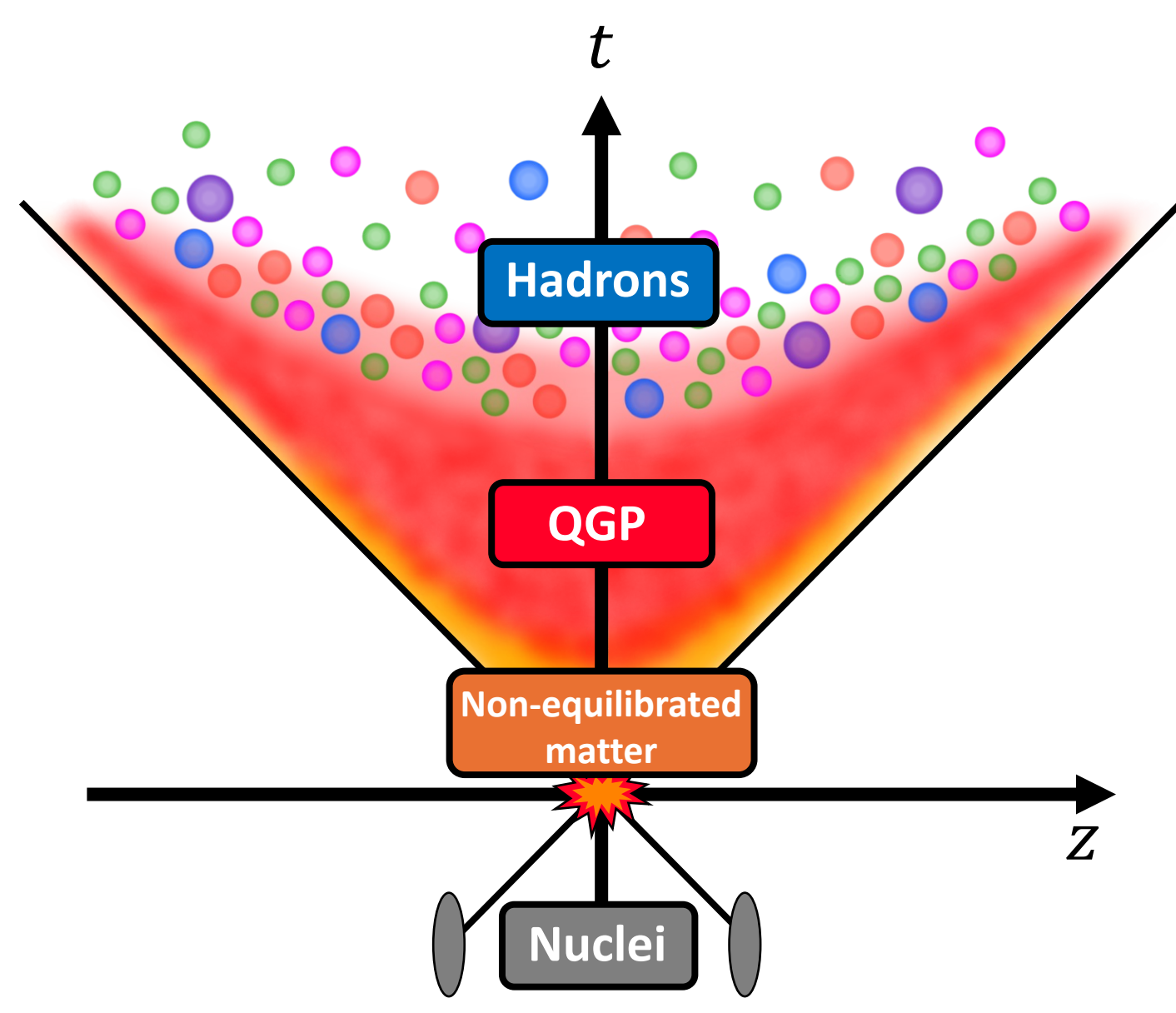


Effects of hydrodynamic fluctuations on 2-particle correlations in an expanding system

1. Introduction



Press release

Discovery of QGP's perfect fluid behavior

2005/04/18

<https://www.bnl.gov/newsroom/news.php?a=110303>

Standard description of space-time evolution by "Relativistic hydrodynamics"

