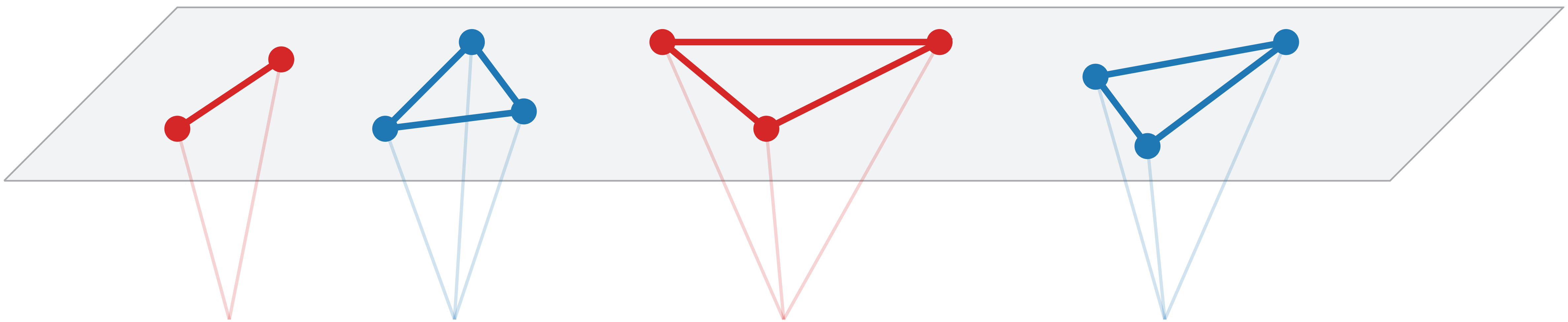
New Techniques for Cosmological Correlators



Going off-shell / Going non-local

Denis Werth

Cosmological Correlators in Taiwan
December 2nd 2024

Based on

DW [2024]
Jazayeri, Renaux-Petel, **DW** [2023]
Jazayeri, Renaux-Petel, Tong, **DW**, Zhu [2023]

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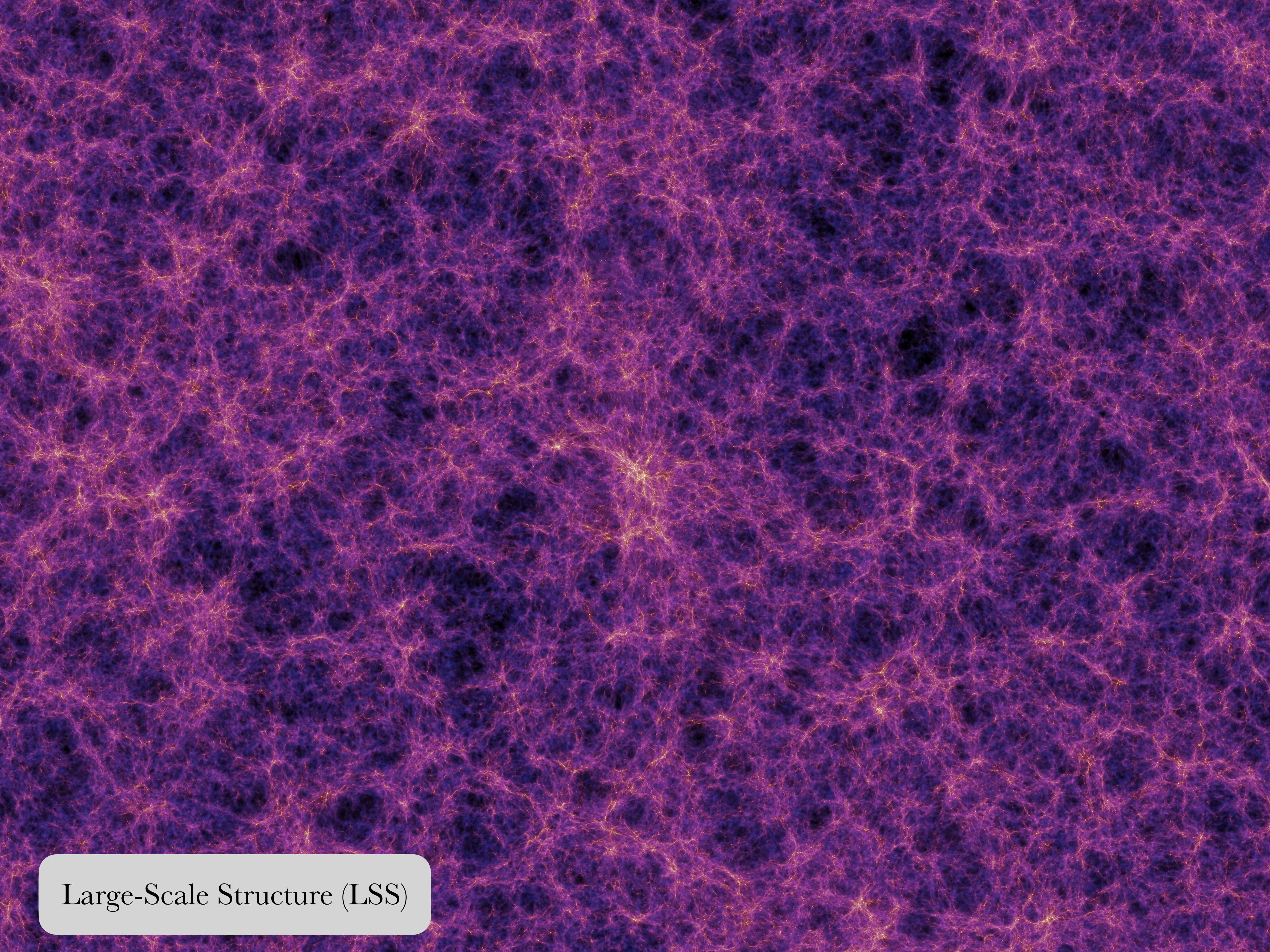
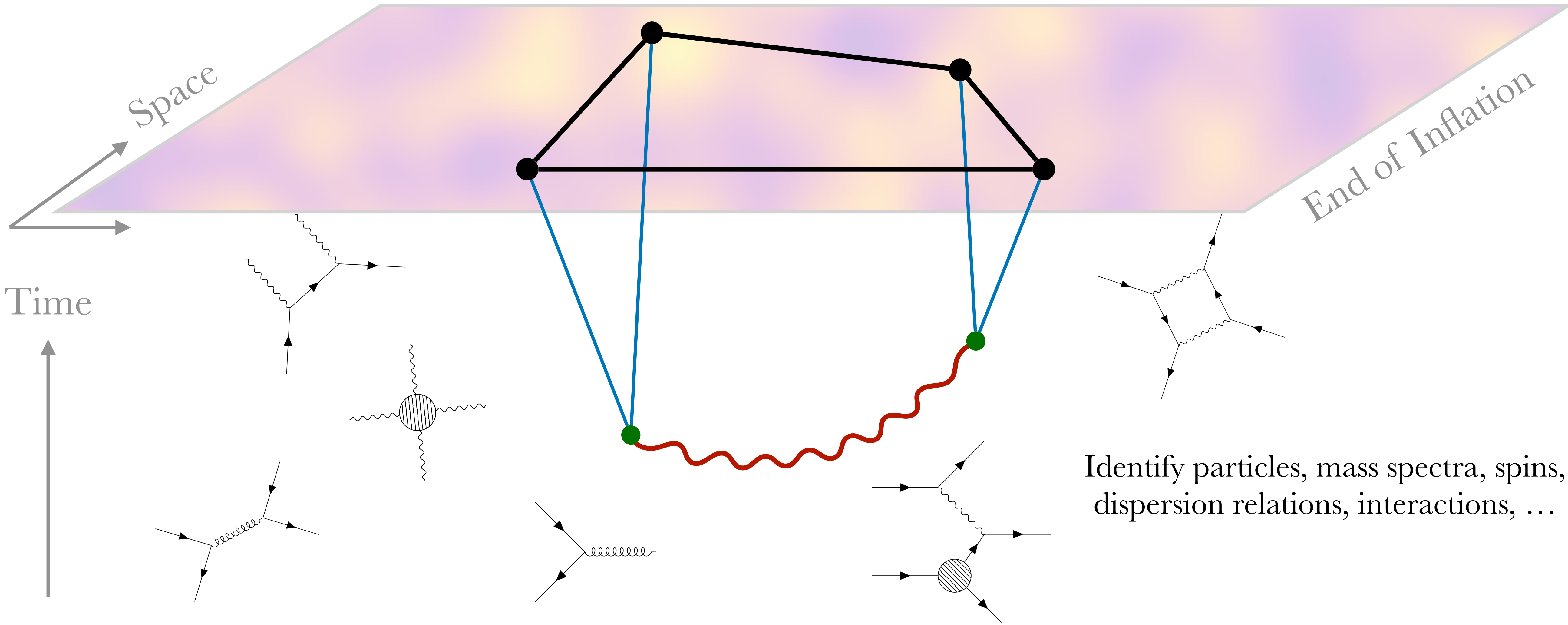


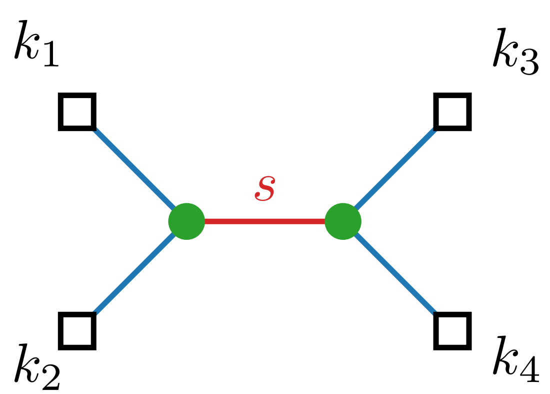
Large-Scale Structure (LSS)

Towards a Standard Model of Inflationary Cosmology

The early Universe is a unique probe of the physics at the highest reachable energies



Identify particles, mass spectra, spins, dispersion relations, interactions, …



$$\langle 0|\hat{\varphi}(\tau_1)\hat{\varphi}(\tau_2)|0\rangle\, \Theta(\tau_1-\tau_2) + (\tau_1 \leftrightarrow \tau_2)$$

