Hydrodynamic initial conditions from non-linear causality

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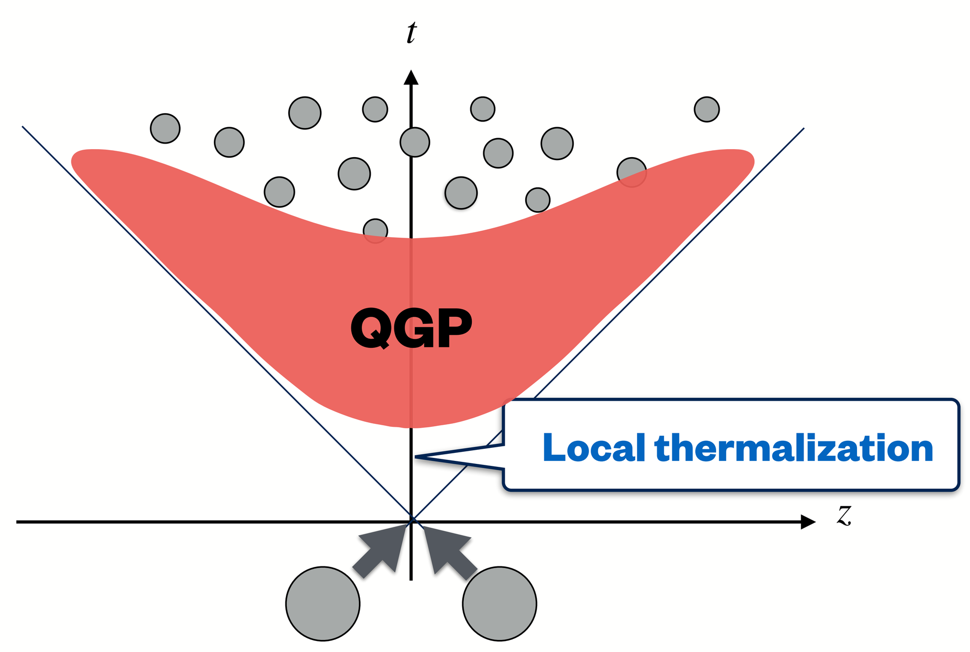
Abstract: It is not at all trivial at which stage after the first contact the fluid picture can be applied. Whether non-linear hydrodynamic equations obey the causality depends on how far the system is away from local thermal equilibrium. Thus, for the system to be causal, initial conditions must be close to the equilibrium state. In this study, we apply the conditions obtained from causality to the conformal theory in a one-dimensionally expanding system, analyze how far the system can be away from local thermal equilibrium and constrain initial conditions so that the system can obey causality during the evolution.

1. Introduction



Press release https://www.bnl.gov/newsroom/news.php?a=110303

Discovery of QGP's behavior like an iedal fluid

Standard description by using relativistic hydrodynamics

At which stage after the first contact can the fluid picture be applied?

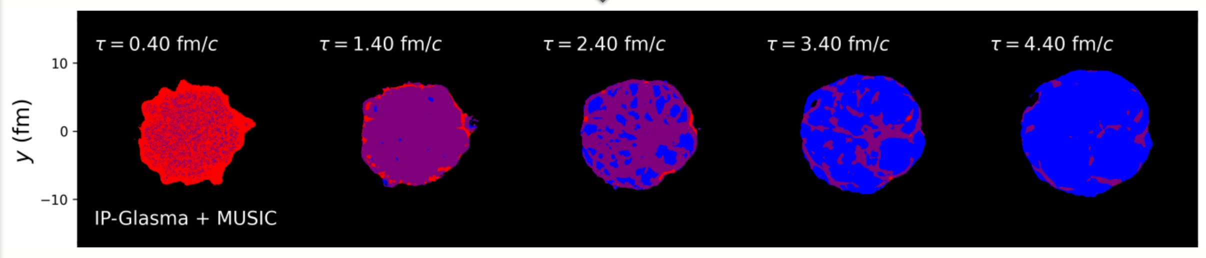
Necessary/sufficient conditions for causality in relativistic hydrodynamic equation

$$F_i(e, P, \Pi, \pi^{\mu\nu}, \ldots) \geq 0 \quad (i = 1, 2, \cdots n)$$

Equilibrium variables   Non-equilibrium variables

F.S. Bemfica *et al.*, Phys. Rev. Lett. **126**,222301 (2021).