Space-time evolution of Heavy Ion Collisions

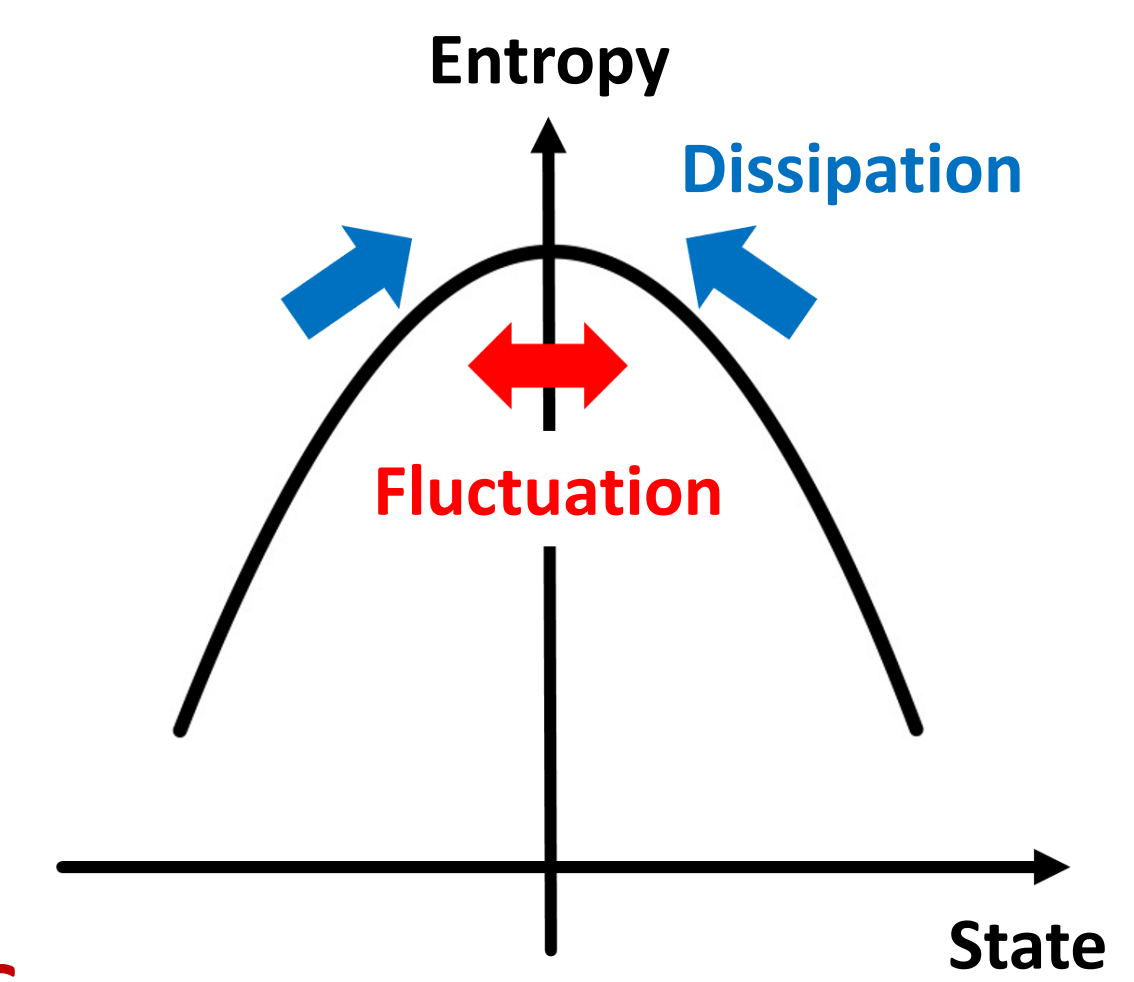
Fluctuation Dissipation Relation (FDR)