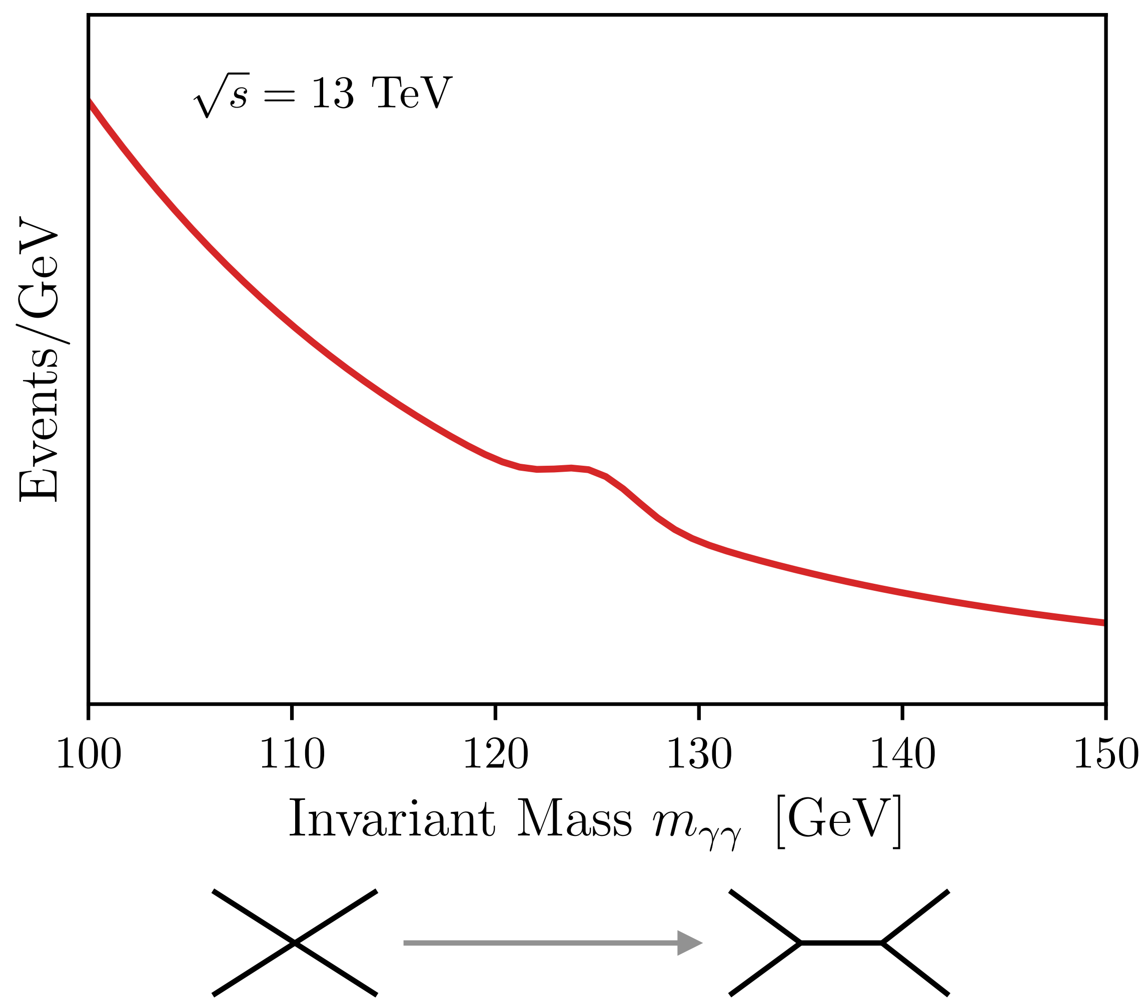
$$\equiv -g^2 \int_{-\infty^+}^{0} \mathrm{d}\tau_1 a^4(\tau_1)\, \mathrm{d}\tau_2 a^4(\tau_2) \mathcal{G}(k_1;\tau_1)\mathcal{G}(k_2;\tau_1)\mathcal{G}(s;\tau_1,\tau_2)\mathcal{G}(k_3;\tau_2)\mathcal{G}(k_4;\tau_2)$$

Difficulties: (i) nested time integrals, (ii) complicated propagators

Signals of New Physics

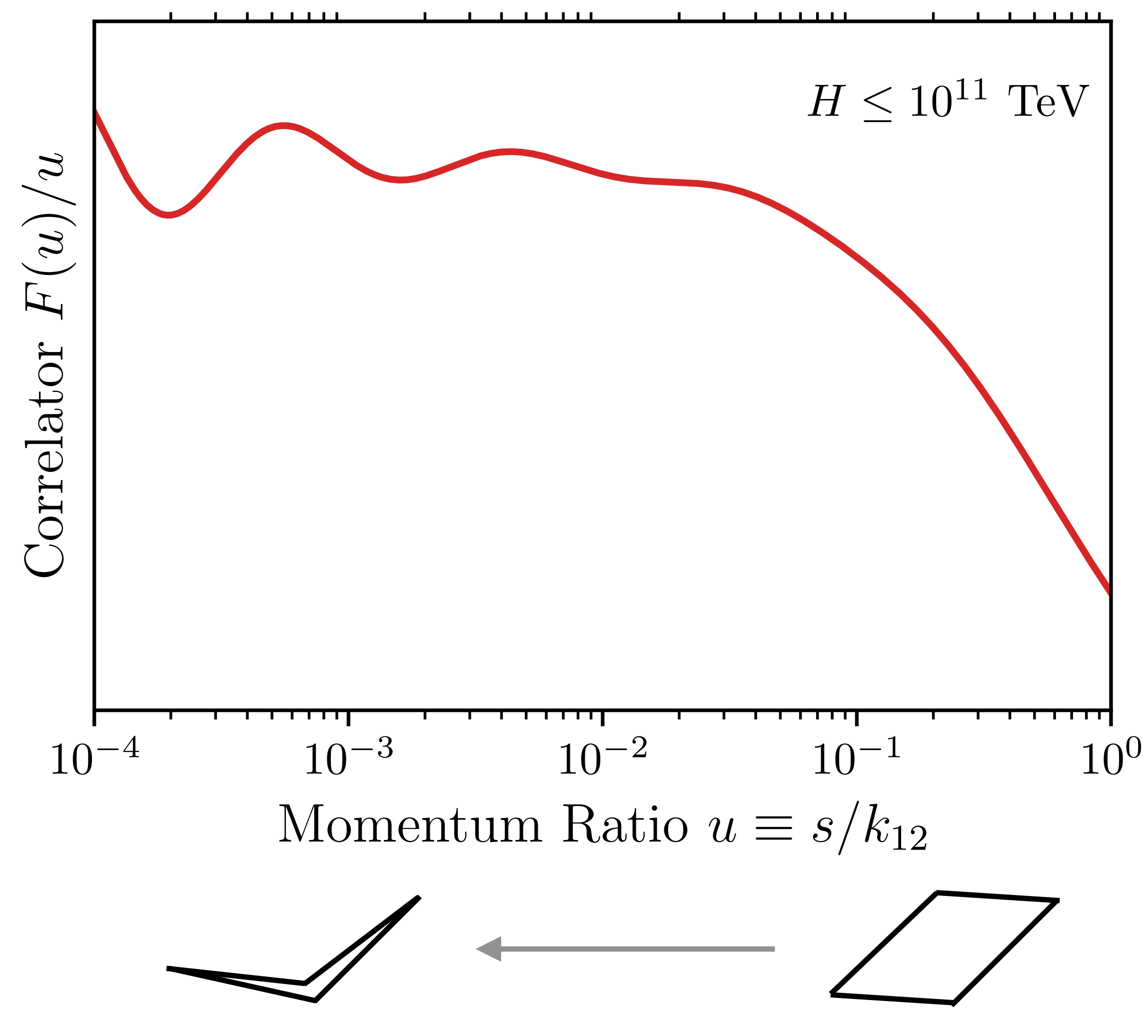
New physics is encoded in soft limits of cosmological correlators

