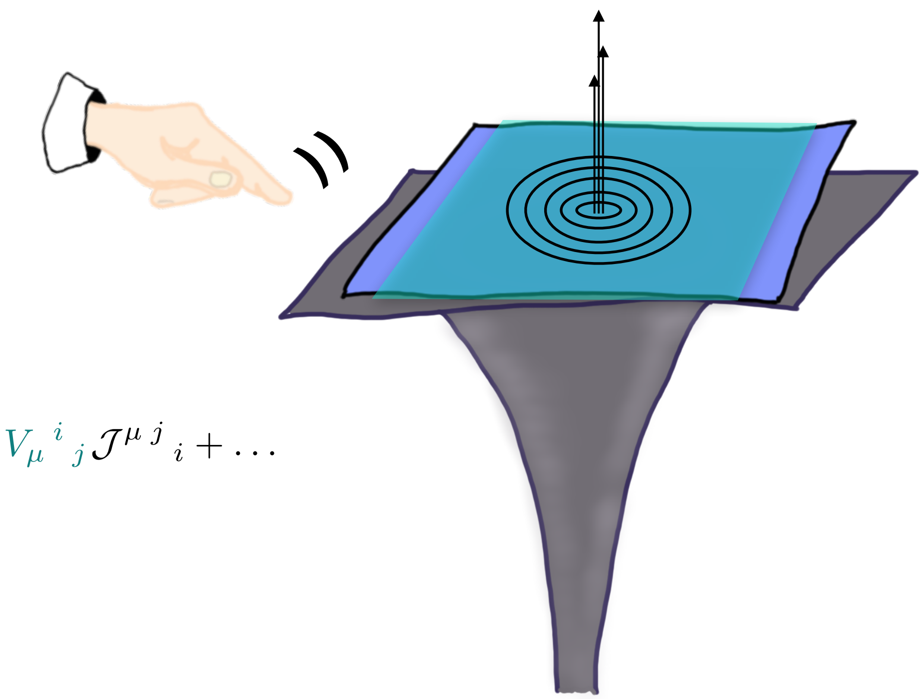
This approach generalises immediately to other theories in other dimensions, where even less is presently known (RG monotones, etc.)

Punchlines

$$S_{\mathrm{CFT}_d} \to S_{\mathrm{CFT}_d} + \int m(x)\mathcal{O}$$

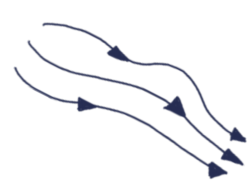


$$\int \mathcal{L}_{\mathcal{N}=4} + \varphi_{ij}\left(D^{ij}{}_{kl} + E^2\right)\varphi^{kl} - \frac{1}{2}\bar{\psi}^i E_{ij}\psi^j + V_\mu{}^i{}_j \mathcal{J}^{\mu j}{}_i + \dots$$

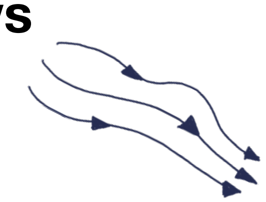


These approaches can be used in concert to inform our understanding of:

Defect/Boundary CFTs



RG flows



QFT

Lots to do, lots to learn…

Thank you!