Interplay between dissipations (viscosities) and fluctuations

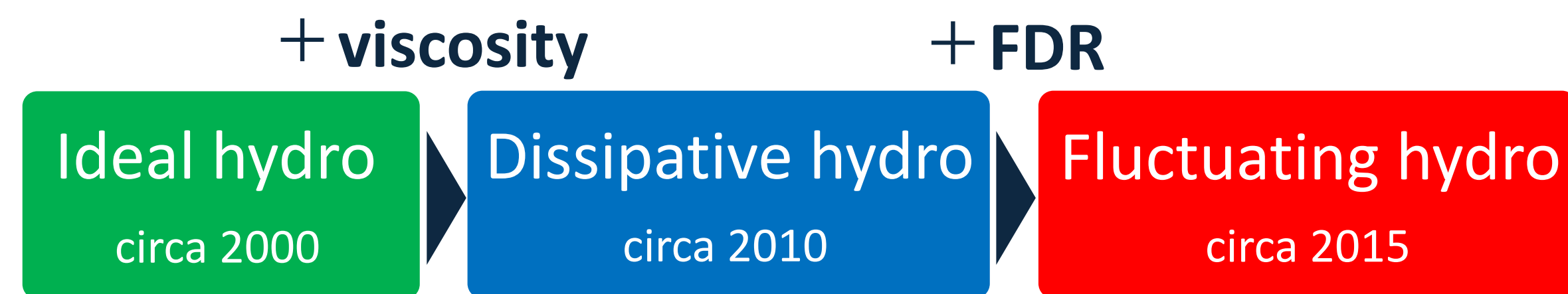
ex.) FDR for Shear stress tensor $\pi^{\mu\nu}$

$$\langle \xi_{\pi}^{\mu\nu}(x) \xi_{\pi}^{\alpha\beta}(x') \rangle = 4\eta T \delta^4(x-x') \Delta^{\mu\nu\alpha\beta}$$

$\xi_{\pi}^{\mu\nu}$: fluctuation η : viscosity



Evolution of relativistic hydrodynamic models



Hot Topic
Research on QGP properties using fluctuations

2. Formalisms

J. D. Bjorken, Phys. Rev. D 27, 140 (1983)

Perturbative expansion around the Bjorken's solution

$$u_{\text{Bj}}^{\mu} = (\cosh \eta_s, 0, 0, \sinh \eta_s) \quad \eta_s = \frac{1}{2} \ln \left(\frac{t+z}{t-z} \right): \text{coordinate rapidity}$$

Small deviations

$$u^{\mu} \rightarrow (\cosh(\eta_s + \delta y(\tau, \eta_s)), 0, 0, \sinh(\eta_s + \delta y(\tau, \eta_s)))$$

$$e \rightarrow e_0(\tau) + \delta e(\tau, \eta_s) \text{ etc.} \quad e: \text{energy density, } \tau = \sqrt{t^2 - z^2}: \text{proper time}$$

Energy-momentum conservation

Balance equation for Background (0th order perturbation)

Balance equation for Fluctuation (1st order perturbation)

Background

$$\frac{d}{d\tau} e_0 + \frac{1}{\tau} (w_0 + \Pi_0 - \pi_0) = 0 \quad (\text{Bjorken equation})$$

$w = e + p$: enthalpy density
 p : hydrostatic pressure
 $\pi \equiv \pi^{00} - \pi^{33}$: shear pressure
 Π : bulk pressure

Fluctuation

$$\frac{\partial}{\partial \tau} (\delta y (w_0 + \Pi_0 - \pi_0)) + \frac{1}{\tau} \frac{\partial}{\partial \eta_s} (\delta y (w_0 + \Pi_0 - \pi_0)) + \frac{1}{\tau} (\delta w + \delta \Pi - \delta \pi) = 0$$

Causal constitutive equations

W. Israel and J. M. Stewart, Annals Phys. 118, 341 (1979)

$$\text{Israel-Stewart equation + noise} \quad (1 + \tau_{\pi} D) \pi = \frac{4\eta}{3} \theta + \xi_{\pi}$$

$D \equiv u^{\mu} \partial_{\mu}$
 $\theta \equiv \partial_{\mu} u^{\mu}$
 τ_{π} : relaxation time
 ξ_{π} : noise

Perturbative expansion

Background

$$(1 + \tau_{\pi} \frac{d}{d\tau}) \pi_0 = \frac{4\eta_0}{3\tau}$$

Fluctuation

$$(1 + \tau_{\pi} \frac{\partial}{\partial \tau}) \delta \pi = -\frac{\delta \tau_{\pi}}{\tau_{\pi}} \left(\frac{4\eta_0}{3\tau} - \pi_0 \right) + \frac{4\eta_0}{3\tau} \frac{\partial}{\partial \eta_s} \delta y + \frac{4\delta \eta}{3\tau} + \xi_{\pi}$$

