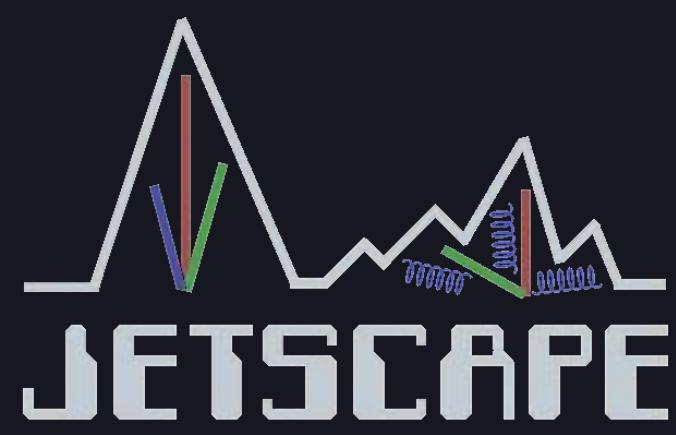




公立大学法人
国際教養大学
Akita International University



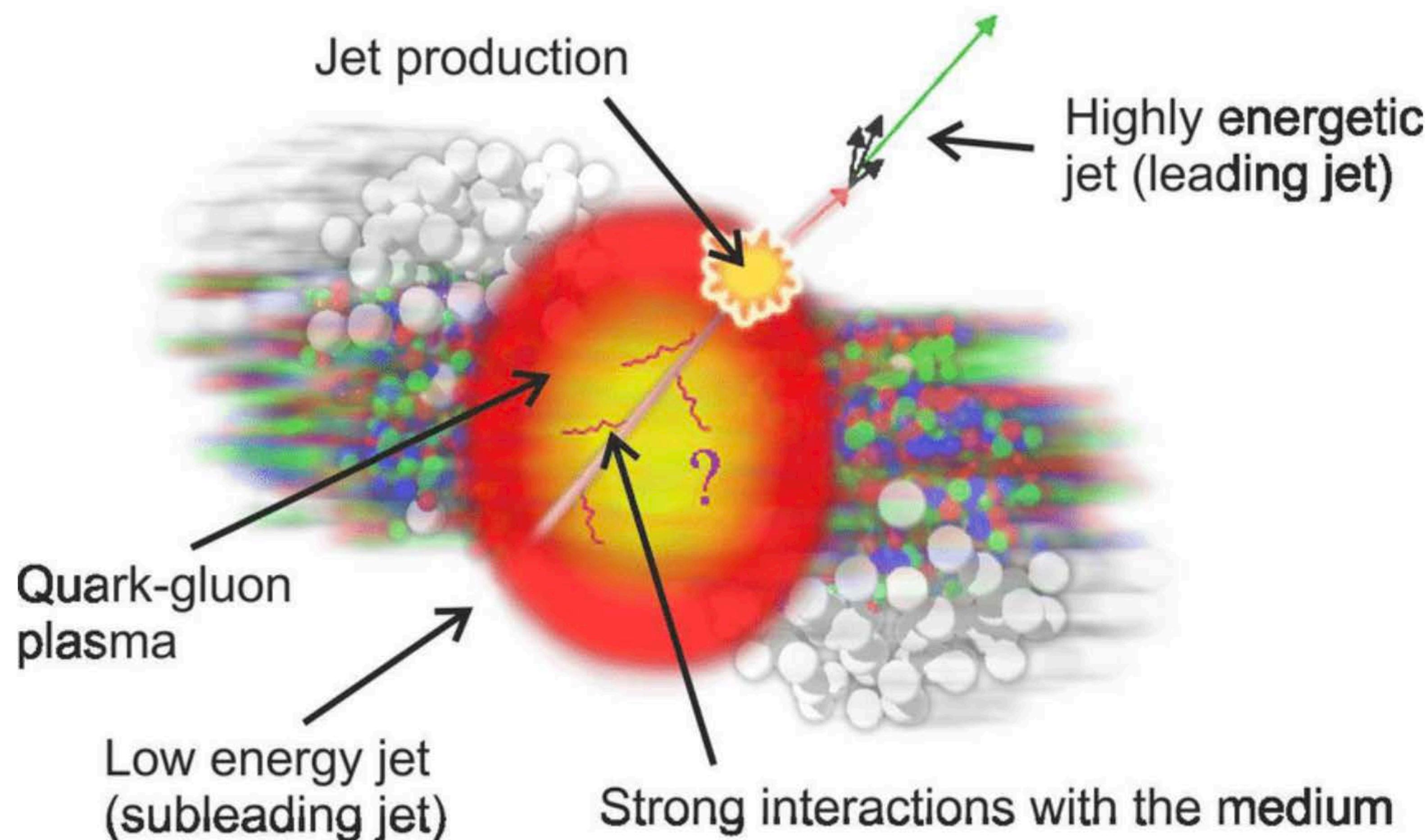
ジェット (概観・理論)

橘 保貴

チュートリアル研究会, 大阪大学, 2024年 8月 6日

モチベーション

重イオン衝突におけるジェット



重イオン衝突におけるジェット

- ジェットを用いて QGP を見る
(ジェットの持つ分解能)
- QGP にジェットを見せる
(QGP の持つ分解能)

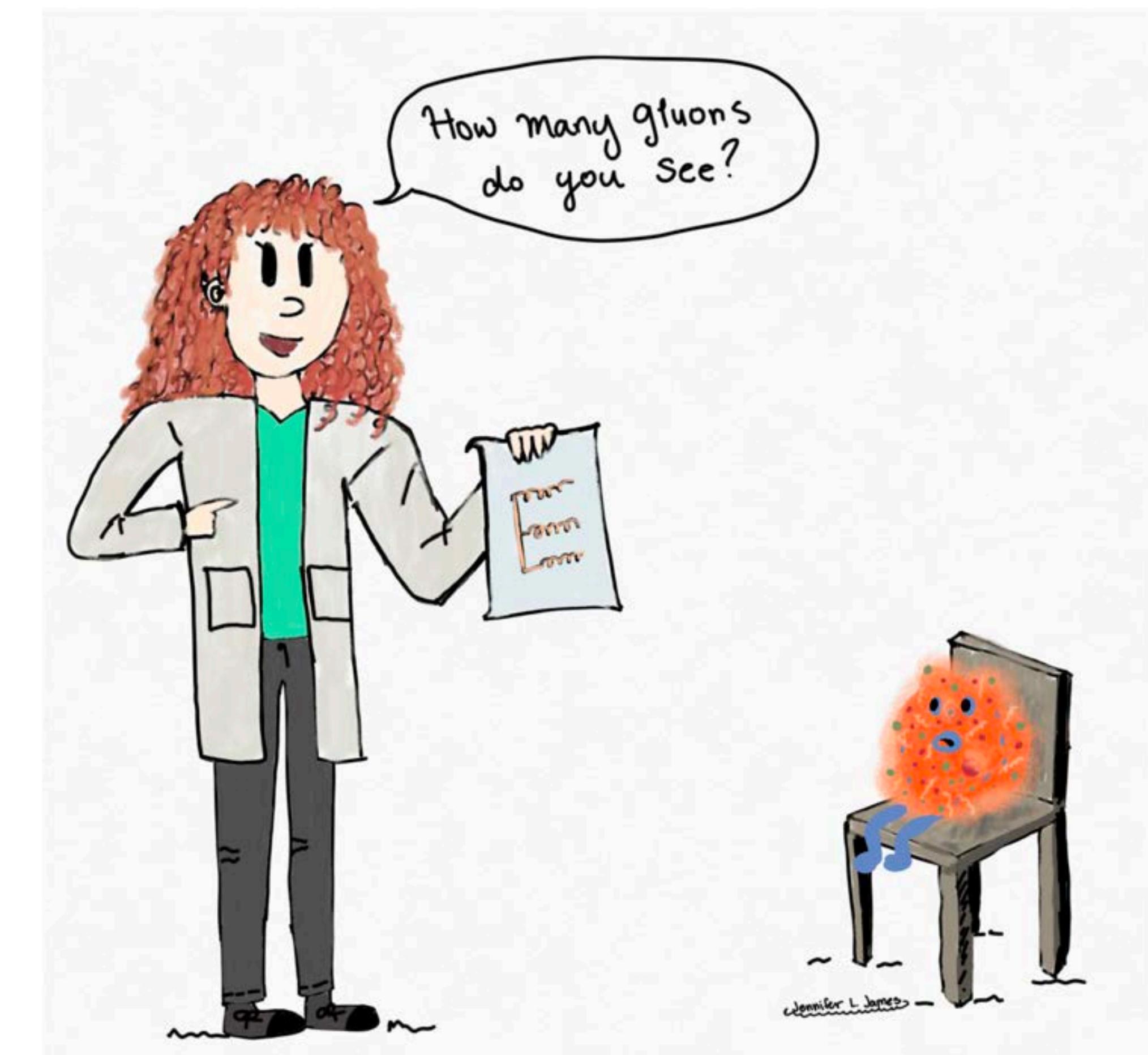
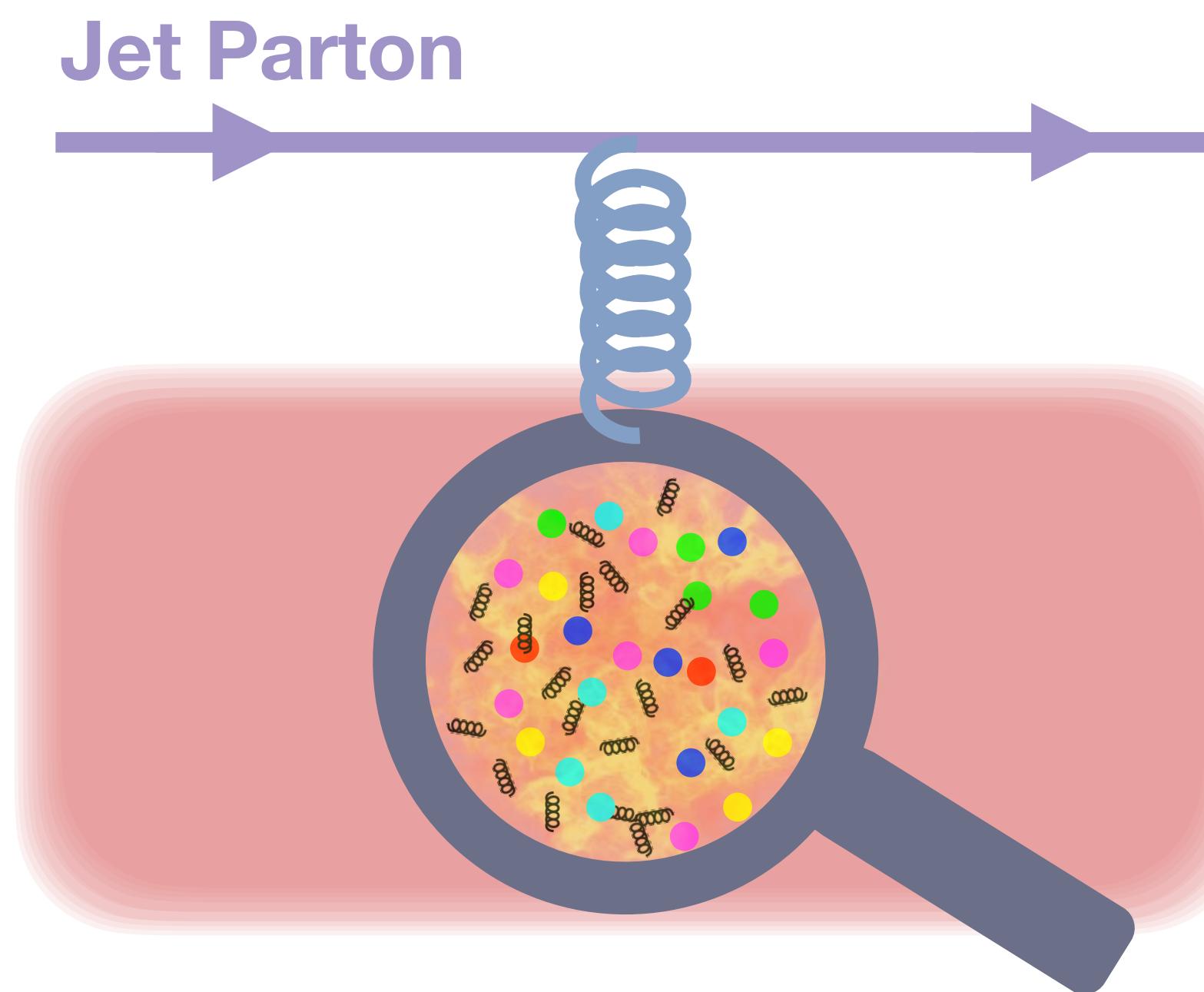
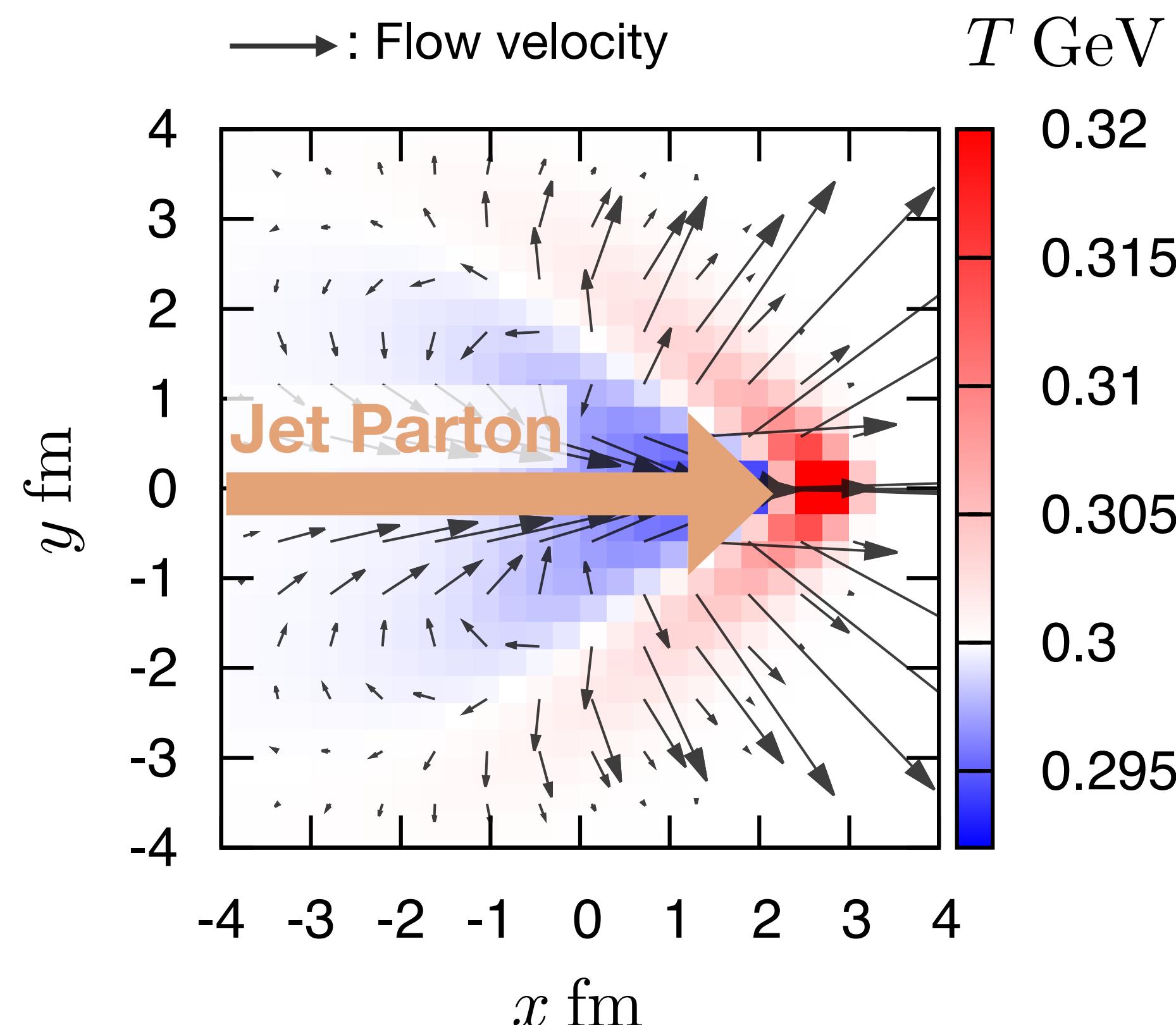


Image credit: Jennifer James (Vanderbilt)

重イオン衝突におけるジェット

- **Collective reaction of QGP**

- In-medium thermalization process
- Bulk property of QGP medium

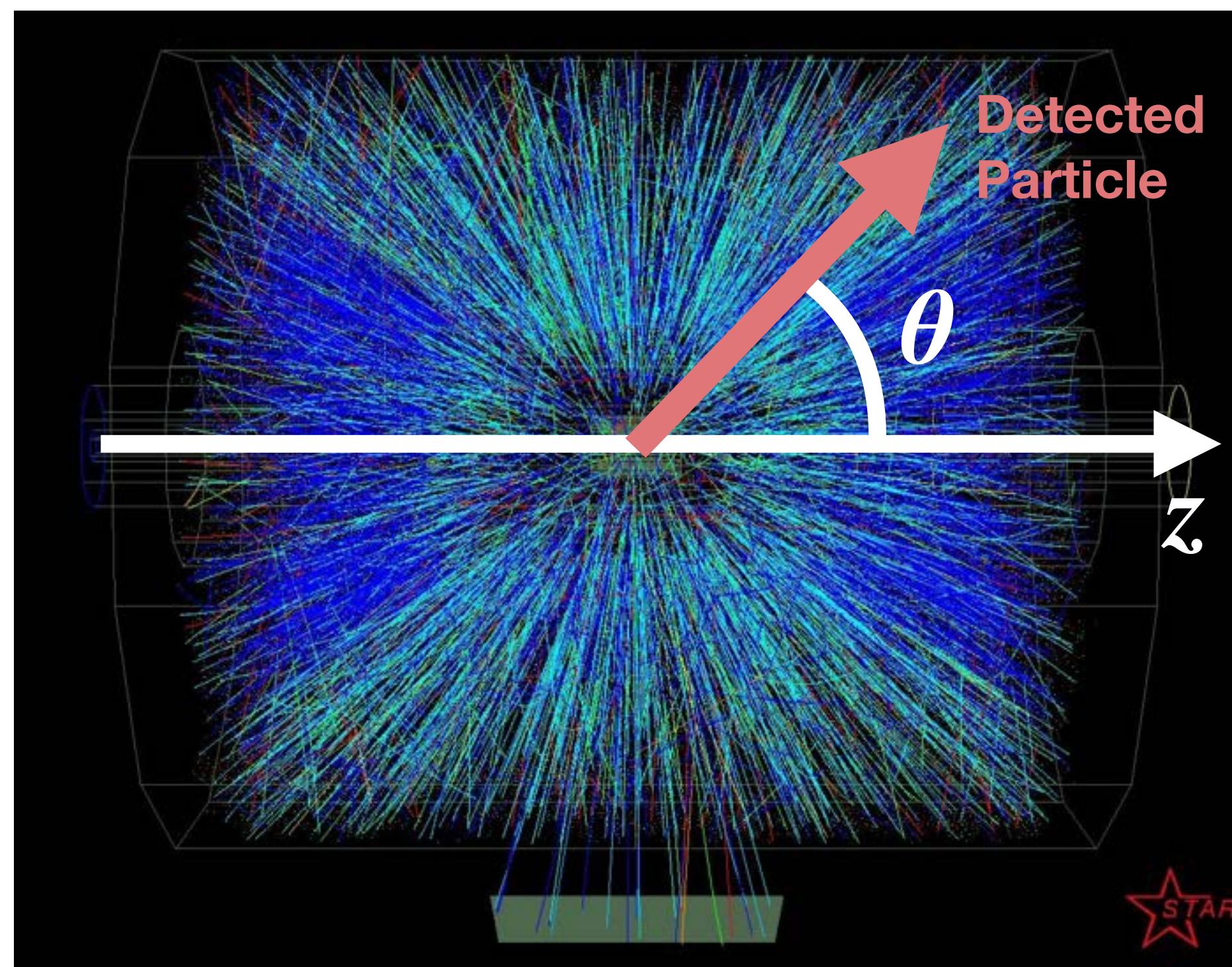


準備

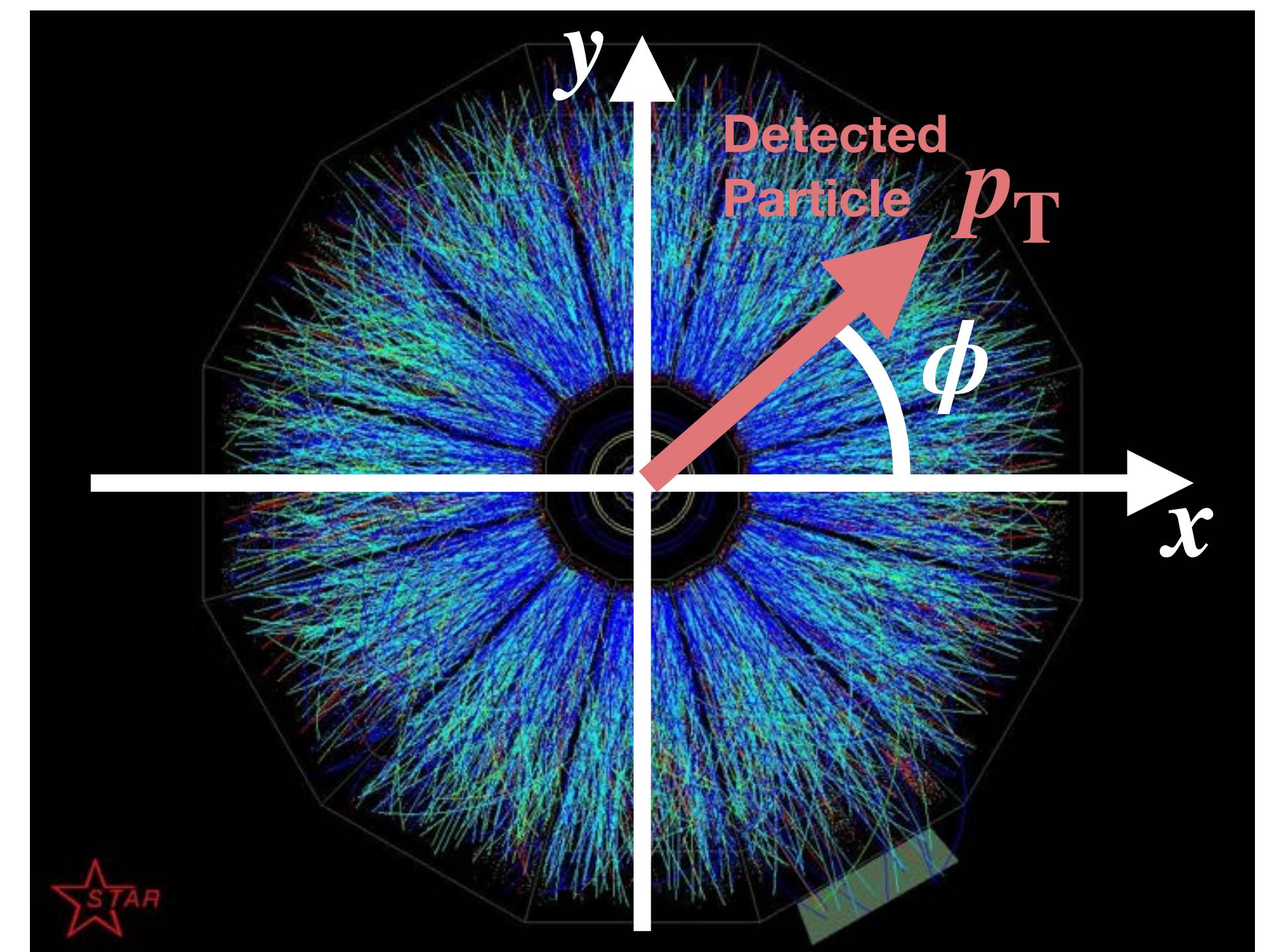
● 衝突実験における座標系

- 衝突軸を z -軸 にとる
- Transverse Momentum: $p_T = \sqrt{p_x^2 + p_y^2} = |\vec{p}| \sin \theta$, Pseudorapidity: $\eta = -\log \left(\tan \frac{\theta}{2} \right)$

Reaction Plane



Transverse Plane



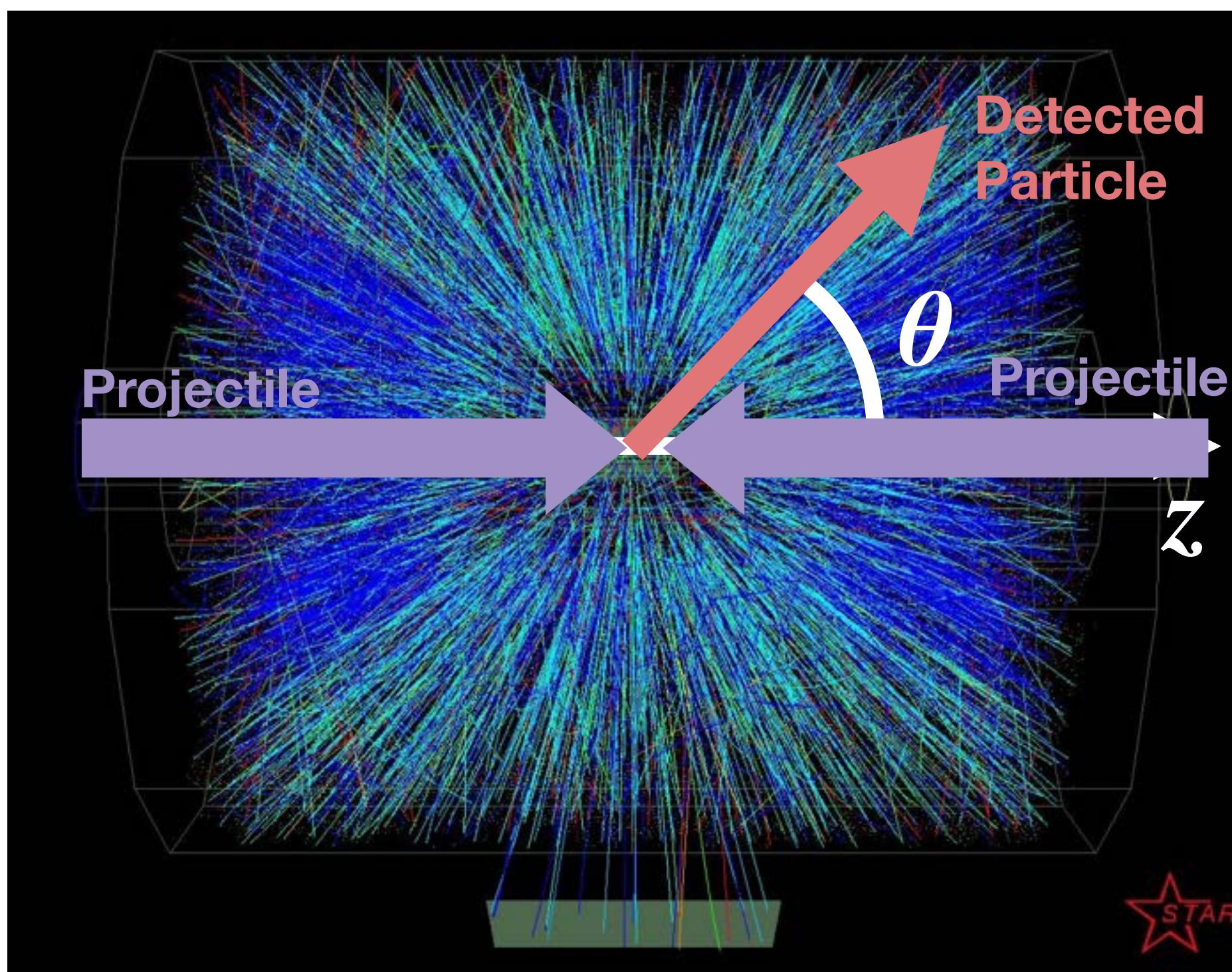
Adapted from <http://www.star.bnl.gov>

準備

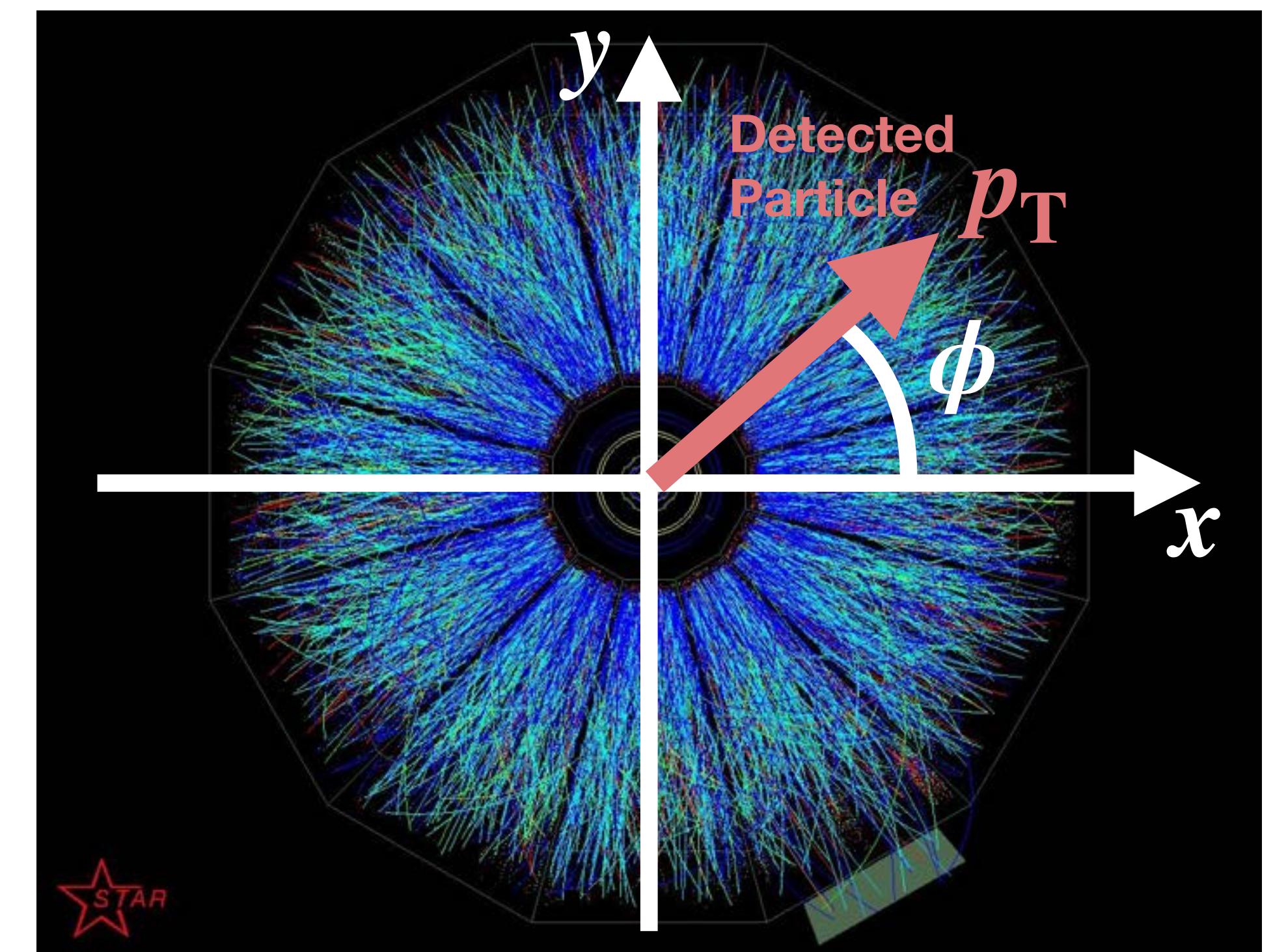
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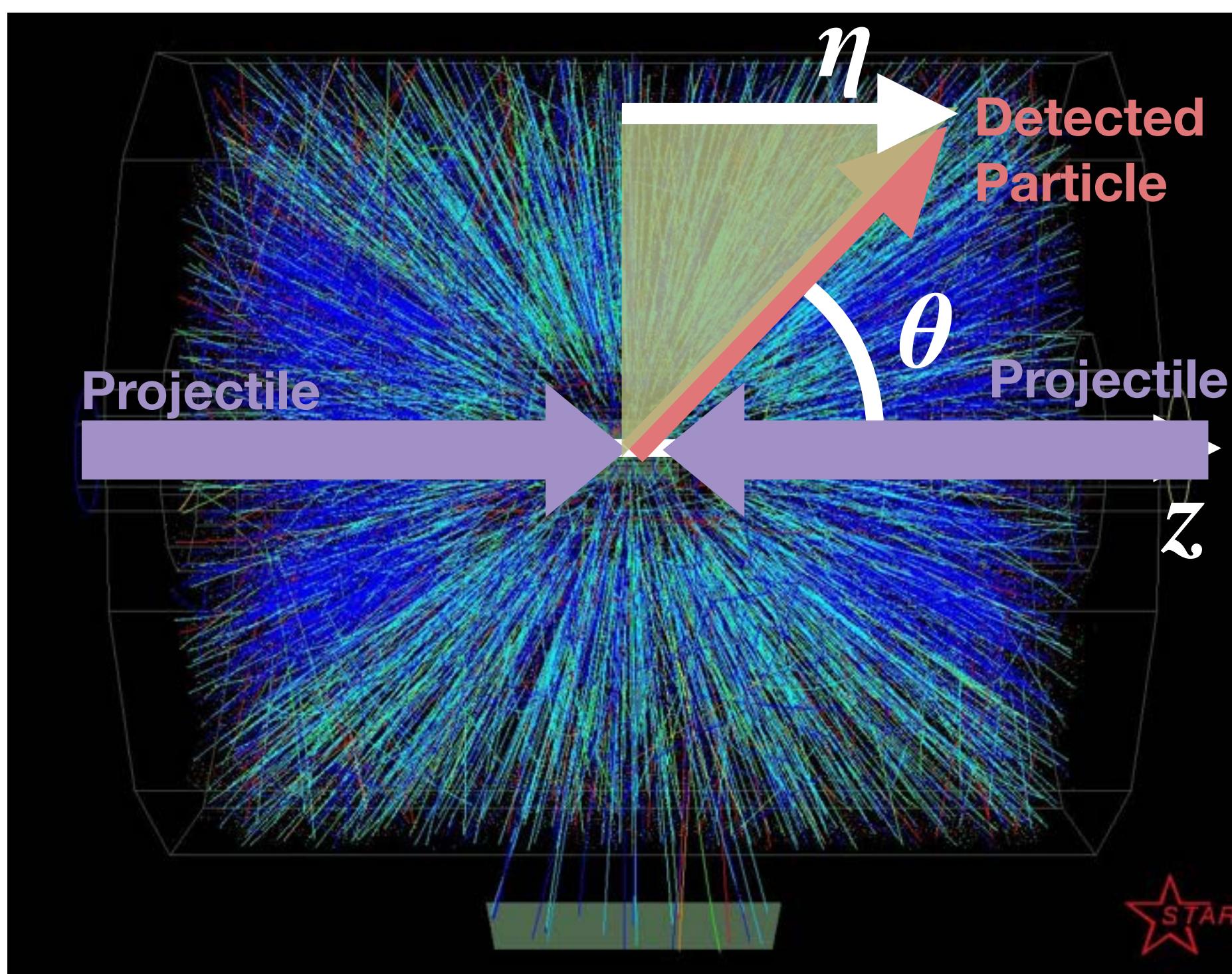
Adapted from <http://www.star.bnl.gov>

準備

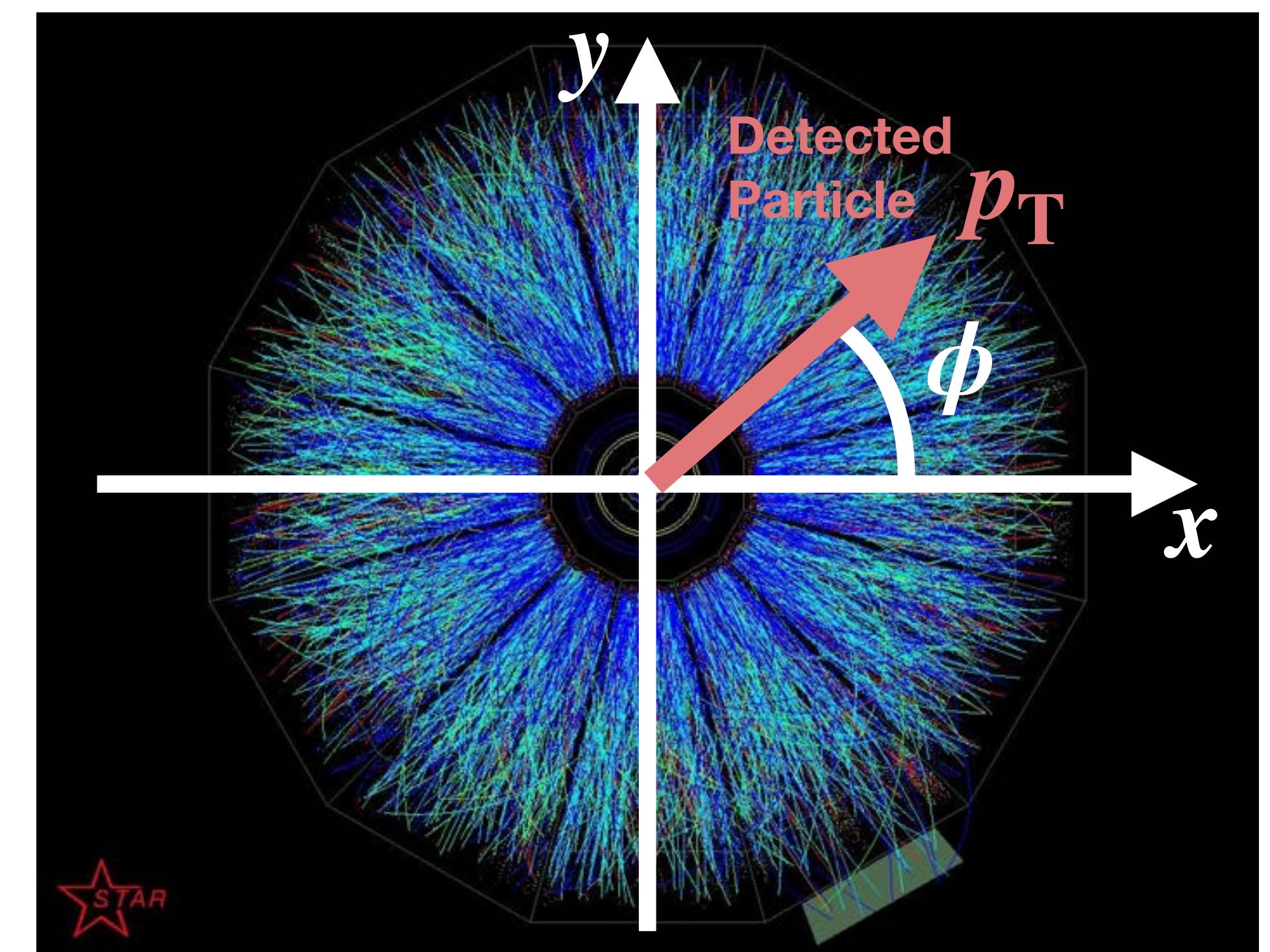
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Reaction Plane



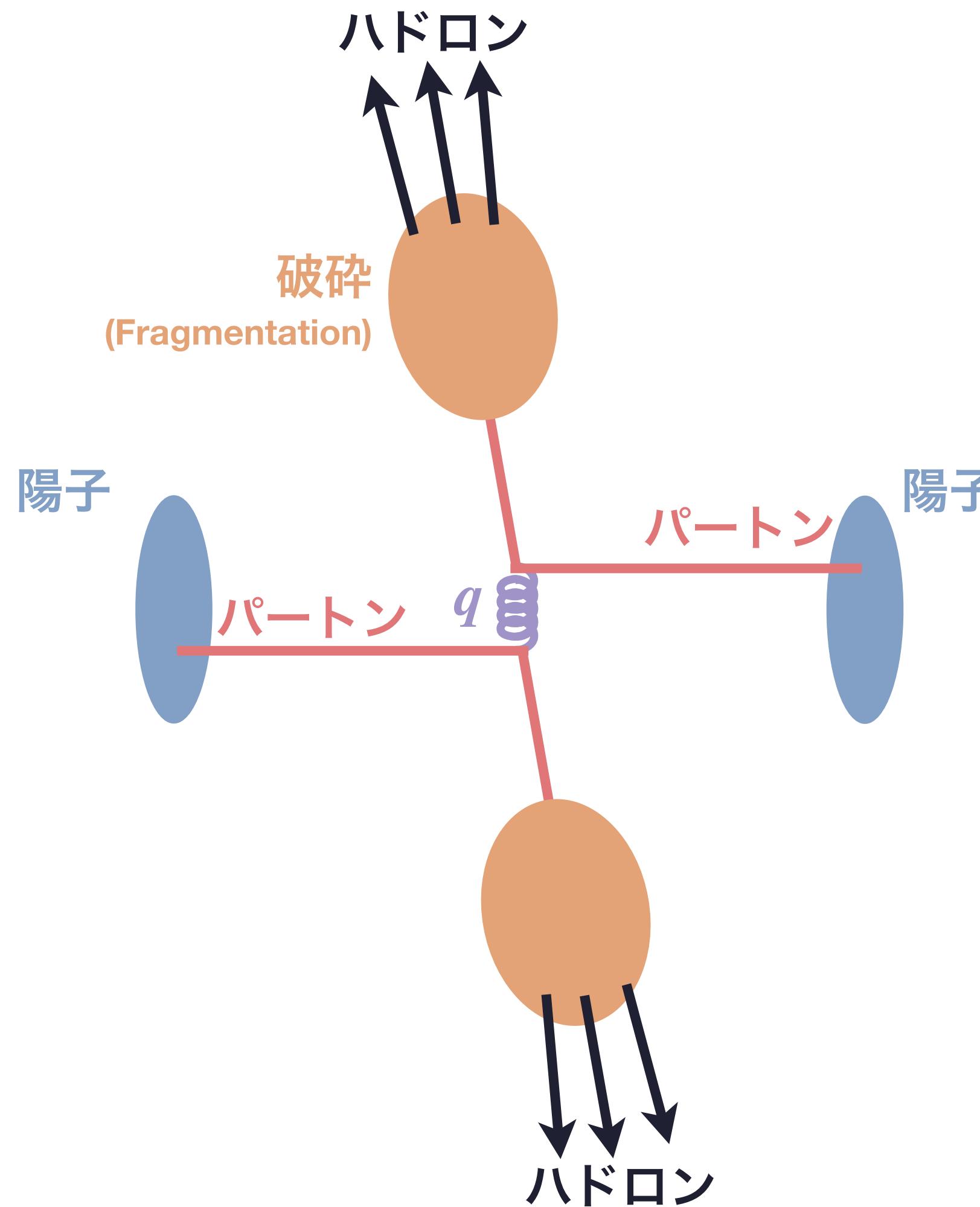
Transverse Plane



ジェットとは

ジェット (事象)

● ジェット事象 (例: 陽子-陽子衝突)

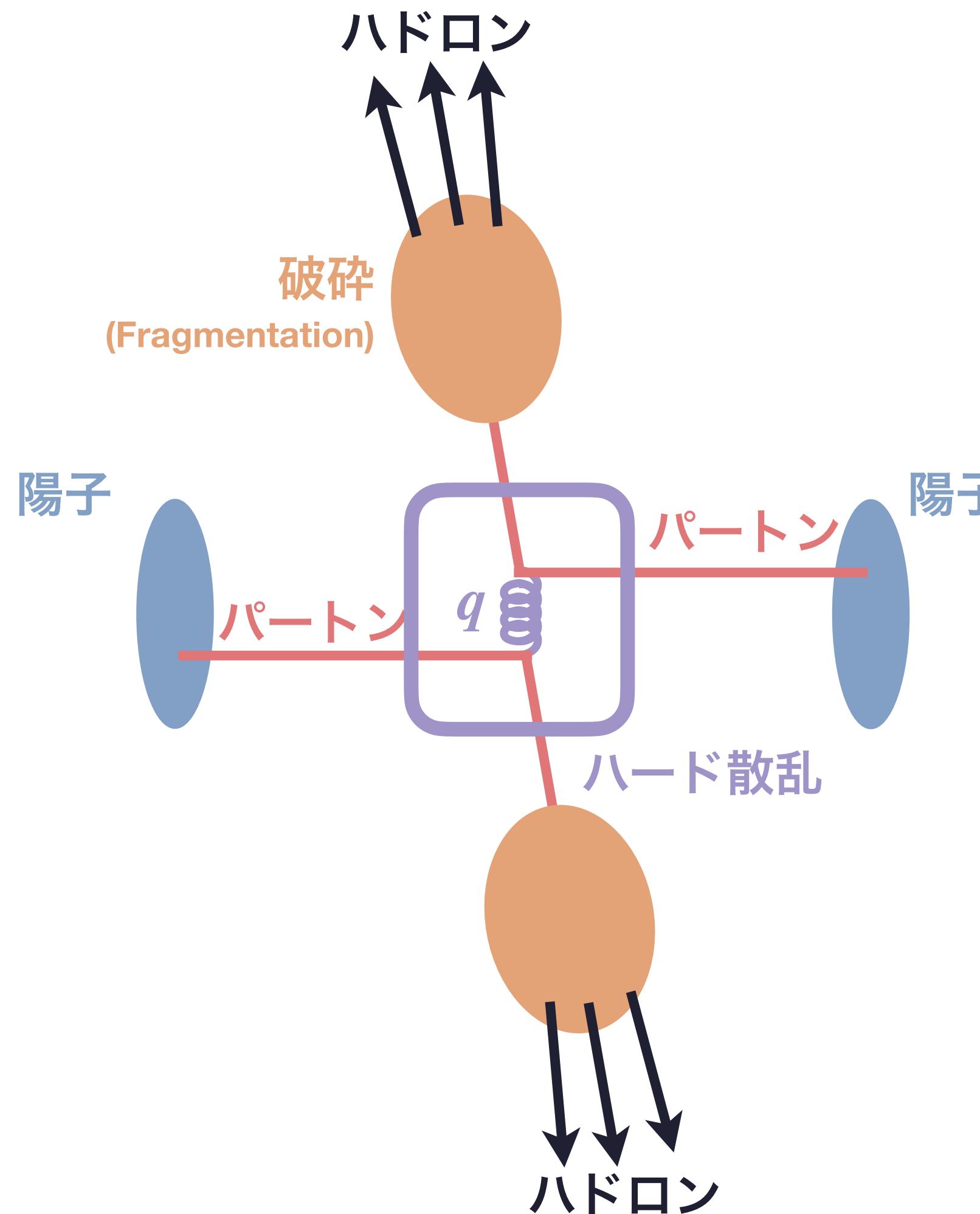


- 終状態でエネルギーが小さな角度領域に集中する事象
- パートンのハード (大きな運動量移行 $-q^2$) 散乱に由来
- コリメートした高- p_T ハドロン粒子の束として観測される*

*ここでハドロンは、光子やレプトンなどパートン以外の粒子全般を指すこともある。

ジェット (事象)

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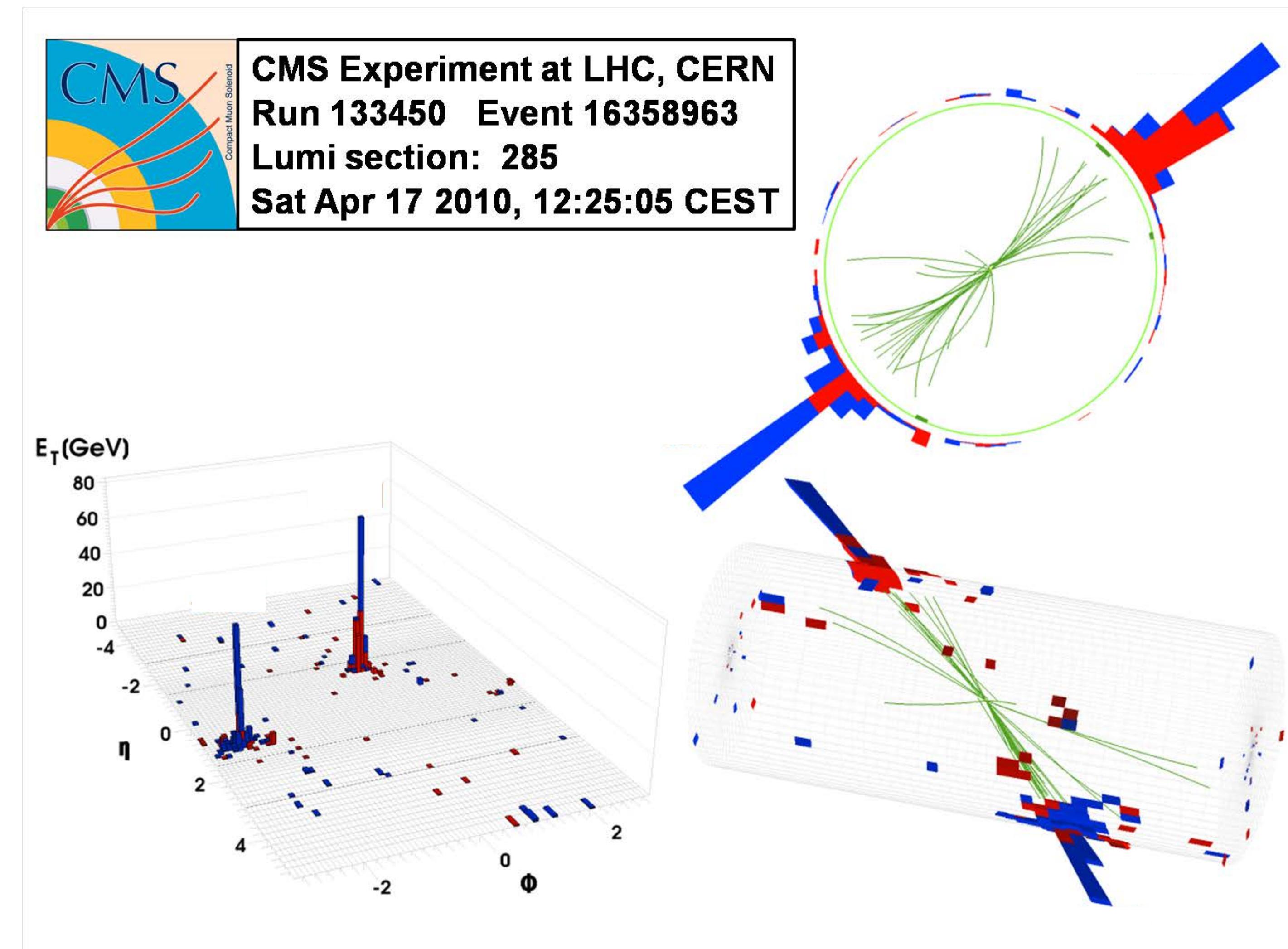


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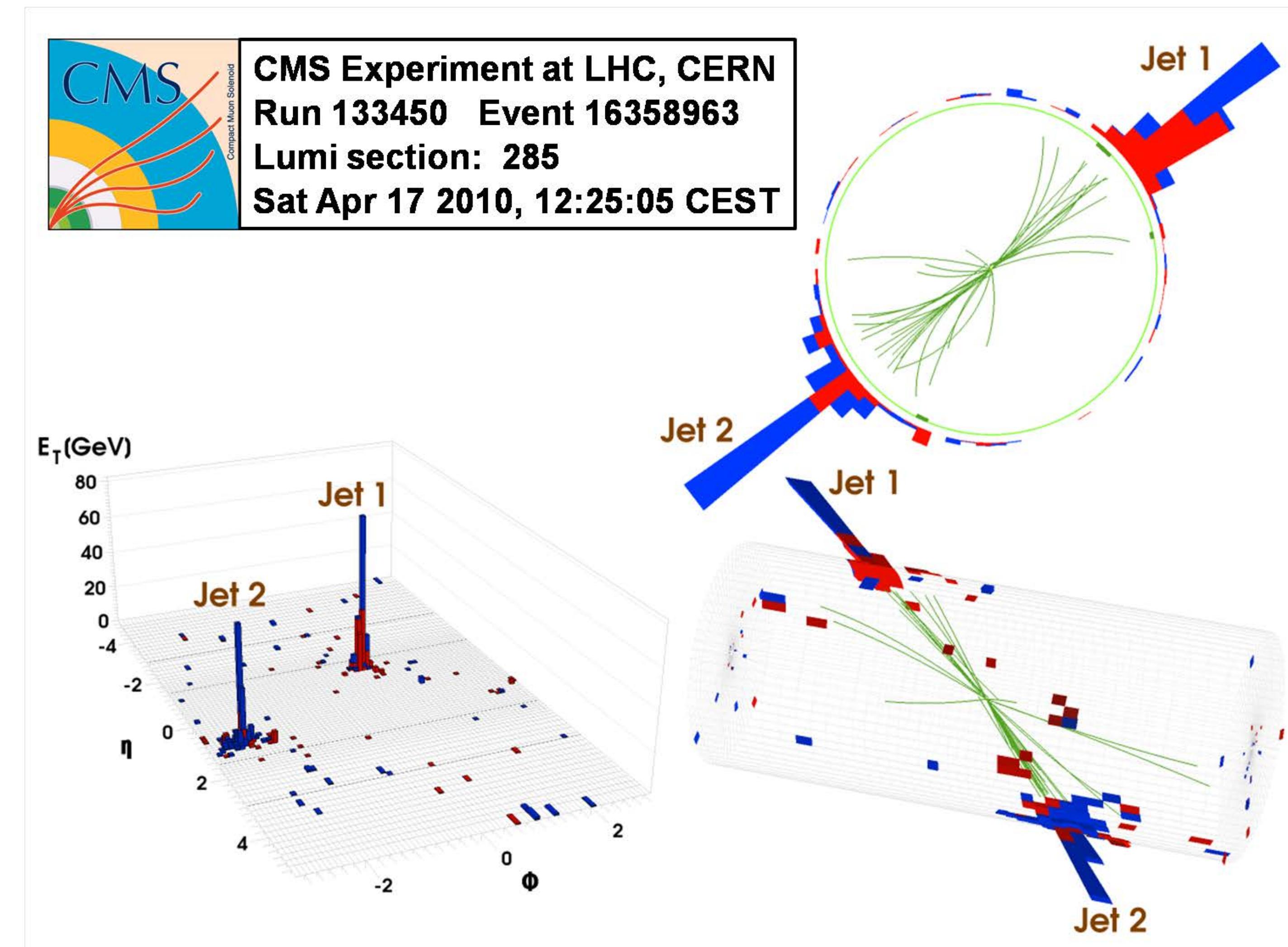
ジェット(事象)

- 実験で観測されたジェット事象



ジェット (事象)

- 実験で観測されたジェット事象



ジェット (測定量)

● 再構成された (reconstructed) ジェット

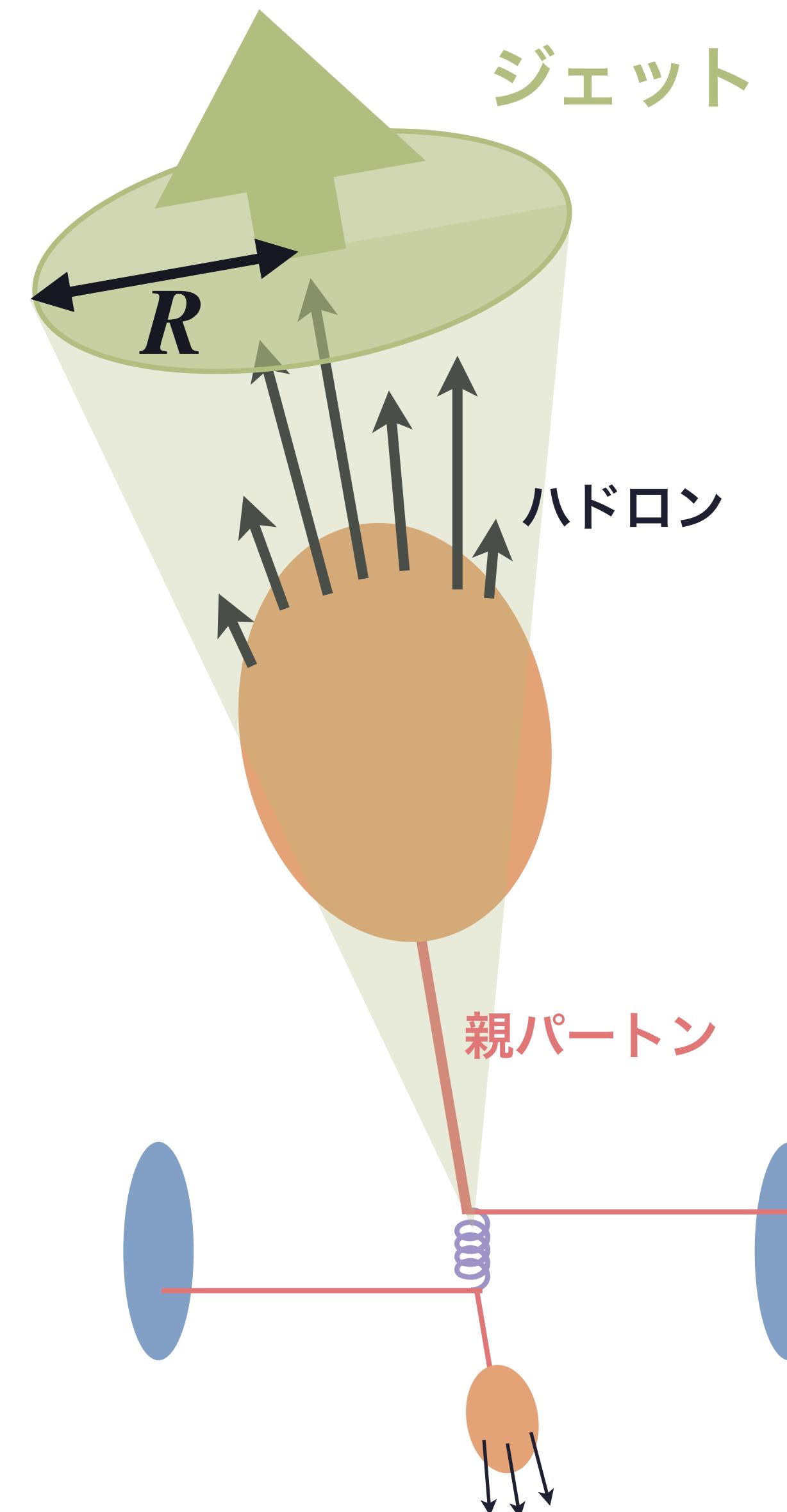
- 複数のハドロンから構成されるコリメートしたもの
- ジェット再構成アルゴリズムによってクラスタリング
- 角度サイズのパラメータ: ジェットコーンサイズ R

ジェットの運動量 (E -scheme)

$$p_{\text{jet}}^{\mu} = \sum_{i \in \text{clustered hadrons}} p_i^{\mu}$$

($\sim p_{\text{parent}}^{\mu}$ と期待)

- ジェットというと通常はこの再構成されたジェットを指す



ジェット (測定量)

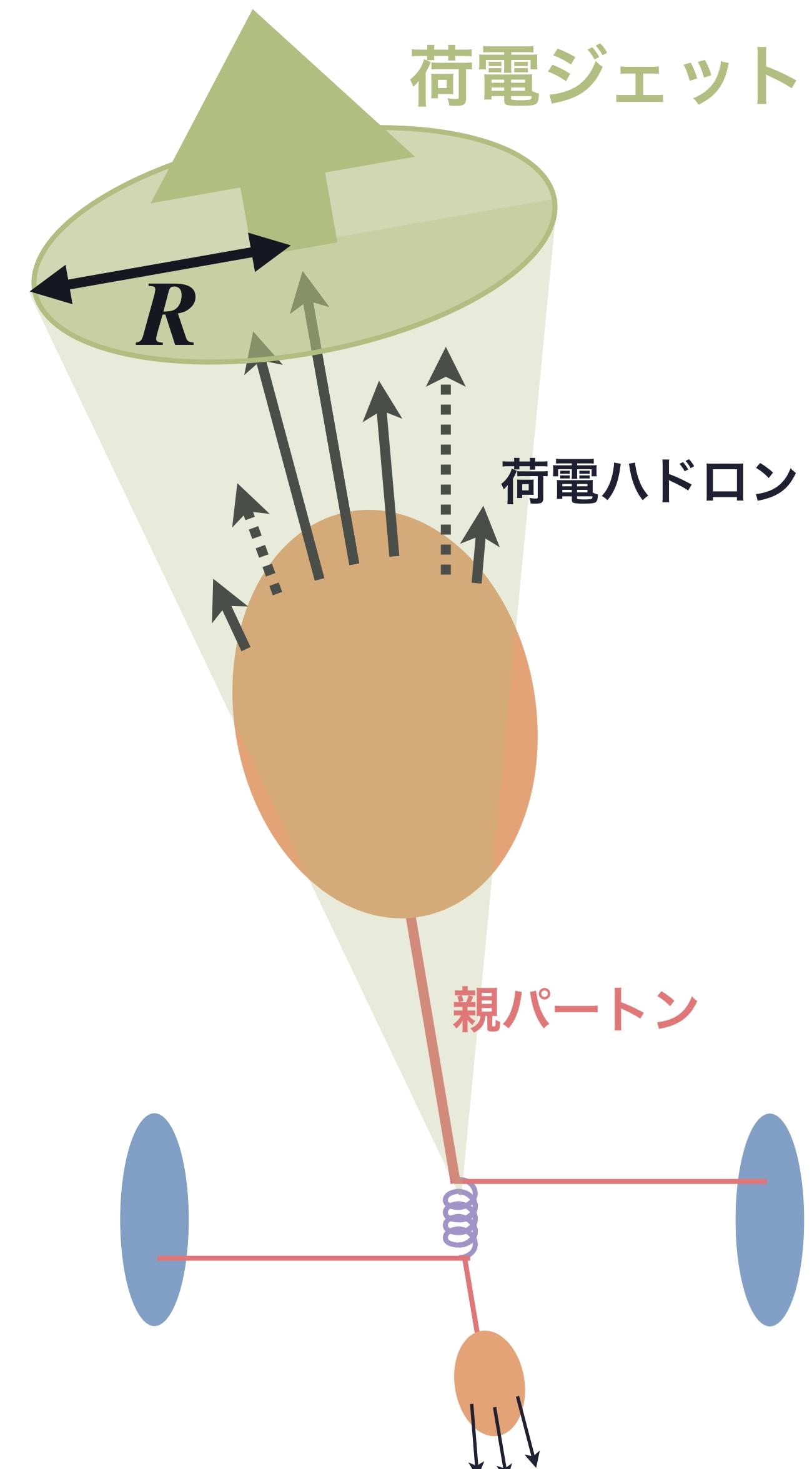
● 荷電 (charged) ジェット

- 荷電ハドロンのみをクラスタリング

ジェットの運動量 (*E*-scheme)

$$P_{\text{ch,jet}}^{\mu} = \sum_{i \in \text{clustered charged hadrons}} p_i^{\mu}, \quad (p_{\text{ch,jet}}^0 < p_{\text{parent}}^0)$$

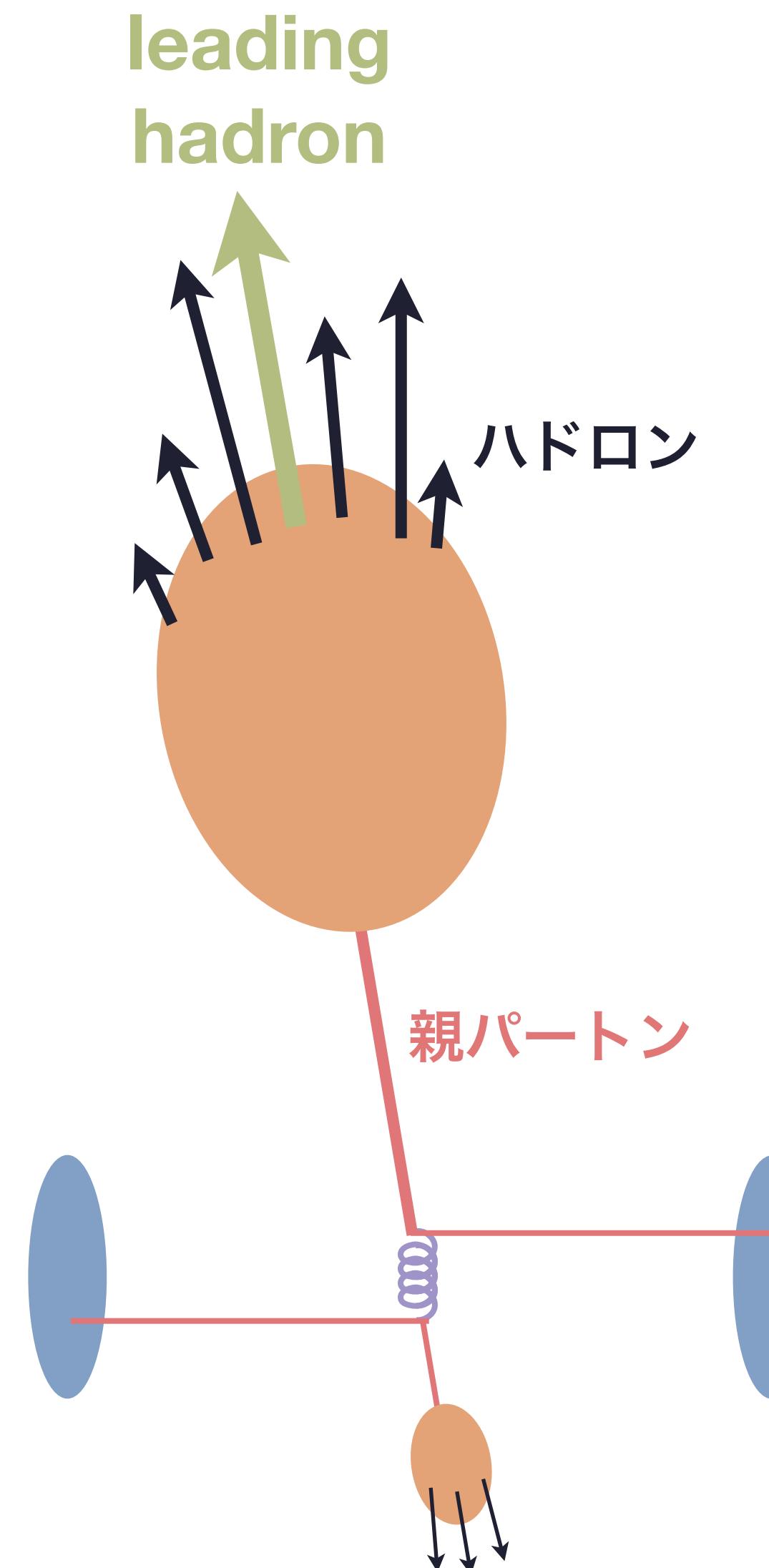
- 対比して中性粒子を含むジェットを full jet とも呼ぶ



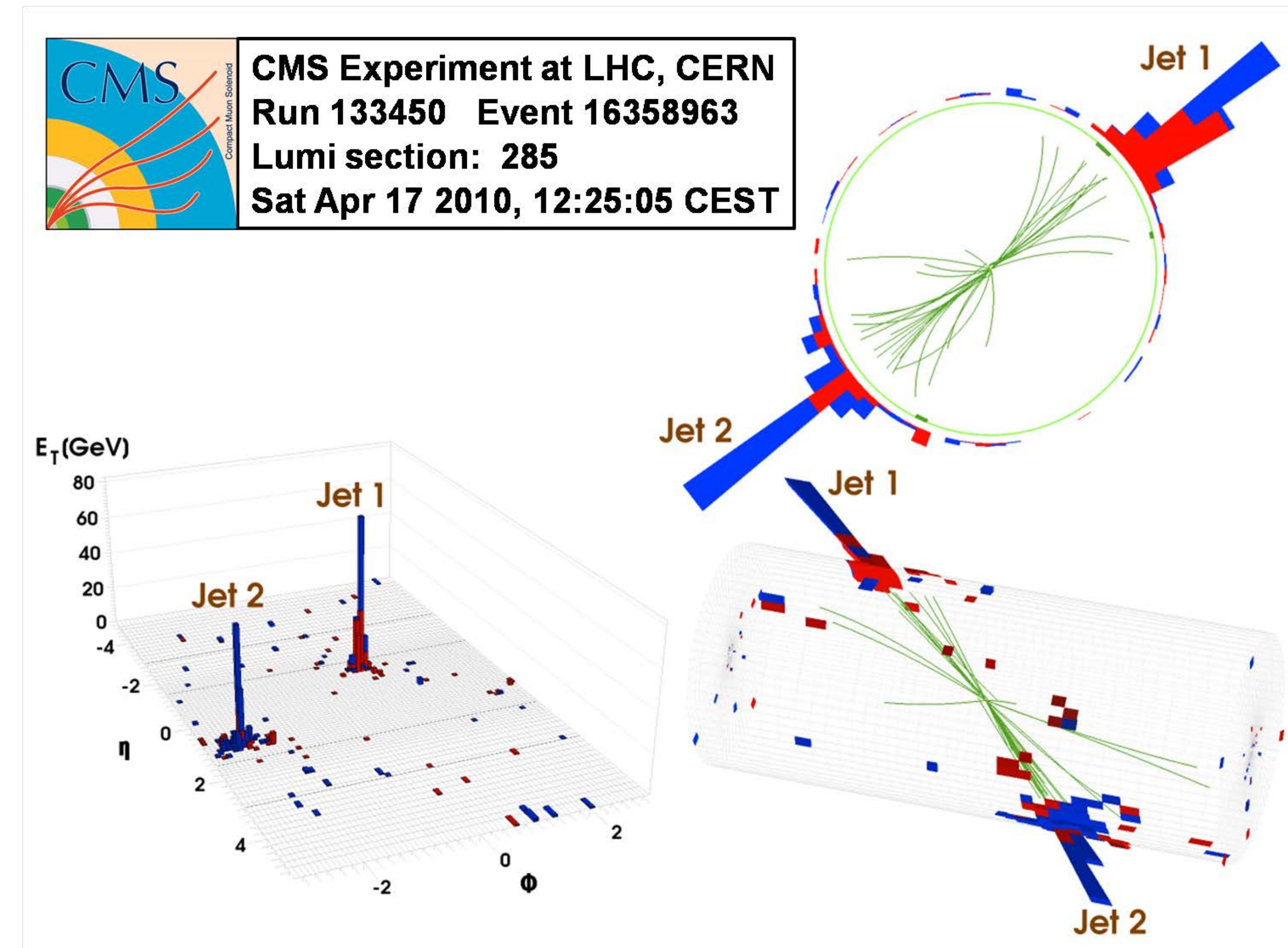
ジェット (測定量)

● 単体の $high-p_T$ 粒子

- 個々のハドロンをそれぞれ測定
- スペクトルは leading (highest- p_T) ハドロンが支配
- ジェットコーンサイズ $R \rightarrow 0$ の極限に対応
- 現在では通常, ハドロン単体をジェットとは呼ばない

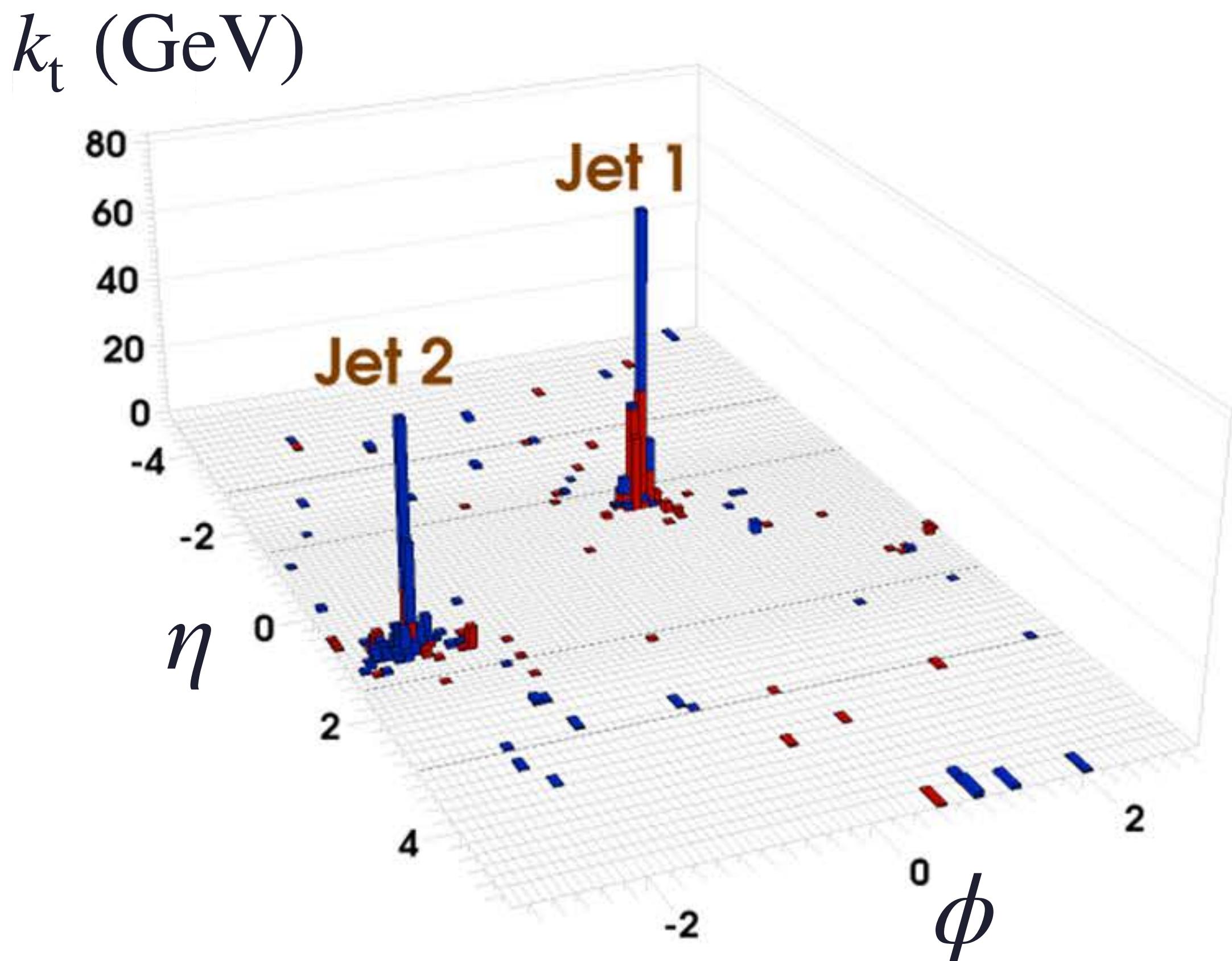


ジェット再構成



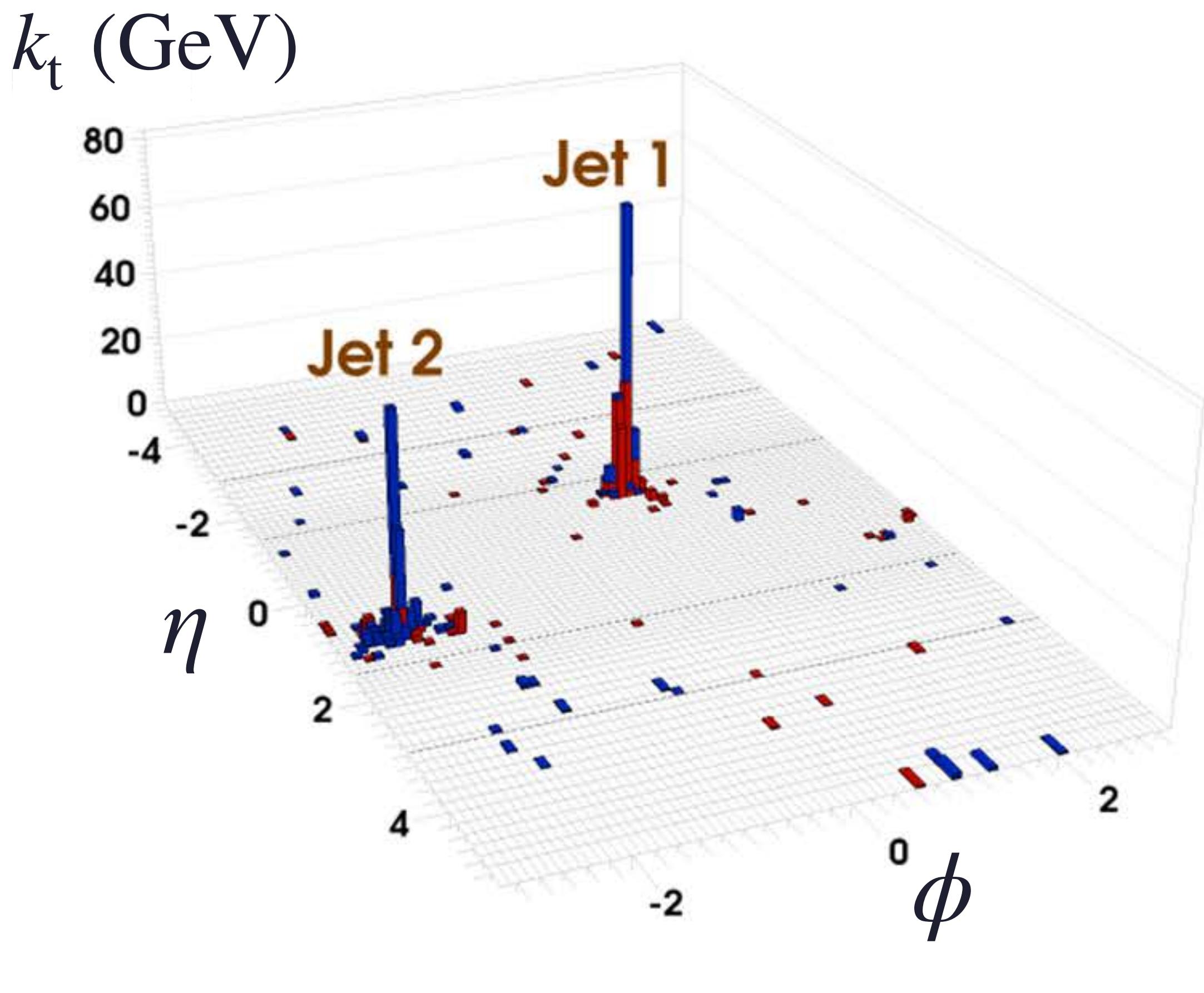
ジェット再構成

- ジェット再構成アルゴリズム



- Event-by-event の解析
- η - ϕ 平面で粒子やカロリーメータタワーを
クラスタリング

ジェット再構成



- ジェット再構成アルゴリズム
 - ジェットコーンサイズパラメータ*: R
*重イオンでは大体 0.2-0.4
 - 以下の d_{ij} が最小となるペア i, j を合成
$$d_{ij} = \Delta r_{ij} \cdot \min(k_{ti}^p, k_{tj}^p),$$
ただし $\Delta r_{ij} = \sqrt{(\phi_i - \phi_j)^2 + (\eta_i - \eta_j)^2} < R$
 - $\Delta r_{ij} < R$ を満たすペアがなくなるまで繰り返す

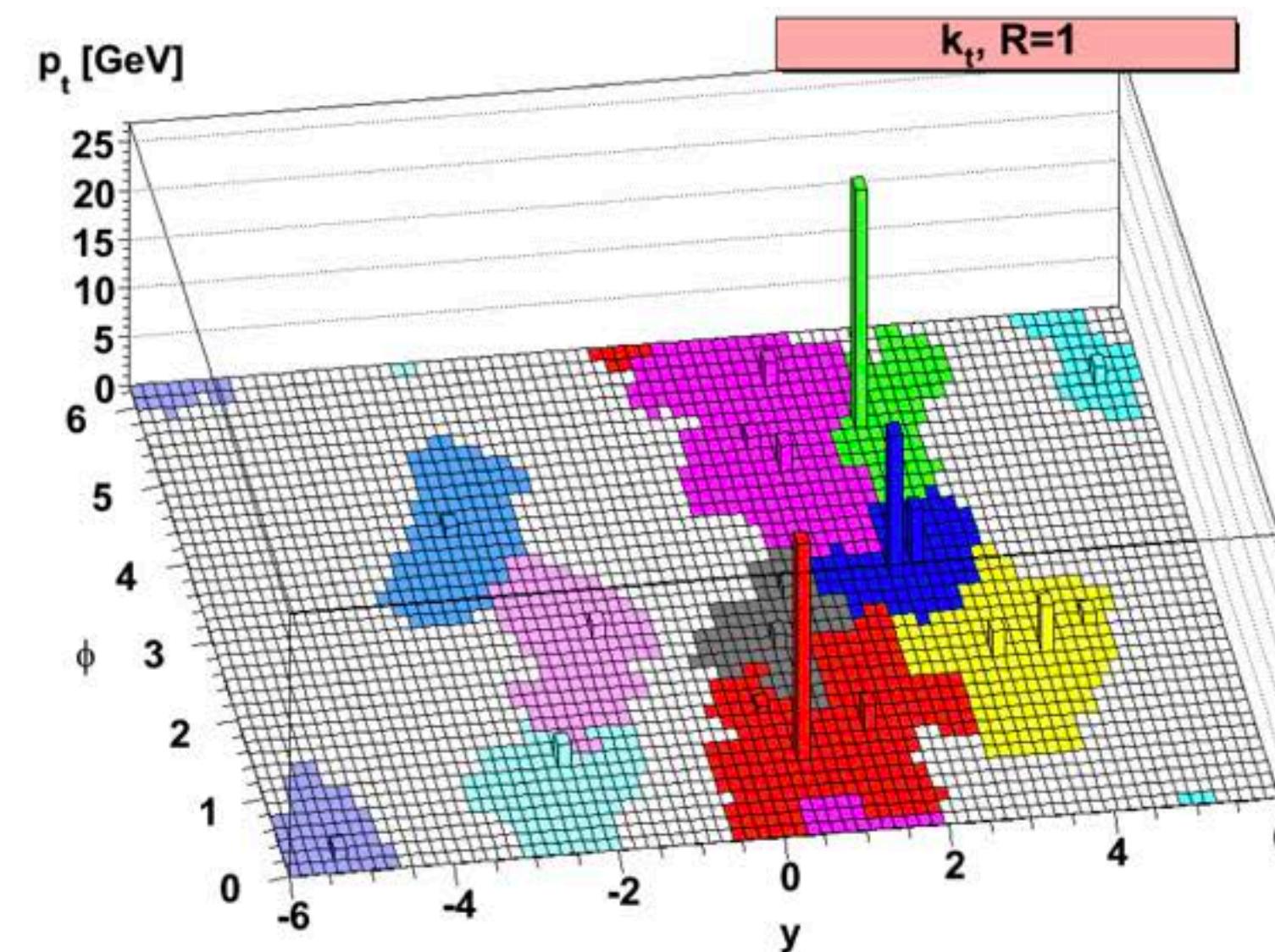
$p = 1 : k_t$ algorithm
 $p = 0 : \text{Cambridge Aachen}$
 $p = -1 : \text{anti-}k_t$ algorithm

ジェット再構成

$$d_{ij} = \Delta r_{ij} \cdot \min(k_{ti}^p, k_{tj}^p), \quad (\Delta r_{ij} = \sqrt{(\phi_i - \phi_j)^2 + (\eta_i - \eta_j)^2} < R)$$

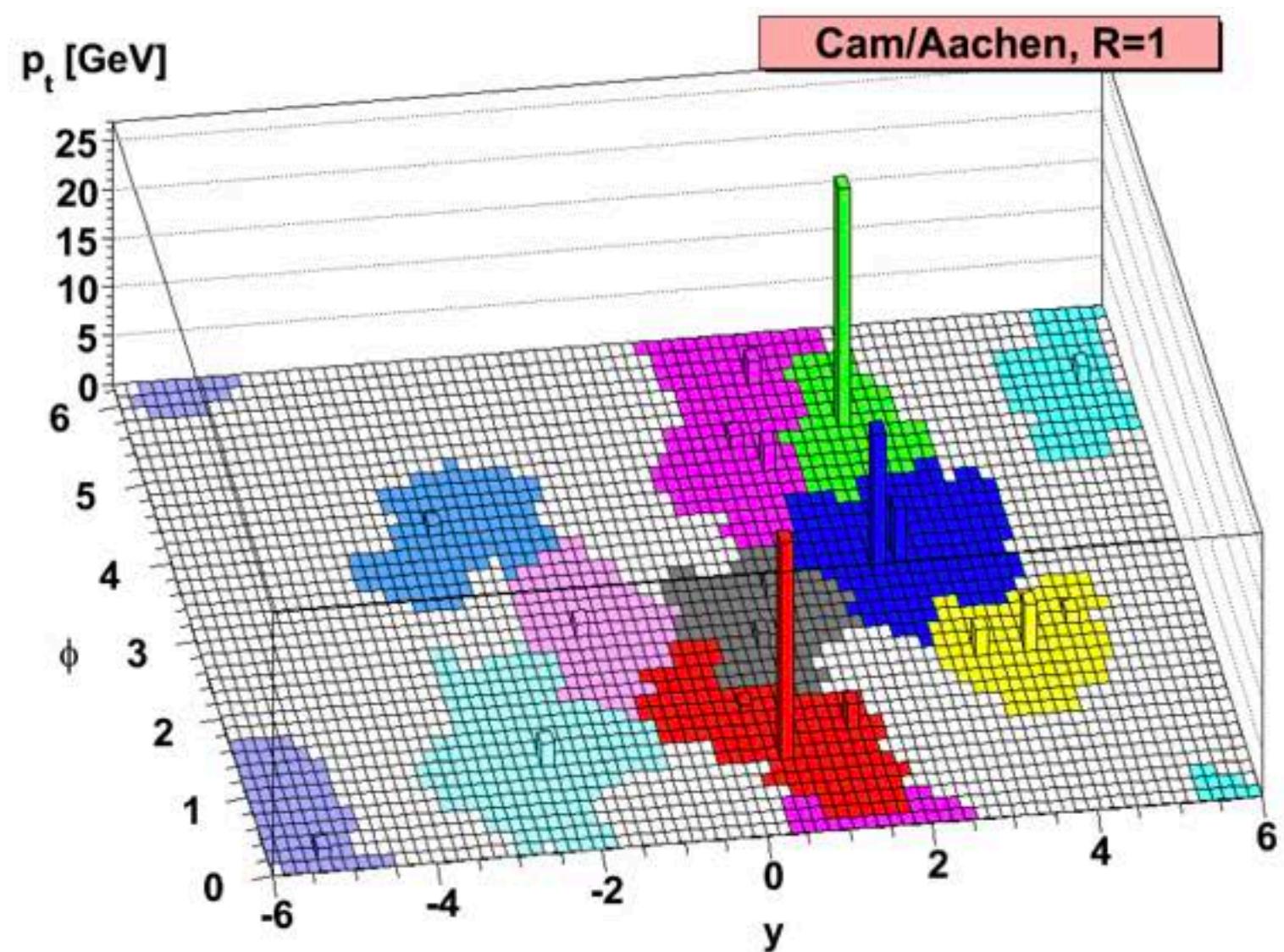
$p = 1: k_t$ algorithm

- soft なものの周りから合成



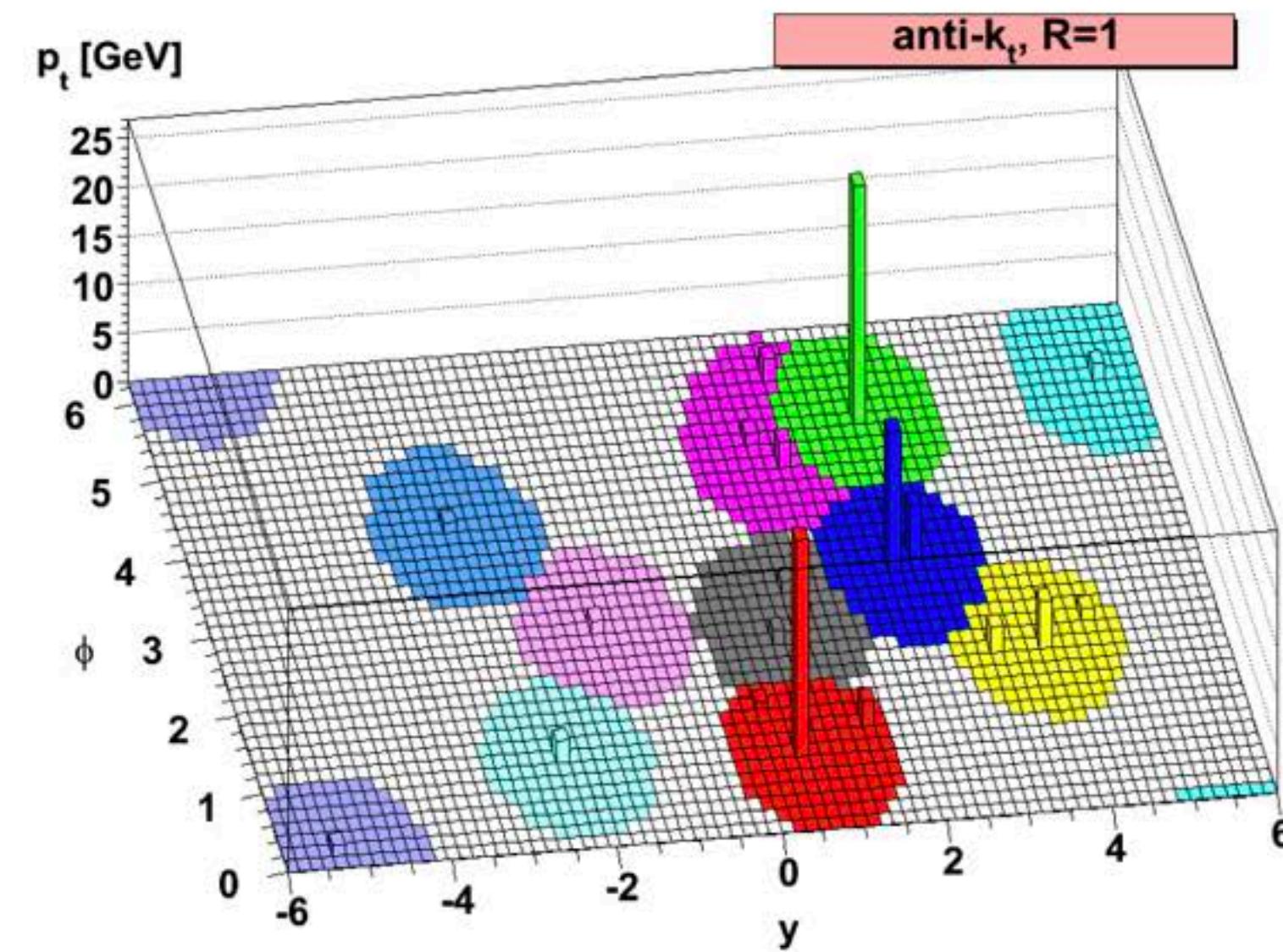
$p = 0:$ Cambridge Aachen

- 角度が近いものから合成



$p = -1:$ anti- k_t algorithm

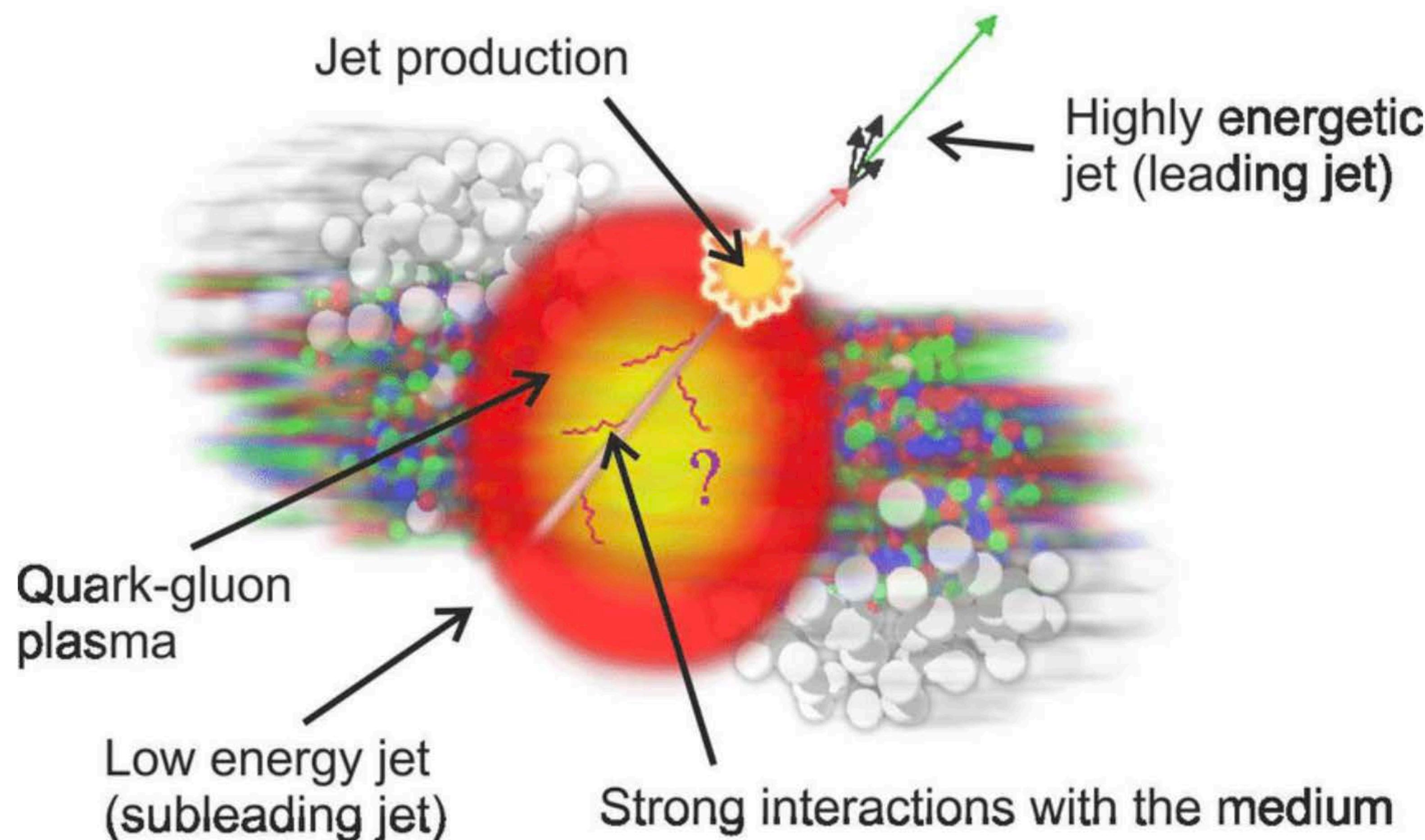
- hard なものの周りから合成



Adapted from M. Cacciari, G. P. Salam and G. Soyez, JHEP 04, 063 (2008)

パートンシャワー発展と ジェット-QGP媒質相互作用

重イオン衝突におけるジェット

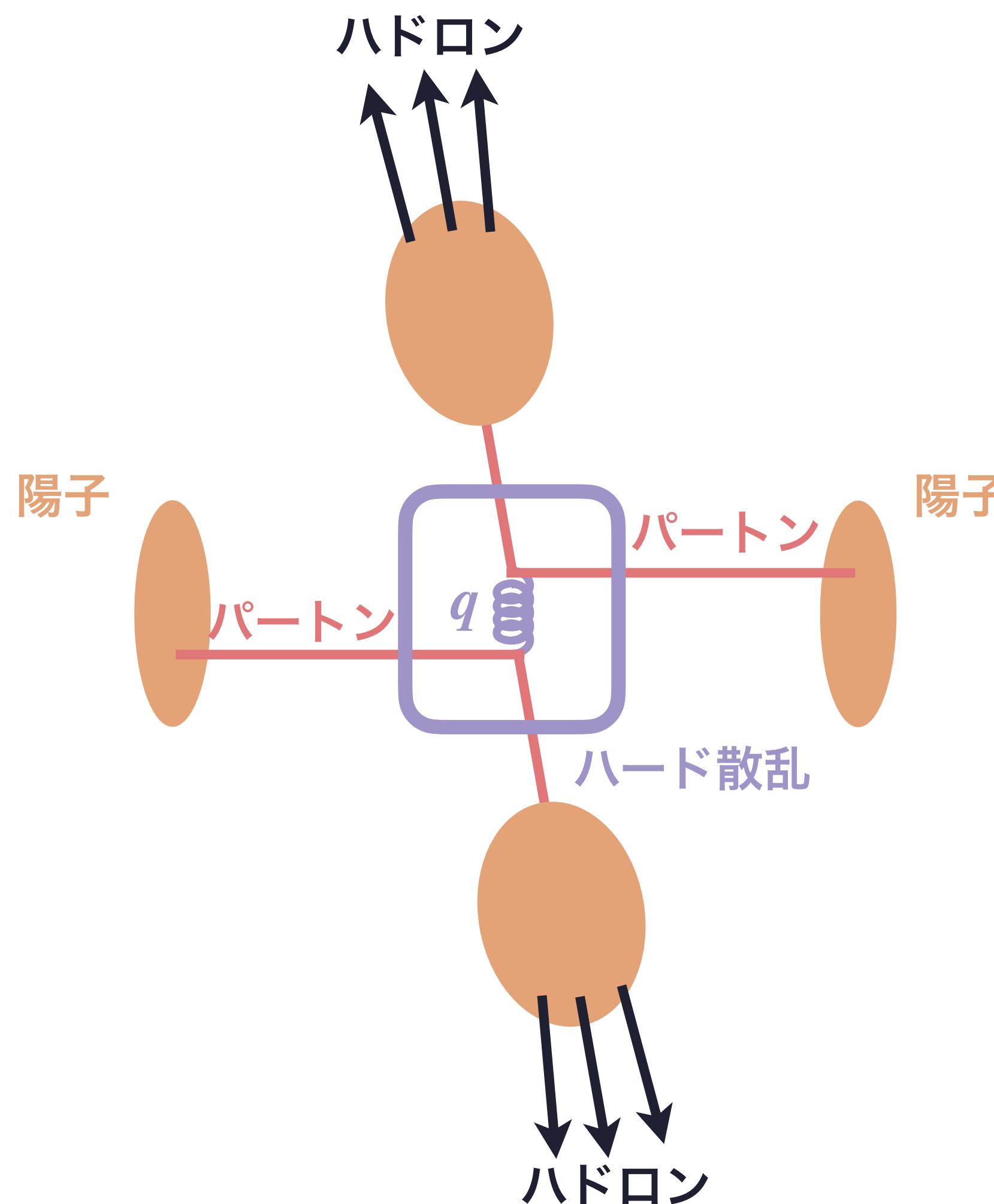


なぜジェットか？

- 真空中（陽子陽子衝突など）では摂動計算が比較的有効
→真空中の結果をベースラインに QGP 媒質効果を議論できる

なぜジェットか？

● Factorization [Collins, Soper, 1987]



- パートンのハード散乱とハドロンのソフトな過程の間のスケールの分離
- ハードプロセスとソフトプロセスの干渉が抑制

$$\frac{1}{\lambda_{\text{hard}}} \gg \frac{1}{\lambda_{\text{soft}}}$$

$$\begin{aligned} & |(H_1 + H_2 + \dots) \cdot (S_1 + S_2 + \dots)|^2 \\ & \sim |H_1 + H_2 + \dots|^2 \cdot |S_1 + S_2 + \dots|^2 \end{aligned}$$

[$(\lambda_{\text{hard}} \Lambda_{\text{QCD}})^2$ で補正は抑制]

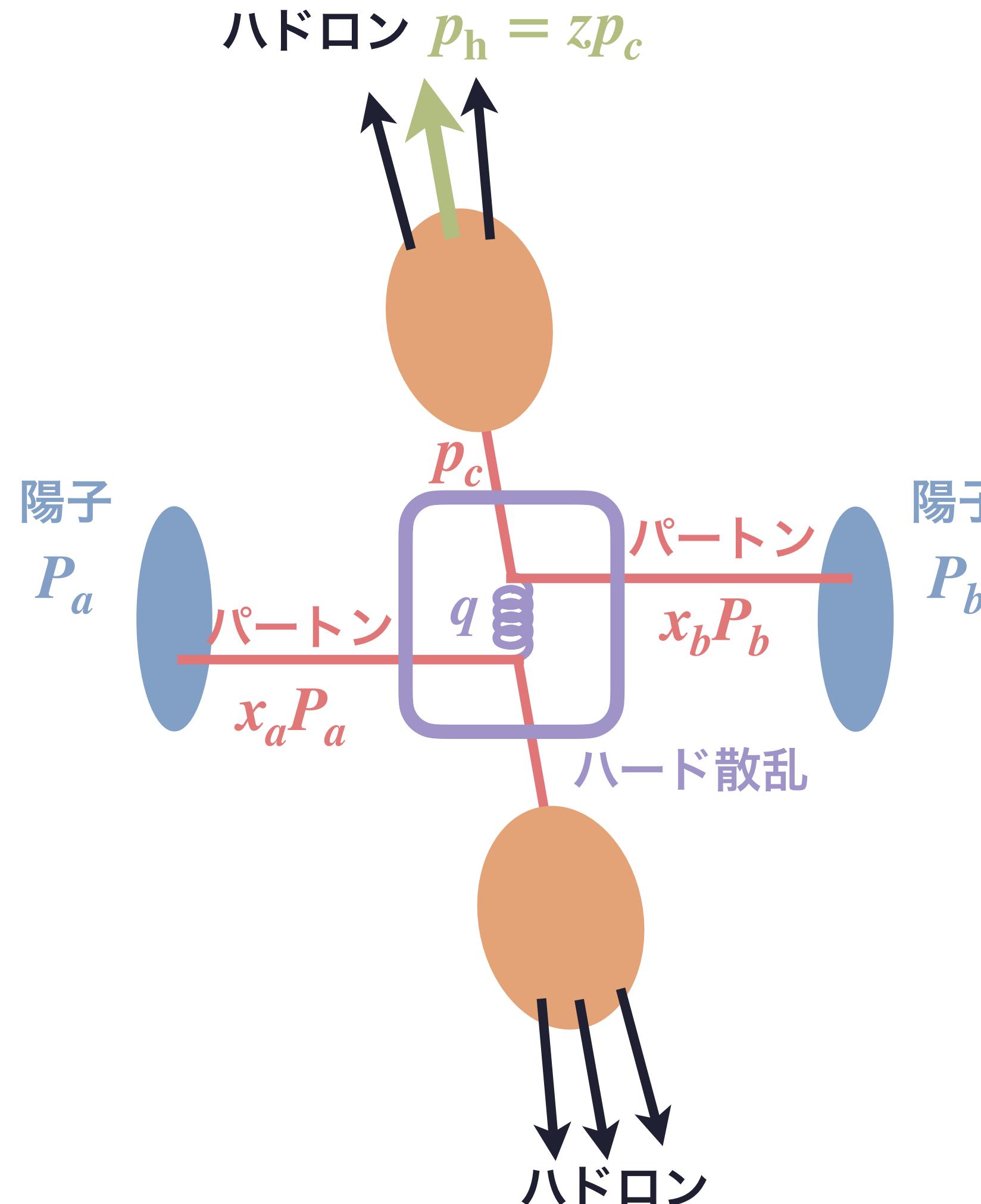
e.g.) high- p_T ハドロンの生成断面積

$$\frac{d\sigma^h}{dp_T dy} \sim \int dx_a dx_b G(x_a) G(x_b) \frac{d\hat{\sigma}}{d\hat{t}} D^h(z)$$

なぜジェットか?