Particle Physics



Breit-Wigner = Pole

Cosmology

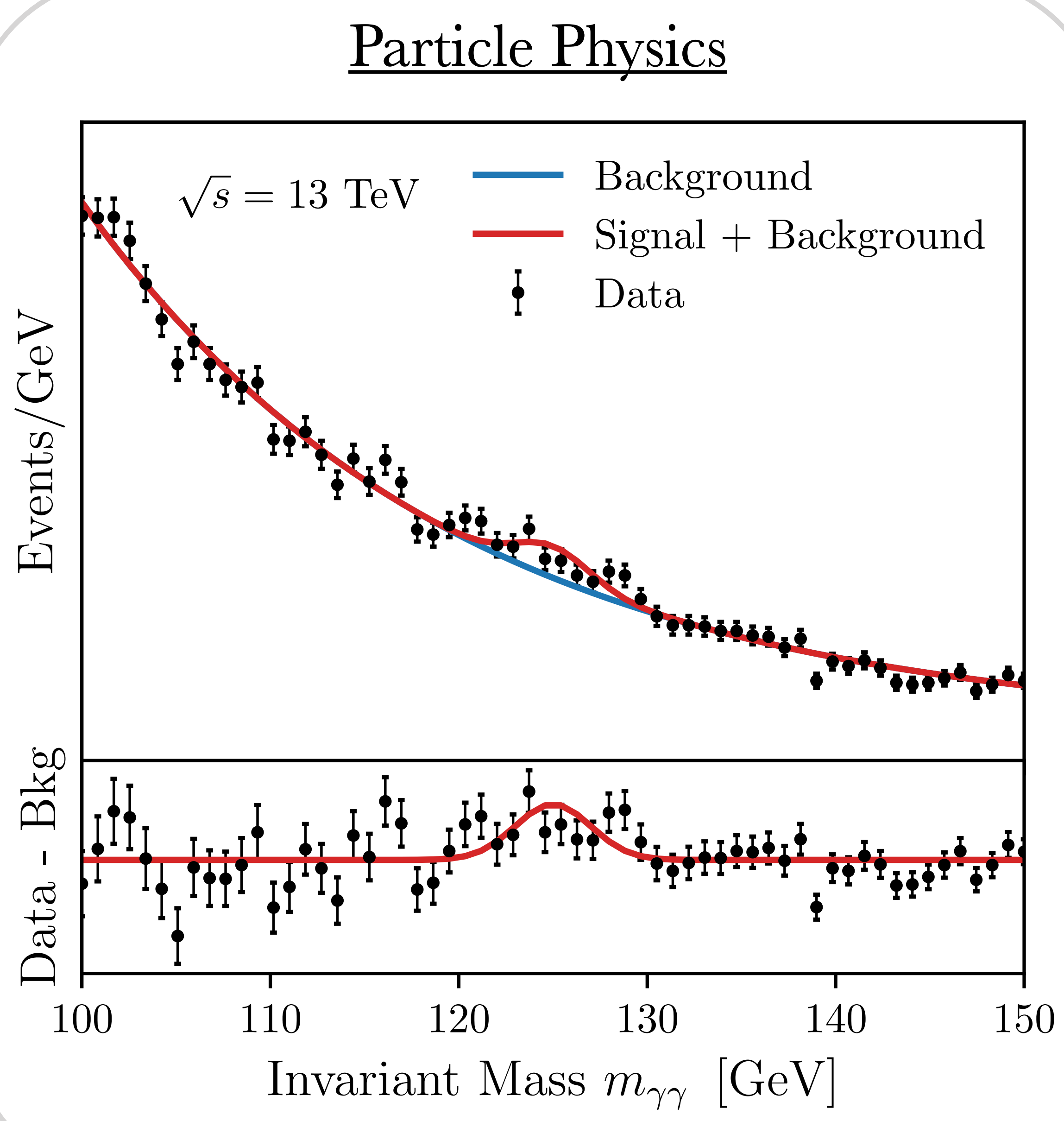


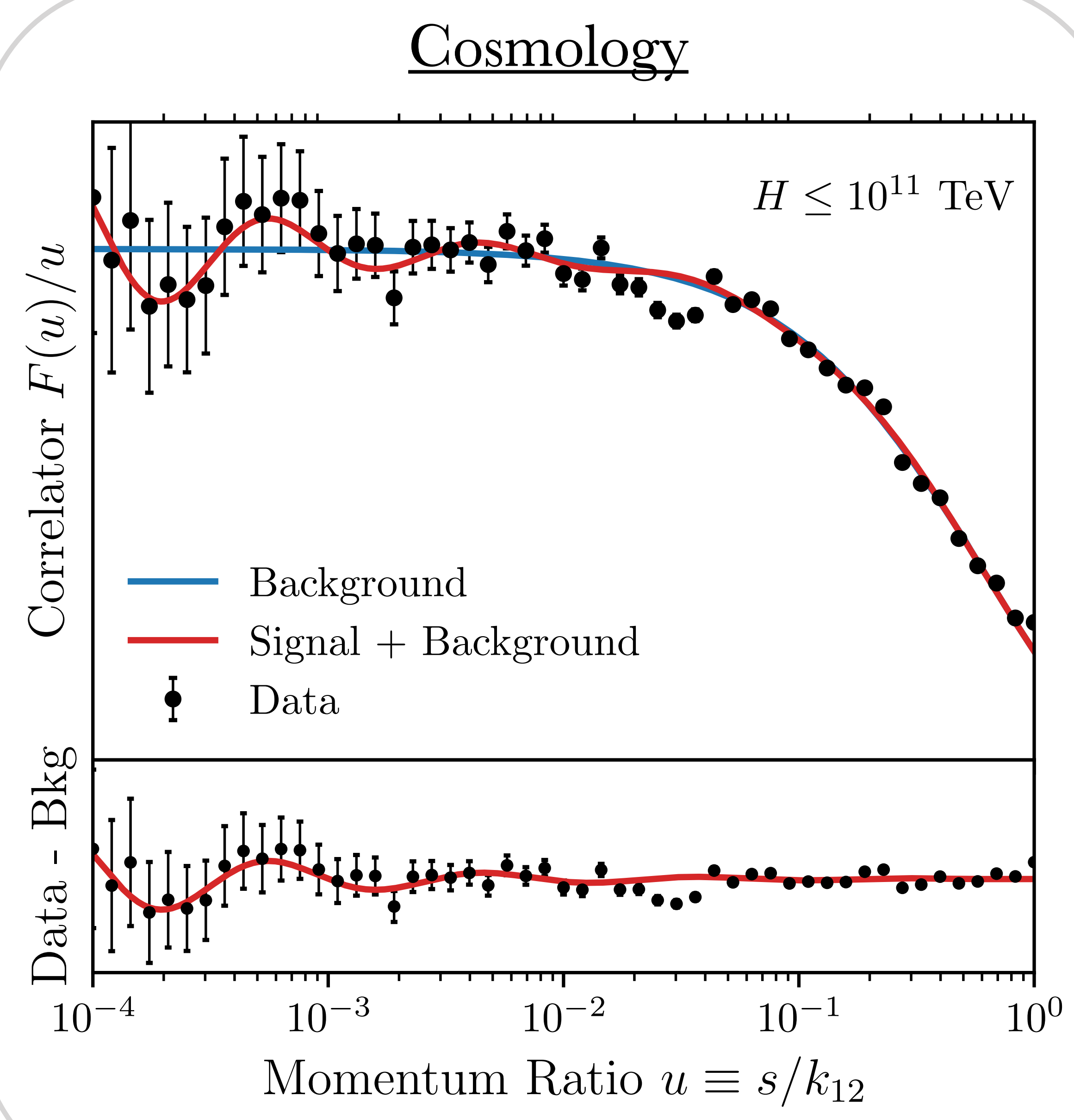
Oscillations = Branch Cut

Observable signatures are imprints of non-analyticities in the complex energy plane

Signals of New Physics

Detecting new physics requires precise subtraction of the background





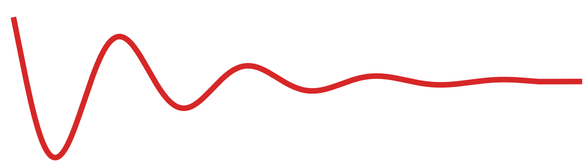
Data will be noisy and limited by cosmic variance. Do we have the necessary theoretical tools ?

Signals of New Physics

The challenge is to probe soft-enough kinematic configurations without being limited by cosmic variance

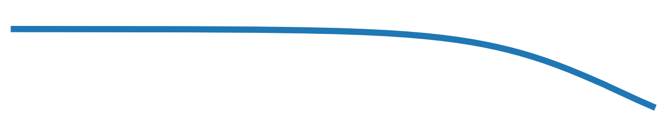
Signal

- Easy analytically
- Hard numerically

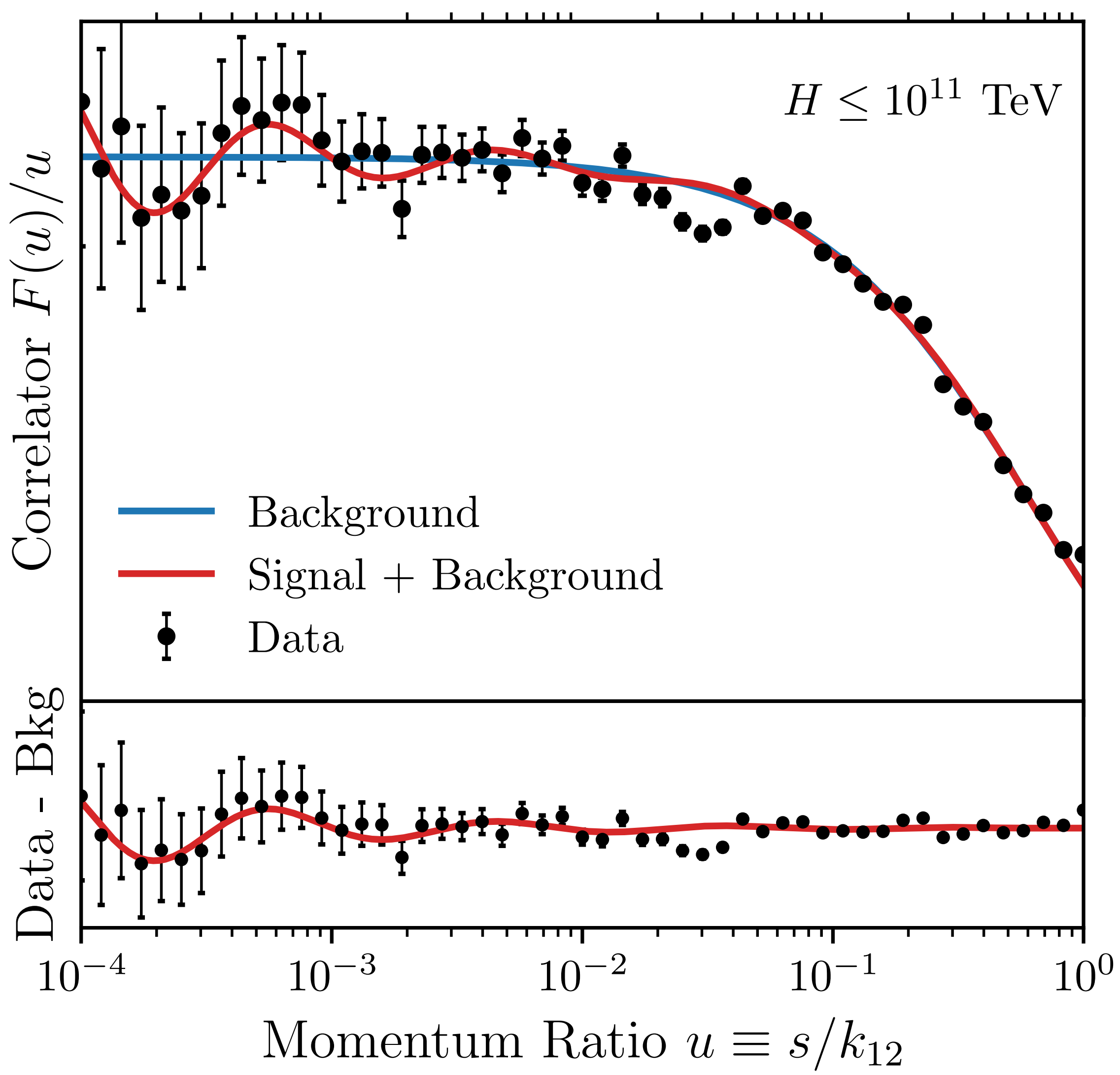


Background

- Easy numerically
- Hard analytically



Cosmology



We need new techniques to distinguish signal/background and to derive simple but precise templates for the background

Going Off-Shell

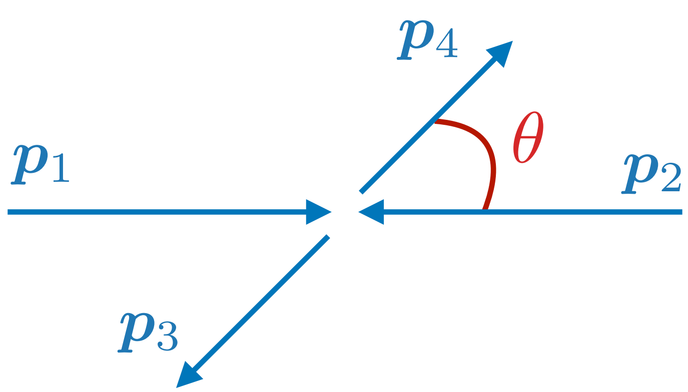
Going Non-Local

Going Off-Shell

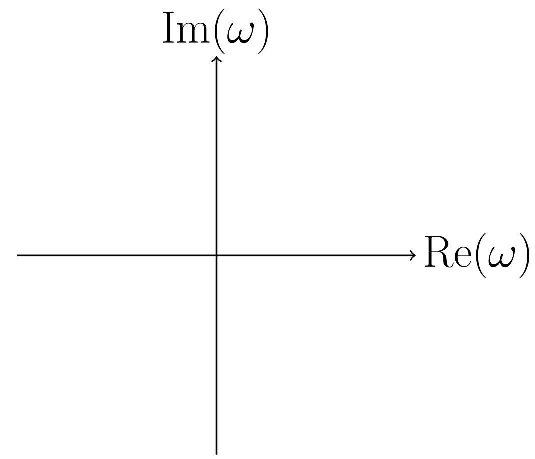
- Massive Fields in de Sitter
- Bootstrapping with the Spectral Representation

Going Off-Shell in One Slide: General Philosophy

Particles collide with real momenta and scatter off with real angles



Uplift scattering amplitudes to the complex momenta/energy plane



<u>Use of Complex Analysis</u>

- (Non)-analyticity (poles/ branch cuts)
- Cauchy's residue theorem

$$\oint \frac{\mathrm{d}z}{z} = 2i\pi$$

<u>Physical Principles</u>

- Causality/Locality/Unitarity
- Internal resonant states as poles
- Bound states as branch cuts
- …

<u>Computational Power</u>

- Crossing symmetry
- Recursion relations
- …

We want to import similar techniques to cosmology as going to the complex plane (i) makes physics manifest, and (ii) makes analytical computations more tractable

Massive Fields in de Sitter

From (non-)Analyticity to Contour Integrals

Free Propagation in de Sitter

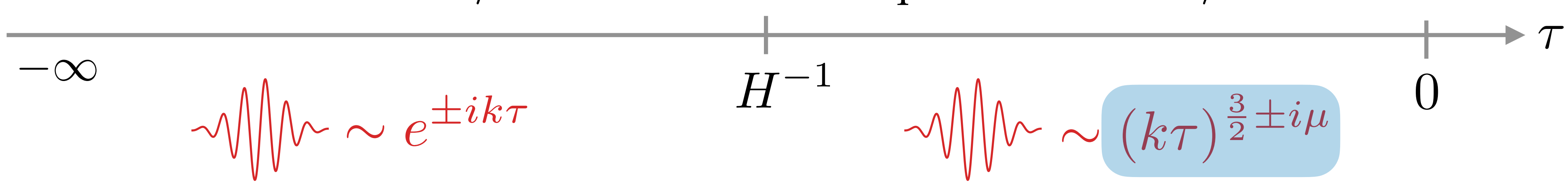
<u>Flat Space</u>

$$(\partial_t + k^2)\sigma_k = 0$$

<u>De Sitter</u>

$$[\partial_\tau^2 + \omega_k^2(\tau)]\sigma_k = 0 \;\text{ with }\; \omega_k^2(\tau) = k^2 + \frac{\mu^2 + \frac{1}{4}}{\tau^2}, \;\; \mu^2 = \frac{m^2}{H^2} - \frac{9}{4}$$

sub-Hubble $k/a \gg H$ super-Hubble $k/a \ll H$



Non-analytic

<u>Particle Production</u>

$\tau \to -\infty$

$$\sigma_k(\tau) \sim e^{-ik\tau}$$

$$\hat{a}_{\boldsymbol{k}} \left|0\right\rangle_a = 0$$

$$_a\langle 0|\hat{b}_{\boldsymbol{k}}^\dagger \hat{b}_{\boldsymbol{k}}|0\rangle_a = |\beta_k|^2 \neq 0$$

$\tau \to 0$

$$\sigma_k(\tau) \sim \alpha_k (k\tau)^{+i\mu} + \beta_k (k\tau)^{-i\mu}$$

$$\hat{b}_{\boldsymbol{k}} \left|0\right\rangle_b = 0$$

"Bessology"