Fluctuations from FDR

$$\langle \xi_{\pi}(\tau, \eta_s) \xi_{\pi}(\tau', \eta_s') \rangle = \frac{8\eta_0 T_0}{3\tau \Delta x \Delta y} \frac{G(\eta_s - \eta_s') \delta(\tau - \tau')}{\text{smeared by gaussian}}$$

η_0 : shear viscosity (background)
 T_0 : temperature (background)
 $\Delta x = \Delta y = 2 \text{ fm}$, $\langle \xi_{\pi}(\tau, \eta_s) \rangle = 0$, $\sigma_{\eta} = 0.5$
 $G(\eta_s - \eta_s') = \frac{1}{\sqrt{2\pi\sigma_{\eta}^2}} \exp \left[-\frac{(\eta_s - \eta_s')^2}{2\sigma_{\eta}^2} \right]$

Particle spectra

$$E \frac{dN}{d^3p} = \frac{d}{(2\pi)^3} \int f(p^{\mu} u_{\mu}, T) p_{\mu} d\sigma^{\mu} \rightarrow \frac{dN}{dY} = \frac{d\tau A}{(2\pi)^2} \int_{-\infty}^{\infty} d\eta_s \int_m^{\infty} dm_{\tau} m_{\tau}^2 \cosh(Y - \eta_s) f$$

F. Cooper and G. Frye, Phys. Rev. D 10, 186 (1974)

d : degrees of freedom, A : transverse area

Distribution function

$$f_{\text{ideal}}(p^{\mu} u_{\mu}, T) = \exp \left(-\frac{p^{\mu} u_{\mu}}{T} \right) = \exp \left(-\frac{m_{\tau} \cosh(Y - \eta_s - \delta y)}{T_0 + \delta T} \right)$$

including fluctuations

Viscous correction

$$f = f_{\text{ideal}} + f_{\text{vis}}$$

1st vis.

Perturbative expansion

$$f = f_{0,\text{ideal}} + f_{0,\text{vis}} + \delta f_{\text{ideal}} + \delta f_{\text{vis}}$$

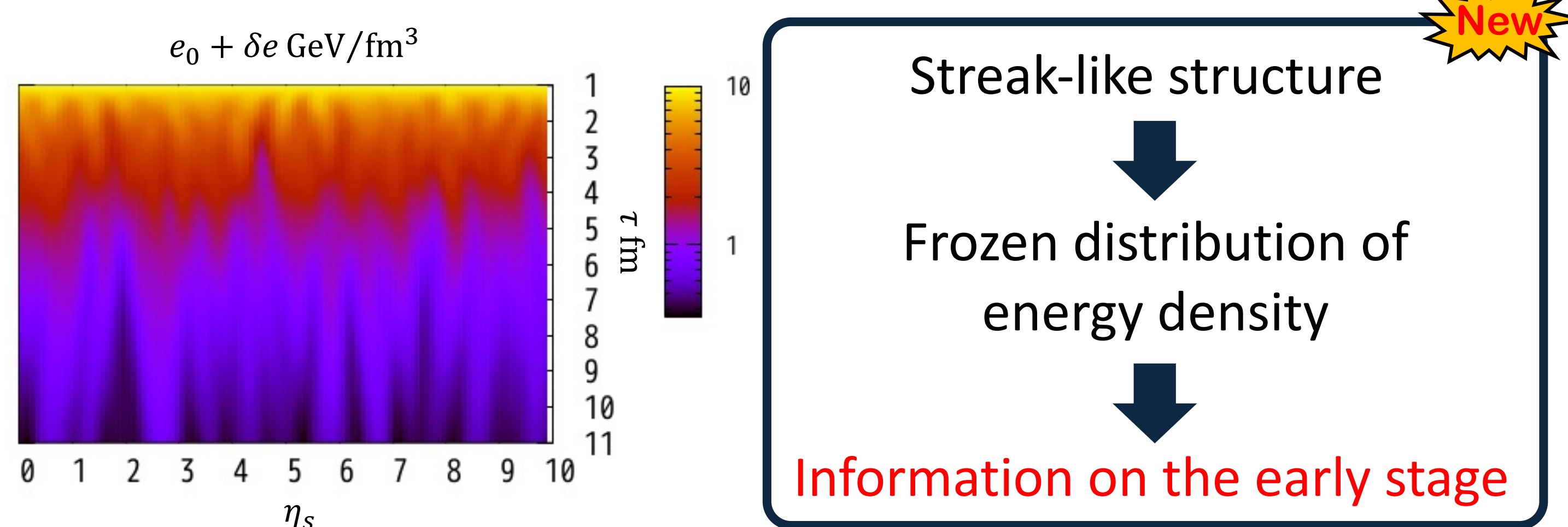
1st per. 1st vis., 1st per.