● Factorization [Collins, Soper, 1987]

e.g.) high- p_T ハドロンの生成断面積



$$\frac{d\sigma^h}{dp_T dy} \sim \int dx_a dx_b G(x_a) G(x_b) \frac{d\hat{\sigma}}{d\hat{t}} D^h(z)$$

- 摂動論計算可能

$\frac{d\hat{\sigma}}{d\hat{t}}$: パートン-パートン散乱断面積

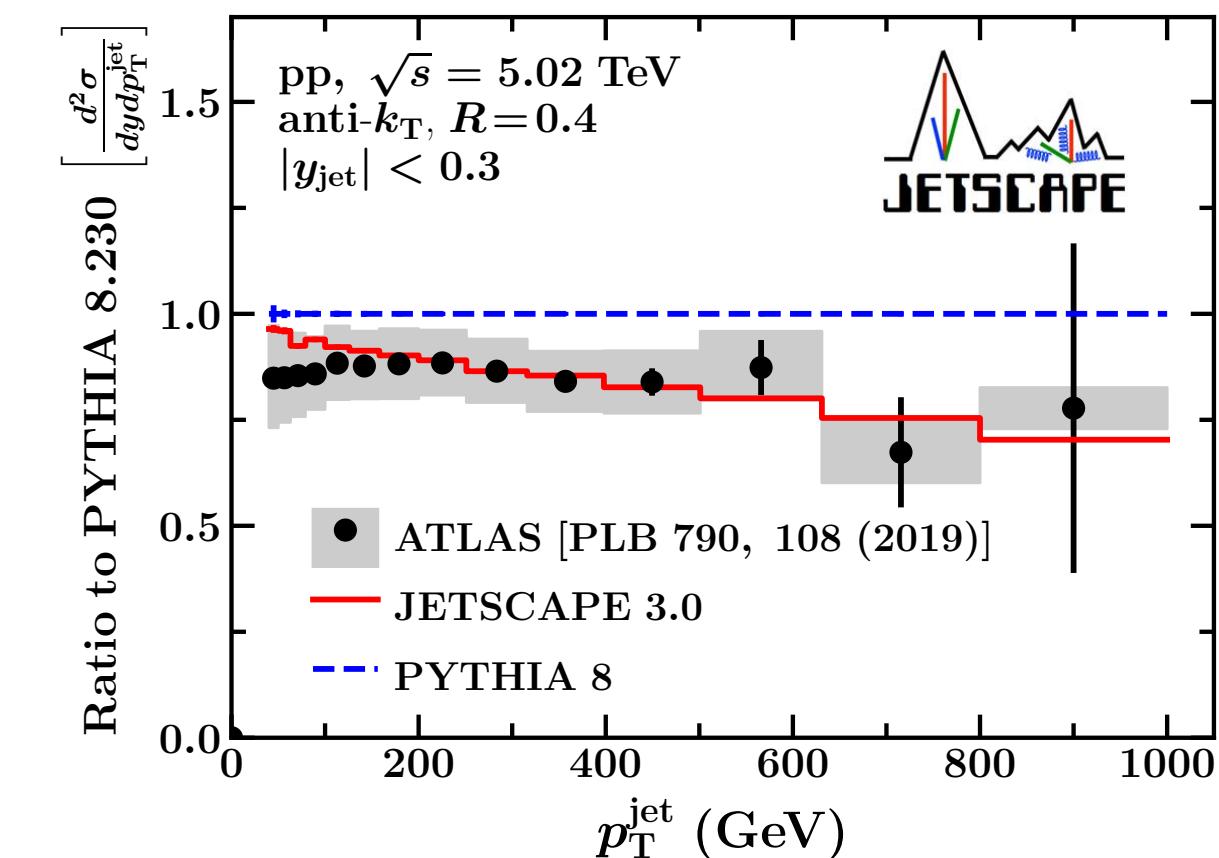
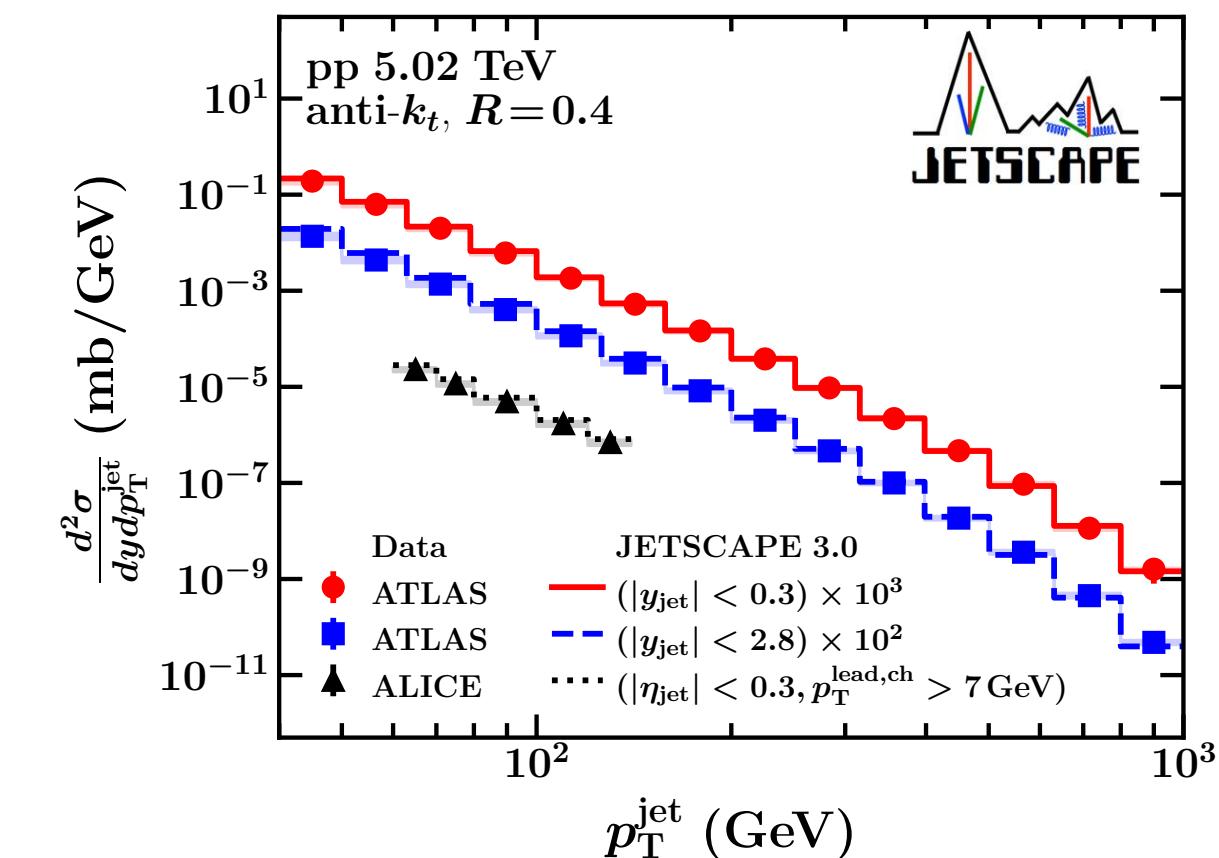
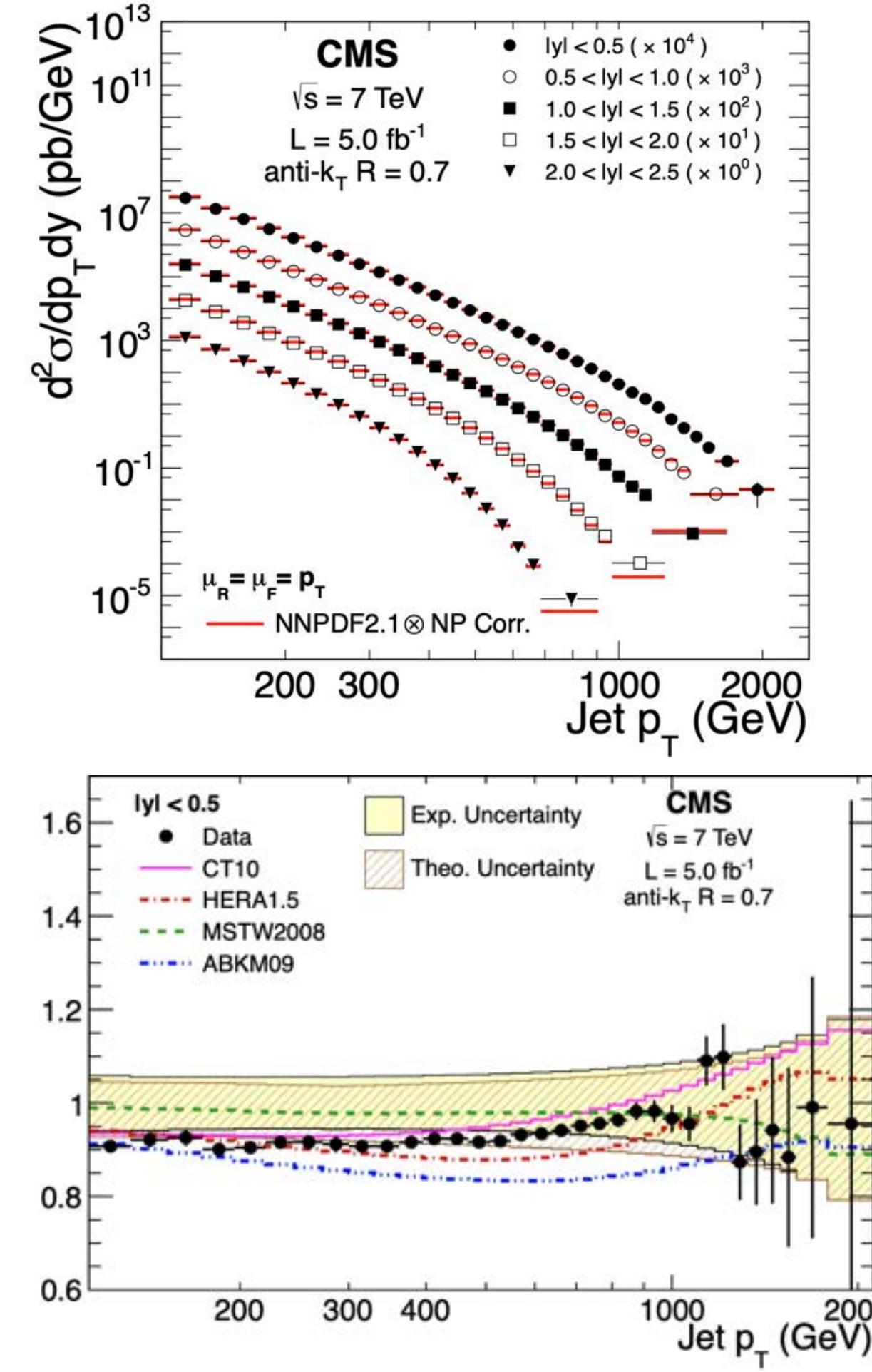
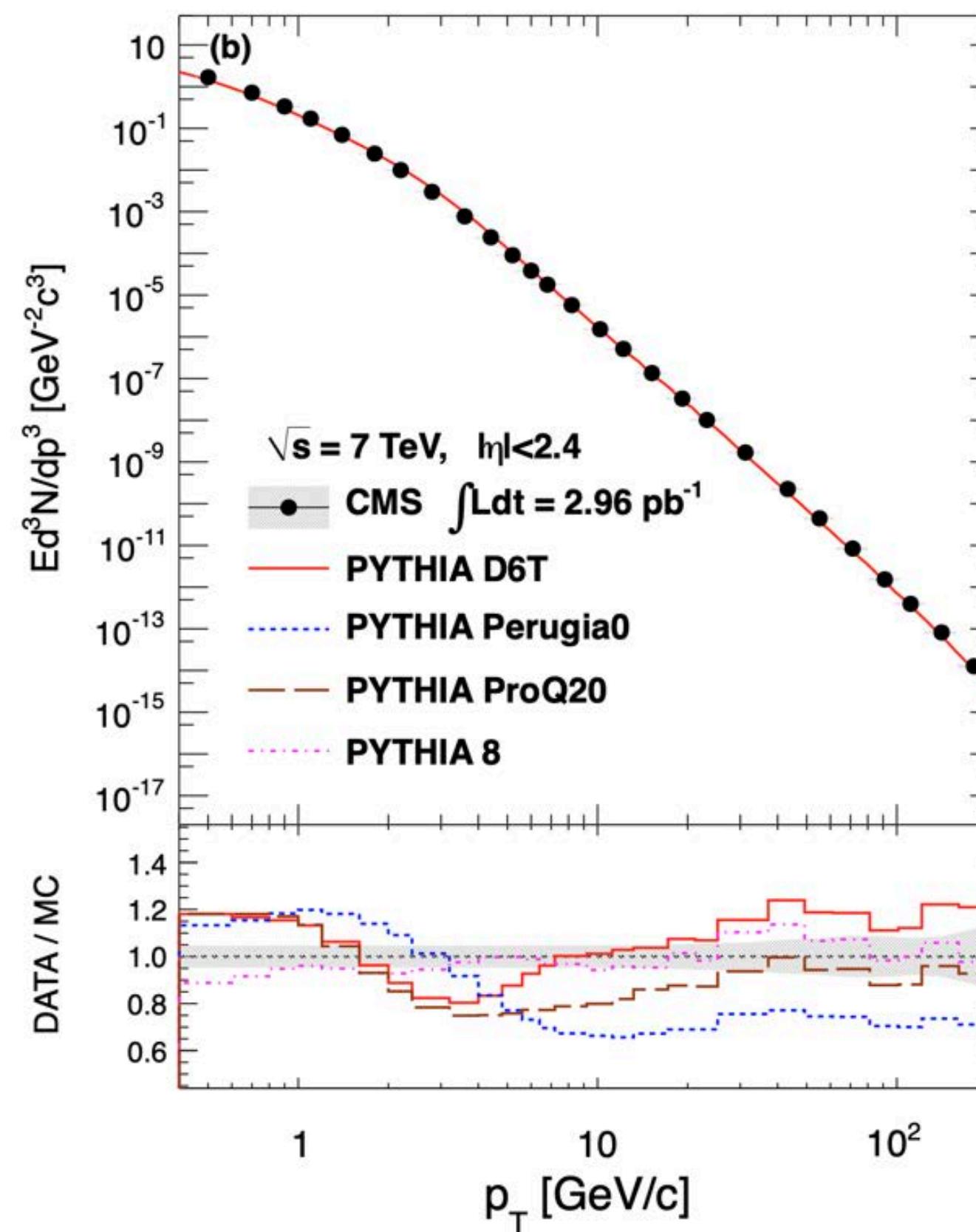
- 他の実験からのインプット (+摂動計算)

$G(x_a)$: 阳子のパートン分布関数 (PDF)

$D^h(z)$: 破碎関数 (Fragmentation function)

なぜジェットか？

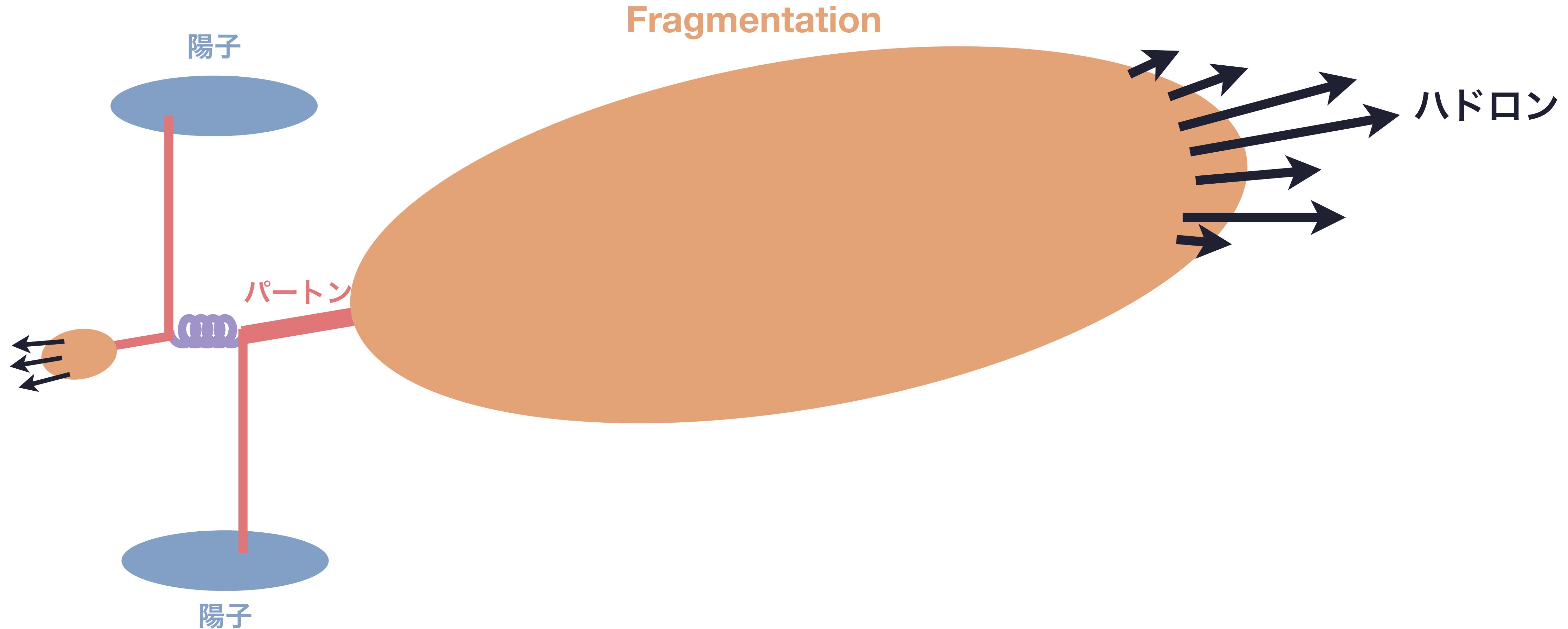
- 理論計算による実験結果の再現



*まだ一部、QGP 媒質効果を調べるのに十分な精度がないものや、理論計算で再現できていないジェット観測量はまだ存在する（真空中のジェットの物理もまだ進行中）。

パートンシャワー（真空中）

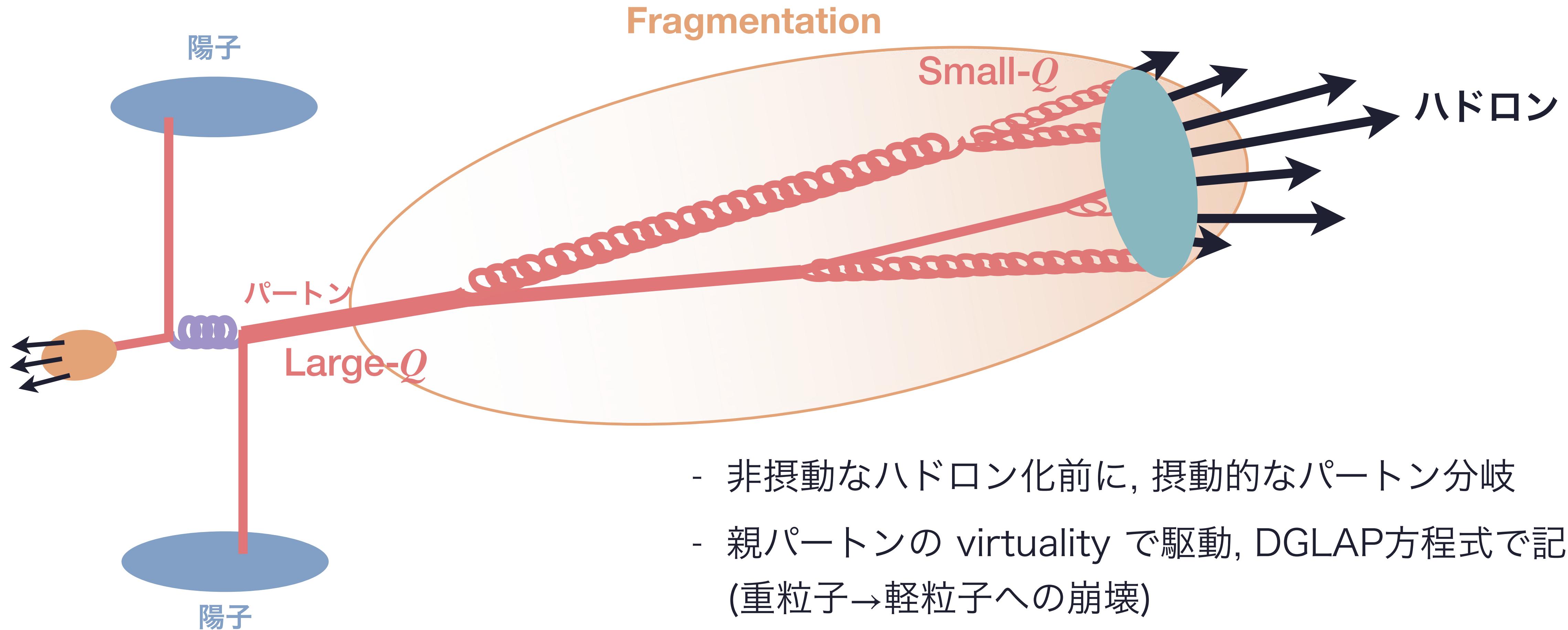
- 破碎過程中のパートン分岐 [V. Gribov and L. Lipatov (1972) G. Altarelli and G. Parisi (1977) Yu. Dokshitzer (1977)]



パートンシャワー (真空中)

● 破碎過程中のパートン分岐

[V. Gribov and L. Lipatov (1972) G. Altarelli and G. Parisi (1977) Yu. Dokshitzer (1977)]



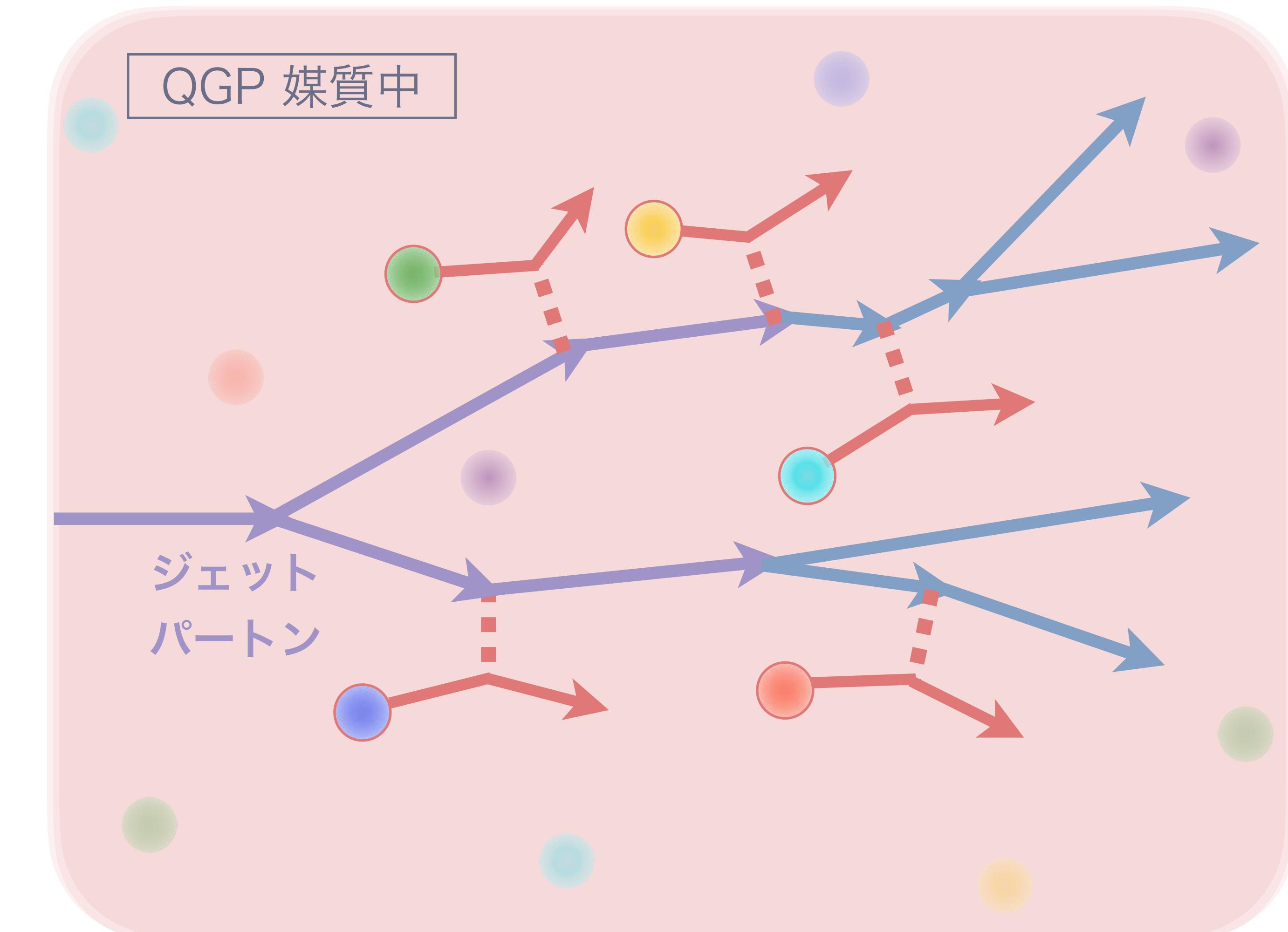
$$\text{Virtuality (off-shellness): } Q^2 = p^\mu p_\mu - m^2$$

重イオン衝突におけるジェットの発展

QGP 中のパートンシャワー

- QGP 媒質中を通過
- QGP との強い相互作用
- パートンシャワー構造が変化
→ ジェット測定量に影響

ジェット-QGP 相互作用の情報



ジェットは QGP と相互作用するのか？

Q. ハードなジェットは、ソフトなQGPに比べ非常に速くに生成するはず。これらスケールは大きく離れているのになぜ相互作用するといえるのか？

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A. QGP のサイズが十分に大きいから

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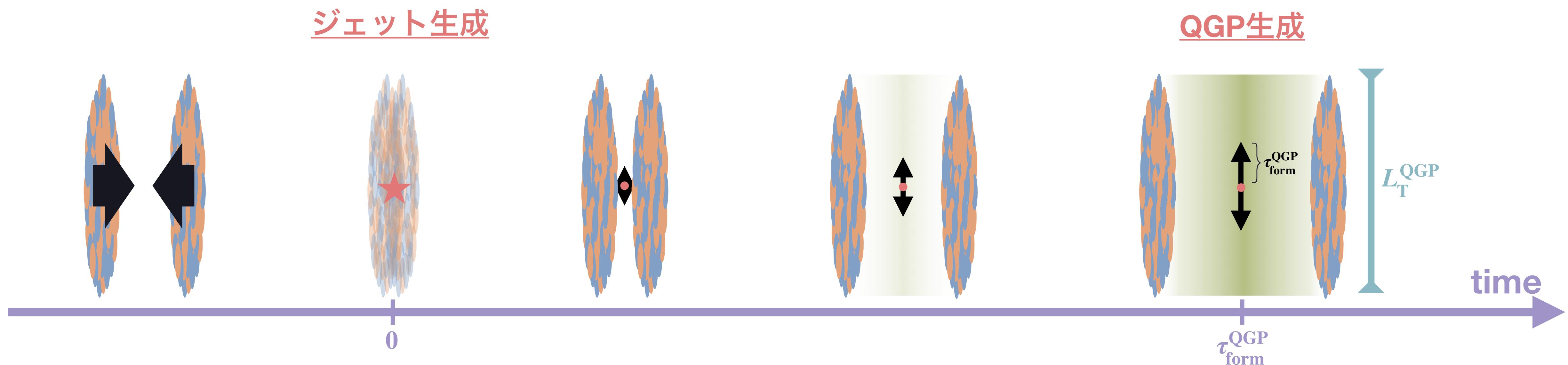
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A. QGP のサイズが十分に大きいから

$$\tau_{\text{form}}^{\text{QGP}} < L_T^{\text{QGP}}$$

$\tau_{\text{form}}^{\text{QGP}} \sim 1 \text{ fm}$: QGP の生成時間

$L_T^{\text{QGP}} \sim 10 \text{ fm}$: QGP の (横) サイズ



ジェットは QGP と相互作用するのか？

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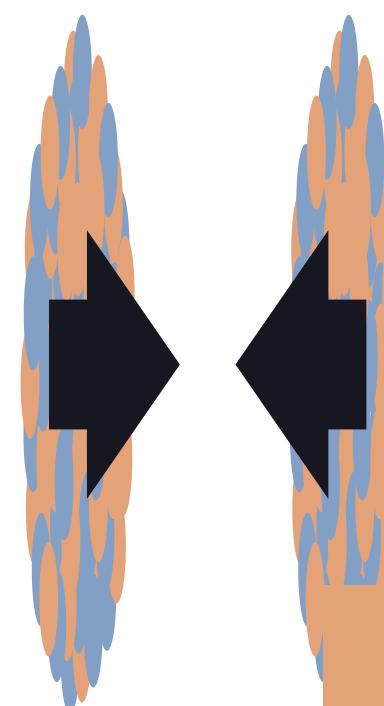
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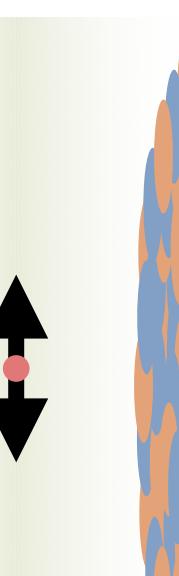
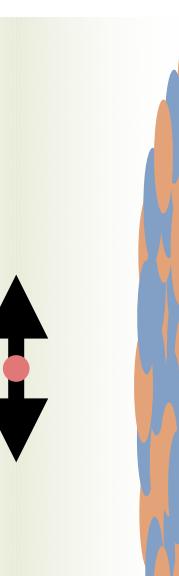
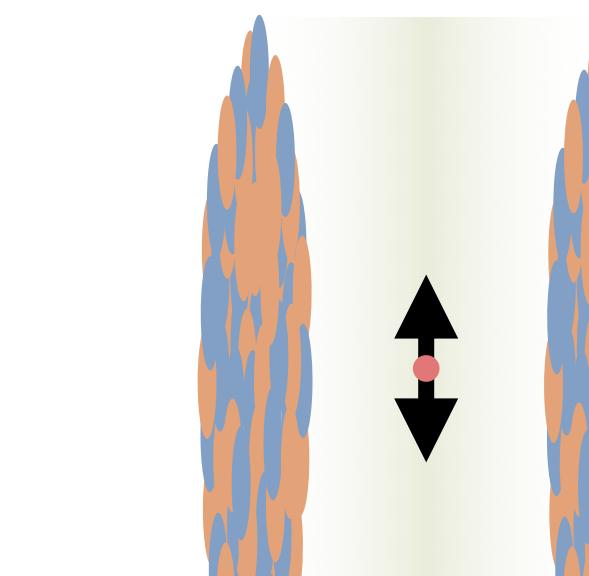
$L_T^{\text{QGP}} \sim 10 \text{ fm}$: QGP の (横) サイズ

ジェット生成



- 重イオンでは時空間での発展が重要

QGP生成



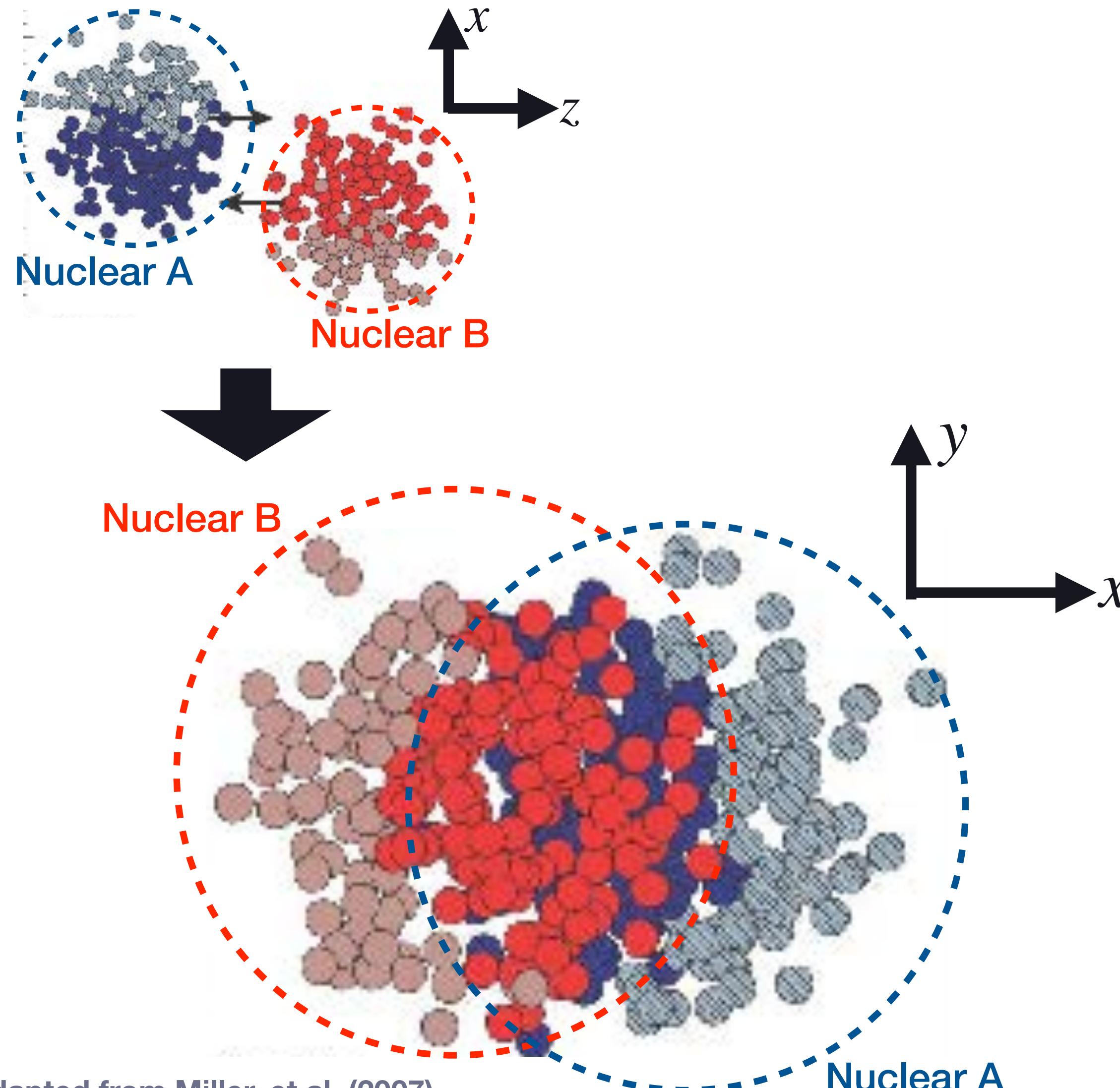
$\tau_{\text{form}}^{\text{QGP}}$

L_T^{QGP}

time

- 小さい系で QGP ができていても、ジェット相互作用があるかは非自明

Space-time Geometry



Position of jet creation

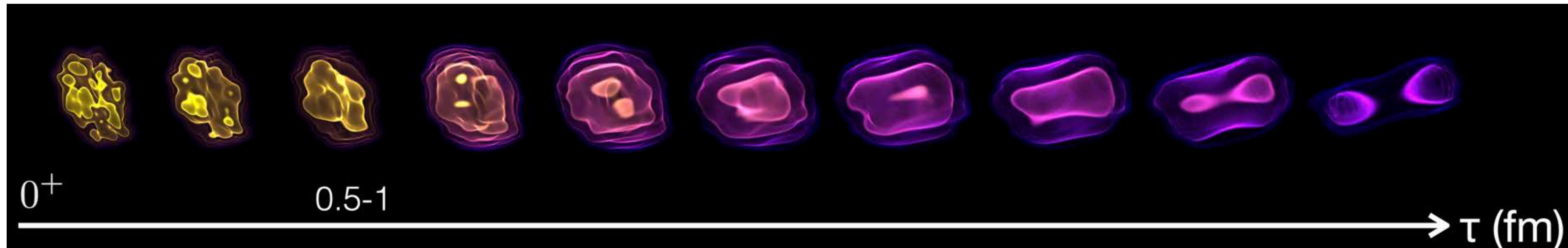
- Transverse distribution of nucleon-nucleon collisions in the nucleus
- Nucleon distribution in a nuclear: Woods-Saxon
- Nucleon positions sampled event-by-event (MC-Glauber)
- Same geometry used to generate the initial profile of bulk QGP fluid

Adapted from Miller, et al. (2007)

Space-time Geometry

● Background QGP medium evolution

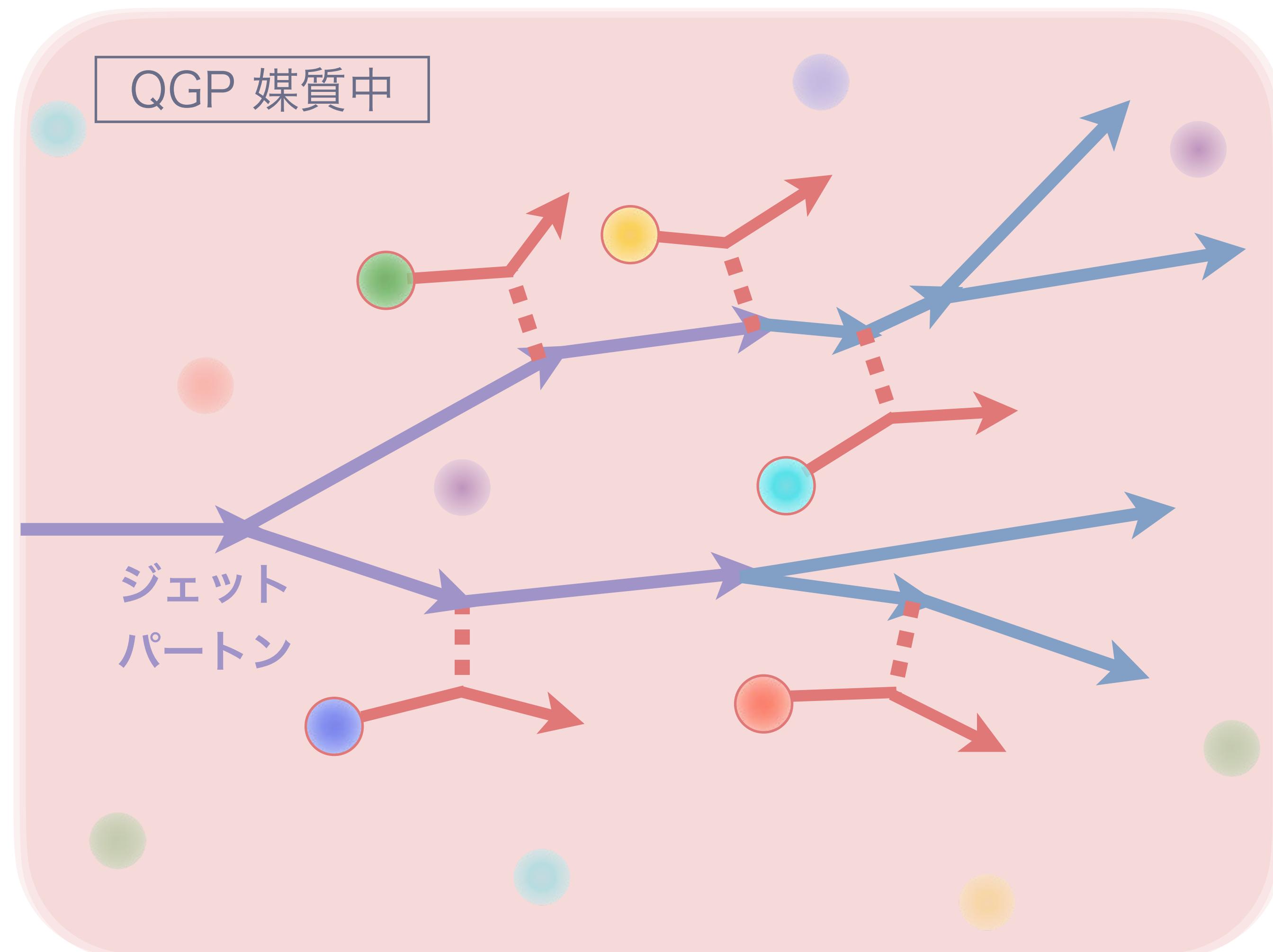
- Non-uniform, non-static → complicated path dependence (e.g. $\langle \Delta E_{\text{rad}} \rangle \propto \hat{q}L^2$)
- Space-time profile, $u^\mu(x)$, $T(x)$, from hydro simulation
- Event-by-event Initial profile correlated with jet production point



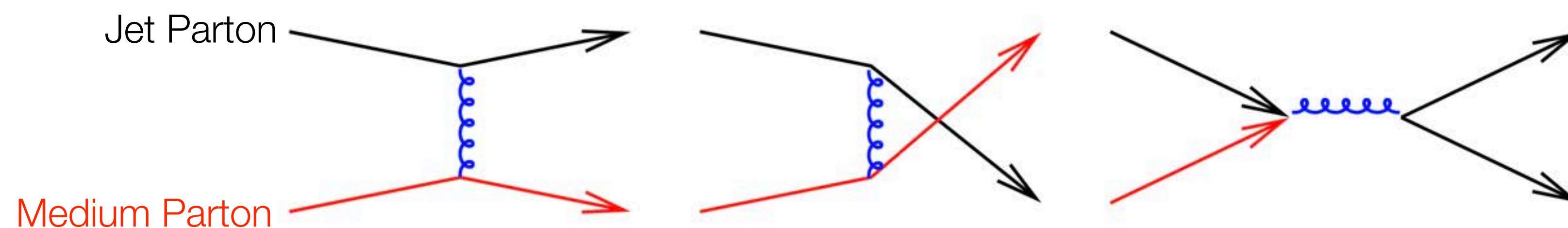
Adapted from Chun Shen

媒質中のジェットシャワー発展

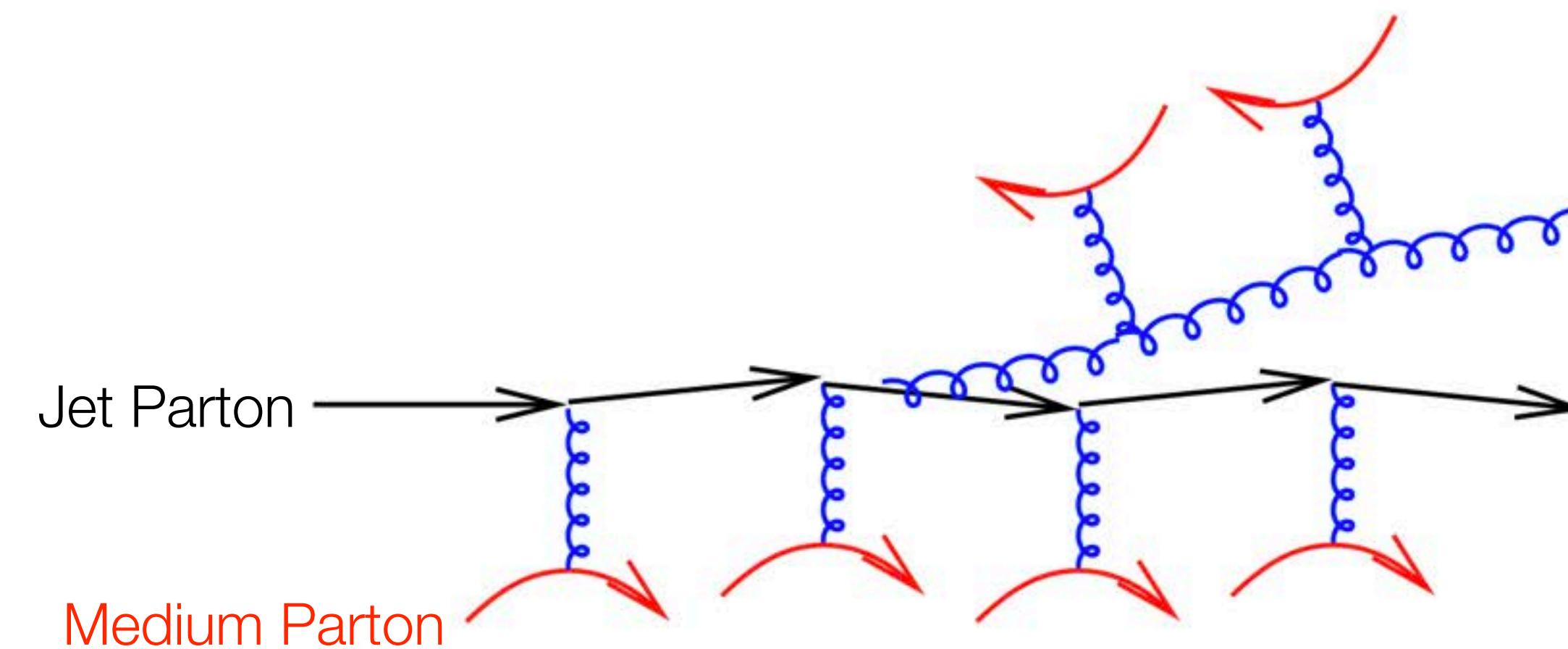
パートン-媒質相互作用



パートン-媒質相互作用



Elastic scatterings with thermal particles



Collinear radiation

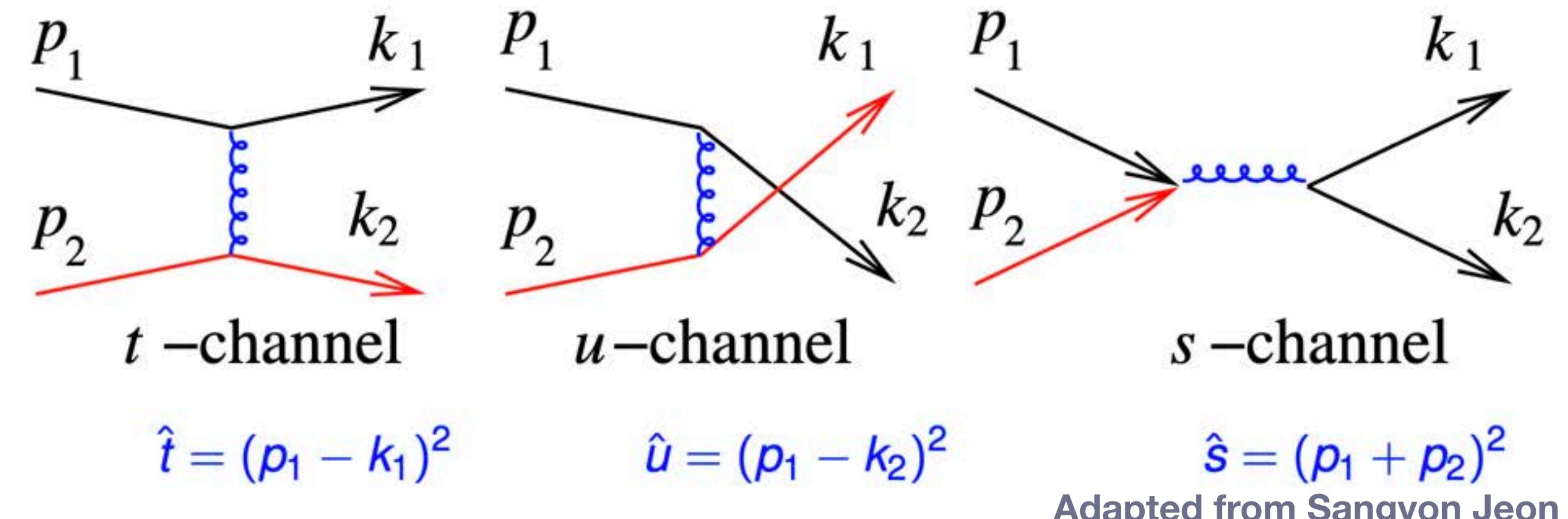
Adapted from Sangyon Jeon

- Modify jet parton's energy and momentum

- Modify and enhance jet parton's radiations

パートン-媒質相互作用

● Elastic component [Bjorken, FERMILAB-PUB-82-059-THY]



Adapted from Sangyon Jeon

$$\Gamma_{\text{el}} = \int \frac{d^3 p_2}{(2\pi)^3 2E_{p_2}} \frac{d^3 k_1}{(2\pi)^3 2E_{k_1}} \frac{d^3 k_2}{(2\pi)^3 2E_{k_2}} f_{\text{th}}(\vec{p}_2) \frac{(2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2)}{2p_1^0} |\mathcal{M}_{p_1 p_2 \rightarrow k_1 k_2}|^2$$

- Calculation performed at the local rest frame of the fluid by boost with flow velocity $u^\mu(x)$
- p_2 is a medium parton sampled from thermal distribution $f_{\text{th}}(p_2)$

パートン-媒質相互作用

● Medium-modified/induced partonic radiation

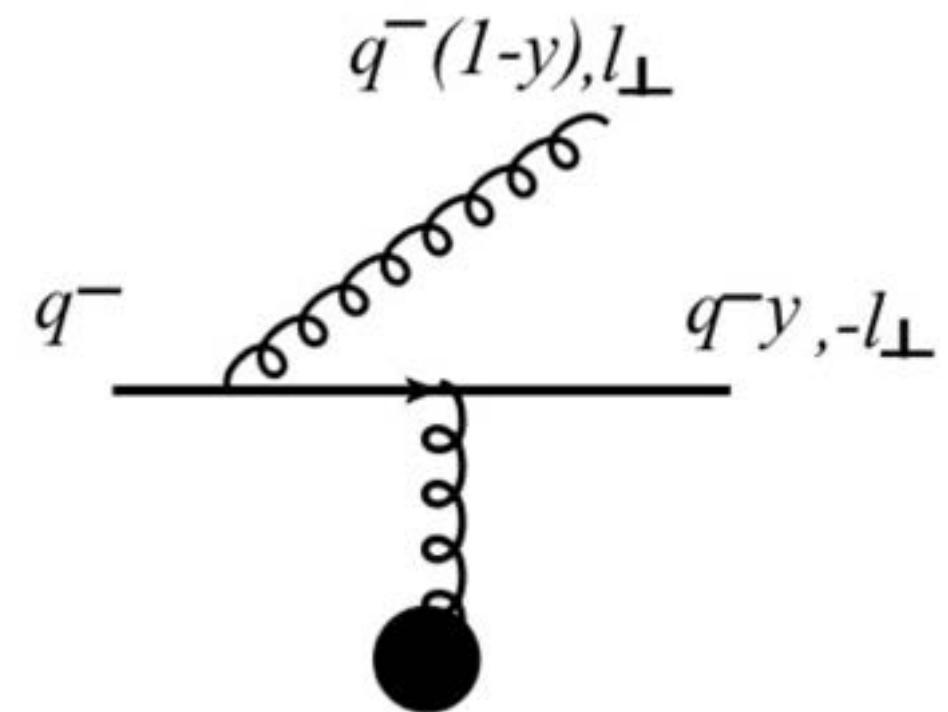
$$\frac{dN_g}{dydQ^2} = \frac{\alpha_s}{2\pi} \frac{P(y, Q^2)}{Q^2} \left[1 + \int_{\xi_o^+}^{\xi_o^+ + \tau^+} d\xi^+ K(\xi^+, \xi_o^+, y, p^+, Q^2) \right]$$

vacuum medium effect

Example:
Higher Twist calculation

$$K_{\text{HT}} = \hat{q} \frac{C \left[2 - 2 \cos \left(\xi^+ - \xi_o^+ / \tau^+ \right) \right]}{y(1-y)Q^2(1+\chi_a)^2}$$

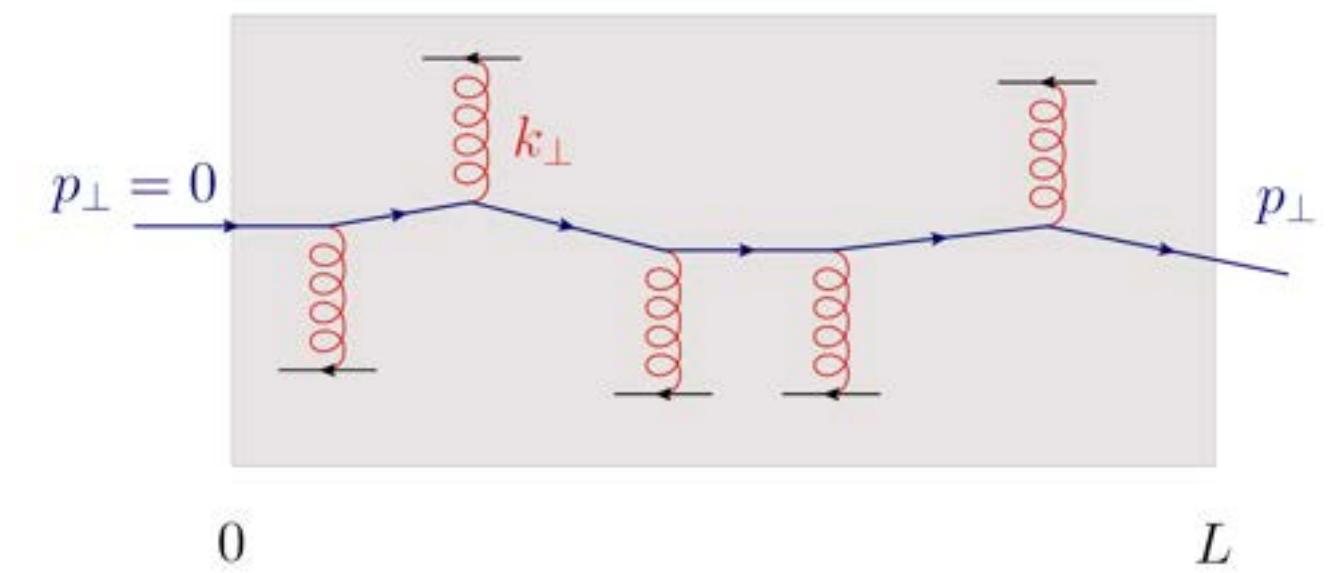
Higher Twist



Adopted from N. Armesto et al., PRC86, 064904 (2012)

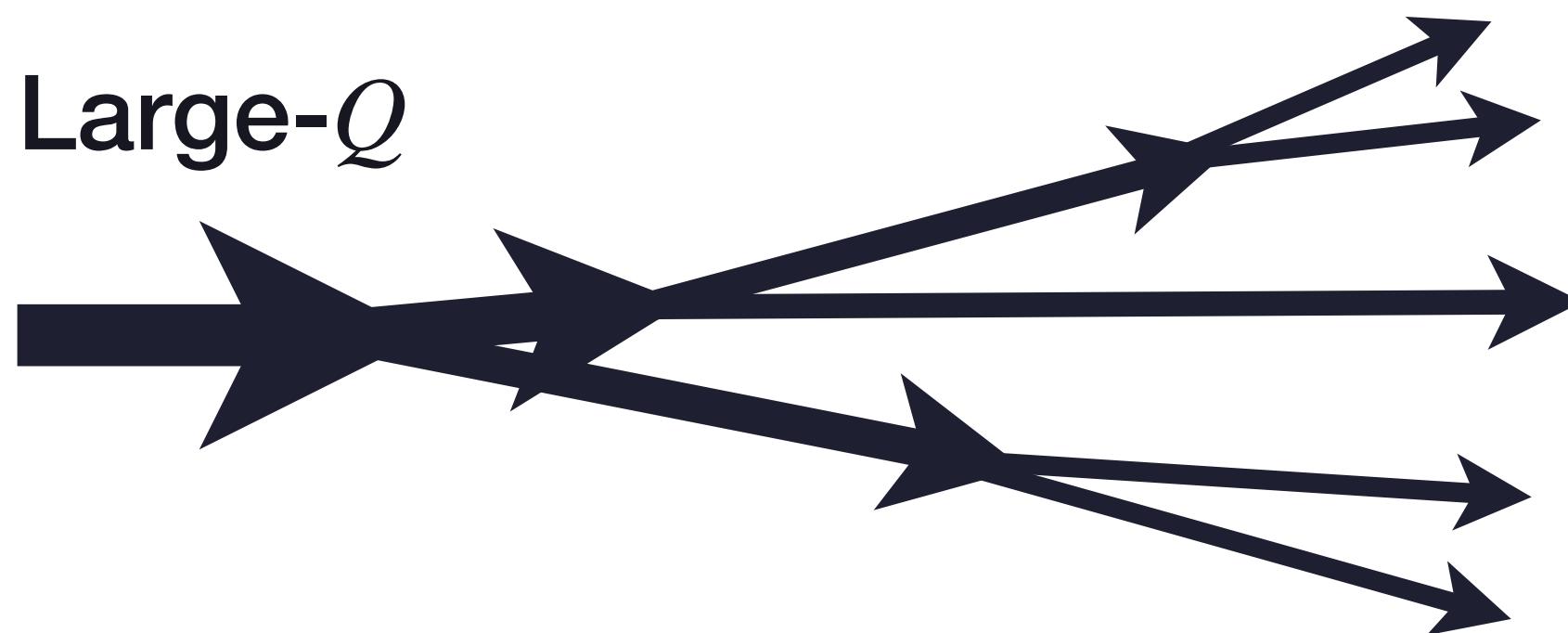
- Radiation spectrum modification controlled by the jet transport coefficient

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L}$$



ジェットの QGP 媒質中の発展

In-vacuum



Small- Q

- **Jet parton-medium interaction**

- Exchange energy and momentum
- Jet structures modification by the medium effects

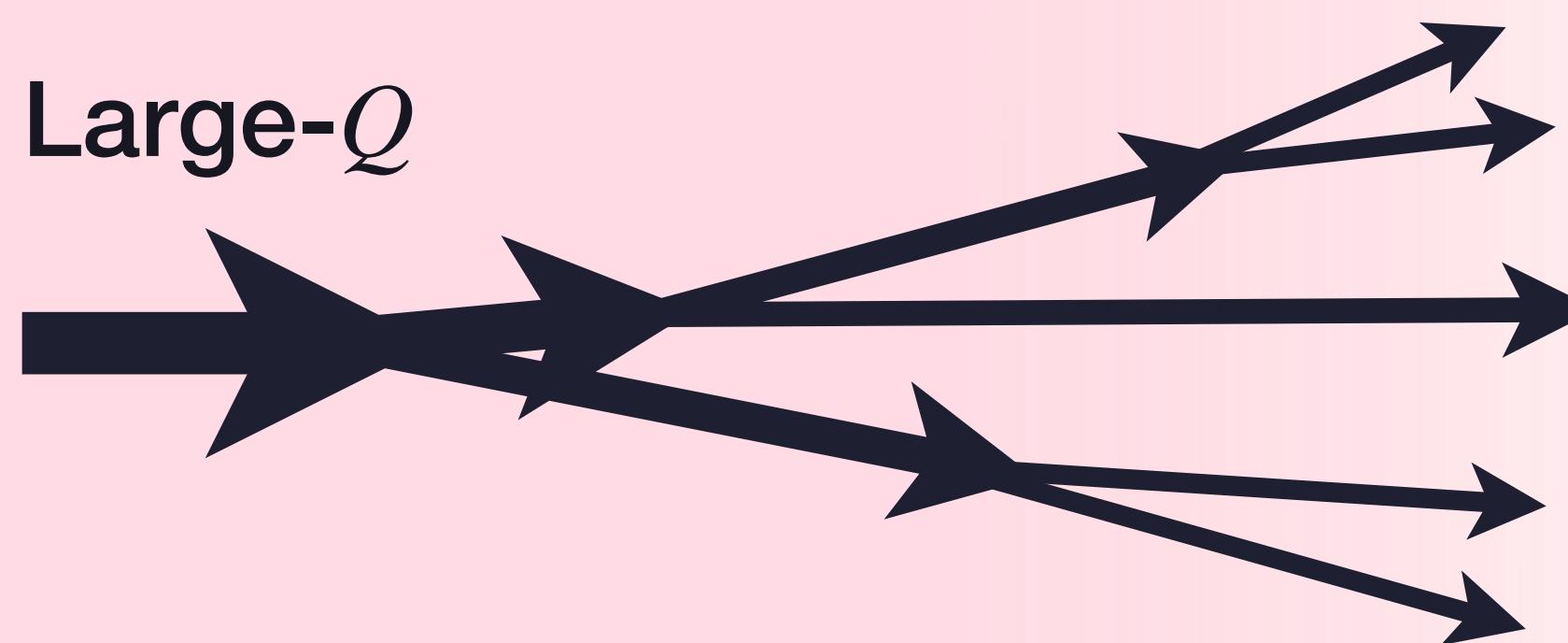
- **Shower pattern modification**

- In-vacuum: Virtuality ordered splitting

ジェットの QGP 媒質中の発展

In-medium

Small- Q

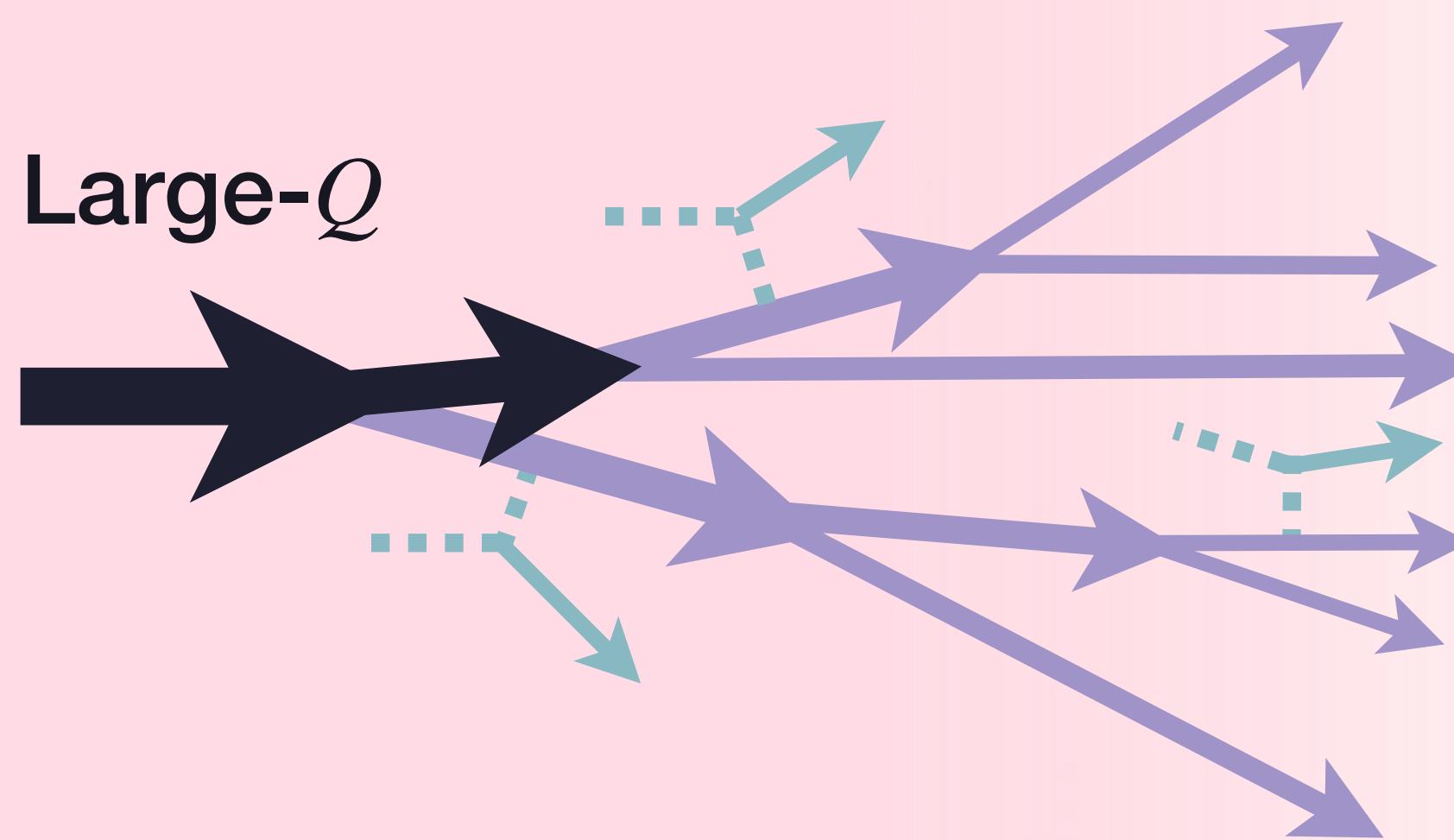


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ジェットの QGP 媒質中の発展

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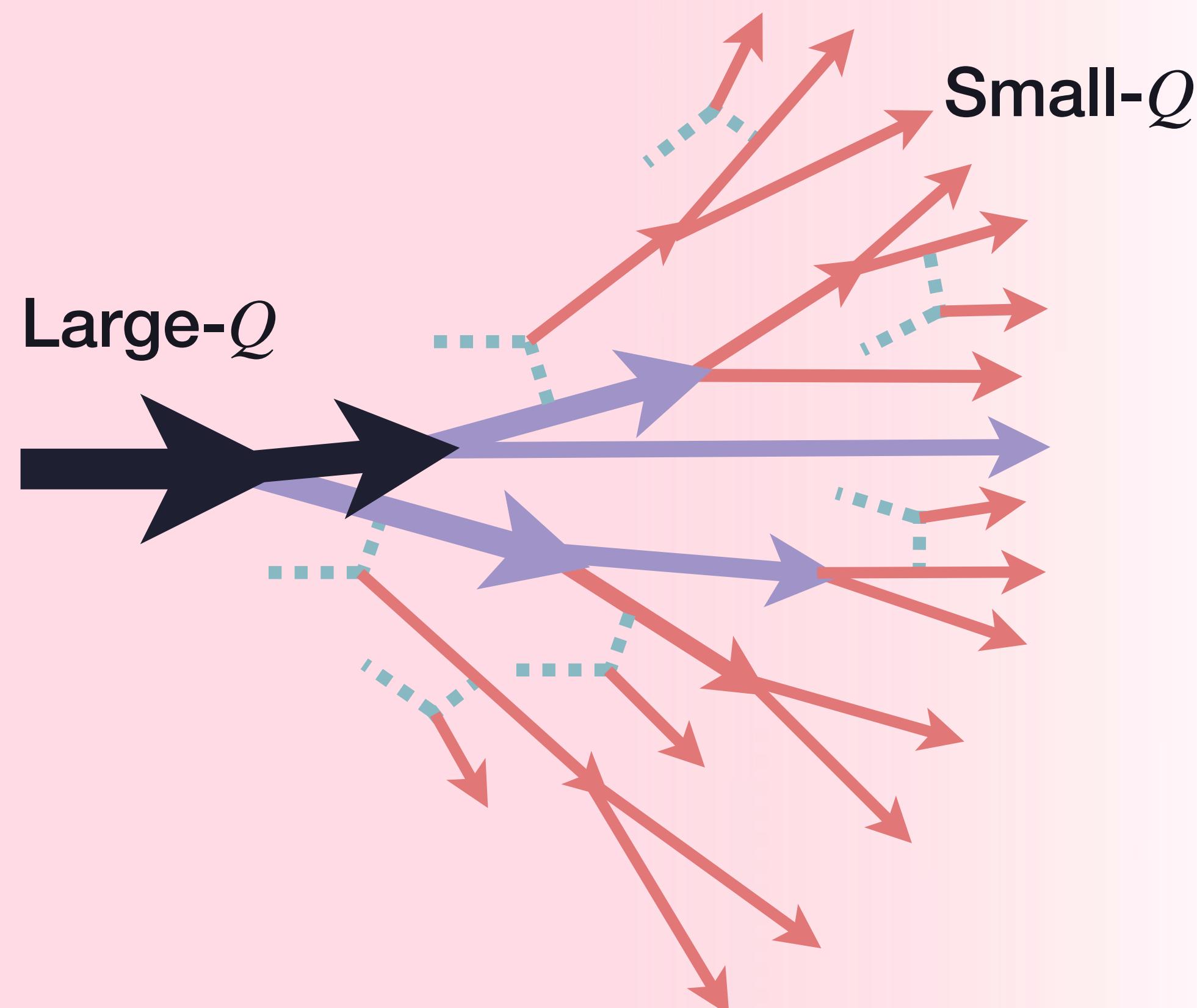
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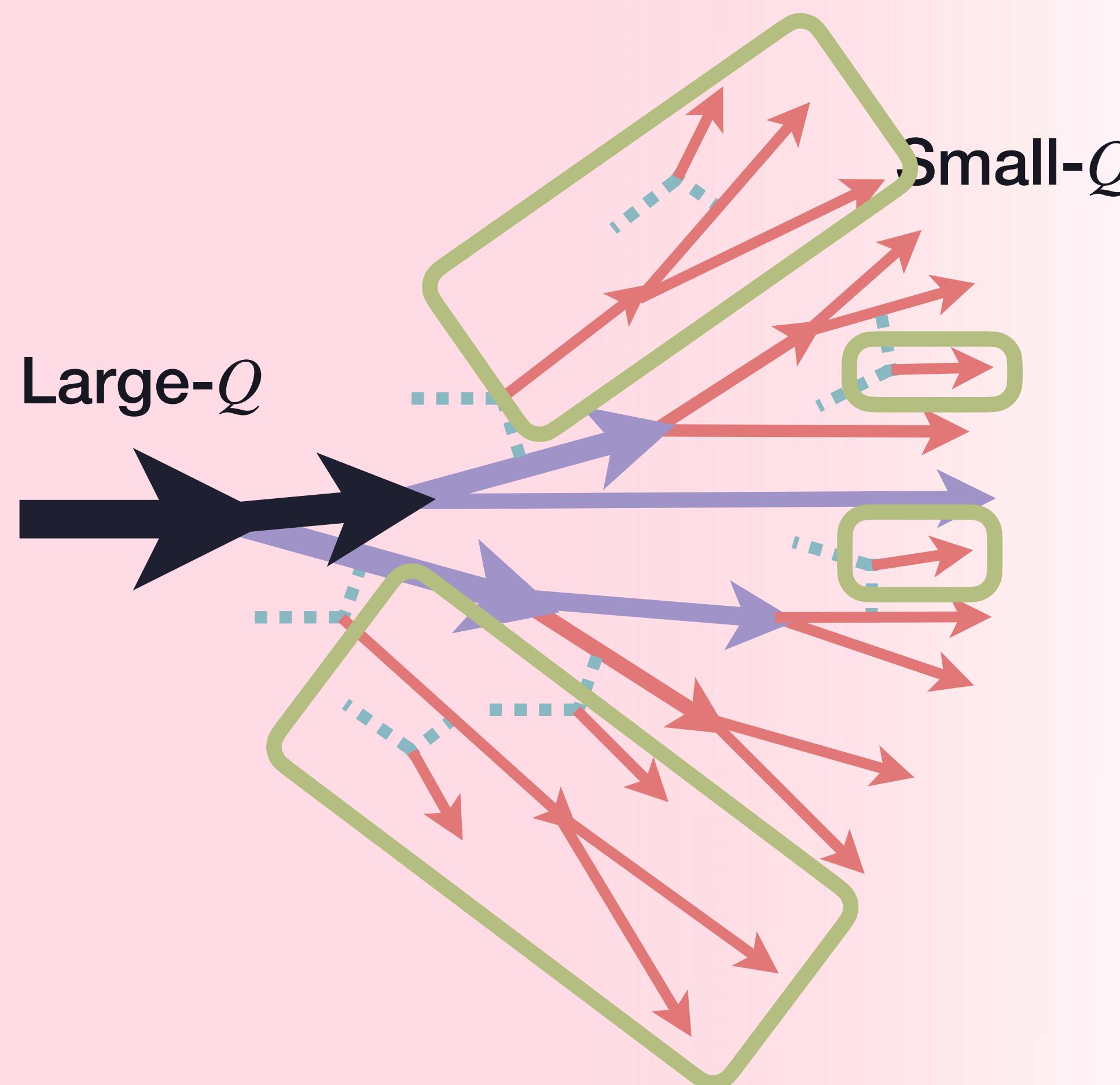
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ジェットの QGP 媒質中の発展

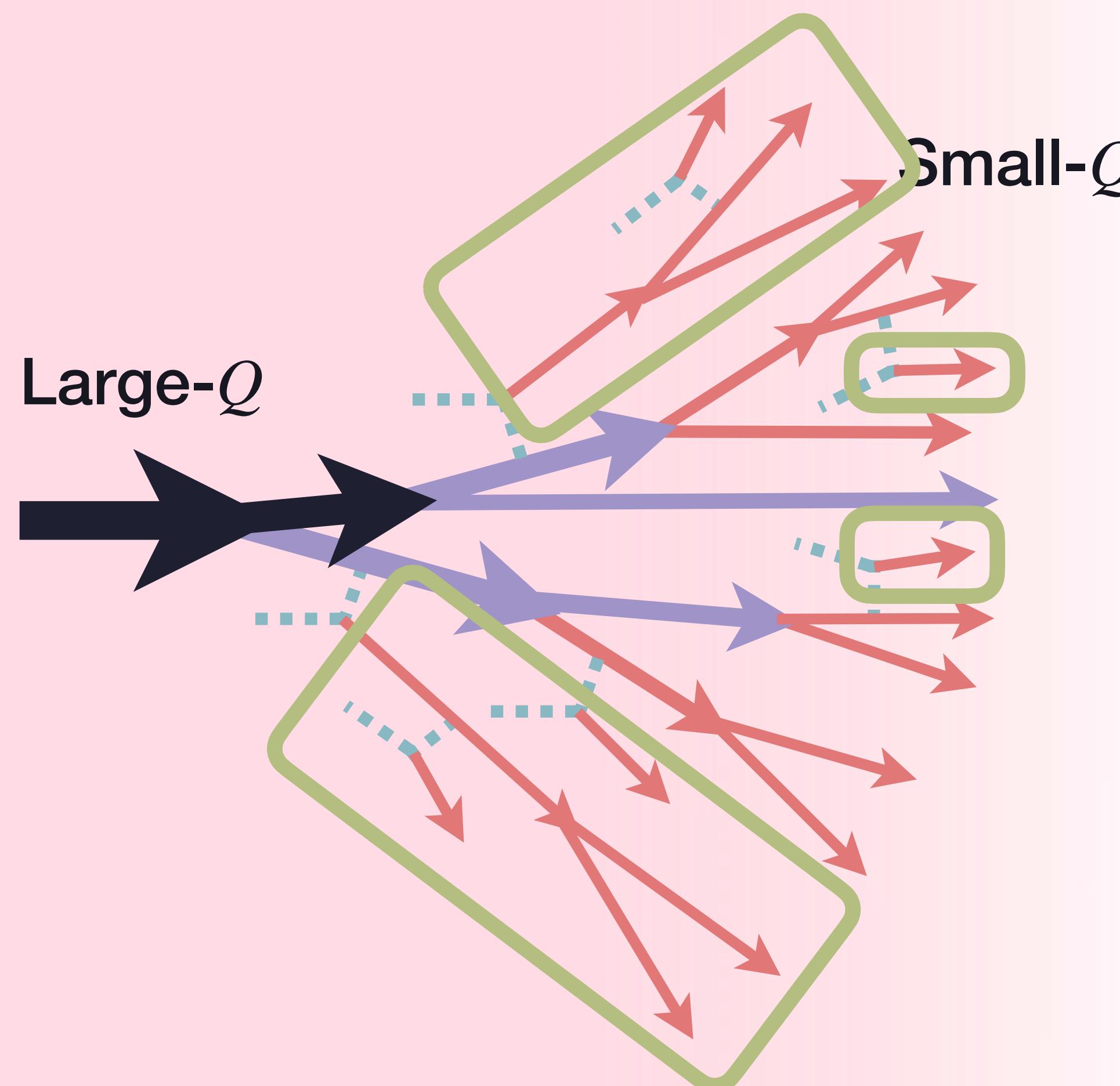
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- **Medium response**
 - Recoils: Medium partons struck by jet partons

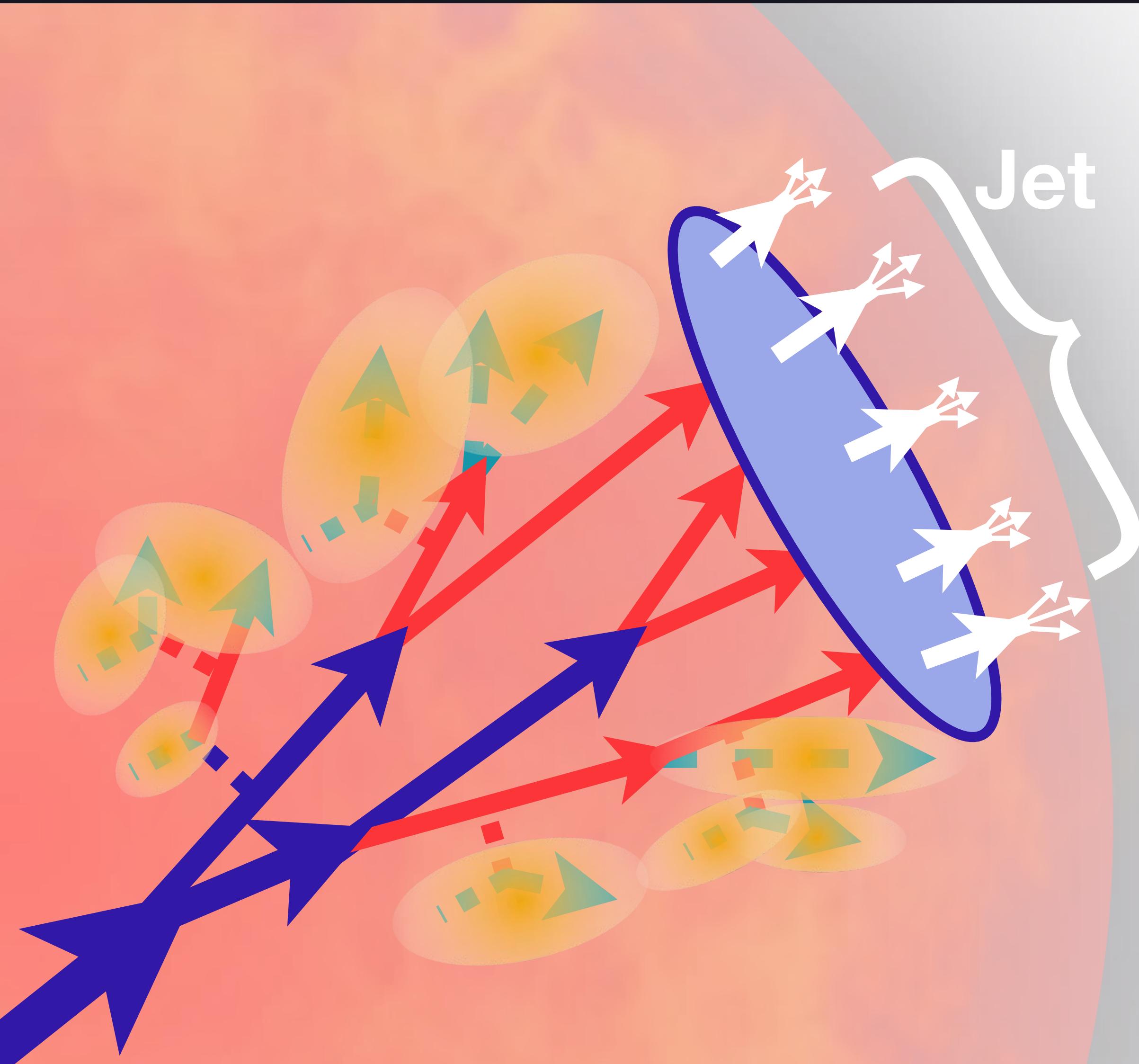
ジェットの QGP 媒質中の発展

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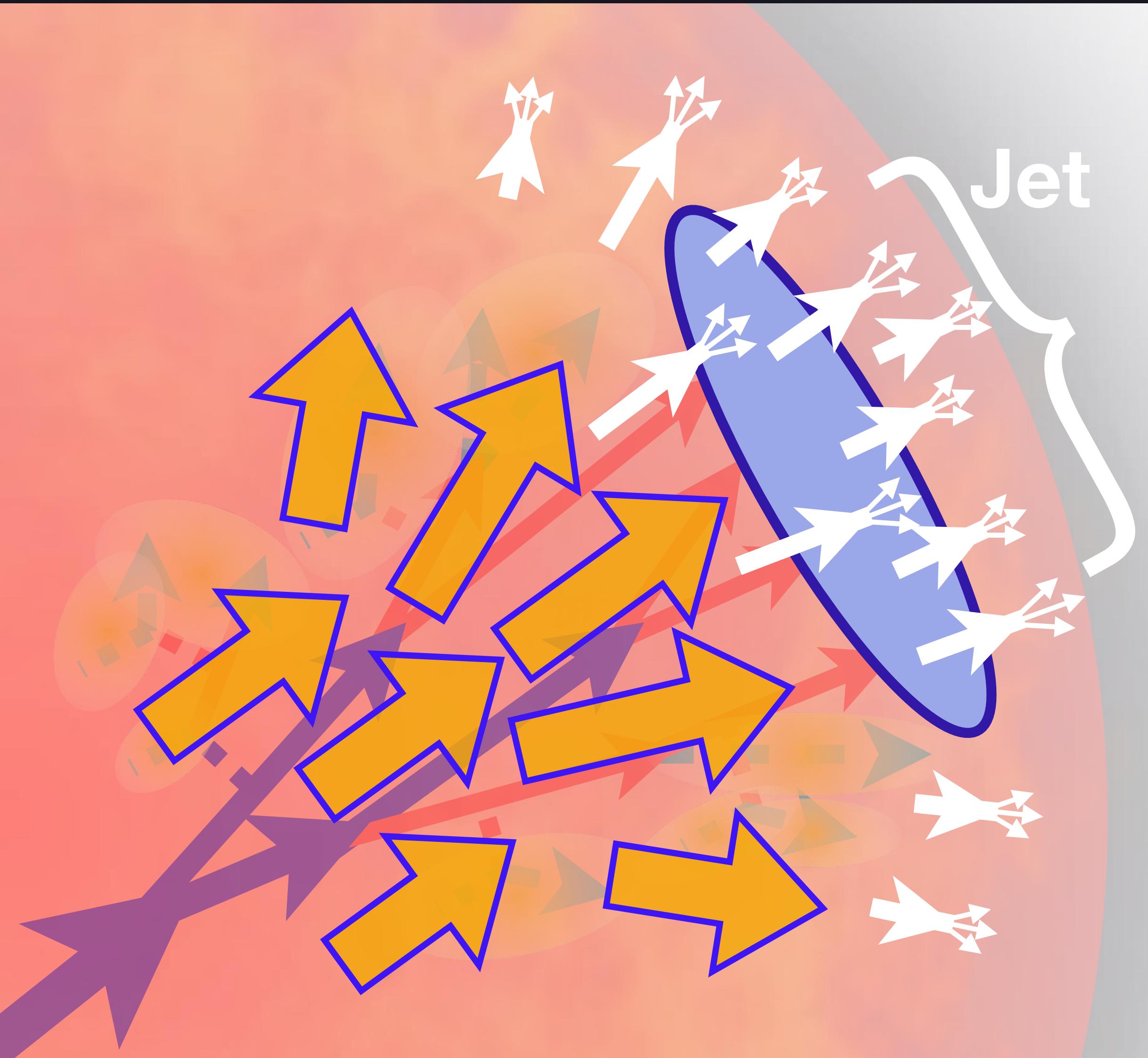
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 - Hydrodynamic response: jet induced flow

Hydro Medium Response to Jet in QGP



- **Jet energy-momentum deposition**
 - Thermalization of some energy and momentum in QGP
- **Medium excitation by the deposition**
 - Hydrodynamic flow induced by jet propagation
 - Jet energy-momentum transport mediated by QGP fluid
- **Hadrons from hydro response**
 - Soft, spread out from jet
 - Jet-correlated, cannot/should not be subtracted
 - Affect structures inside/around jet

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Hydro Medium Response to Jet in QGP

● 湧き出し項つき相対論的流体模型

- 媒質に流入したジェットのエネルギー運動量の流体発展を記述

$$\partial_\mu T^{\mu\nu}(x) = J^\nu(x)$$

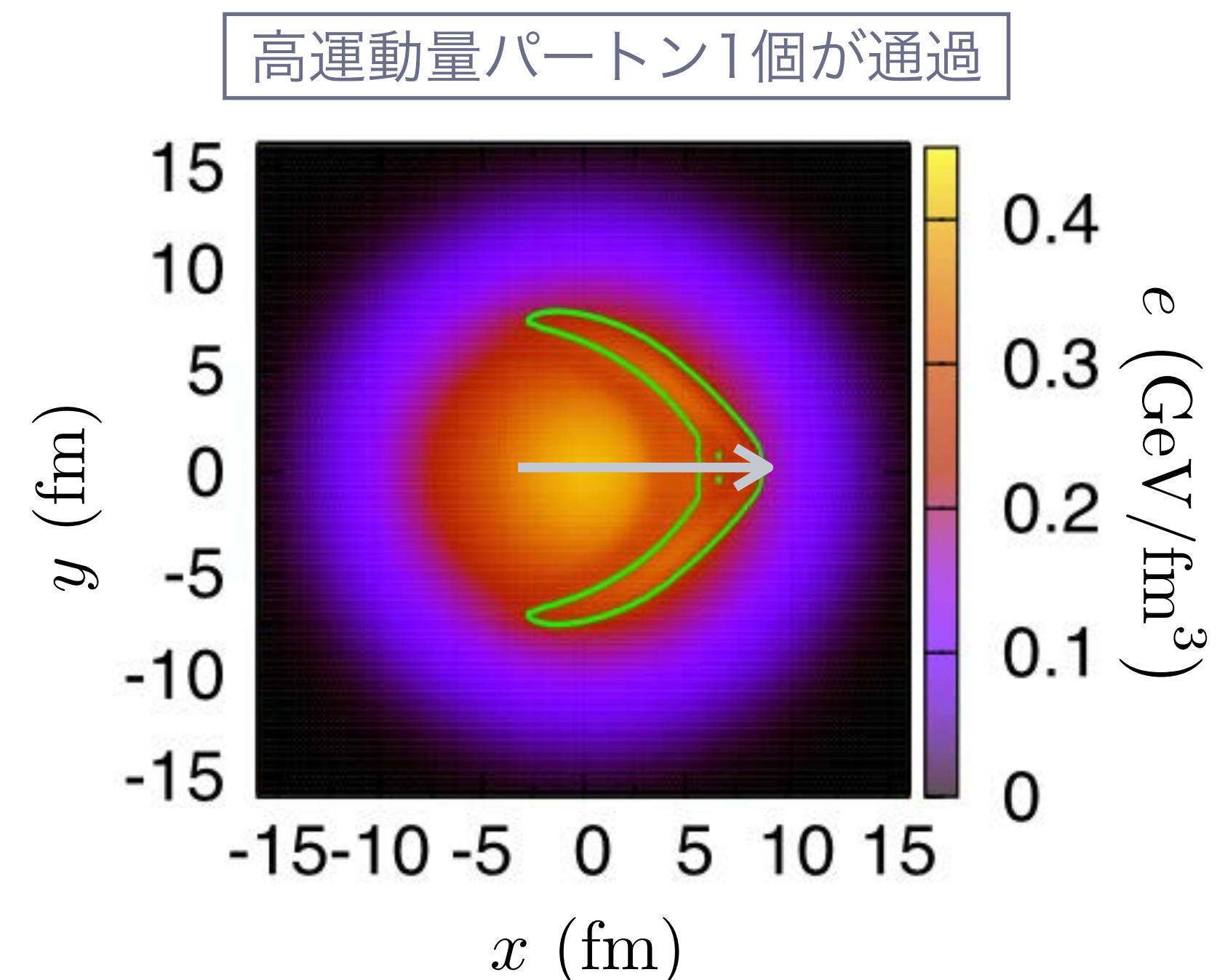
QGP 流体のエネルギー運動量テンソル

湧き出し項: ジェットから流入する4元運動量密度

● ジェットが誘起するフロー

- 衝撃波 (マッハコーン) を形成
- 流体から出てくる終状態のハドロン分布に影響

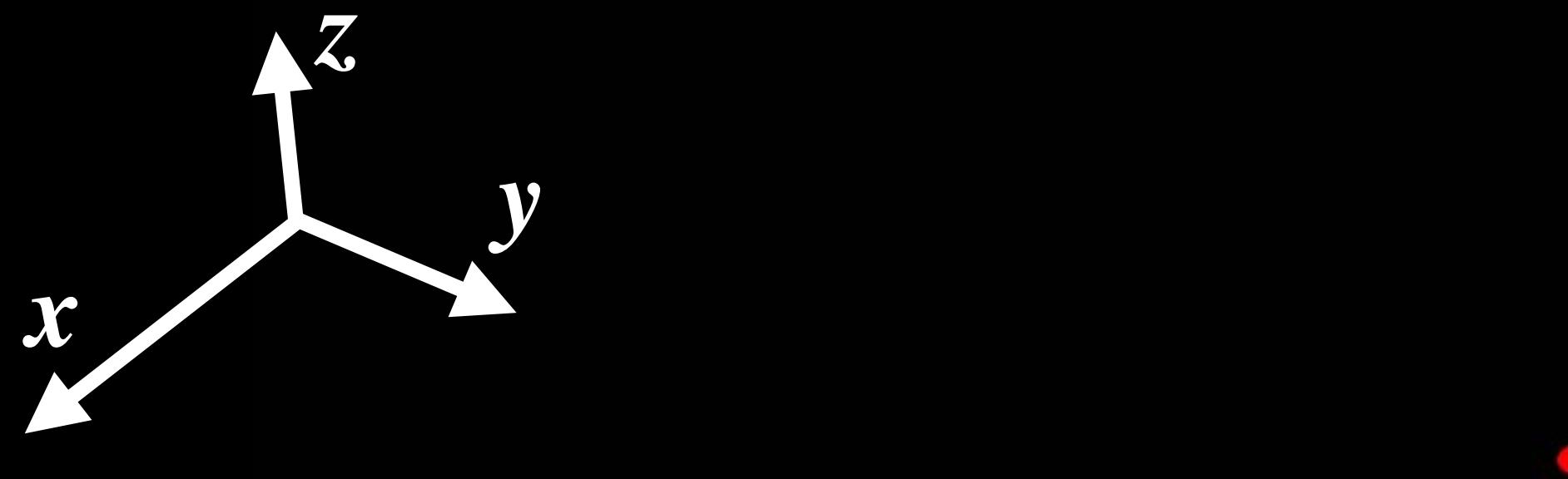
YT, Hirano, PRC 90 (2014); PRC 93, 054907 (2016)