Application to numerical simulation



Evolution in the transverse plane

Red: acausal ≈ Away from equilibrium
Blue: causal ≈ Close to equilibrium
Purple: indeterminate

C. Plumberg *et al.*, Phys. Rev. C **105**, L061901 (2022)

Violation of causality far away from equilibrium (?)

Purpose of this study Constrain initial conditions in a one-dimensionally expanding conformal system from a view point of causality

2. Model

Conditions from causality

Violate necessary condition: acausal
Satisfy sufficient condition: causal

Derivation of conditions for conformal and Bjorken system

An example of the necessary conditions:

$$e + P + \Pi - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}\Lambda_3 \geq 0$$

Λ_a : An eigenvalue of π^μ_ν

Local rest frame: $\pi^{\mu\nu}_{\mathrm{Bj,LRF}} = \mathrm{diag}\left(0, \frac{\phi}{2}, \frac{\phi}{2}, -\phi\right)$

$\phi = \pi^{00} - \pi^{33}$

$$\frac{\phi}{e+p} \leq \left(1 - \frac{\eta}{s}\frac{1}{\tau_\pi T}\right)\frac{4\tau_\pi}{\tau_{\pi\pi}}$$

$\tau_{\pi\pi}$: $\frac{1}{2\pi T}$ (BRSSS) のはず

NEW **Conditions for dissipative quantity** under the Bjorken expansion

Hydrodynamic model

Hydrodynamic equations under the **Bjorken** expansion

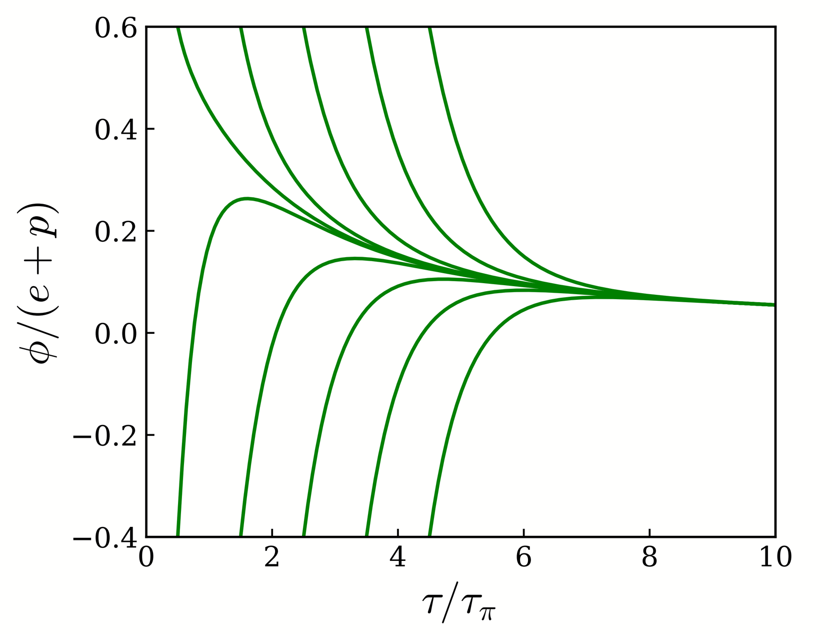
J. D. Bjorken, Phys. Rev. D 27, 140 (1983).

$$\frac{d}{d\tau}e = -\frac{e + p - \phi}{\tau}$$

$$\left(1 + \tau_\pi \frac{d}{d\tau}\right)\phi = \frac{4\eta}{3\tau} - \frac{4\tau_\pi}{3\tau}\phi - \frac{1}{2\eta}\frac{C_{\lambda_1}}{T}\phi^2$$

R. Baier *et al.*, JHEP 0804, 100 (2008).

e: energy density, $\phi = \pi^{00} - \pi^{33}$, $\tau = \sqrt{t^2 - z^2}$: proper time, T: temperature
τ_π: relaxation time, η: shear viscosity, C_{λ_1}: dimensionless constants



Inverse Reynolds number

R. Baier *et al.*, Phys. Rev. C **73** 064903 (2006)

$$Re_\pi^{-1} = \frac{|\phi|}{e+p}$$

3. Results

Transport coefficients from AdS/CFT
P. Kovtun *et al.*, Phys. Lett. 94, 11601 (2005).
R. Baier *et al.*, JHEP 0804, 100 (2008).