3. Results

Lattice EoS: A. Bazavov et al., Phys. Rev. D 90, 094503 (2014)

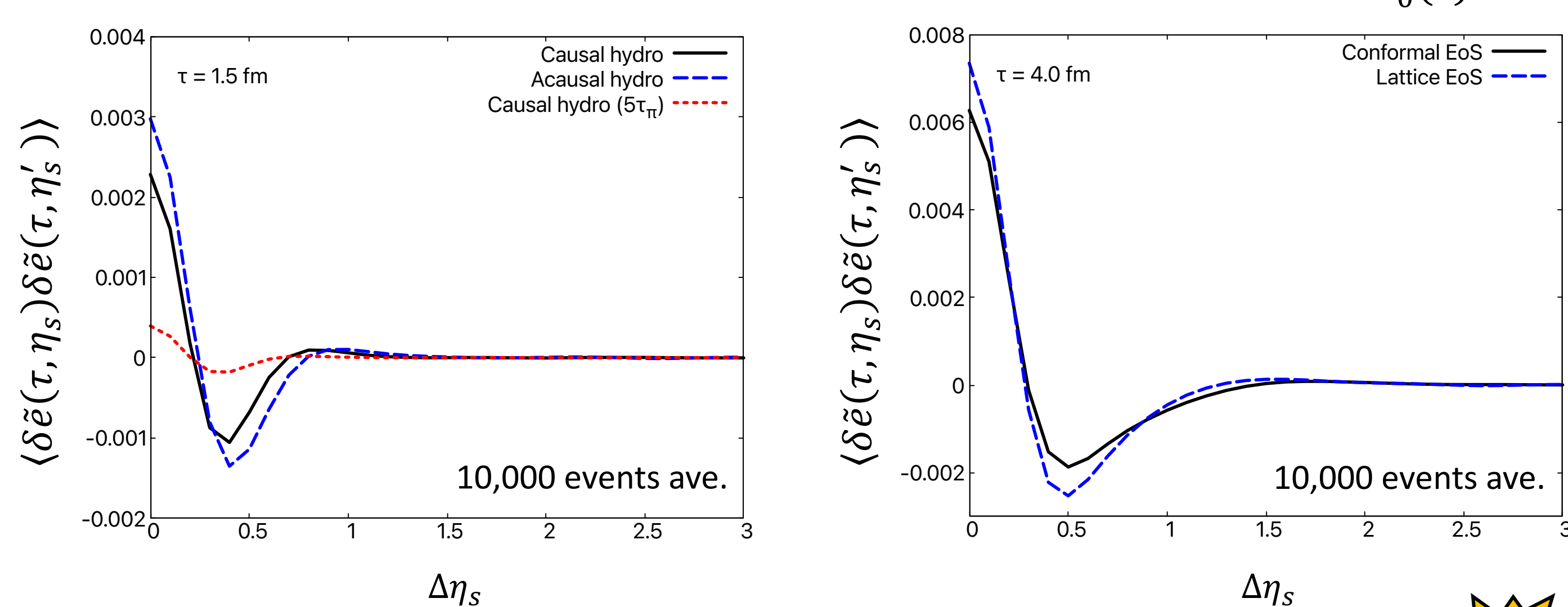
Space-time evolution of energy density

Initial conditions $\tau_0 = 1 \text{ fm}$, $e_0(\tau_0) = 10 \text{ GeV/fm}^3$, $\pi_0(\tau_0) = \frac{4\eta}{3\tau}$, $\delta e(\tau_0) = \delta y(\tau_0) = \delta \pi(\tau_0) = 0$