Evolution of Hydrodynamic Medium Response

MATTER + LBT + Causal Diff. + Ideal Hydro [Static Brick, $T_{\text{brick}} = 250 \text{ MeV}$]
YT, C. Shen, A. Majumder, PRC 106, L021902 (2022)

- Jet-Induced flow induced by a parton shower propagating in the x direction

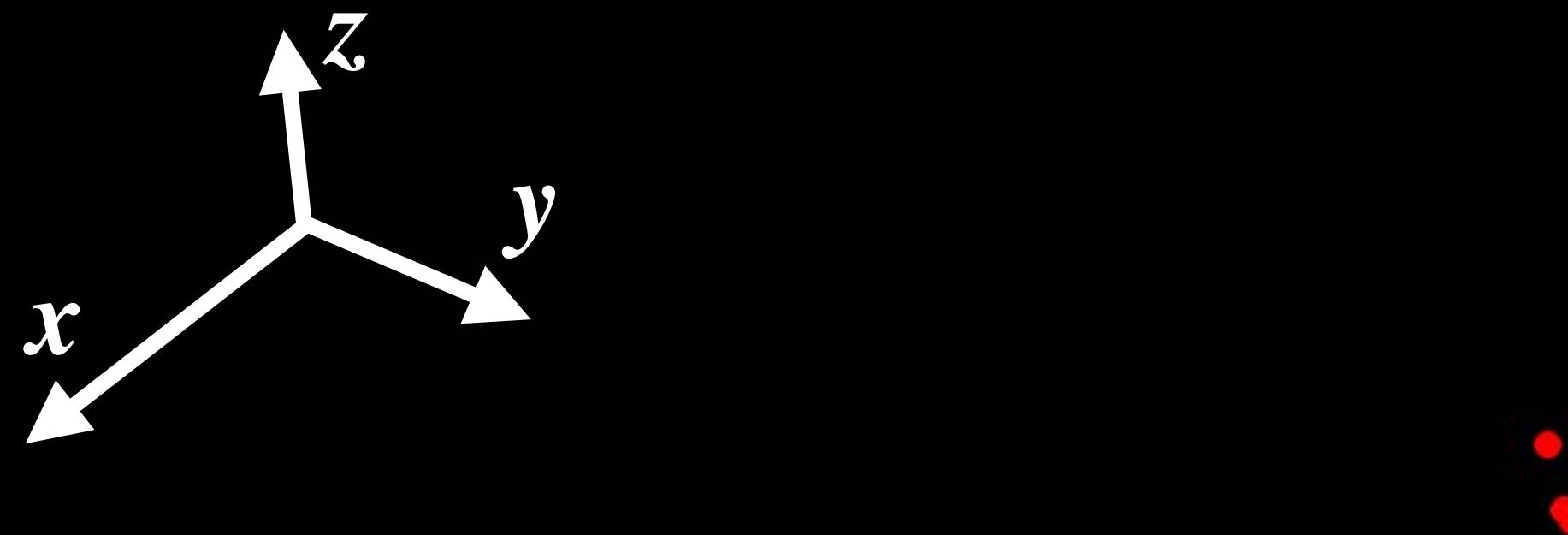


Orange: Region with $T > 250 \text{ MeV}$
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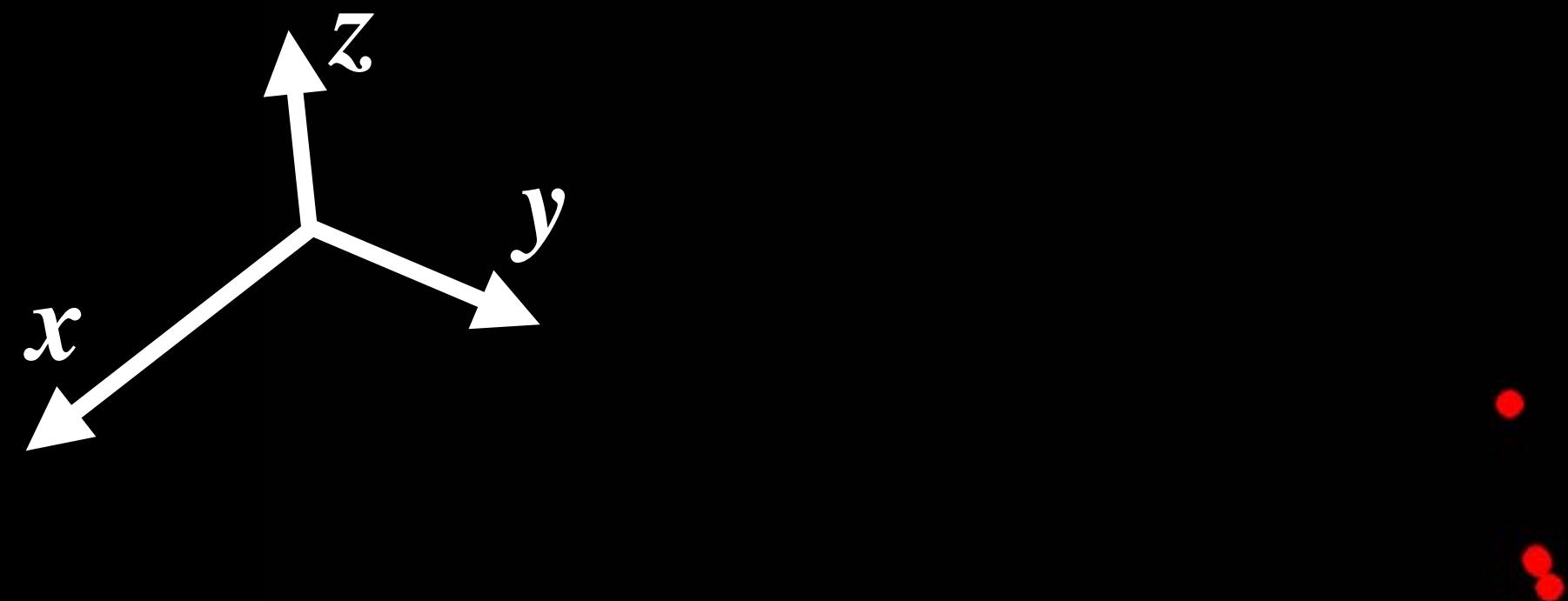


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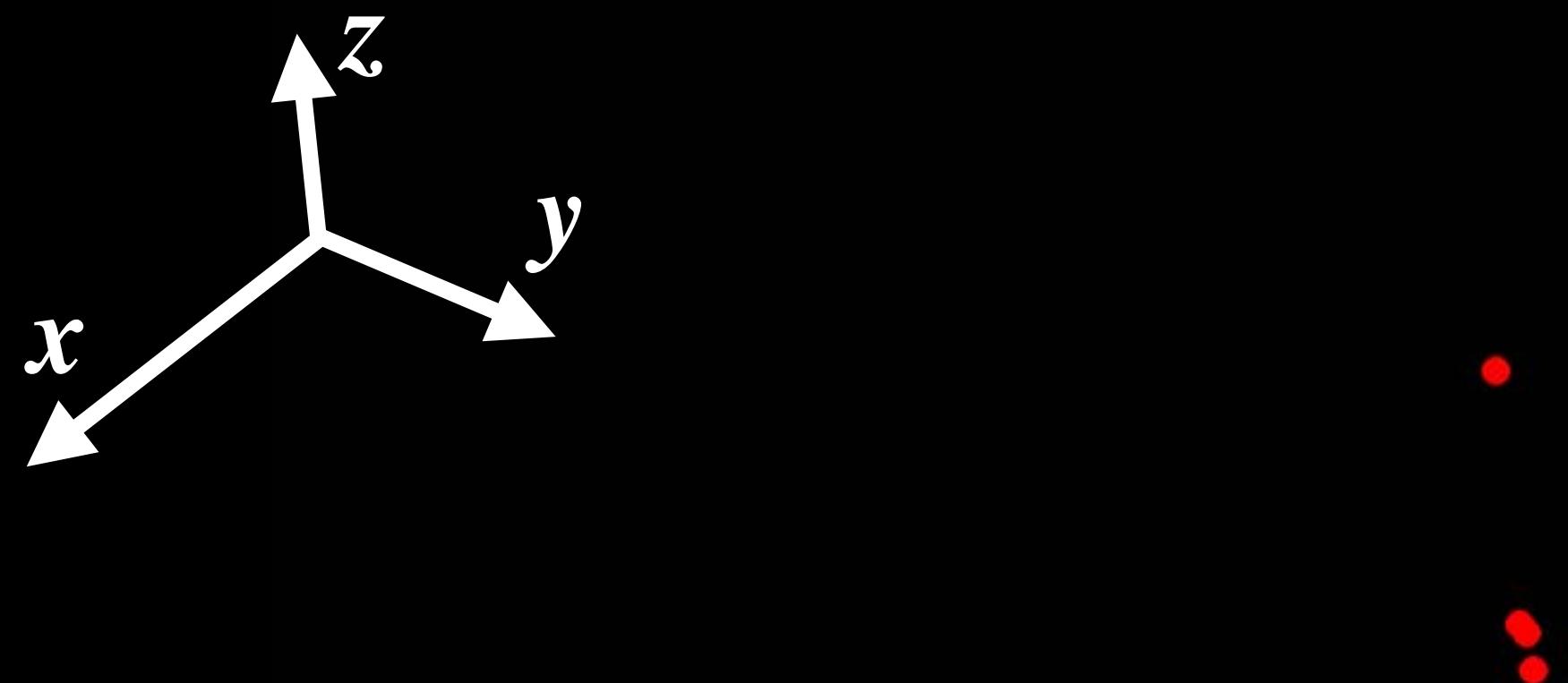


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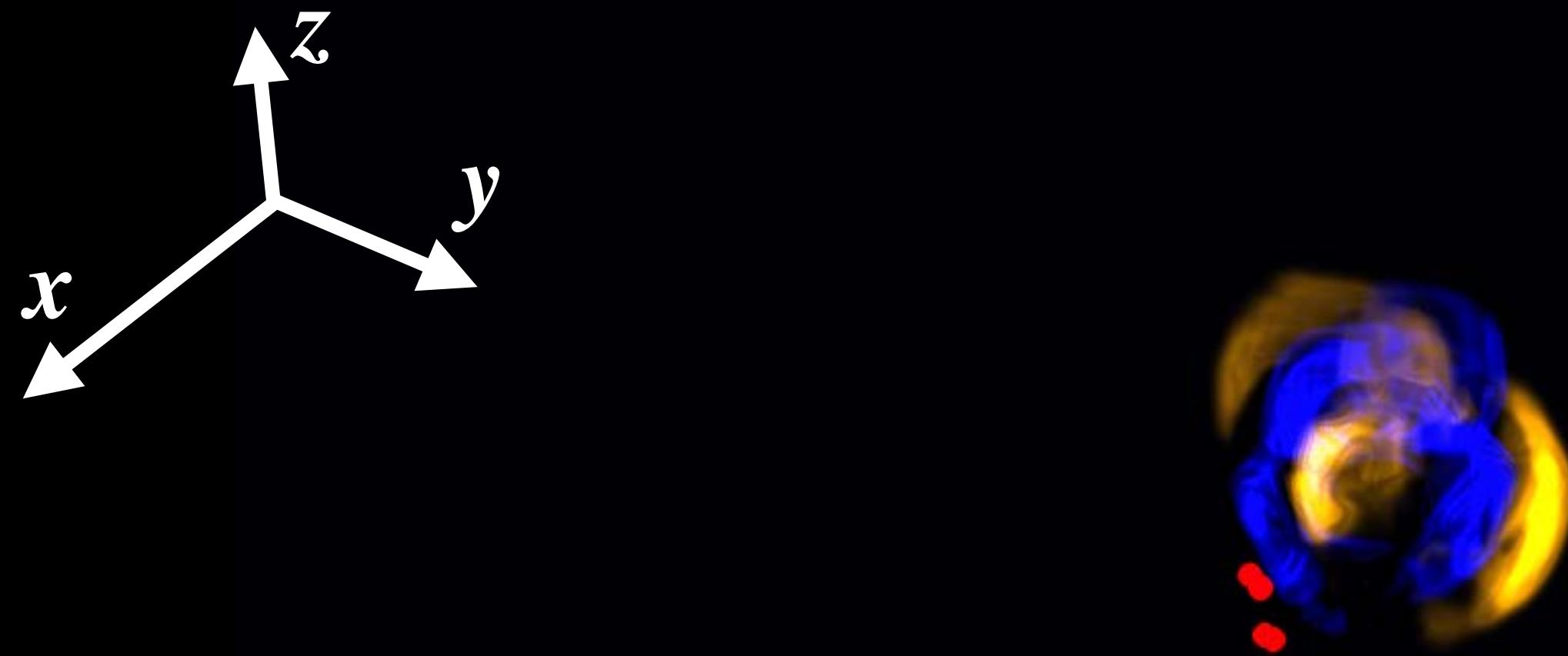


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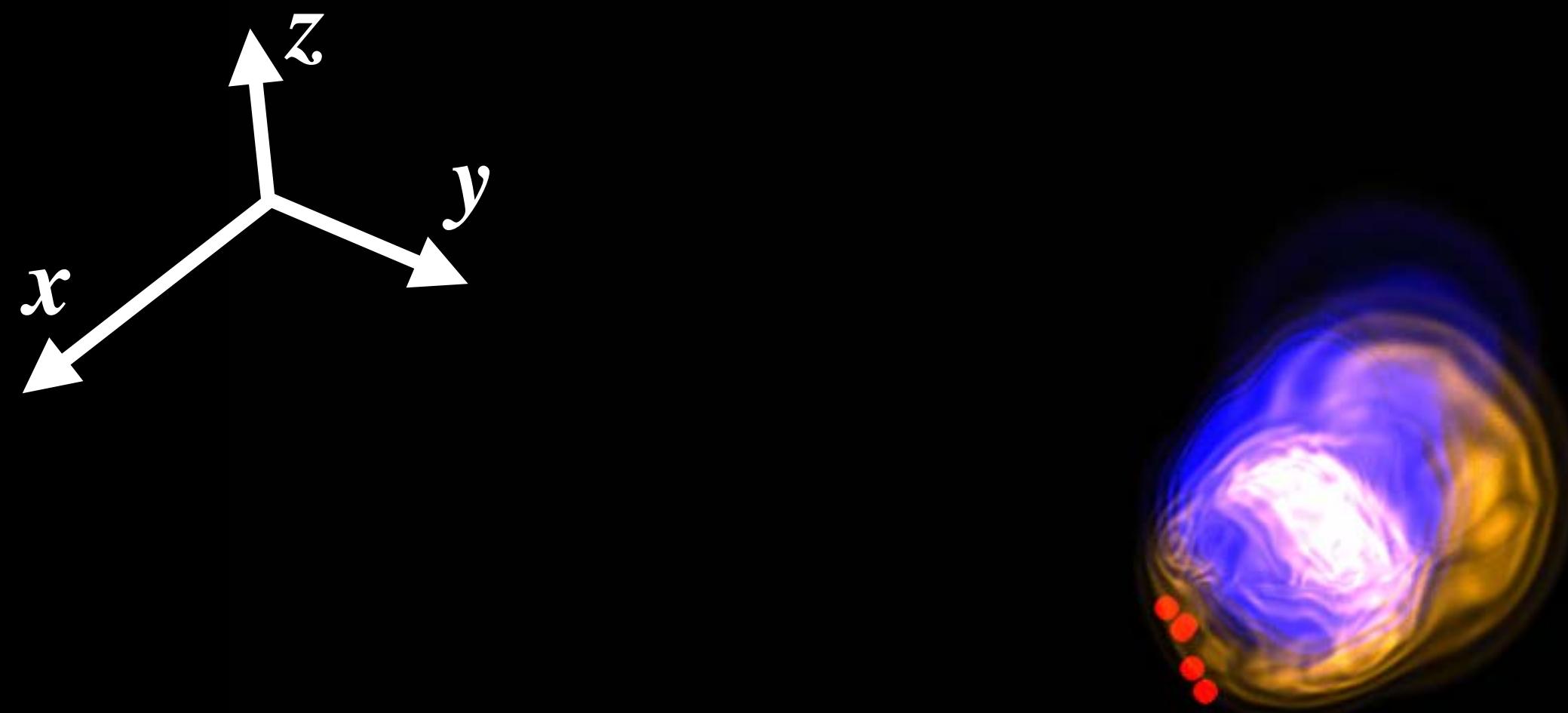


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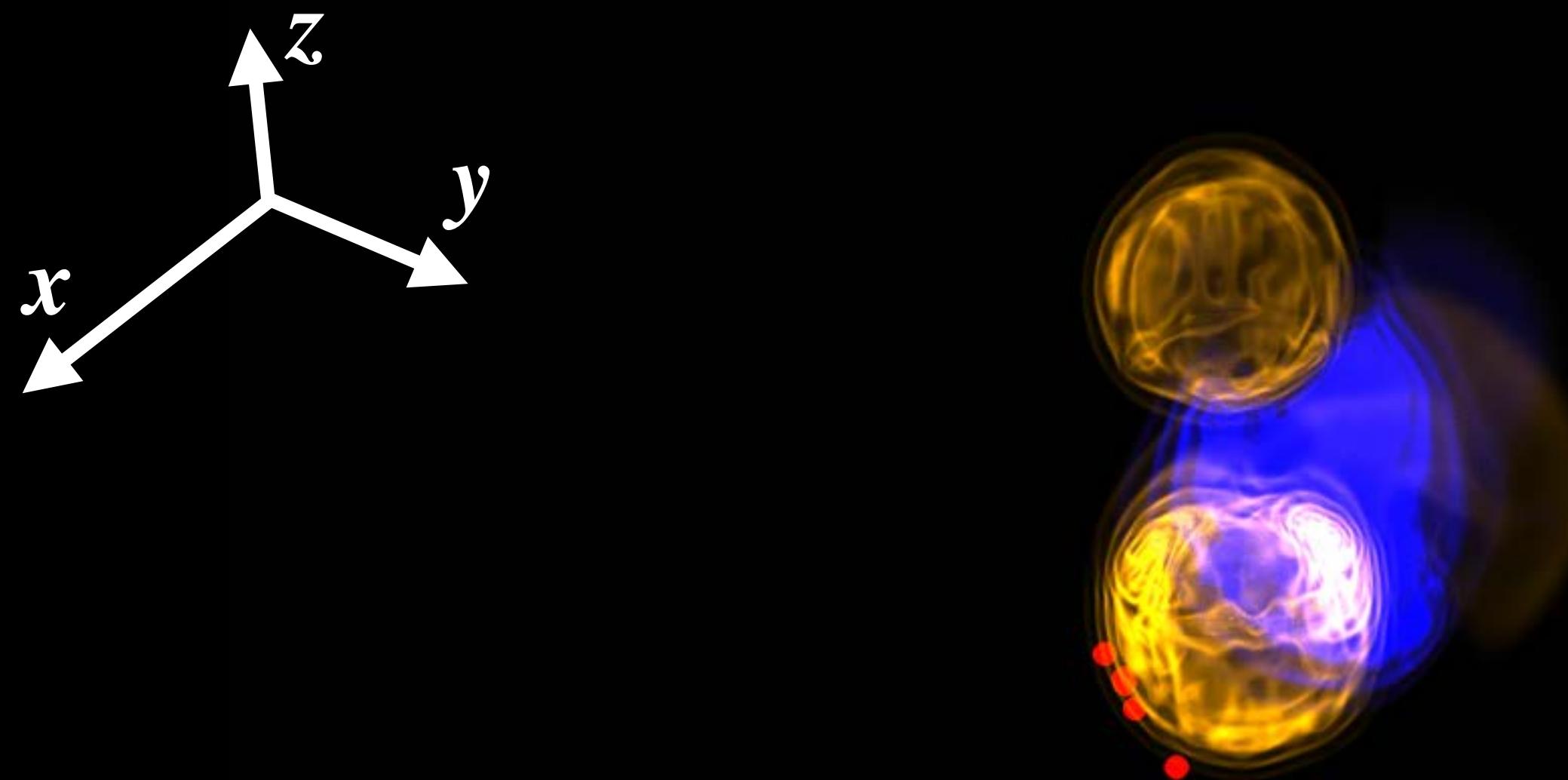


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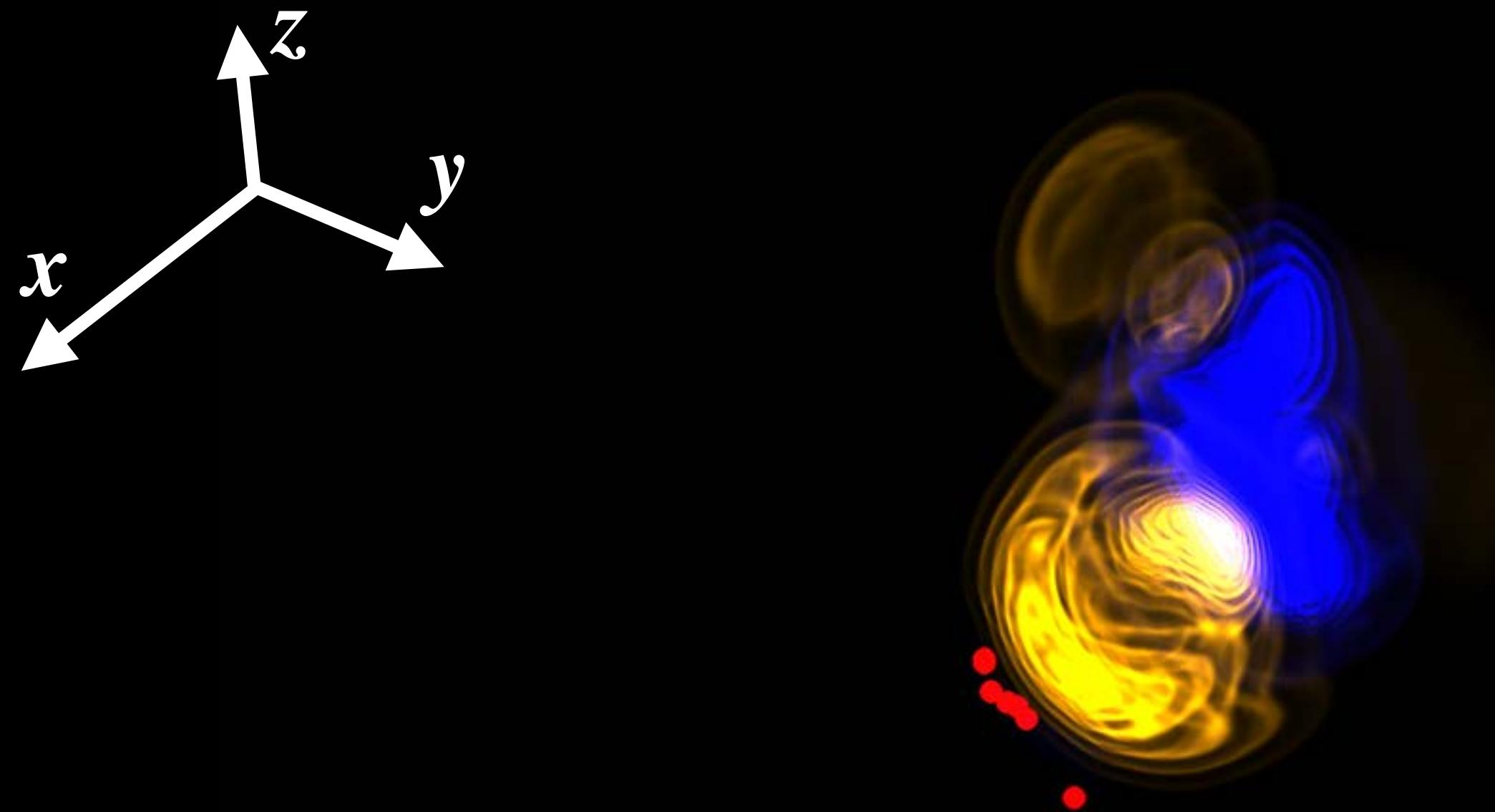


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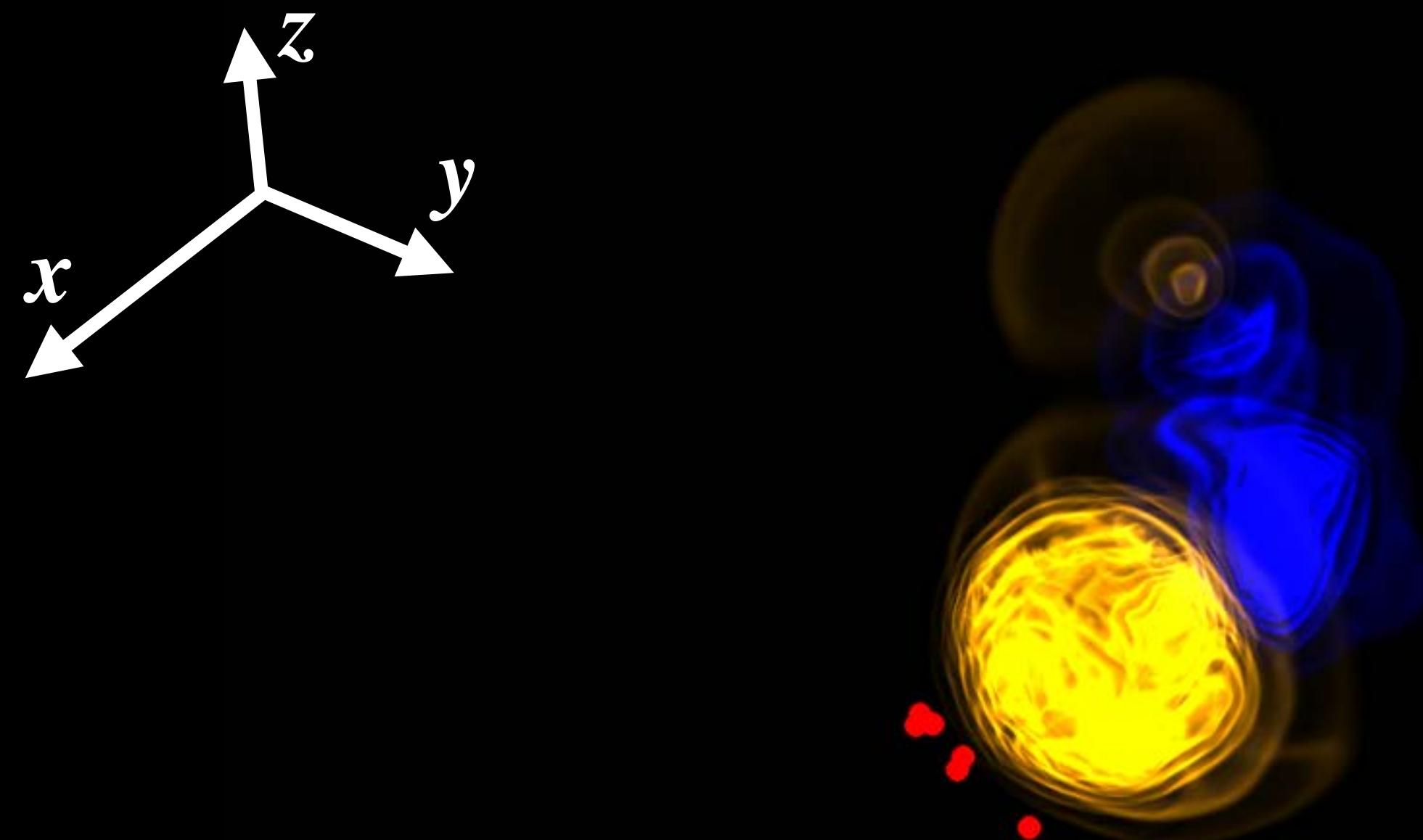


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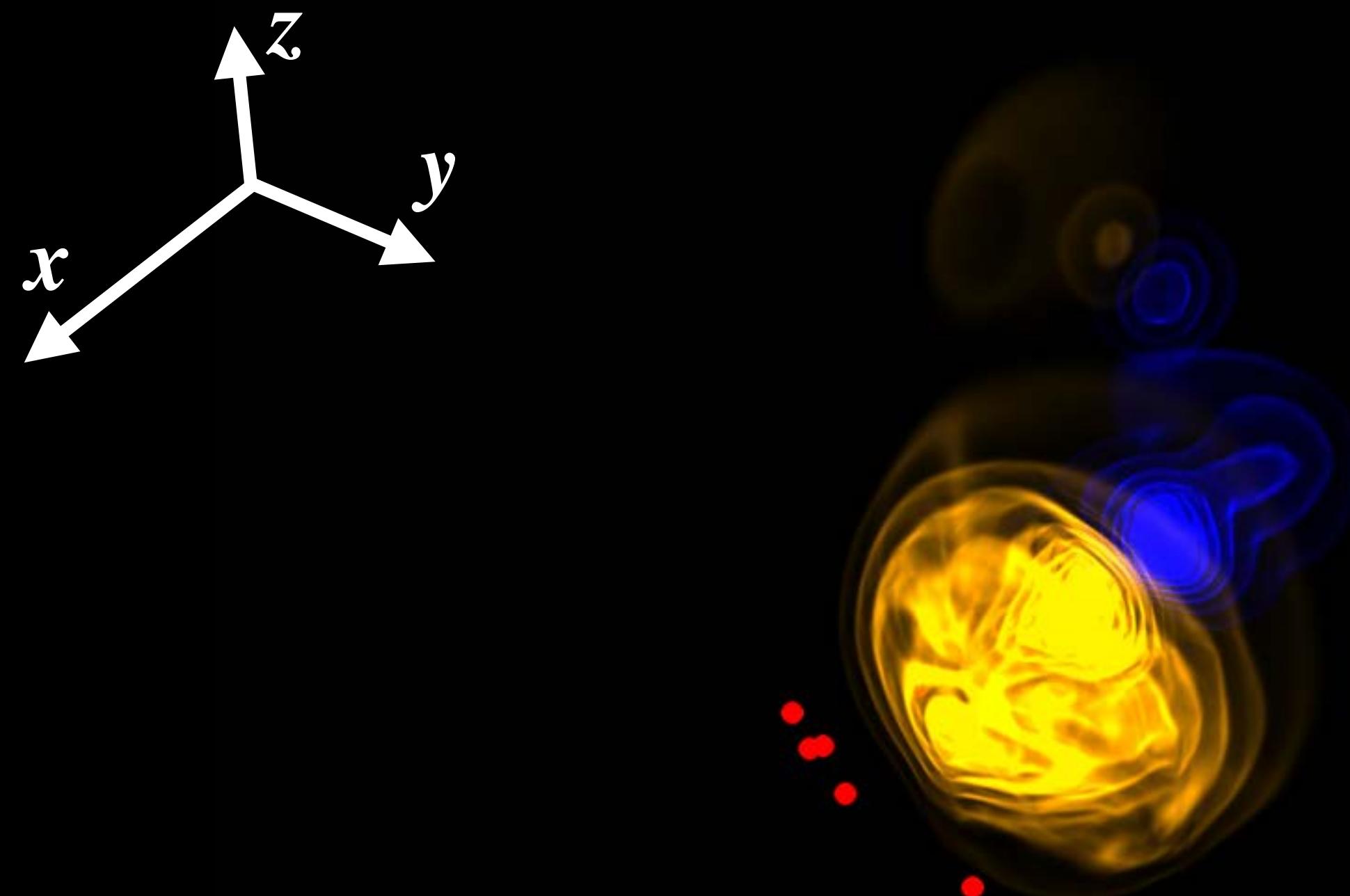


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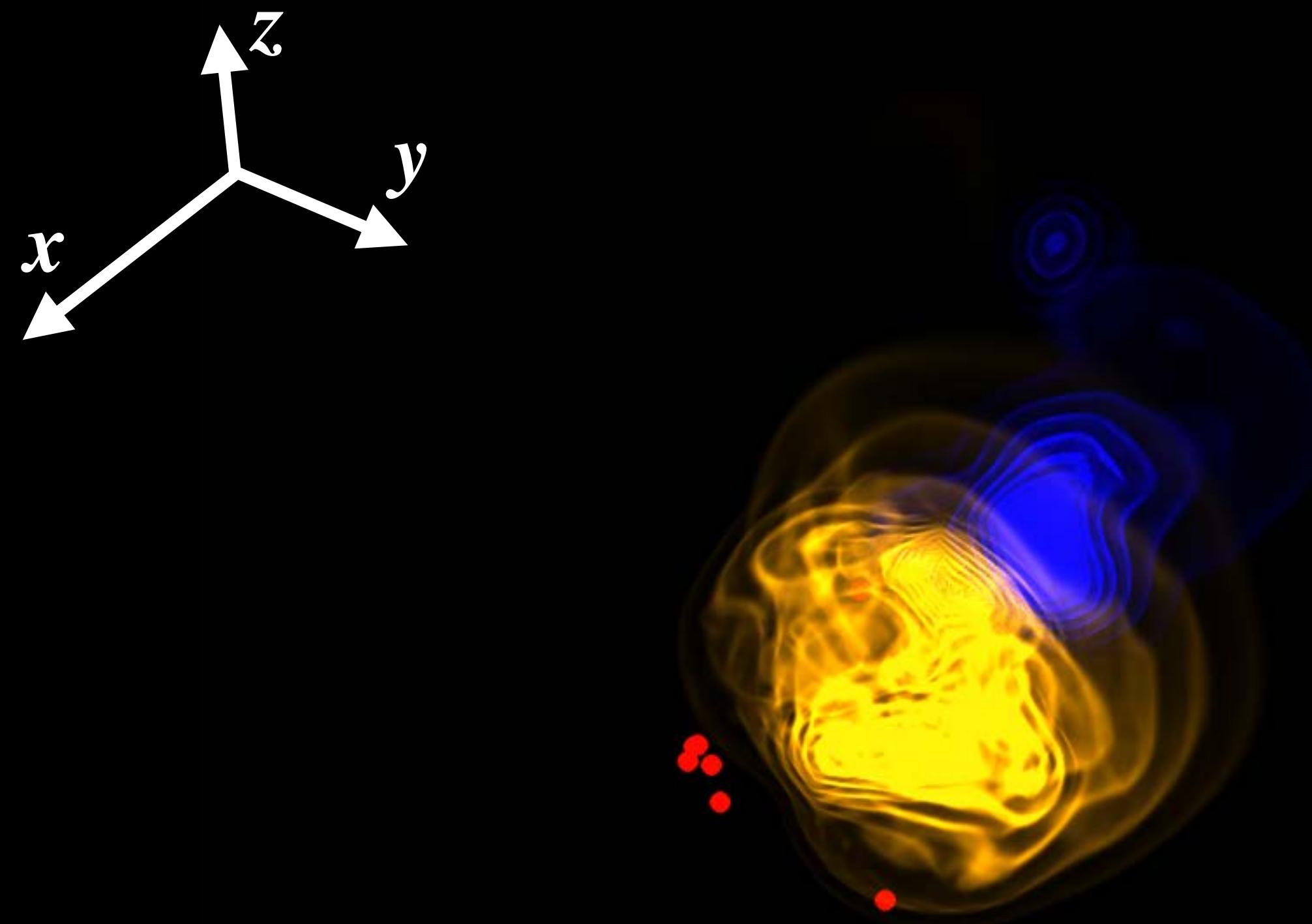


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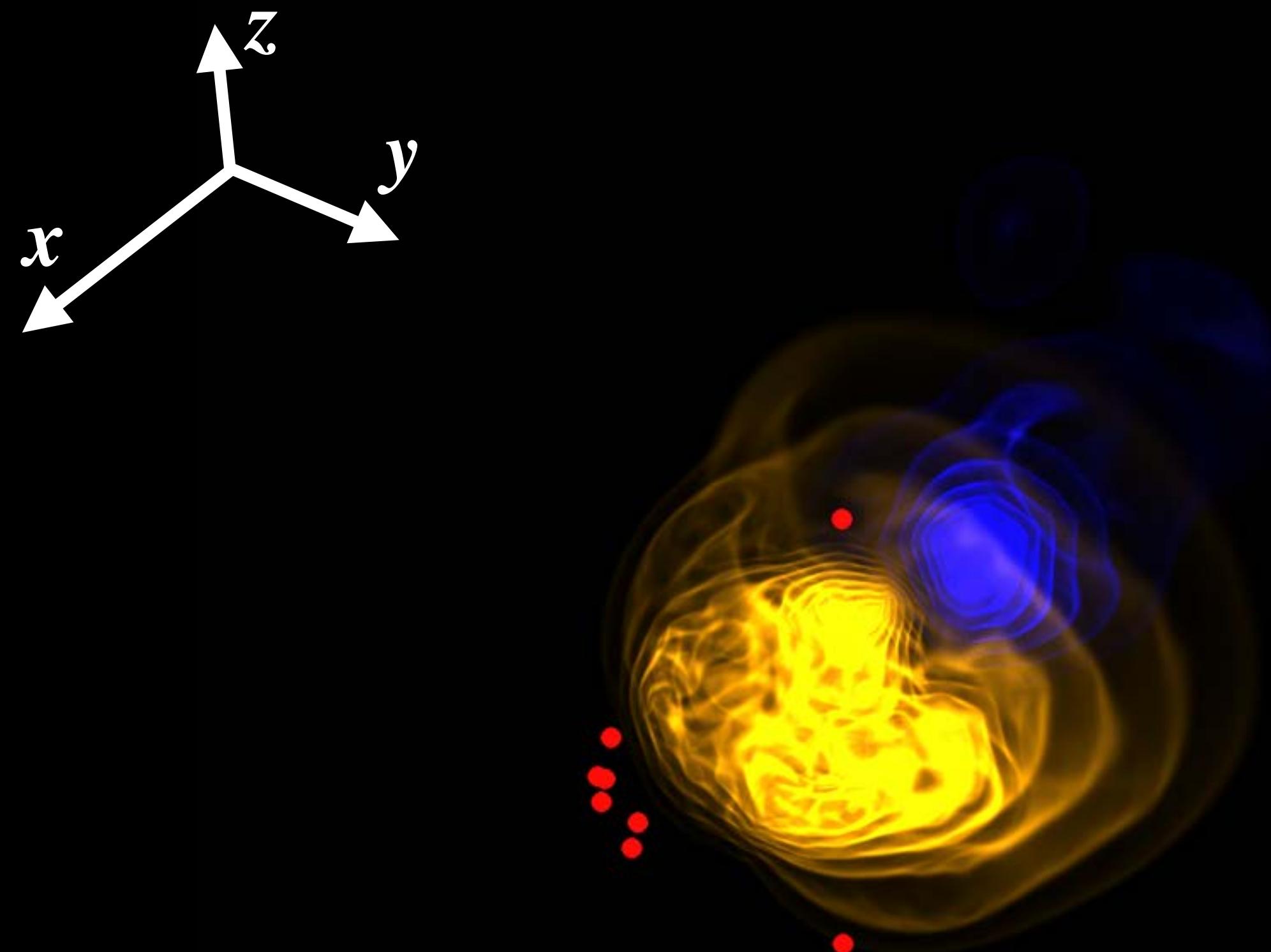


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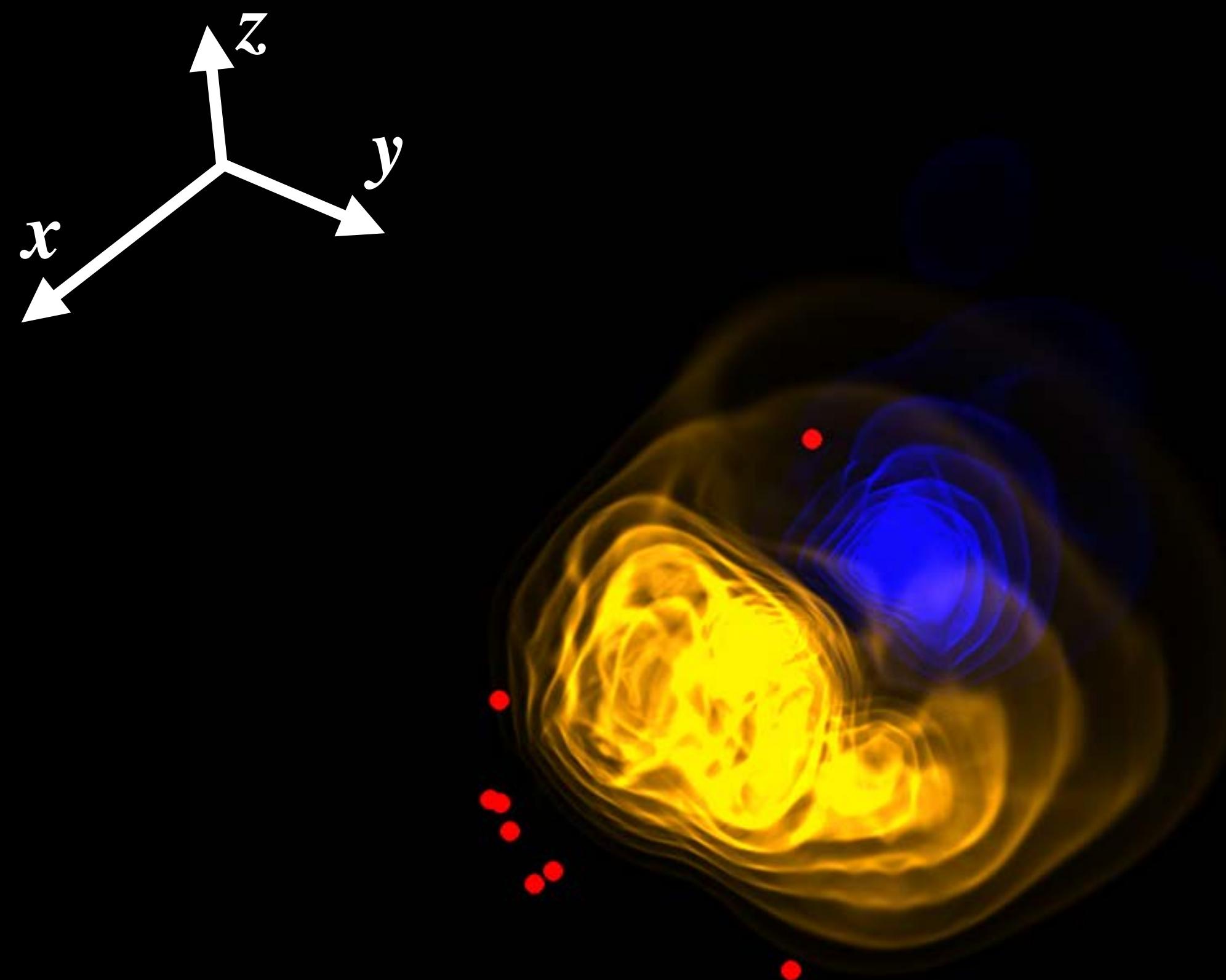


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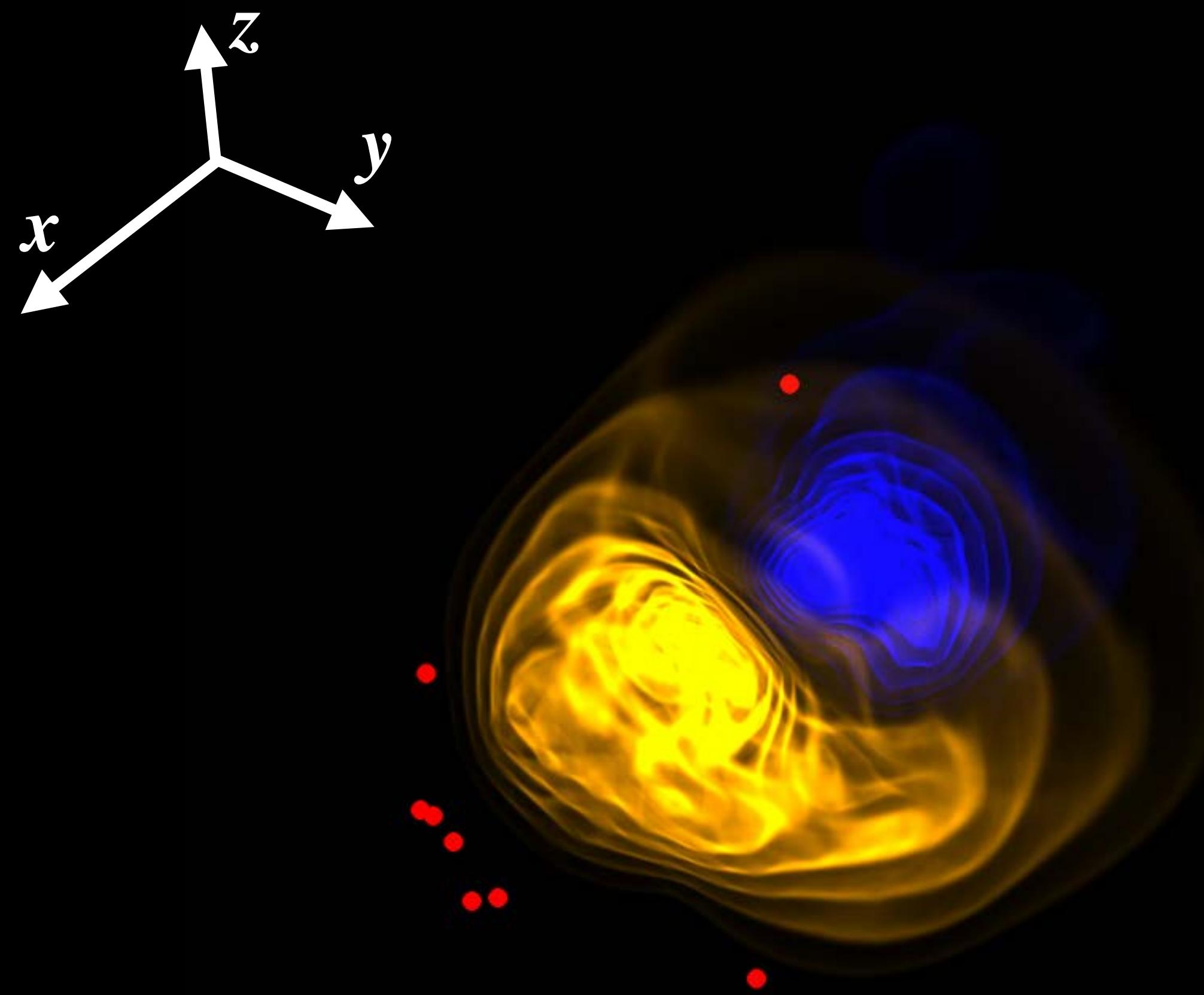


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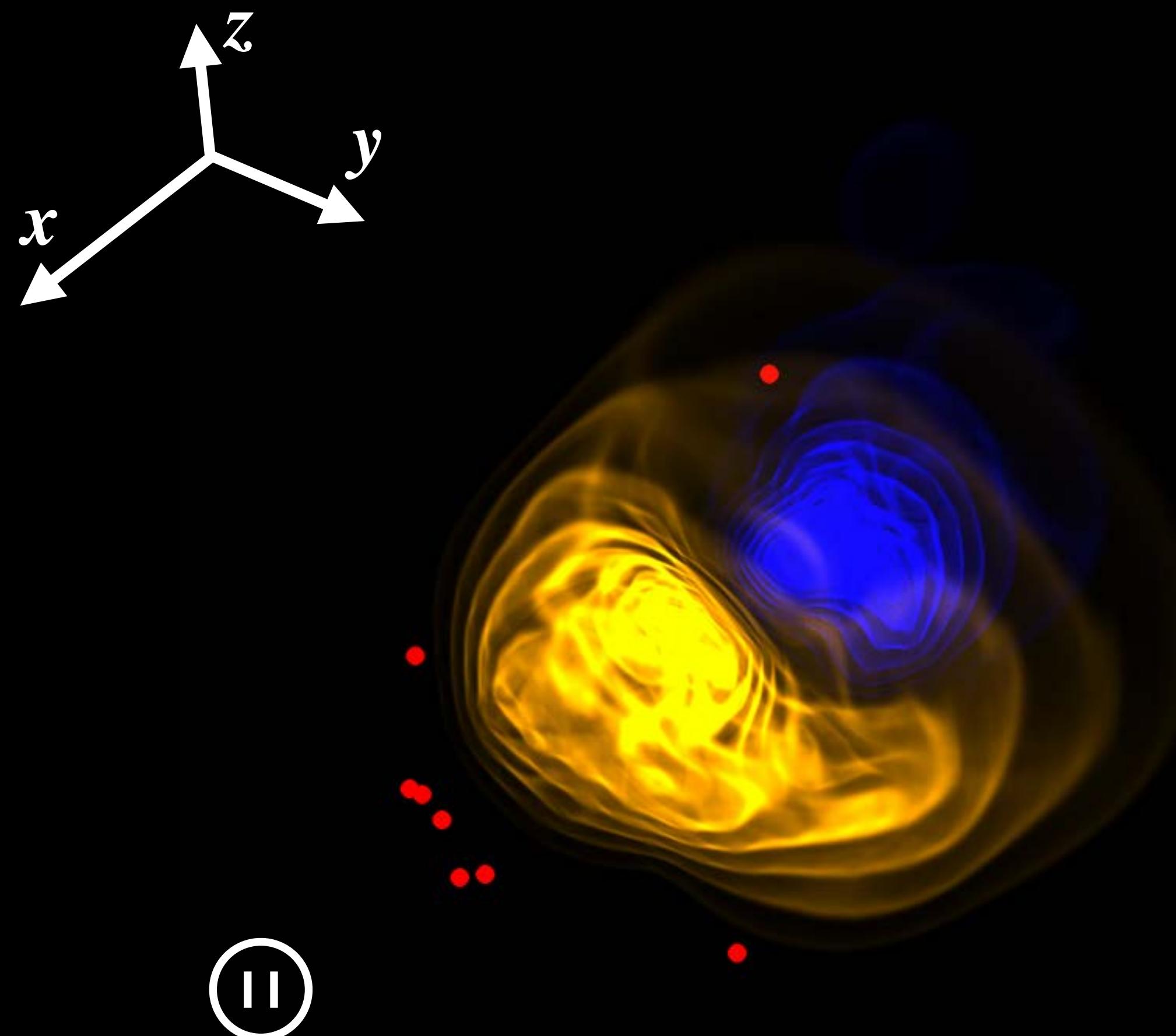


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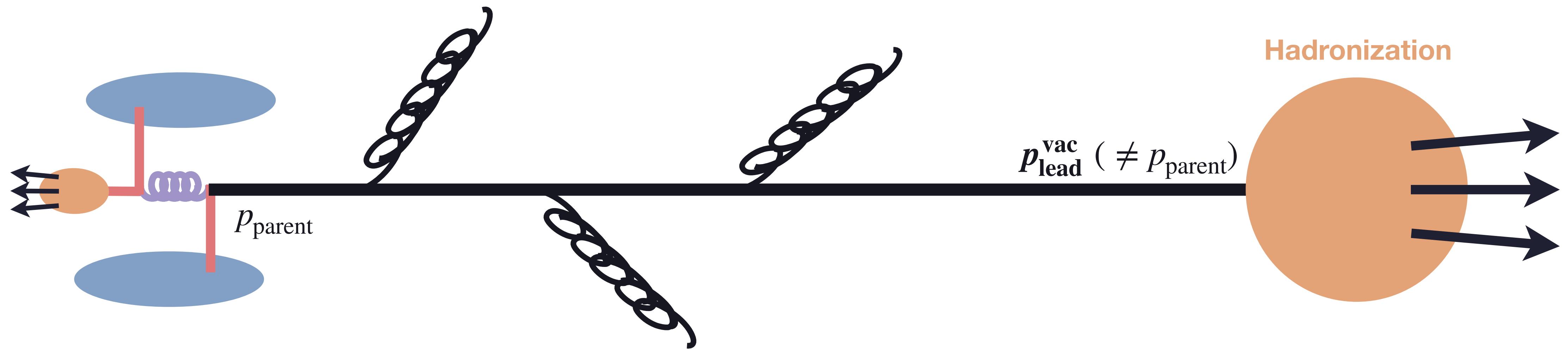


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媒質効果の測定量への影響

—エネルギー損失と R_{AA} —

パートンエネルギー損失



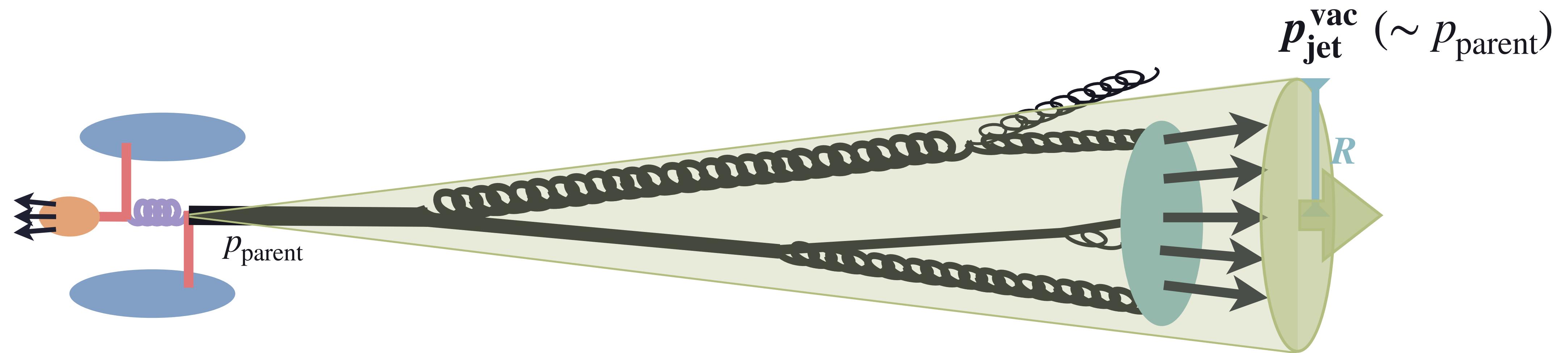
パートンエネルギー損失



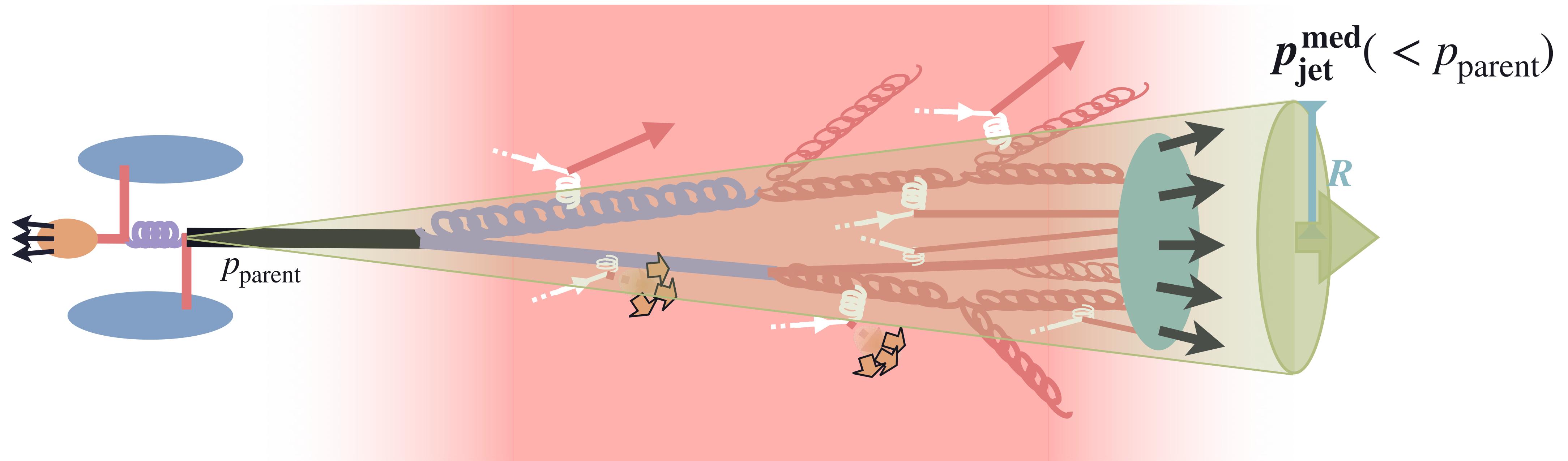
- More energy carried away via radiations enhanced by the medium (dominant)
- Drag force due to the elastic component of parton-medium interaction
- Mechanism of the energy loss of hadron from the leading parton

実験で直接計測はできない (leading hadron spectrum に間接的に見える)

ジェットエネルギー"損失"



ジェットエネルギー"損失"



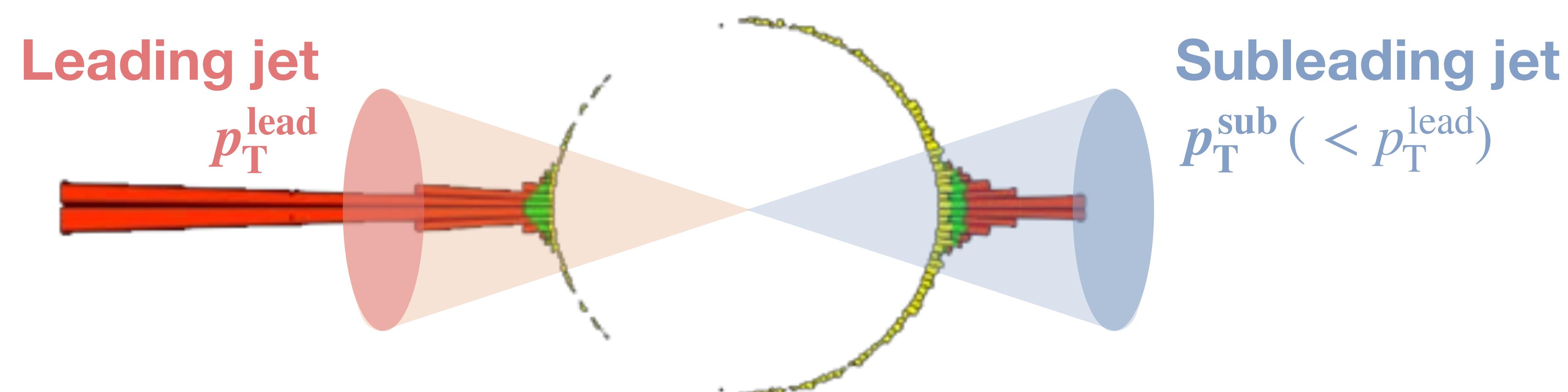
- Lost energy = Energy carried away from the jet cone
- Dominated by (daughter) parton-medium interaction at large angles
- Almost directly affect on the energy loss of reconstructed (hadron) jet

エネルギー損失というより、構造・分布の変化(ブロードニング)

ジェットエネルギー"損失"

CMS ('11)

- >8 GeV
- 4-8 GeV
- 1-4 GeV



● Full picture of jet quenching in heavy-ion collisions

- Re-distribution of the jet energy and momentum
- Essentiality of energy-momentum conservation through the whole process

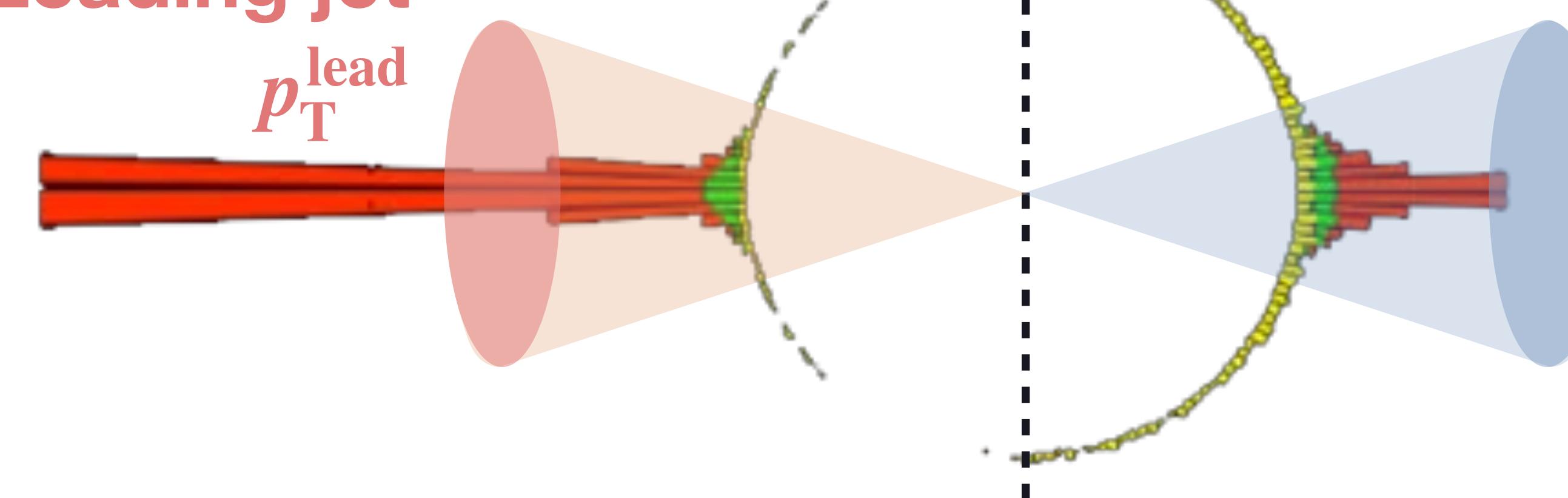
ジェットエネルギー"損失"

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Leading jet

p_T^{lead}



Balance

$$\sum_{i \in H_{\text{lead}}} \vec{p}_i \simeq - \sum_{i \in H_{\text{sub}}} \vec{p}_i$$

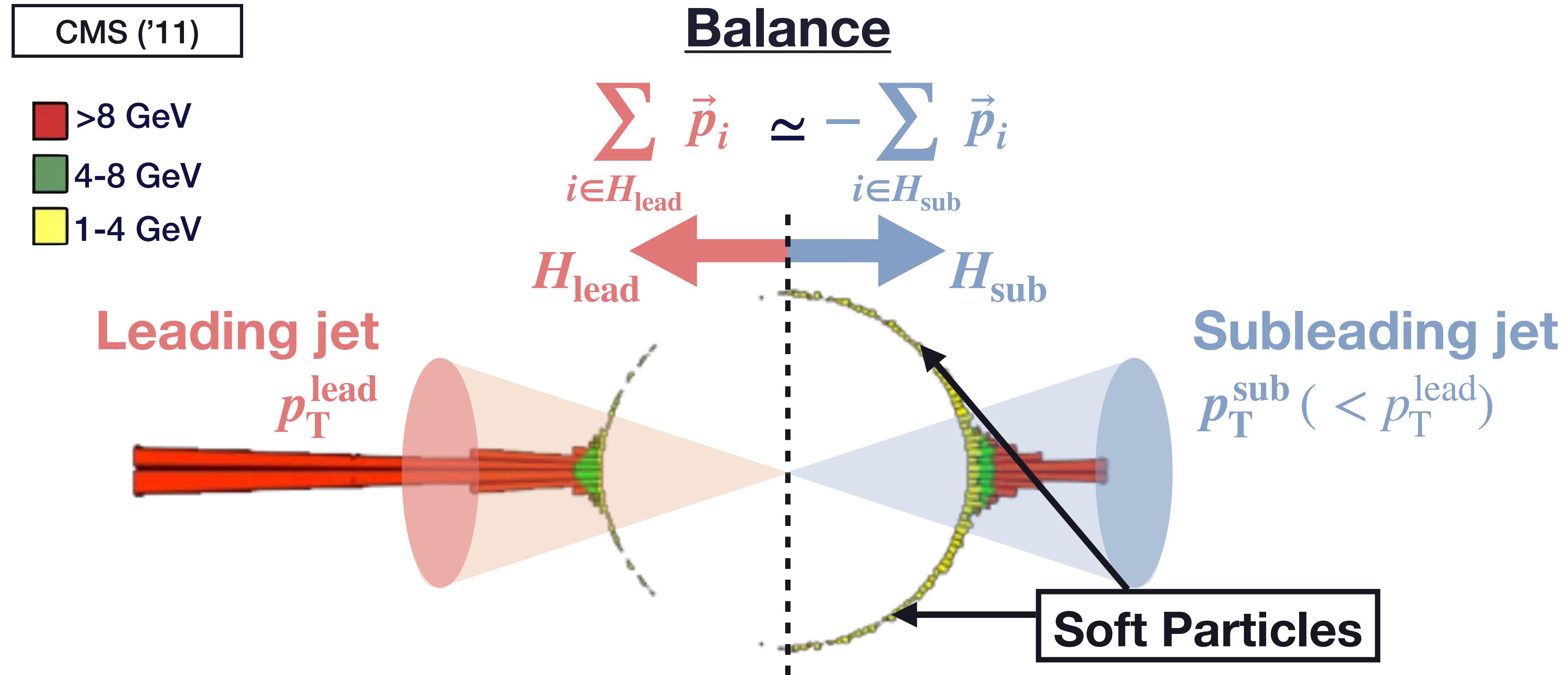
Subleading jet

$p_T^{\text{sub}} (< p_T^{\text{lead}})$

● Full picture of jet quenching in heavy-ion collisions

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ジェットエネルギー"損失"

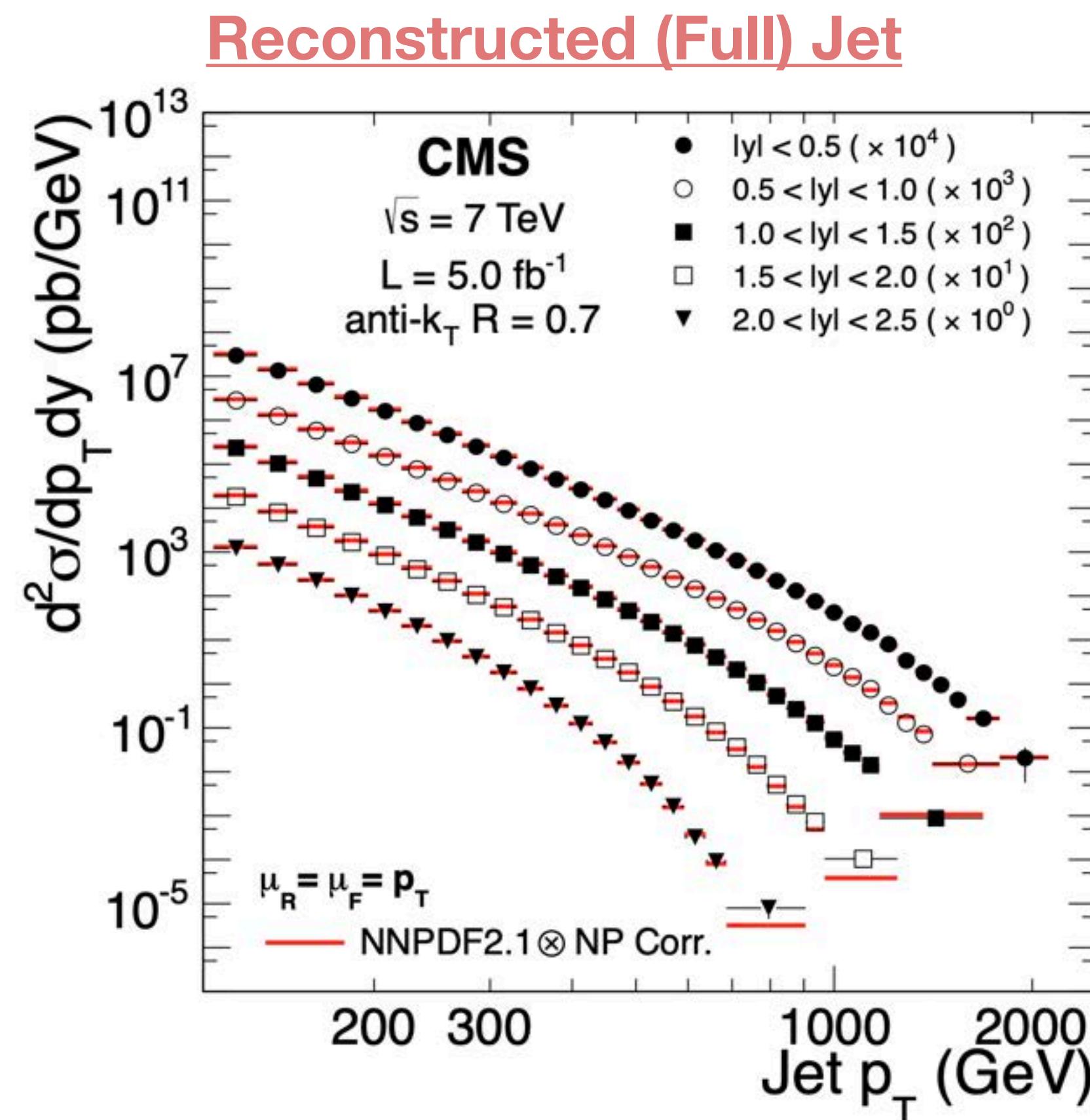
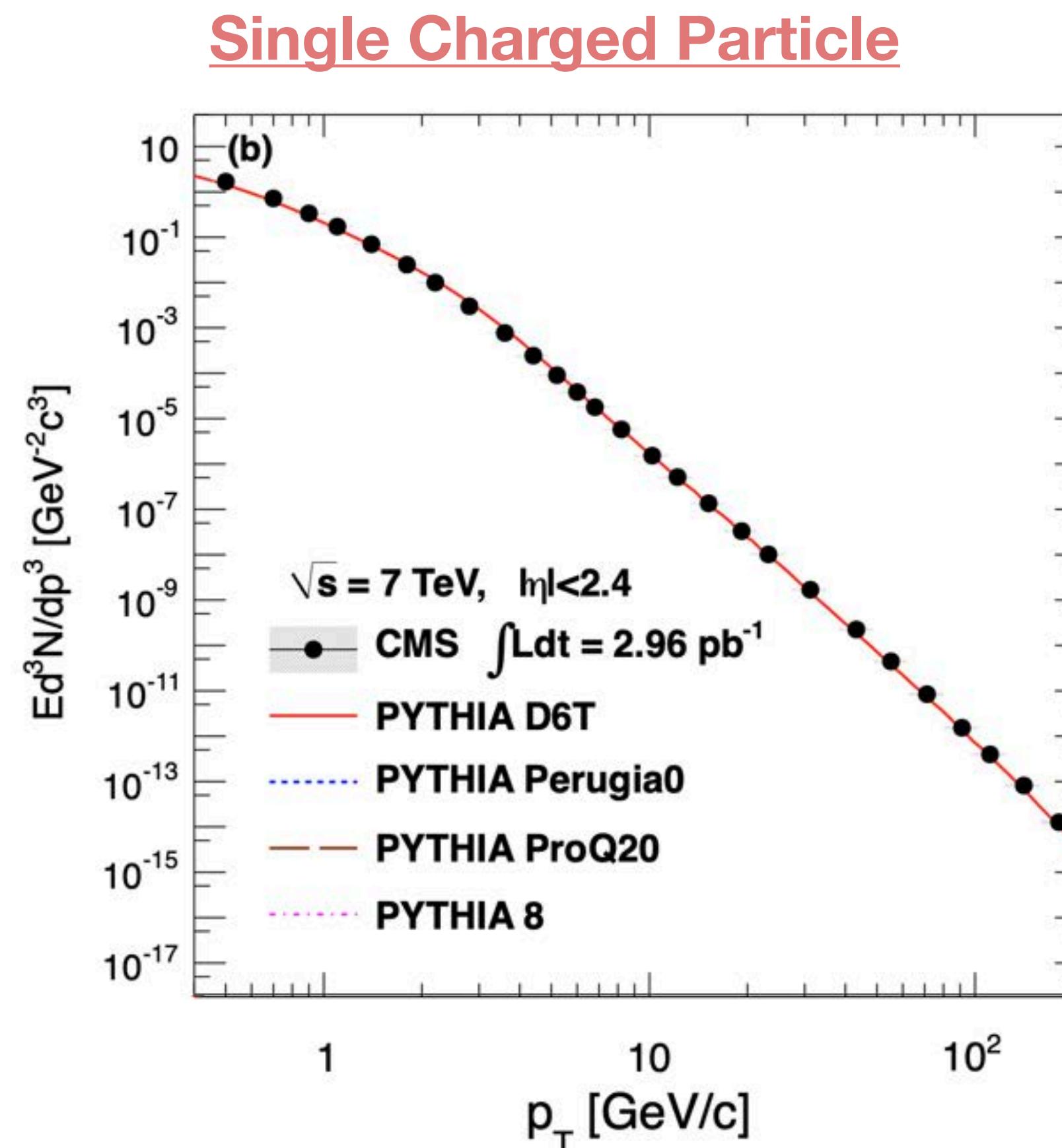


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Nuclear Modification Factor R_{AA}

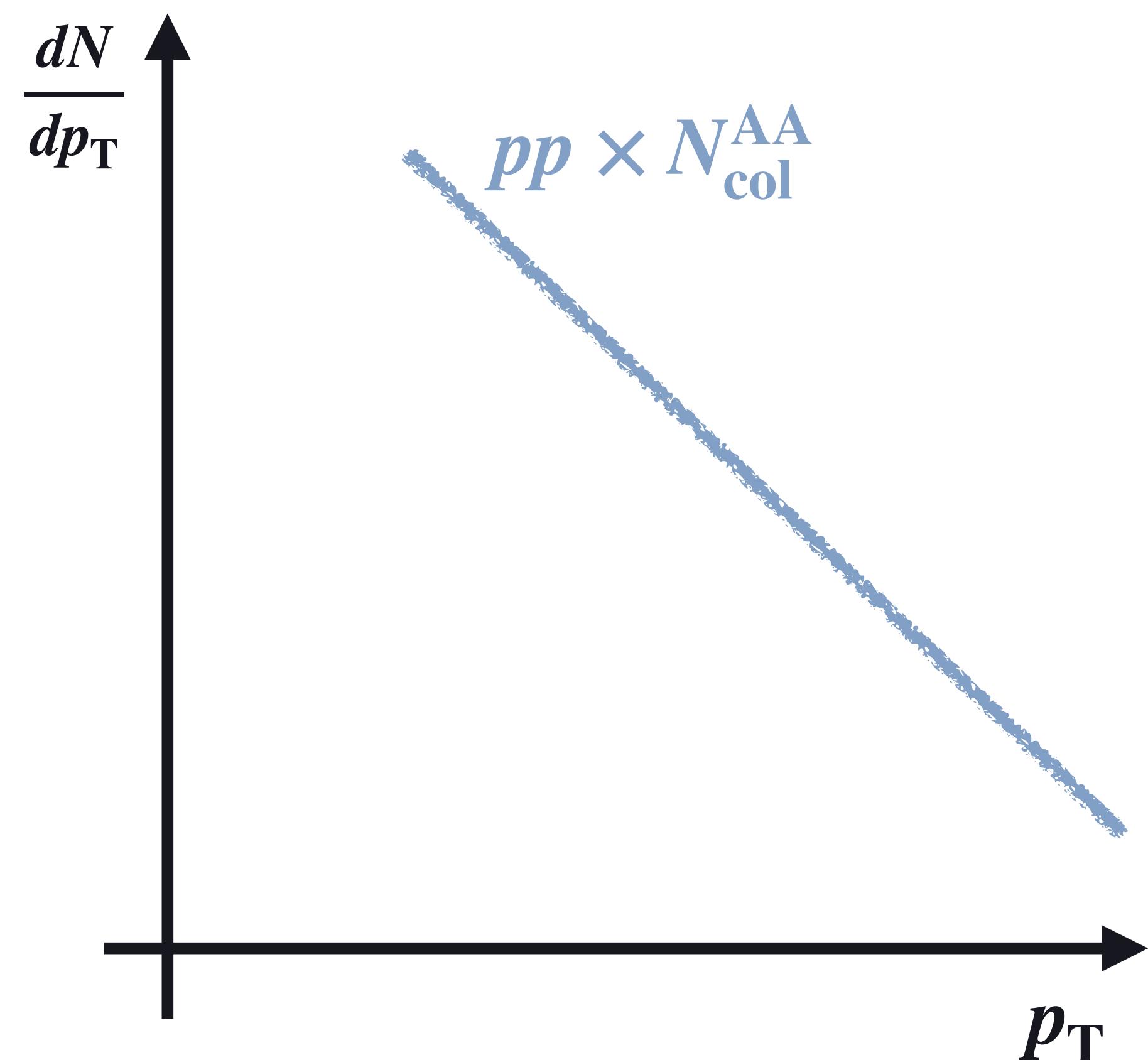
- Spectra of jets and high- p_T hadrons



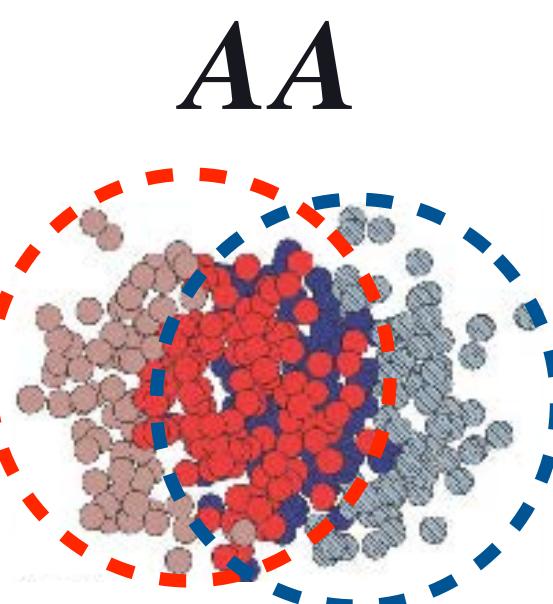
Steeply decreasing functions of p_T

Nuclear Modification Factor R_{AA}

- Nuclear modification factor R_{AA}

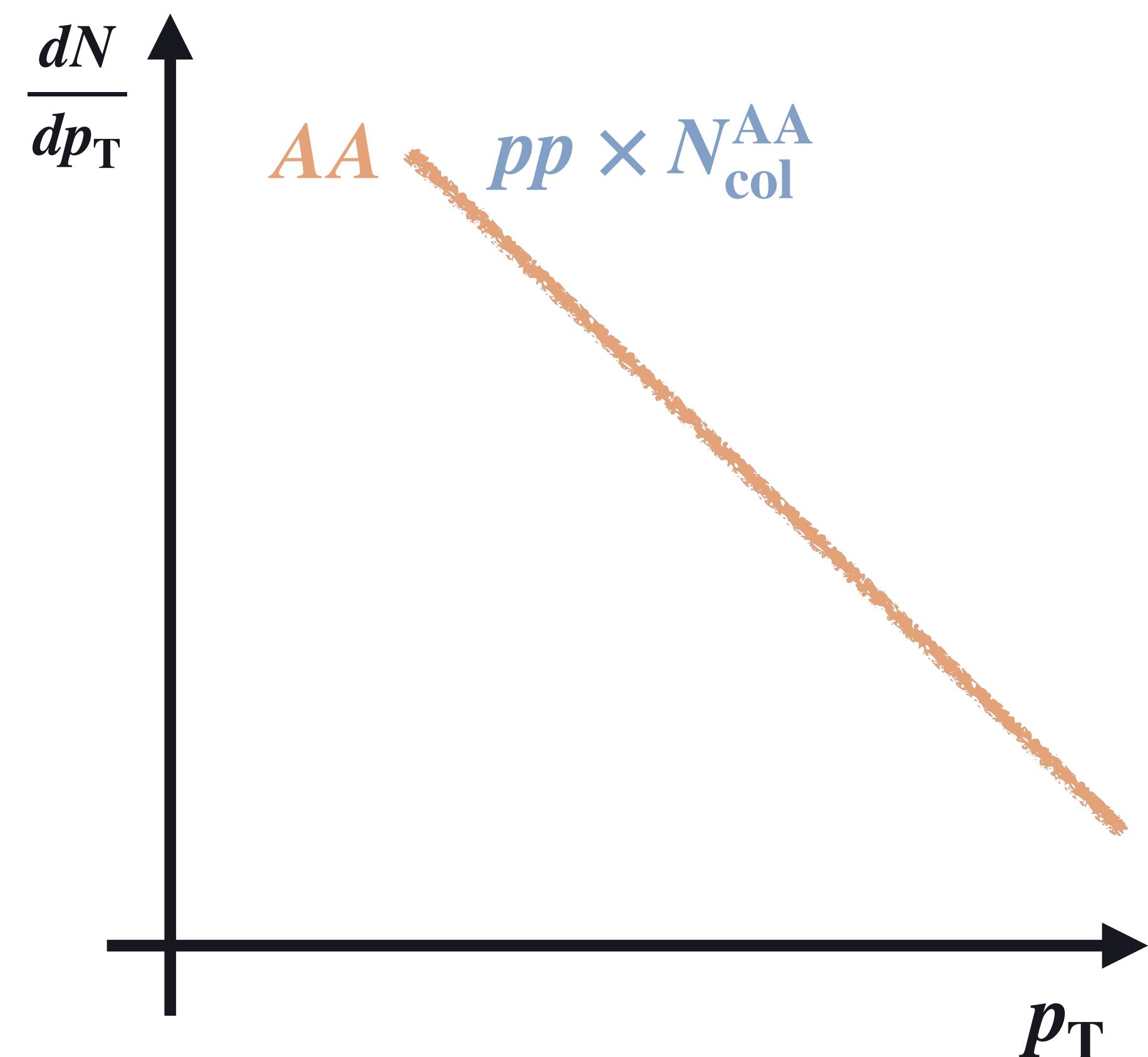


$N_{\text{coll}}^{\text{AA}}$: Number of nucleon-nucleon collisions in an AA collision

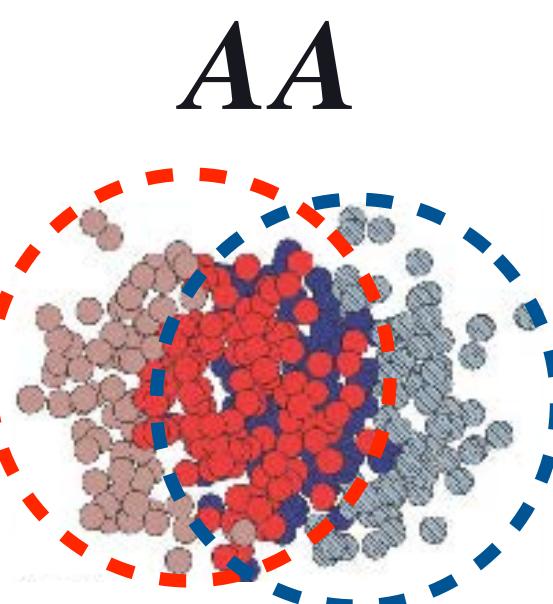


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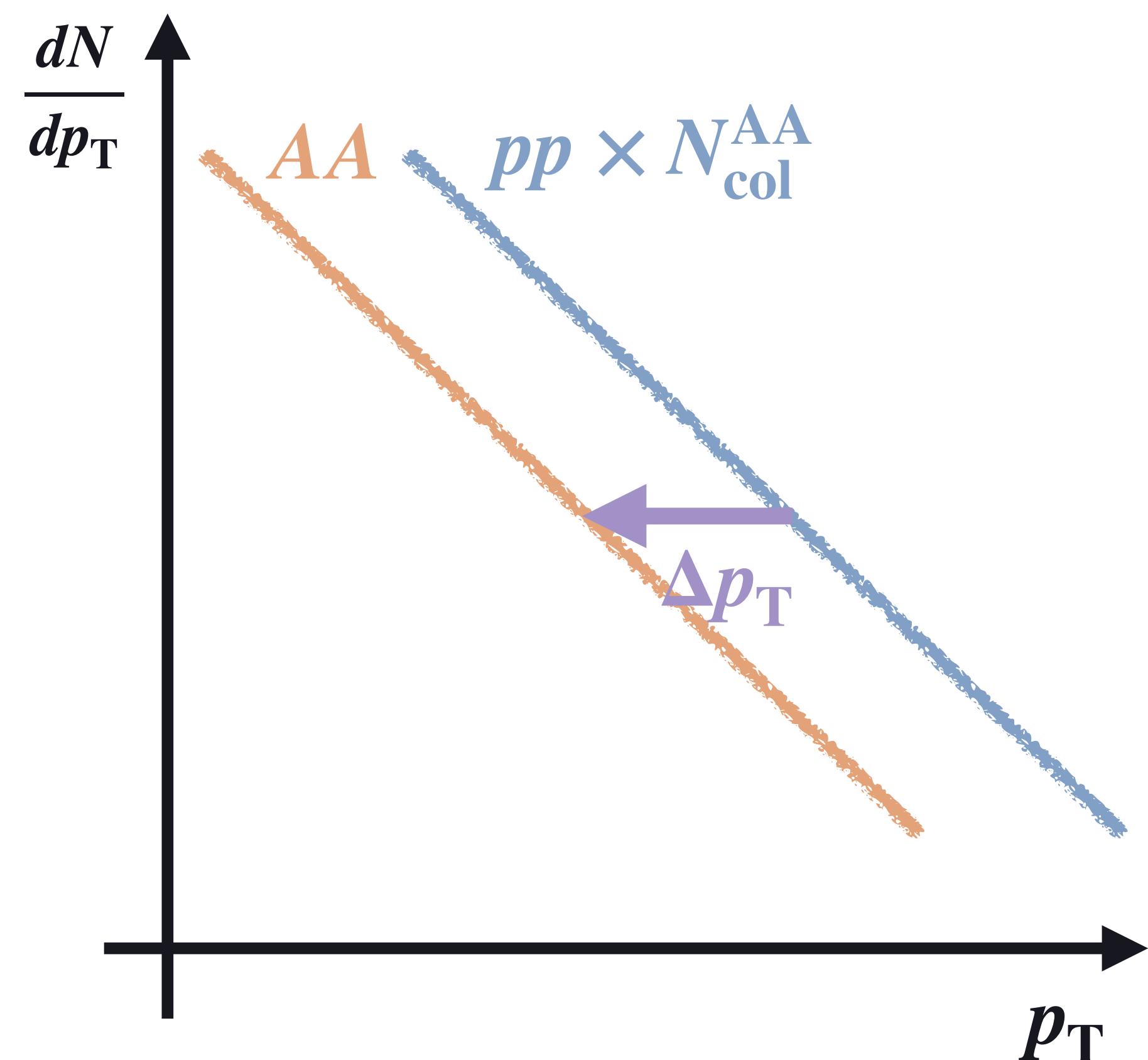


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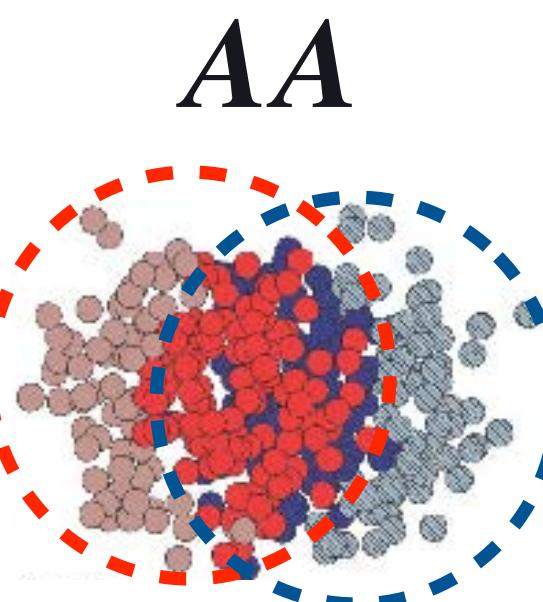


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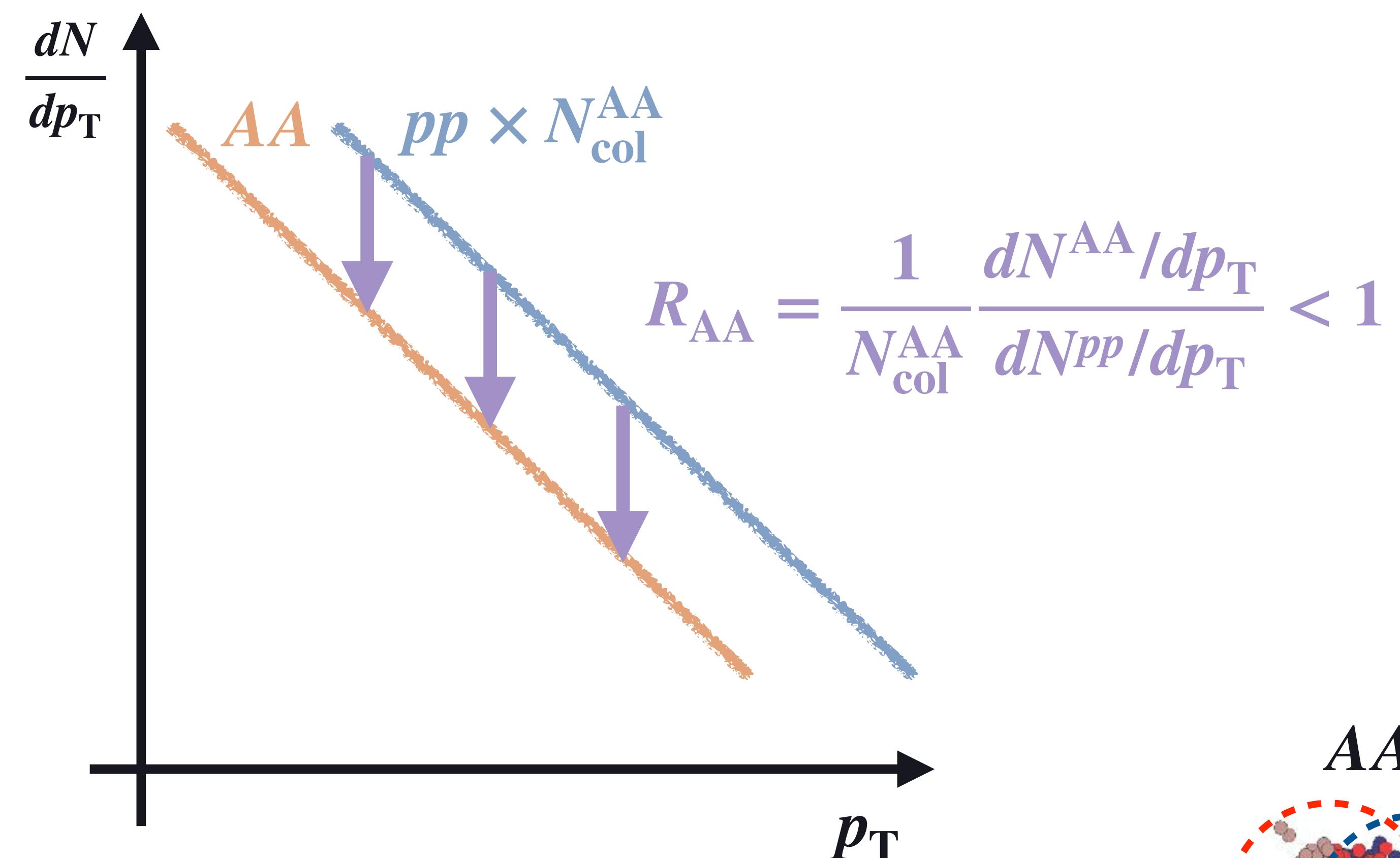


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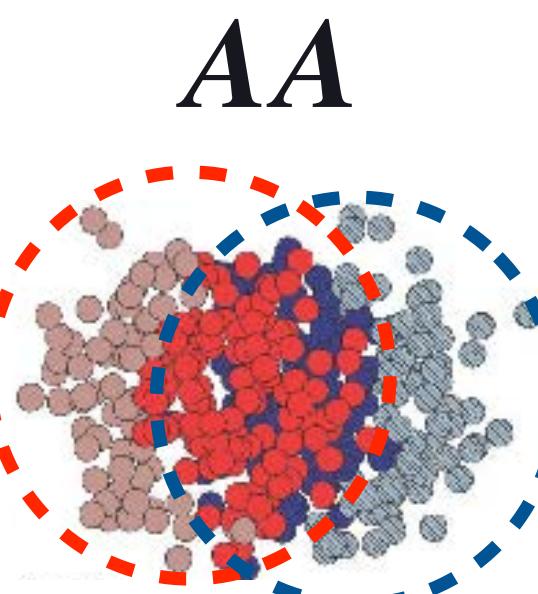


Nuclear Modification Factor R_{AA}

- Nuclear modification factor R_{AA}

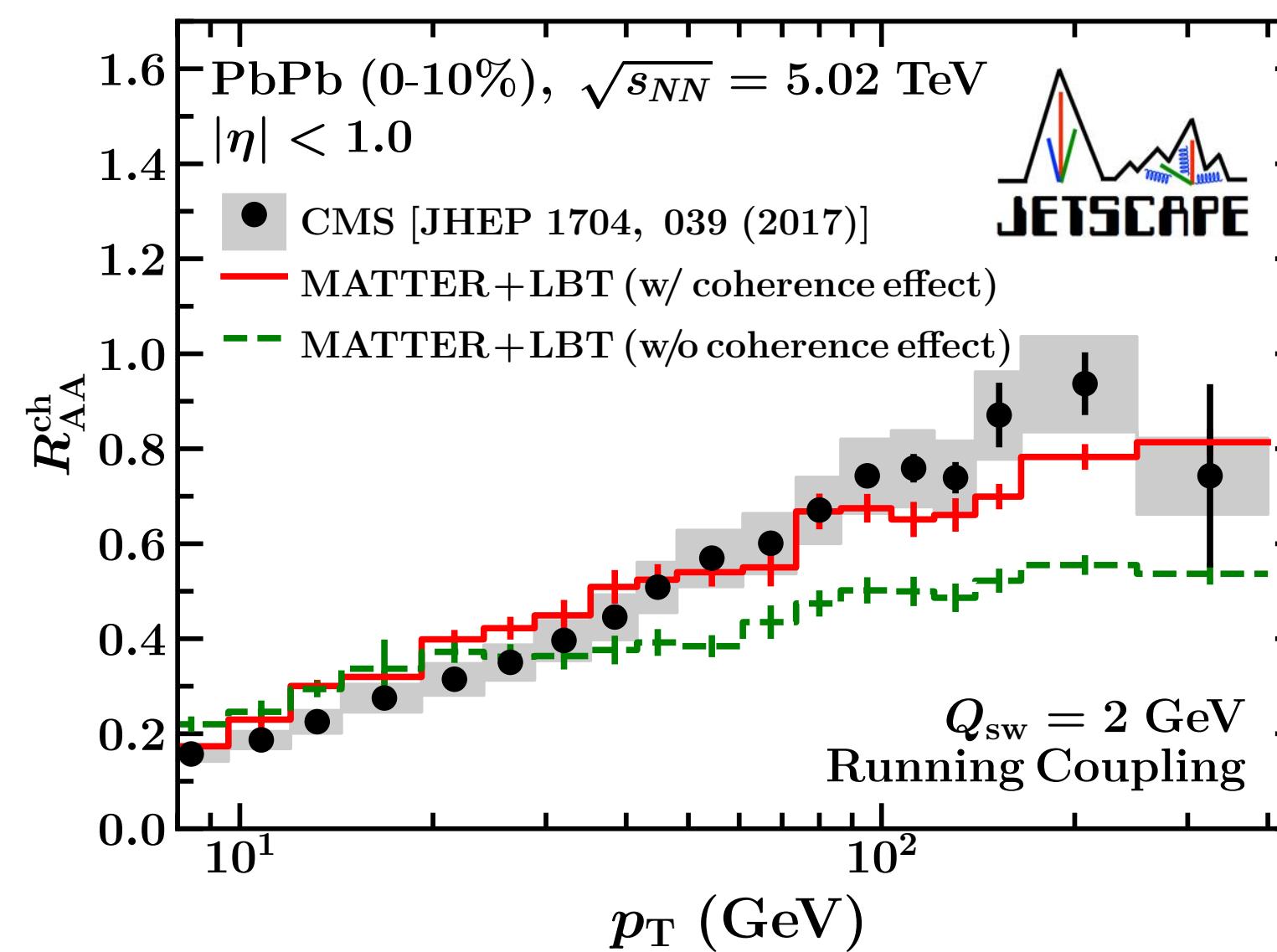


$N_{\text{coll}}^{\text{AA}}$: Number of nucleon-nucleon collisions in an AA collision