$$\tau_\pi T = \frac{2 - \ln 2}{2\pi}, \quad \frac{\eta}{s} = \frac{1}{4\pi}$$

Acausal :

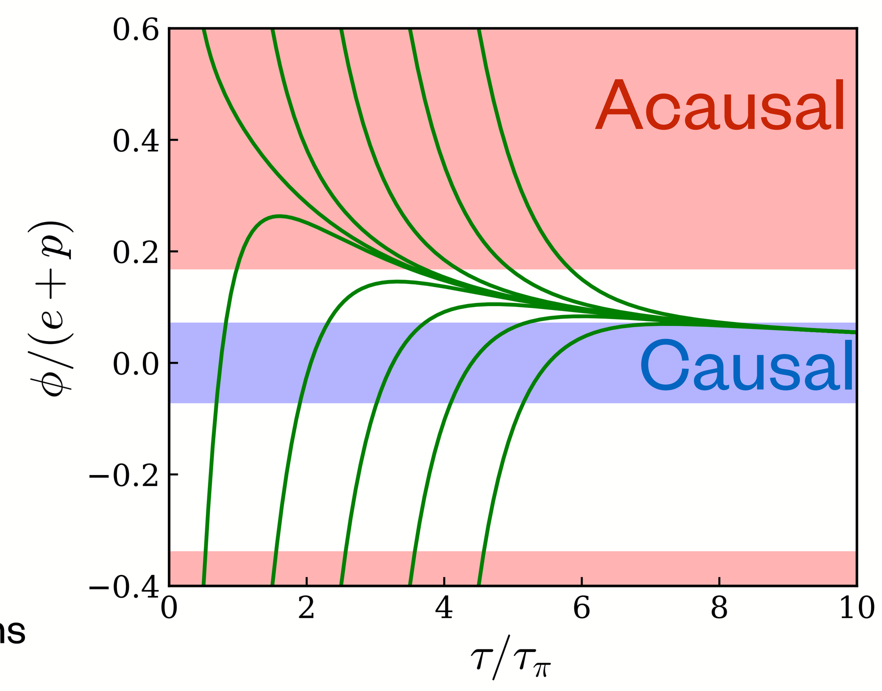
$$\frac{\phi}{e+p} < -0.34, \quad \frac{\phi}{e+p} > 0.17$$

Causal :

$$-0.07 \leq \frac{\phi}{e+p} \leq 0.07$$

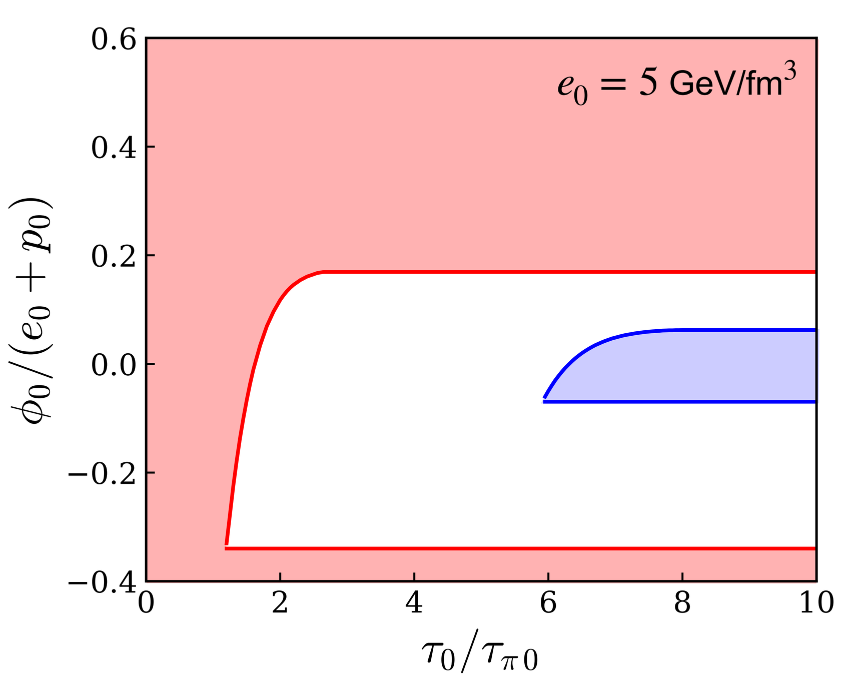
Others :