Bessel equation : $[\partial_\tau^2 + \omega_k^2(\tau)]\sigma_k = 0$ with $\omega_k^2(\tau) = k^2 + \frac{\mu^2 + \frac{1}{4}}{\tau^2}$

$(\tau \to -\infty, \mu \to 0)$

$(\tau \to 0, \mu \to +\infty)$

Analytic Continuation

$H_{+i\mu}(z)$

Positive Frequency

$J_{+i\mu}(z)$

Connection Formula

$\sinh(\pi\mu) H_{i\mu}(z) = e^{+\pi\mu} J_{+i\mu}(z) - J_{-i\mu}(z)$

$H_{-i\mu}(z)$

Negative Frequency

$J_{-i\mu}(z)$

Analytic Continuation

Asymptotic Behaviour

$\sim e^{\pm iz}$

Asymptotic Behaviour

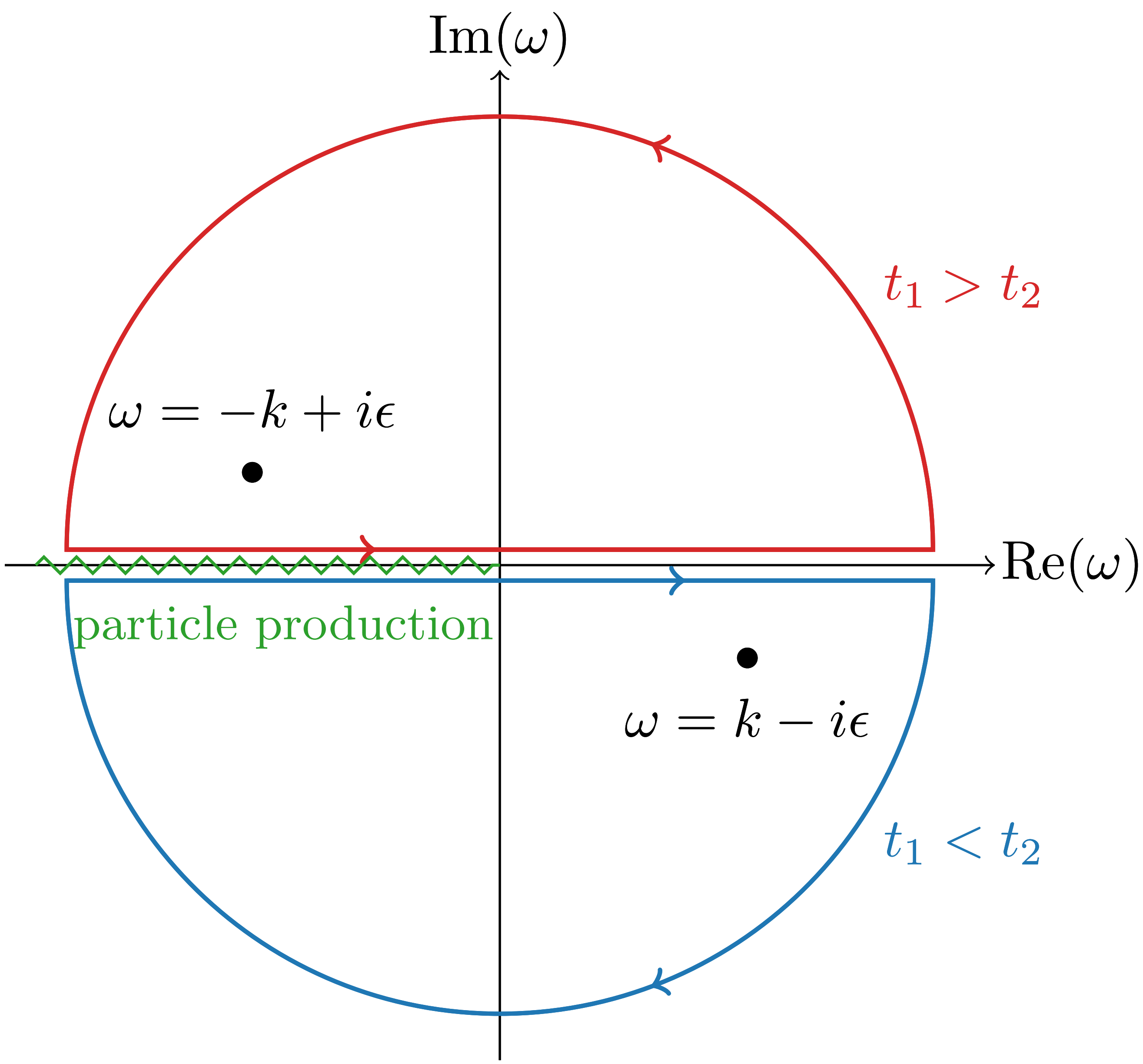
$\sim z^{\pm i\mu}$

Particle Production

$${}_a\langle 0|\hat{b}_{\boldsymbol{k}}^\dagger \hat{b}_{\boldsymbol{k}}|0\rangle_a = |\beta_k|^2 = \frac{1}{e^{2\pi\mu} - 1} \quad \text{with} \quad T_{\text{dS}} = H/2\pi$$

Flat-space Feynman Propagator

Time ordering can be traded for a contour integral in the complex energy domain



$\varphi_k(t) \sim e^{-ikt}$ ← Analytic

$$\mathcal{G}(k;t_1,t_2) = \varphi_k(t_1)\varphi_k^*(t_2)\Theta(t_1-t_2) + (1 \leftrightarrow 2)$$
$$= i\int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{2\pi}\frac{\varphi_\omega^*(t_1)\varphi_\omega(t_2)}{(\omega^2-k^2)_{i\epsilon}}$$

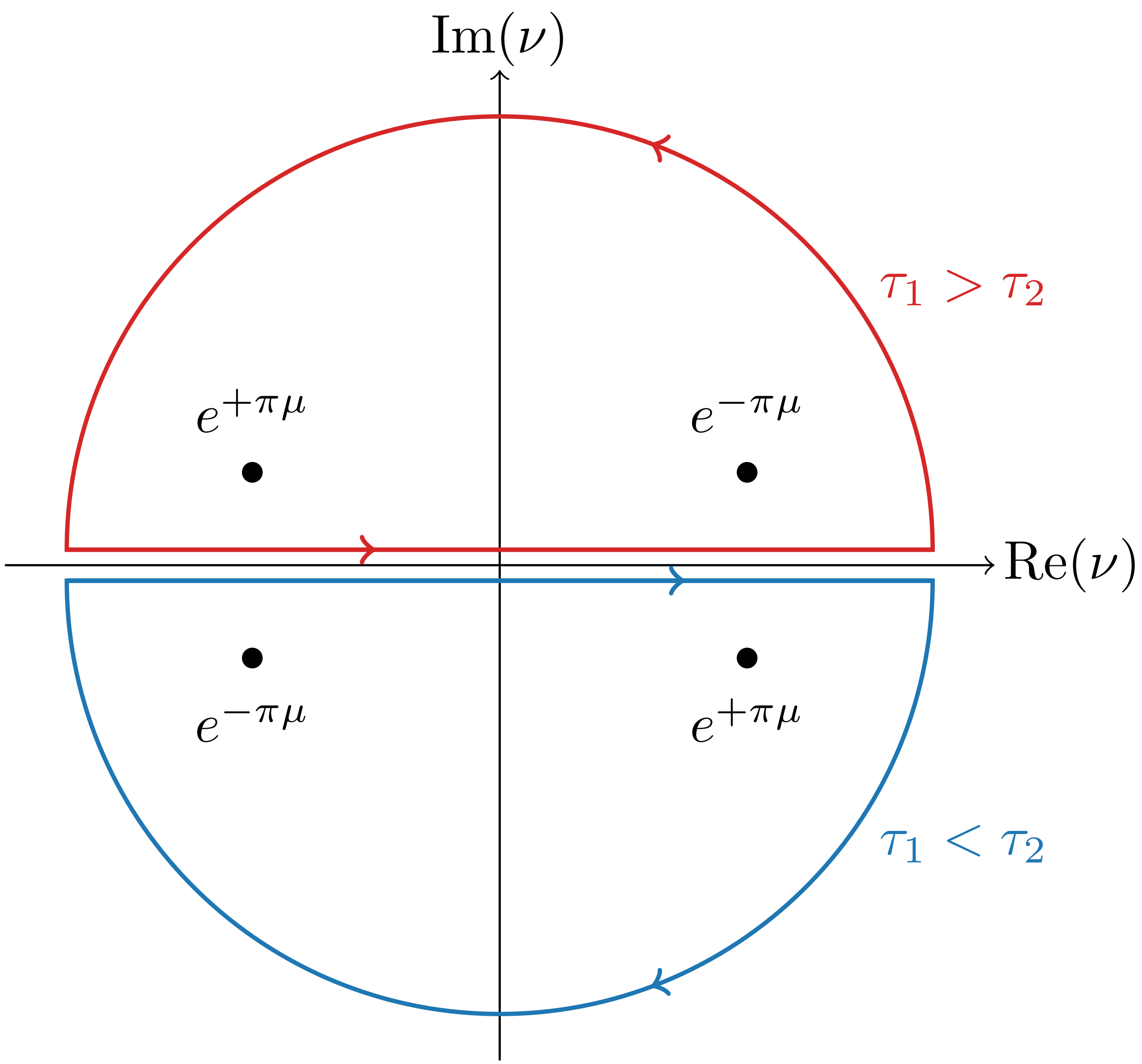
with $\frac{1}{(\omega^2-k^2)_{i\epsilon}} \equiv \frac{1}{\omega^2-k^2+i\epsilon}$

↑ Pole Prescription

In de Sitter, particle production leads to a branch cut in the energy domain

Massive Propagator in de Sitter

Time ordering can be traded for a contour integral in the complex mass domain



<u>Contour Integral</u>

$$\mathcal{G}(k;\tau_1,\tau_2) = i\int_{-\infty}^{+\infty} \mathrm{d}\nu \mathcal{N}_\nu \frac{\varphi_k^*(\tau_1,\nu)\varphi_k^*(\tau_2,\nu)}{(\nu^2-\mu^2)_{i\epsilon}}$$

$$\varphi_k^*(\tau,\nu) \sim H_{-i\nu}(-k\tau)$$

<u>Pole Prescription</u>

$$\frac{1}{(\nu^2-\mu^2)_{i\epsilon}} \equiv \frac{1}{2\sinh(\pi\mu)}\left[\frac{e^{+\pi\mu}}{\nu^2-\mu^2+i\epsilon} - \frac{e^{-\pi\mu}}{\nu^2-\mu^2-i\epsilon}\right]$$