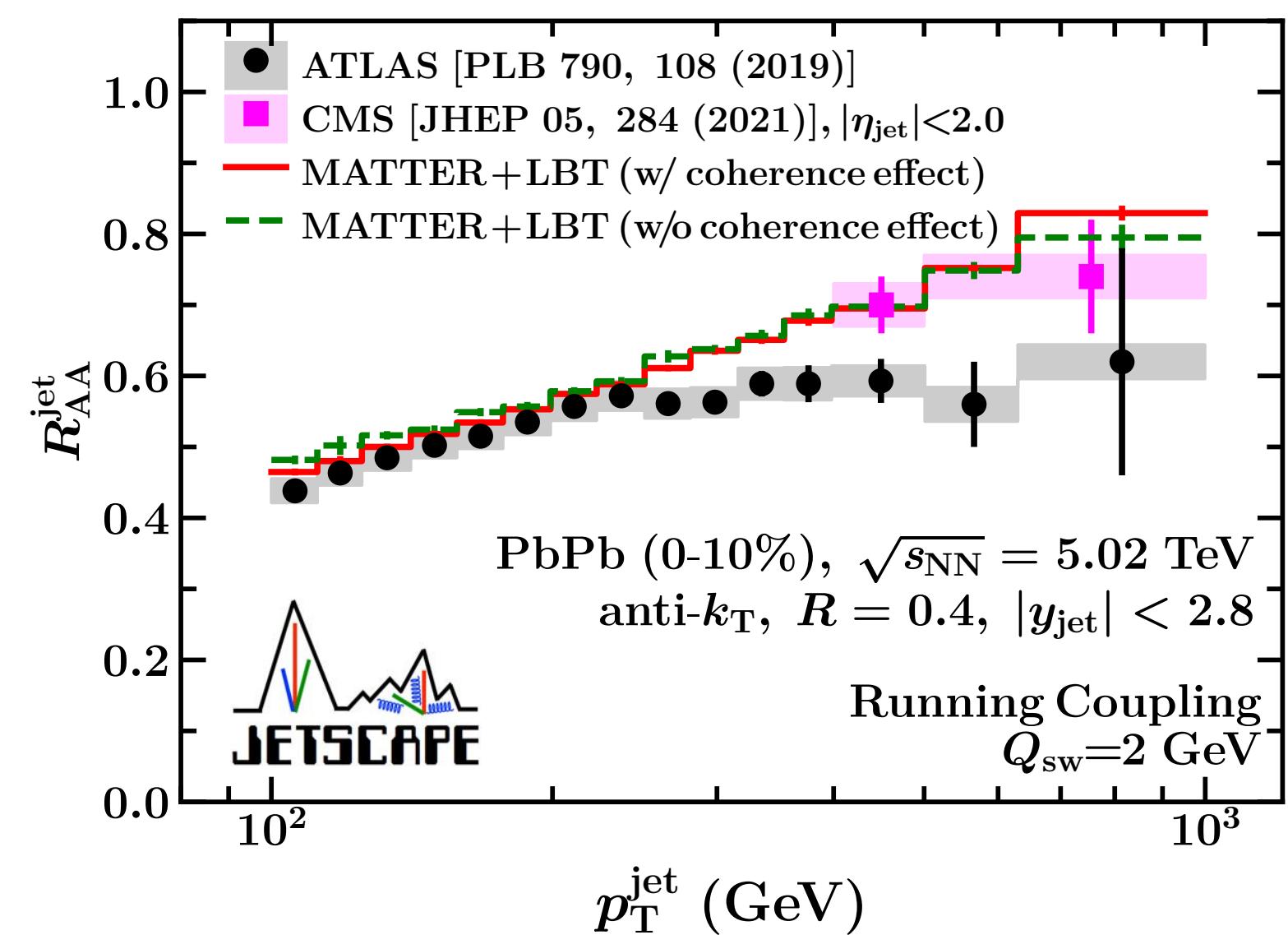


Nuclear Modification Factor R_{AA}

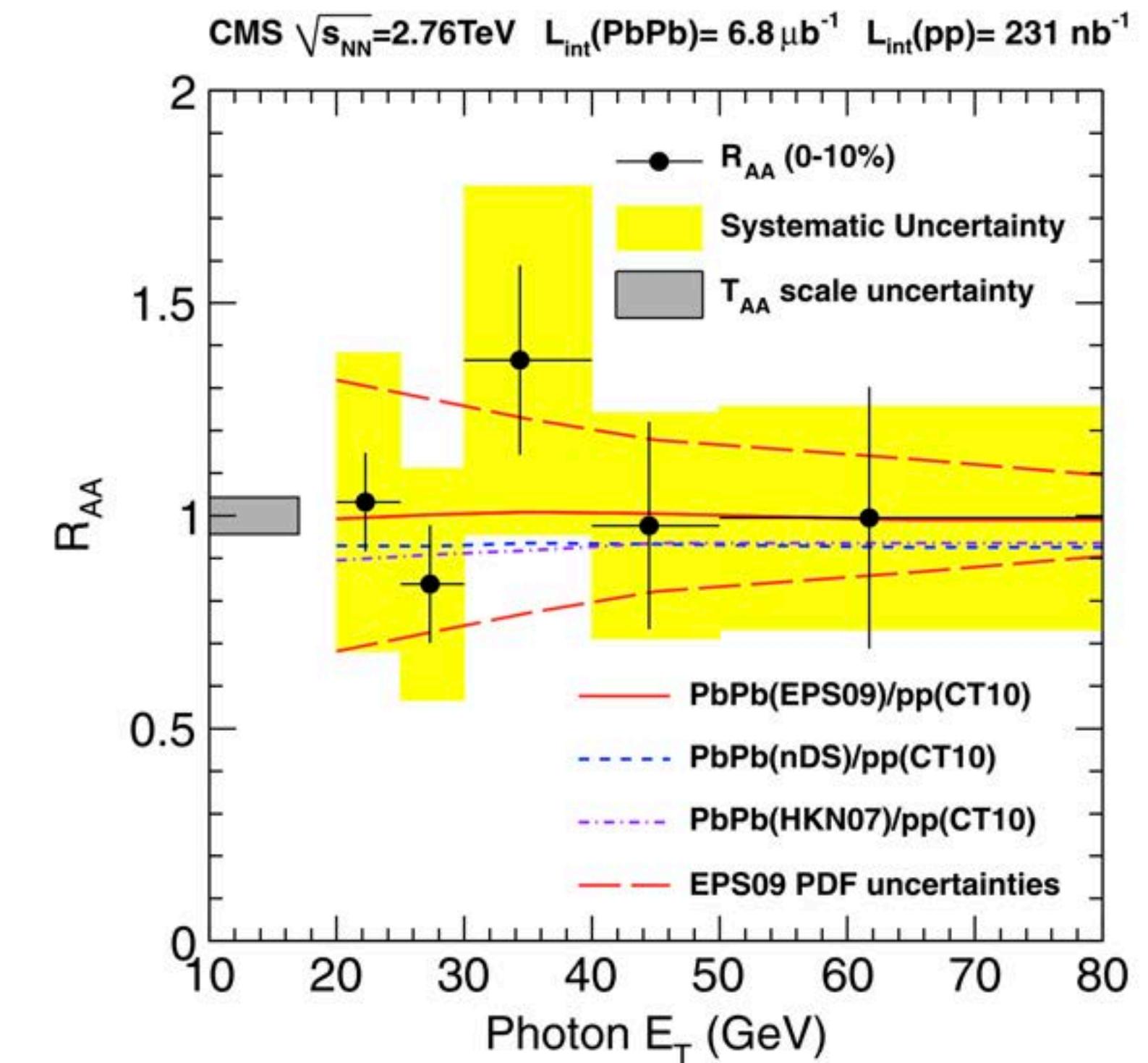
Single Charged Particle



Reconstructed (Full) Jet



(Isolated) Photon



媒質効果の測定量への影響

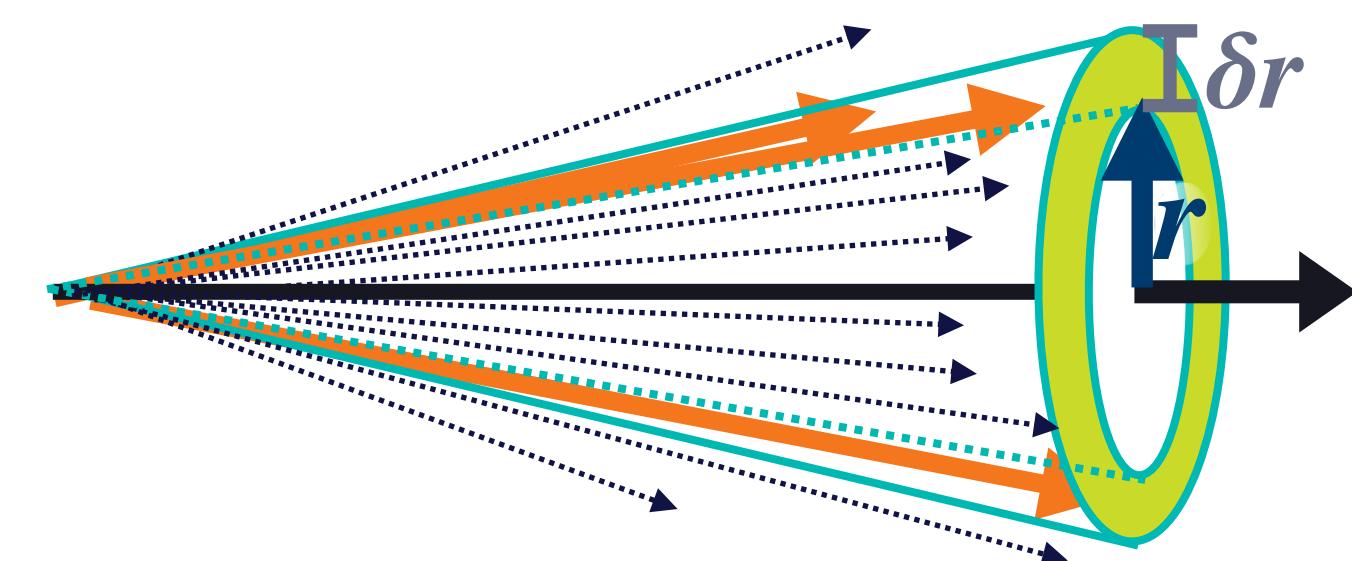
—ジェット内部構造—

Jet Shape

- Angular distribution of momentum in jets

$$\rho(r) = \frac{1}{\delta r} \frac{\sum_{i \in (r-\delta r/2, r+\delta r/2)} p_T^i}{\sum_{i \in (0, R)} p_T^i}$$

$$(r = \sqrt{(\eta_p - \eta^{\text{jet}})^2 + (\phi_p - \phi^{\text{jet}})^2})$$



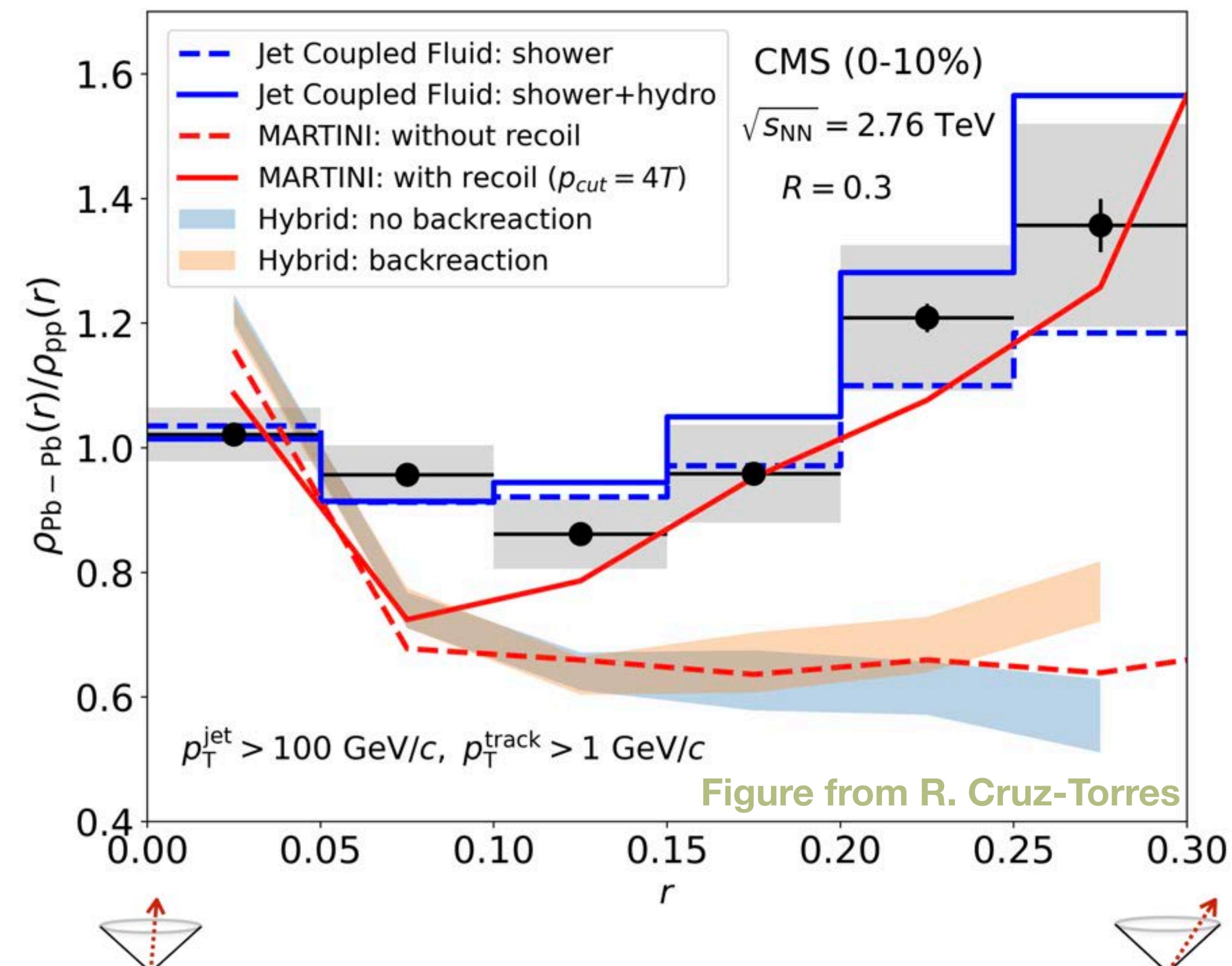
Enhancement at large angles

The same trend seen in other model calculations:

JEWEL [Kunnawalkam Elayavalli, Zapp, JHEP 1707, 141 (2017)],

LBT [Luo et al, PLB782, 707-716(2018)], etc.

Couple jet-fluid: YT, Chang, Qin, PRC 95, 044909 (2017)
MARTINI: Park, Jeon, Gale, NPA 982, 643 (2019)
Hybrid: Pablos et al, JHEP 03, 135 (2017)

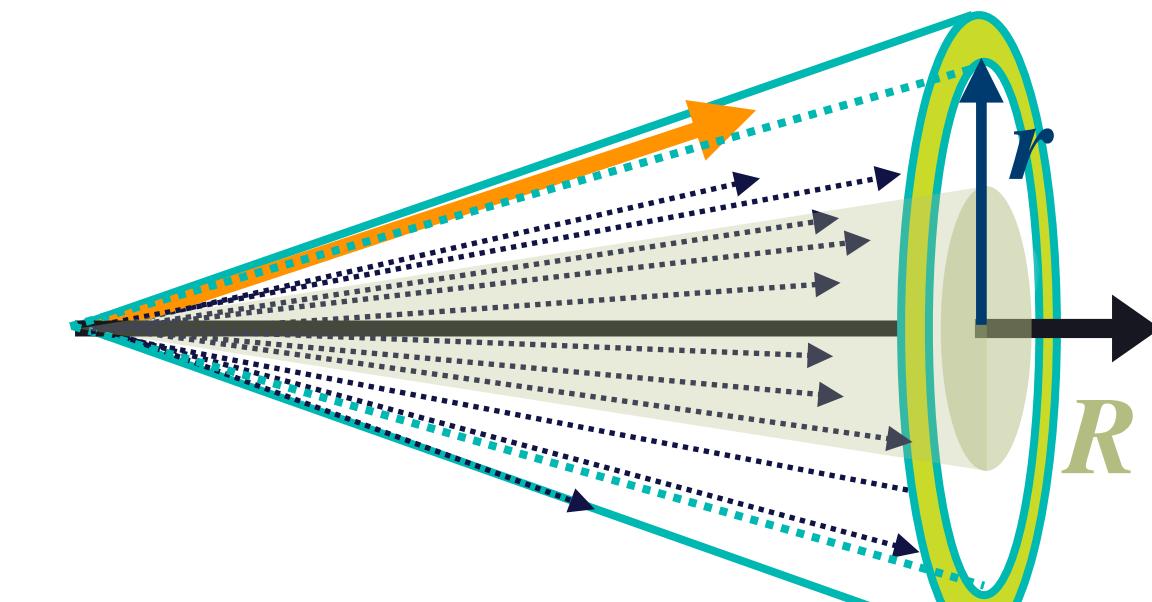
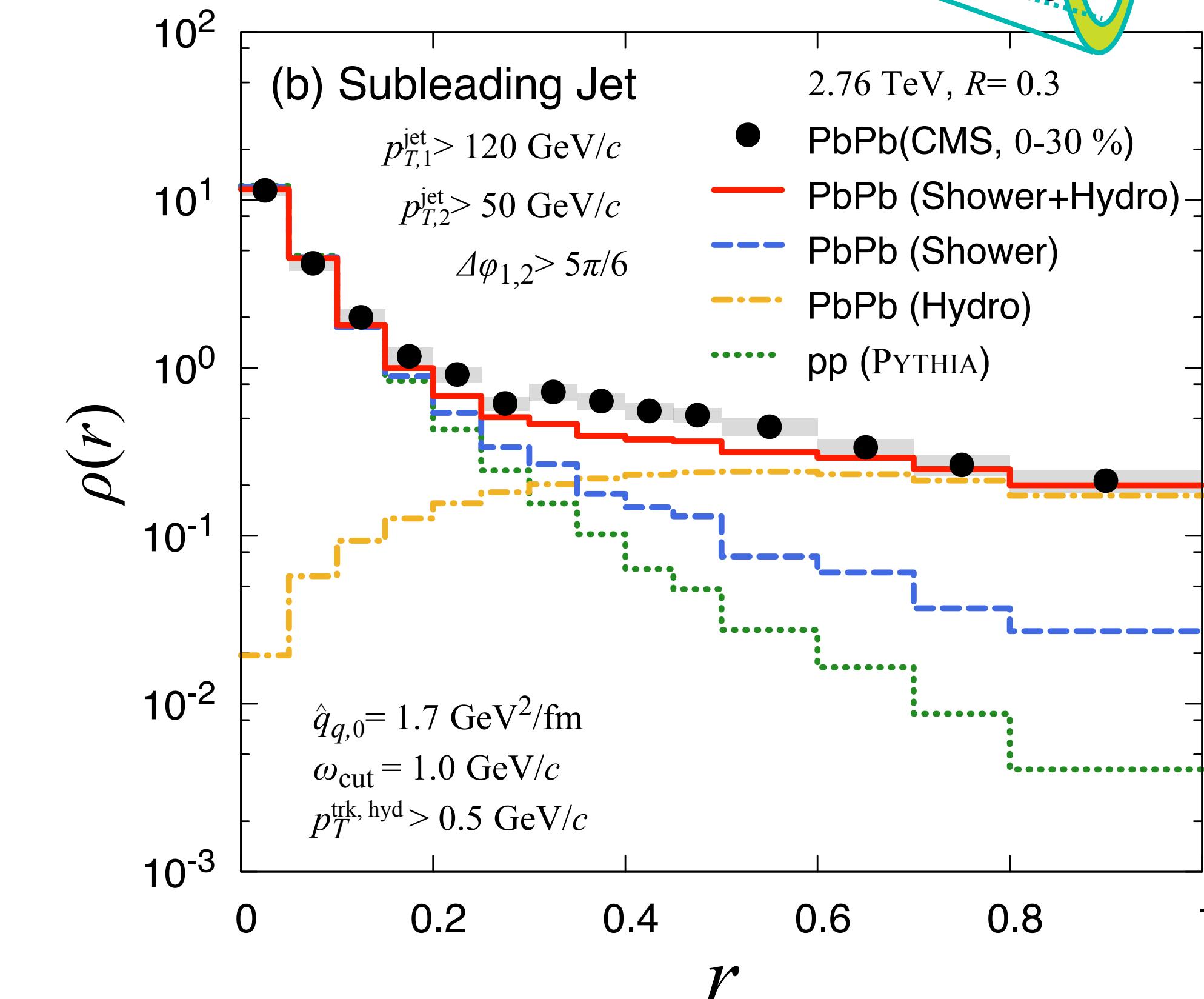
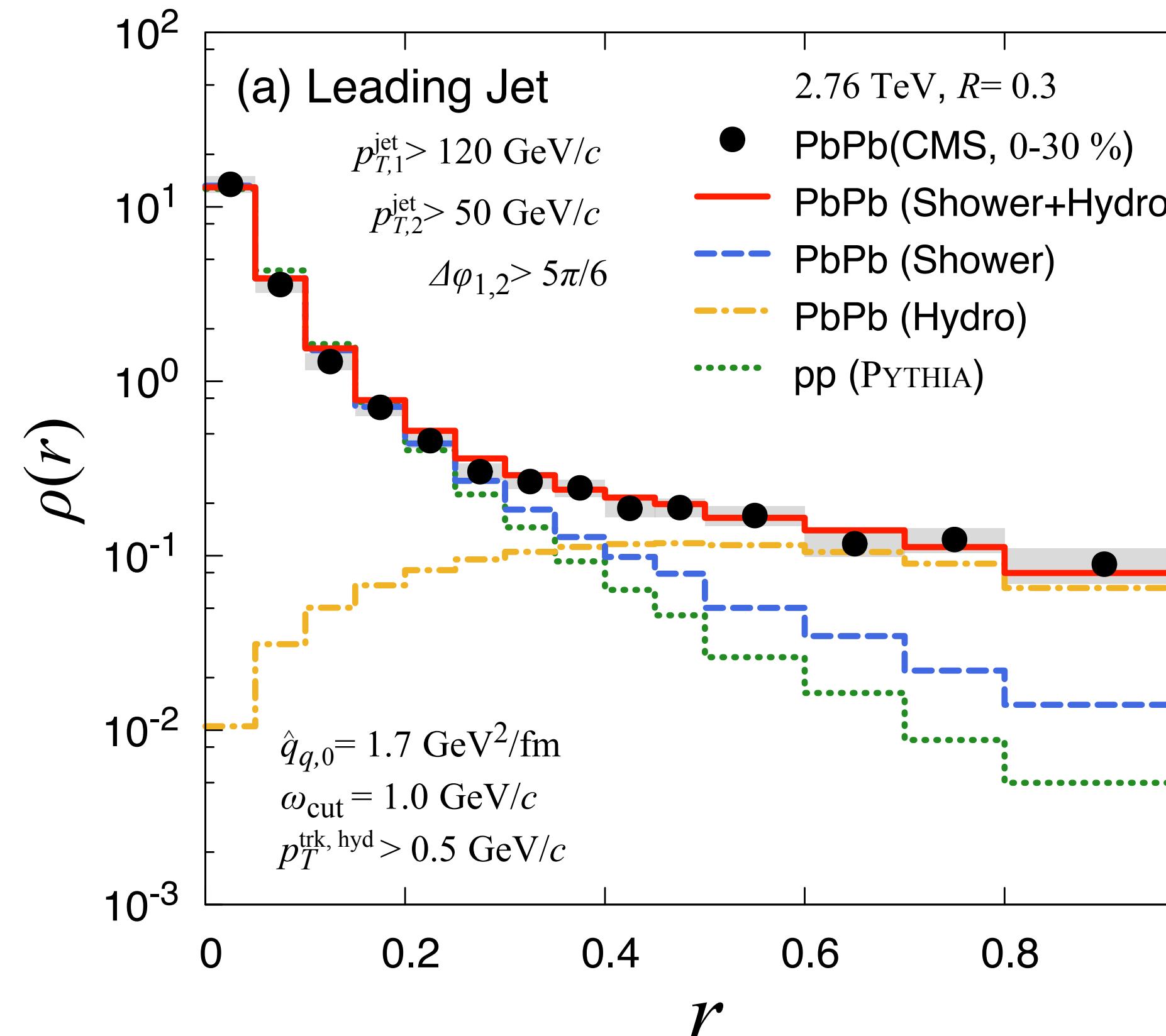


Jet Shape

Couple jet-fluid: YT, Chang, Qin, PRC 95, 044909 (2017)

- Extended to out-cone region

Enhancement at large angles

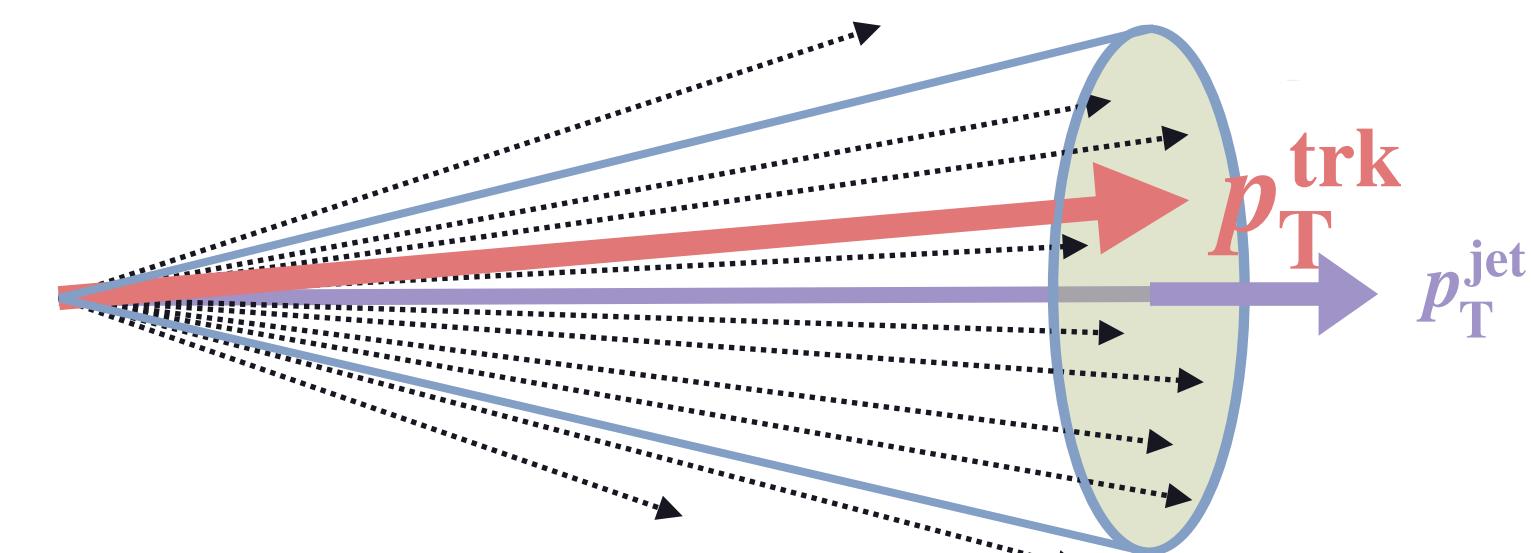


Jet Fragmentation Function

- Momentum distribution in jets

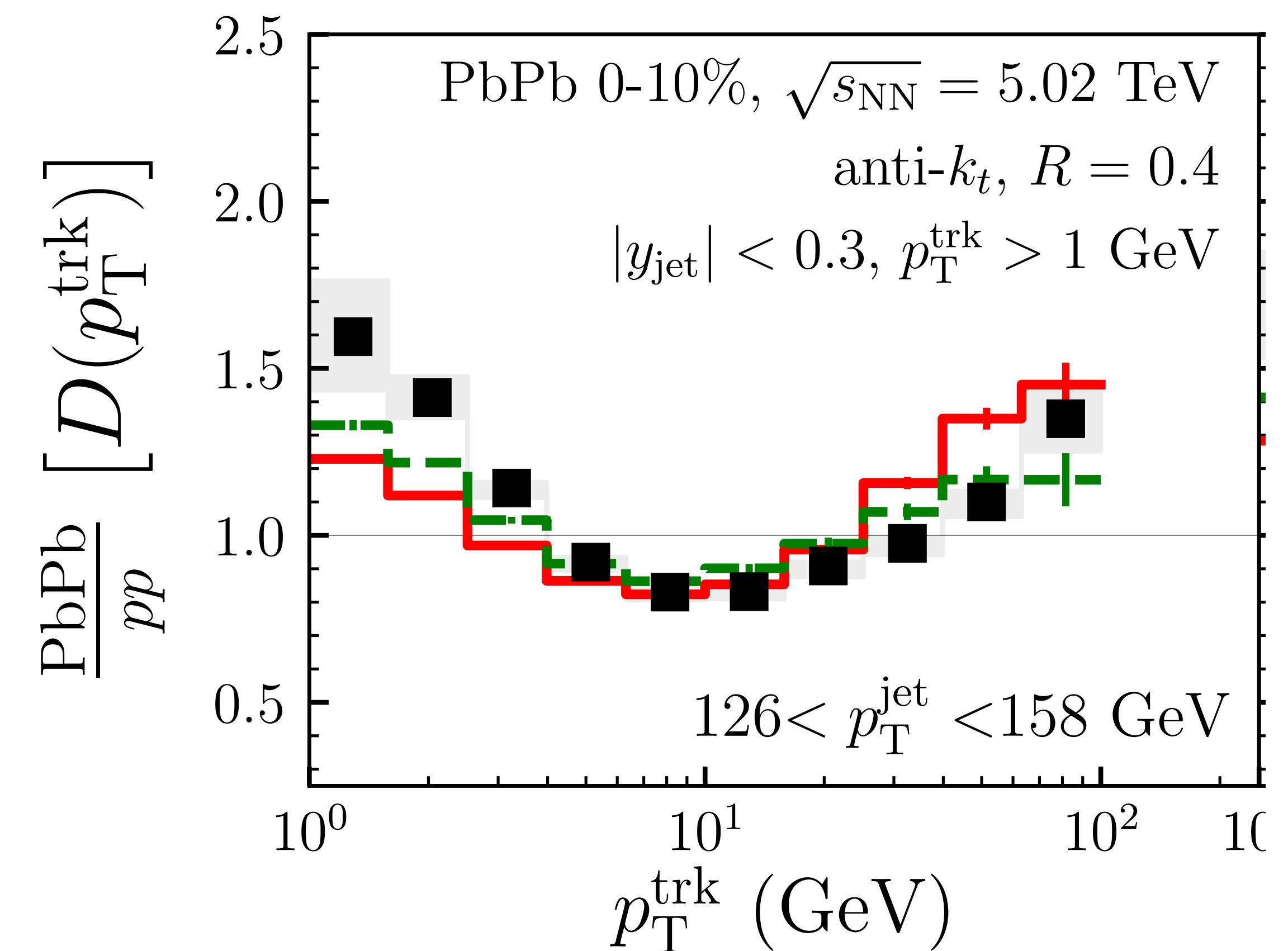
$$D(p_T^{\text{trk}}) = \frac{1}{N_{\text{jet}}} \sum_{\text{jet}} \left. \frac{dN_{\text{trk}}}{dp_T^{\text{trk}}} \right|_{\text{in jet}}$$

$$D(z) = \frac{1}{N_{\text{jet}}} \sum_{\text{jet}} \left. \frac{dN_{\text{trk}}}{dz} \right|_{\text{in jet}} \quad z = \frac{p_T^{\text{trk}}}{p_T^{\text{jet}}}$$



Enhancement of soft hadrons

— JETSCAPE
 [MATTER+LBT (w/ mod. coh.)]
— JETSCAPE
 [MATTER+LBT (w/o coh.)]
■ ATLAS
 [PRC 98, no.2, 024908 (2018)]

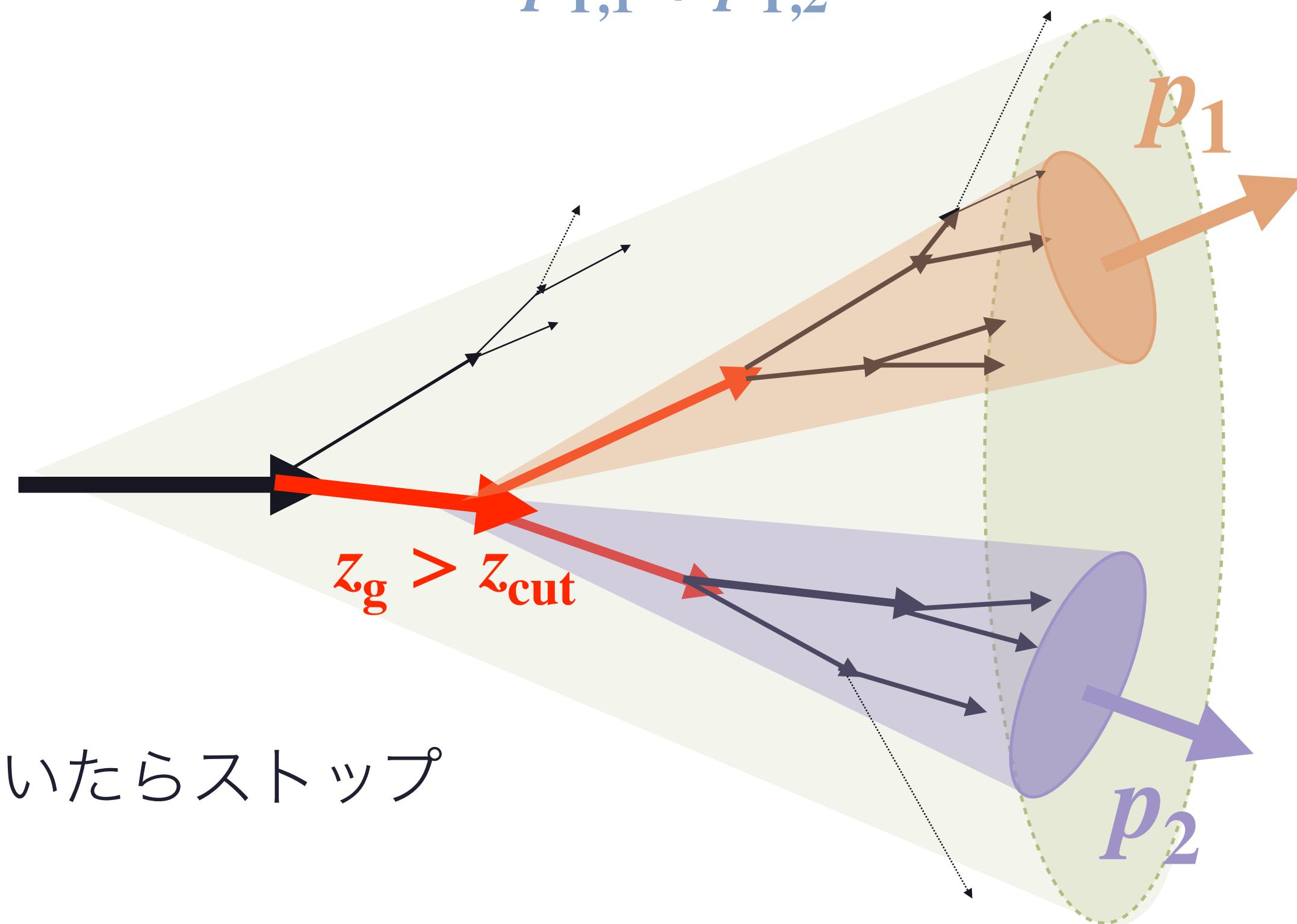


Groomed Jet Observables

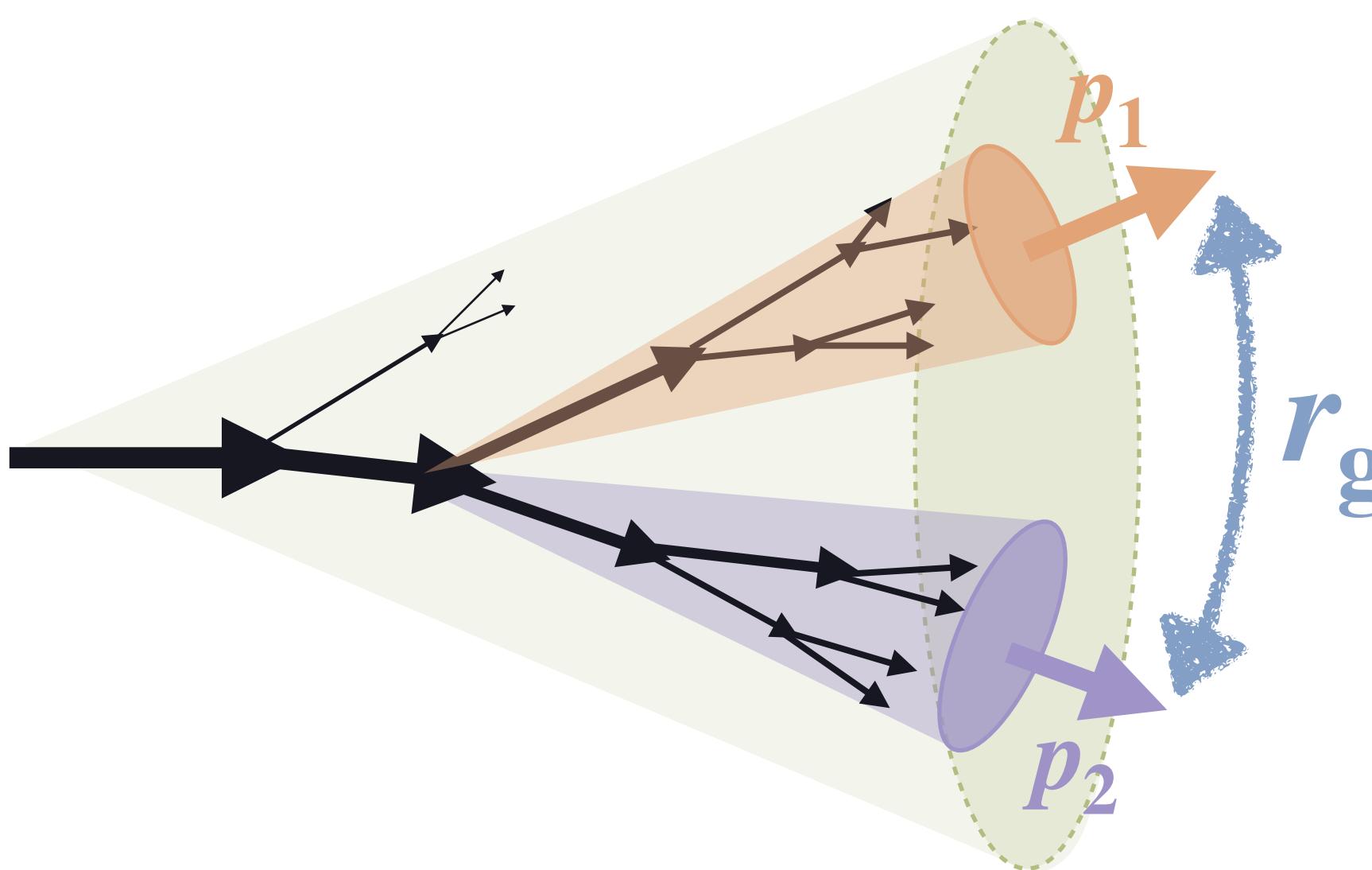
● Soft Drop Grooming

- 一旦 anti- k_t でジェットを見つける
- ジェットの構成要素を再度 Cambridge Aachen ($p = 0$) で再度ジェット構成する(Angular ordered にする)
$$d_{ij} = \Delta r_{ij} \cdot \min(k_{ti}^p, k_{tj}^p),$$
- Cambridge Aachen で構成したジェットをほぐしていく(大角度の merge からほぐす)
- ほぐした時にできた prong 間の運動量比が z_{cut} を超えていたらストップ

$$z_g = \frac{\min\{p_{T,1}, p_{T,2}\}}{p_{T,1} + p_{T,2}}$$

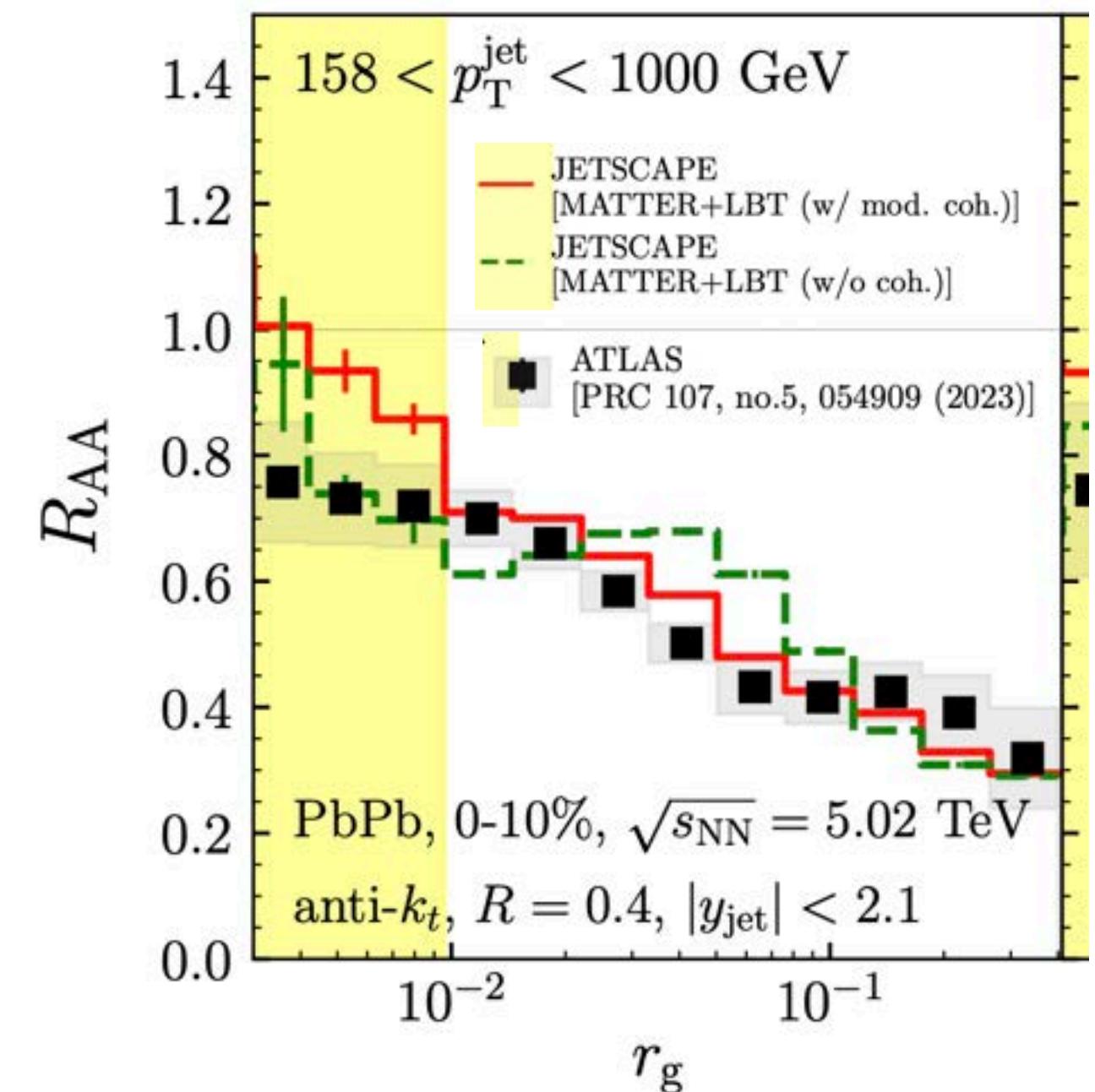
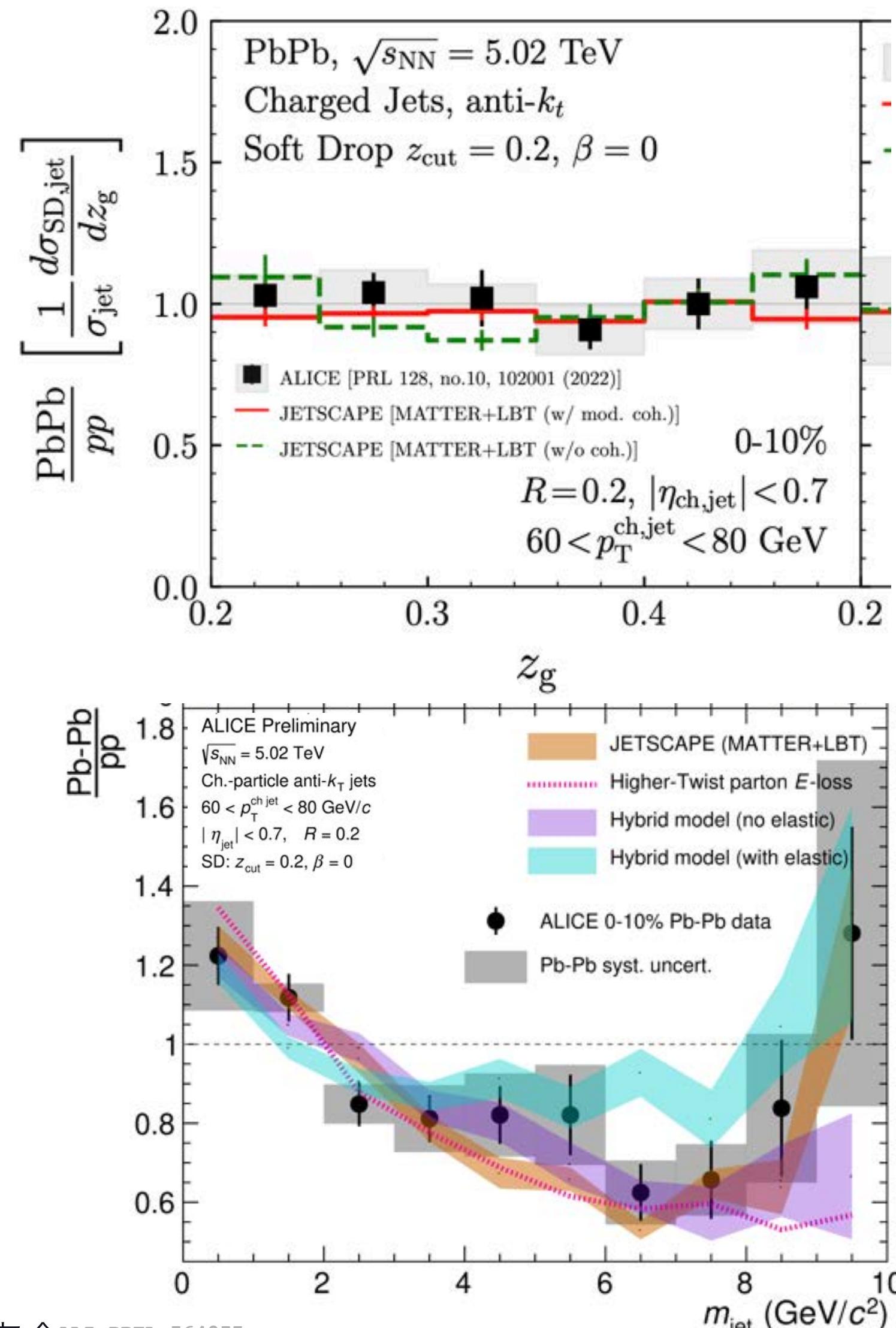


Groomed Jet Observables



$$z_g = \frac{\min\{p_{T,1}, p_{T,2}\}}{p_{T,1} + p_{T,2}}$$

$$m_g = \sqrt{(p_1 + p_2)_\mu (p_1 + p_2)^\mu}$$



- Small z_g modification
- Narrower and lighter splittings in triggered inclusive jets

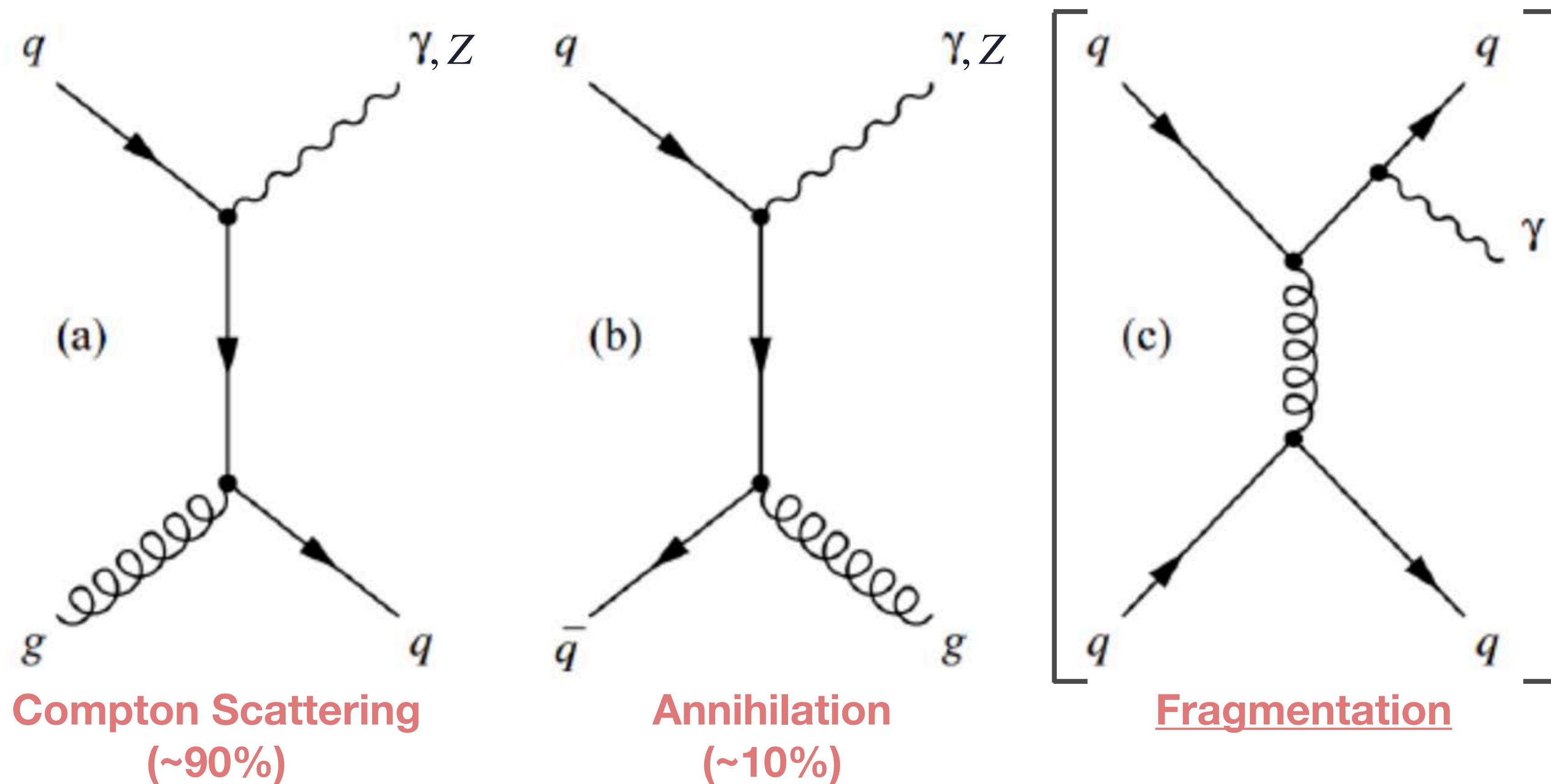
- Selection bias? (E-loss)
- Structure modification?

媒質効果の測定量への影響

—タグ付きジェット—

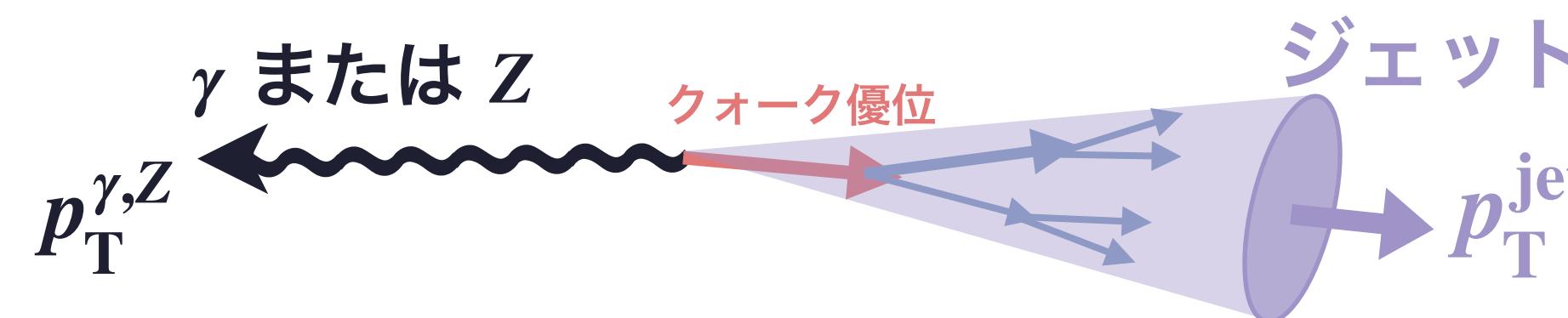
γ/Z -tagged Jet

- Prompt γ/Z production at hard scattering

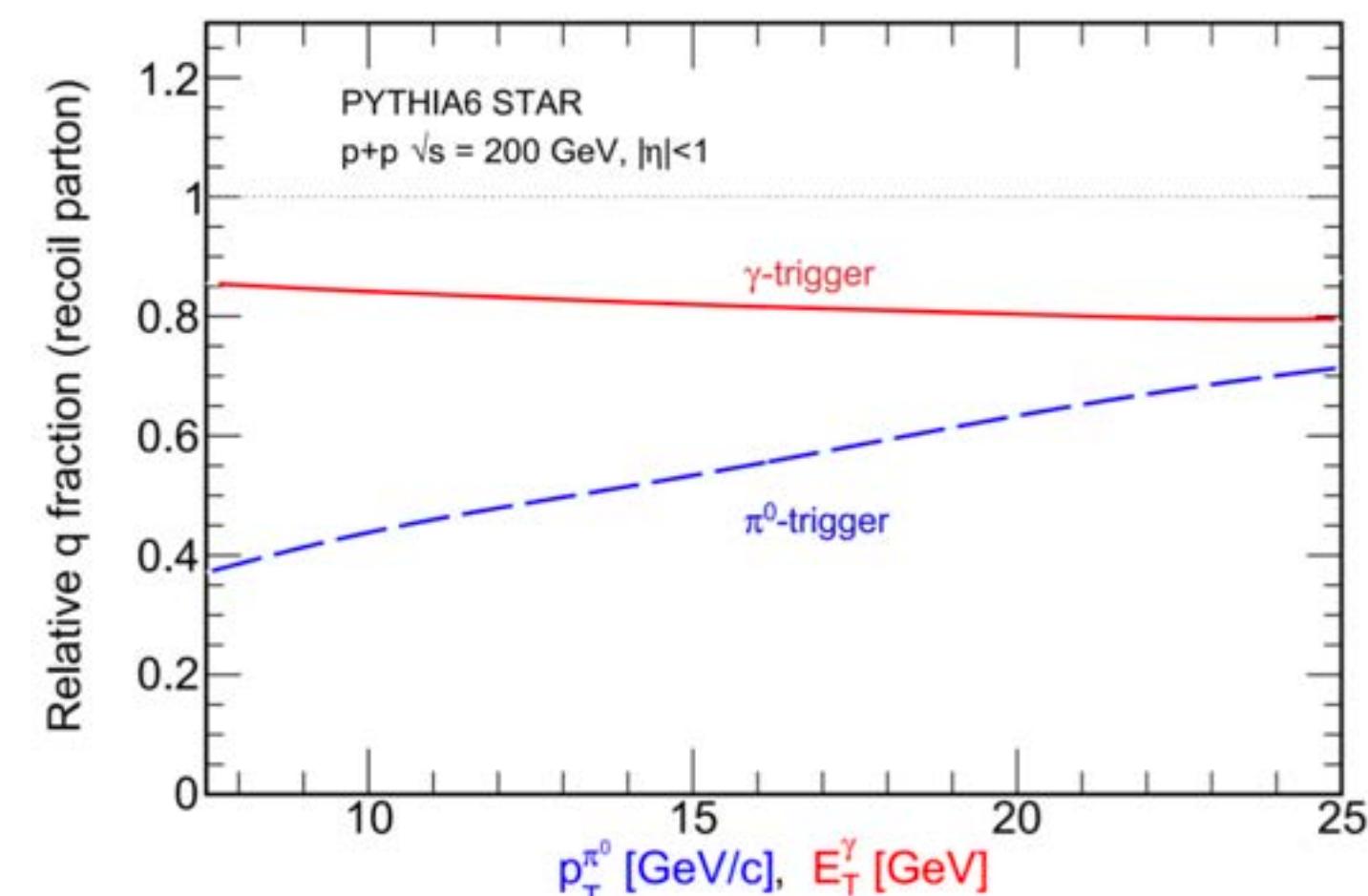
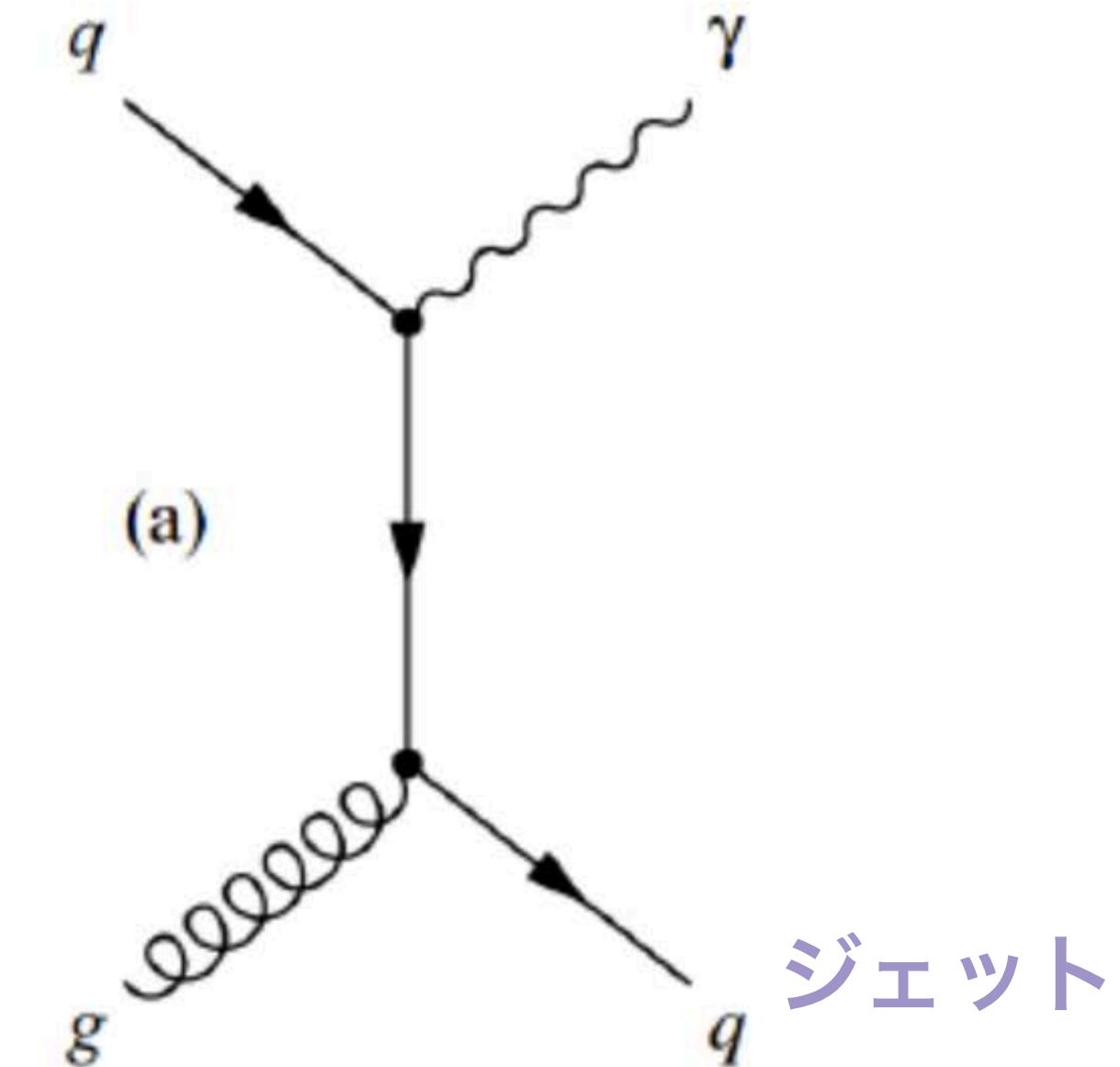


γ/Z -tagged Jet

- γ/Z -tagged jets



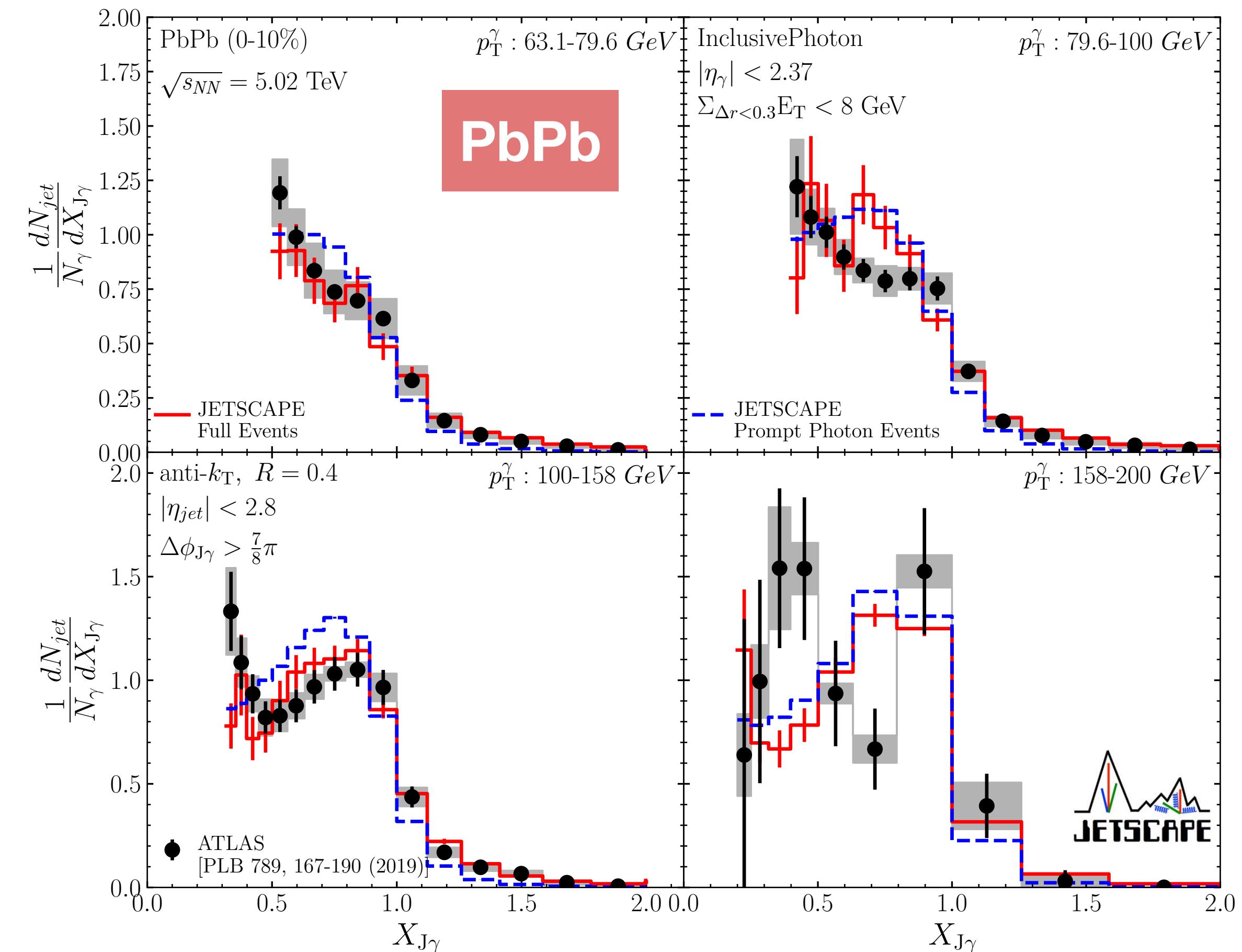
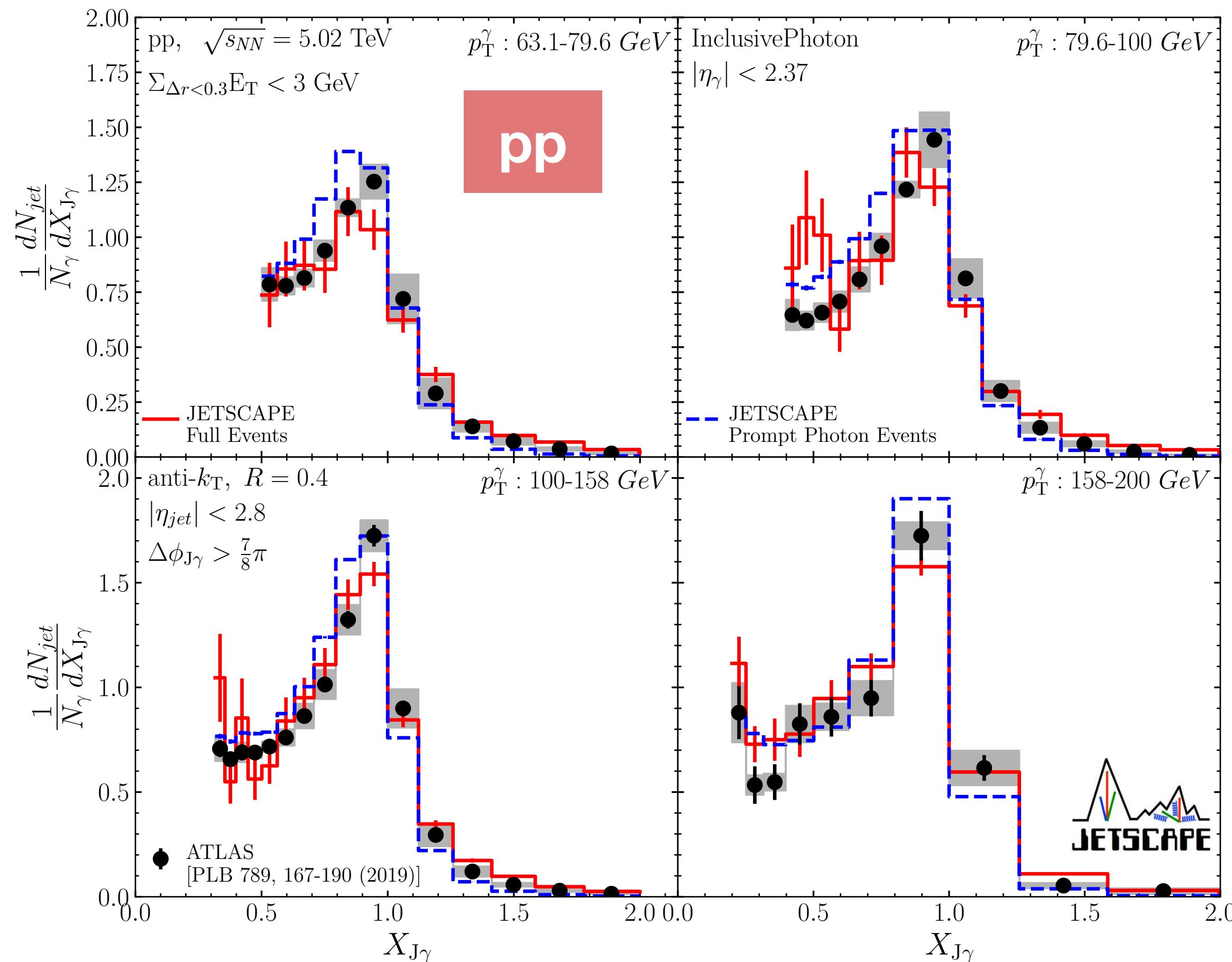
- γ, Z は媒質と相互作用しない
($p_T^{\gamma,Z} - p_T^{\text{jet}}$ ~ ジェットのエネルギー損失)
- クォークジェットが優位
- Z にはフラグメンテーションからの寄与無し
- 通常相対方位角で back-to-back に制限



γ/Z -tagged Jet

- γ -jet asymmetry

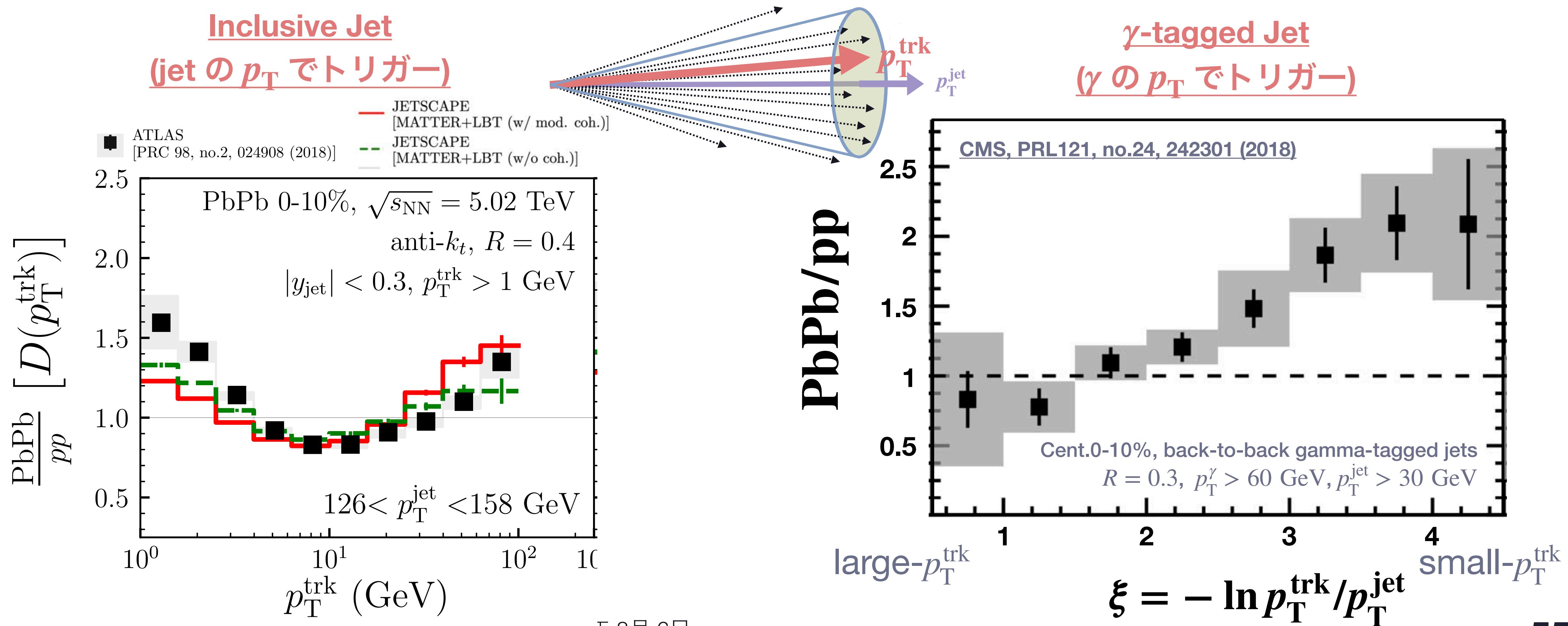
$$X_{J\gamma} = p_T^{\text{jet}}/p_T^\gamma$$



Substructures of γ/Z -tagged Jet

- Energy loss による selection bias 効果の抑制, Flavor 依存性

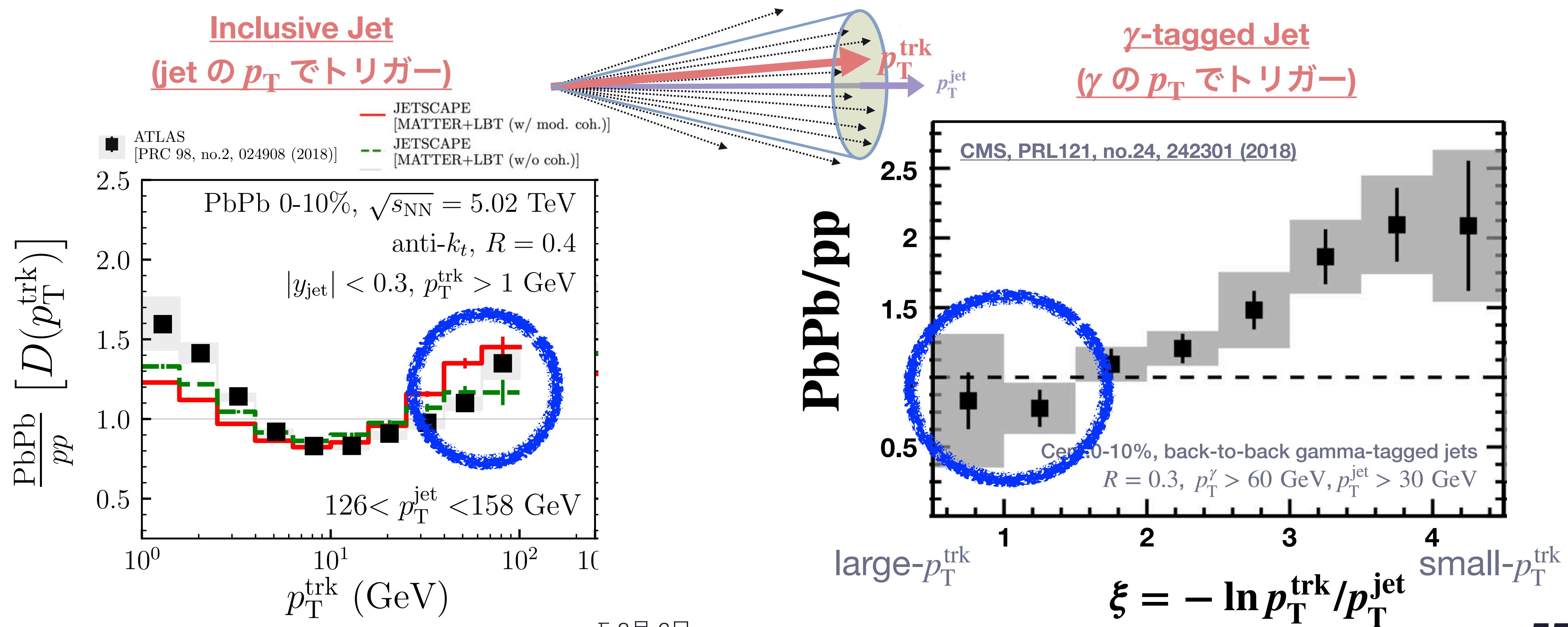
- pp と AA で親パートンの横運動量が近いと思われるジェットを比較できる



Substructures of γ/Z -tagged Jet

- Energy loss による selection bias 効果の抑制, Flavor 依存性

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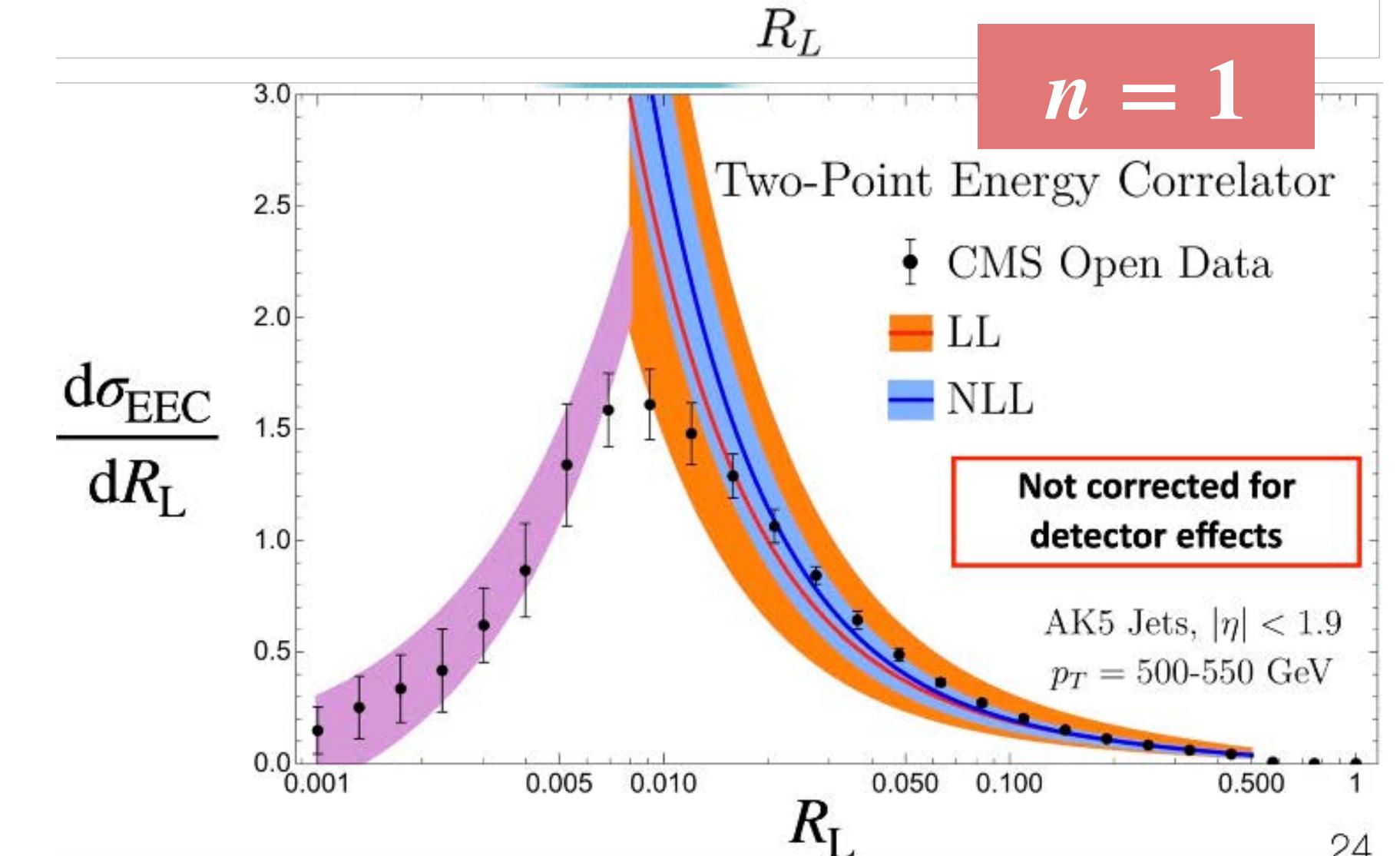
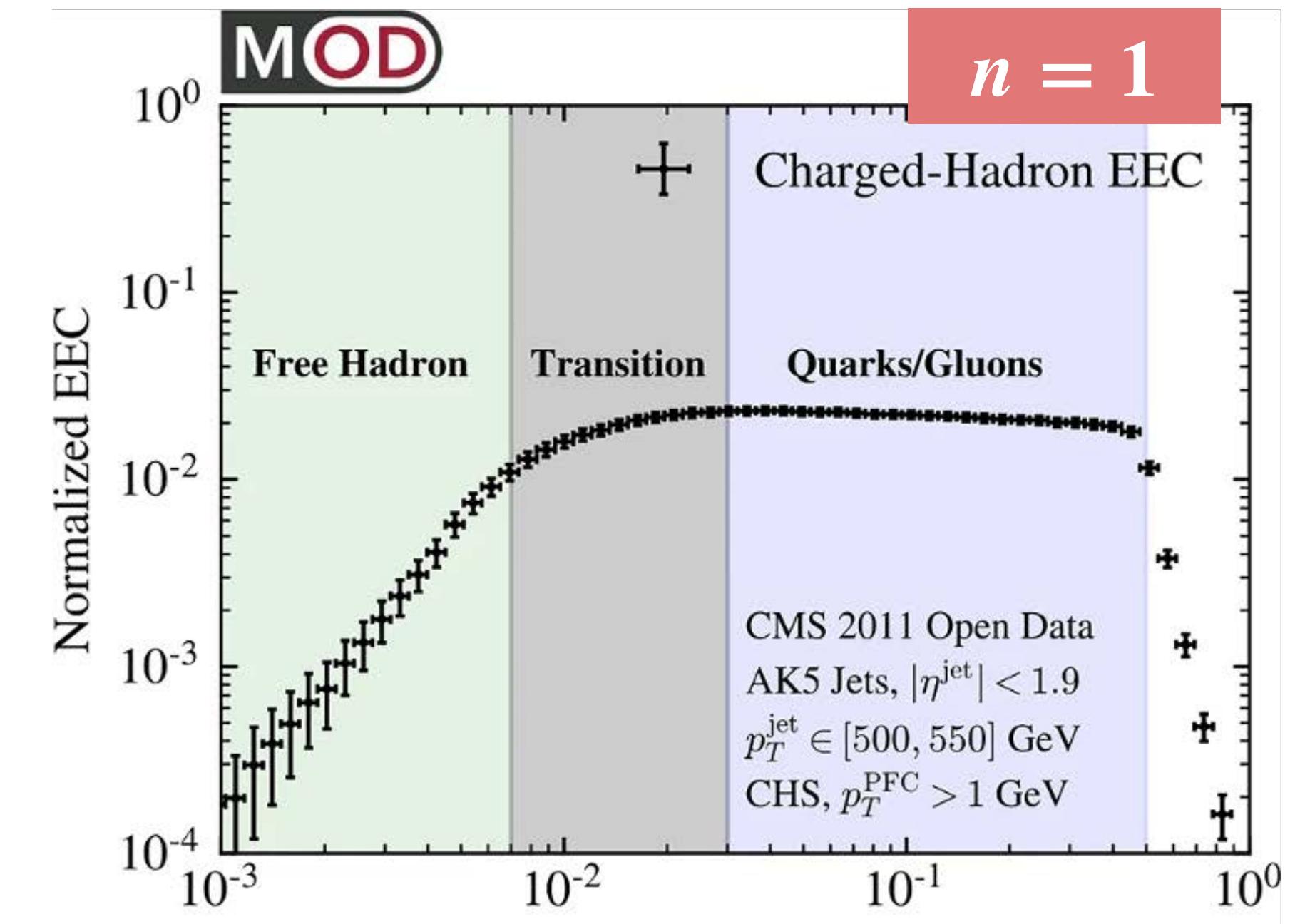
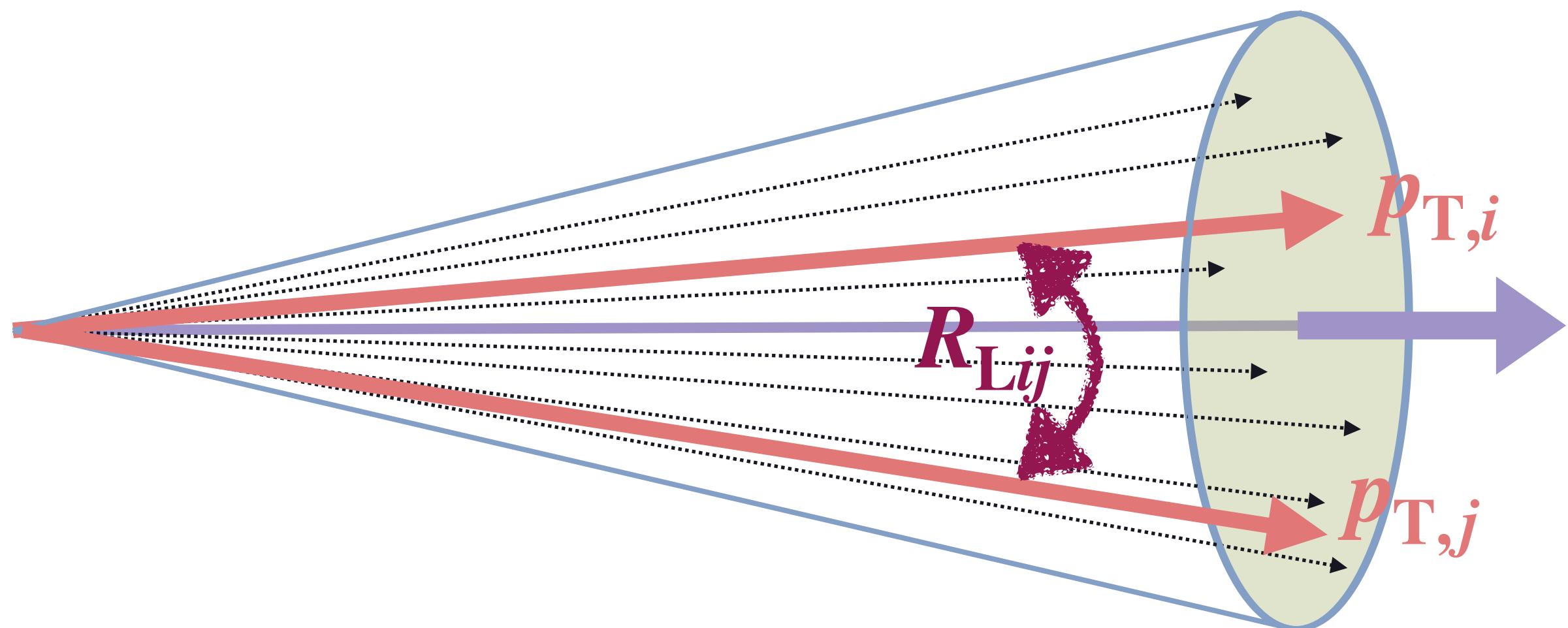


Energy-energy Correlator

Energy-energy Correlator

$$\frac{dN_{\text{EEC}}}{dR_L} = \sum_{|R_L - R_{Lij}| < \delta R/2} \frac{(p_{T,i} p_{T,j})^n}{\delta R}$$

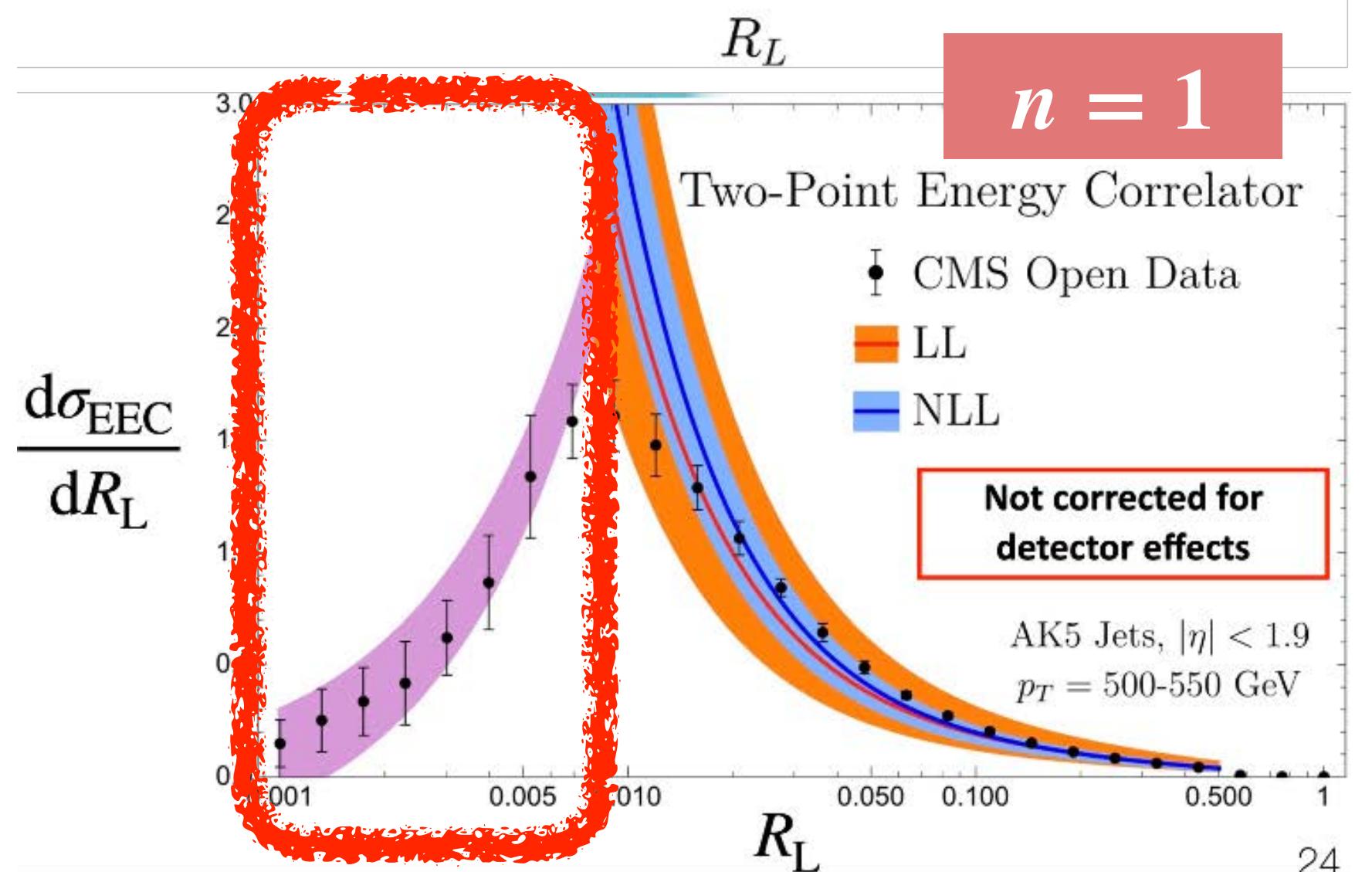
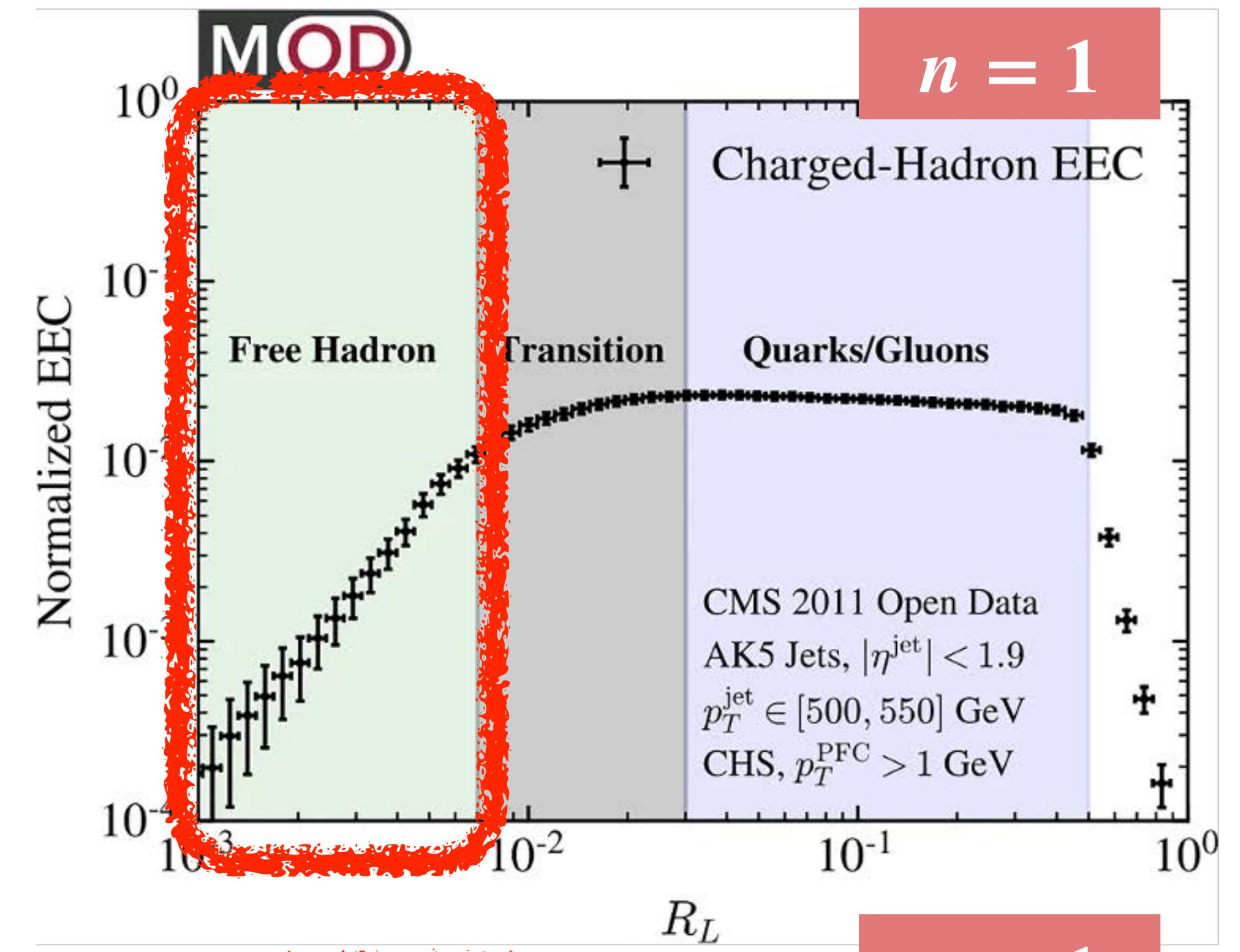
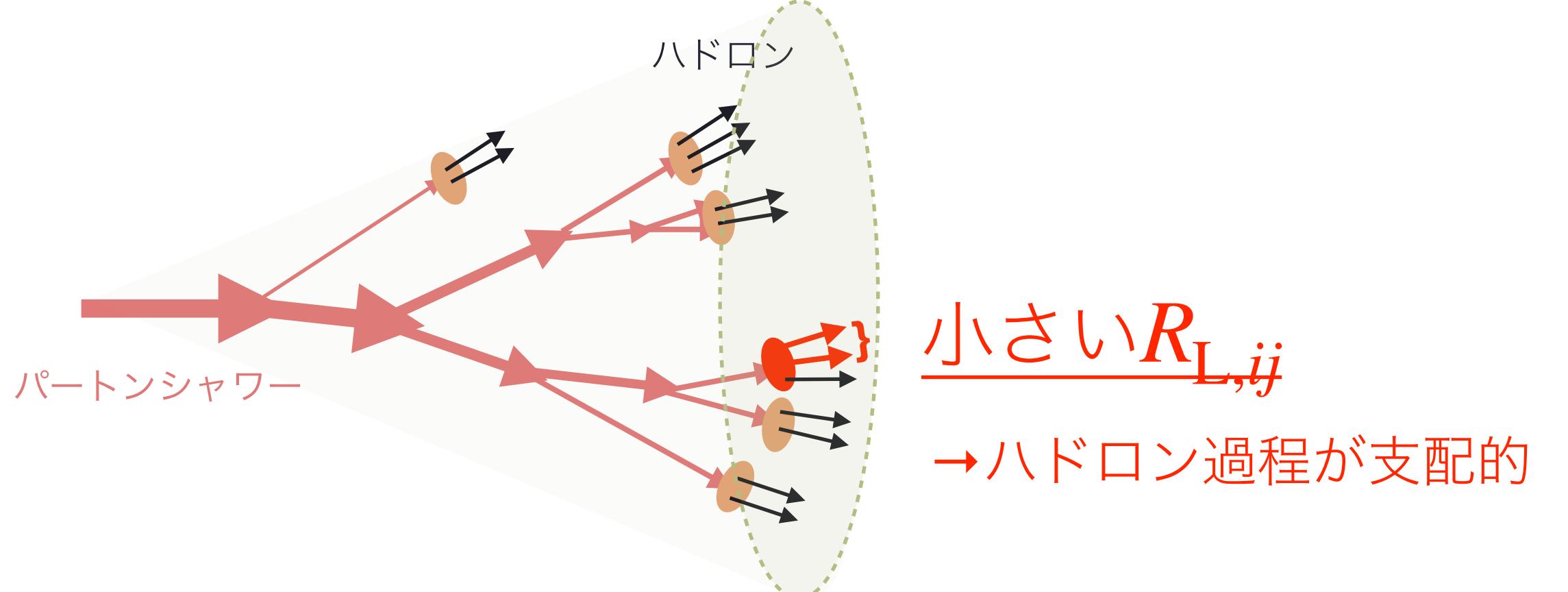
$$R_{L,ij} = \sqrt{(\phi_i - \phi_j)^2 + (\eta_i - \eta_j)^2}$$