Not determined solely from the conditions

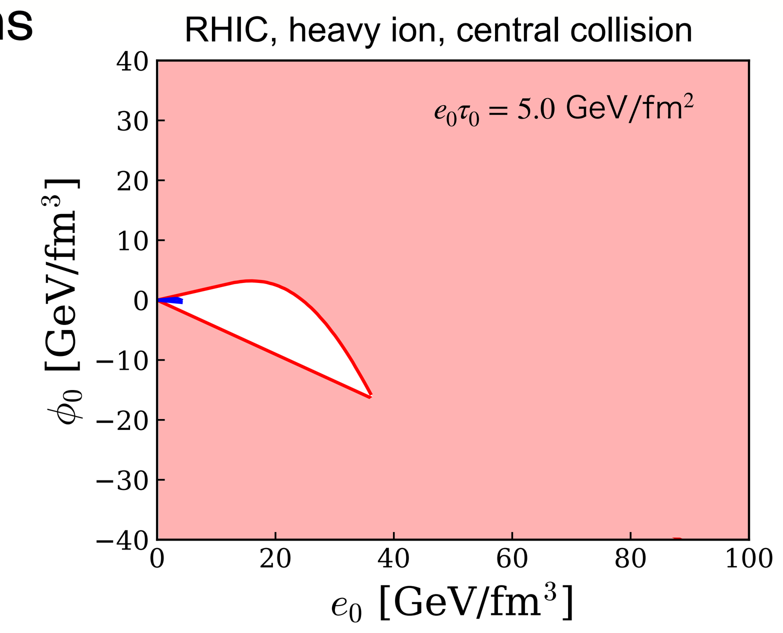


- **Acausal** for the large inverse Reynolds number
- Any solutions pass through the acausal area are **not** acceptable.

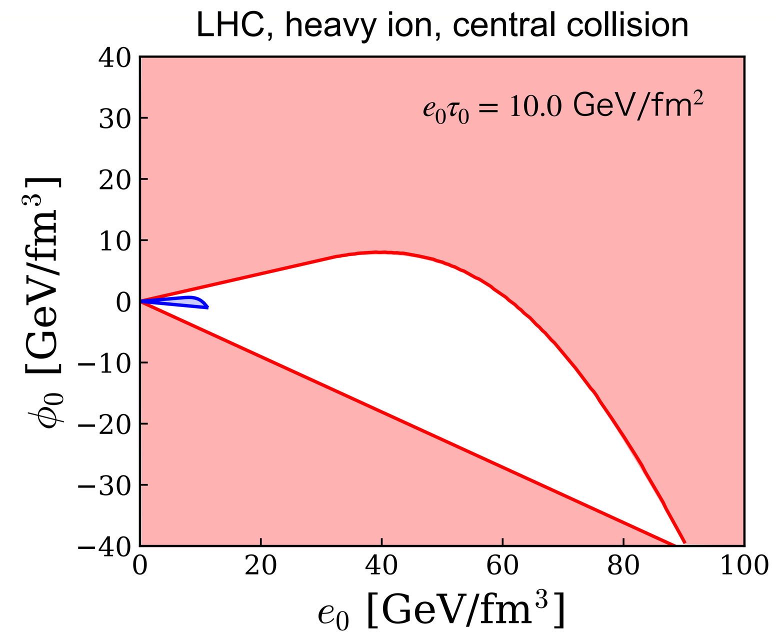
Available regions of the initial conditions



RHIC, heavy ion, central collision



LHC, heavy ion, central collision



Bjorken energy density

$$e_{\mathrm{Bj}} = \frac{dE_T}{dy}\frac{1}{\tau A}$$

J. D. Bjorken, Phys. Rev. D 27, 140 (1983).

4. Summary

- We analyzed how far the one-dimensionally expanding system can be away from local thermal equilibrium from the causality.
- We **constrained the initial condition** of thermodynamic and dissipative variables in conformal theory under Bjorken expansion.
  - Possibilities to pass through the acausal area even starting from local equilibrium state
  - Little room of initial conditions for the system to strictly obey the causality