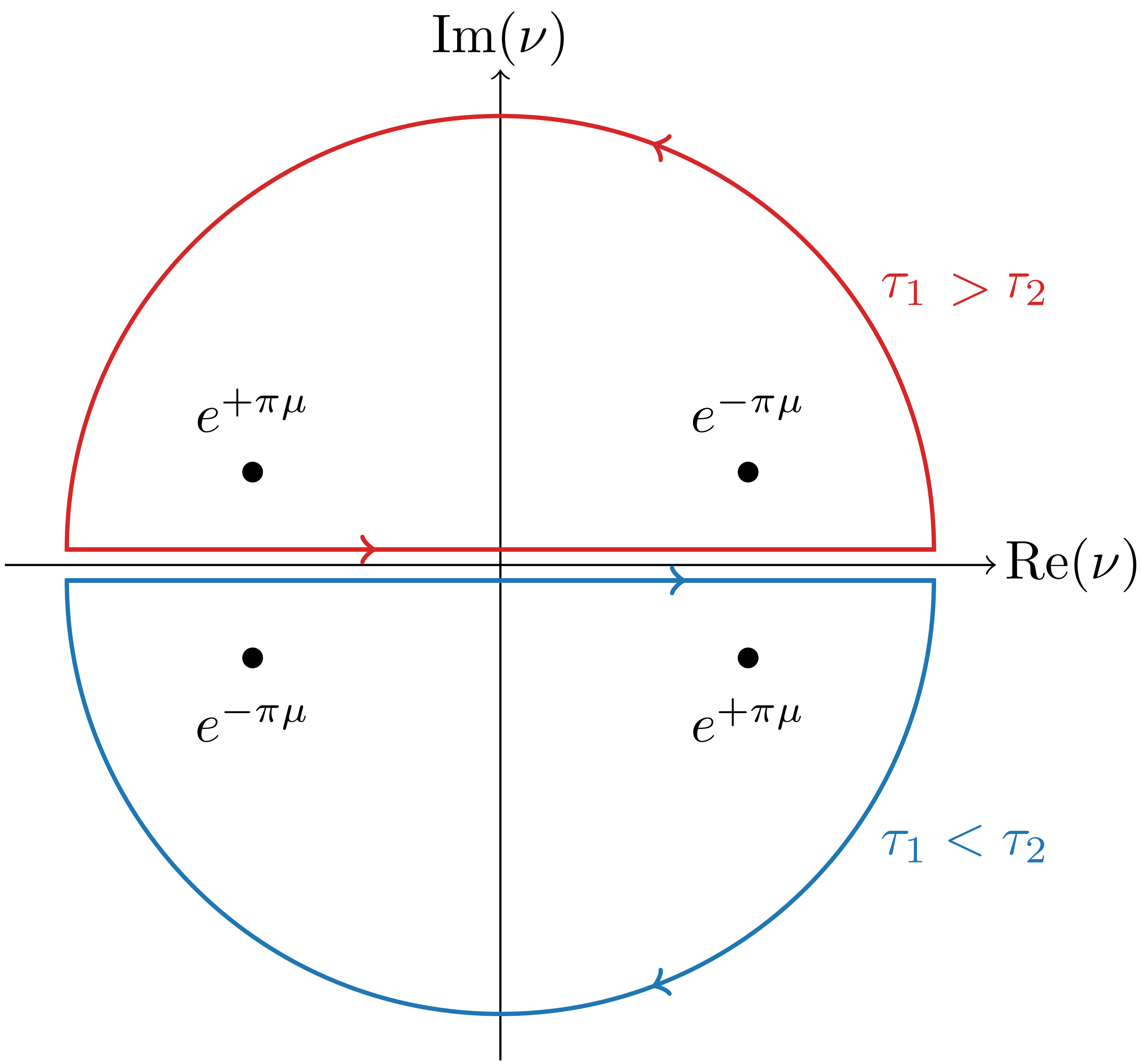
Projects outgoing particle states onto ingoing ones

Melville, Pimentel [2024]

$H_{-i\mu}$ → Connection Formula (from "Bessology") → $J_{\pm i\mu}$ $(\nu \to \infty)$ → Connection Formula (from pole prescription) → $H_{+i\mu}$

Massive Propagator in de Sitter

The spectral representation makes the origin of certain limits manifest



$$\frac{1}{(\nu^2-\mu^2)_{i\epsilon}} \equiv \frac{1}{2\sinh(\pi\mu)}\left[\frac{e^{+\pi\mu}}{\nu^2-\mu^2+i\epsilon} - \frac{e^{-\pi\mu}}{\nu^2-\mu^2-i\epsilon}\right]$$

<u>Flat-space Limit</u> $(H \to 0, \mu \to \infty)$

$$\frac{1}{(\nu^2-\mu^2)_{i\epsilon}} \to \frac{1}{\nu^2+\mu^2+i\epsilon}$$

Select one pole

$$\mathcal{G}(k;\tau_1,\tau_2) = J_{+i\mu}(-k\tau_1)J_{-i\mu}(-k\tau_2)$$

$$\sim \left(\frac{\tau_1}{\tau_2}\right)^{+i\mu} \sim e^{-i\mu(t_1-t_2)}$$

Gradients have redshifted away

$$\omega^2(\boldsymbol{k}) = k^2 + \mu^2 \to \mu^2$$

<u>Soft Limit</u> $(k \to 0)$

$$\mathcal{G}(k;\tau_1,\tau_2) \sim i\int_{-\infty}^{+\infty} \mathrm{d}\nu \mathcal{N}_\nu \frac{(k^2\tau_1\tau_2)^\nu}{(\nu^2-\mu^2)_{i\epsilon}}$$

$$\sim \delta(\mu)[k^2\tau_1\tau_2]^{+i\mu} + \delta^*(\mu)[k^2\tau_1\tau_2]^{-i\mu}$$

$$\sim e^{-\pi\mu}\cos(\mu\log k)$$

Signal of new physics

Bootstrapping with the Spectral Representation

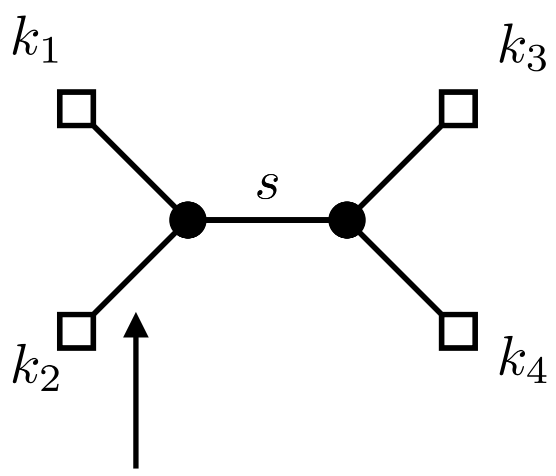
Summing Towers of Residues

Massive Exchange Correlators in de Sitter

The spectral representation of massive propagators factorises nested time integrals

Exchange Diagram

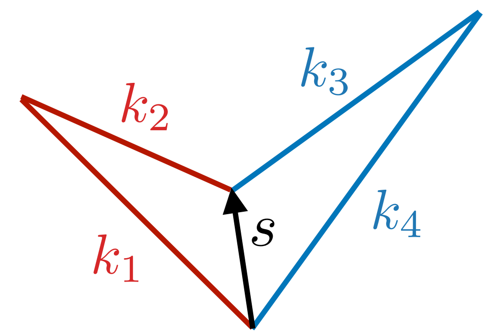
$s = |\boldsymbol{k}_1 + \boldsymbol{k}_2|$



$$= \int_{-\infty}^{+\infty} \mathrm{d}\nu \mathcal{N}_\nu \, \frac{F^{(3)}(u^{-1},\nu) F^{(3)}(v^{-1},\nu)}{(\nu^2 - \mu^2)_{i\epsilon}}$$

Conformally Coupled External Field $(m = \sqrt{2}H)$

Kinematics

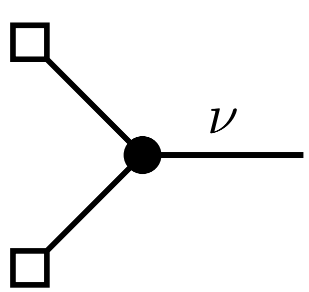


$$u \equiv \frac{s}{k_1 + k_2} \qquad v \equiv \frac{s}{k_3 + k_4}$$

$$0 \leq u, v \leq 1$$

Off-Shell 3pt Function

$F^{(3)}(z,\nu) \equiv$



$$= |\Gamma(\tfrac{1}{2} + i\nu)|^2 P_{i\mu - 1/2}(z) \sim \int_{-\infty^+}^{0} \frac{\mathrm{d}\tau \, e^{ik\tau}}{(-\tau)^{1/2}} H_{i\nu}(-s\tau)$$

Tower of Poles

Legendre Polynomial (“hypergeometric ${}_2F_1$”)

"Legendrology" ("Bessology" with higher transcendality)

Legendre polynomial basis have different scaling behaviours at small and large masses

"Hypergeometric" $P_{\pm i\mu-1/2}(z), Q_{\pm i\mu-1/2}(z) \sim \int H_{\pm i\mu}(z), J_{\pm i\mu}(z)$

$(z \to -\infty, \mu \to 0)$ $(z \to 0, \mu \to +\infty)$

$P_{+i\mu-1/2}(z)$ $Q_{+i\mu-1/2}(z)$

Connection Formula

$$P_{i\mu-1/2}(z) \propto Q_{-i\mu-1/2}(z) - Q_{+i\mu-1/2}(z)$$

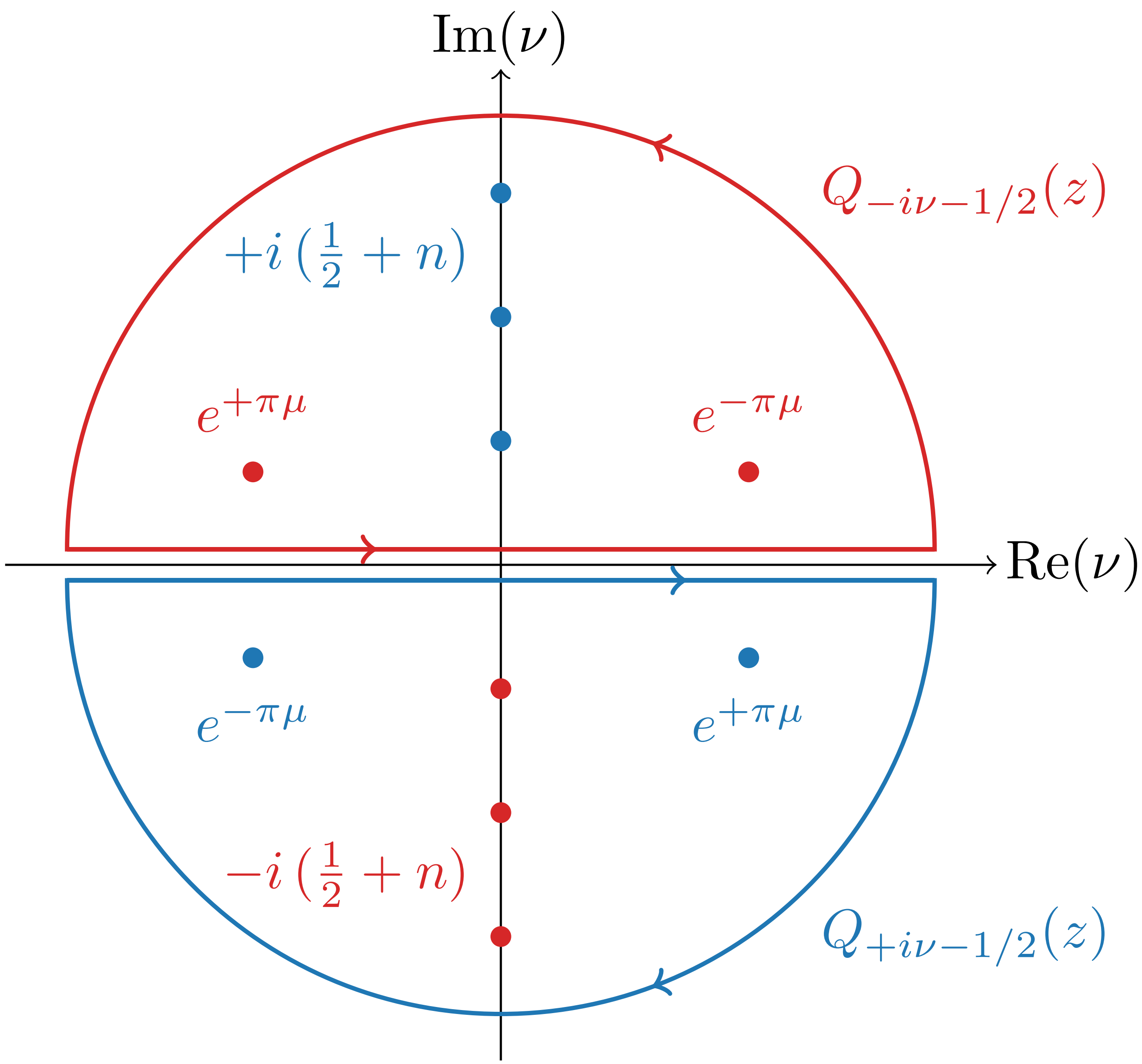
Analytic Continuation

$$P_{i\mu-1/2}(e^{+i\pi}z) \propto e^{+\pi\mu}Q_{+i\mu-1/2}(z) - e^{-\pi\mu}Q_{-i\mu-1/2}(z)$$