Energy-energy Correlator

$$\frac{dN_{\text{EEC}}}{dR_L} = \sum_{|R_L - R_{Lij}| < \delta R/2} \frac{(p_{T,i} p_{T,j})^n}{\delta R}$$

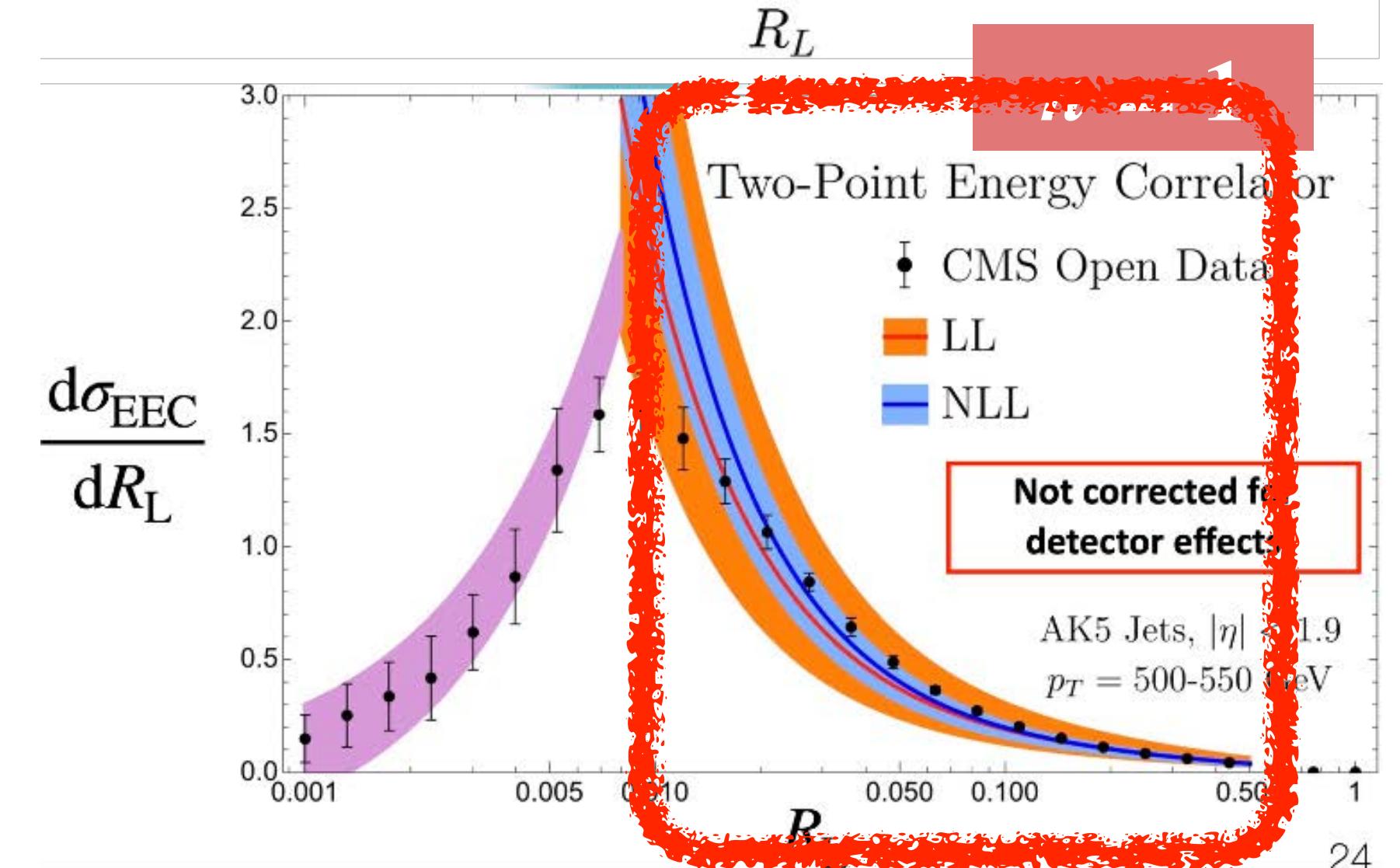
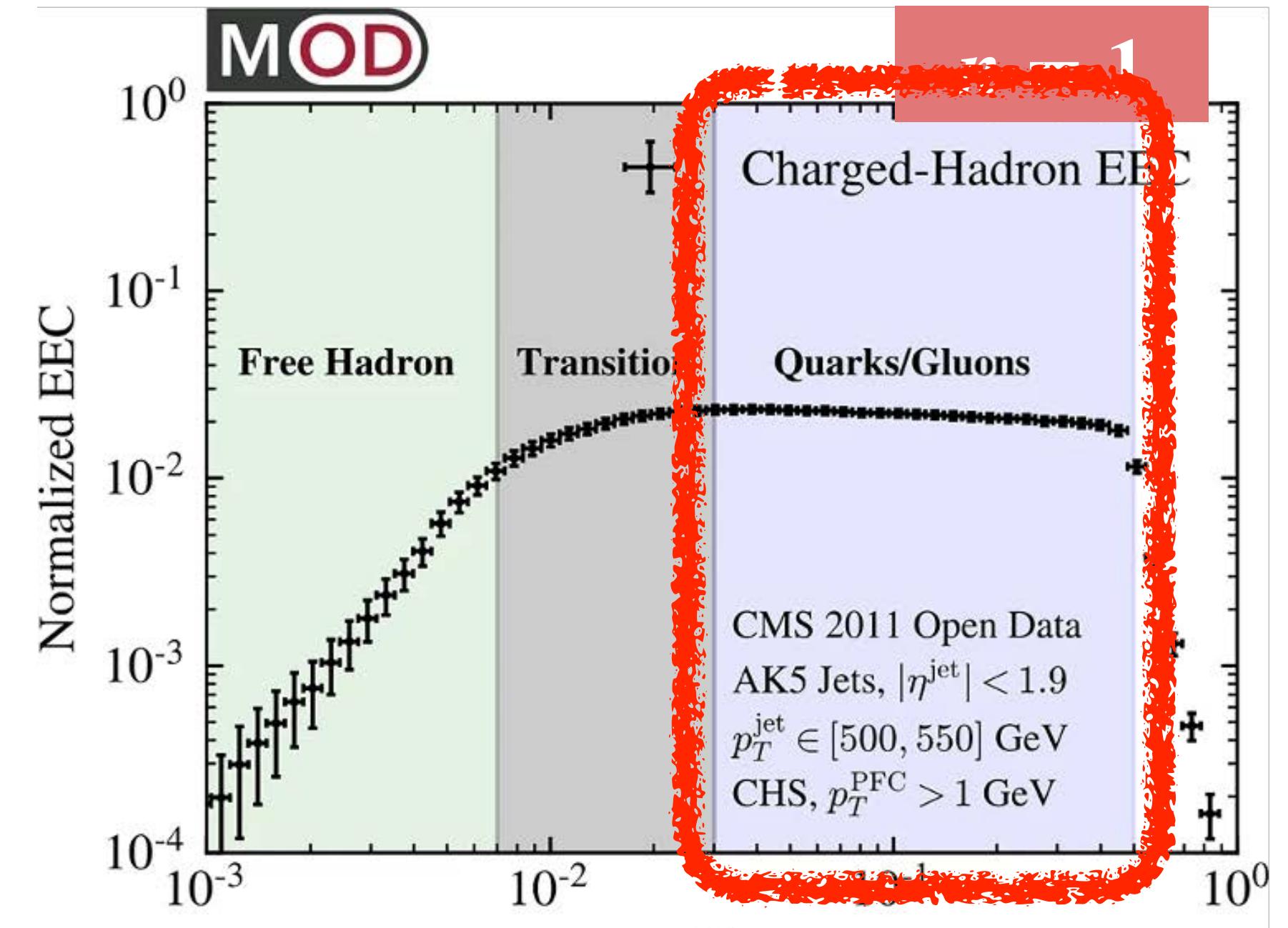
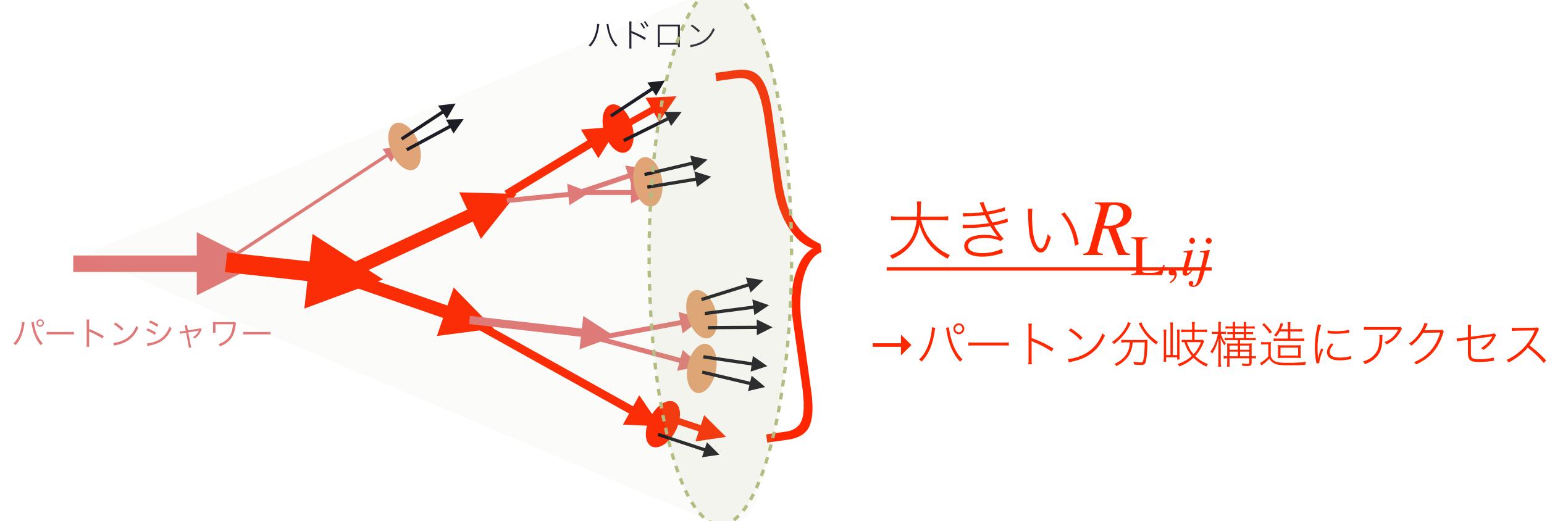
$$R_{L,ij} = \sqrt{(\phi_i - \phi_j)^2 + (\eta_i - \eta_j)^2}$$



Energy-energy Correlator

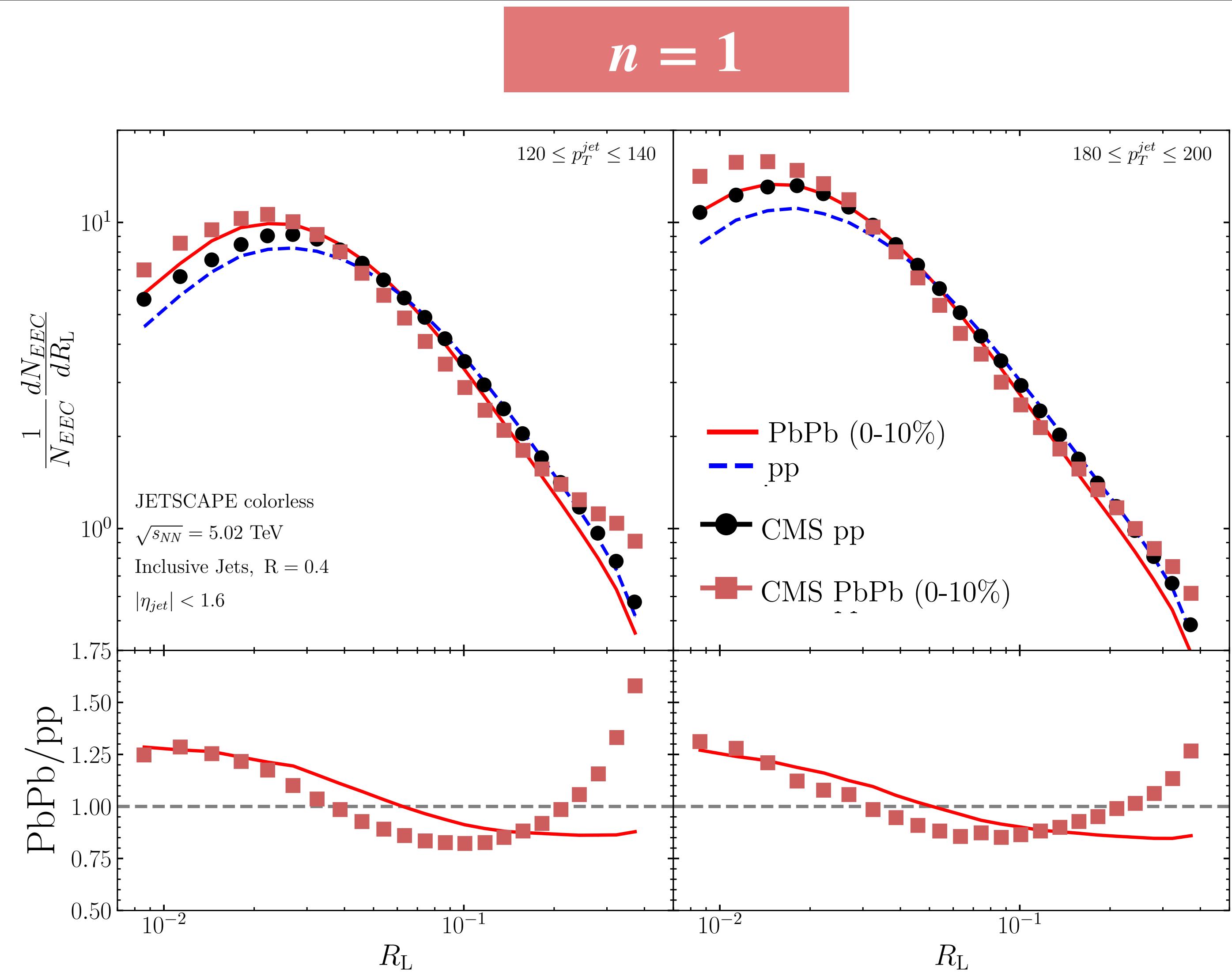
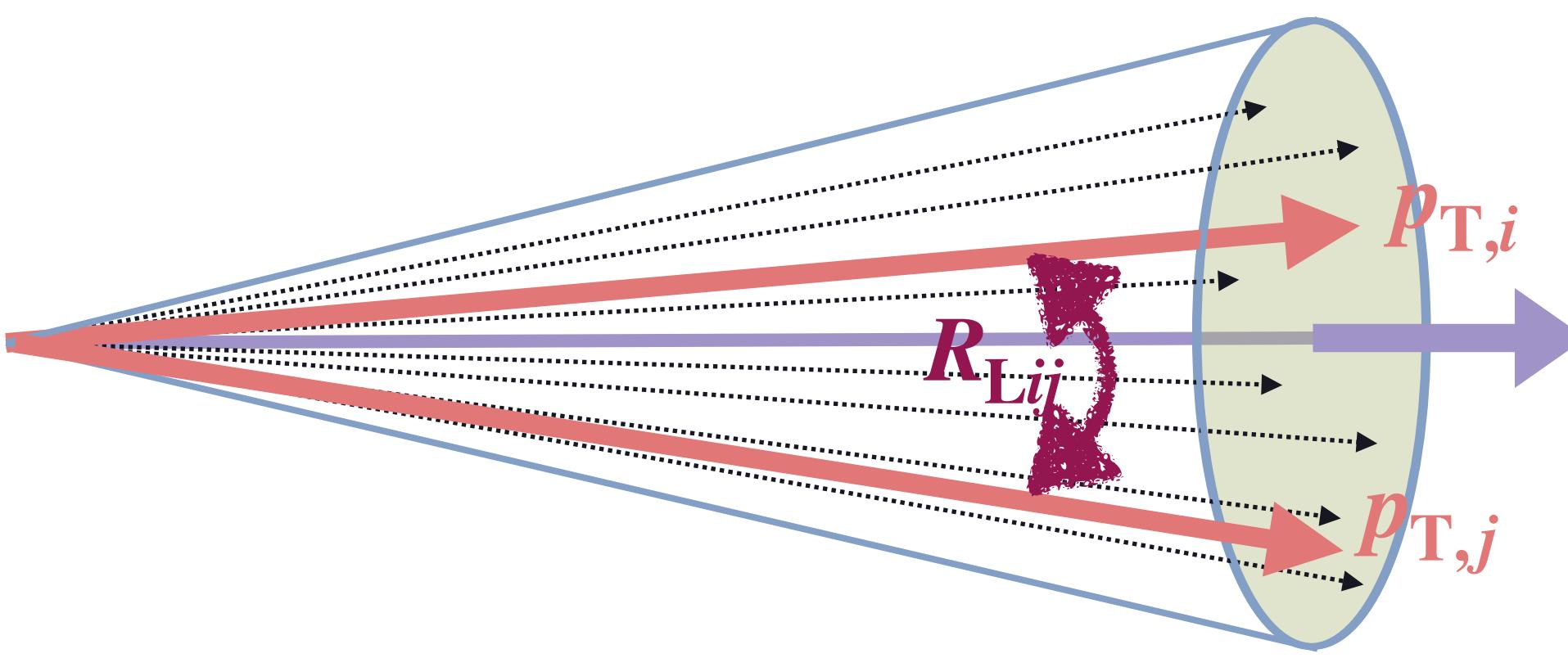
$$\frac{dN_{\text{EEC}}}{dR_L} = \sum_{|R_L - R_{Lij}| < \delta R/2} \frac{(p_{T,i} p_{T,j})^n}{\delta R}$$

$$R_{L,ij} = \sqrt{(\phi_i - \phi_j)^2 + (\eta_i - \eta_j)^2}$$



Energy-energy Correlator

$$\frac{dN_{\text{EEC}}}{dR_L} = \sum_{|R_L - R_{Lij}| < \delta R/2} \frac{(p_{T,i} p_{T,j})^n}{\delta R}$$

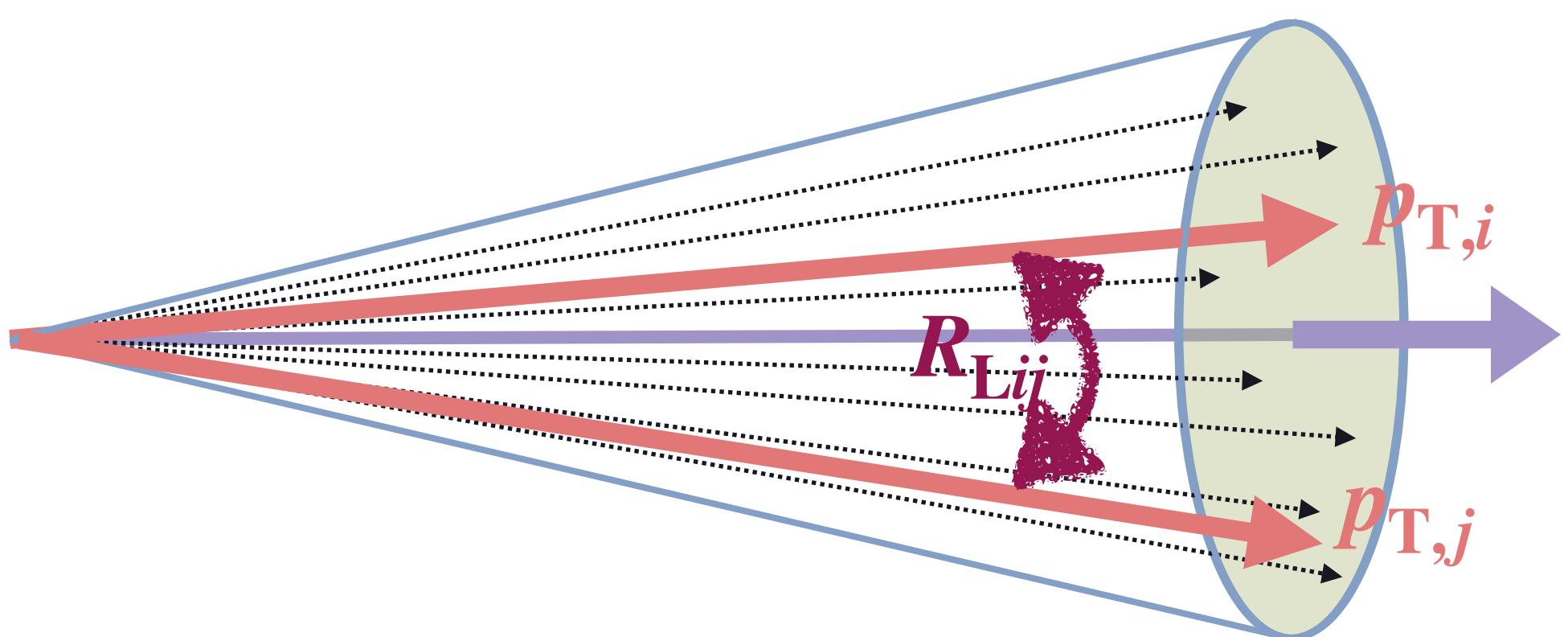


大角度で enhancement

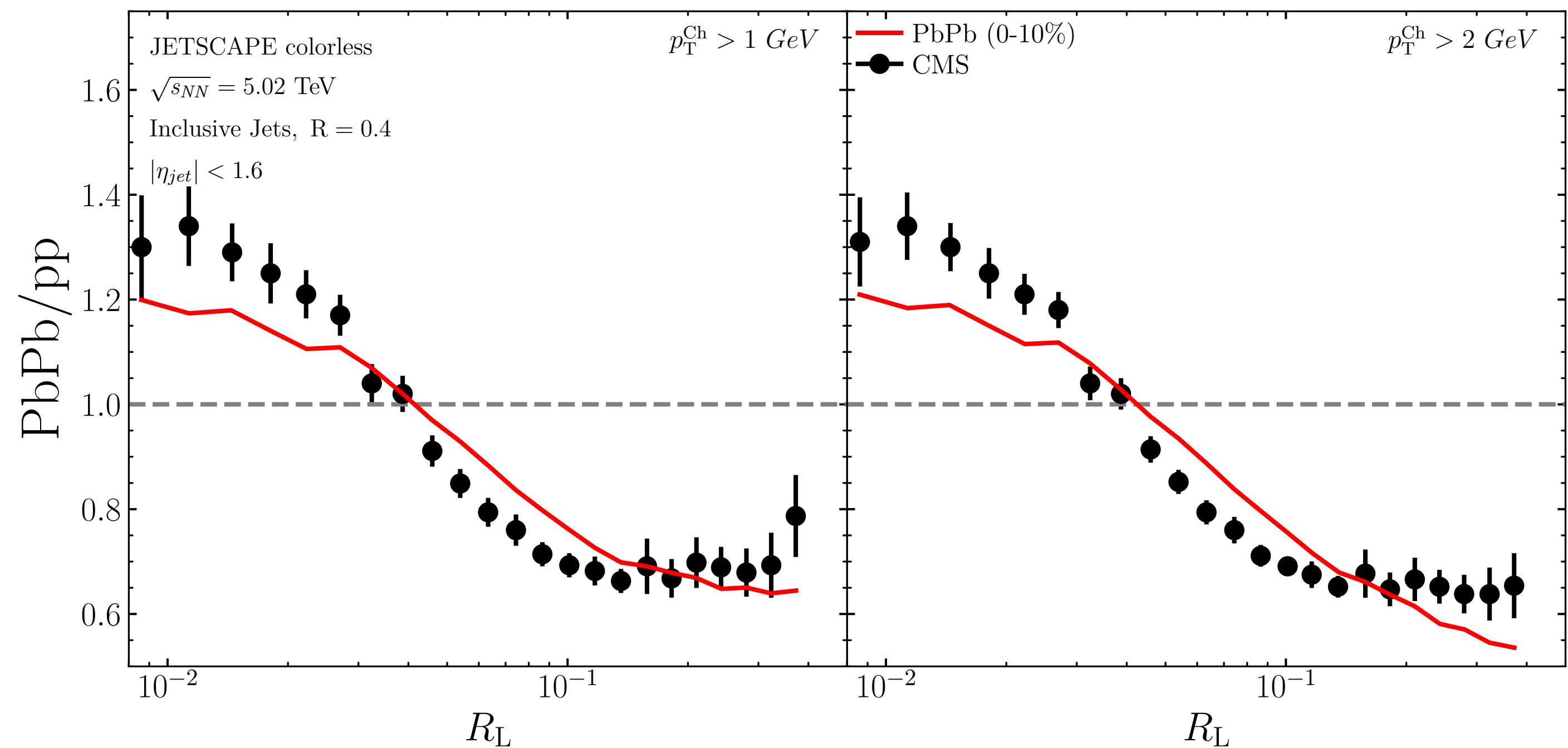
Energy-energy Correlator

$n = 2$

$$\frac{dN_{\text{EEC}}}{dR_L} = \sum_{|R_L - R_{Lij}| < \delta R/2} \frac{(p_{T,i} p_{T,j})^n}{\delta R}$$



- Soft contribution is further suppressed



$n = 1$ での大角度の enhancement
はソフトな成分による

まとめ

まとめ

● ジェット

- ハードなパートン散乱により生成されるハドロンの束
- Jet reconstruction algorithm ($\text{anti-}k_t$ など) で見つける・構成する
- 摂動論が比較的有効, pp が真空中のジェットとしてのベースラインに

● ジェット @ 重イオン

- パートンシャワー発展中に QGP 媒質と強い相互作用 (elastic, radiative)
- シャワー構造が変化→終状態の構造変化, QGP との相互作用の情報
- ジェットエネルギー"損失"=ジェットの運動量の角度分布 (広がり) の変化
- 媒質応答 (衝撃波など) とその測定量への寄与

今何がわかっている（いない）のか

- ジェット内部のハードな成分

- 媒質効果による構造変化は小さい (真空中のジェットの構造を強く保つ)

- ジェット内部のソフトな成分

- 媒質効果による構造変化が大きい
- この大きな変化を, ハードの小さな変化を説明しつつ, 記述するのは難しい
- Medium response, 媒質中のハドロン化過程, 実験でのbackground subtraction, etc.

- Energy-energy correlator

- ジェット内部のハドロン化とパートン分岐構造の寄与がはっきり見える
- 重イオン測定でこれまで以上の新しい物理が見えているかはわかっていない
- ソフトな成分の大きな構造変化がみえている (\rightarrow medium response)

今何がわかっている（いない）のか

● ジェットの内部構造 @ pp

- 媒質効果を詳しく議論できるほど、理論模型の記述の精度がないものがまだ多い
- ハドロン化など、現象論模型

● 媒質形状などの相関 (トモグラフィー)

- 実験では reconstructed jet の大きな ν_2 が見えている
- R_{AA} や dijet/ γ -jet asymmetry を説明するモデルでも説明困難
- バルク側のモデル（初期条件）に対する強い制限をかけるツールになり得る？
- バルク側の異方性により敏感なジェット-QGP 相互作用の機構が必要？

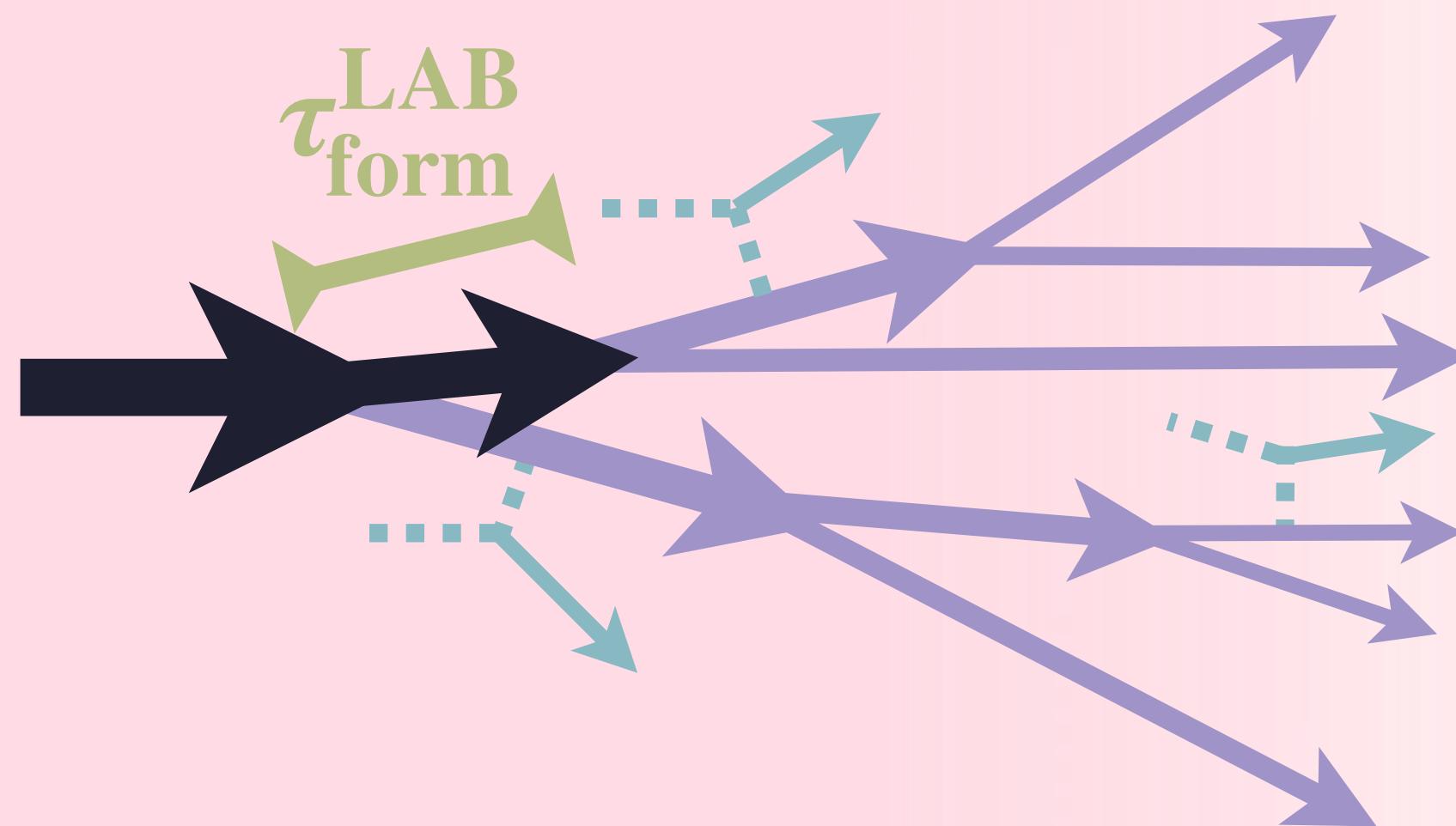
● Color (de)coherence や LPM 効果, energy loss モデルの間の差などは見えているのか

- はっきりと見えている強い証拠はない
- これら効果に本当に敏感なジェット内部構造測定量
- 現実的な計算・シミュレーションによる複数の測定量の説明

This is the last slide.

おまけ

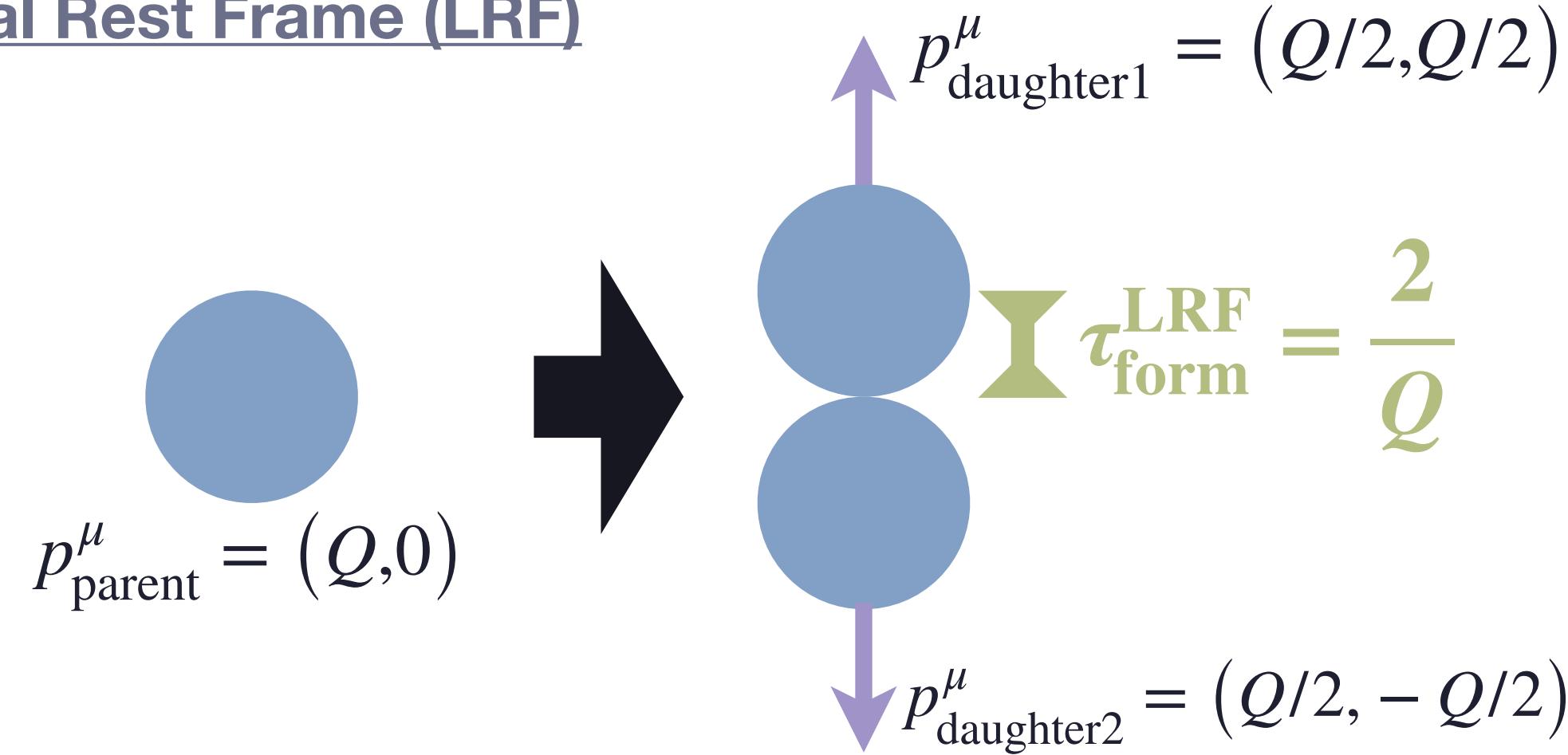
Formation time of parton splitting



Jet parton paths in space-time

- Local interactions with medium evolving in space-time [Inputs from hydro: $u^\mu(x)$, $T(x)$]
- Need space-time information of splittings
- Time for a virtuality-driven splitting:

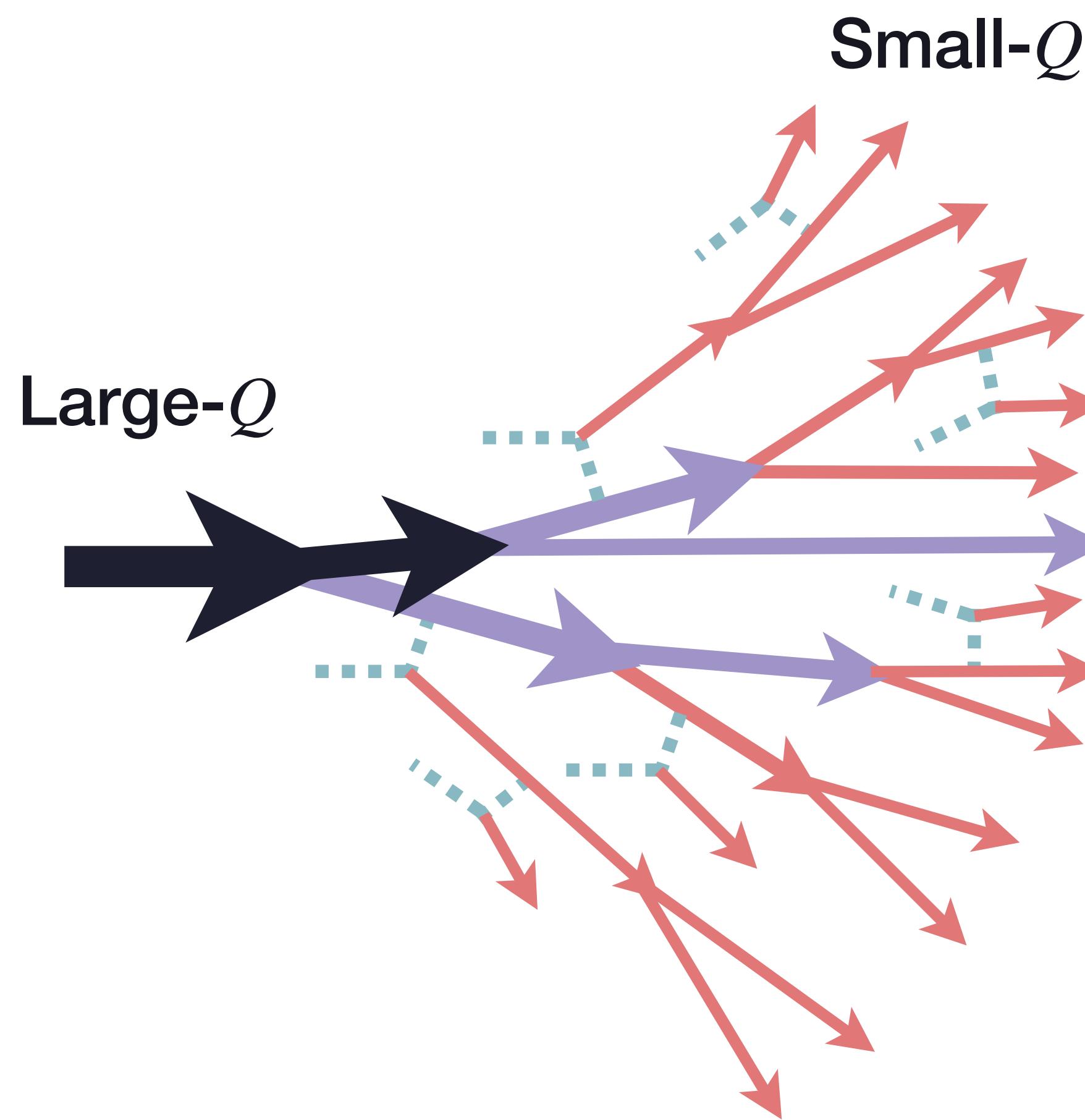
Local Rest Frame (LRF)



$$\tau_{\text{form}}^{\text{LRF}} \sim \frac{2}{Q}$$

$$\tau_{\text{form}}^{\text{LAB}} = \gamma \tau_{\text{form}}^{\text{LRF}} \sim \frac{2E}{Q^2}$$

Formation time of parton splitting



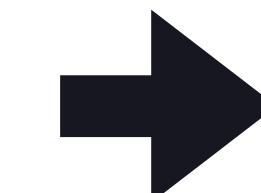
- **High virtuality phase**

- Time for a virtuality-driven splitting: $\tau_{\text{form}} \sim \frac{2E}{Q^2}$

- **Low virtuality phase**

- Virtuality gain from the medium: $\Delta Q_{\text{med}}^2 \sim \hat{q}\tau_{\text{form}}$
- Formation time for medium induced radiation:

$$\tau_{\text{form}} \sim \frac{2E}{\Delta Q_{\text{med}}^2} \sim \frac{2E}{\hat{q}\tau_{\text{form}}}$$



$$\tau_{\text{form}} \sim \sqrt{\frac{2E}{\hat{q}}}$$

どちらの Phase を前提とした計算・議論なのか気をつける必要あり

High and low virtuality phase

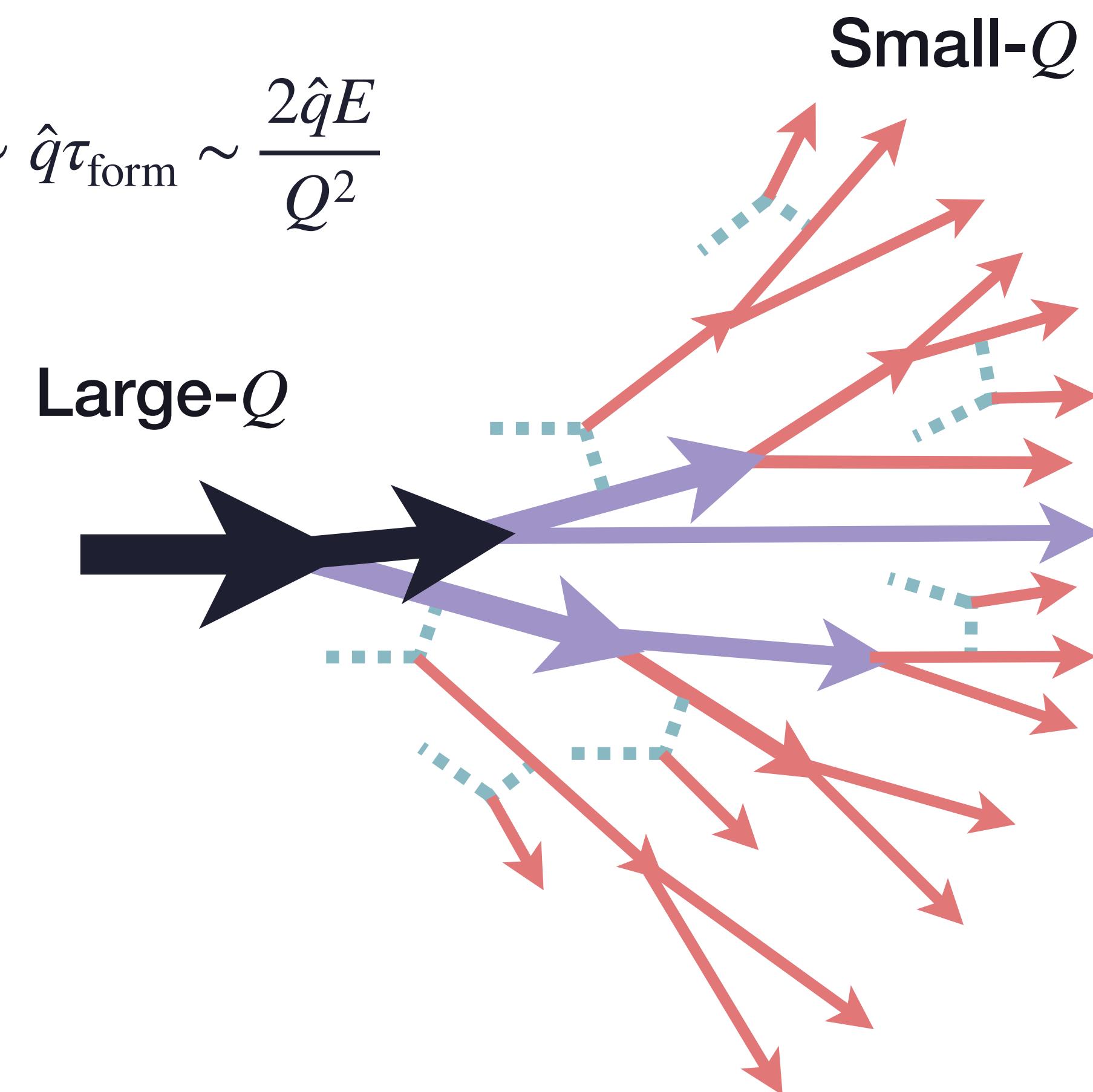
- Formation time: $\tau_{\text{form}} \sim \frac{2E}{Q^2}$
- Virtuality gain from the medium: $\Delta Q_{\text{med}}^2 \sim \hat{q}\tau_{\text{form}} \sim \frac{2\hat{q}E}{Q^2}$

• High virtuality phase

$$Q^2 \gg \Delta Q_{\text{med}}^2 \sim \frac{2\hat{q}E}{Q^2} \rightarrow Q^2 \gg \sqrt{2\hat{q}E}$$

• Low virtuality phase

$$Q^2 \ll \Delta Q_{\text{med}}^2 \rightarrow Q^2 \ll \sqrt{2\hat{q}E}$$

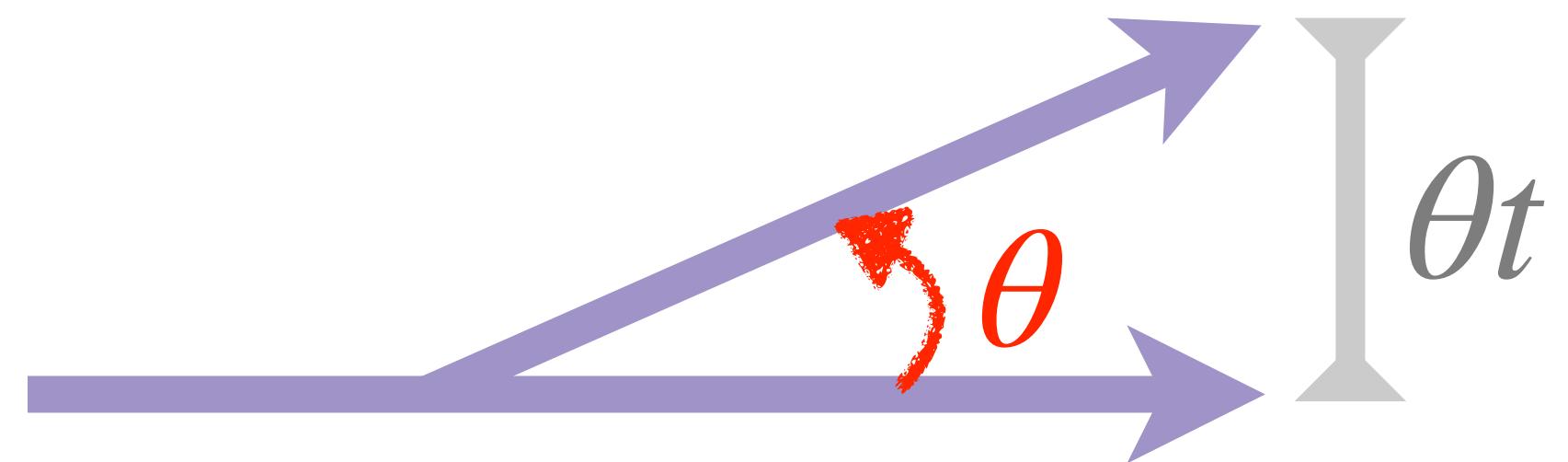


どちらの Phase を前提とした計算・議論なのか気をつける必要あり

Color (de)coherence

Y. Mehtar-Tani, C. A. Salgado, and K. Tywoniuk (2011), J. Casalderrey-Solana and E. Iancu (2011)

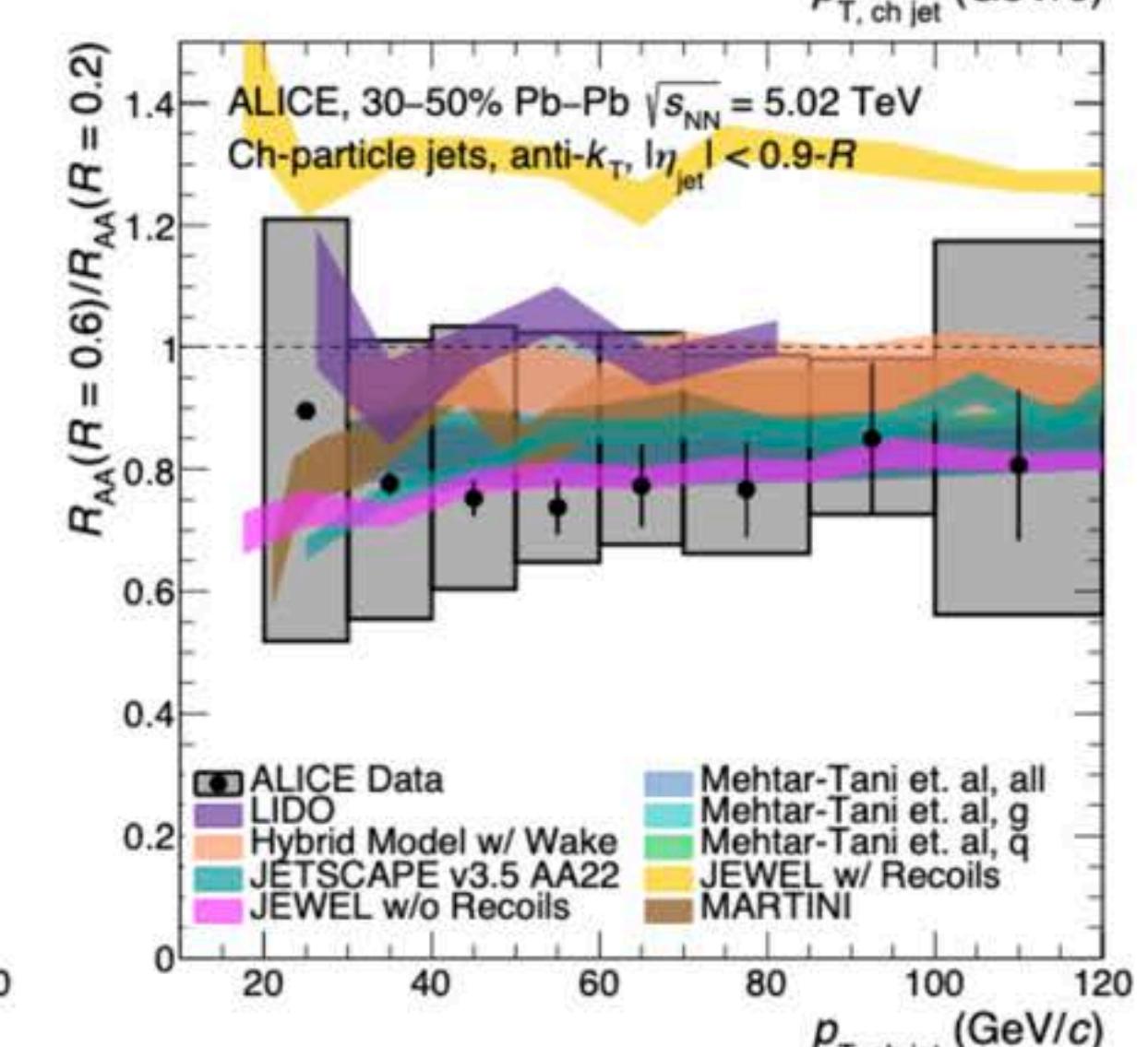
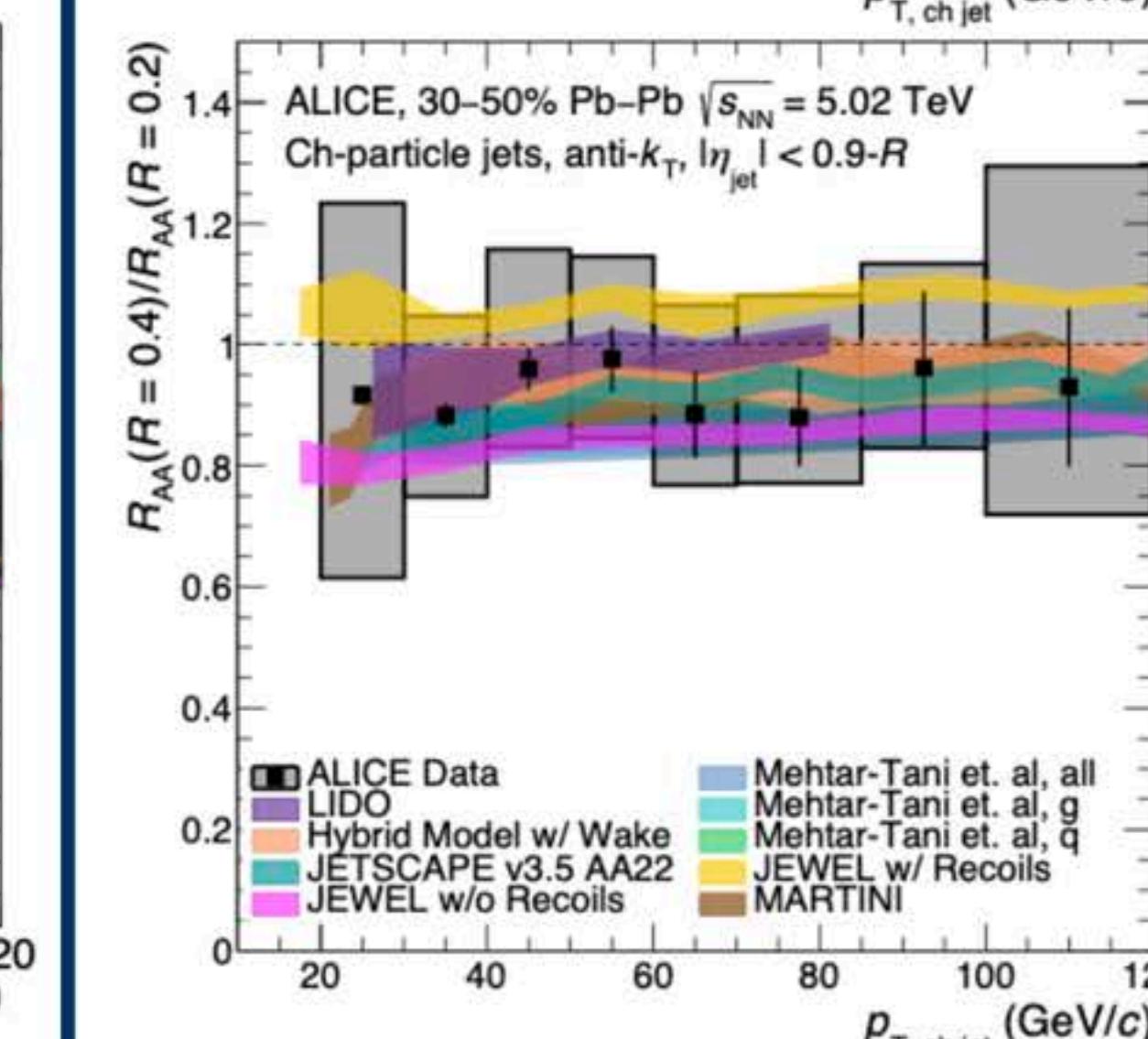
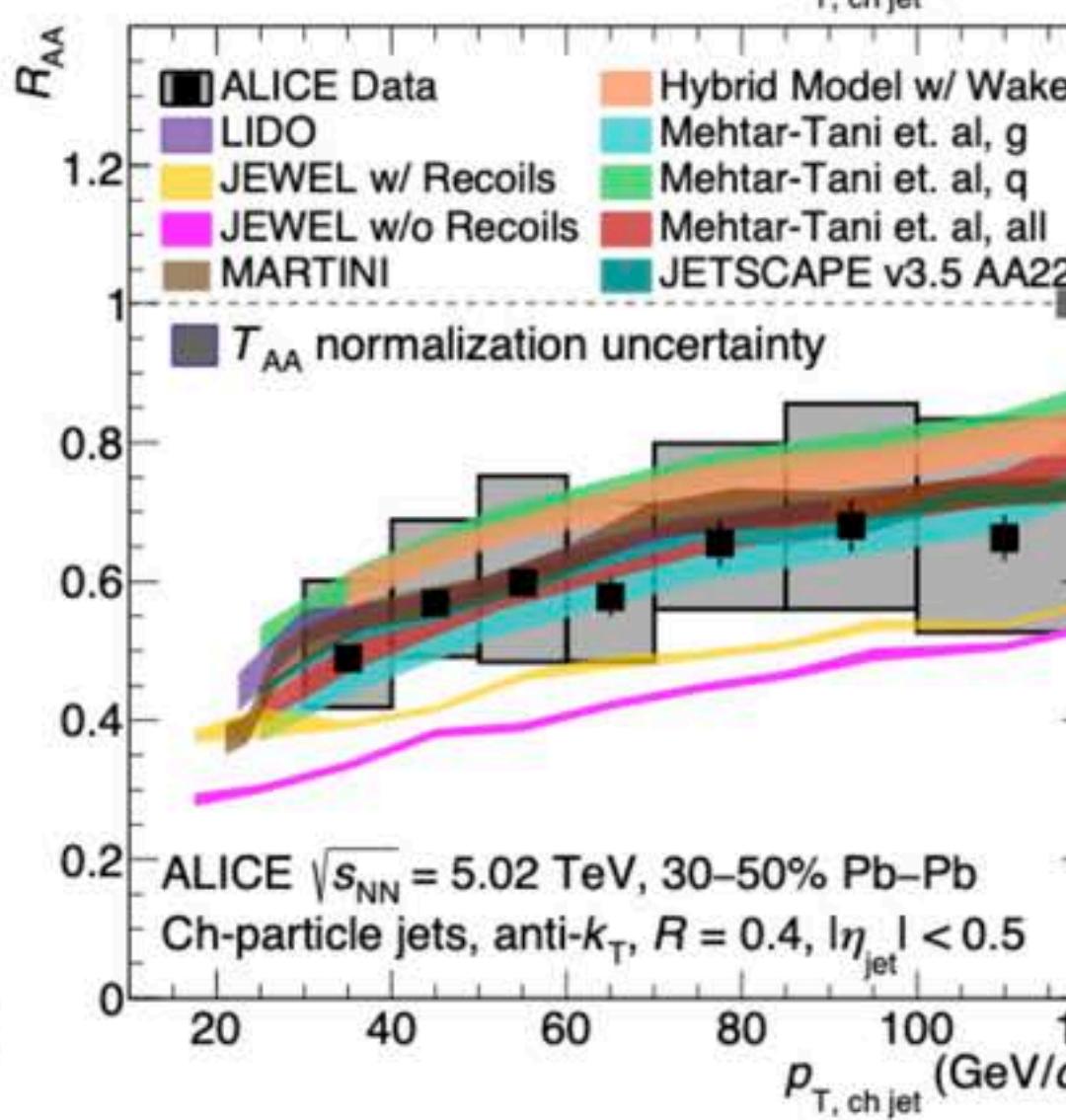
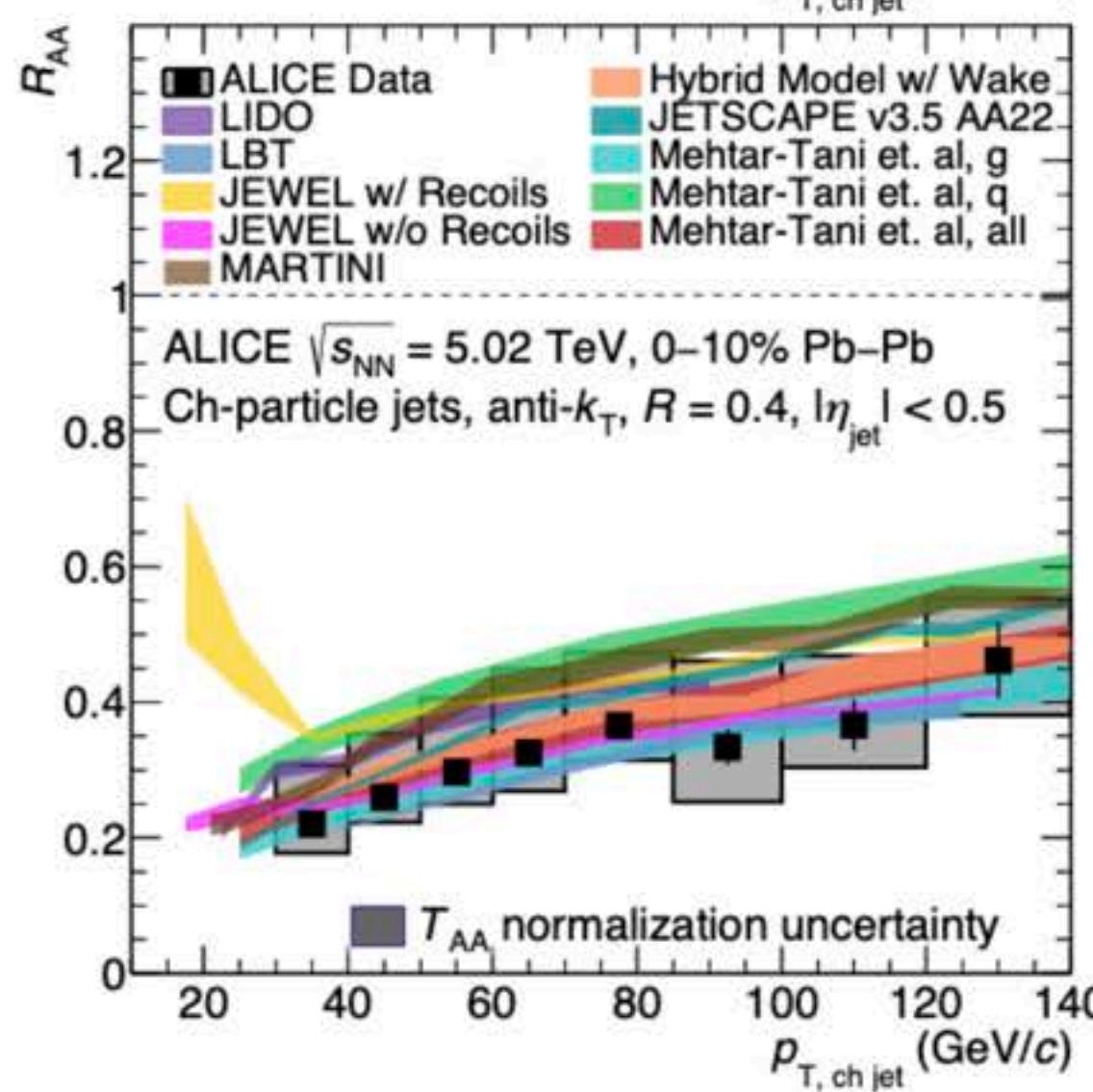
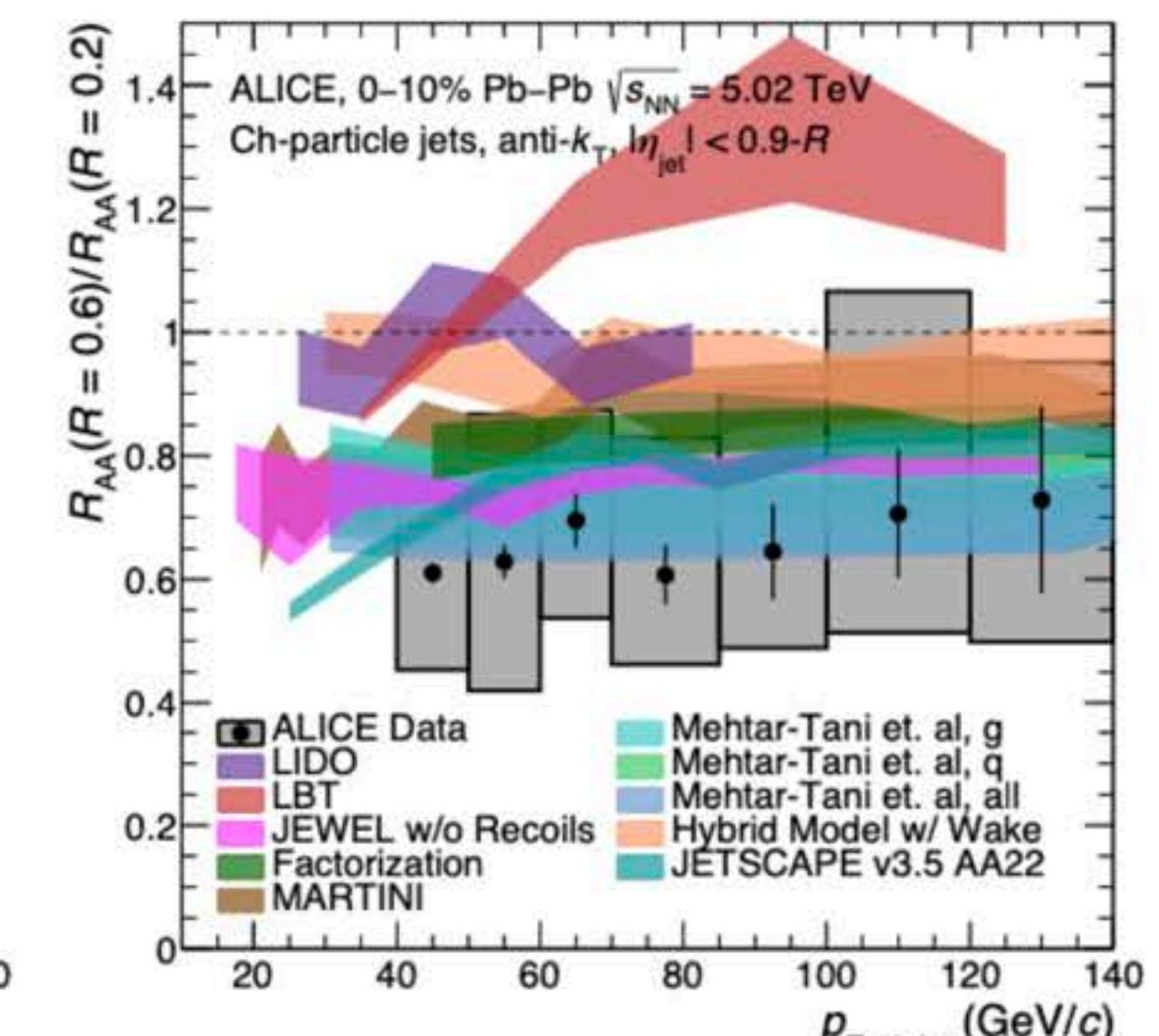
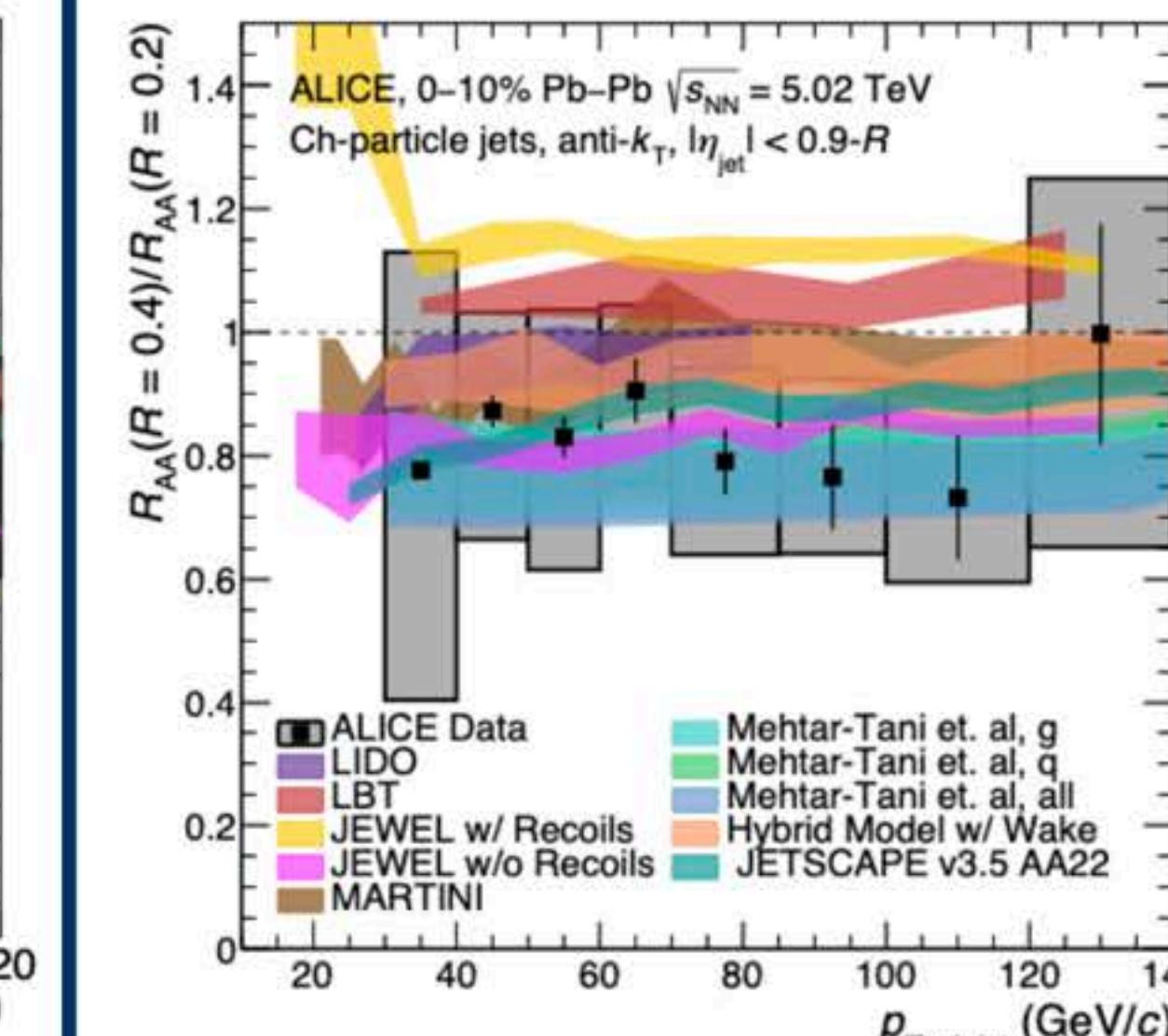
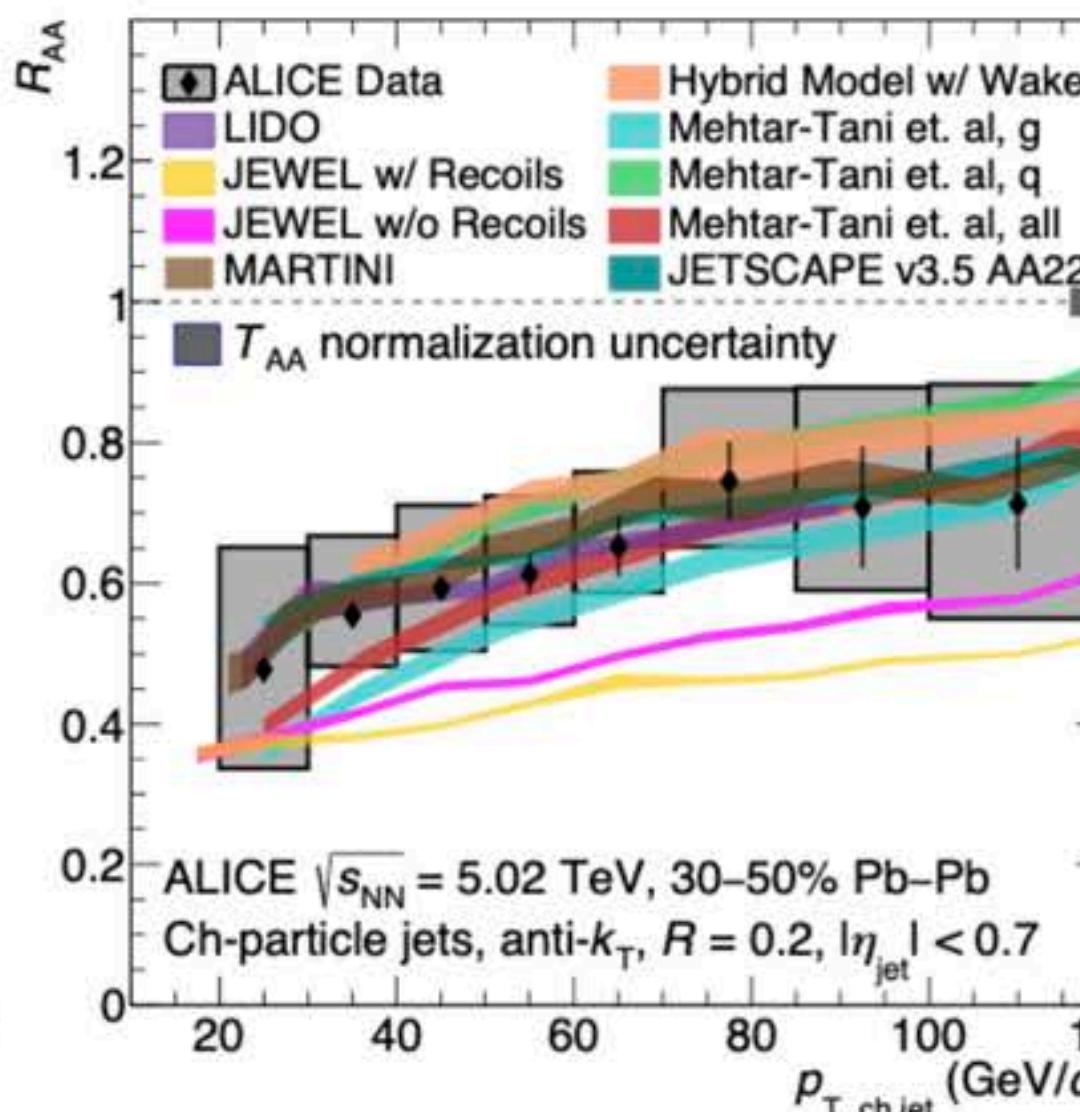
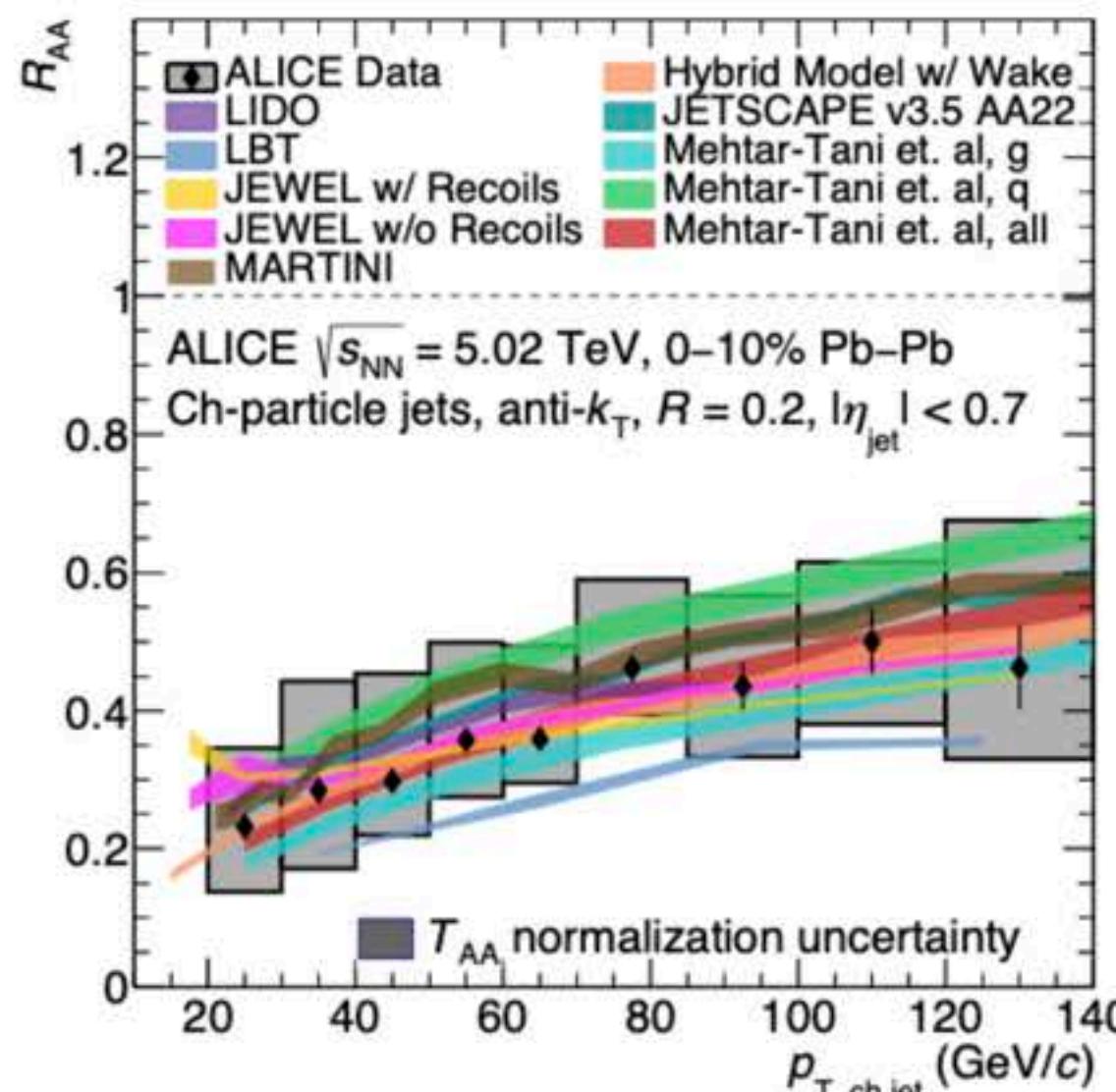
- parton splitting が formation time 後でも, 媒質に分解されるまでに時間がかかることがある



- 媒質の分解能スケール: $\frac{1}{Q_s} \sim \frac{1}{\sqrt{\hat{q}t}}$
- 媒質に分解されるようになる時間 (デコヒーレンス時間): t_d

$$\theta t_d \sim \frac{1}{Q_s} \sim \sqrt{\frac{1}{\hat{q}t_d}} \rightarrow t_d \sim \left[\frac{1}{\hat{q}\theta^2} \right]^{\frac{1}{3}}$$

Conesize Dependence (Jet Shape)

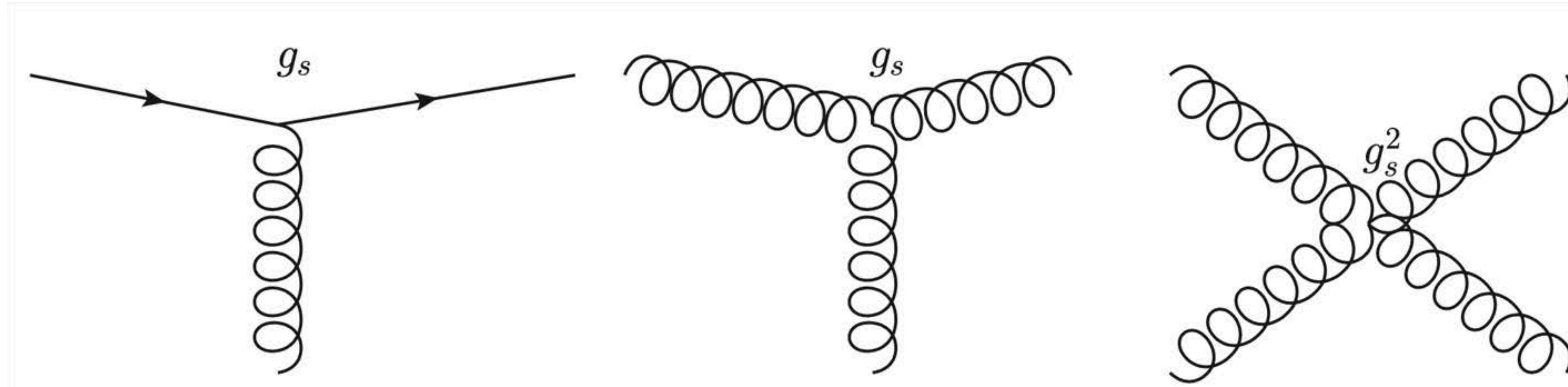


Quantum Chromodynamics (QCD)

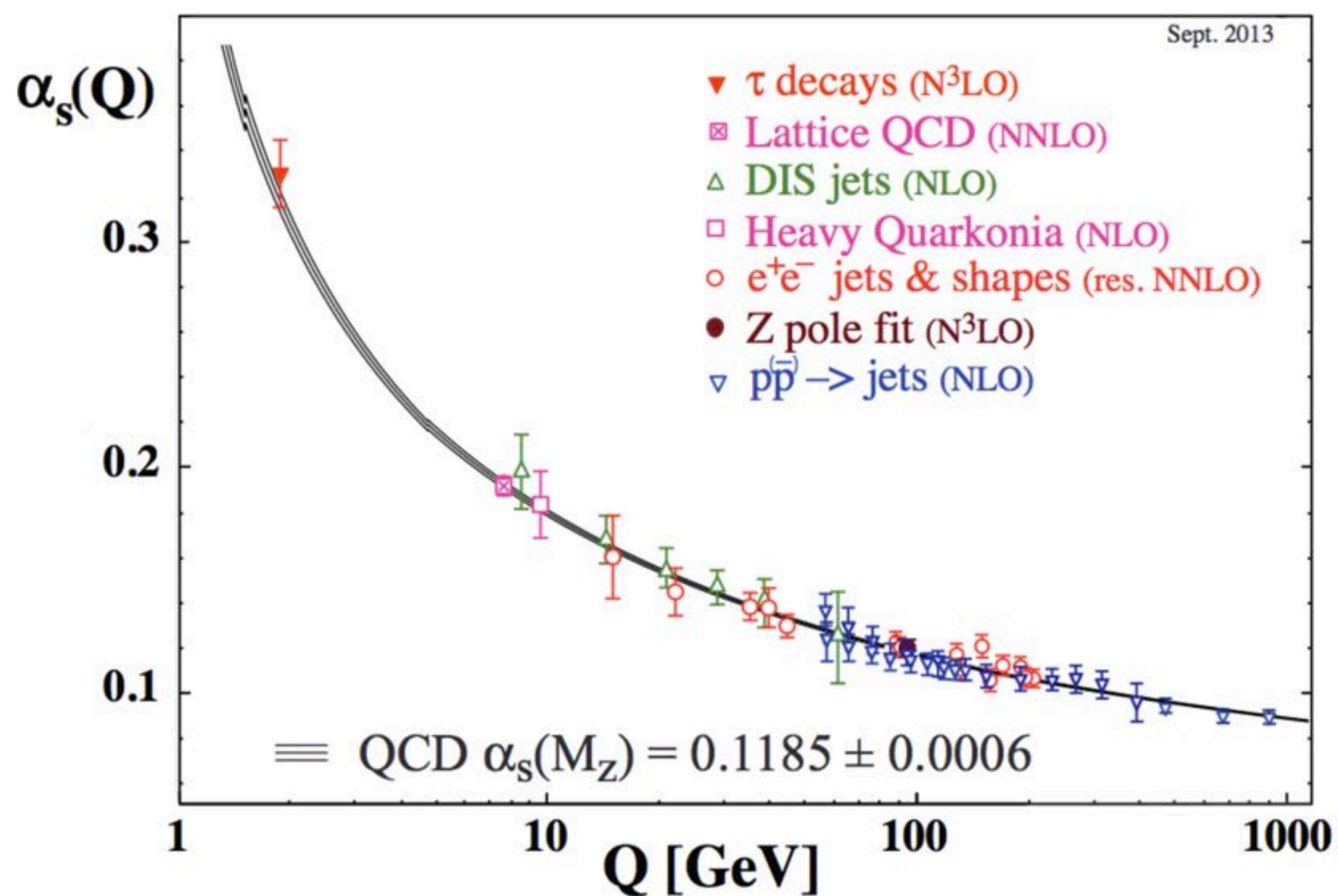
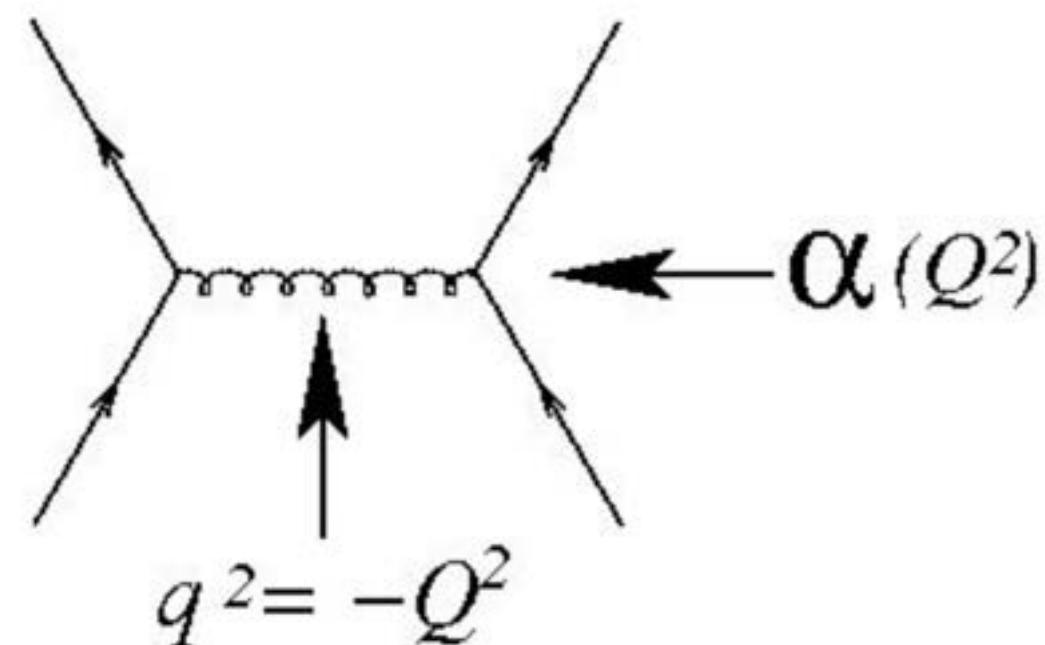
- SU(3) gauge theory (non-Abelian)
- Strong interactions between partons with color charge: quarks and gluons

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (iD - m_f) \psi_f - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\nu^b A_\nu^c$$



Asymptotic Freedom

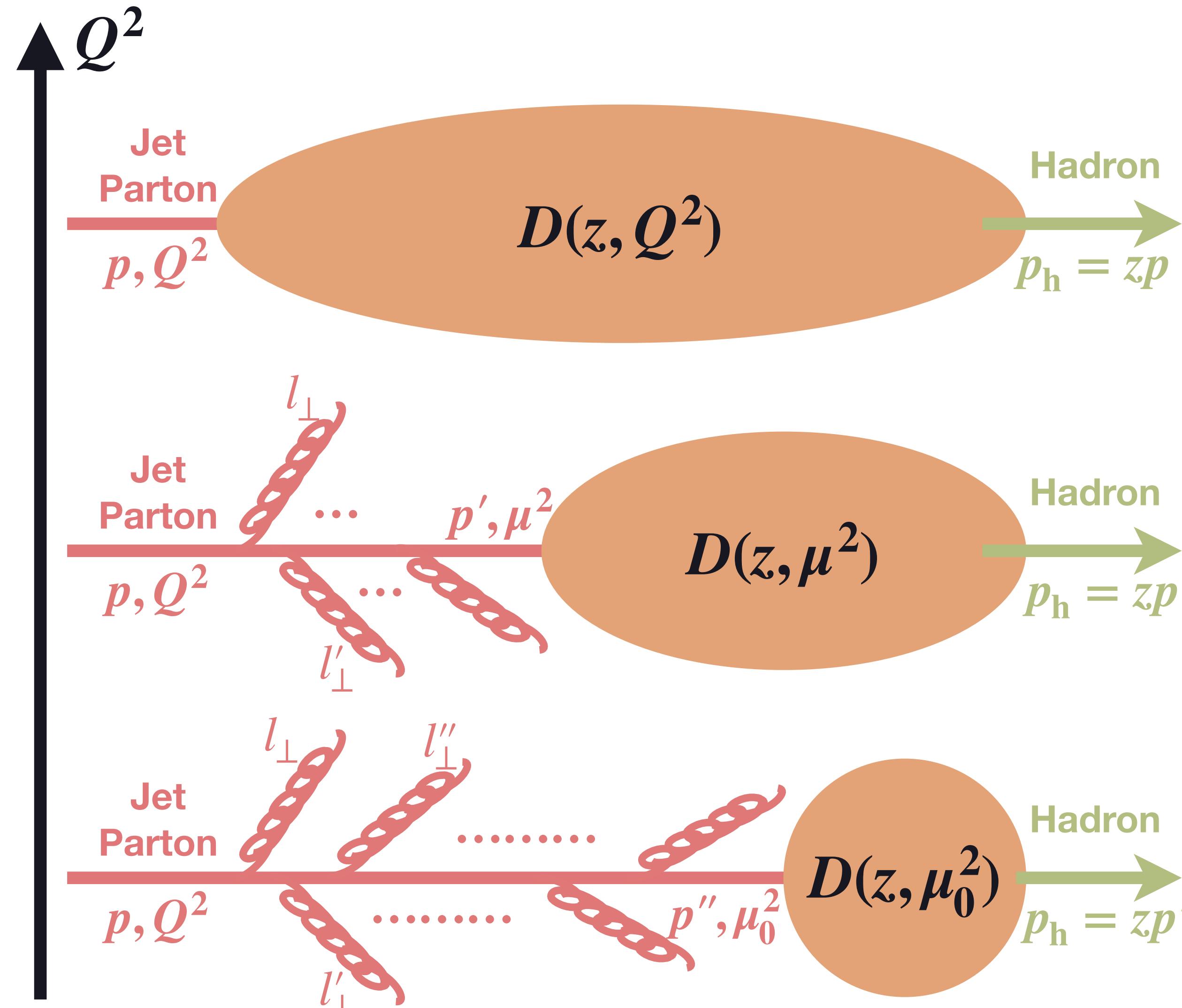


$$\alpha_s(Q^2) = \frac{[g_s(Q^2)]^2}{4\pi} = \frac{12\pi}{(11N_c - 2n_f) \log \frac{Q^2}{\Lambda_{\text{QCD}}^2}}$$
$$\alpha_s(Q^2) \rightarrow +\infty \quad (Q^2 \searrow \Lambda_{\text{QCD}}^2)$$

- Weaker coupling at large momentum transfer
→ Perturbative for hard processes ($Q^2 \gg 1 \text{ GeV}^2$)

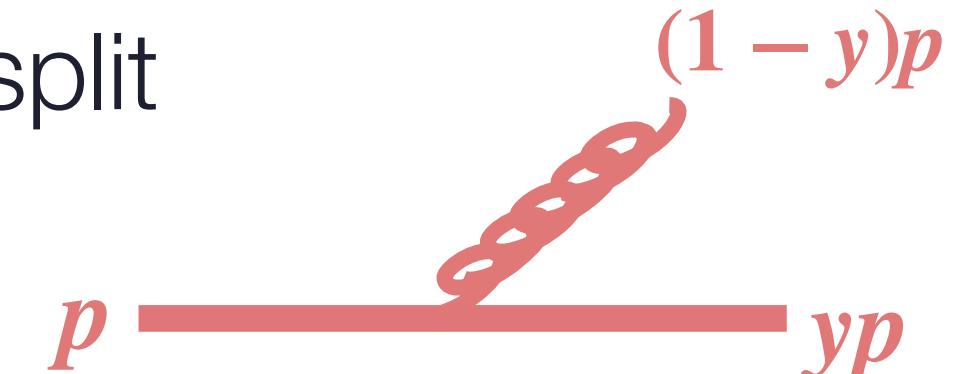
Parton Shower (in Vacuum)

- **DGLAP Evolution** [V. Gribov and L. Lipatov (1972) G. Altarelli and G. Parisi (1977) Yu. Dokshitzer (1977)]



- Probability of a perturbative split

$P(y)$: **splitting function**



- Scale evolution of FF

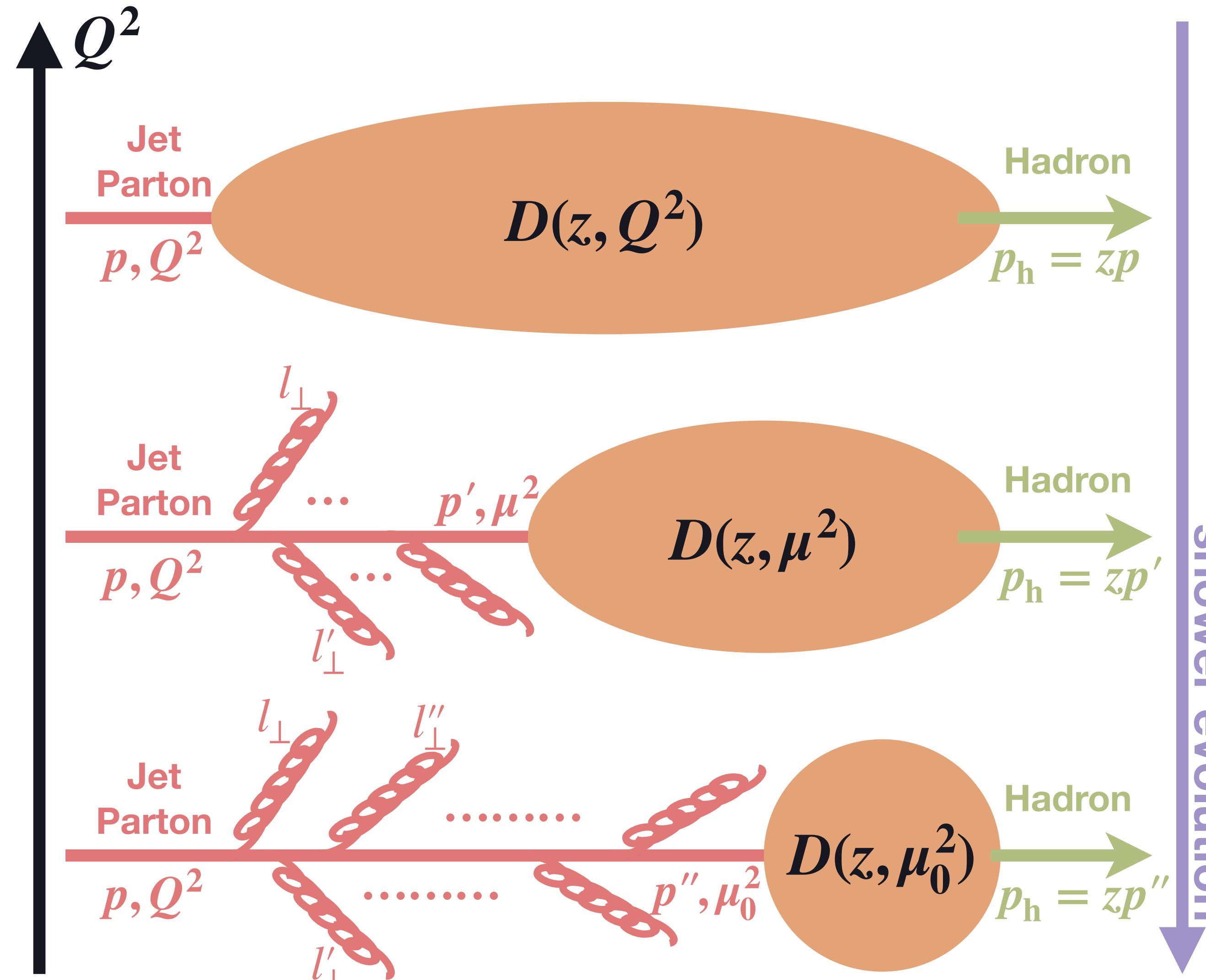
$$D(z, Q^2) = D(z, \mu^2) + \frac{\alpha_s}{2\pi} \int_{\mu^2}^{Q^2} \frac{dl_\perp}{l_\perp^2} \int_z^1 dy \frac{1}{y} P(y) D\left(\frac{z}{y}, \mu^2\right) \\ + \left(\frac{\alpha_s}{2\pi}\right)^2 \int_{\mu^2}^{Q^2} \frac{dl_\perp}{l_\perp^2} \int_{\mu^2}^{l_\perp^2} \frac{dl_\perp}{l_\perp^2} \int_z^1 dy \int_{z/y}^1 \frac{dy'}{y'} \frac{1}{y'} P(y) P(y') D\left(\frac{z}{yy'}, \mu^2\right) \\ + \dots$$