Correlations of energy density fluctuations

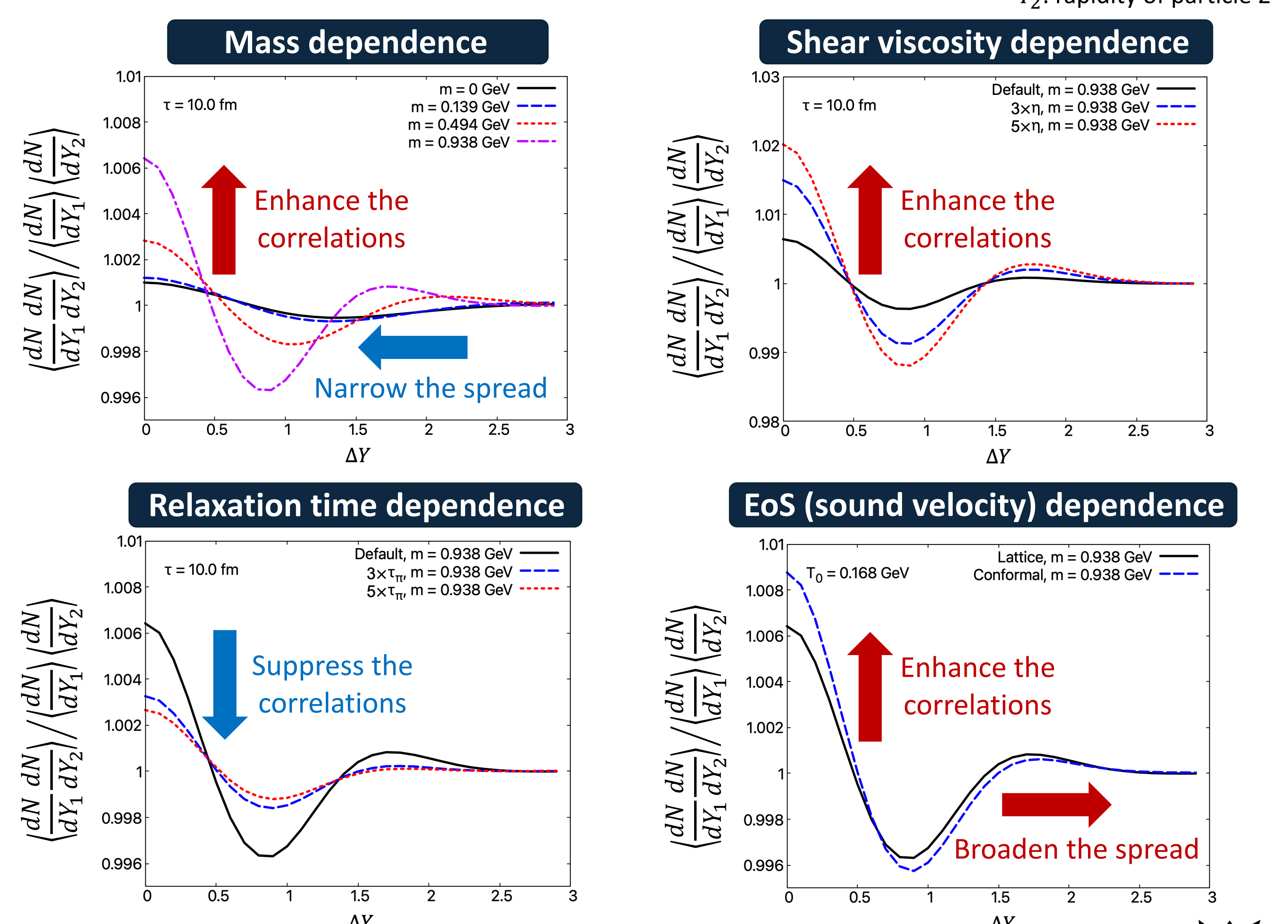
$$\delta \tilde{e}(\tau, \eta_s) \equiv \frac{\delta e(\tau, \eta_s)}{e_0(\tau)}$$



Effects of bulk and transport properties of the medium on energy density correlations

2-particle correlations

Y_1 : rapidity of particle 1
 Y_2 : rapidity of particle 2



- Heavier hadrons as better probes of correlations
- Opposite behavior between shear viscosity and relaxation time
- Effects of EoS (sound velocity) on correlations

Extract the properties of the medium from 2-particle correlations!

4. Summary

- We developed a framework which deals with causal hydrodynamic fluctuations in 1-dimensional expanding system.
- We observed a streak-like structure through the time evolution of energy density caused by a freeze of distribution.
- We found behaviors of correlations of thermodynamic variables and particles are closely related to the properties of the medium.