$P_{-i\mu-1/2}(z)$ $Q_{-i\mu-1/2}(z)$

Off-Shell 3p Function

Closing the contour in the complex mass plane requires changing Legendre basis



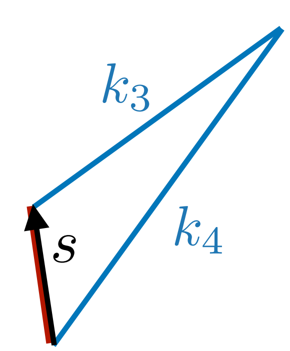
Connection Formula

$$P_{+i\nu-1/2}(z) \propto Q_{+i\nu-1/2}(z) - Q_{-i\nu-1/2}(z)$$

$$\int_{-\infty}^{+\infty} \mathrm{d}\nu \mathcal{N}_\nu \frac{F^{(3)}(z,\nu)}{(\nu^2-\mu^2)_{i\epsilon}}$$

Pole Prescription
= Analytic Continuation

Kinematics



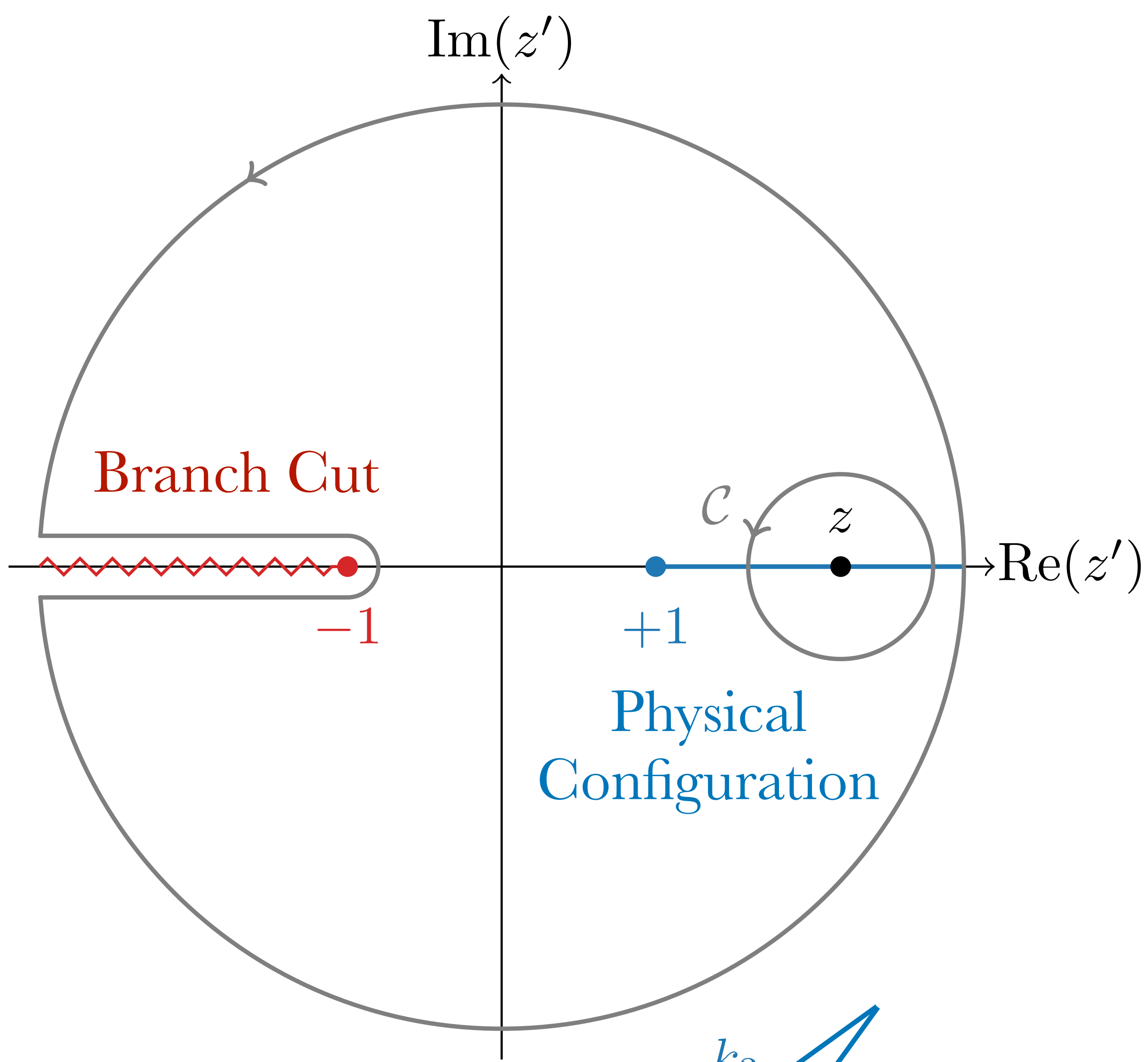
$$u \equiv \frac{s}{k_1+k_2} \to 1$$

$$v \equiv \frac{s}{k_3+k_4} = z^{-1}$$

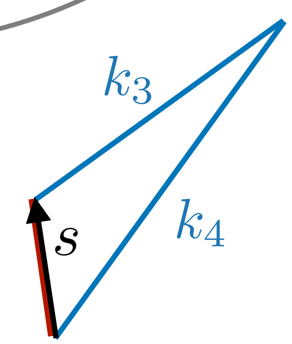
Non-Analyticity in the Energy Domain

Particle production poles in the complex mass plane rotate the kinematics in the energy domain

$$F^{(3)}(z,\mu) \to F^{(3)}(e^{+i\pi}z,\mu)$$



$$z \equiv \frac{k_3+k_4}{s} \geq 1$$



Discontinuity along the Branch Cut

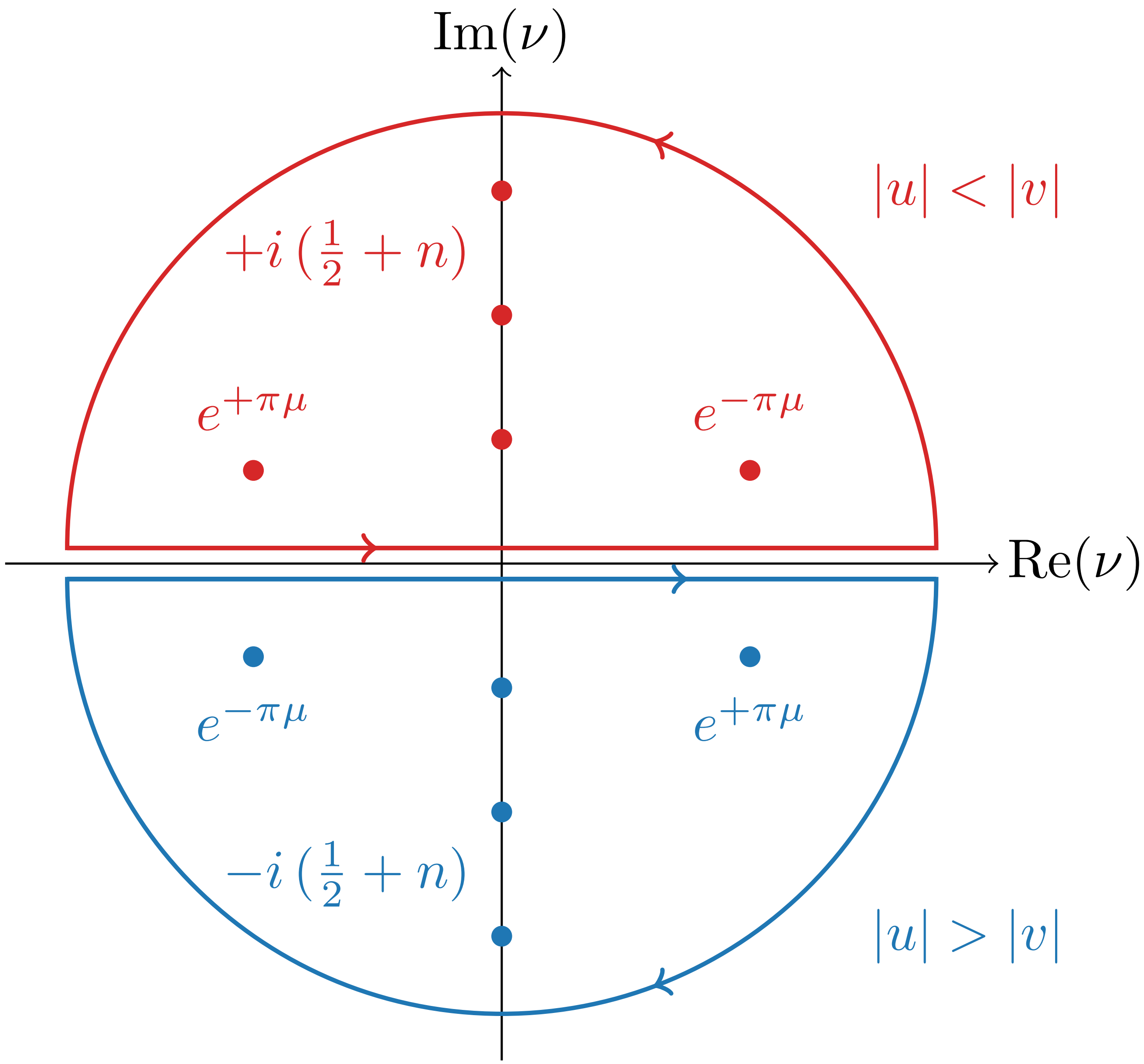
$$\operatorname*{Disc}_{z}\left[F^{(3)}(z,\mu)\right] \propto F^{(3)}(e^{+i\pi}z,\mu)$$

Dispersive Integral

$$\begin{aligned} F^{(3)}(z,\mu) &= \oint_{\mathcal{C}} \frac{\mathrm{d}z'}{2i\pi}\frac{F^{(3)}(z',\mu)}{z'-z} \\ &= \int_{-\infty}^{-1}\frac{\mathrm{d}z'}{2i\pi}\frac{\mathrm{Disc}_{z'}\left[F^{(3)}(z',\mu)\right]}{z'-z} \\ &\propto \int_{-\infty}^{-1}\frac{\mathrm{d}z'}{2i\pi}\frac{F^{(3)}(e^{+i\pi}z',\mu)}{z'-z} \end{aligned}$$

(Residue Theorem)

Massive Exchange Correlators in de Sitter



Summing Residues

$$\int_{-\infty}^{+\infty} \mathrm{d}\nu \mathcal{N}_\nu \frac{F^{(3)}(u^{-1},\nu)F^{(3)}(v^{-1},\nu)}{(\nu^2-\mu^2)_{i\epsilon}}$$

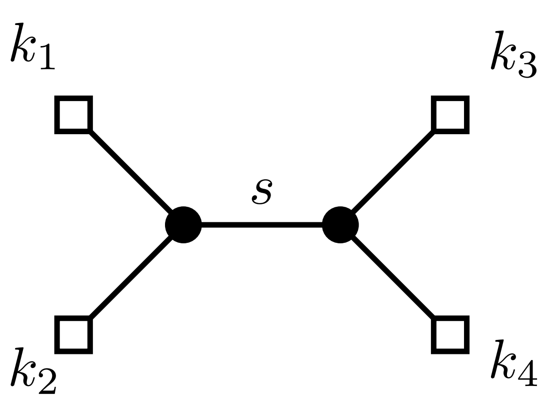
$$\downarrow$$

$$Q_{\pm i\nu-1/2}(u^{-1})Q_{\pm i\nu-1/2}(v^{-1})$$
$$Q_{\pm i\nu-1/2}(u^{-1})Q_{\mp i\nu-1/2}(v^{-1})$$

$$\downarrow$$

Connection Formula +
Analytical Continuation

Full Result (e.g. $|u|<|v|$)