↓

$$Q^2 \frac{d}{dQ^2} D(z, Q^2) = \frac{\alpha_s}{2\pi} \int_z^1 \frac{dy}{y} P(y) D\left(\frac{z}{y}, Q^2\right)$$

Parton Shower (in Vacuum)

- Monte-Carlo simulation of in-vacuum parton shower



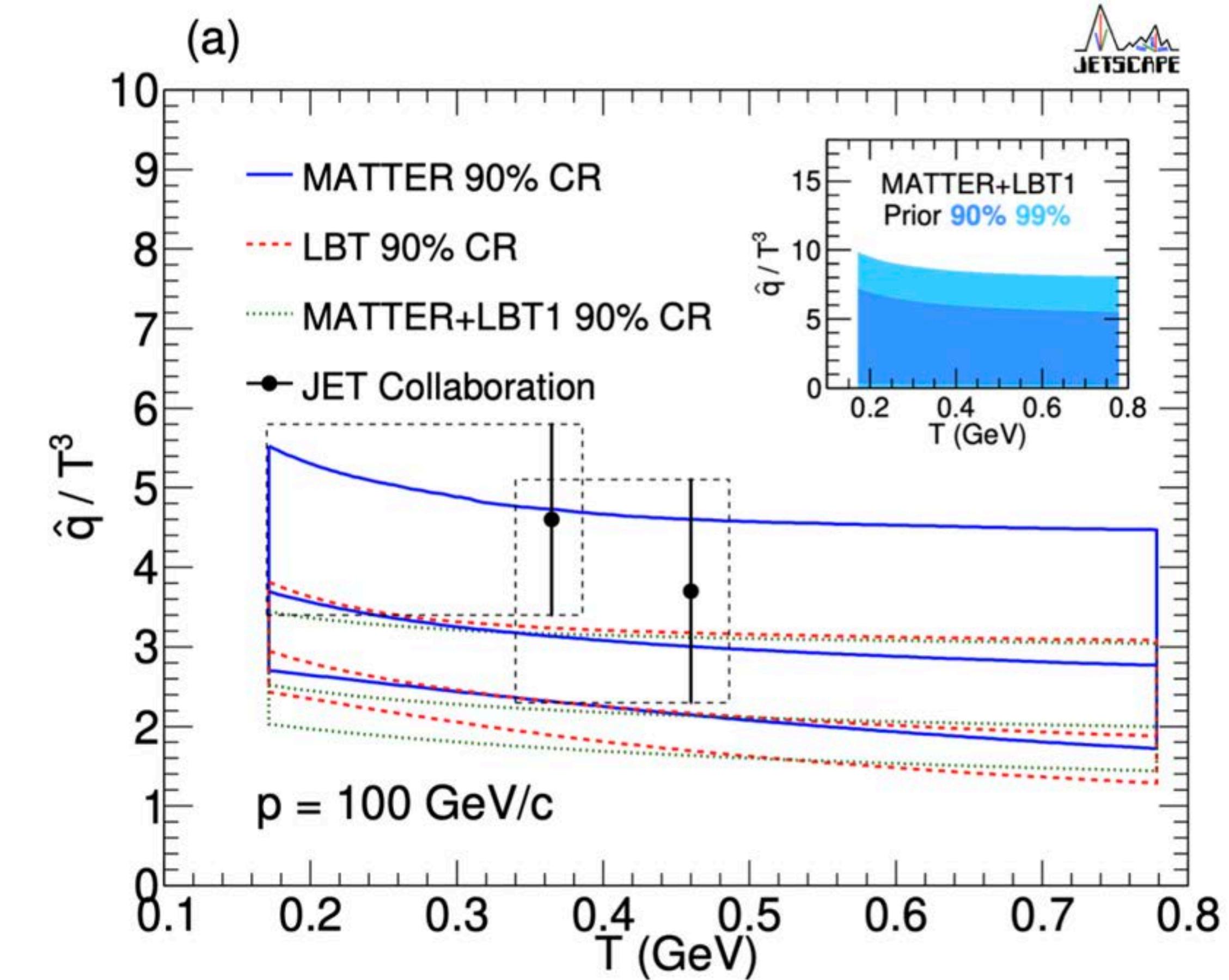
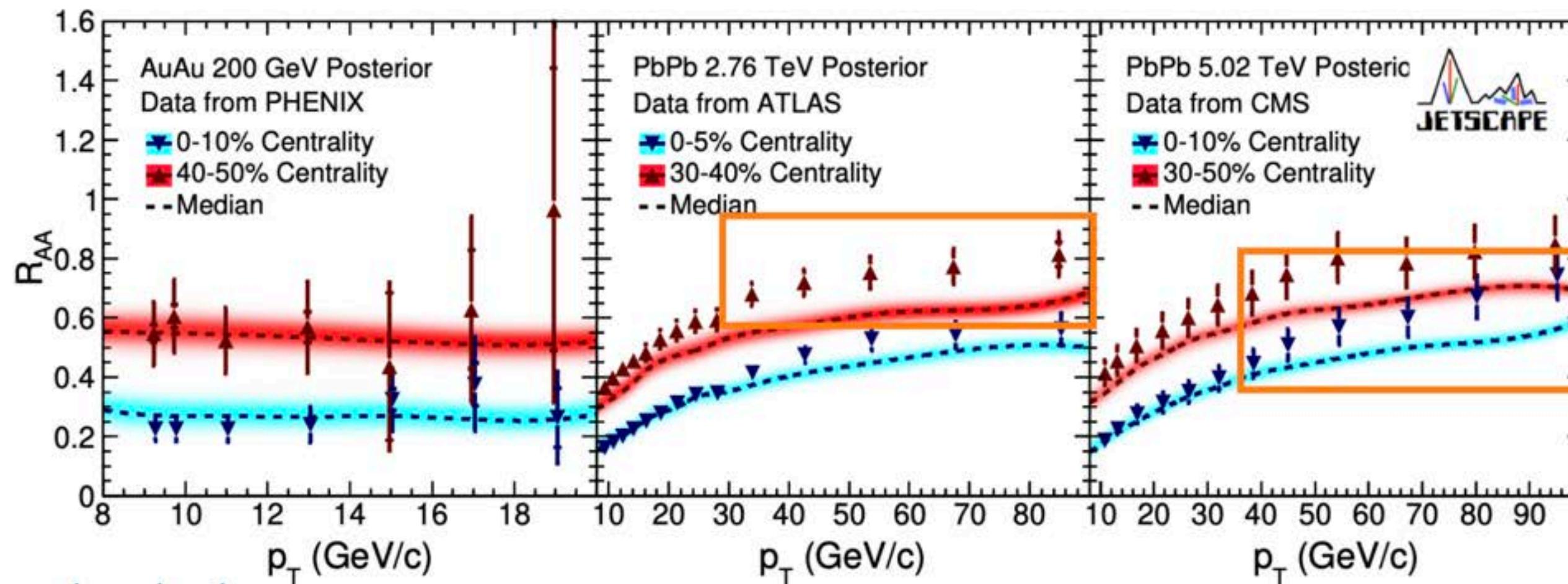
- DGLAP-like evolution in *negative* Q^2 direction (not in time)
- Generate splittings via DGLAP based formalism (Sudakov form factor)
- Sampled radiated daughters also evolve in Q^2 with further splittings
- Switch to hadronization model* when Q^2 reaches a value predetermined in the model

*Hadronization is usually done by a phenomenological model tuned to reproduce subset of experimental data

Parton-medium Interactions

● Bayesian analysis for the extraction of \hat{q} [JETSCAPE, PRC 104, 024905 (2021)]

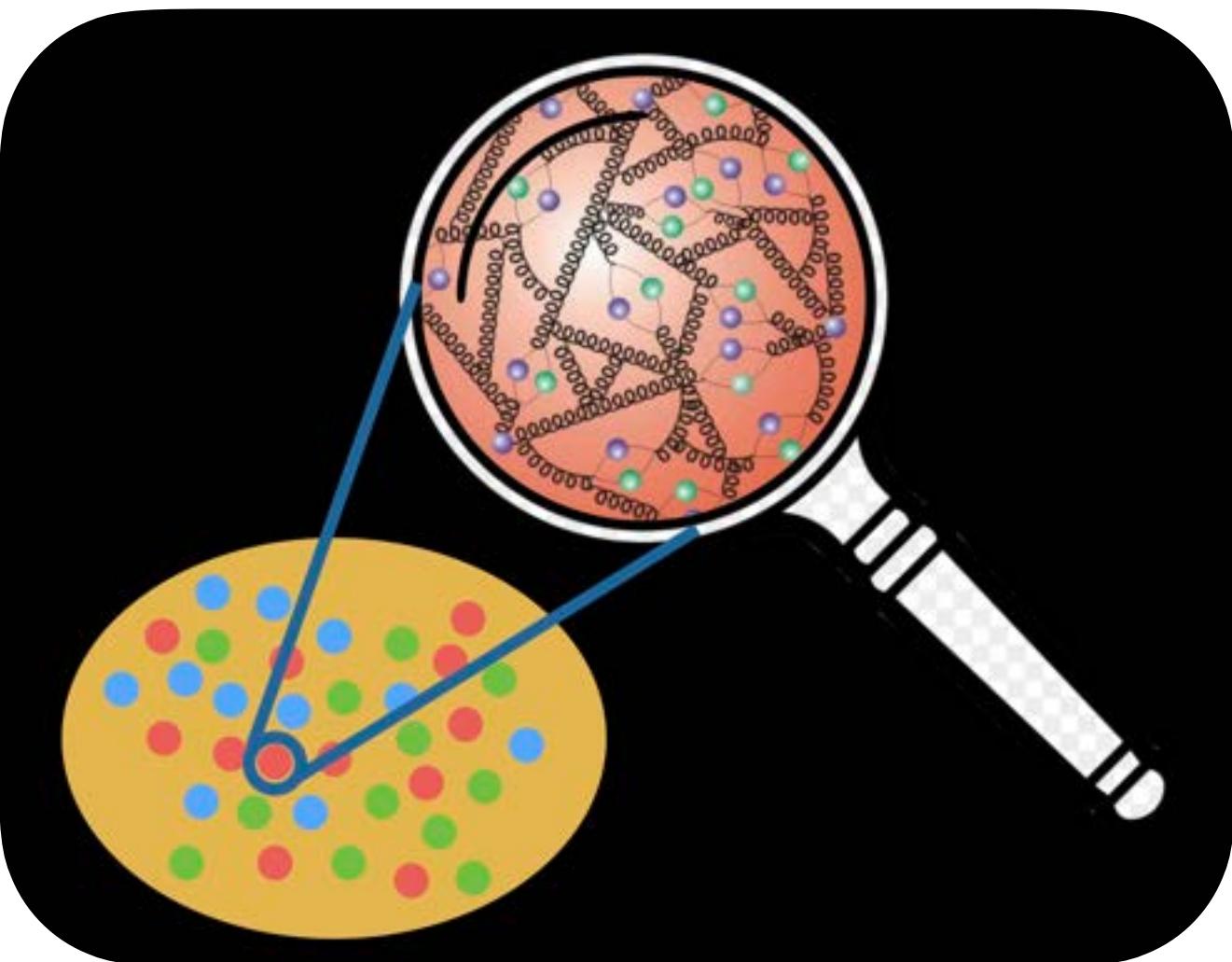
- Fit single hadron R_{AA} at LHC and RHIC
- Tension at high- p_T



Motivations of Studies of Jets in QGP

● Dynamical probing of the QGP medium

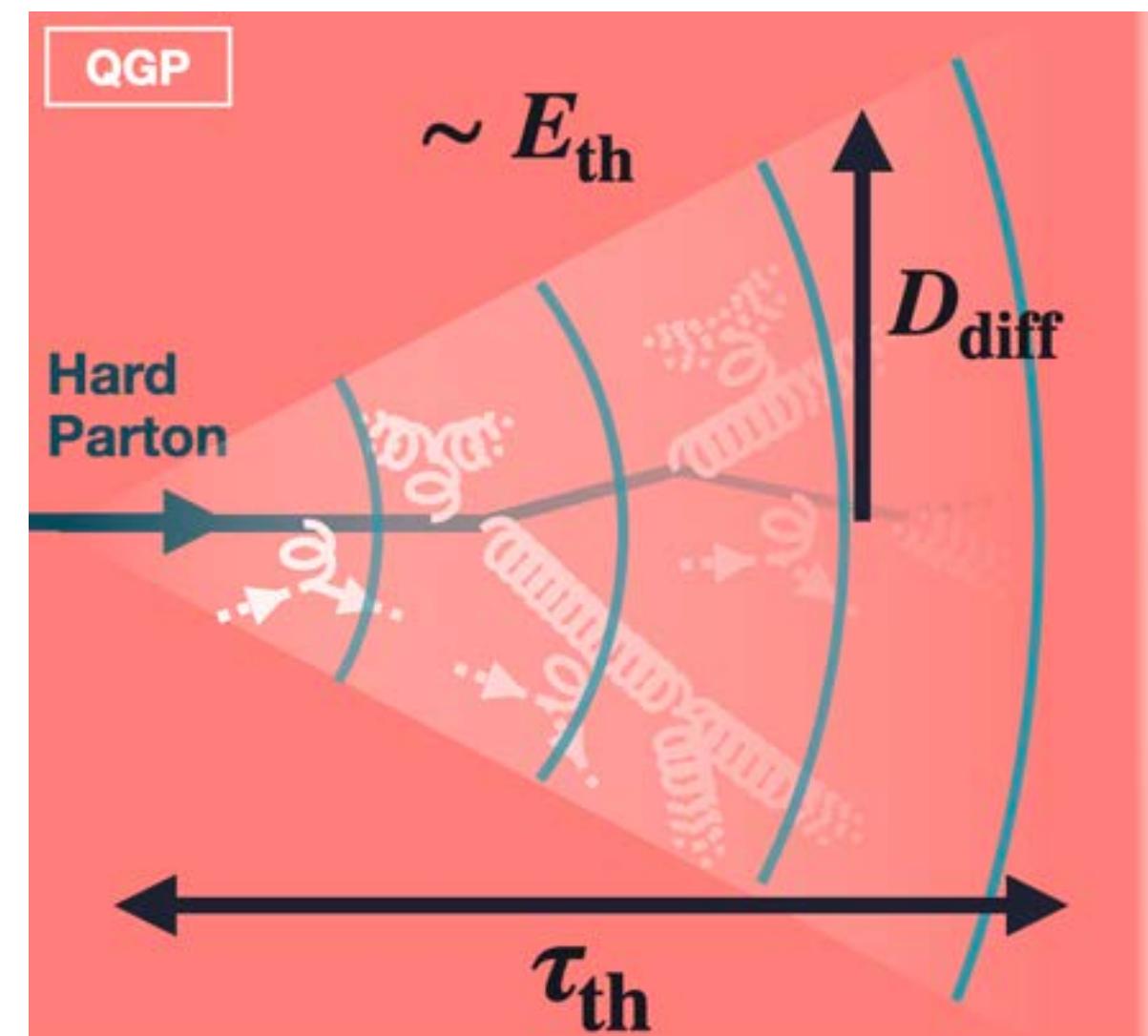
- Interaction strength between jet and medium
- Parton distribution in the QGP medium
- Jet virtuality and energy dependence
(Multi-resolution scale)



Adapted from Chun Shen

● In-medium thermalization of partons

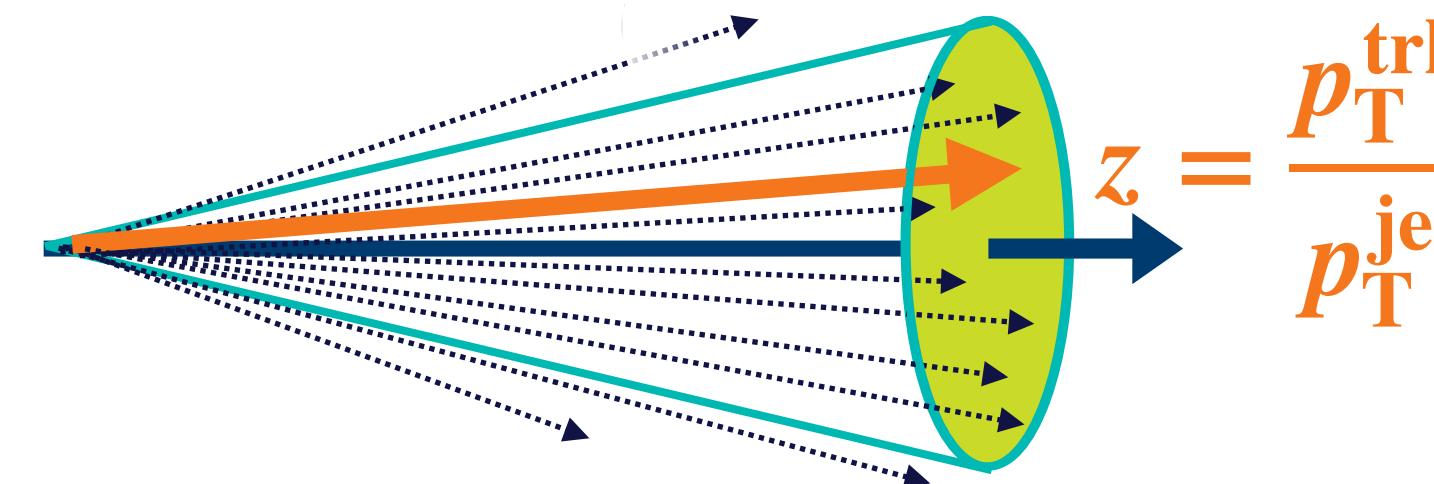
- Clue to understand the QGP formation



More Jet Observables

- **Jet fragmentation**

$$D(z) = \frac{1}{N_{\text{jet}}} \sum_{\text{jet}} \left. \frac{dN_{\text{trk}}}{dz} \right|_{\text{in jet}}$$

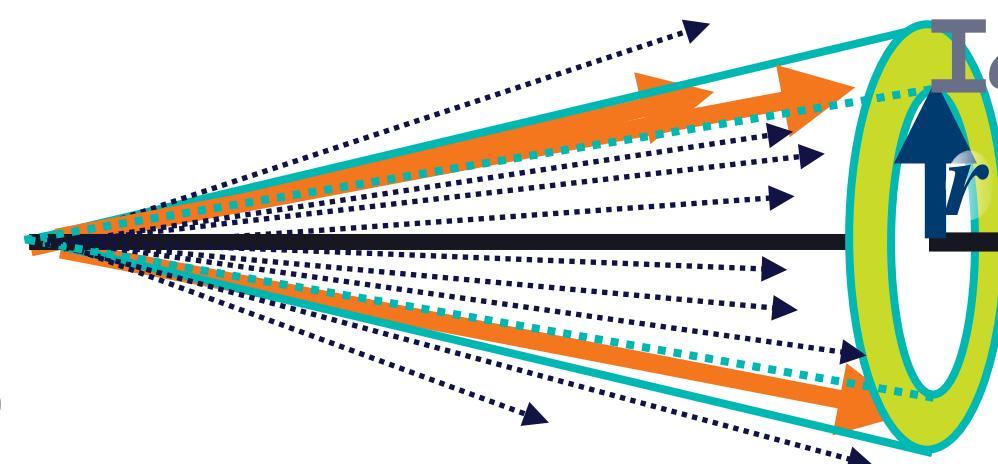


$$z = \frac{p_T^{\text{trk}}}{p_T^{\text{jet}}}$$

- **Jet Shape**

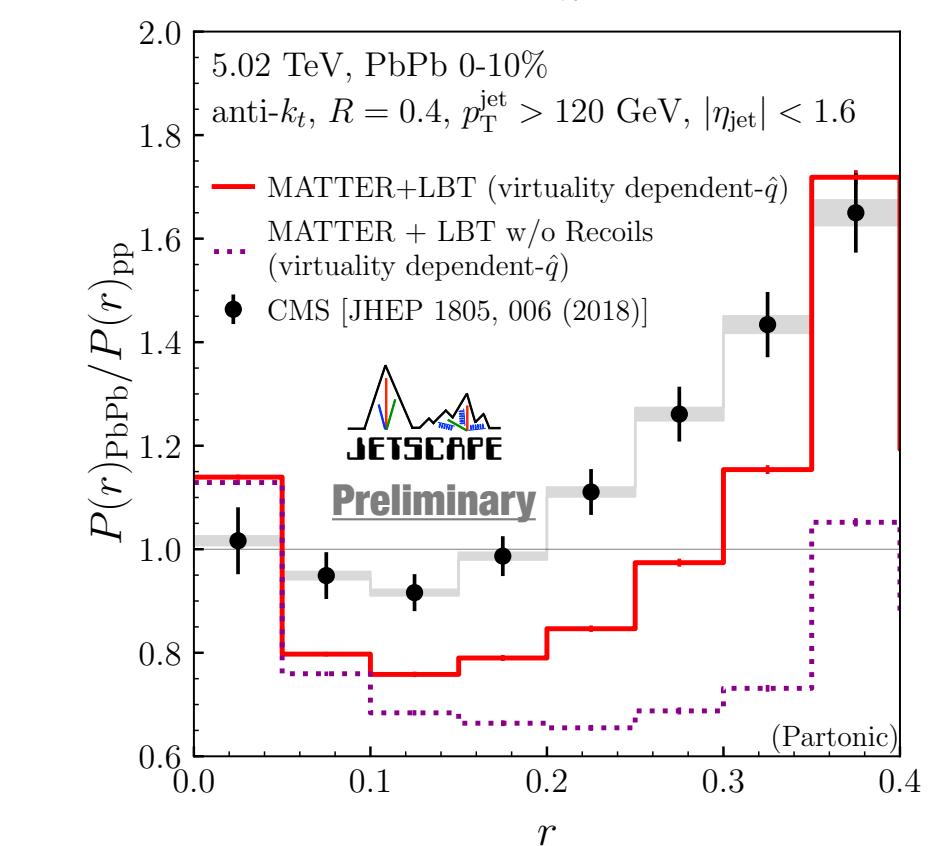
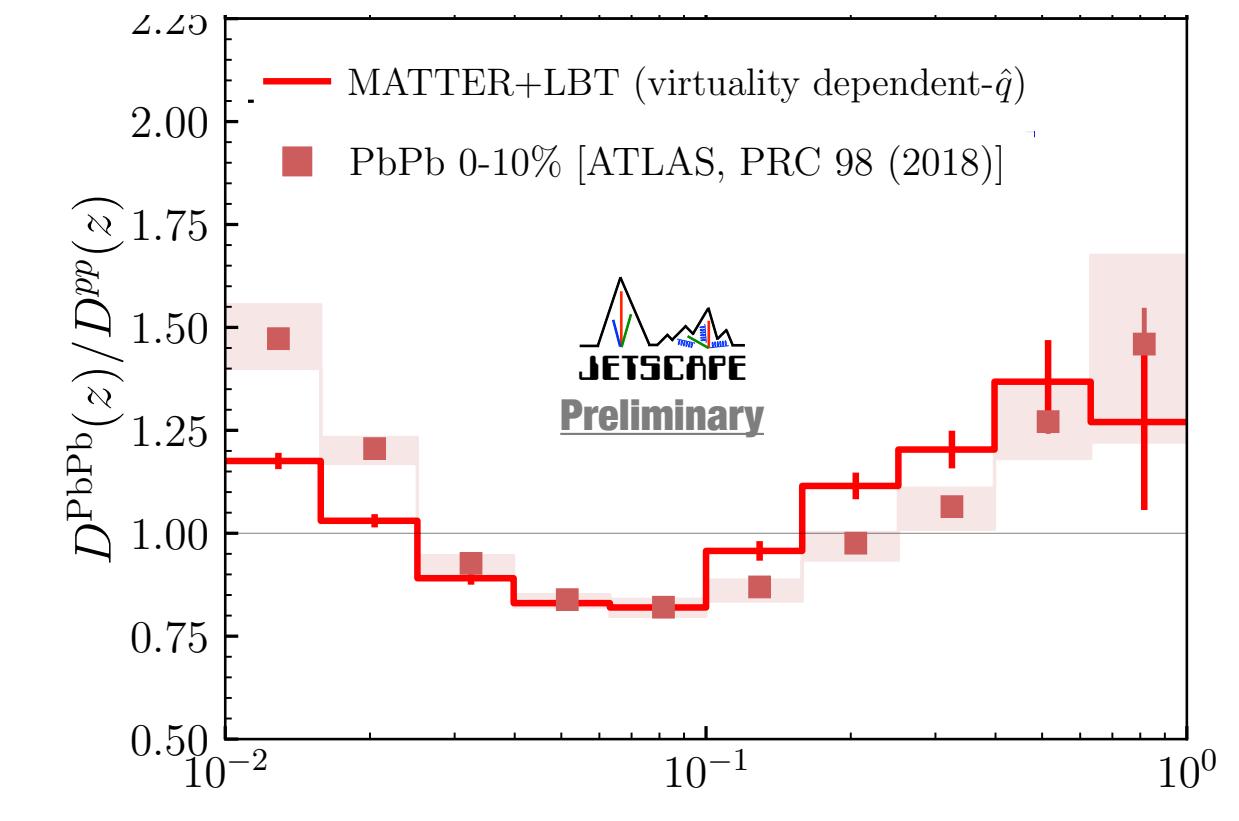
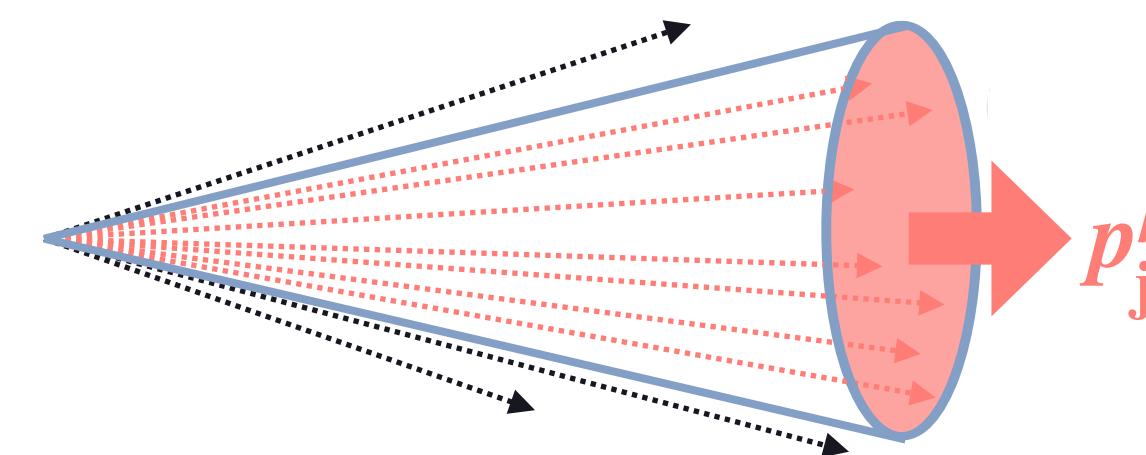
$$\rho(r) = \frac{1}{\delta r} \frac{\sum_{i \in (r-\delta r/2, r+\delta r/2)} p_T^i}{\sum_{i \in (0, R)} p_T^i}$$

$$(r = \sqrt{(\eta_p - \eta^{\text{jet}})^2 + (\phi_p - \phi^{\text{jet}})^2})$$



- **Jet Mass**

$$M_{\text{jet}} = \sqrt{p_\mu^{\text{jet}} p^\mu_{\text{jet}}}$$

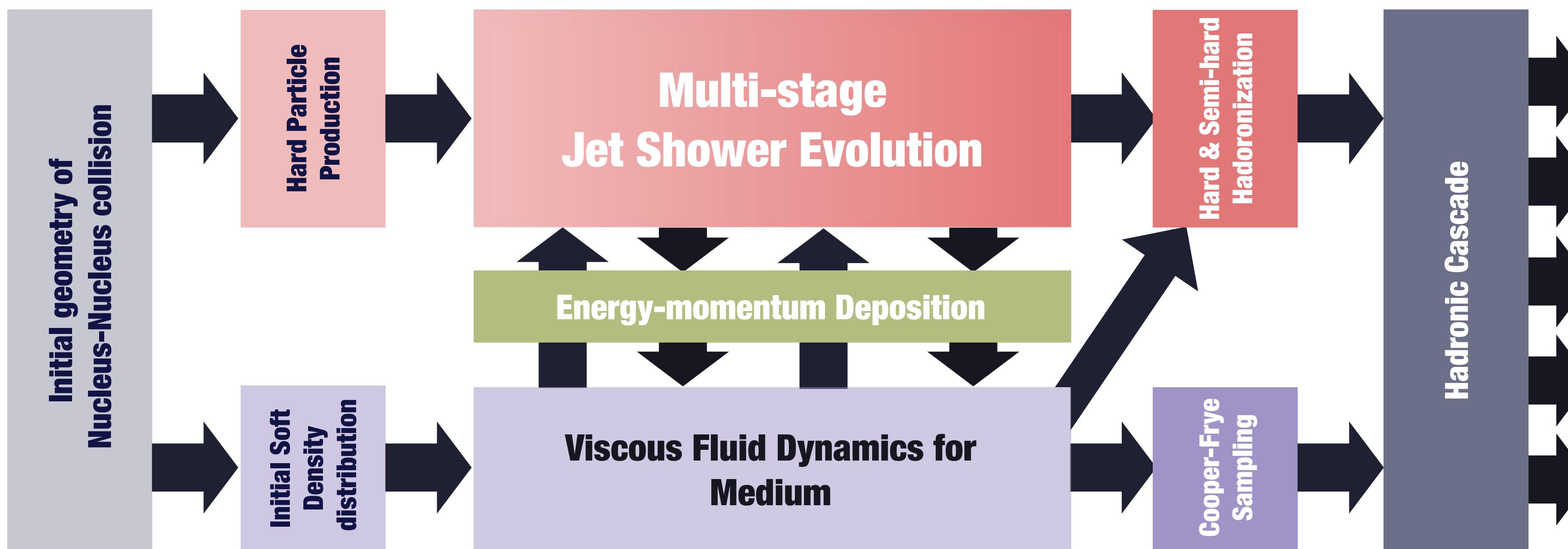


And there are many more.

Monte-Carlo Simulation for Heavy-ion Collisions

● Physics elements involved in a heavy-ion collision event

- Geometric initial conditions
- Hydrodynamic evolution of the bulk QGP,
- Production by hard particles
- Jet parton shower evolution, etc.

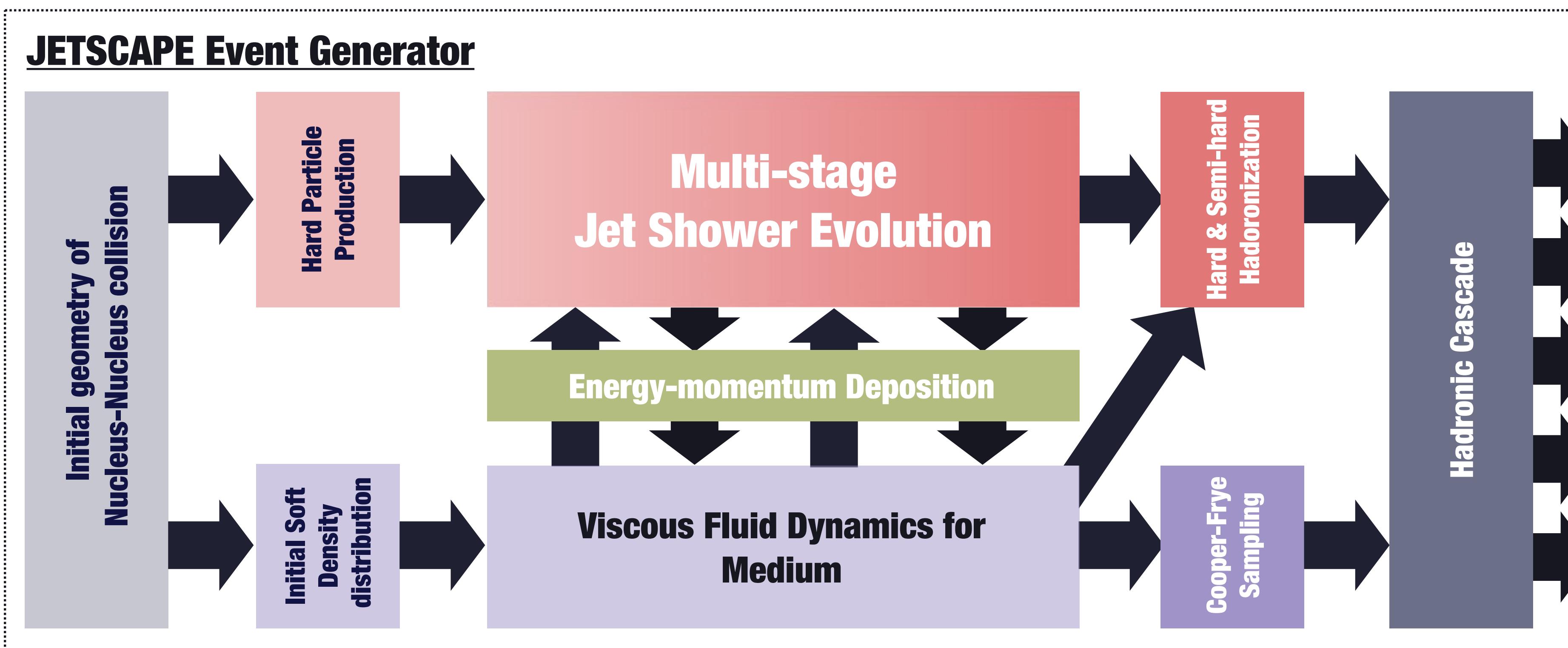


Example: JETSCAPE framework

JETSCAPE, arXiv:1903.07706

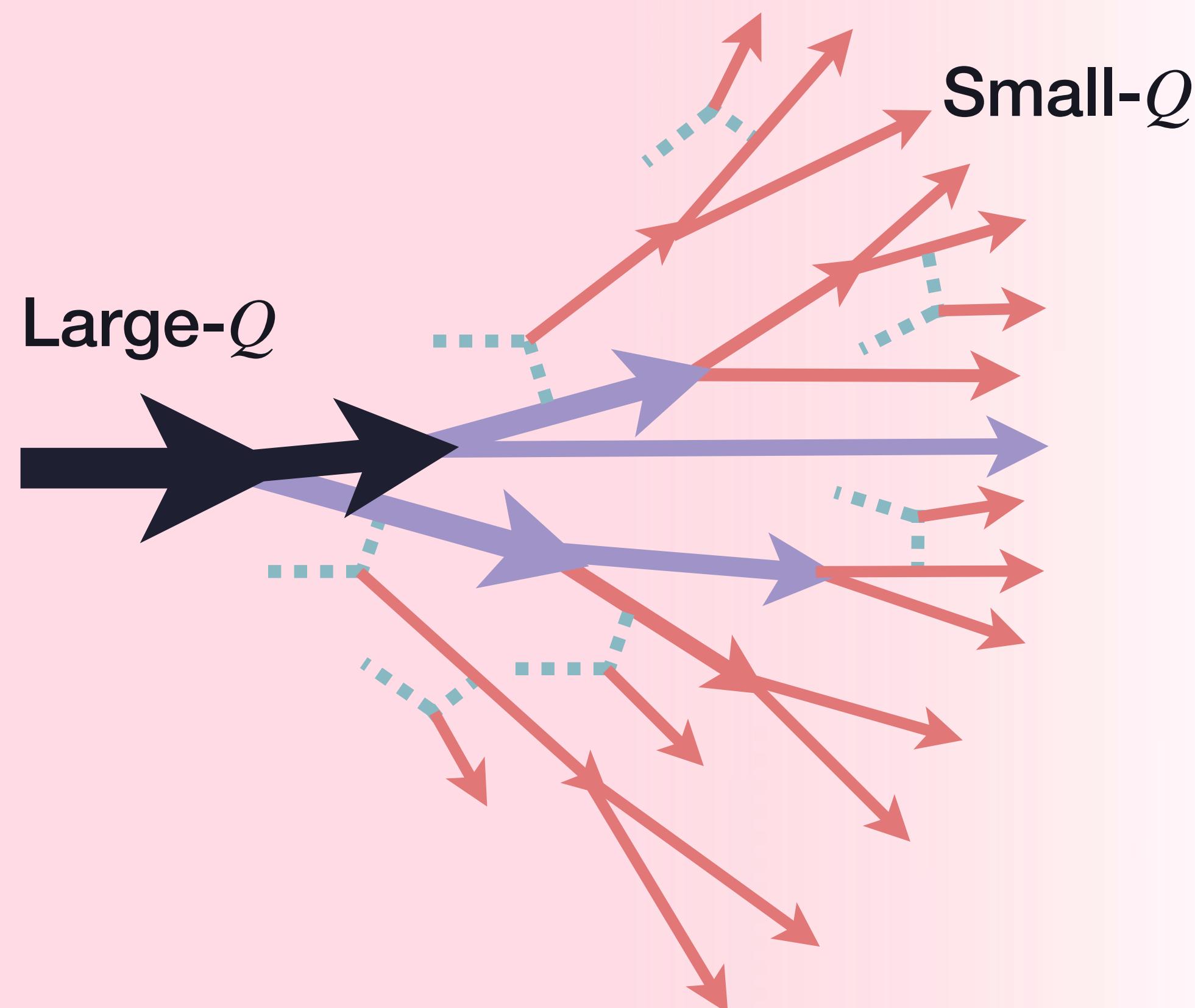
- **MC event generator package for heavy ion collisions**

- General, modular and *customizable* (users can add their own modules)
- Support communications between modules
- Available on  github.com/JETSCAPE



Multi-stage Parton Shower Evolution

In-medium

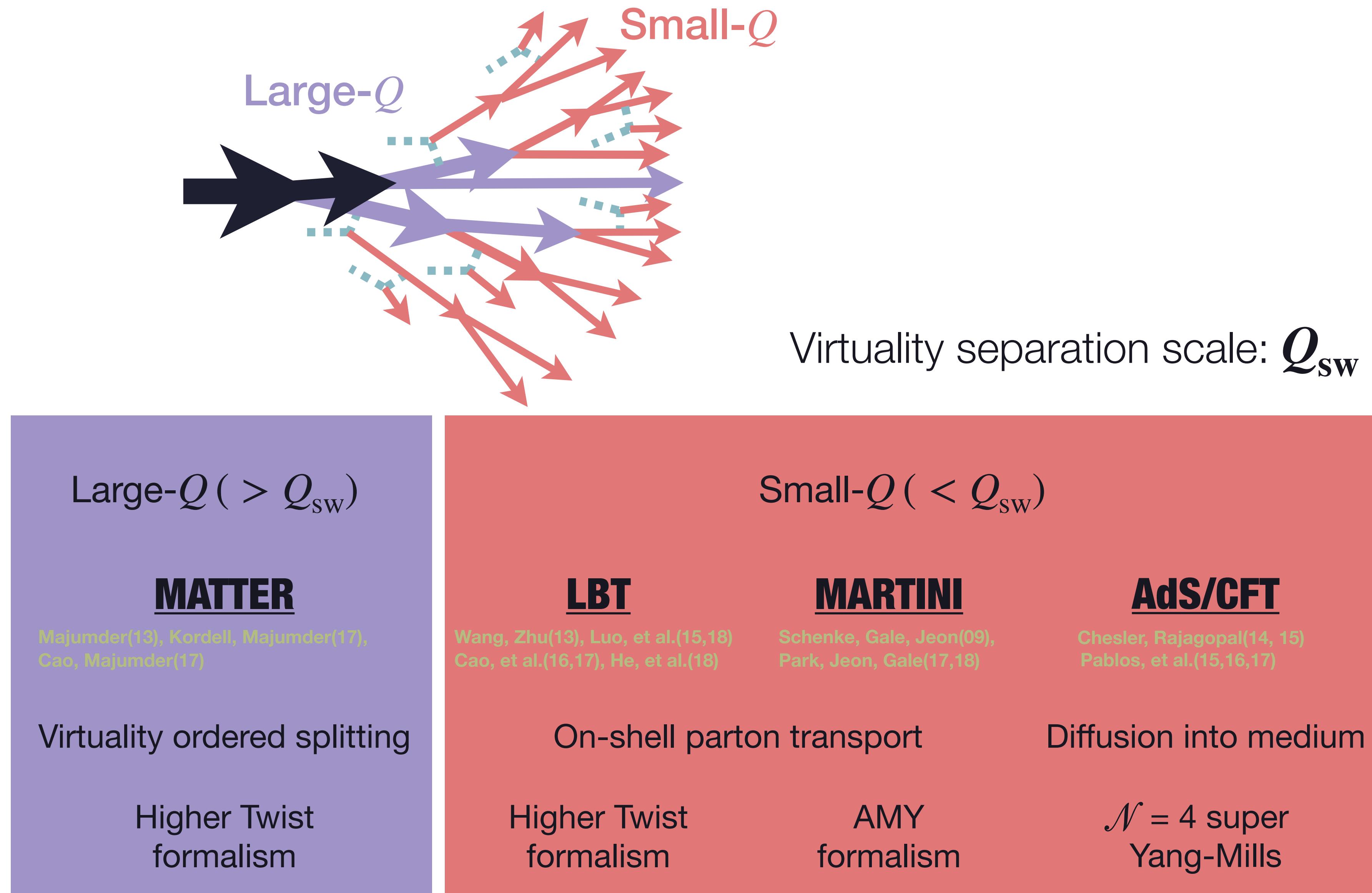


- In-vacuum: Virtuality ordered splitting
- Large- Q : Medium effect on top of in-vacuum splitting
- Small- Q : Splitting driven almost purely by medium effects

**- Radiation triggered by different mechanisms
- Cannot be described by a single model
→ Combination of multiple models**

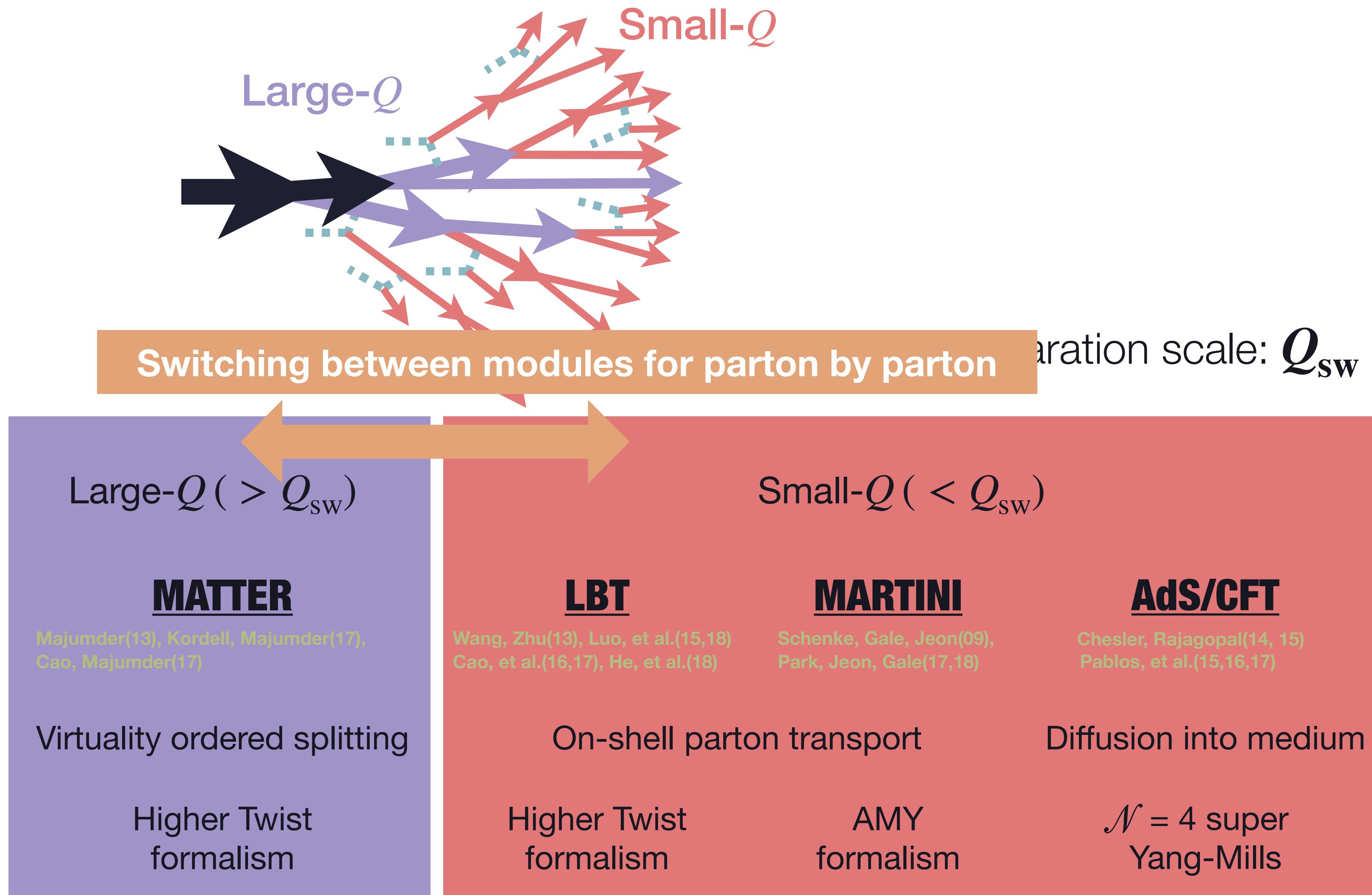
Multi-stage Parton Shower Evolution in JETSCAPE

JETSCAPE, PRC96, 024909 (2017)



Multi-stage Parton Shower Evolution in JETSCAPE

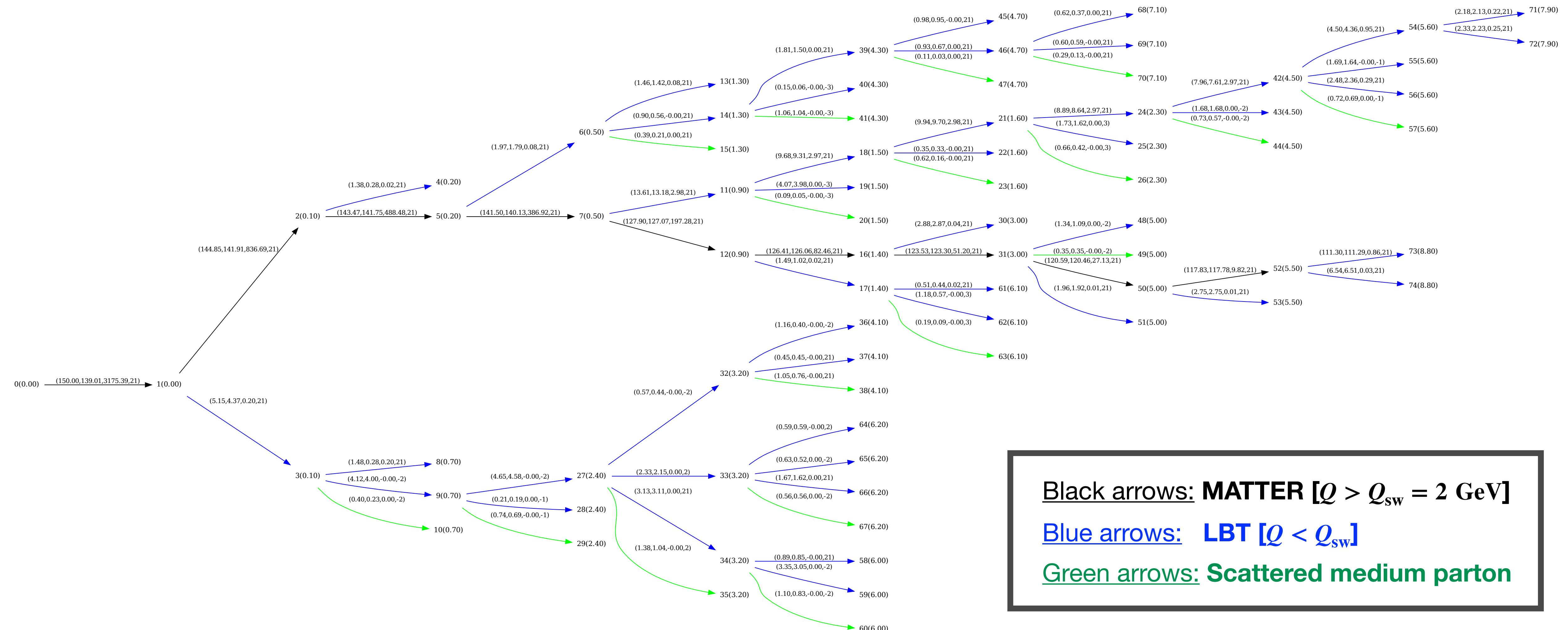
JETSCAPE, PRC96, 024909 (2017)



Multi-stage Parton Shower Evolution in JETSCAPE

JETSCAPE, PRC96, 024909 (2017)

- Graph of parton shower generated by JETSCAPE



High Virtuality Parton Shower [example: MATTER]

A. Majumder, PRC88 014909 (2013), A. Majumder, S. Cao, PRC101, 024903 (2020)

- Virtuality-ordered splitting with medium effect correction

- Assume medium effects small enough to keep the virtualiy ordering structure

$$Q^2 > Q_{\text{med}}^2 \sim \hat{q}\tau_{\text{form}} = \hat{q} \frac{2E}{Q^2} \quad \rightarrow \quad Q^2 > \sqrt{2\hat{q}E}$$

- Jet parton path estimated by sampling with the formation time $\tau_{\text{form}} = 2E/Q^2$
- DGLAP evolution with medium-modified splitting function calculated in Higher Twist

R. Abir, G. D. Kaur, A. Majumder, PRC94, 054902 (2016)

$$Q^2 \frac{d}{dQ^2} D(z, Q^2) = \frac{\alpha_s}{2\pi} \int_z^1 \frac{dy}{y} P_{\text{med}}(y, Q^2) D\left(\frac{z}{y}, Q^2\right)$$

$$P_{\text{med}}(y, Q^2) = P(y) + \tilde{P}_{\text{med}}(y, Q^2), \quad \tilde{P}_{\text{med}}^{\text{HT}}(y, Q^2) = P(y) \left\{ 1 + \int_0^{\tau_{\text{form}}^+} d\xi^+ \hat{q}_{\text{HTL}}^a \frac{c_q^a f(Q^2) \left[2 - 2 \cos(\xi^+/\tau_{\text{form}}^+) \right]}{y(1-y)Q^2(1+\chi_a)^2} \right\}$$

- Solve DGLAP event-by-event using the Sudakov form factor in Monte Carlo
- Elastic energy loss with recoils taken into account at the same time

Other MC models for high virtuality parton shower: Q-Pythia, Ya-JEM, JEWEL, Hybrid

High Virtuality Parton Shower [example: MATTER]

A. Majumder, PRC88 014909 (2013), A. Majumder, S. Cao, PRC101, 024903 (2020)

Coherence effects

Y. Mehtar-Tani, C. A. Salgado, K. Tywoniuk, PLB707, 156-159 (2012)
J. Casalderrey-Solana, E. Iancu, JHEP08, 015 (2011)

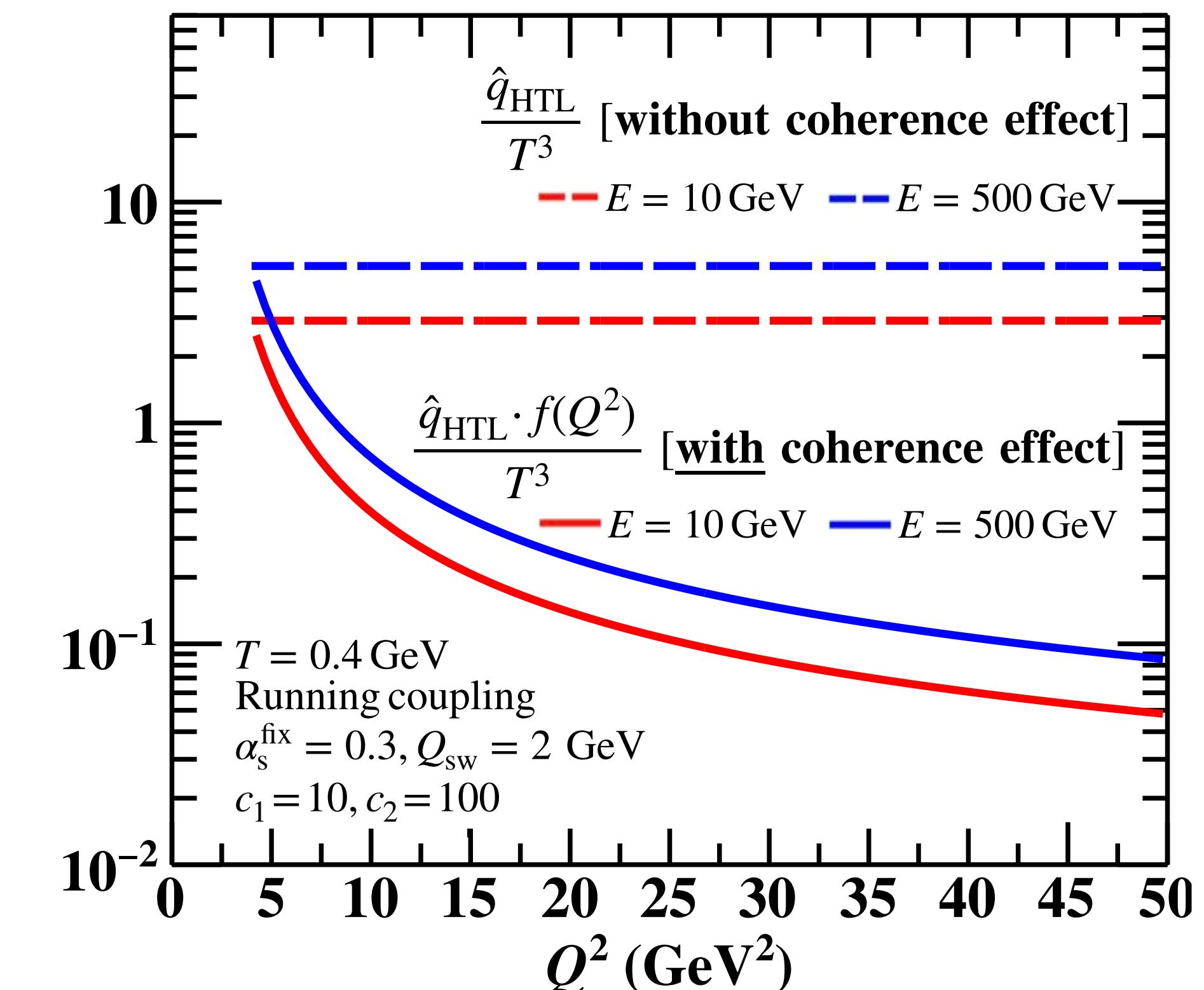
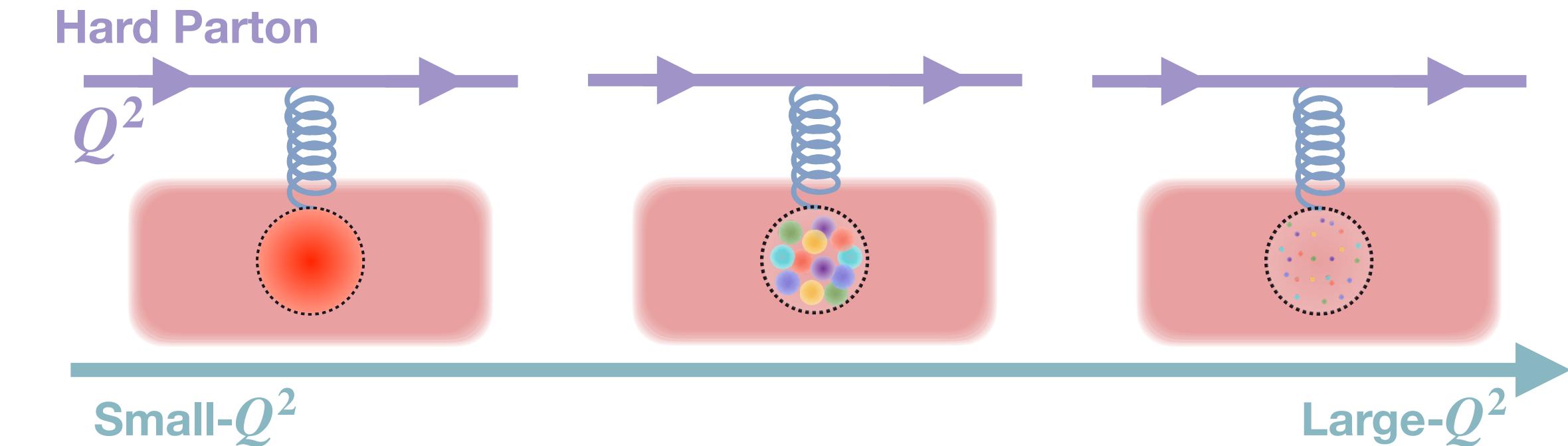
- Scale evolution of QGP constituent distribution
Kumar, Majumder, Shen, PRC101, 034908 (2020)
- Less interaction for large- Q^2 partons
→ Implemented in MATTER

Effective jet-quenching strength

$$\hat{q}_{\text{HTL}} \cdot f(Q^2)$$

$$f(Q^2) = \frac{1 + c_1 \ln^2(Q_{\text{sw}}^2) + c_2 \ln^4(Q_{\text{sw}}^2)}{1 + c_1 \ln^2(Q^2) + c_2 \ln^4(Q^2)}$$

$$\hat{q}_{\text{HTL}} = C_a \frac{42\zeta(3)}{\pi} \alpha_s^{\text{run}} \alpha_s^{\text{fix}} T^3 \ln \left[\frac{2ET}{6\pi T^2 \alpha_s^{\text{fix}}} \right]$$



Low Virtuality Parton Shower [example: LBT]

Y. He, T. Luo, X.-N. Wang, and Y. Zhu, PRC91, 054908 (2015), S. Cao, T. Luo, G.-Y. Qin, and X.-N. Wang, PRC94, 014909 (2016), ...

- **Kinetic theory-based approach (time ordered)**

- Assume medium effects large enough to be dominating sources of radiations

$$Q^2 < Q_{\text{med}}^2 \sim \hat{q}\tau_{\text{form}} = \hat{q} \frac{2E}{Q^2} \quad \rightarrow \quad Q^2 < \sqrt{2\hat{q}E}$$

- Solve Linearized Boltzmann equations in Monte Carlo

$$p^\mu \partial_\mu f(x, p) = p^0 (\mathcal{C}_{\text{el}} + \mathcal{C}_{\text{inel}})$$

- Collision integrals for elastic (\mathcal{C}_{el}) and inelastic ($\mathcal{C}_{\text{inel}}$) scatterings estimated using the rates

$$\Gamma_a^{\text{el}} = \sum_{b,c,d} \frac{1}{2E_a} \int \prod_{i=b,c,d} \frac{d^3 p_i}{(2\pi)^3 2E_i} f_b^{\text{th}}(\vec{p}_b) (2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_d) |\mathcal{M}_{ab \rightarrow cd}|^2$$

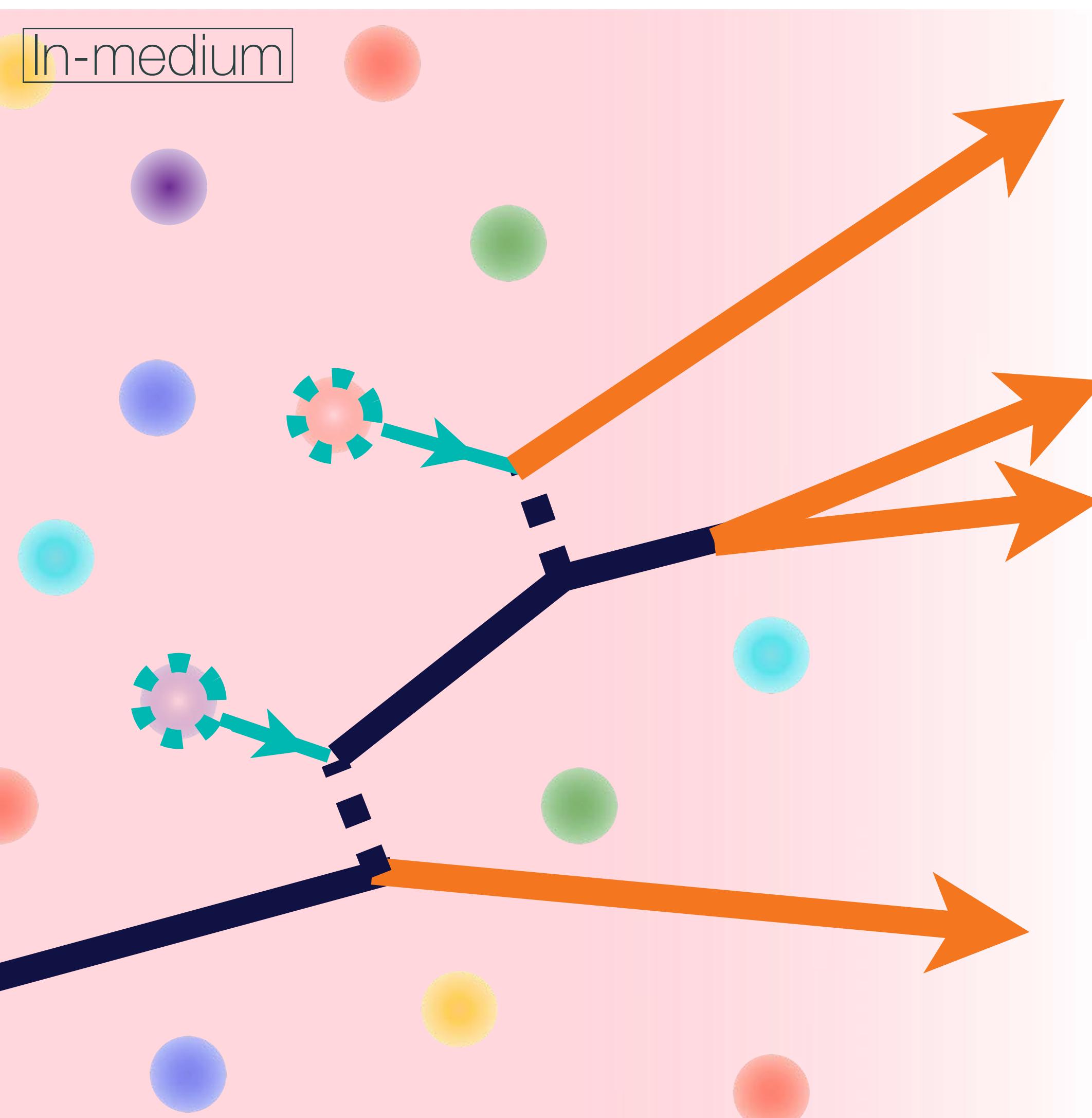
$$\Gamma_a^{\text{inel}} = \frac{1}{1 + \delta_g^a} \int dy dl_\perp^2 \left. \frac{dN_g^a}{dy dl_\perp^2 dt} \right|_{\text{HT}} = \frac{1}{1 + \delta_g^a} \int dy dl_\perp^2 \frac{2\alpha_s(l_\perp^2) P_a(y) l_\perp^4}{\pi(l_\perp^2 + y^2 m_a^2)^4} \hat{q}_{\text{HTL}}^a \sin^2 \left(\frac{t - t_i}{2\tau_f} \right)$$

- Recoils generated in the elastic processes

Other MC models for low virtuality parton shower: MARTINI, LIDO, JetMed, TEQUILA, (Hybrid, JEWEL)

Weakly-coupled Medium Response: Recoils

Zapp, Krauss, Wiedemann ('13), Wang, Zhu(13), Luo, et al.(15,18), Park, Jeon, Gale(18), Cao, Majumder (18)



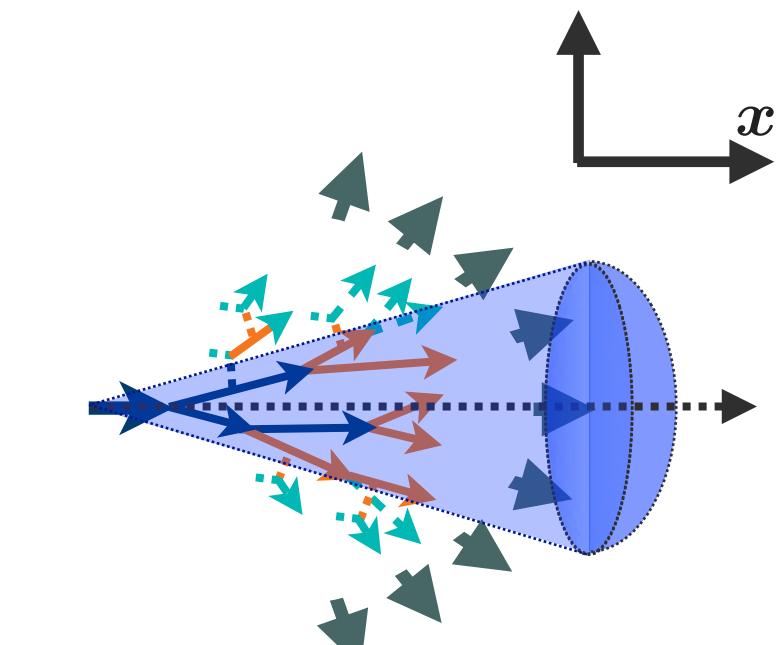
- **Recoil Partons**

- Medium partons kicked out by jet parton
- Propagate as a parton in jet shower
- Wakes induced by successive scatterings

- **Hole: Picked up energy and momentum**

- Sampled from thermal medium
- Freestreaming
- Subtracted from final signal

$$\left. \frac{dp^\mu}{d\theta} \right|_{\text{signal}} = \left. \frac{dp^\mu}{d\theta} \right|_{\text{jet shower}} - \left. \frac{dp^\mu}{d\theta} \right|_{\text{hole}}$$



Jet simulation with JETSCAPE

- **$p+p$ simulation setup** JETSCAPE PRC102, 054906 (2020)

Jet simulation with JETSCAPE

- **$p+p$ simulation setup** JETSCAPE PRC102, 054906 (2020)

Jet Shower

Hard Scattering: Pythia8 (w/ ISR and MPI)

Parton Shower: MATTER (vacuum)

Hadronization: Lund String

Jet simulation with JETSCAPE

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JETSCAPE PP19 tune [jetscape_user_PP19.xml]

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JETSCAPE PP19 tune [jetscape_user_PP19.xml]

- **A+A simulation setup** [JETSCAPE, arXiv:2204.01163](#)

Jet simulation with JETSCAPE

- **$p+p$ simulation setup** [JETSCAPE PRC102, 054906 \(2020\)](#)

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- **A+A simulation setup** [JETSCAPE, arXiv:2204.01163](#)

Jet Shower

Hard Scattering: Pythia8 (w/ ISR and MPI)

Parton Shower: MATTER+LBT (recoil on, $Q_{\text{sw}} = 2 \text{ GeV}$)

Hadronization: Lund String

Jet simulation with JETSCAPE

- **$p+p$ simulation setup** JETSCAPE PRC102, 054906 (2020)

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Jet Shower

Hard Scattering: Pythia8 (w/ ISR and MPI)

Parton Shower: MATTER+LBT (recoil on, $Q_{\text{sw}} = 2 \text{ GeV}$)

Hadronization: Lund String

Initial Condition
 $T(x)$
 $u^\mu(x)$

Bulk Medium

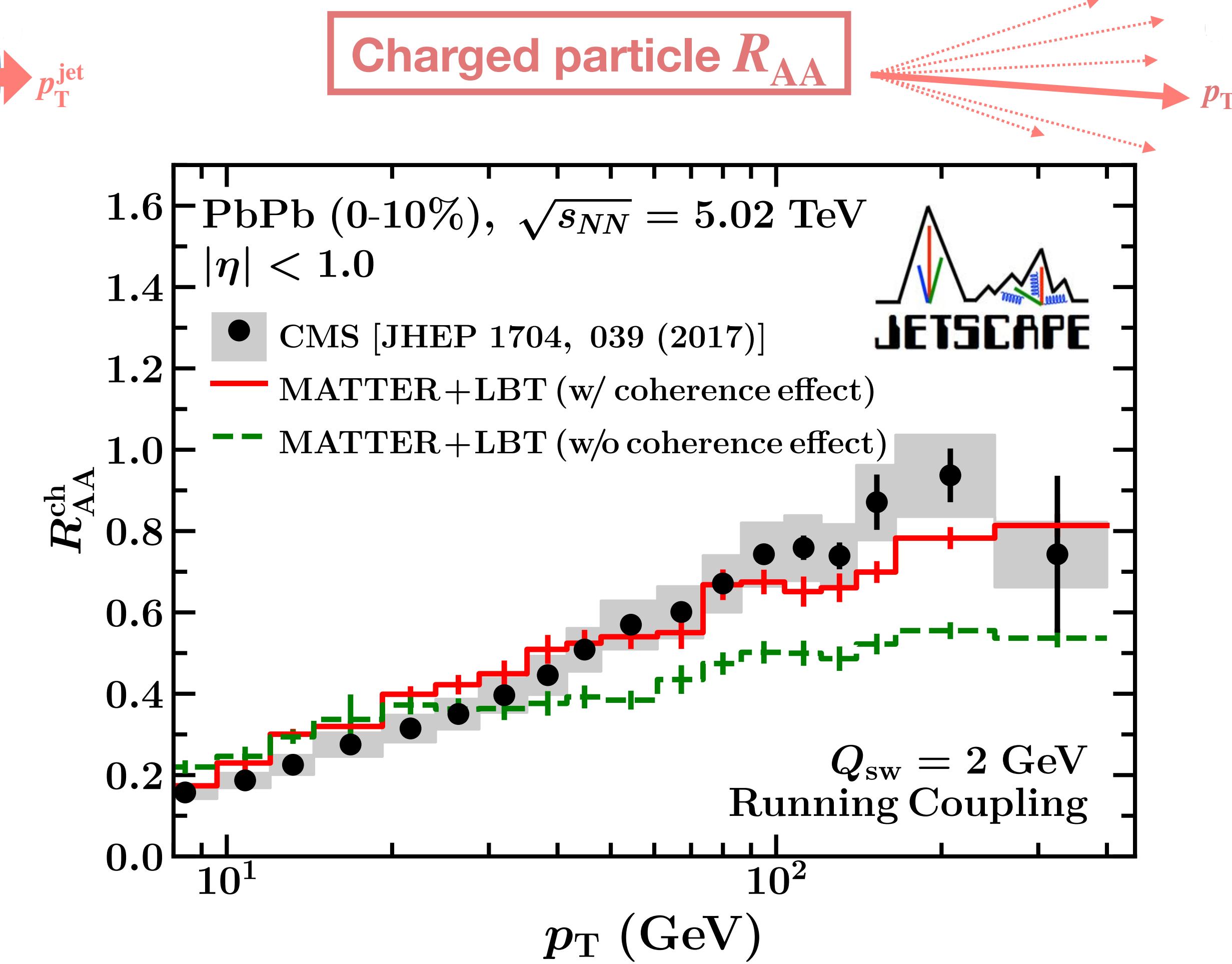
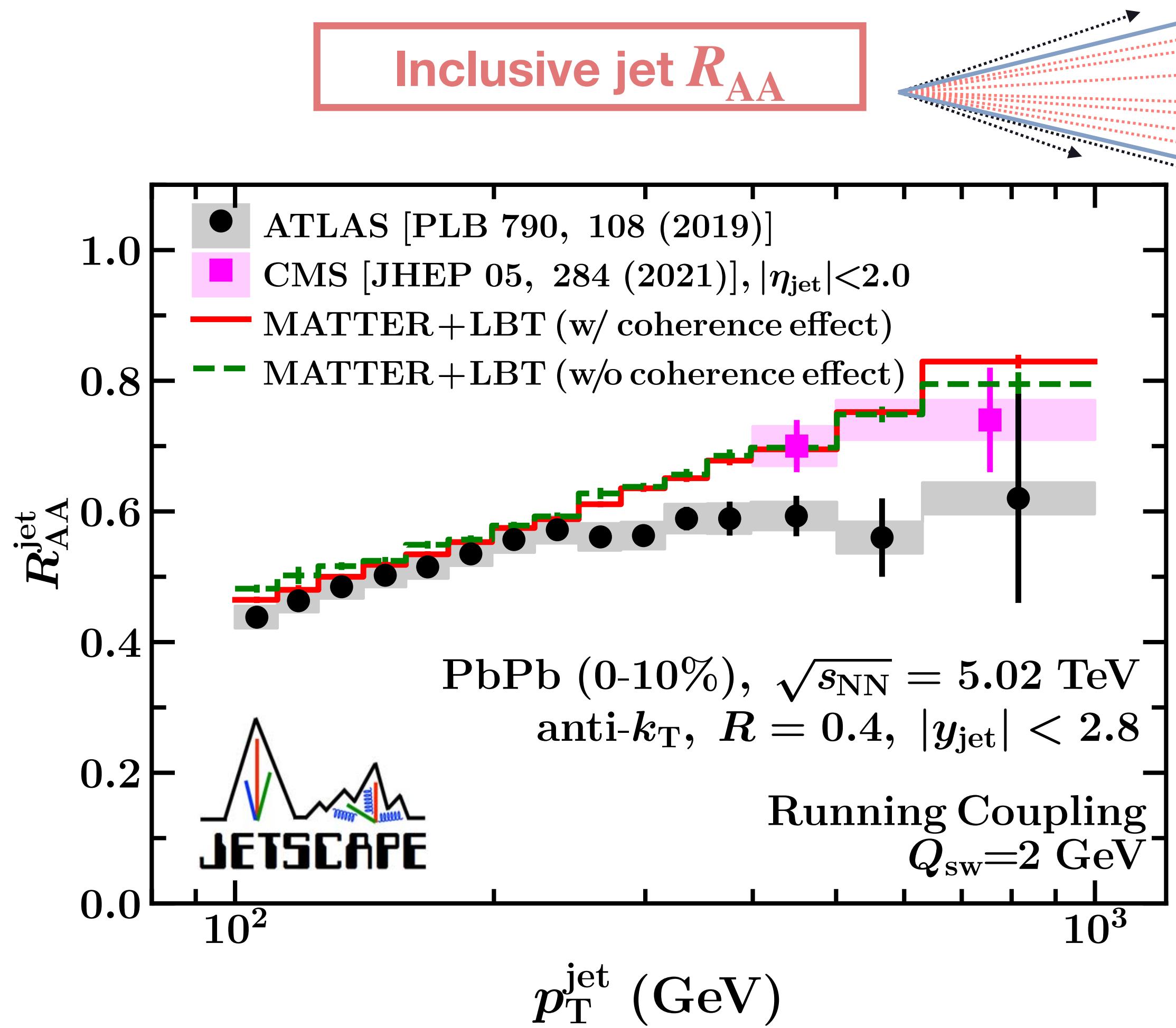
Initial Condition: TRENTo+Freestreaming
 Moreland, Bernhard, Bass (14) , Liu, Shen, Heinz(15)

Hydro Evolution: VISHNU (2+1D viscous)
 Shen, Qiu, Song, Bernhard, Bass, Heinz(16)

Light flavor particles and jet R_{AA}

JETSCAPE, arXiv:2204.01163

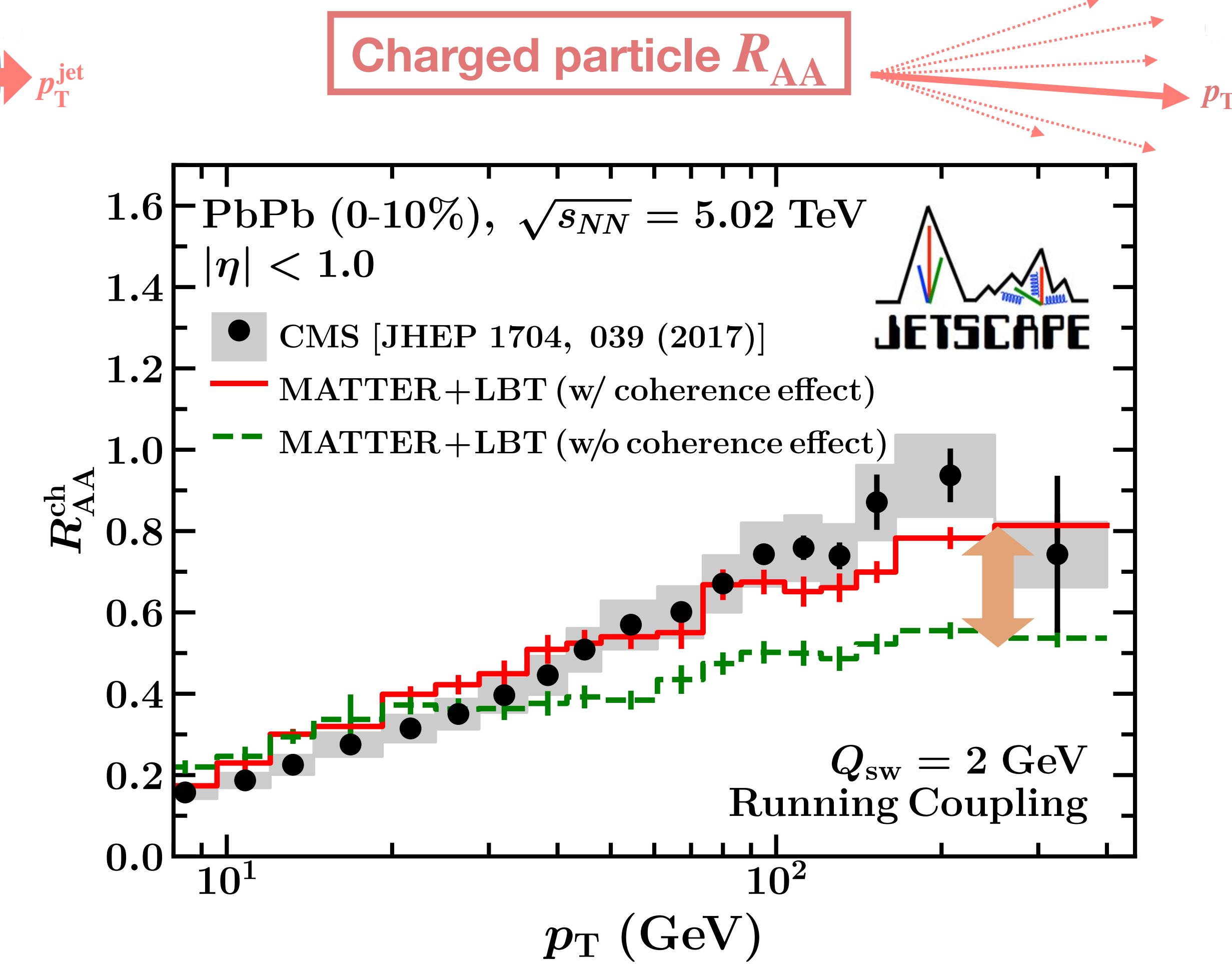
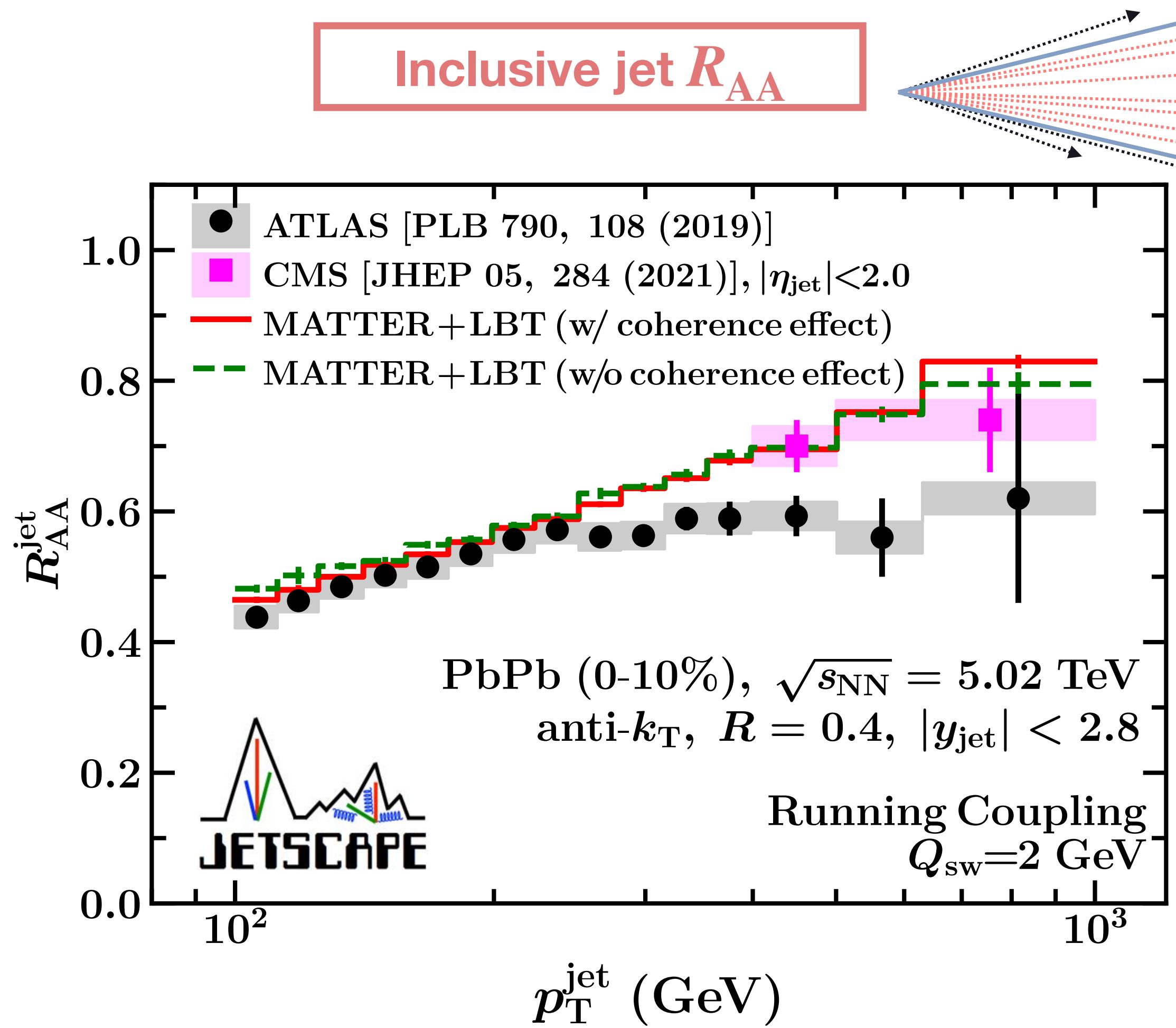
- Jet and high- p_T particle energy loss for PbPb@5.02 TeV



Light flavor particles and jet R_{AA}

JETSCAPE, arXiv:2204.01163

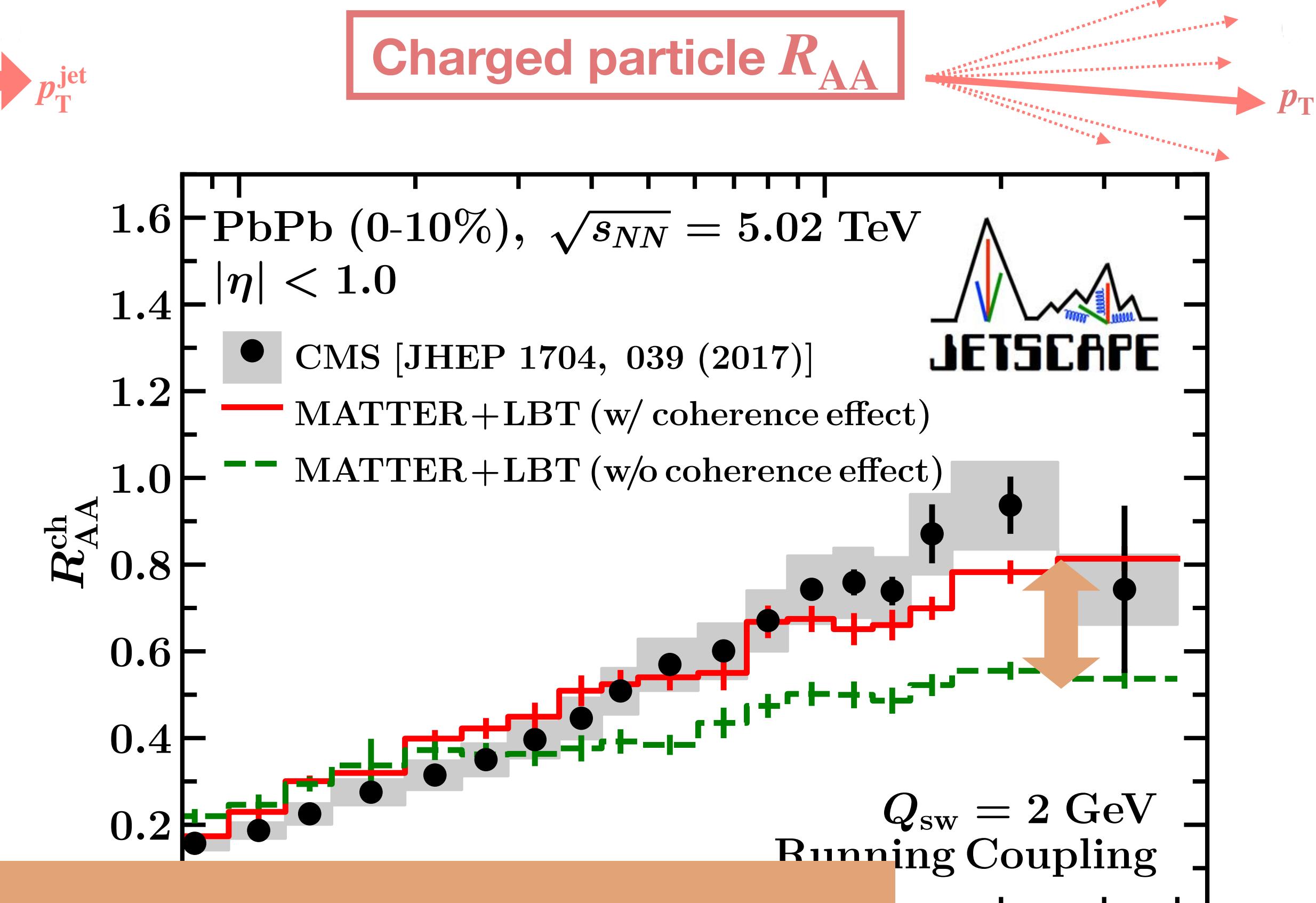
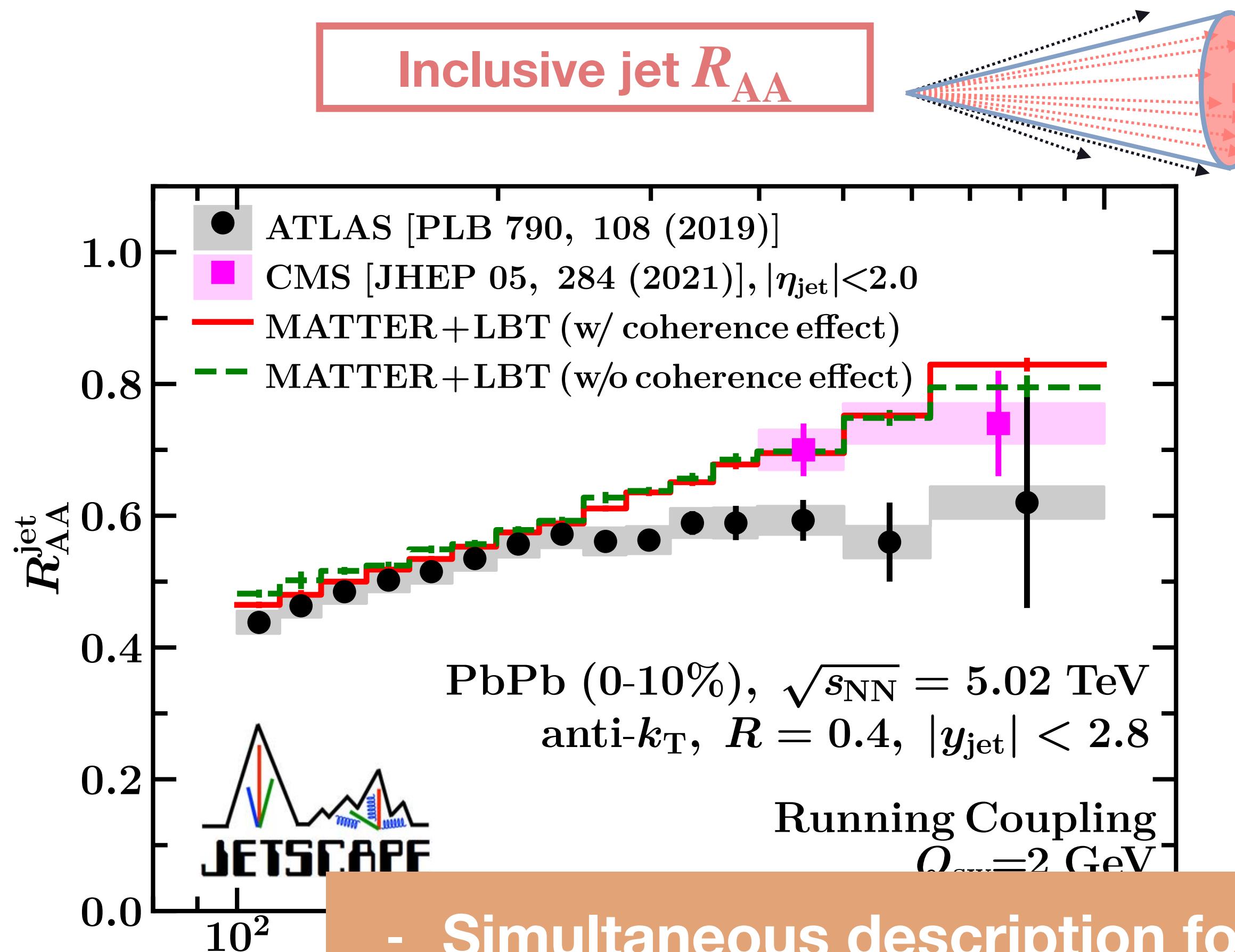
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Light flavor particles and jet R_{AA}

JETSCAPE, arXiv:2204.01163

- Jet and high- p_T particle energy loss for PbPb@5.02 TeV



- Simultaneous description for jet and single particle
- Significant coherence effect in single particle energy loss

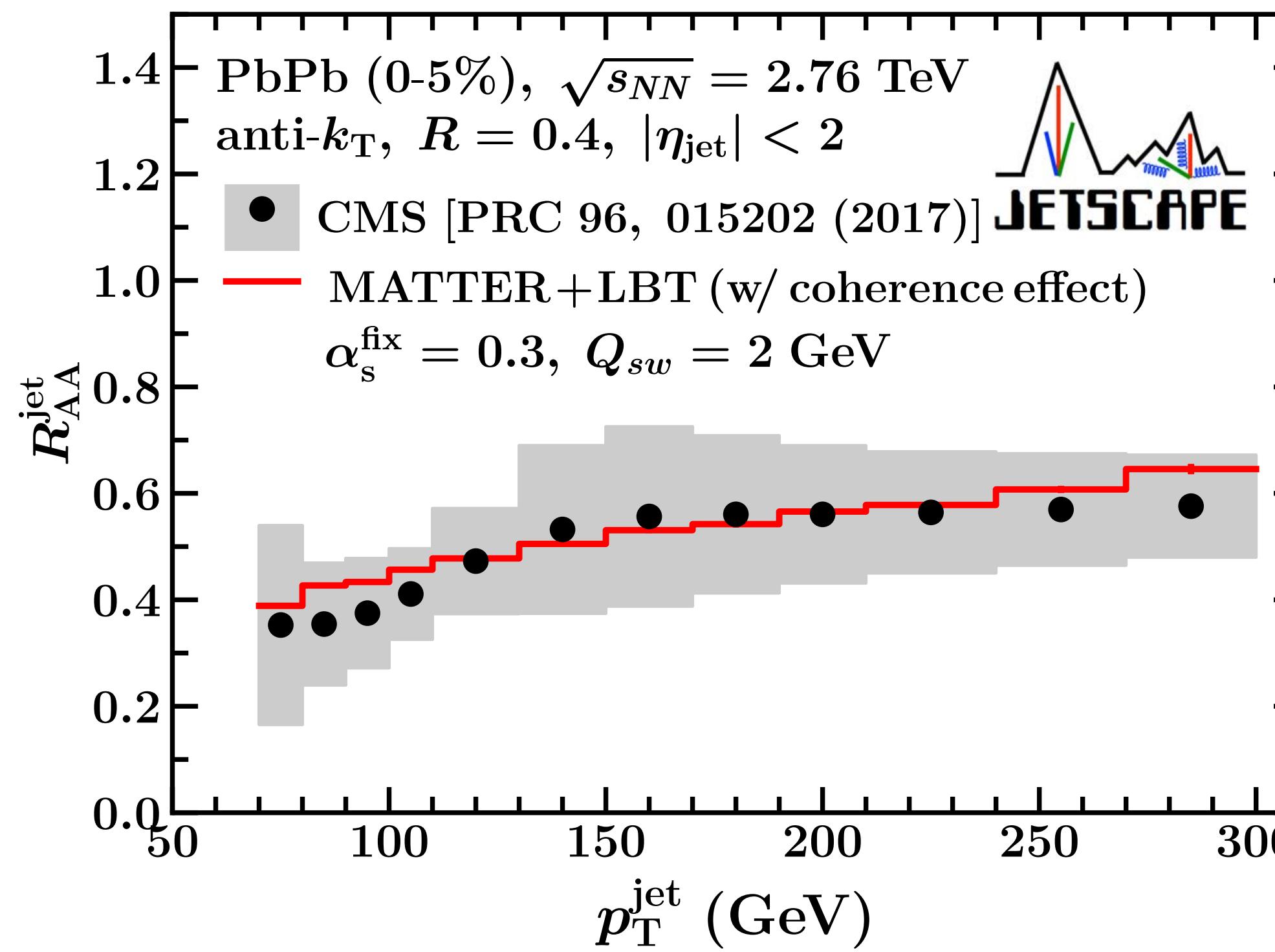
Light flavor particles and jet R_{AA}

JETSCAPE, arXiv:2204.01163

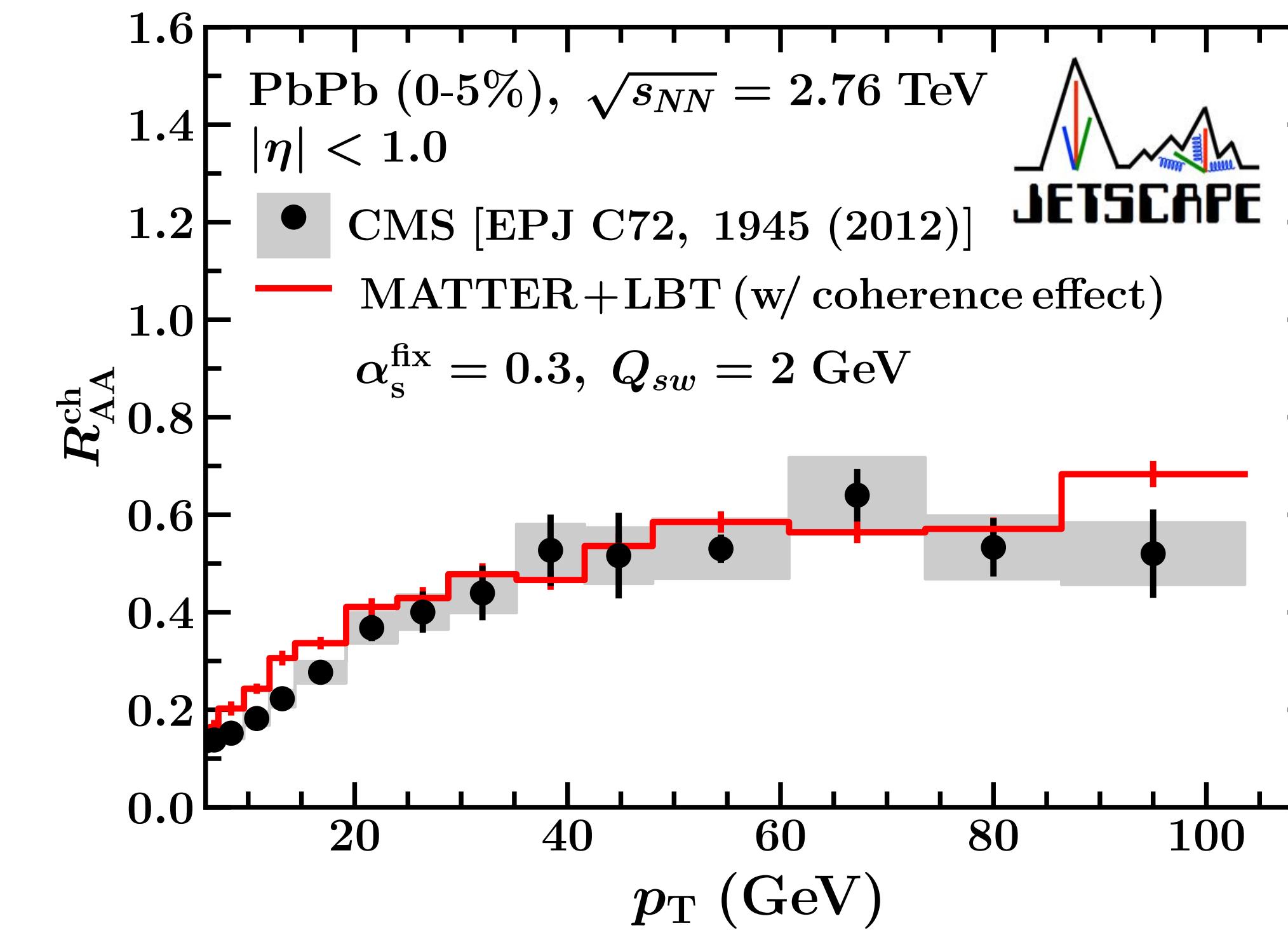
- Jet and high- p_T particle energy loss for PbPb@2.76 TeV

The same parameter set as 5.02 TeV is used

Inclusive jet R_{AA}



Charged particle R_{AA}



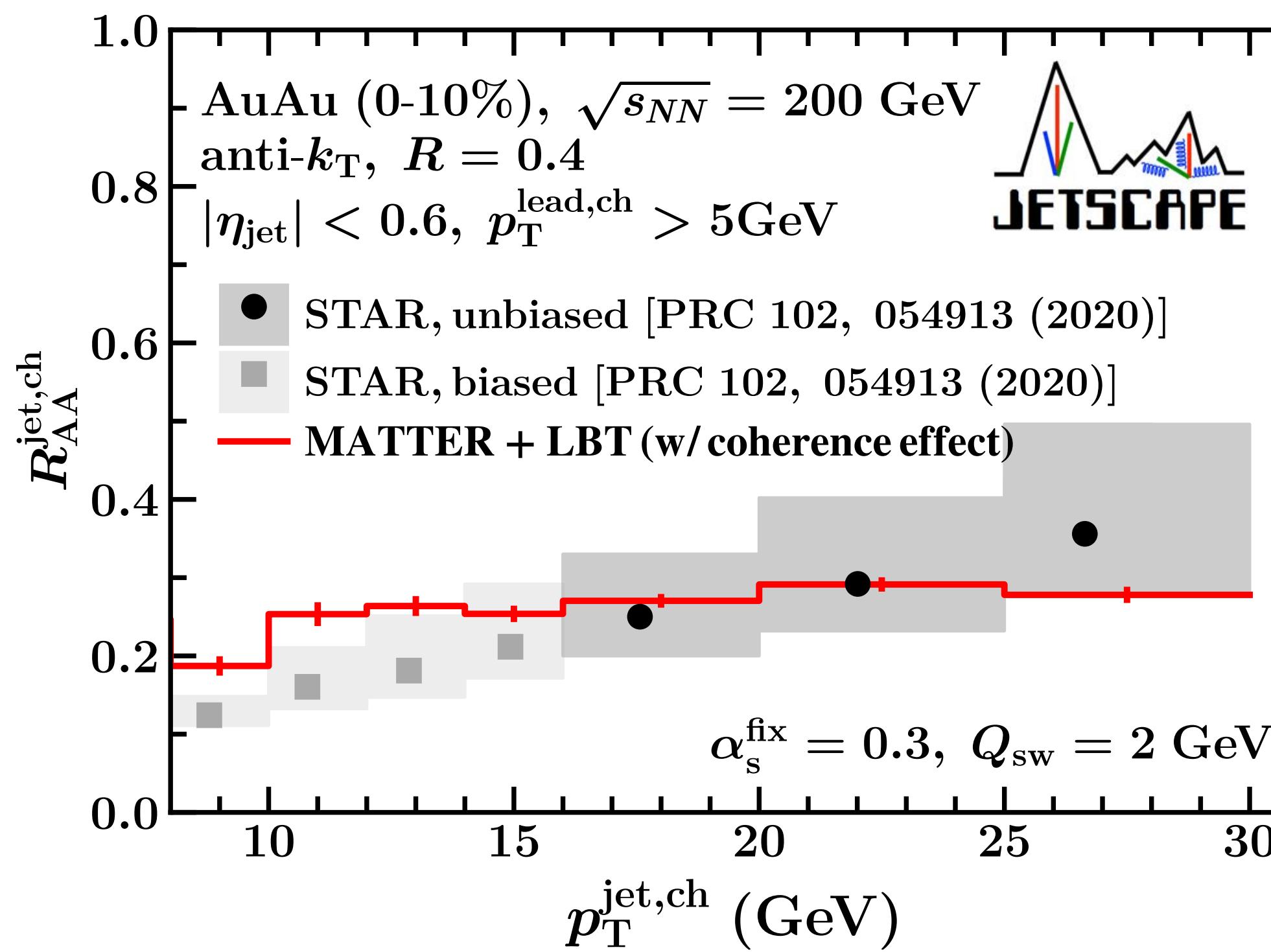
Light flavor particles and jet R_{AA}

JETSCAPE, arXiv:2204.01163

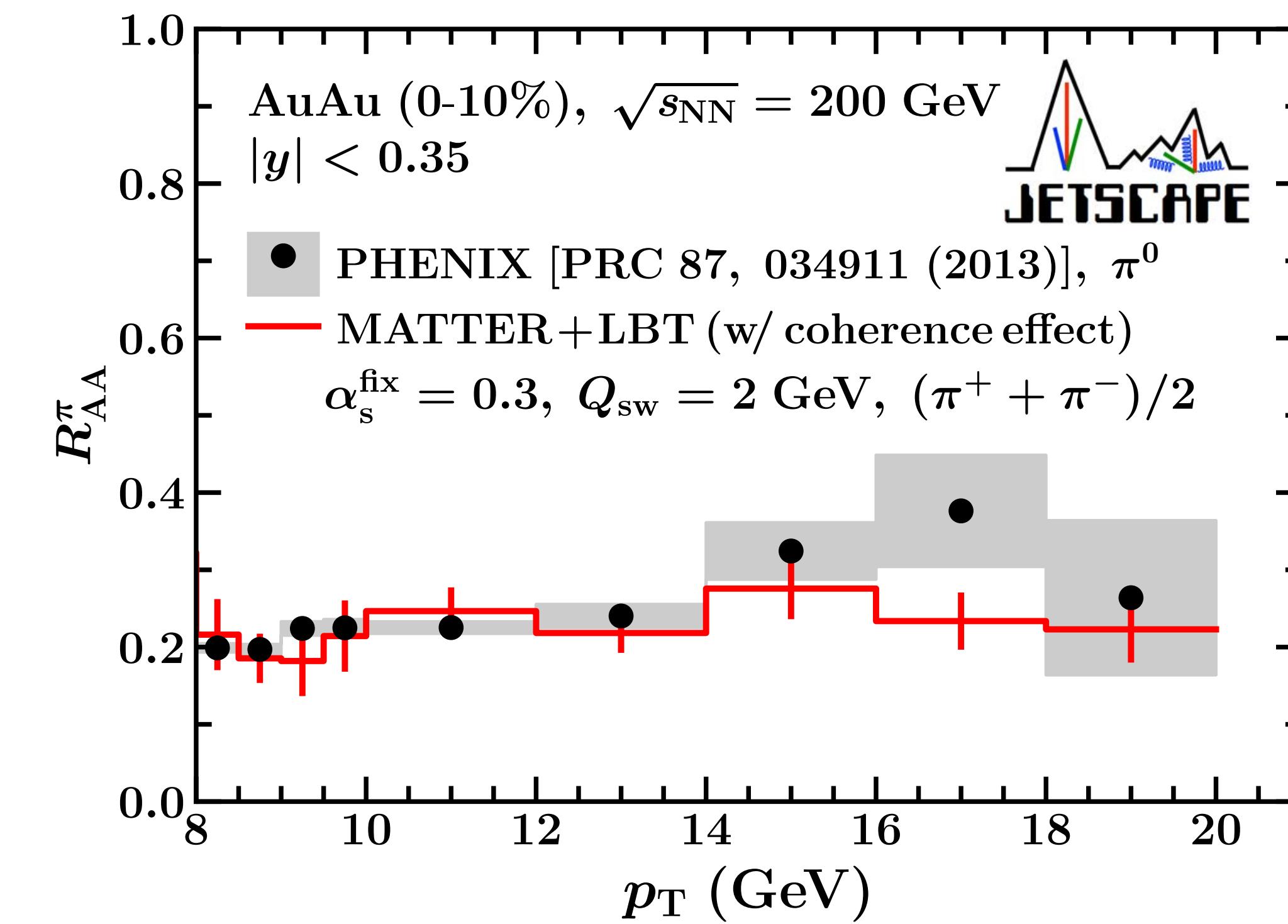
- Jet and high- p_T particle energy loss for AuAu@200 GeV

The same parameter set as 5.02 TeV is used

Charged jet R_{AA}



Pion R_{AA}



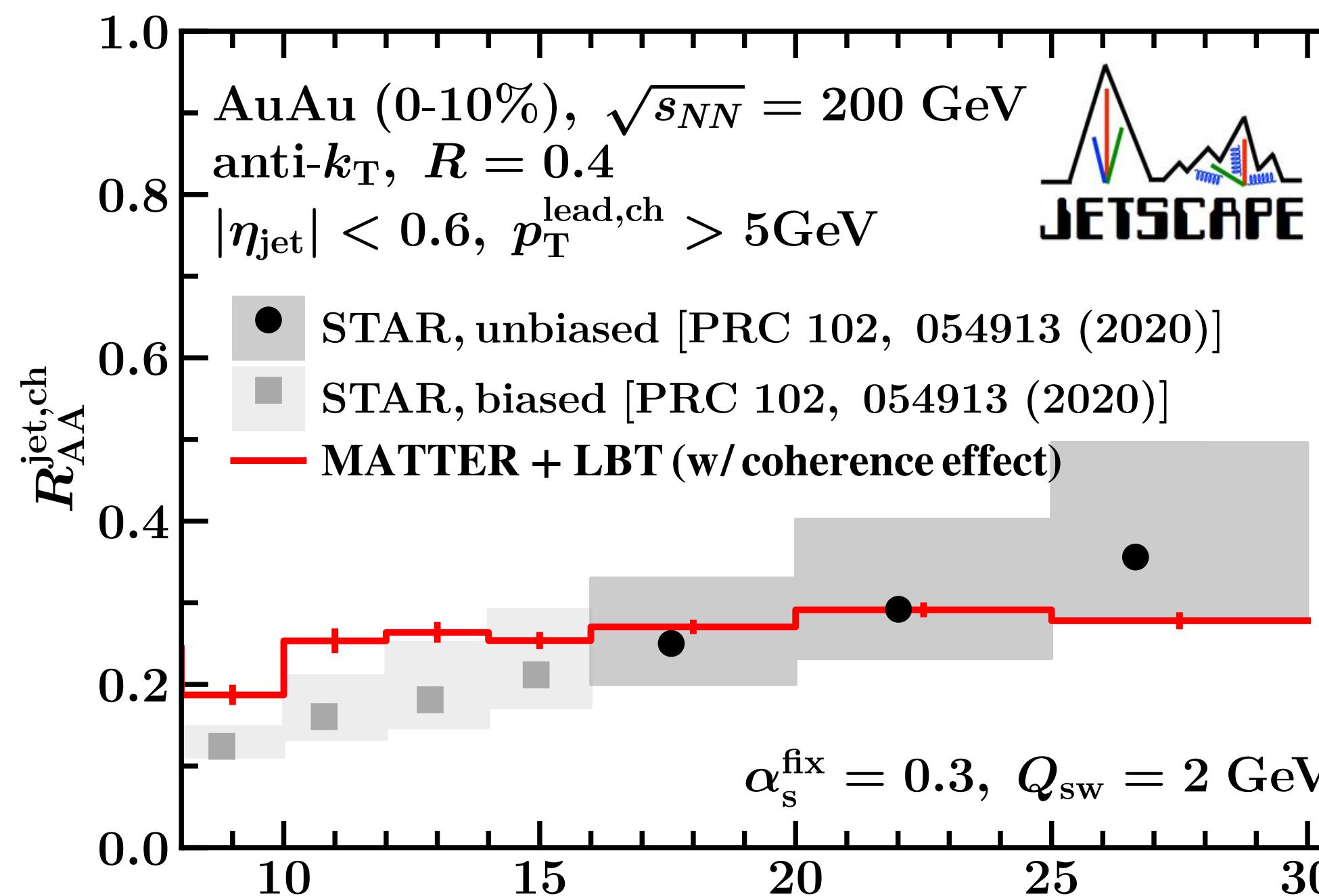
Light flavor particles and jet R_{AA}

JETSCAPE, arXiv:2204.01163

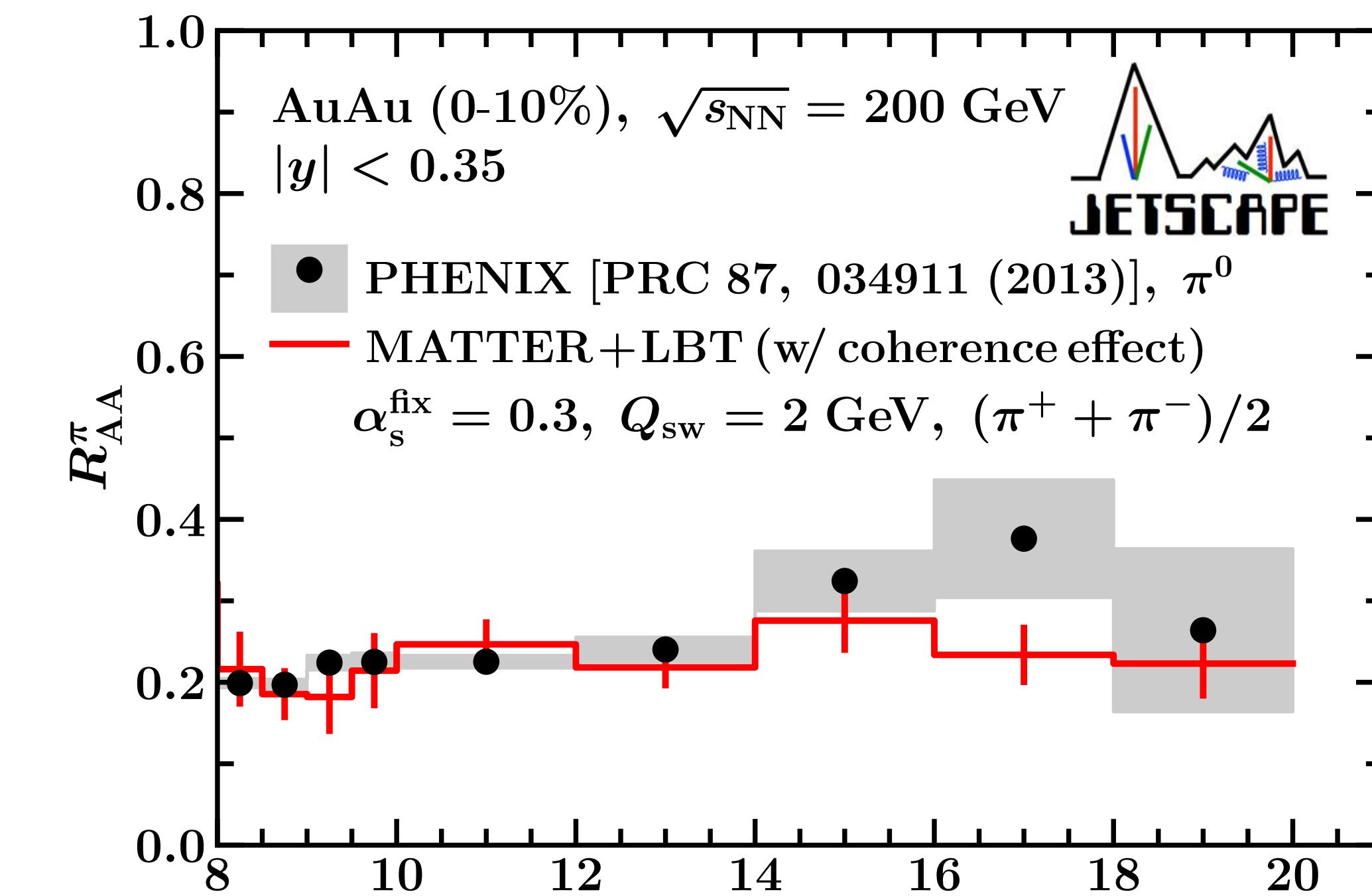
- Jet and high- p_T particle energy loss for AuAu@200 GeV

The same parameter set as 5.02 TeV is used

Charged jet R_{AA}



Pion R_{AA}

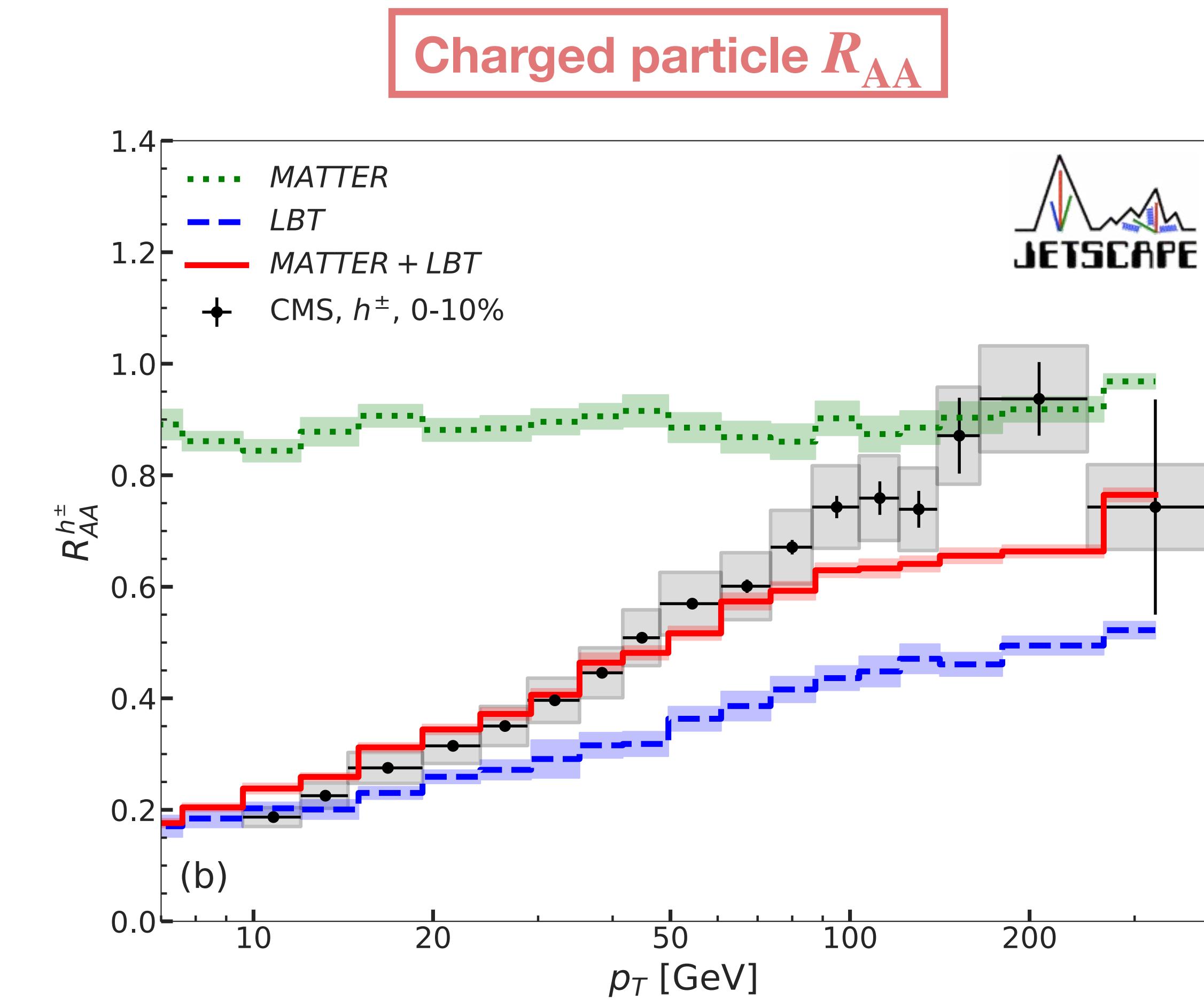
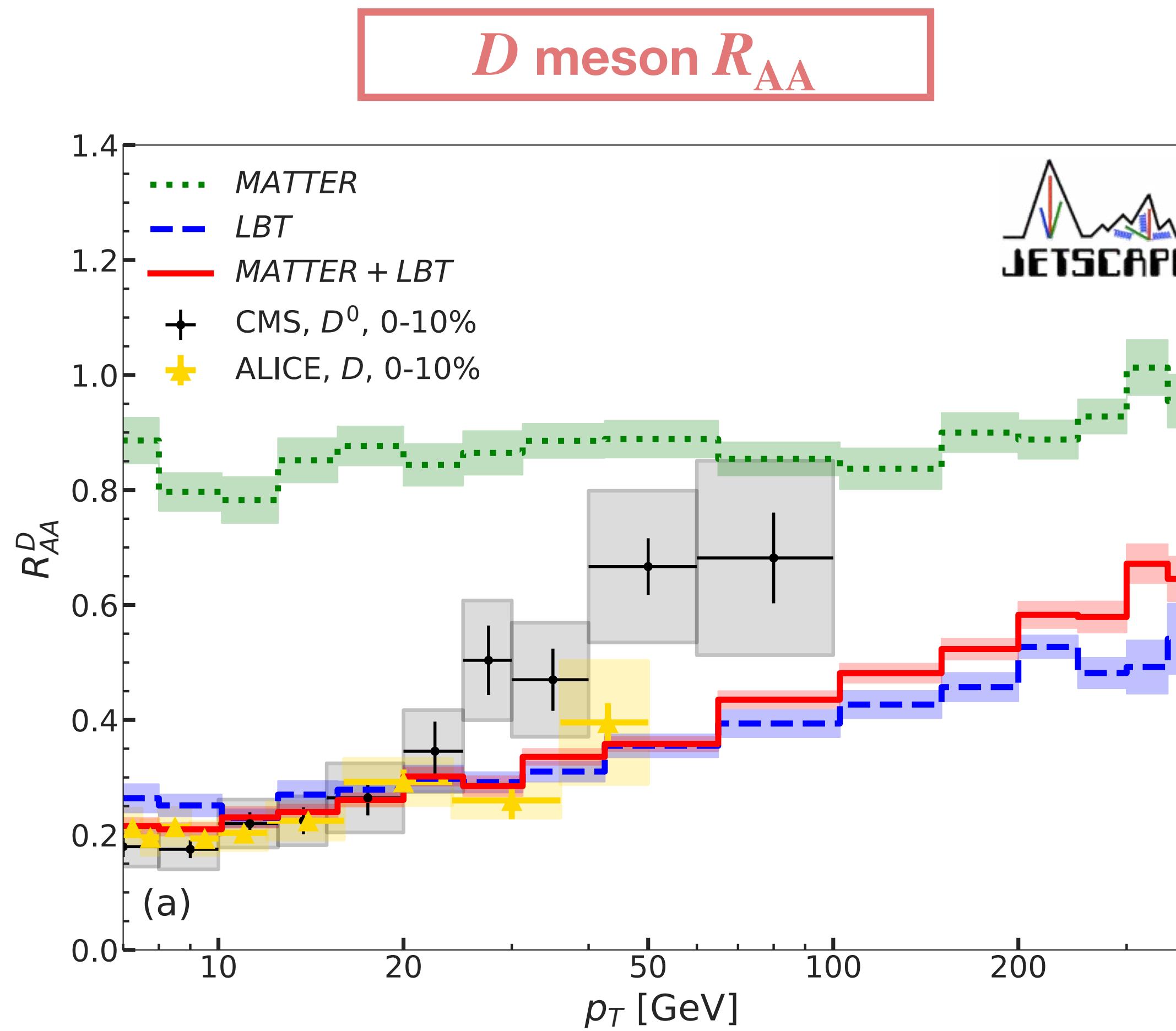


- Simultaneous description of different $\sqrt{s_{NN}}$ with the same parameter set

Charm hadron R_{AA}

JETSCAPE, arXiv:2208.00983

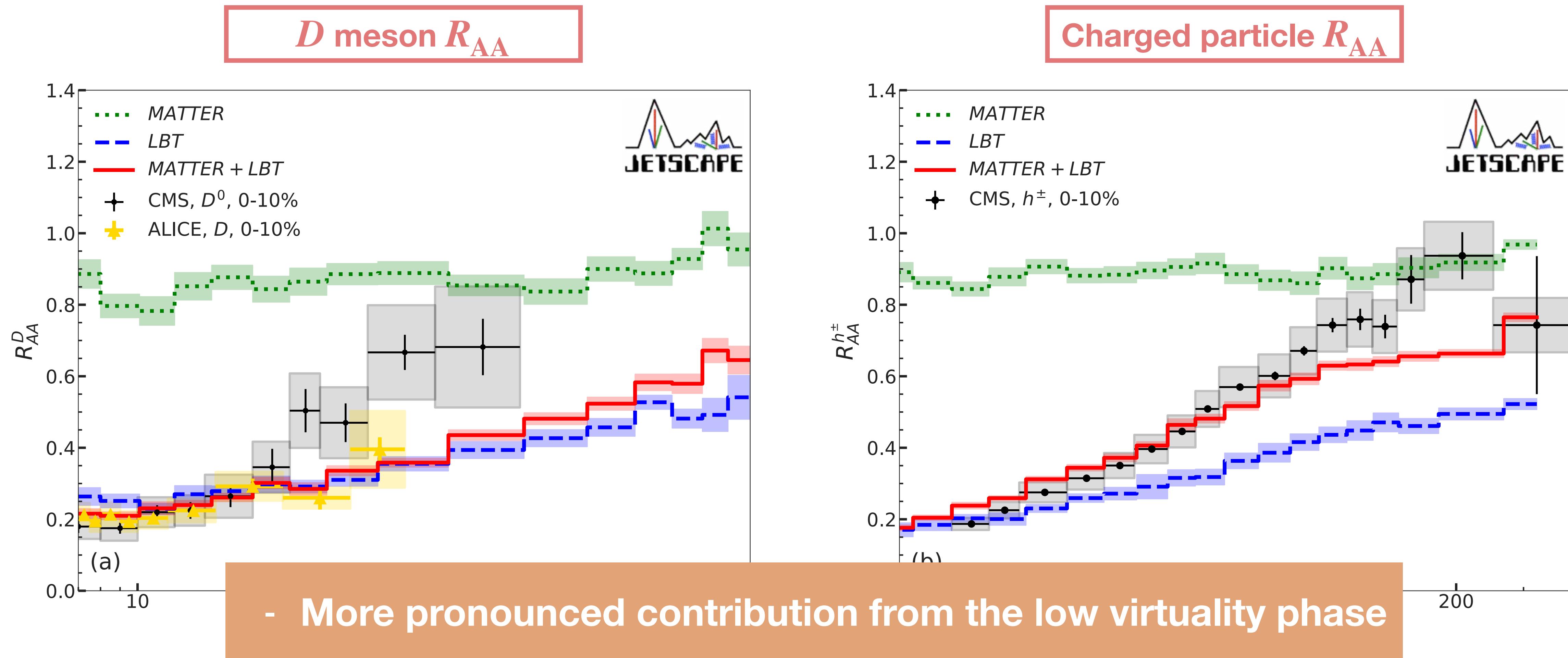
- Role of multi-stage description for energy loss



Charm hadron R_{AA}

JETSCAPE, arXiv:2208.00983

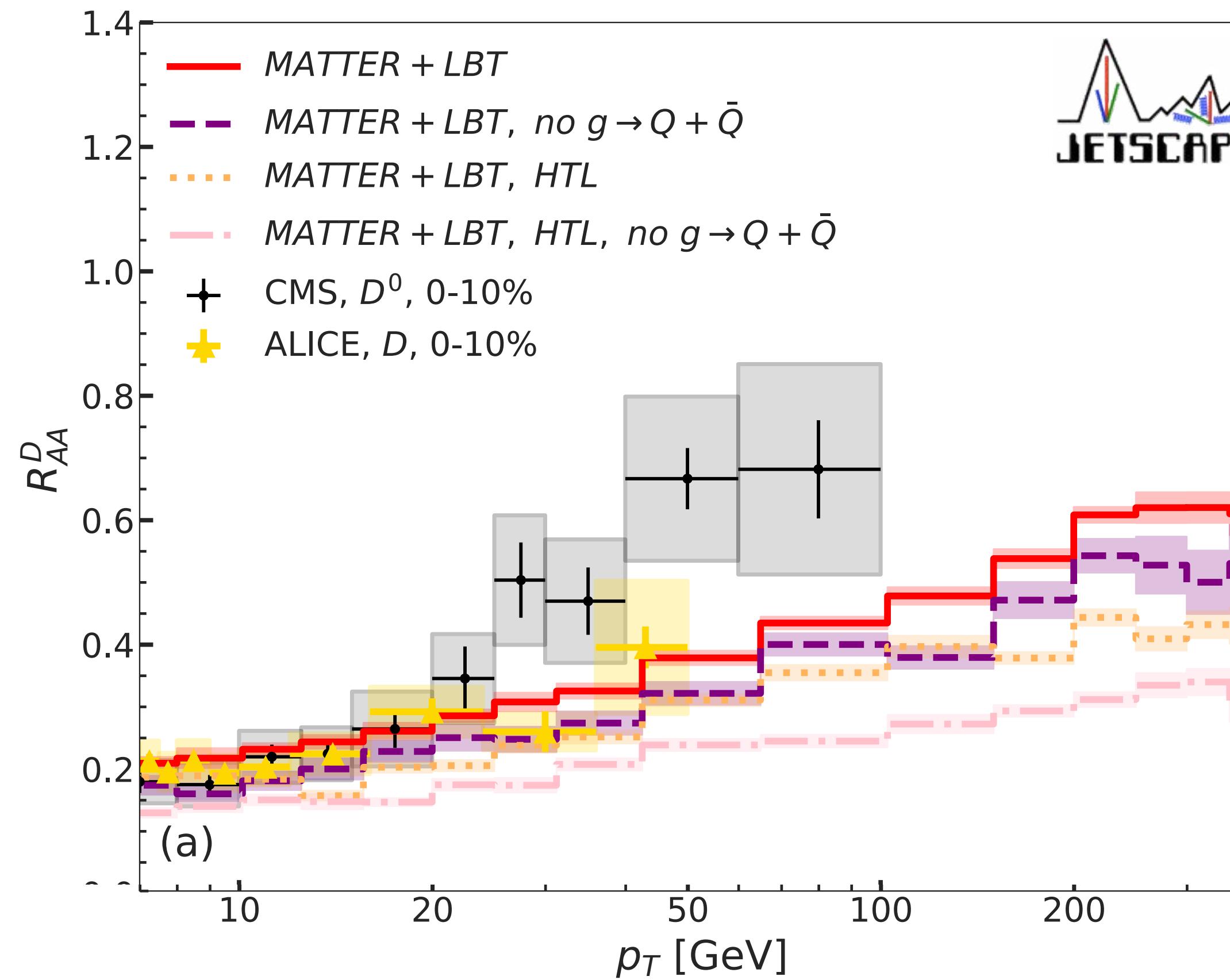
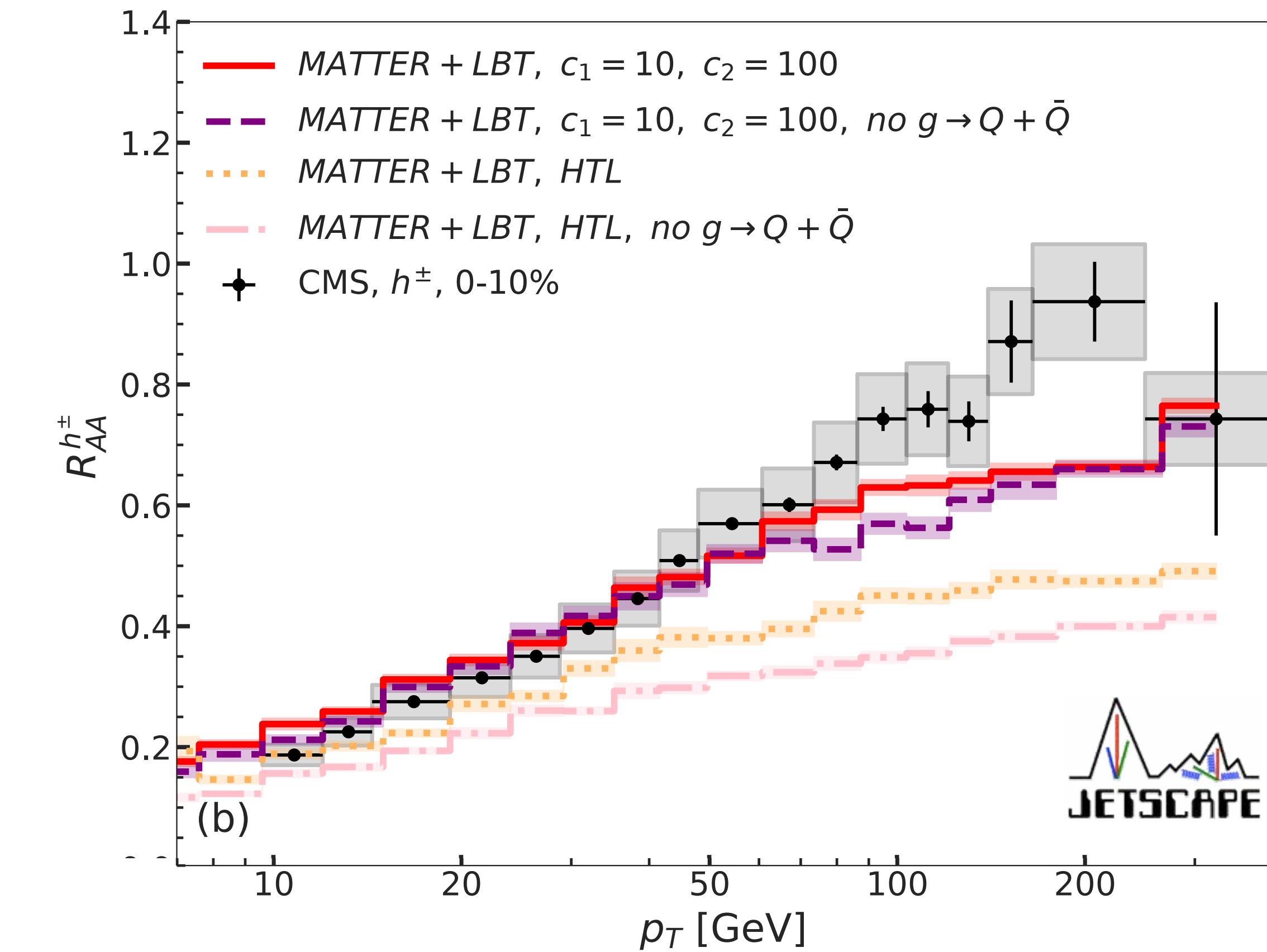
- Role of multi-stage description for energy loss



Charm hadron R_{AA}

JETSCAPE, arXiv:2208.00983

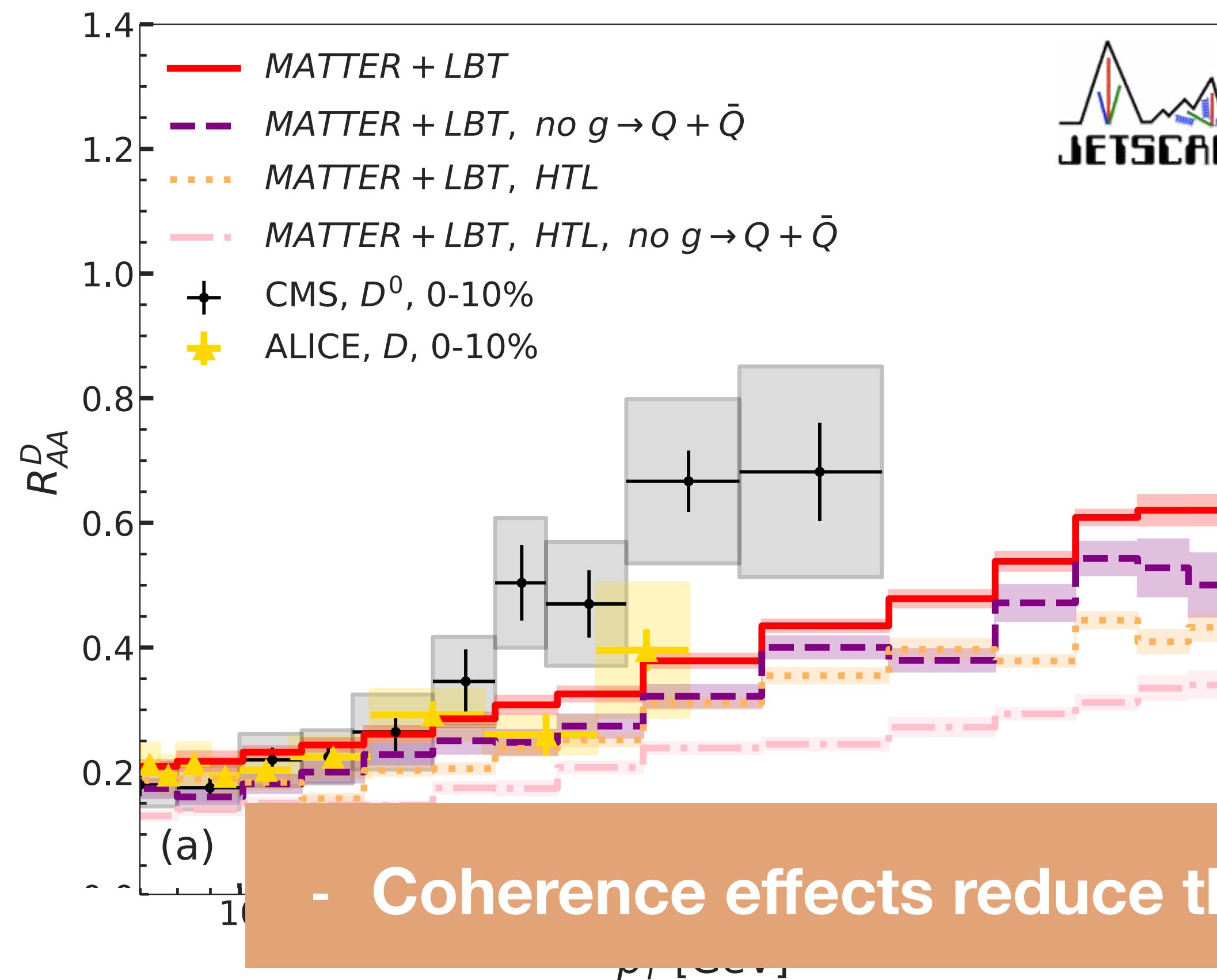
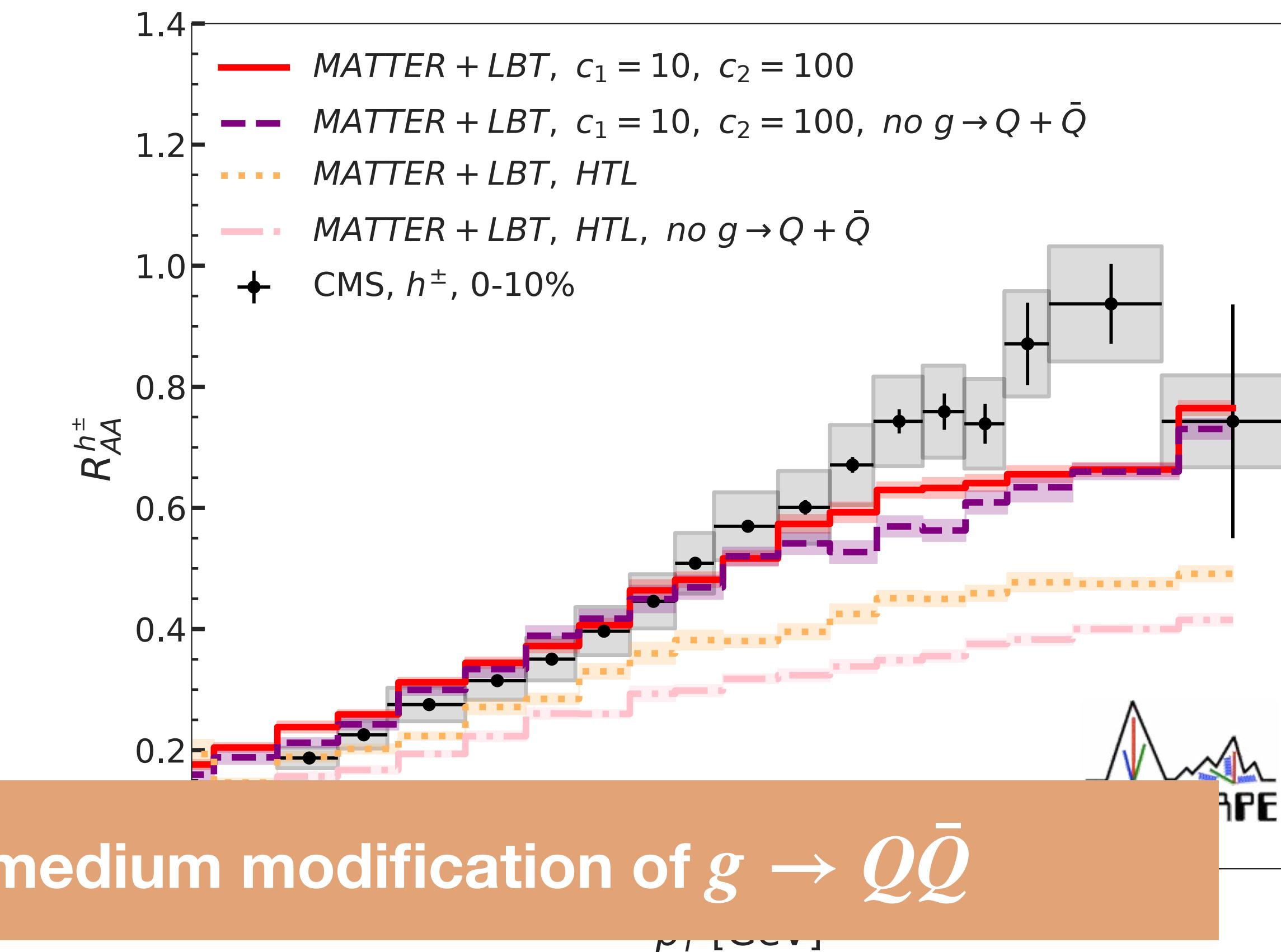
- Coherence Effects on $g \rightarrow Q\bar{Q}$

 D meson R_{AA} **Charged particle R_{AA}** 

Charm hadron R_{AA}

JETSCAPE, arXiv:2208.00983

- Coherence Effects on $g \rightarrow Q\bar{Q}$

***D* meson R_{AA}** **Charged particle R_{AA}** 

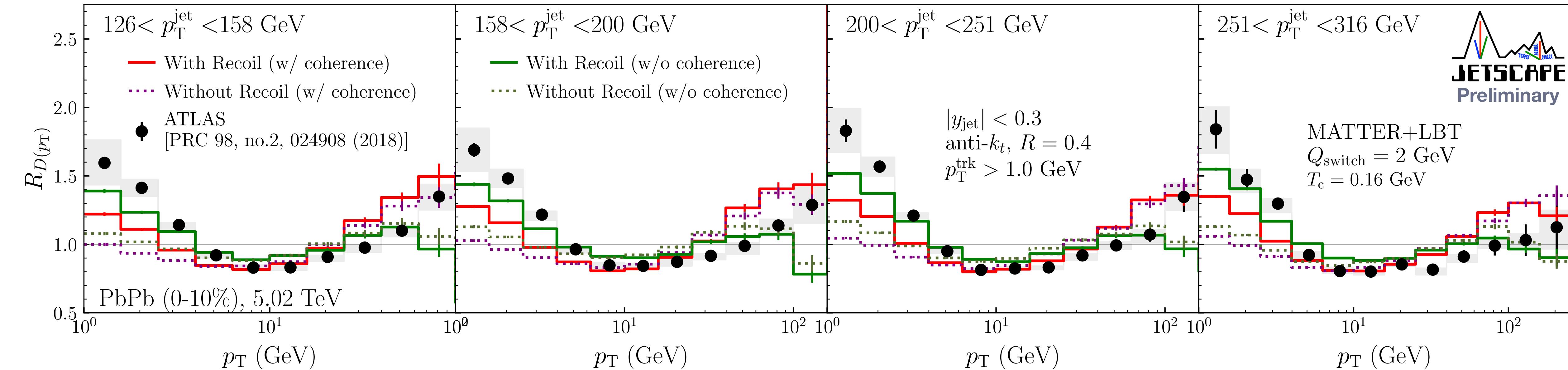
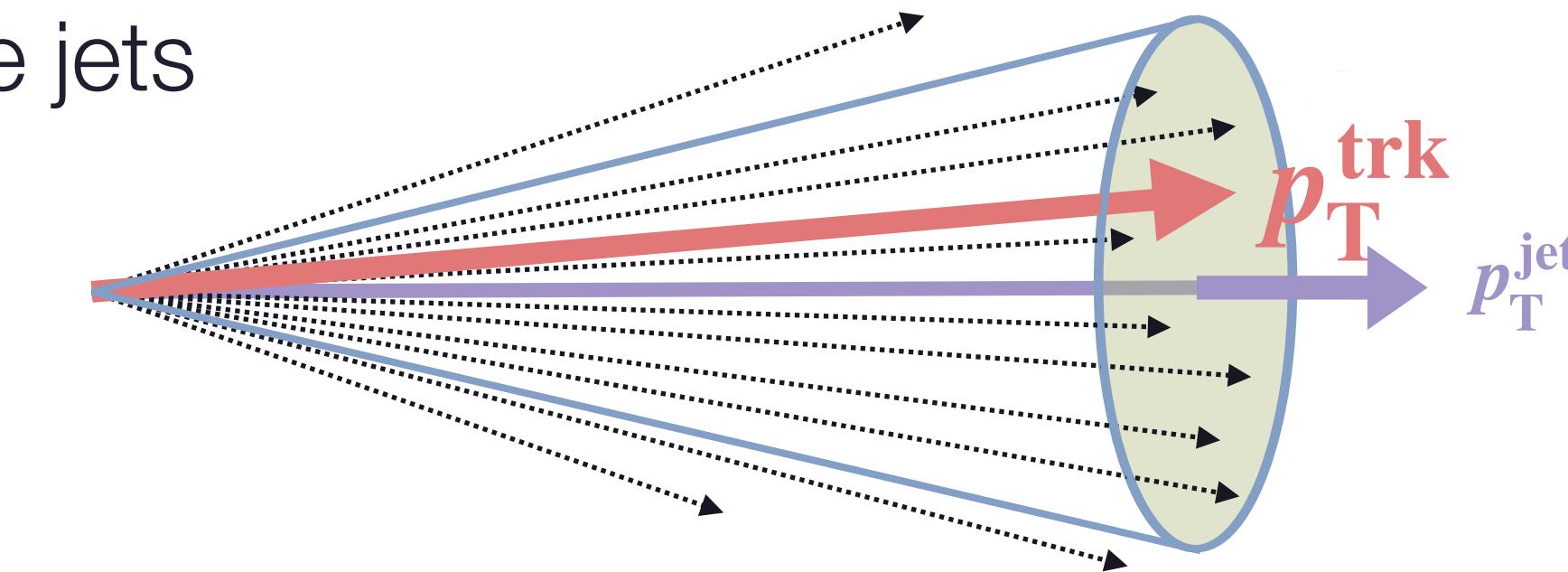
- Coherence effects reduce the medium modification of $g \rightarrow Q\bar{Q}$

Jet Substructures

- **Jet Fragmentation Function**

- p_T distribution of charged track particle inside jets

$$D(p_T^{\text{trk}}) = \frac{1}{N_{\text{jet}}} \sum_{\text{jet}} \left. \frac{dN_{\text{trk}}}{dp_T^{\text{trk}}} \right|_{\text{in jet}}$$

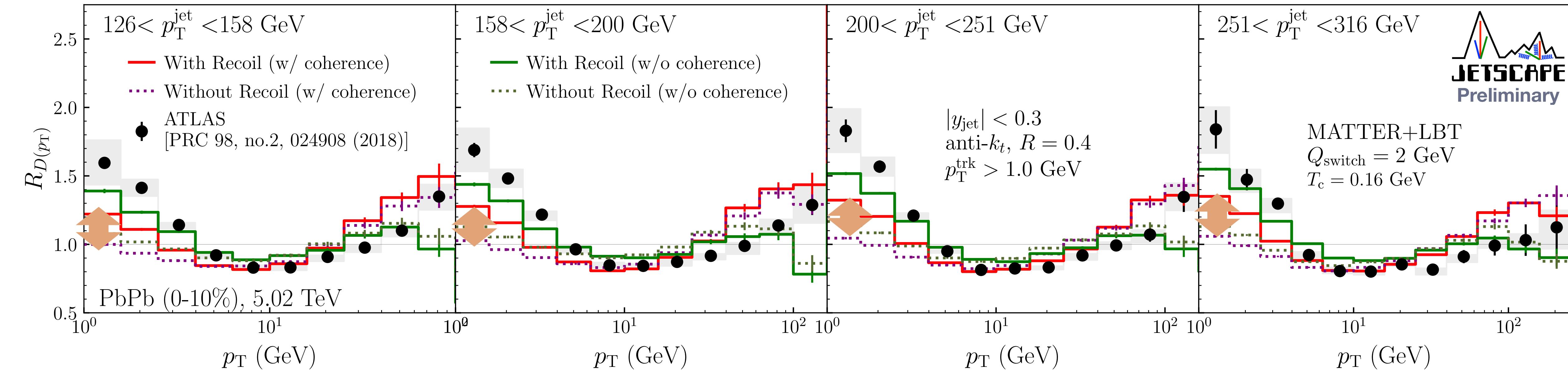
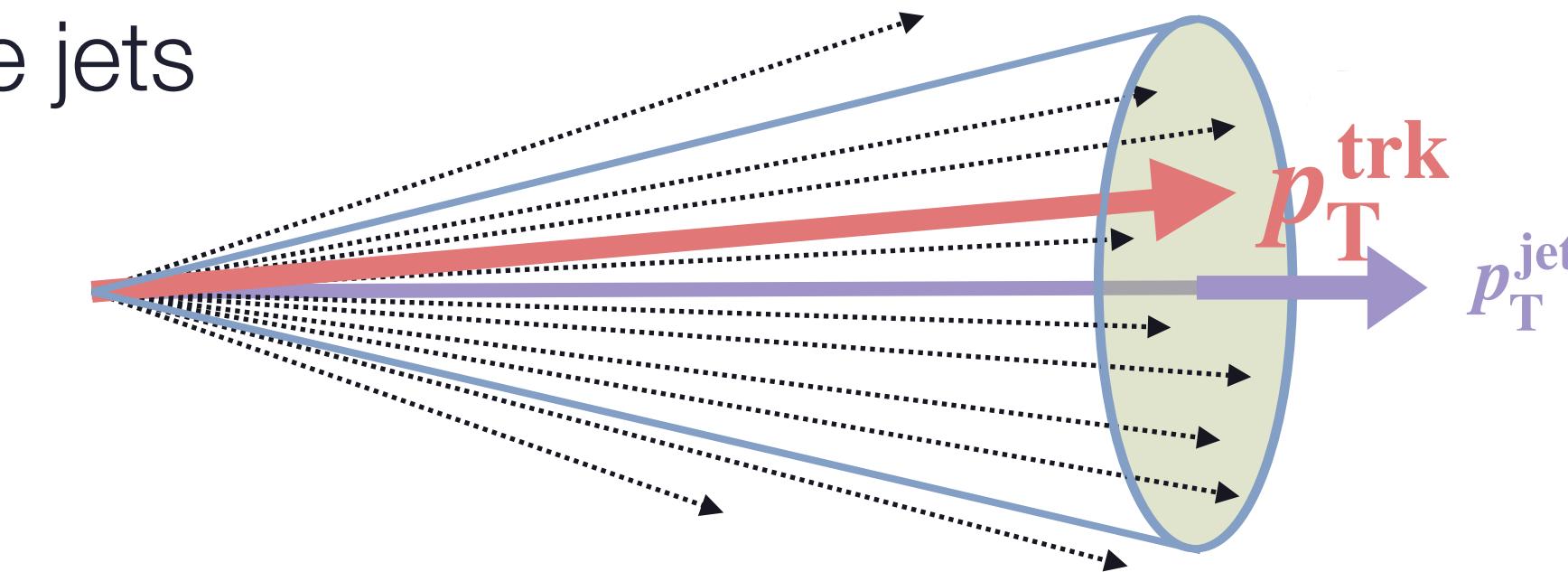


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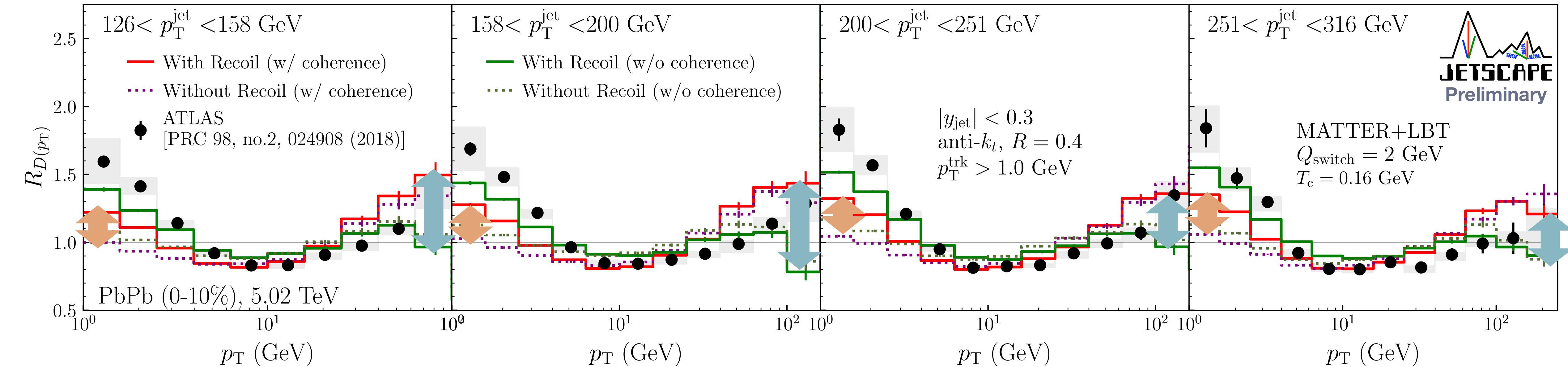
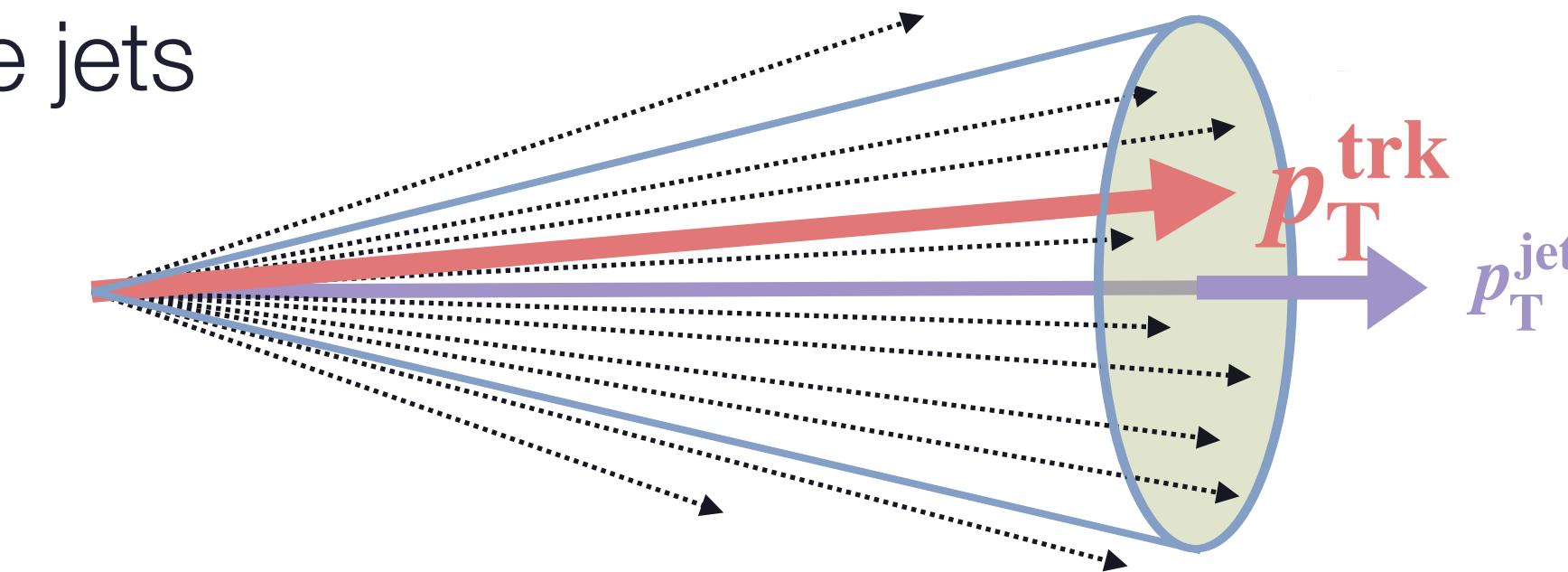


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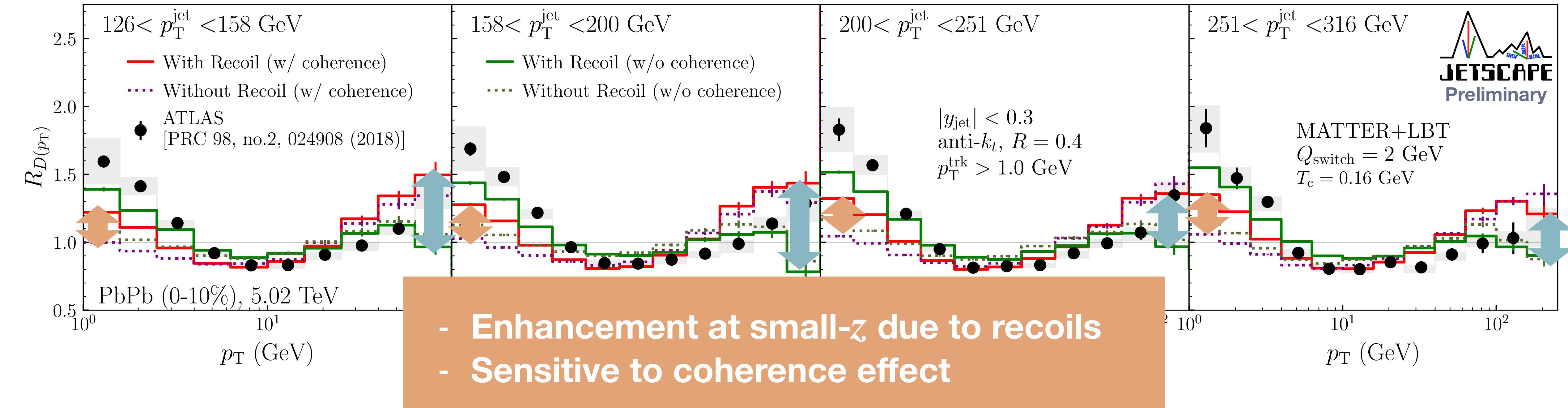
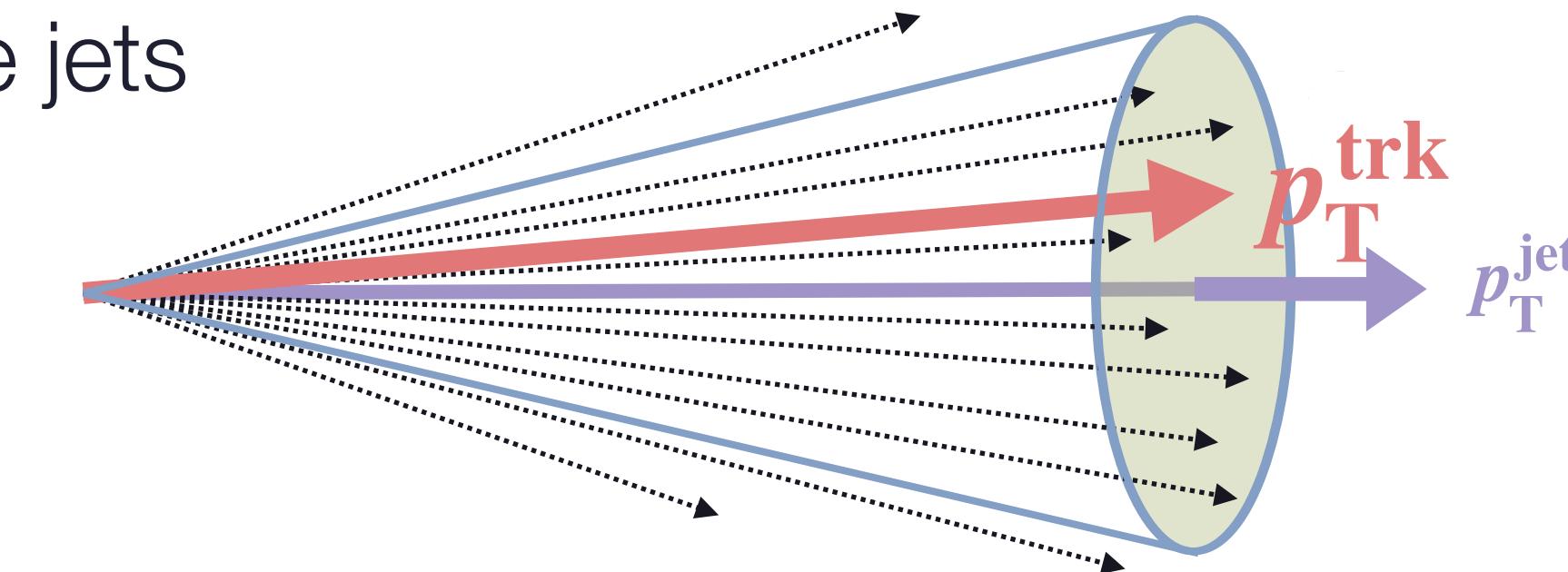


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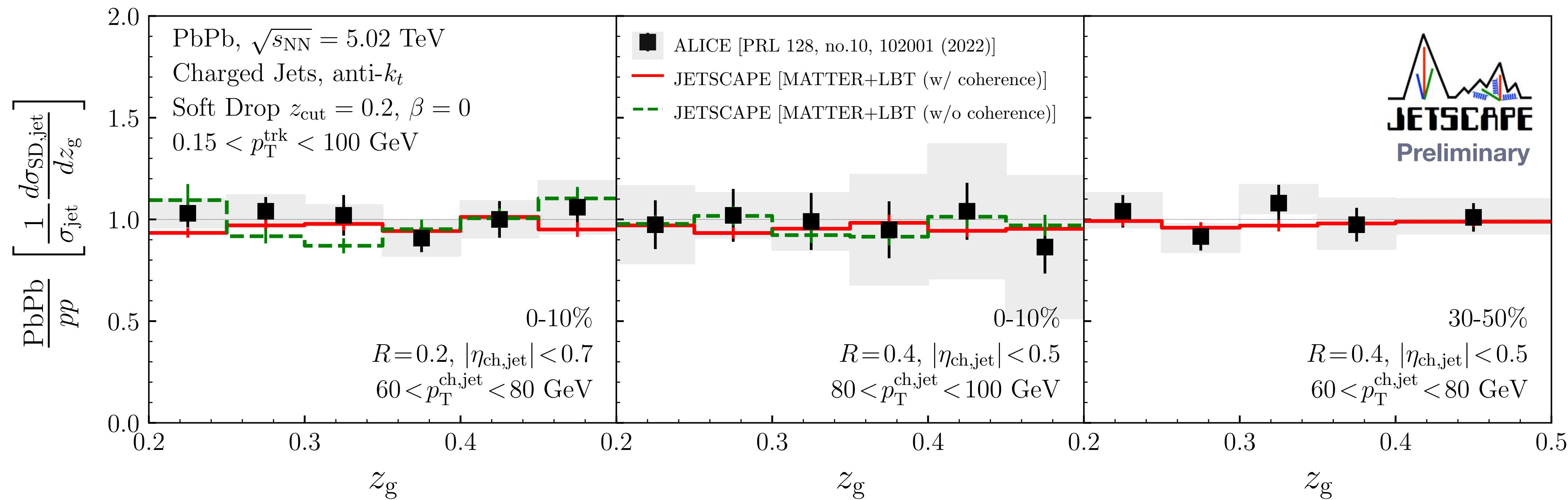
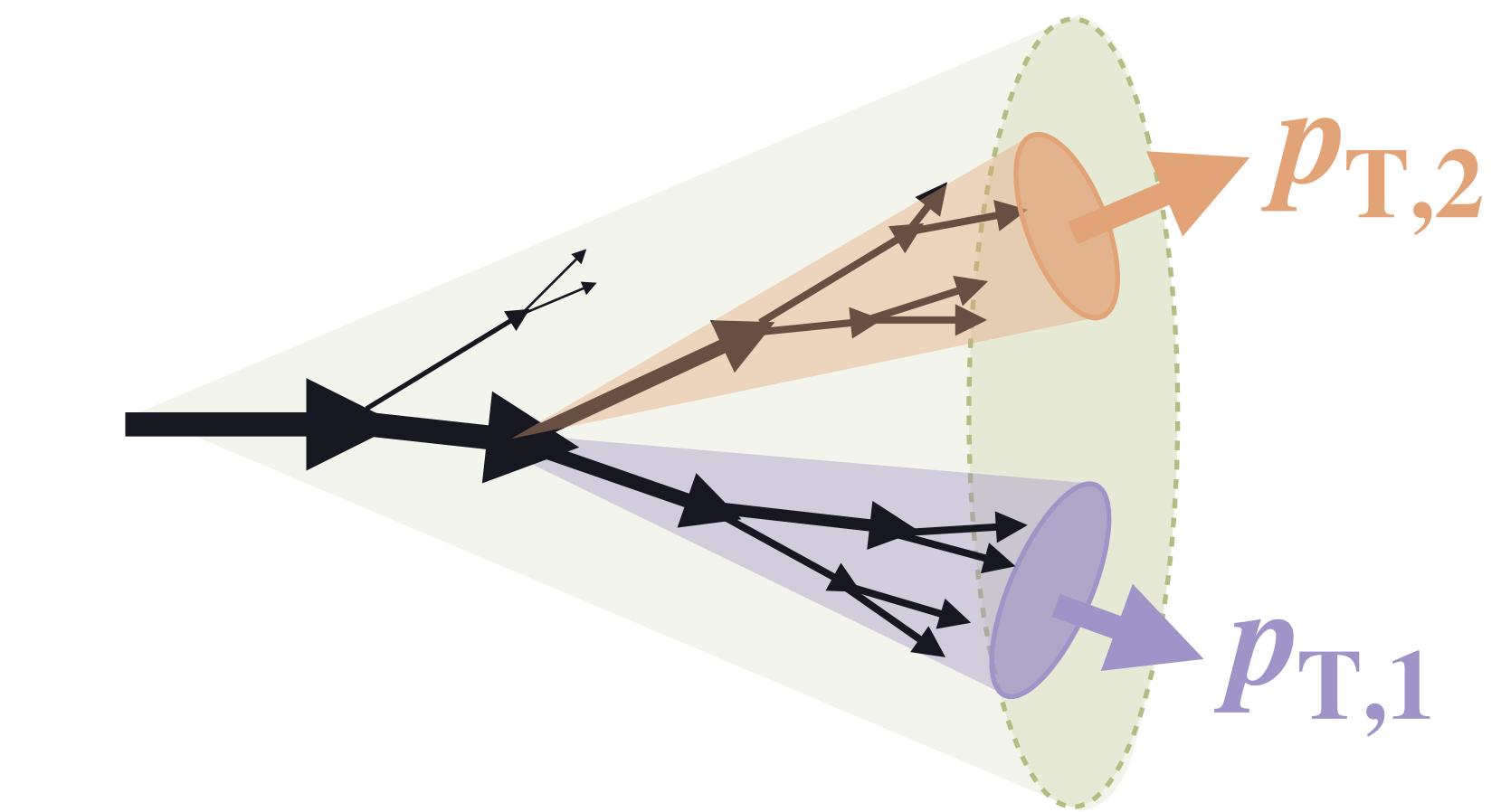


Jet Substructures

- **Jet splitting function**

- Hardest splitting determined by Soft Drop
- Momentum fraction in the hardest splitting of jet (z_g)

$$z_g = \frac{\min(p_{T,1}, p_{T,2})}{p_{T,1} + p_{T,2}}$$

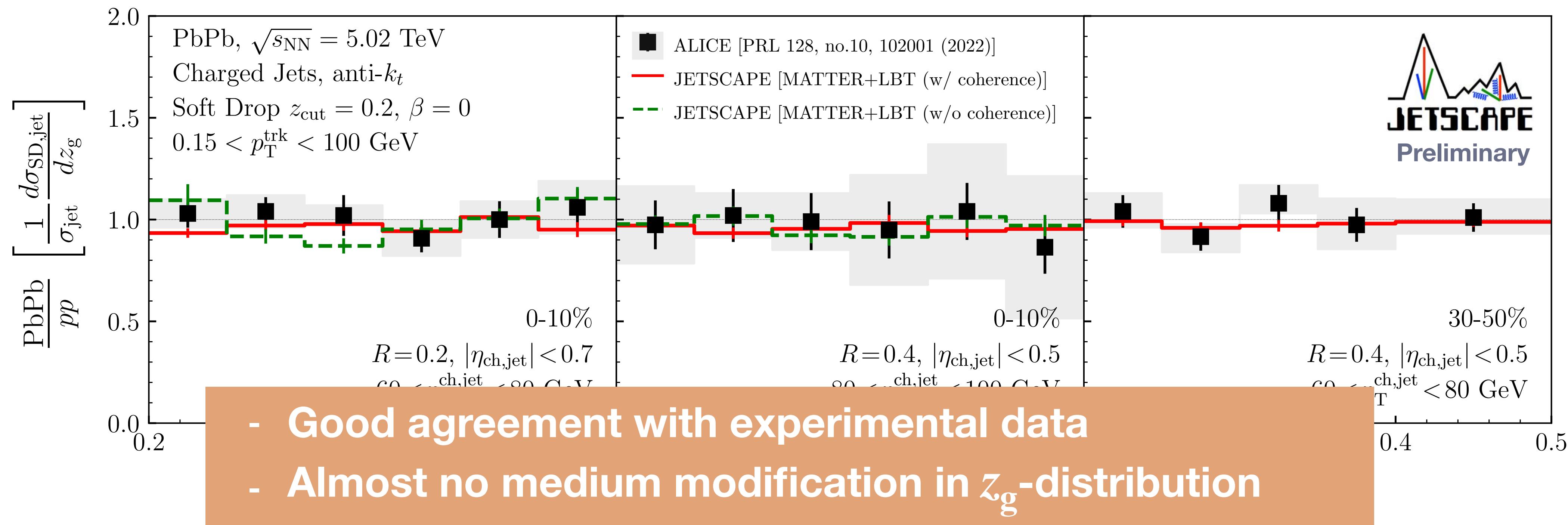
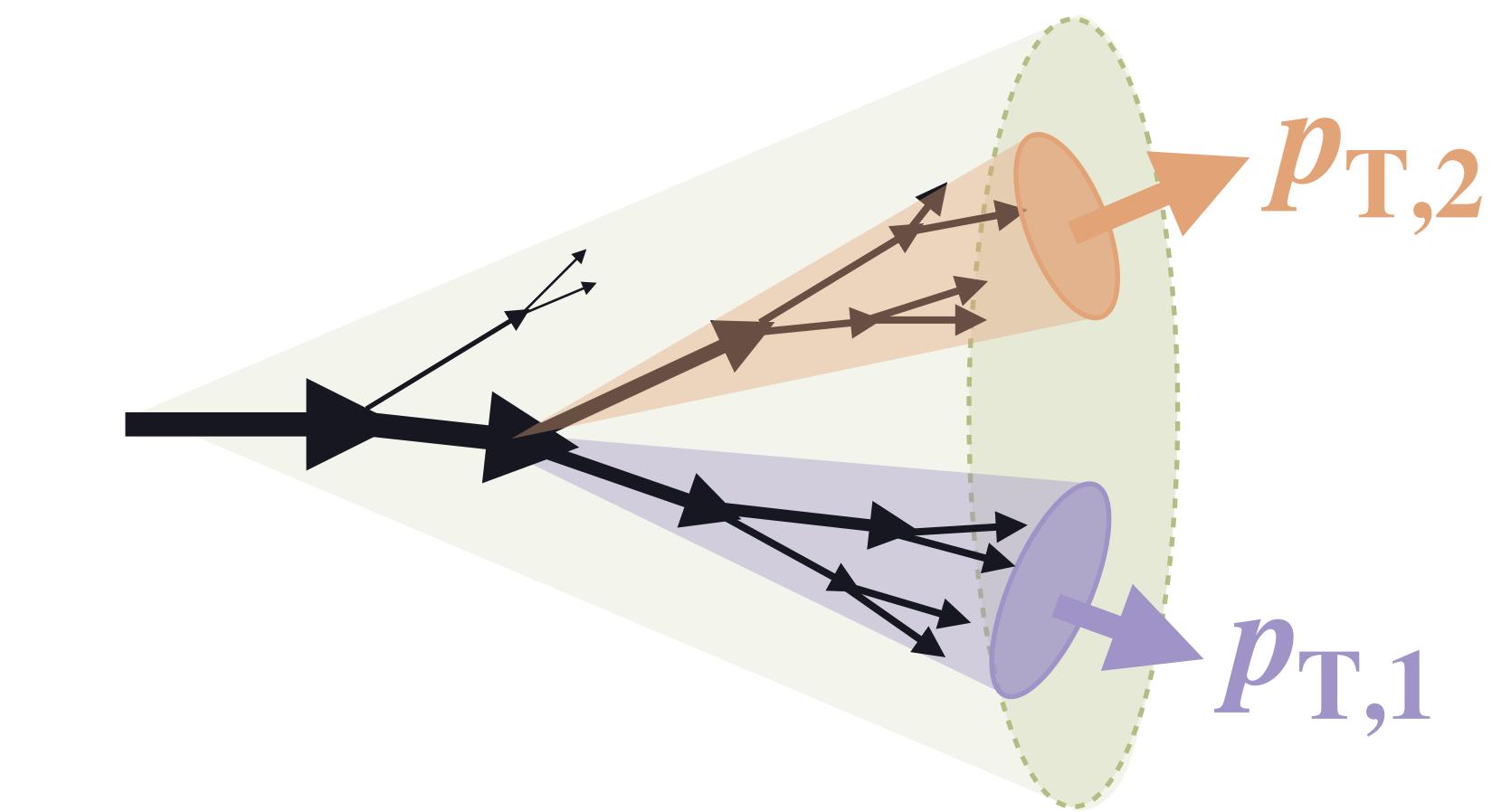


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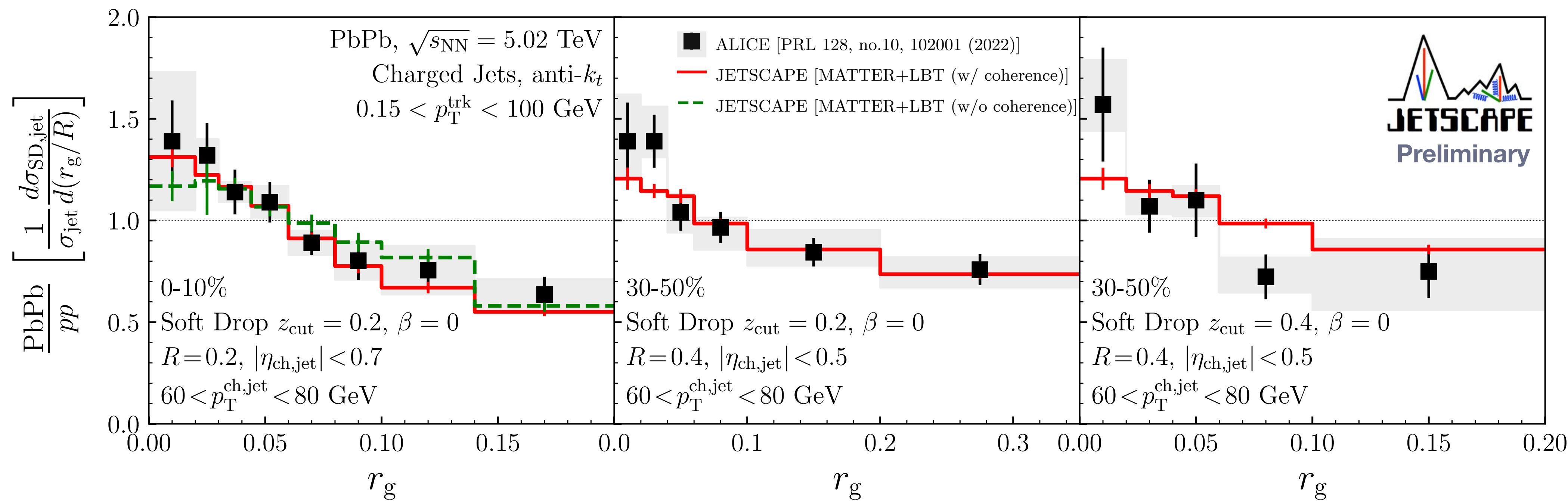
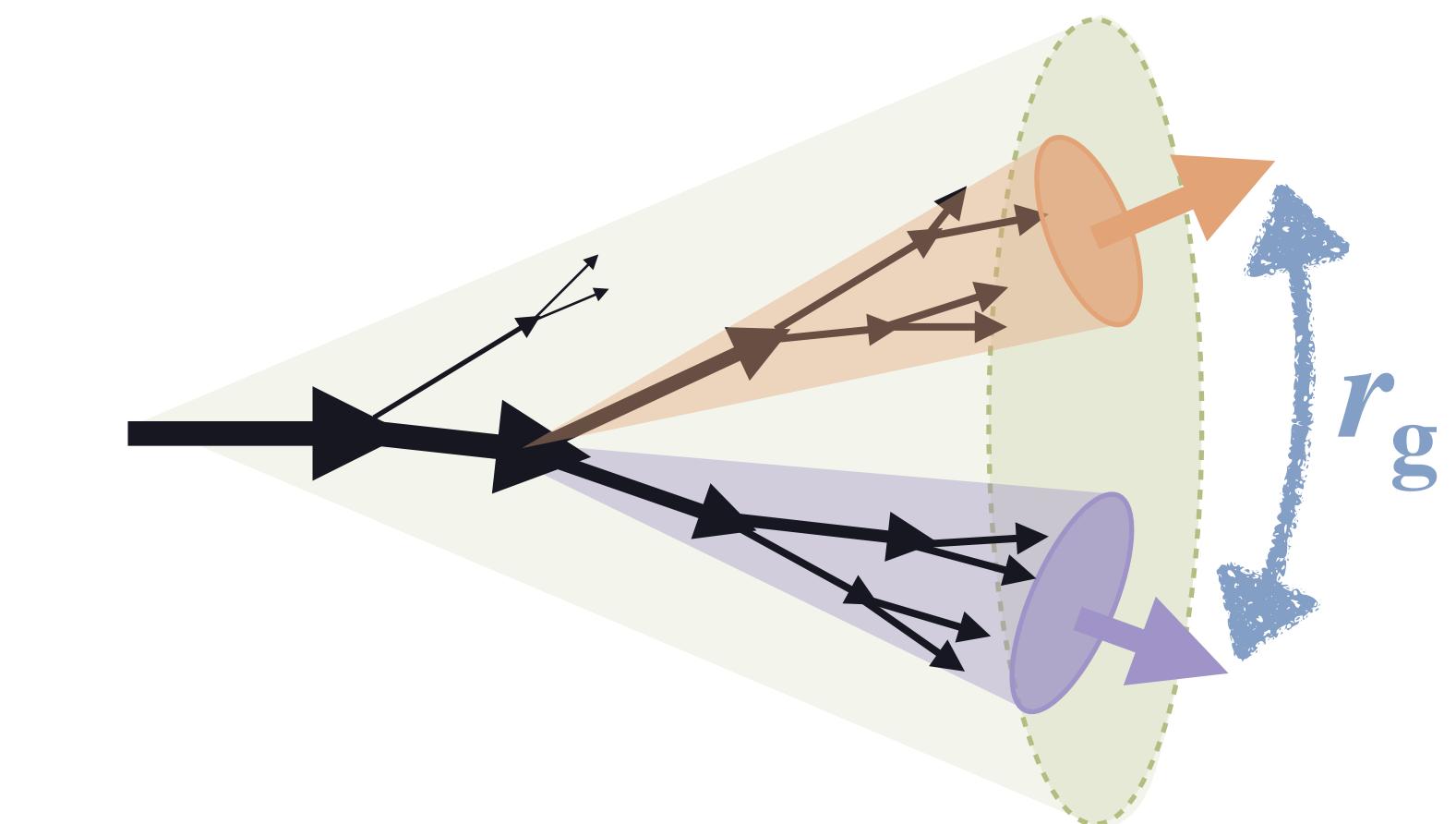
$$z_g = \frac{\min(p_{T,1}, p_{T,2})}{p_{T,1} + p_{T,2}}$$



Jet Substructures

- **Splitting radial distance distribution**

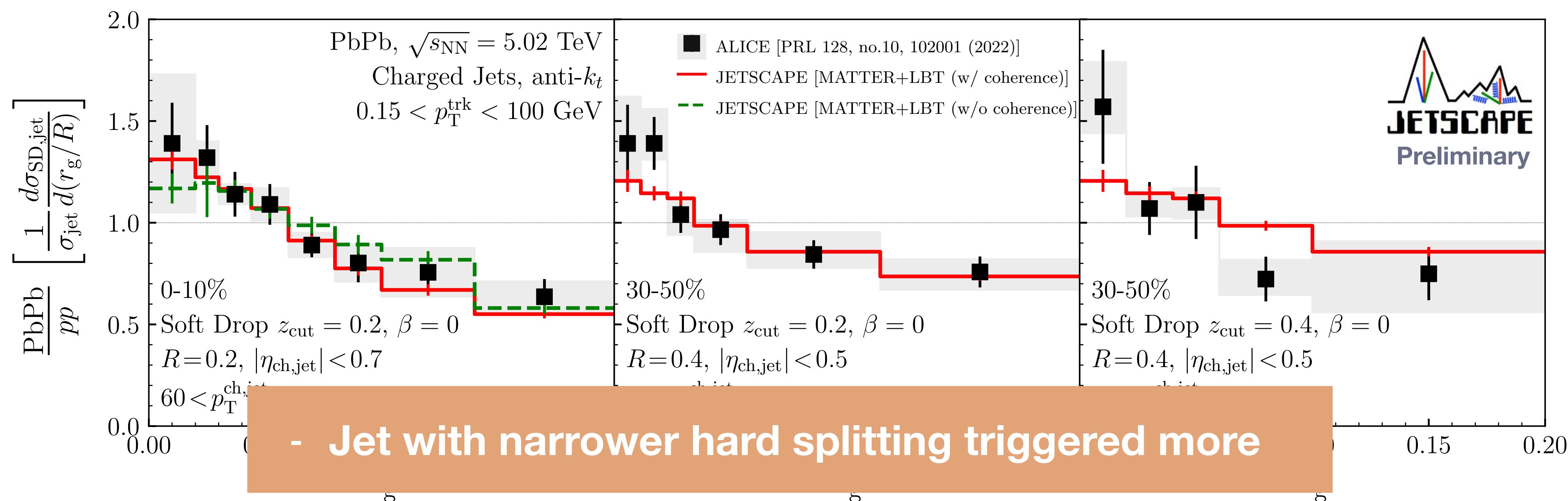
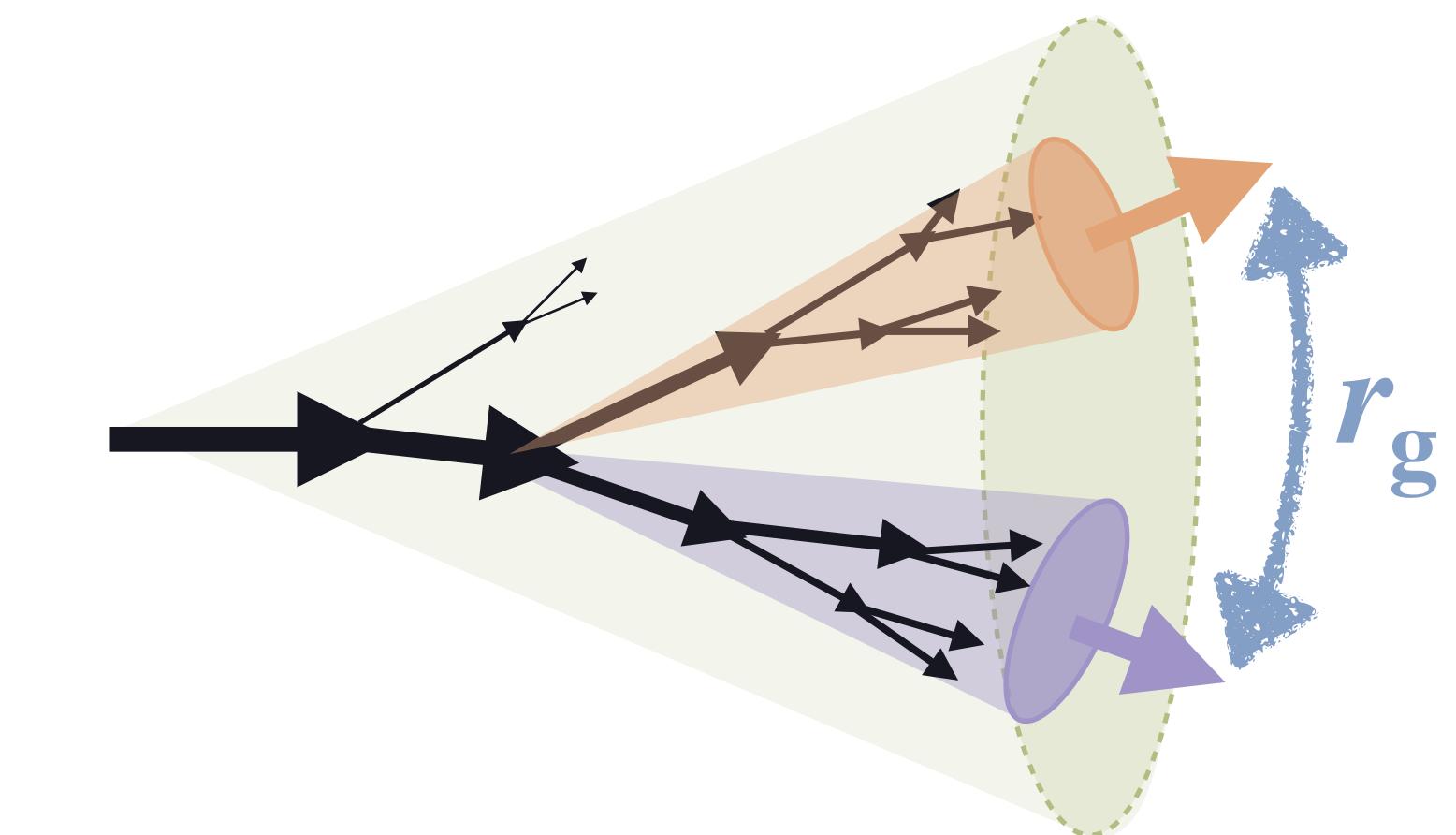
- Hardest splitting determined by Soft Drop
- r_g : the opening radial distance in the hardest splitting



Jet Substructures

- **Splitting radial distance distribution**

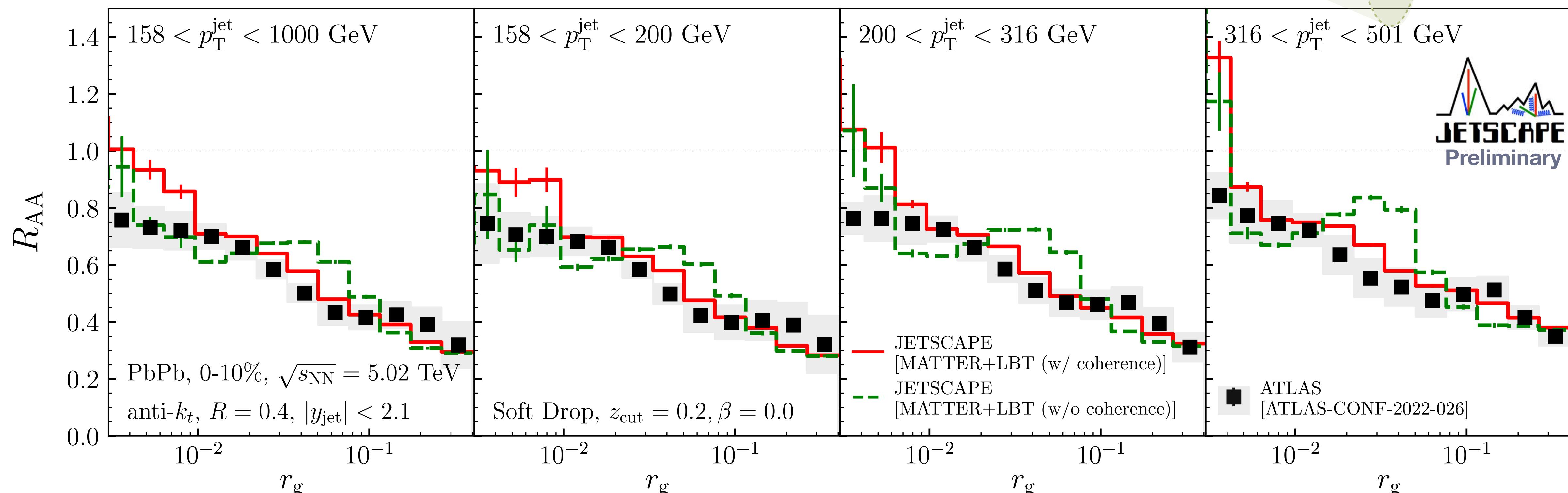
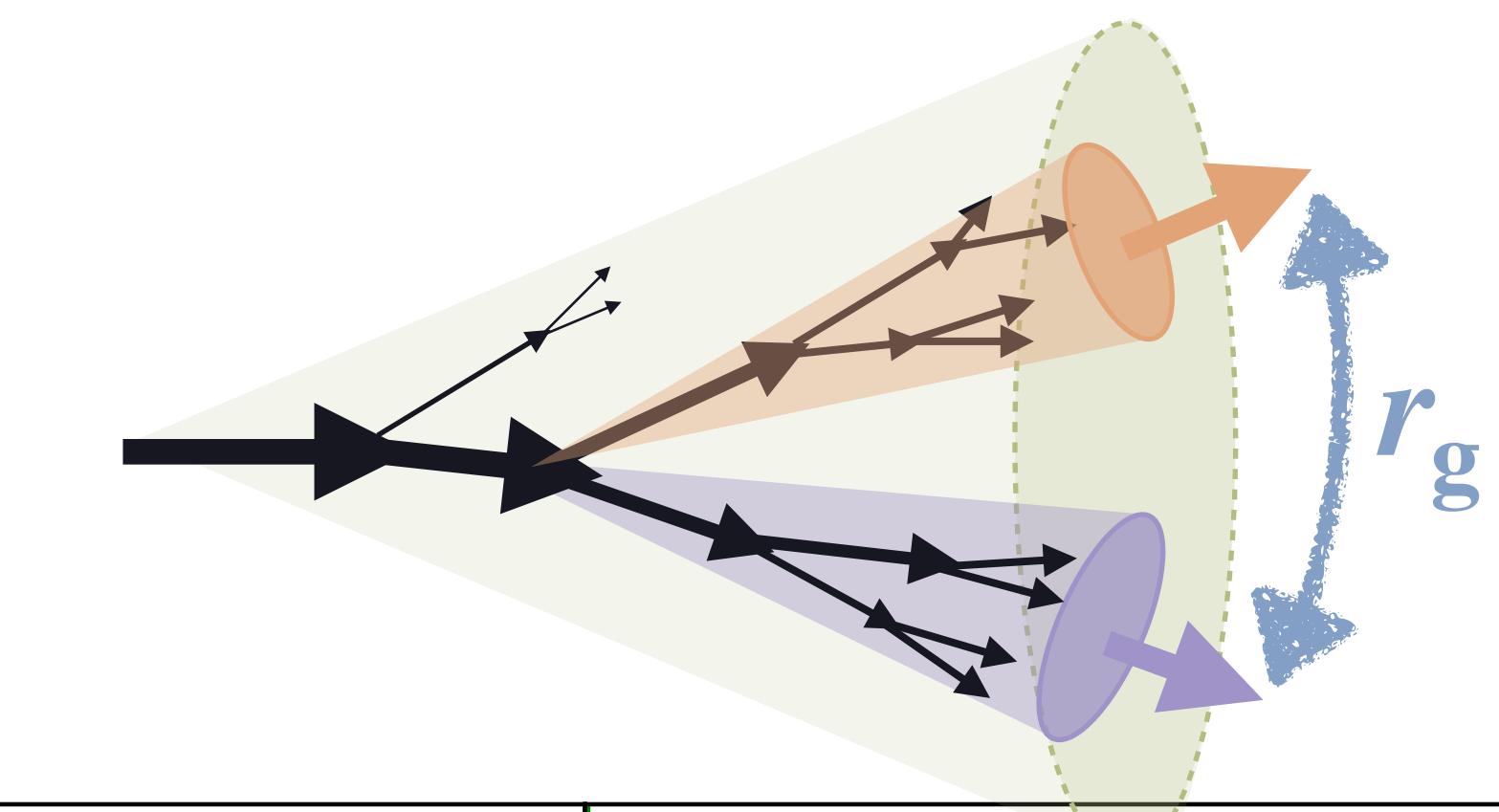
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Jet Substructures