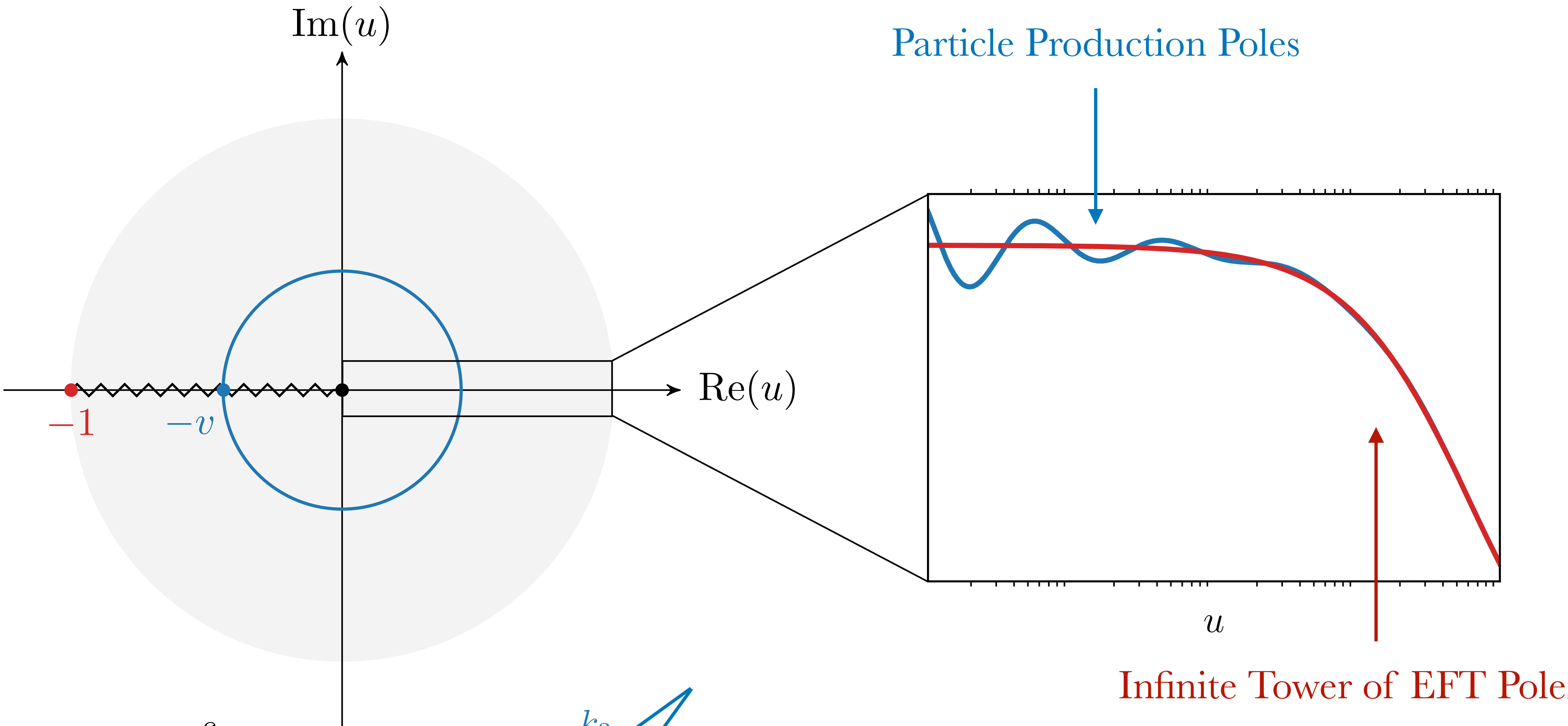
$$= F^{(3)}(-u^{-1})F^{(3)}(v^{-1}) + u\sum_{n=0}^{+\infty}\frac{(-1)^n}{(n+\frac{1}{2})^2+\mu^2}\left(\frac{u}{v}\right)^n {}_2F_1(\#|u^2)\,{}_2F_1(\#|v^2)$$

Particle Production Poles

Infinite Tower of EFT Poles

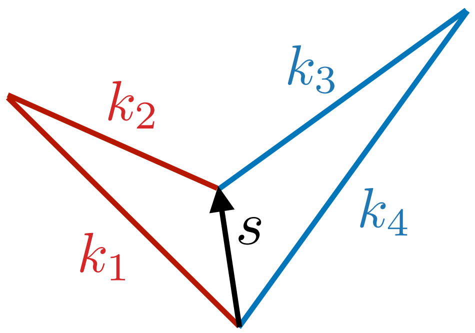
Analytic Structure in the Energy Domain

Signals of new physics are images of branch cuts just like Breit-Wigner resonances are images of poles



$$u \equiv \frac{s}{k_1 + k_2}$$

$$v \equiv \frac{s}{k_3 + k_4}$$

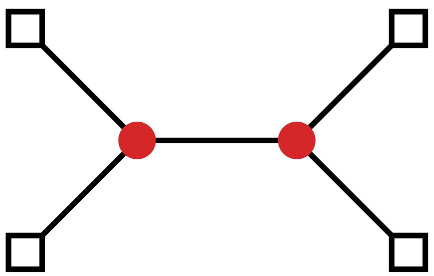


The signal and the background are linked by a dispersive integral

Going Non-Local

Going Non-Local in One Slide: General Philosophy

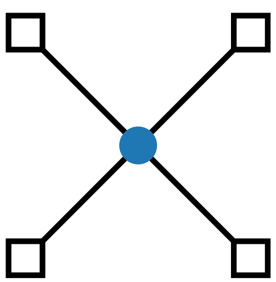
We want to accurately model the background to derive ready-to-use simple templates



Soft Configurations

$$(-\Box + m^2)\sigma = J(\varphi)$$

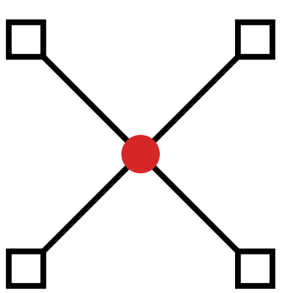
(local)



Mildly-Soft Configurations

$$\sigma = (-\partial_i^2 + m^2)^{-1} J(\varphi)$$

(non-local)

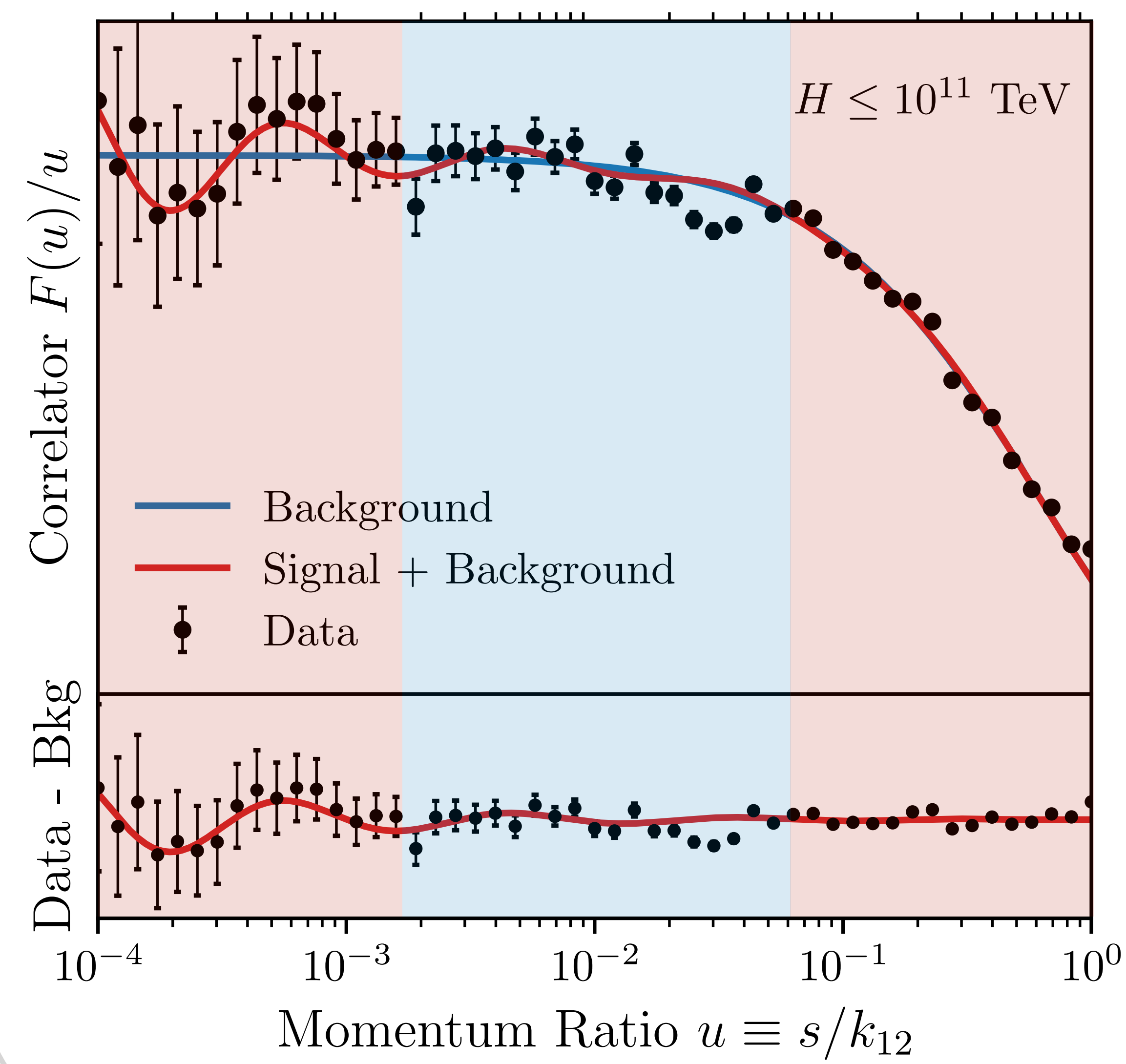


Regular Configurations

$$\sigma = m^{-2} J(\varphi)$$

(local)

Cosmology

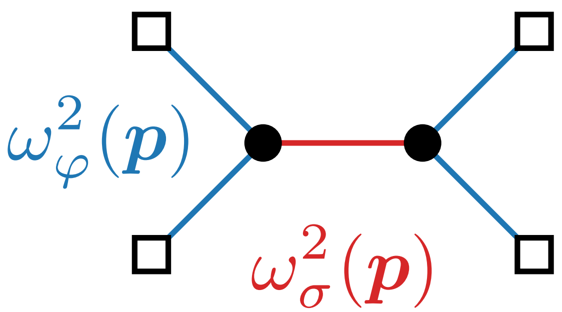


Locally integrating out heavy fields is not precise enough to correctly subtract the background, we need to go non-local

Non-Local EFT

In the long wavelength regime, the system is populated by particle states with low frequencies

Spectrum of Propagating Modes



$$\omega_\varphi^2(\boldsymbol{p}) = c_s^2 \boldsymbol{p}^2 \quad \to 0 \quad \leftarrow \text{Gapless}$$

$$\omega_\sigma^2(\boldsymbol{p}) = \boldsymbol{p}^2 + m^2 \to m^2 \quad \leftarrow \text{Massive}$$

Mildly-Long Wavelength Regime $\boldsymbol{p}^2 \lesssim m^2 \to \omega_\varphi^2(\boldsymbol{p}) \lesssim \omega_\sigma^2(\boldsymbol{p})$

$$\frac{1}{\omega^2 - \boldsymbol{p}^2 - m^2} \sim \frac{-1}{\boldsymbol{p}^2 + m^2} \quad \leftarrow \text{Non-dynamical}$$