- Substructure Dependence on Jet Energy Loss

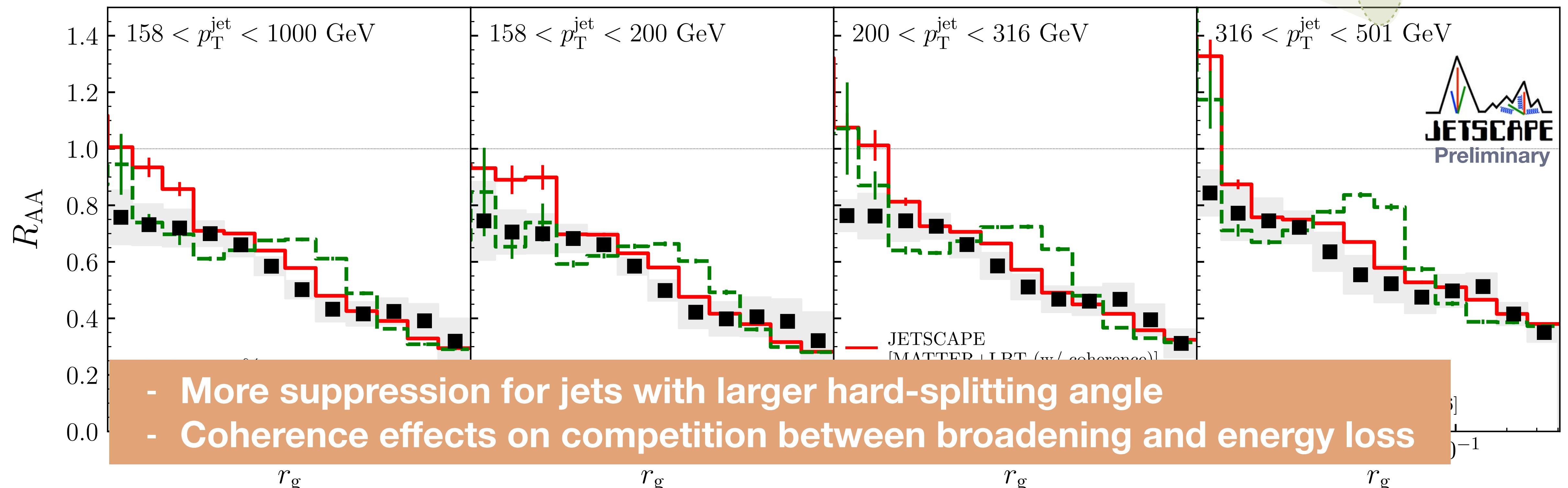
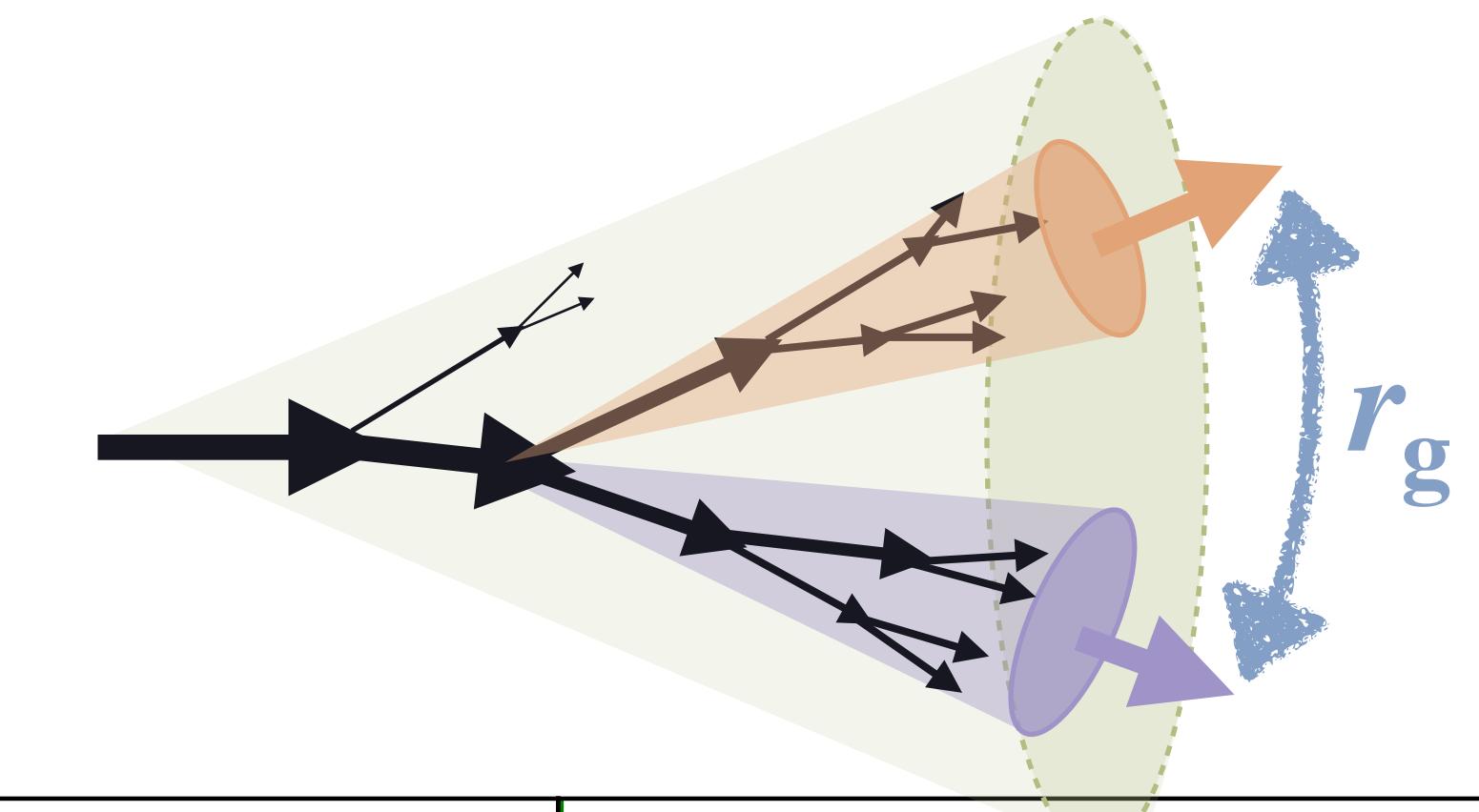
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Jet Substructures

- Substructure Dependence on Jet Energy Loss

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Summary

● Monte-Carlo Simulation for Heavy-ion Collisions

- Many physics components are involved
- Communication between components

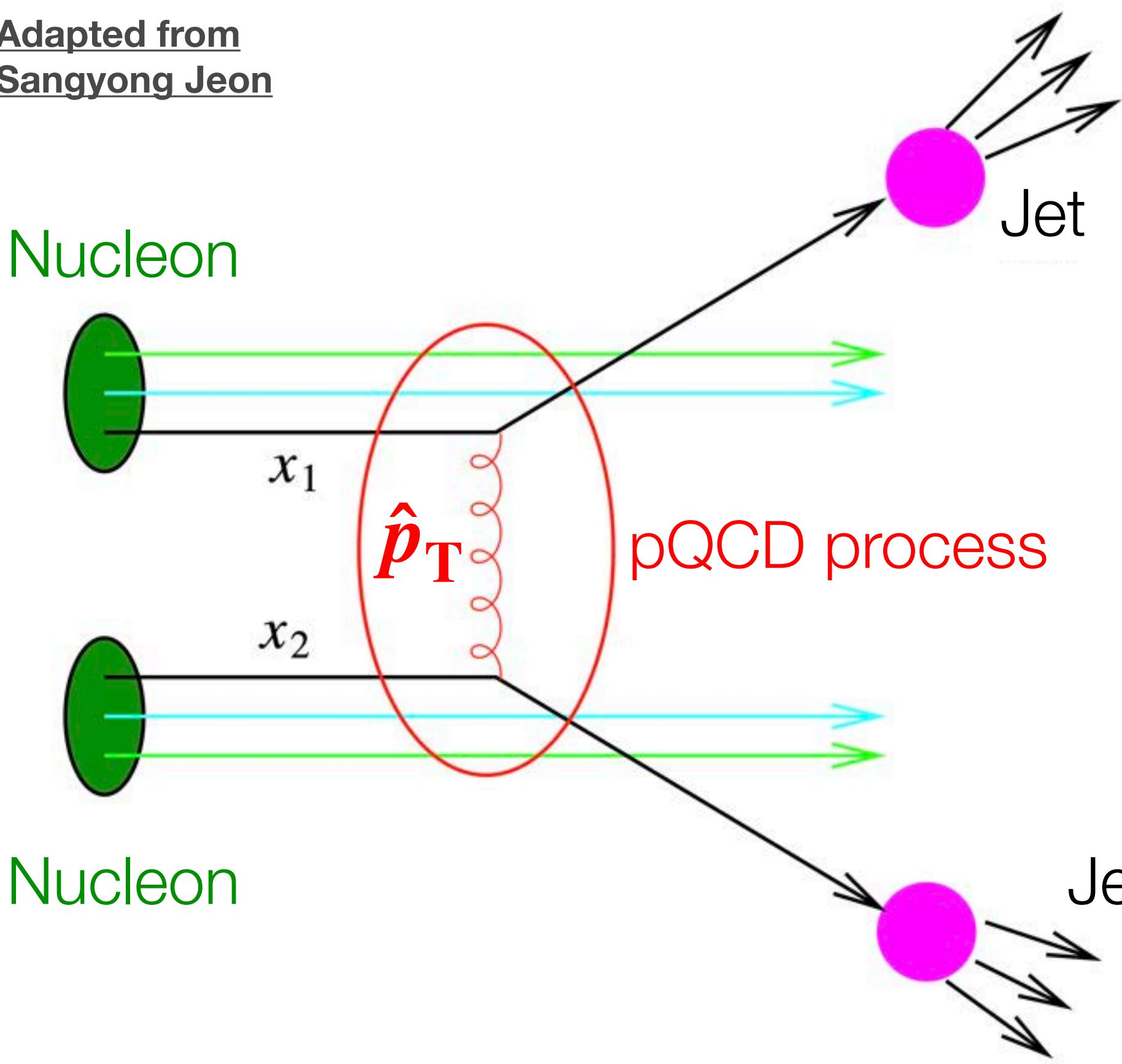
● Multi-stage evolution of jet shower

- Switching description of in-medium parton shower evolution depending on Q^2
- Extension of multi-stage jet energy loss by coherence effect (Q^2 -dependence)
- Simultaneous description of jet and single particle at various $\sqrt{s_{\text{NN}}}$
- Details of interaction encoded in jet substructures (coherence, recoils)

Simulation Setup in the Session

● Hard scatterings at 5.02 TeV by PythiaGun

Adapted from
Sangyong Jeon



- Generate only scatterings with $100 < \hat{p}_T < 160$ GeV.
- Same number of hard scatterings for pp and PbPb (not the number of collision events)

XML File

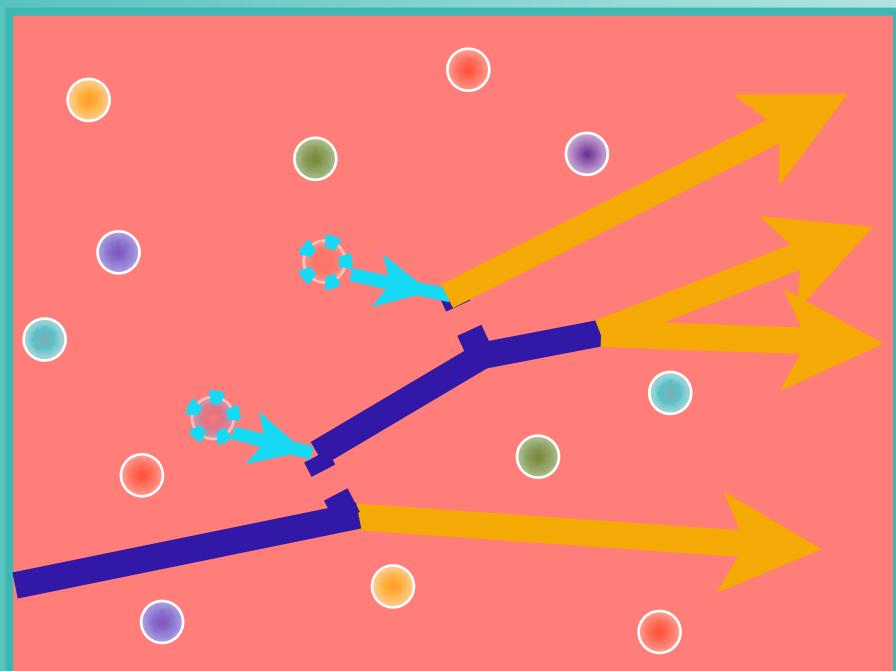
```

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24    <Hard>
25      <PythiaGun>
26        <pTHatMin>100</pTHatMin>
27        <pTHatMax>160</pTHatMax>
28        <eCM>5020</eCM>
29      </PythiaGun>
30    </Hard>
31

```

Description of Medium Response

Recoil



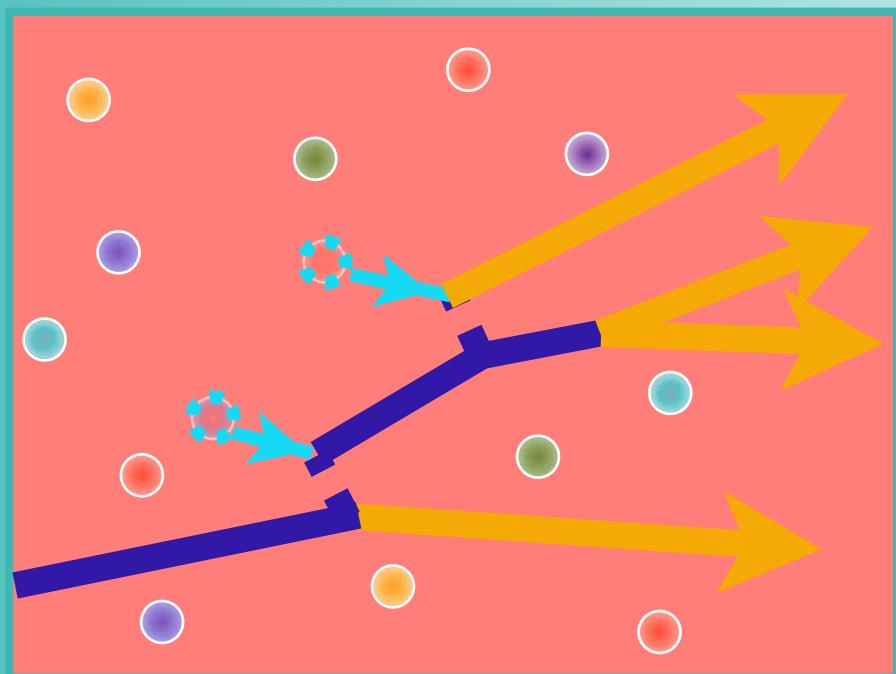
Weakly-coupled ($> E_{\text{med}}$)

$\sim E_{\text{th}} \gtrsim E_{\text{med}}$

Strongly-coupled ($\sim E_{\text{med}}$)

Description of Medium Response

Recoil



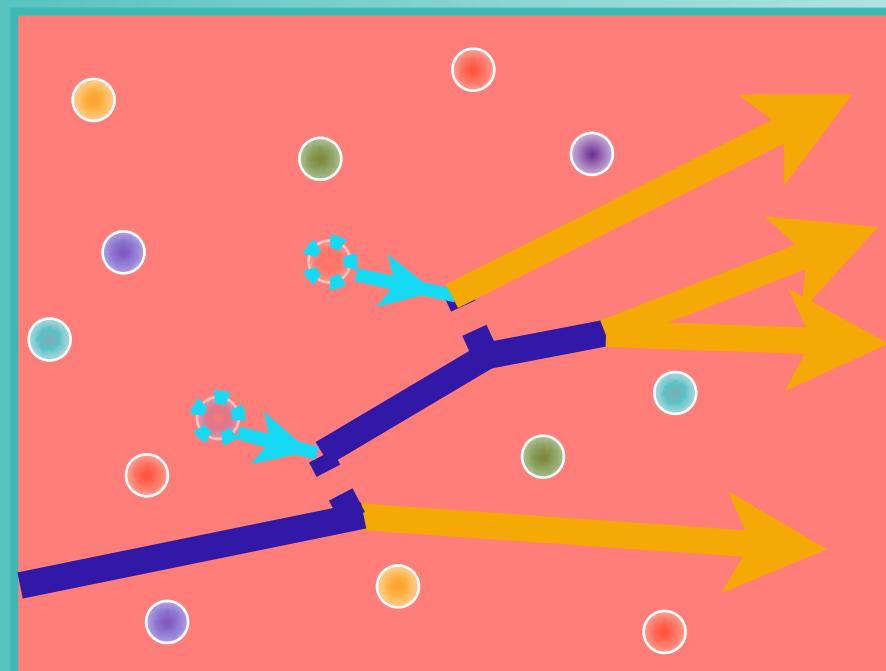
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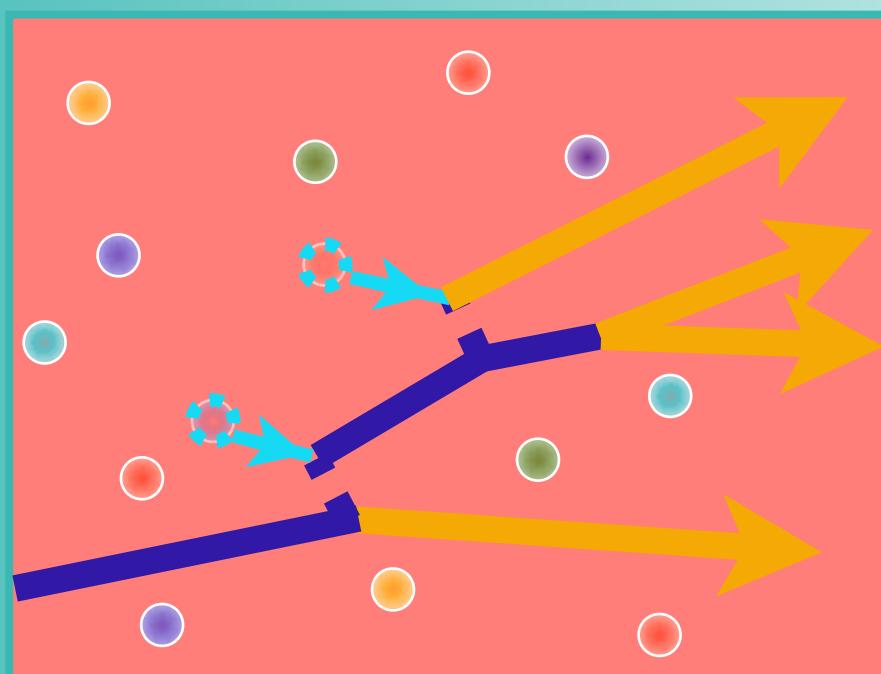
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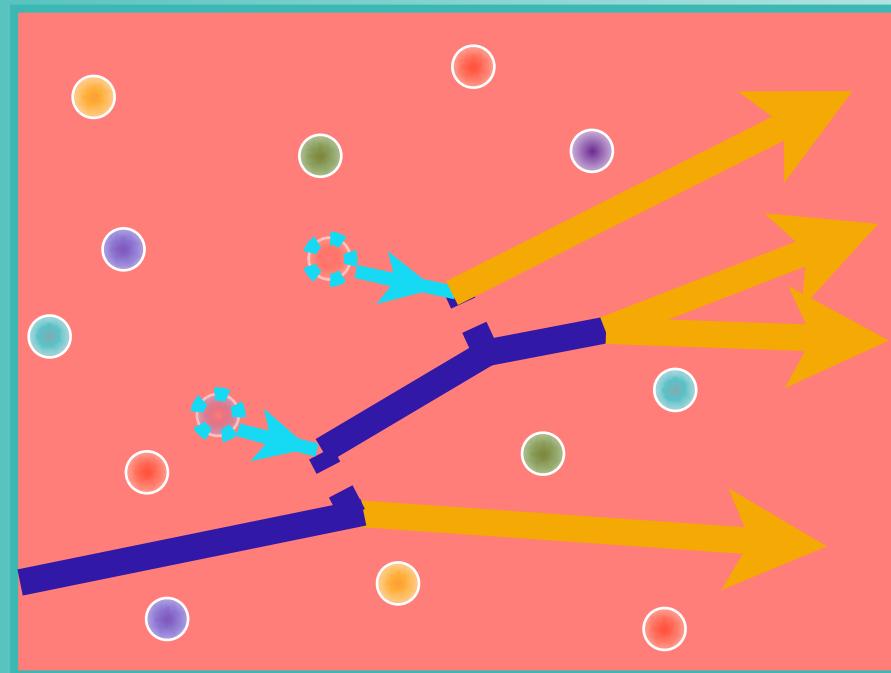
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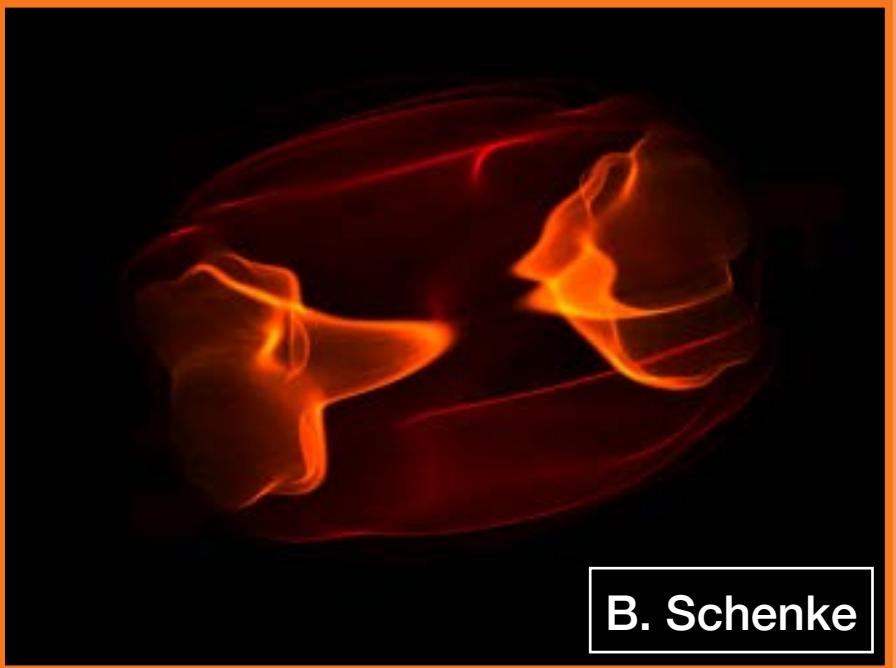


Description of Medium Response

Recoil



Hydrodynamics



Weakly-coupled ($> E_{\text{med}}$)

$\sim E_{\text{th}} \gtrsim E_{\text{med}}$

Strongly-coupled ($\sim E_{\text{med}}$)

Description of Hydro Medium Response to Jets

● Medium fluid evolution with energy-momentum deposition

Coupled Jet-Fluid (YT, N.-B. Chang, G.-Y. Qin), CoLBT-hydro (W. Chen, X.-N. Wang, Z. Yang,...), EPOS3-HQ (I. Karpenko,...), JETSCAPE (JETSCAPE), LEXUS+MUSIC (C. Shen, B. Schenke,...), JAM (Y. Nara), DCCI2 (Y. Kanakubo, YT, T. Hirano,...), Hybrid+MUSIC (D. Pablos, M. Singh...), BESHYDRO (L. Du, U. Heinz,...),...

- Hydrodynamic transport of jet energy-momentum via thermal partons
- (3+1)D evolution together with the bulk QGP fluid

Hydrodynamic equation with source term

$$\nabla_\mu T_{\text{med}}^{\mu\nu}(x) = J_{\text{jet}}^\nu(x)$$

Energy-momentum tensor
of the QGP fluid

Energy and momentum
deposited into the fluid

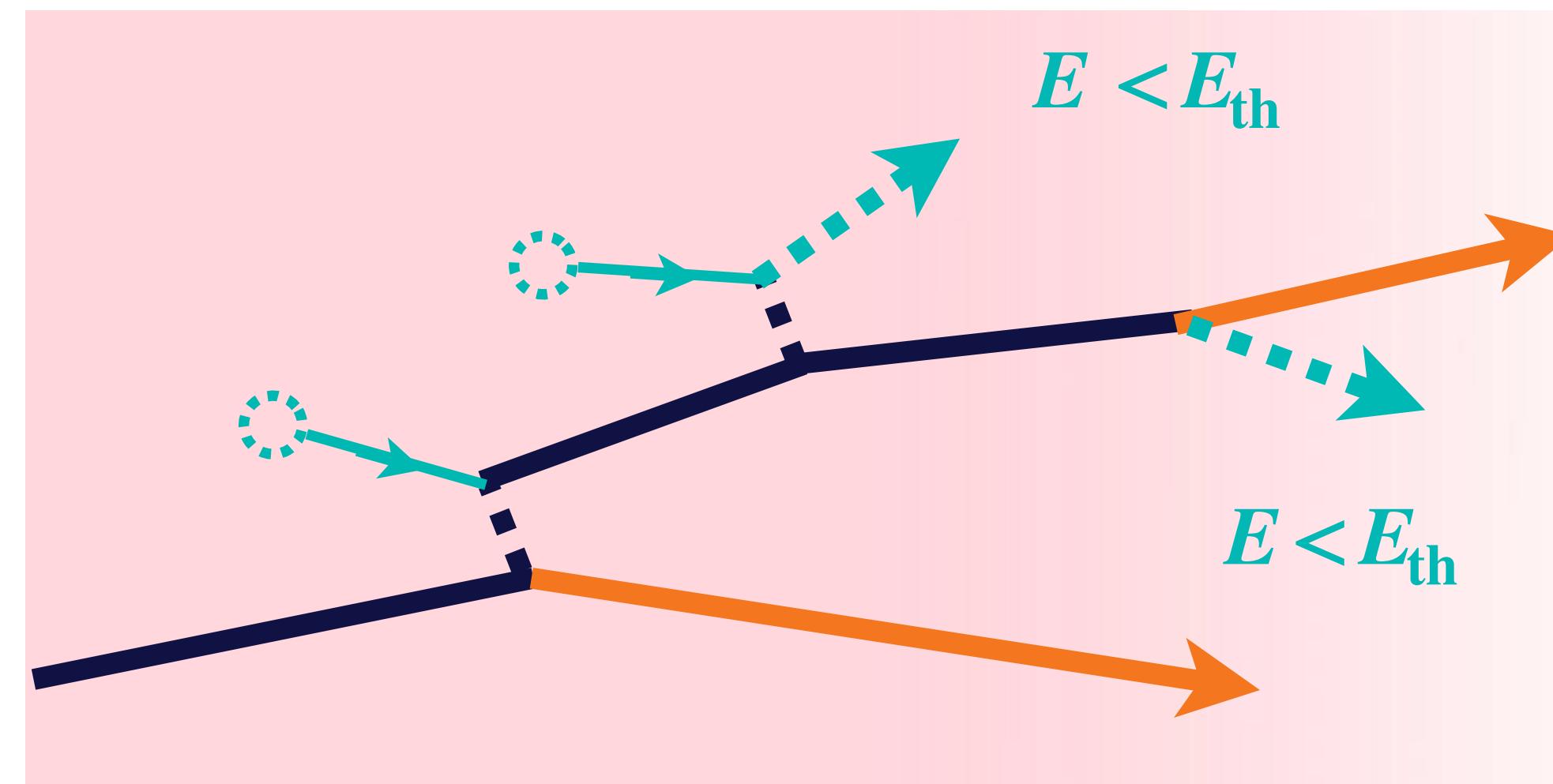
- Source term J_{jet}^ν constructed from jet-shower evolution calculation
- Bulk part particle with hydro response obtained via the Cooper-Frye

Recoil+Hydro Response Description in MC Approach

● Energy-momentum deposition

CoLBT-hydro (W. Chen, X.-N. Wang et al.)

- Soft partons
- Holes' energy and momentum



● Causal source profile

JETSCAPE (JETSCAPE)

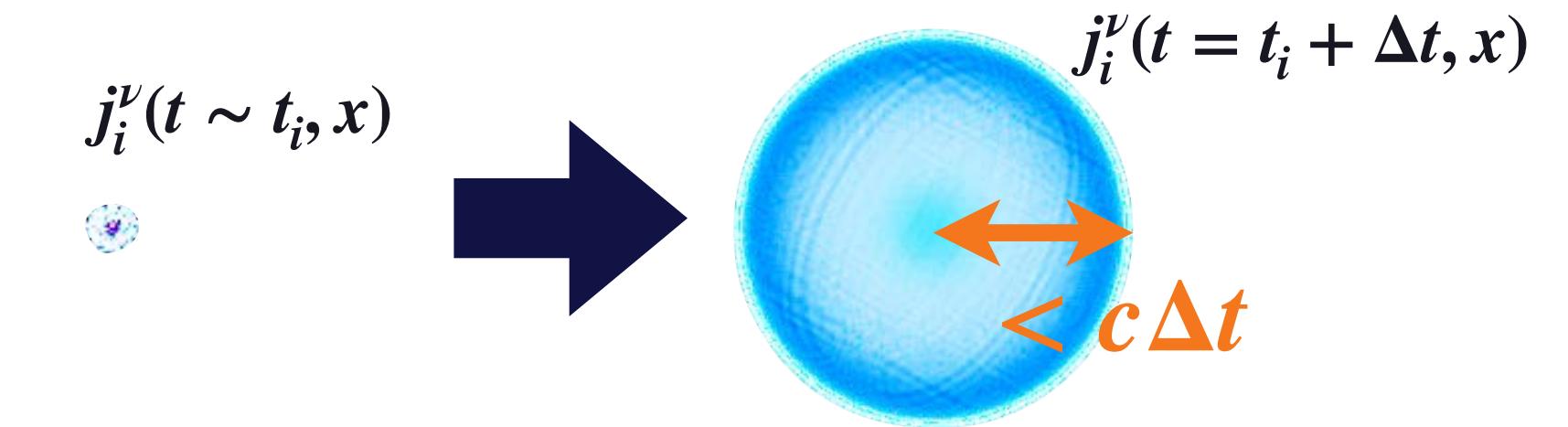
- Relativistic diffusion equation

$$\left[\frac{\partial}{\partial t} + \tau_{\text{diff}} \frac{\partial^2}{\partial t^2} - D_{\text{diff}} \nabla^2 \right] j^\nu(x) = 0$$

with initial condition*

$$j^\nu(t = t_{\text{dep}}, \vec{x}) = p_{\text{dep.}}^\nu \delta^{(3)}(\vec{x} - \vec{x}_{\text{dep}})$$

*massless particle-like dispersion relation



Parameters

E_{th} : Energy scale for in-medium thermalization

Δt : Timescale for in-medium thermalization

D_{diff} : Diffusion coefficient

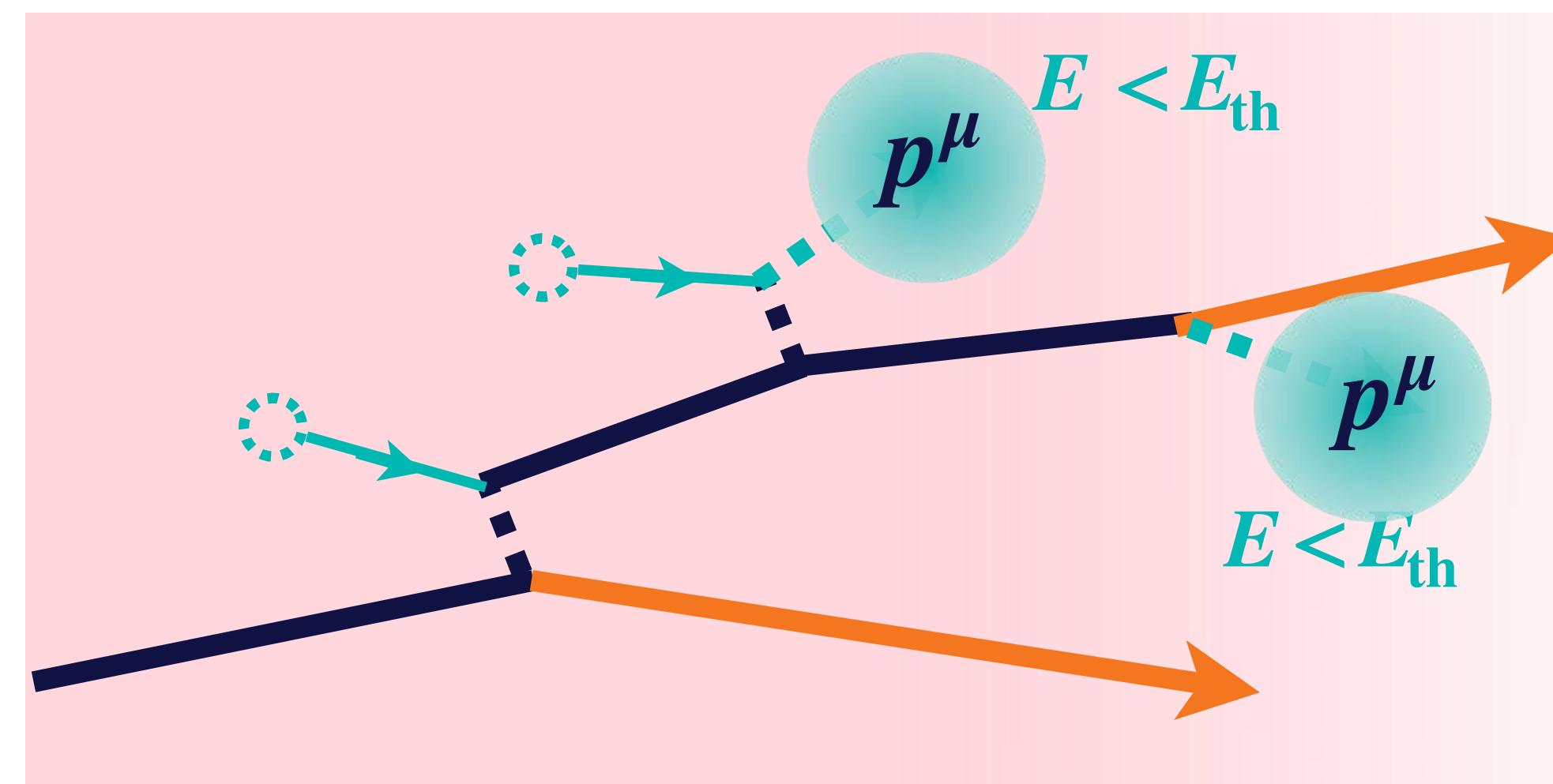
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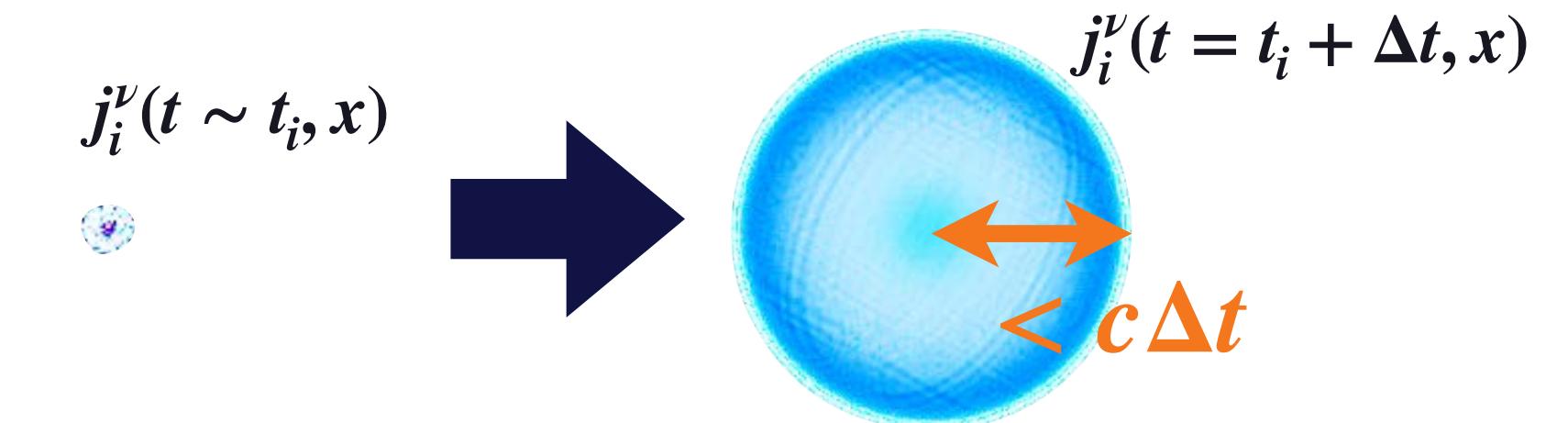
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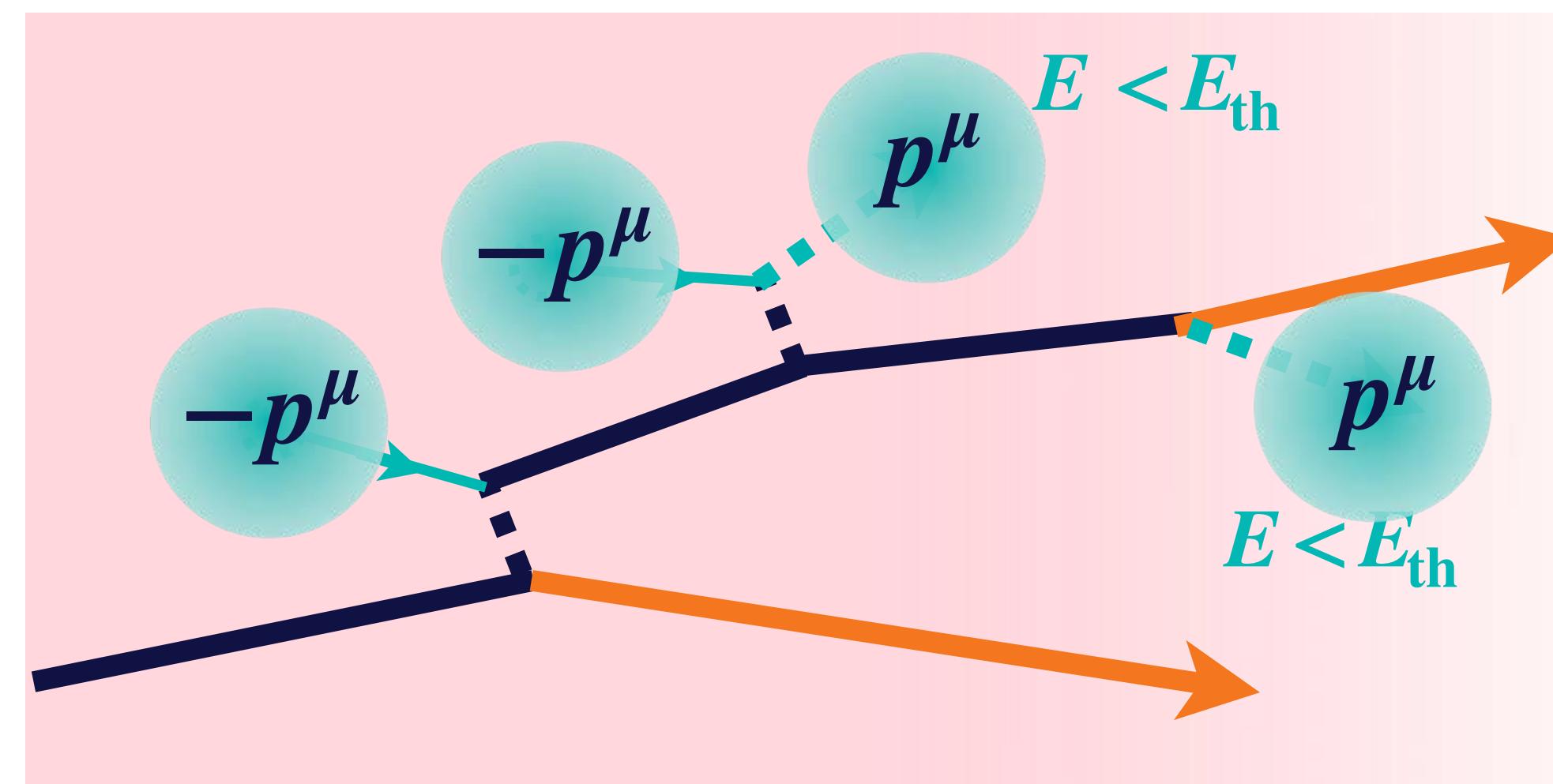
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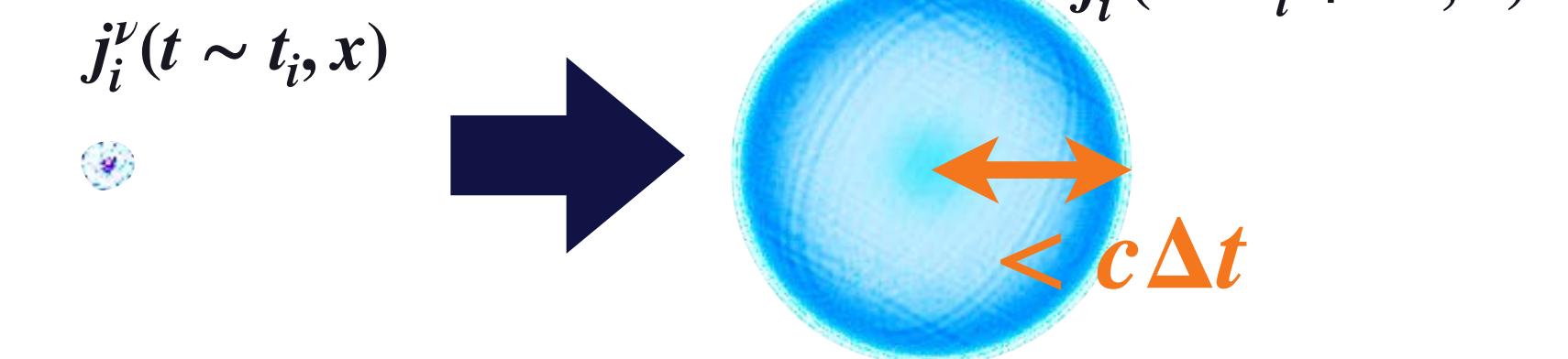
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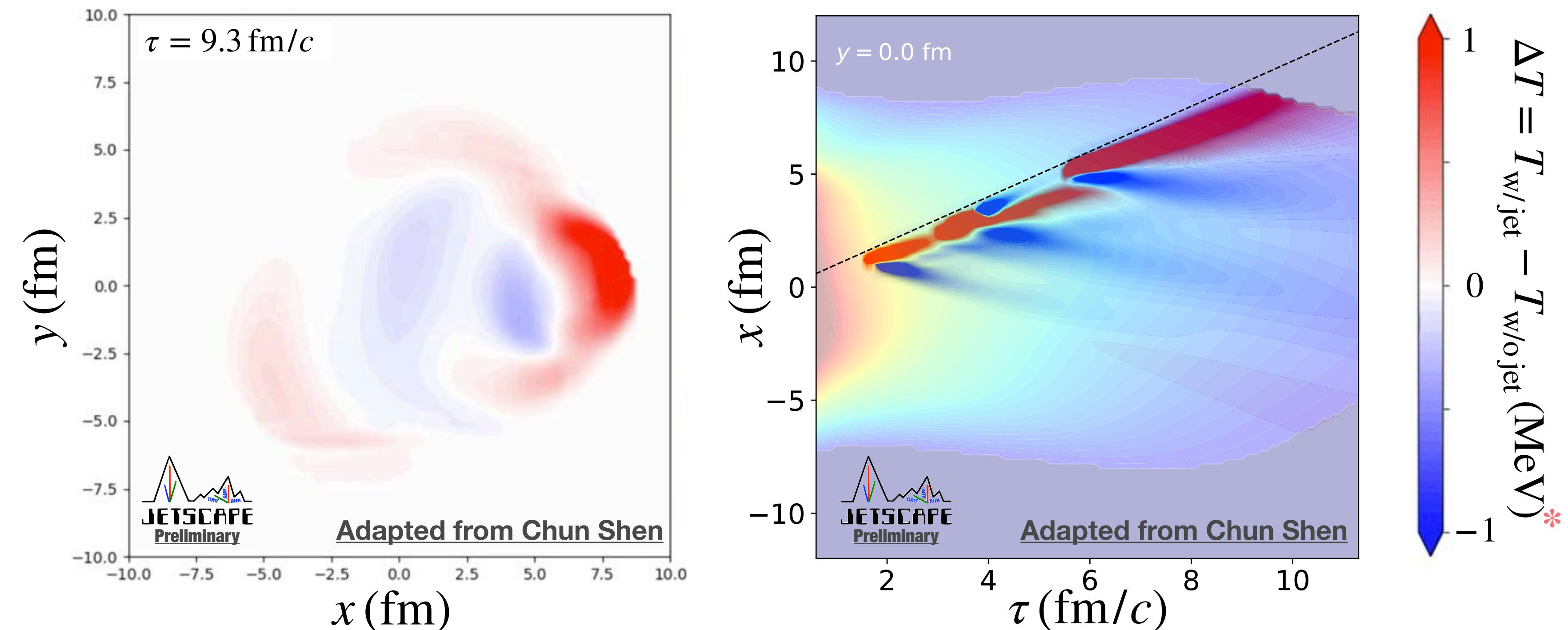
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Evolution of Hydrodynamic Medium Response

JETSCAPE [MATTER+LBT (Recoil ON) + Causal Diffusion + MUSIC Viscous Hydro]

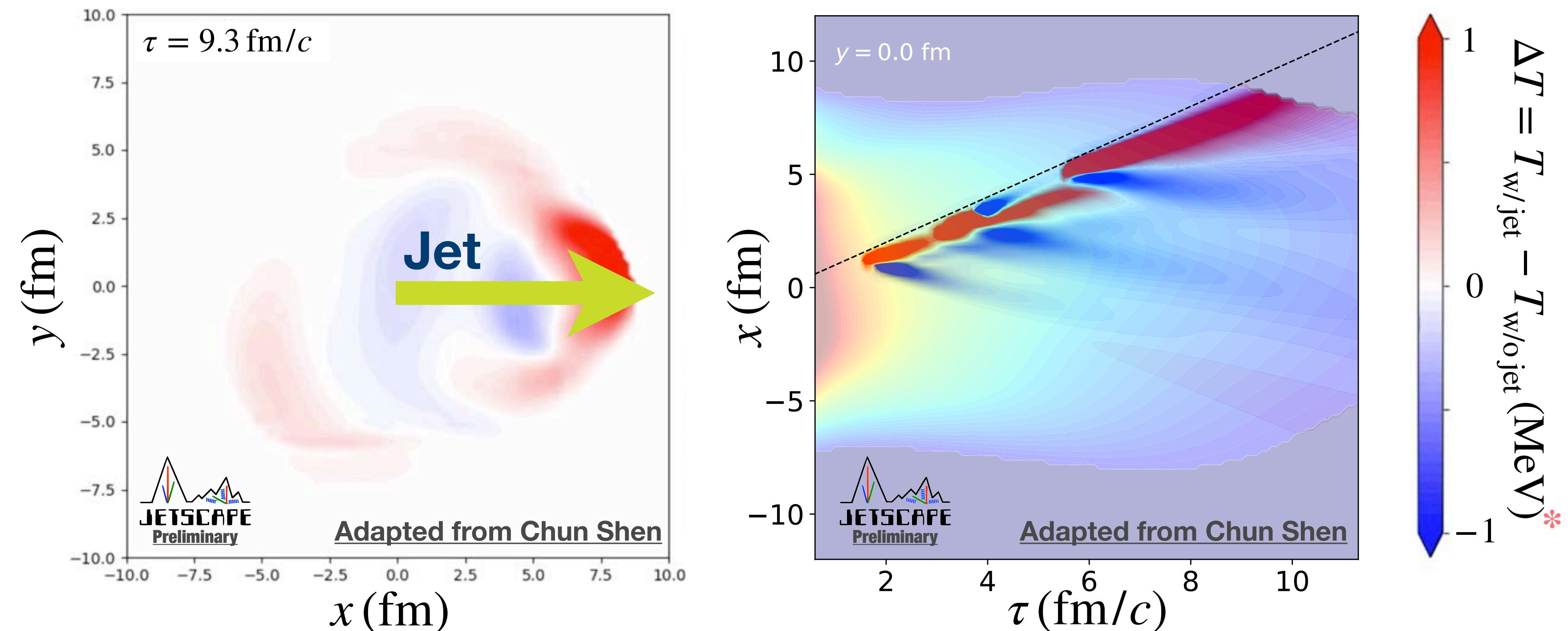


*These are results from a test simulation with an unrealistic configuration

- Jet following flow by energy-momentum deposition
- Diffusion wakes (negative contribution)

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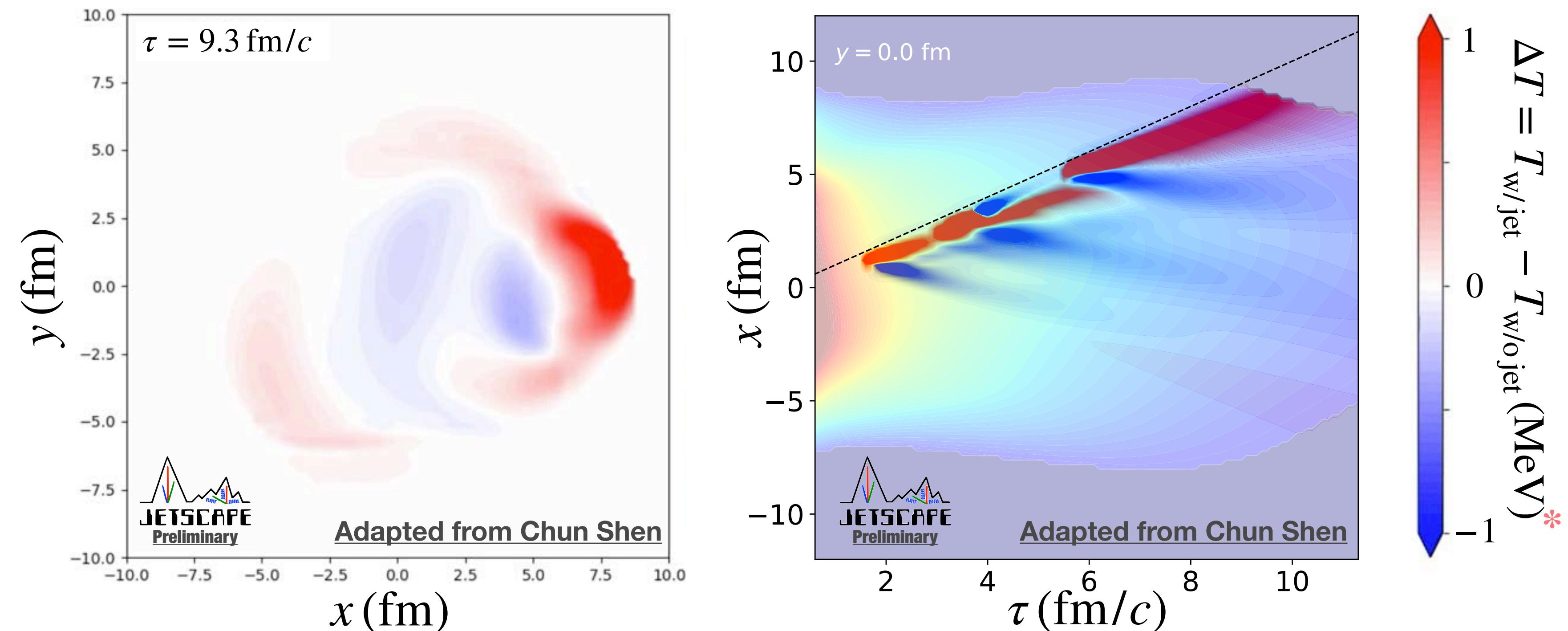


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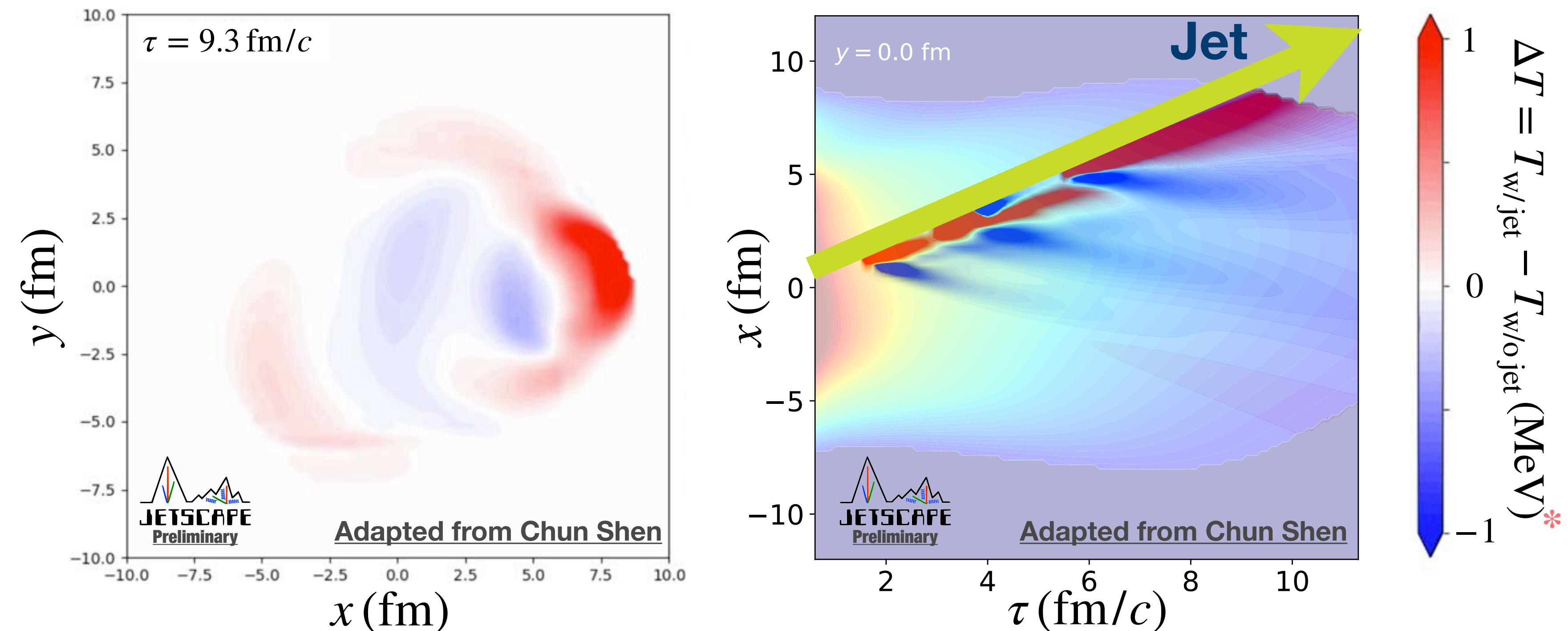


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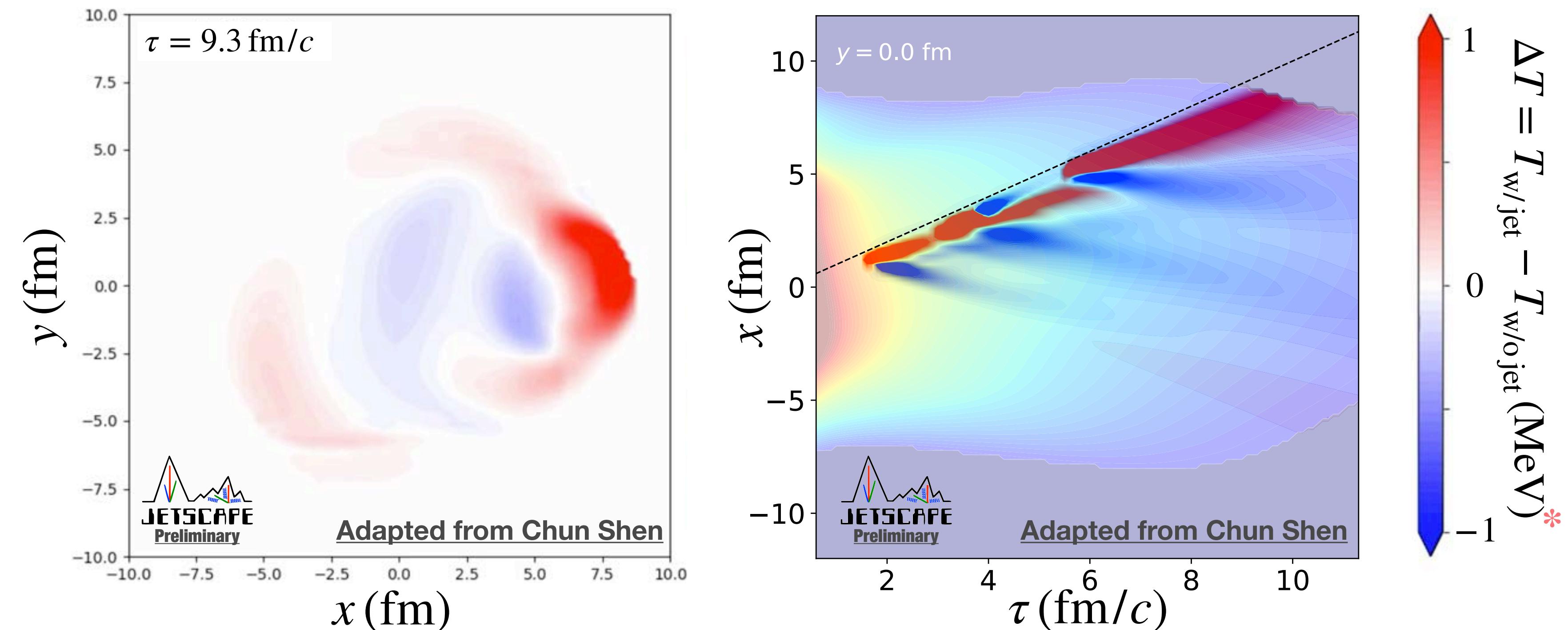


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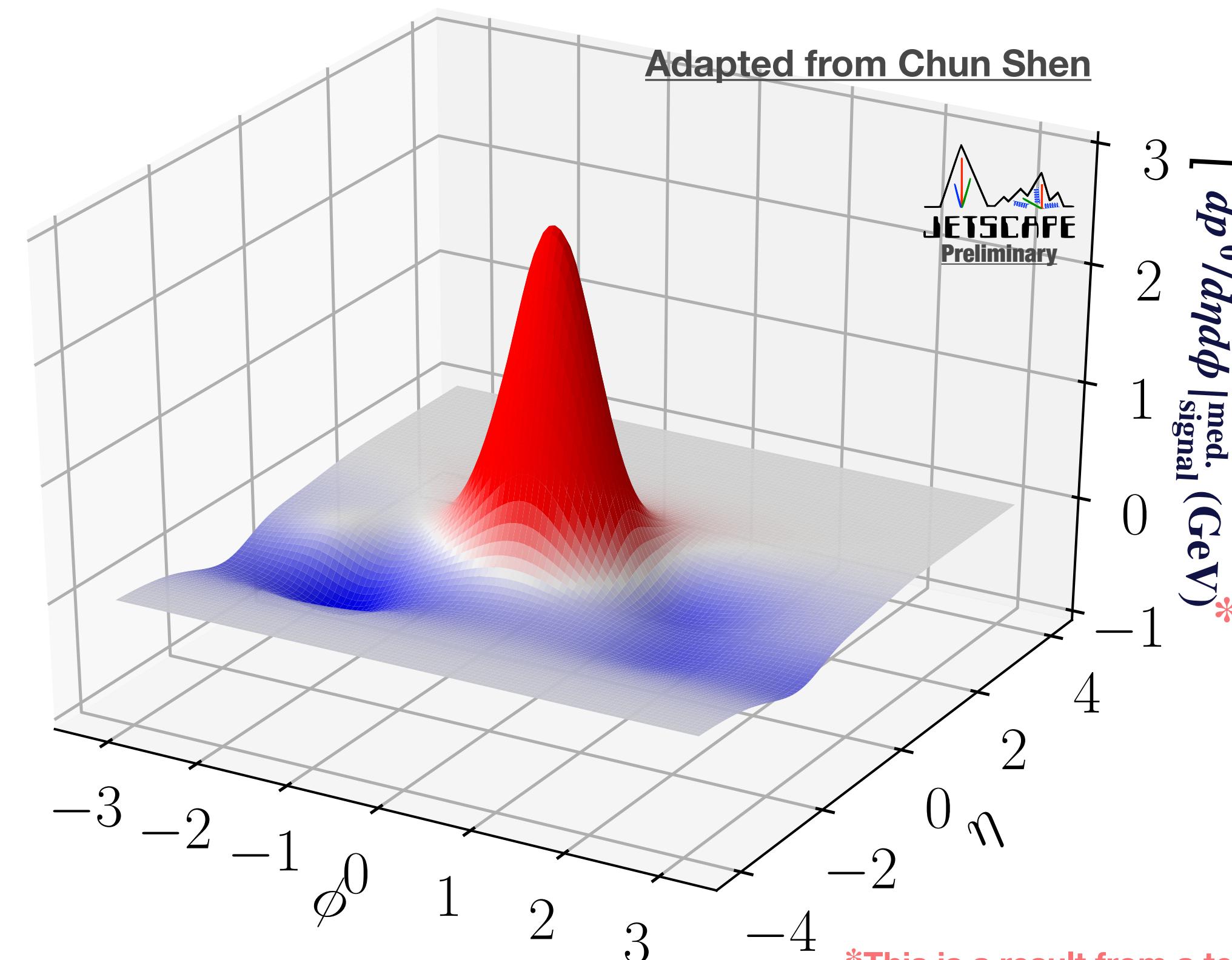
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Signal of hydro medium response in the final state

- Jet-modified spectrum of bulk medium (single event example)

$$\left. \frac{dp^\mu}{d\eta d\phi} \right|_{\text{signal}} = \left. \frac{dp^\mu}{d\eta d\phi} \right|_{\text{w/jet}} - \left. \frac{dp^\mu}{d\eta d\phi} \right|_{\text{w/o jet}}$$

JETSCAPE [MATTER+LBT (Recoil ON) + Causal Diffusion + MUSIC Viscous Hydro]



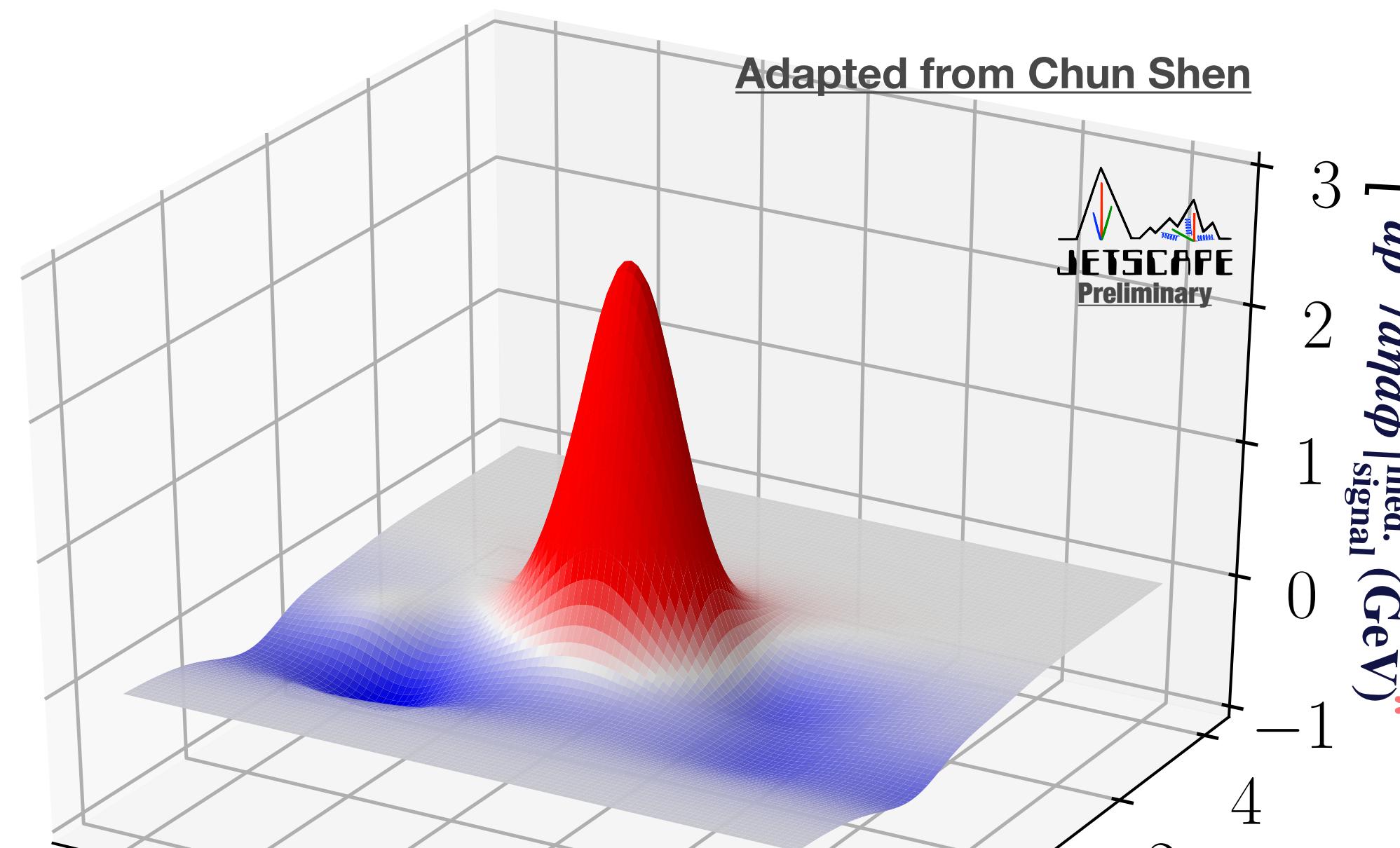
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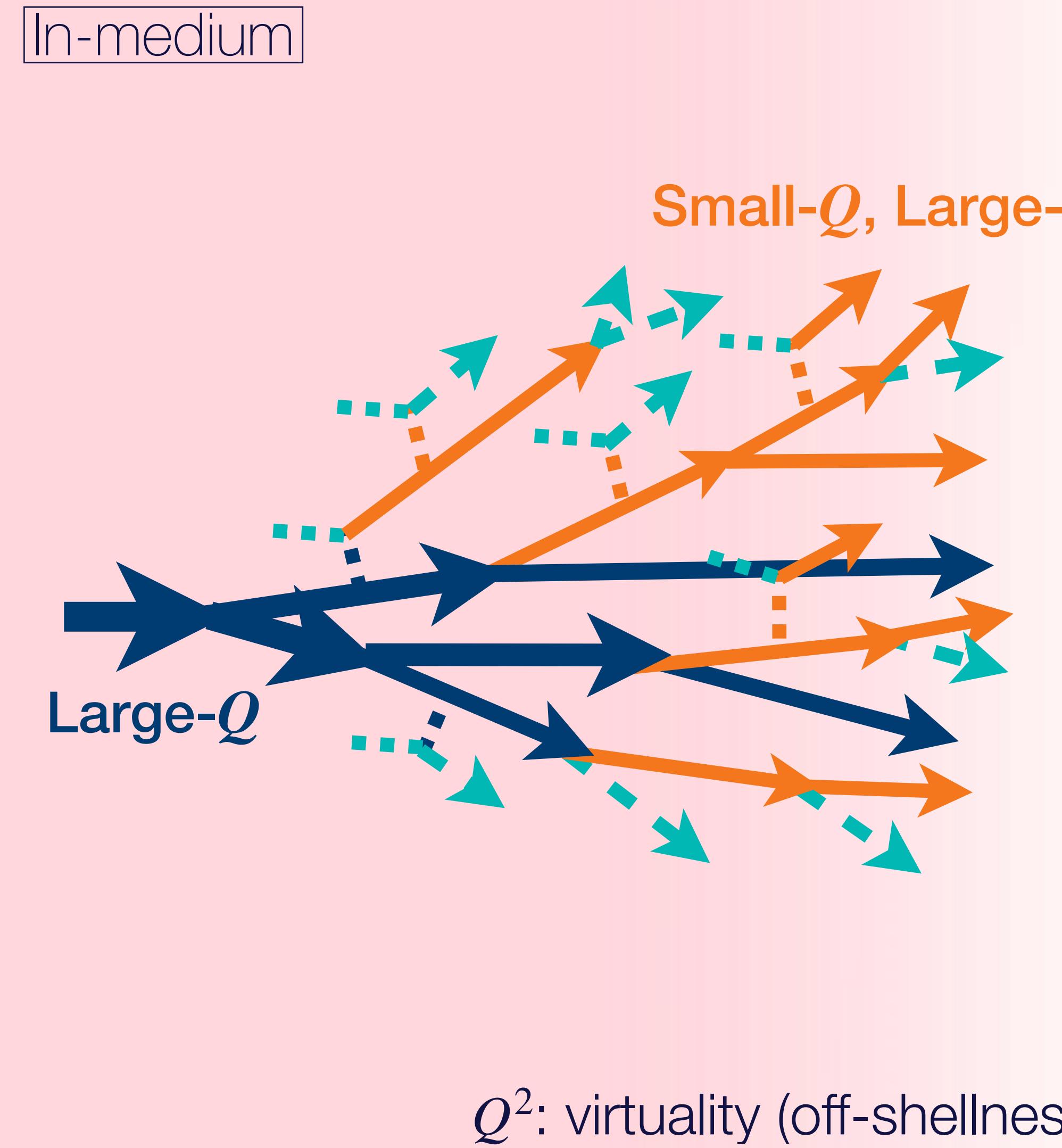
JETSCAPE [MATTER+LBT (Recoil ON) + Causal Diffusion + MUSIC Viscous Hydro]



- Jet-correlated structure in hadron emission from the bulk medium
- Negative contribution by medium response

This is a result from a test simulation with an unrealistic configuration

Simulation with JETSCAPE (In-medium Jet Evolution)



Large- Q

Virtuality ordered splittings with small medium effect

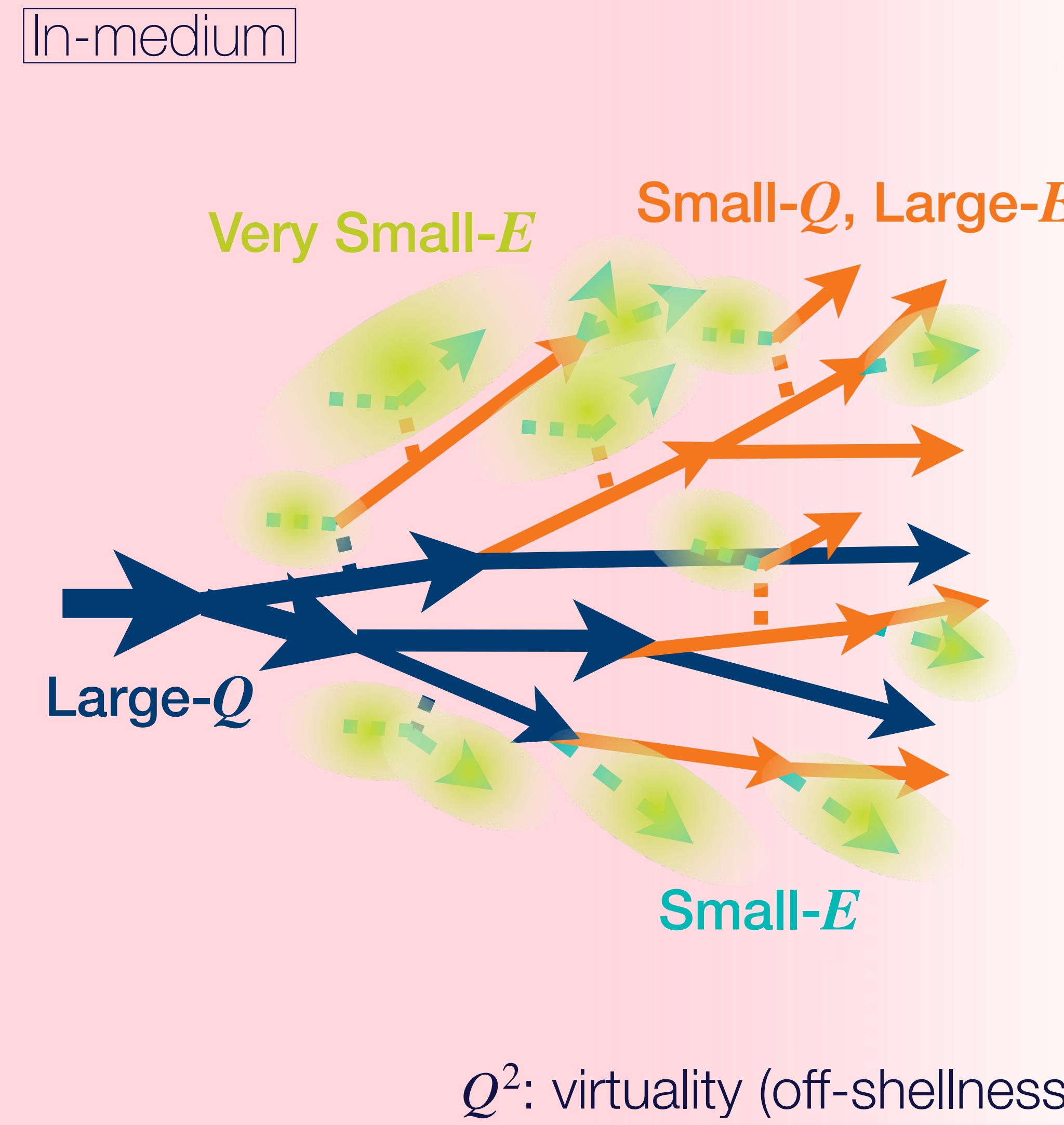
Model: Medium-modified Sudakov (**MATTER**)

Small- Q , Large- $E (> E_{\text{th}})$

Splittings driven by in-medium scatterings

Models: Kinetic Theory (**LBT**, **MARTINI**)

Simulation with JETSCAPE (In-medium Jet Evolution)



Large- Q

Virtuality ordered splittings with small medium effect

Model: Medium-modified Sudakov (**MATTER**)

Small- Q , Large- $E(> E_{\text{th}})$

Splittings driven by in-medium scatterings

Models: Kinetic Theory (**LBT**, **MARTINI**)

Small- $E(\leq E_{\text{th}})$

Energy-momentum diffusion into medium

Model: Causal Diffusion (**Causal Liquefier**)

Very Small- $E(\sim E_{\text{med}})$

Hydrodynamical evolution with bulk medium

Model: Hydrodynamics (**MUSIC**)

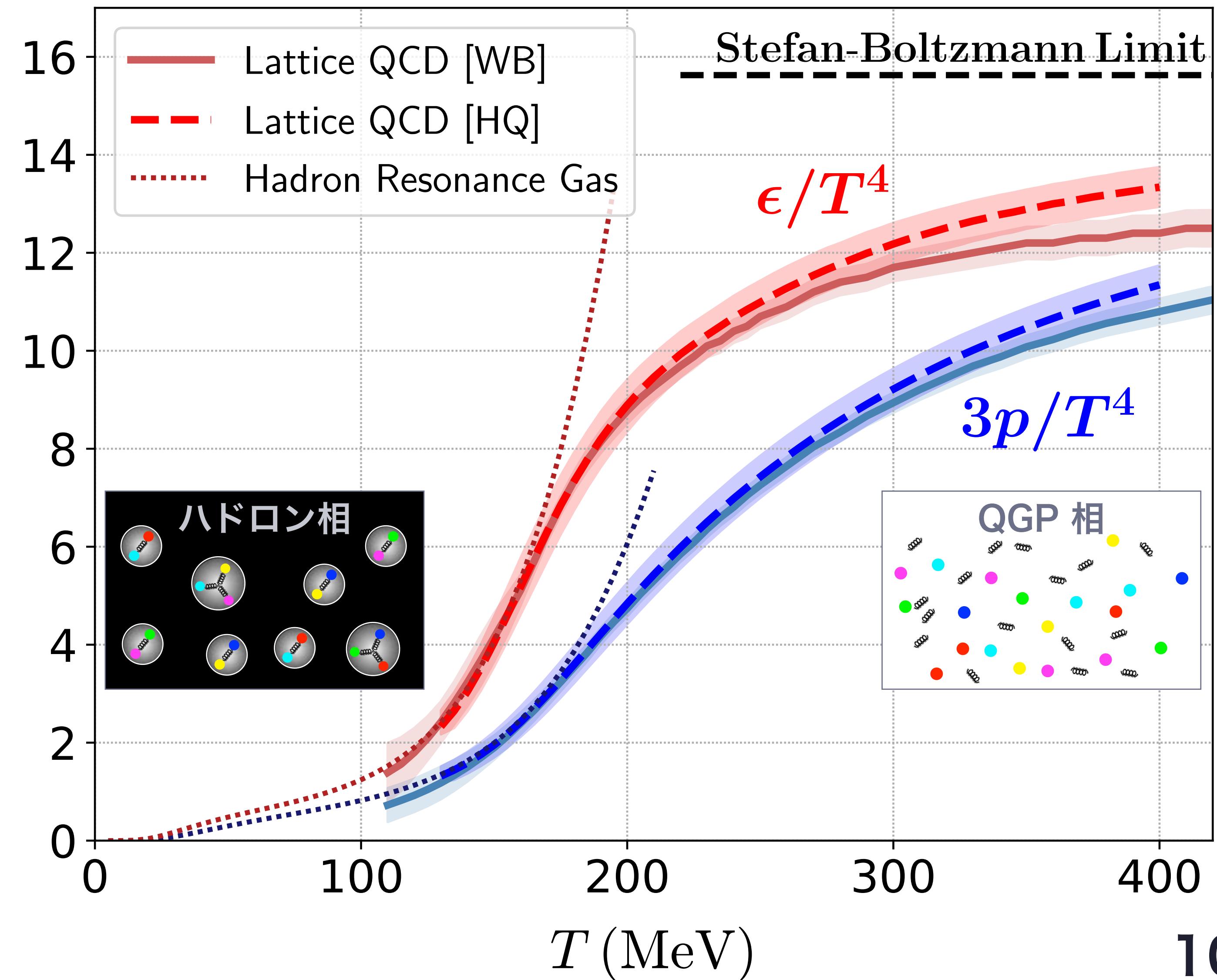
背景: クオーケ・グルーオン・プラズマ (QGP)

図: <https://github.com/MasakiyoK/Saizensen.git>

格子計算 (2+1 flavors): WB, PLB 730 (2014), HQ, PRD 90, 094503 (2014)

● クオーケとグルーオン

- ハドロンを形成、単独の観測不可
(カラー閉じ込め)
- 超高温では非閉じ込め示唆
→ QGP の実現

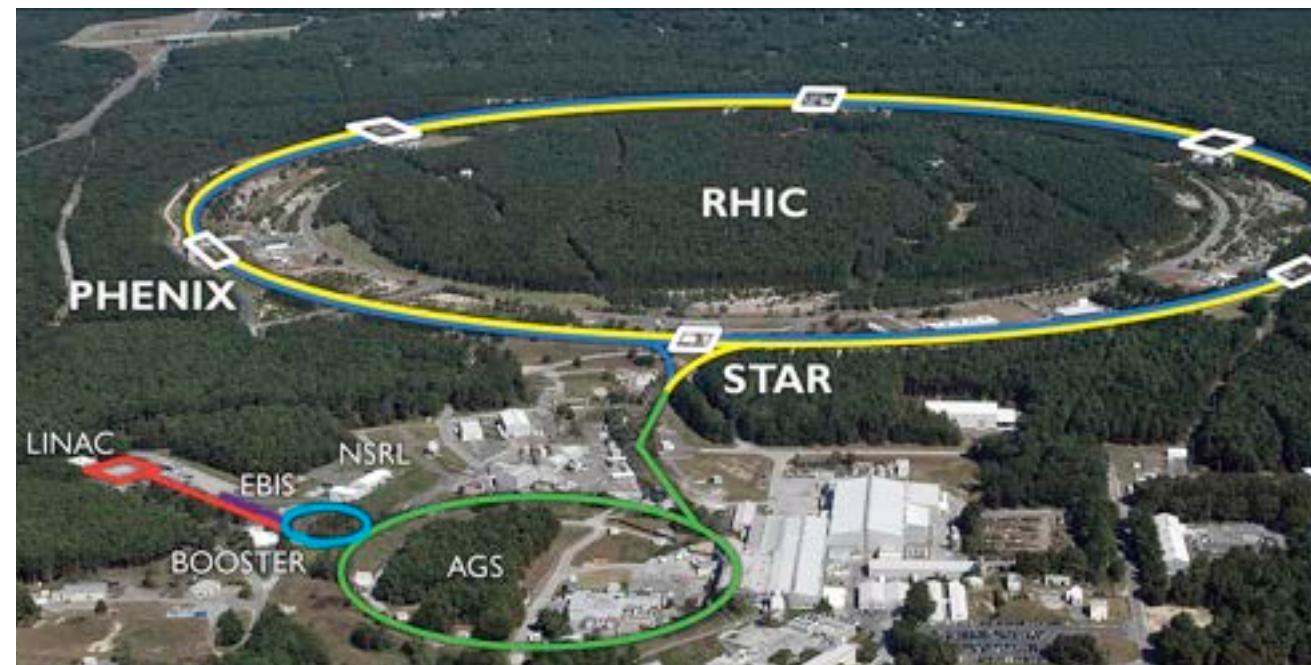


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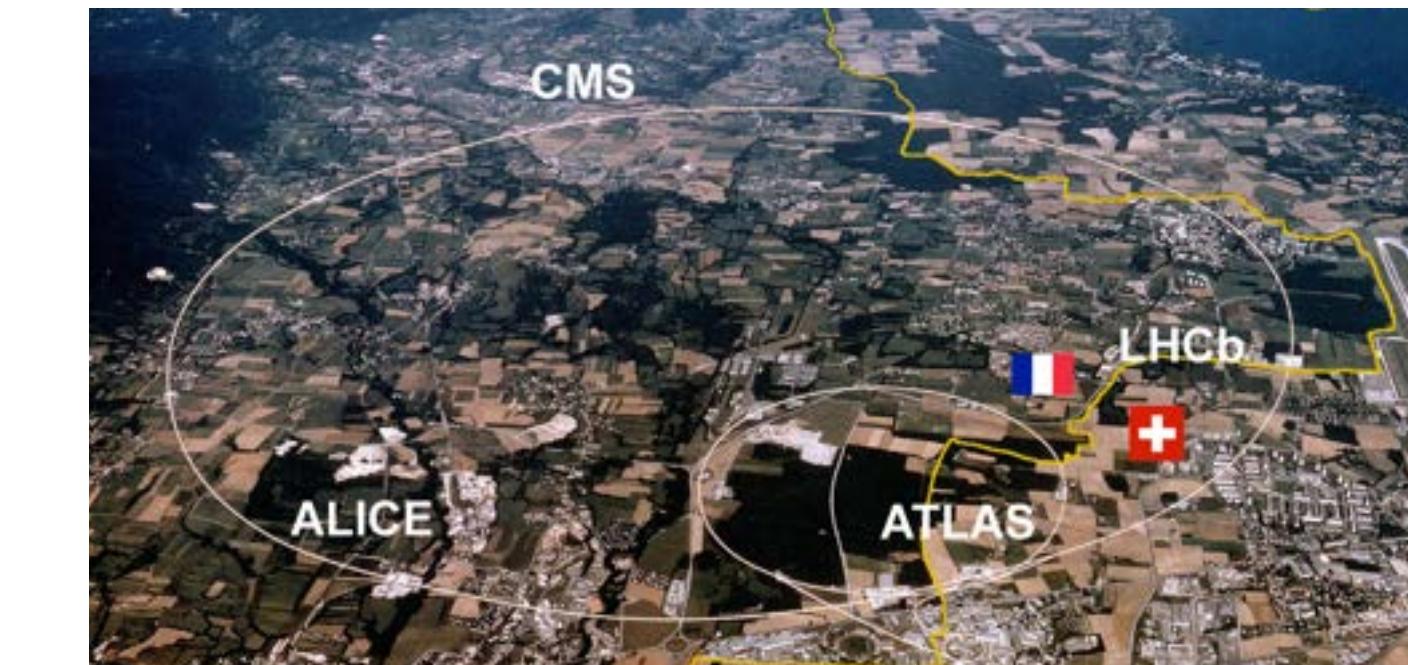
● QGP の実現

- ビッグバン直後~ $10\mu\text{秒}$ の高温初期宇宙
- 大型加速器での高エネルギー重イオン衝突実験

RHIC@BNL



LHC@CERN



● QGP 特性

- QCD を基本法則とした高温多体系の理解
- 再加熱過程など初期宇宙のダイナミクスの解明
→ 温度発展, 暗黒物質やバリオン非対称性生成への影響

背景: 高エネルギー重イオン衝突実験

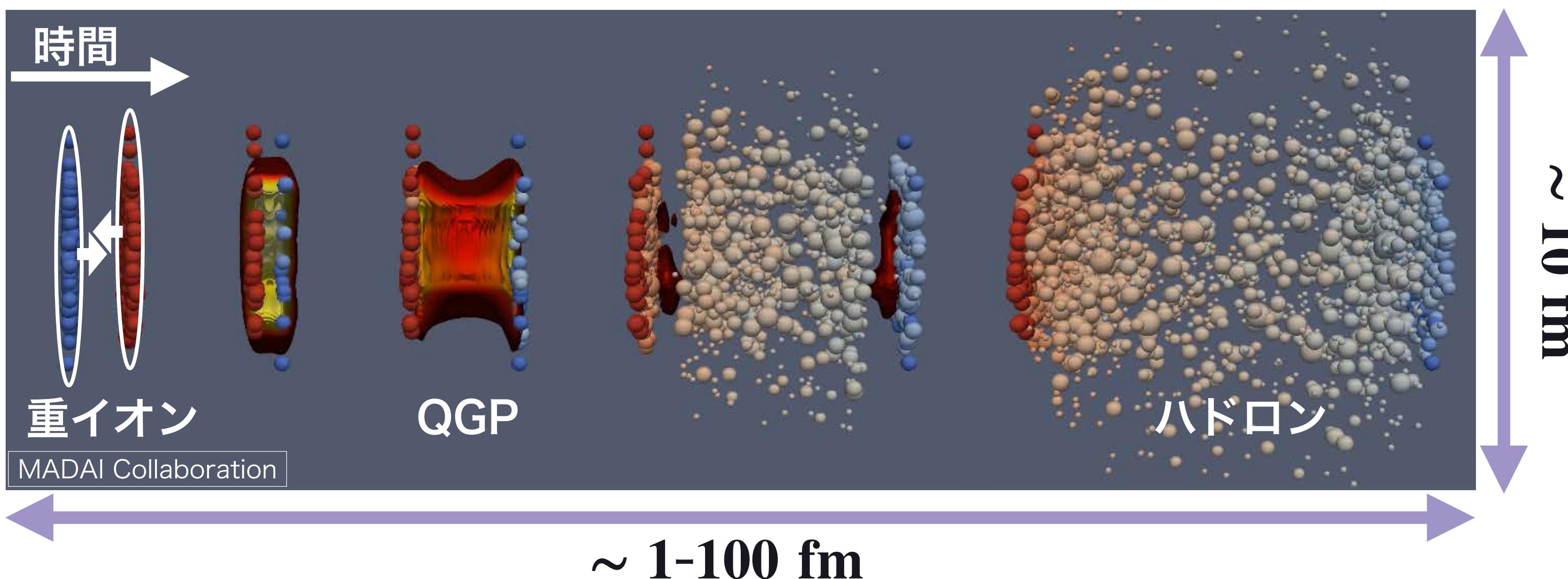
- ほぼ光速まで加速させた重イオン同士を衝突

CERN LHC: 鉛-鉛衝突, 最大 5.02 TeV*

BNL RHIC: 金-金衝突, 最大 200 GeV*

*核子-核子衝突あたりのエネルギー

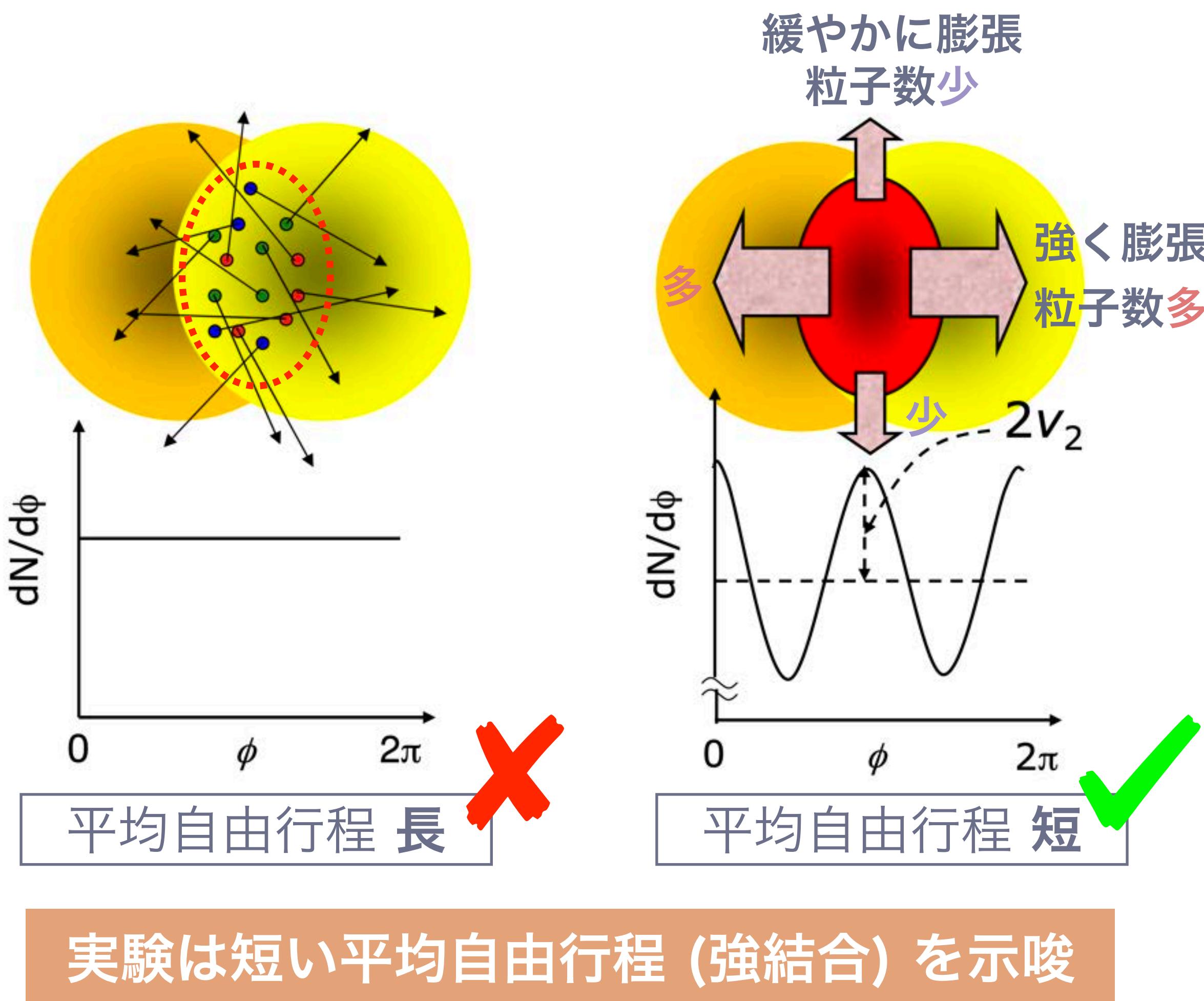
- 超高温を実現 → QGP を生成



背景: QGP 流体

● 粒子数の方位角異方性 (橍円フロー)

図: T. Hirano, et al, Lect.Notes Phys. 785 (2010)

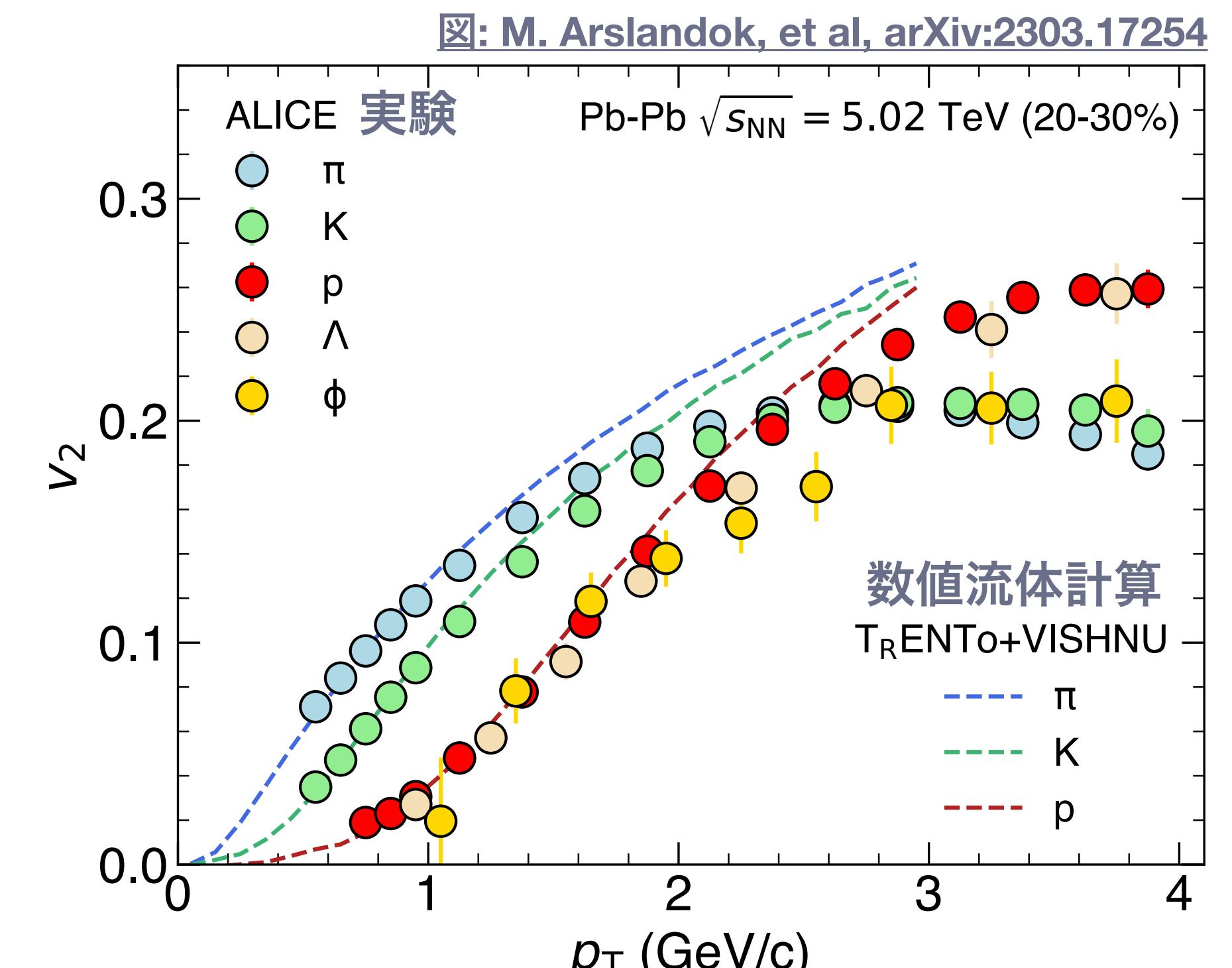


● 相対論的流体模型

- 相対論的流体方程式で膨張を記述

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad (+ \text{ 格子QCD EoS})$$

($T^{\mu\nu}$: エネルギー運動量テンソル)



背景: ジェット-QGP 相互作用

- ジェット

- ハード (大運動量移行 $-q^2$) 散乱で生成するパートンシャワー
- ハドロンのクラスタとして測定
- 摂動計算が比較的有効