High-frequency particle states fastly decay into low-frequency ones

Real Space

$$\mathcal{G}(\boldsymbol{x}, t; \boldsymbol{y}, t') \to \delta(t - t') \frac{e^{-m|\boldsymbol{x}-\boldsymbol{y}|}}{4\pi|\boldsymbol{x} - \boldsymbol{y}|} \quad \leftarrow \text{Yukawa-type interaction}$$

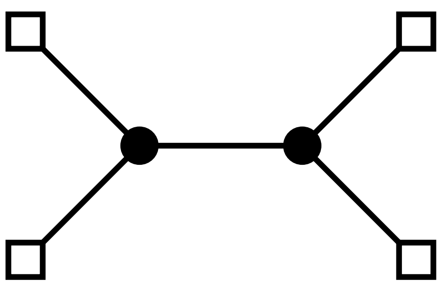
Instantaneous propagation (→ $\delta(t - t')$)

Cosmological Correlator Background Made Simple

Integrating out heavy fields in a non-local manner yields precise and simple templates

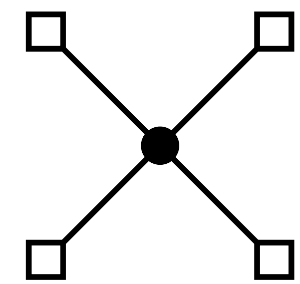
$$\int_{-\infty^+}^{0} \mathrm{d}\tau_1 \mathrm{d}\tau_2 \, J(\tau_1)\mathcal{G}(\boldsymbol{s};\tau_1,\tau_2)J(\tau_2)$$

$$\int_{-\infty^+}^{0} \mathrm{d}\tau \, \frac{J^2(\tau)}{s^2+m^2}$$



Time-derivative expansion

$$\frac{1}{-\Box + m^2} = \mathcal{D}^{-1}\sum_n (-1)^n \left[\partial_\tau^2 \mathcal{D}\right]^n$$



<u>Building Block</u> $\mathcal{D}^{-1} \equiv (k^2+m^2)^{-1}$

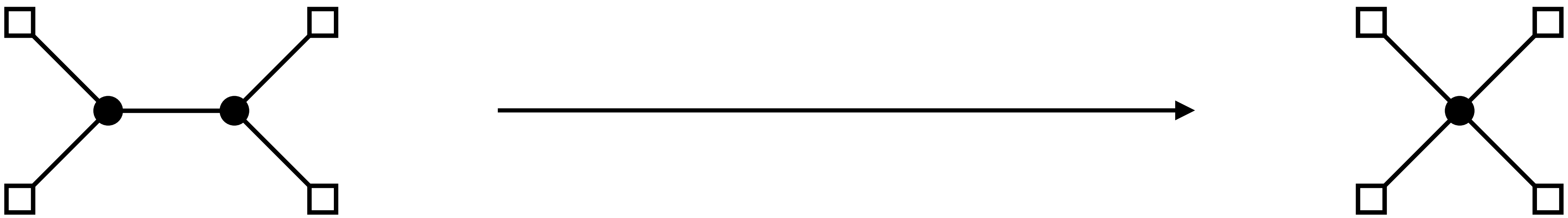
Fully non-local : $\mathcal{D}^{-1} = \frac{1}{k^2}\sum_n \left(\frac{-m^2}{k^2}\right)^n$

Fully local : $\mathcal{D}^{-1} = \frac{1}{m^2}\sum_n \left(\frac{-k^2}{m^2}\right)^n$

- Single-field theory with simple mode functions
- Contact diagrams
- Corrections to leading-order non-local EFT can be computed systematically
- Very general / Very precise

Emergent Non-Locality as a Residue: Example

Background signal arises from a single pole of the non-local operator instead of the infinite tower of poles

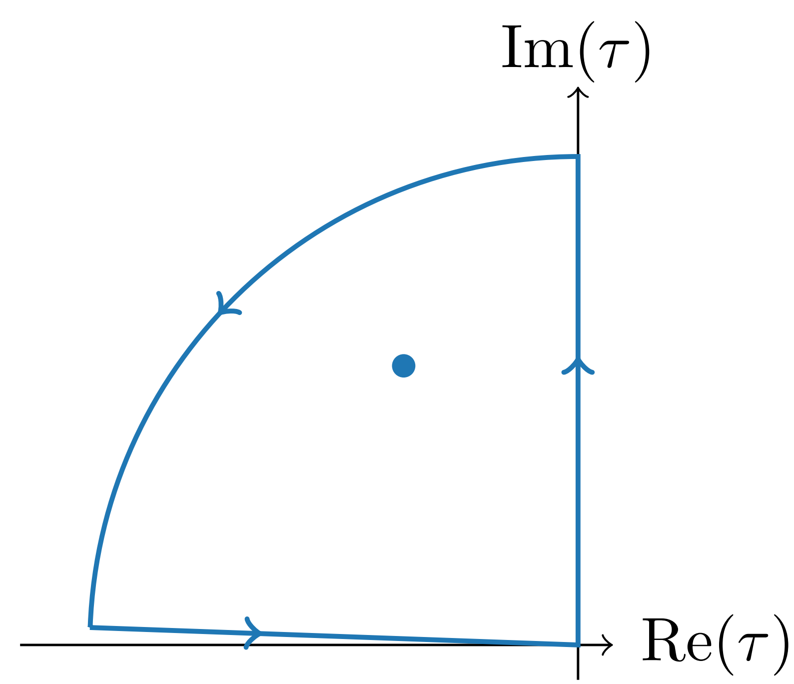


$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{2}m^2\sigma_\mu^2 + \tfrac{\kappa t}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

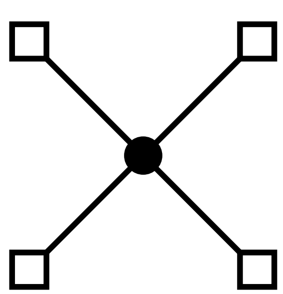
Helical chemical potential $\sigma_+ \neq \sigma_-$

$$\int_{-\infty^+}^{0} \mathrm{d}\tau\, J_i(\tau)[\mathcal{D}]_{ij}^{-1} J(\tau)$$

with $[\mathcal{D}]_{ij}^{-1} \equiv \dfrac{\kappa\,\epsilon_{ijk}\partial_k}{(-\partial_i^2 + m^2)^2 + \kappa^2\partial_i^2}$



<u>Precise & Simple Template</u>

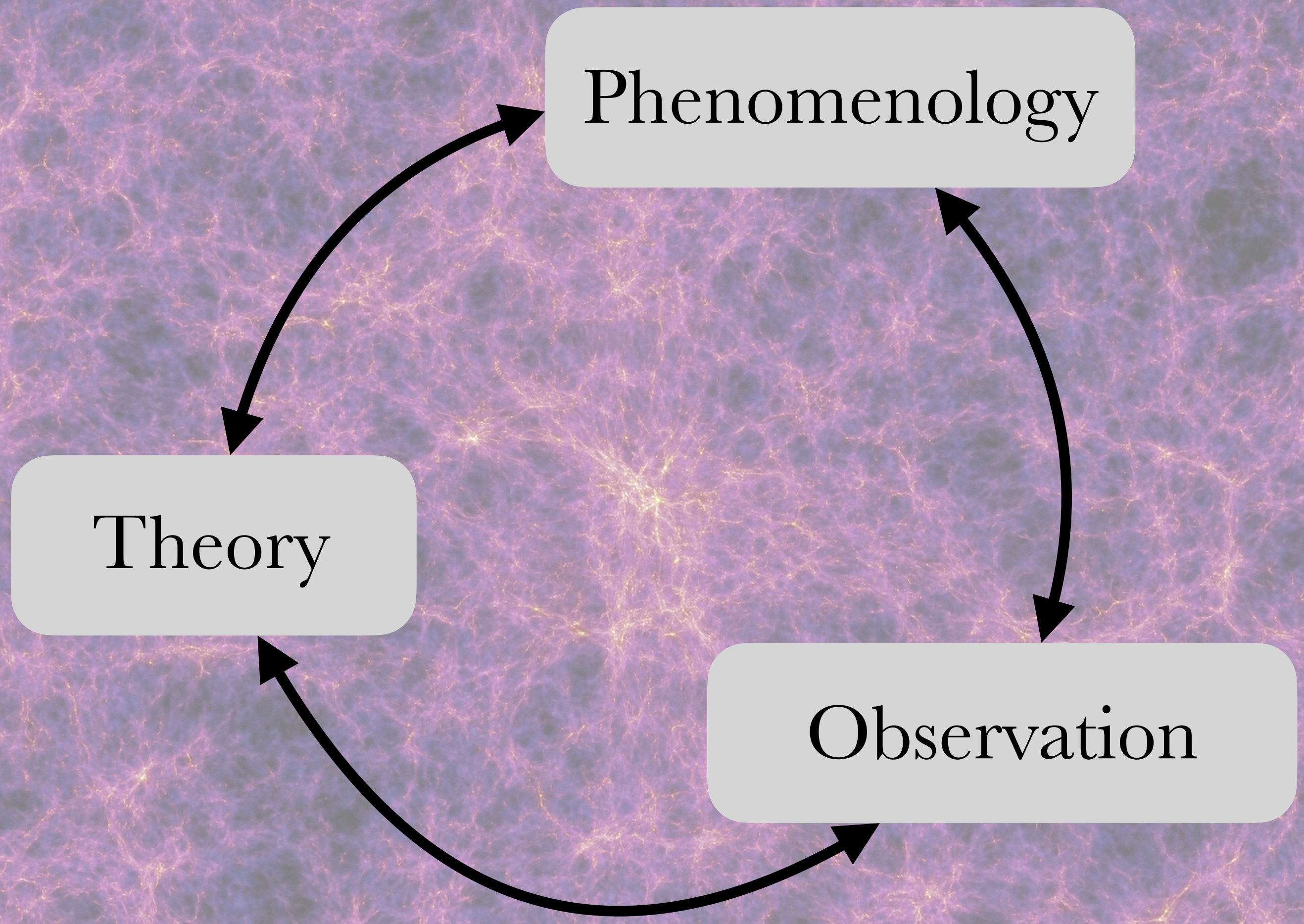


$= \Pi(\{\boldsymbol{k}_i\})\, F(\{\boldsymbol{k}_i\}, s)$

Form factor (elementary functions)

Conclusion

- Explore the space of observational signatures
- Build a complete theory/observable dictionary

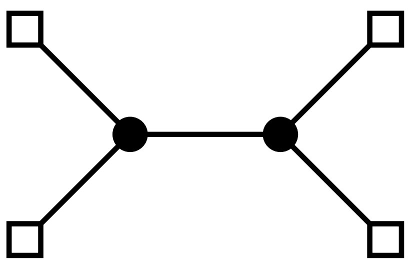


- Explore/construct the space of theories
- Develop new techniques for cosmological correlators

- Numerous/precise future data
- Need accurate templates

Take-Home Messages

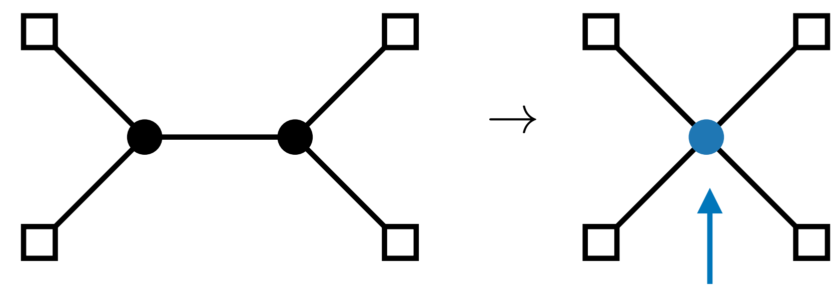
Spectral Representation



$$= \int_\nu \frac{F^{(3)}(\nu)F^{(3)}(\nu)}{(\nu^2-\mu^2)_{i\epsilon}}$$

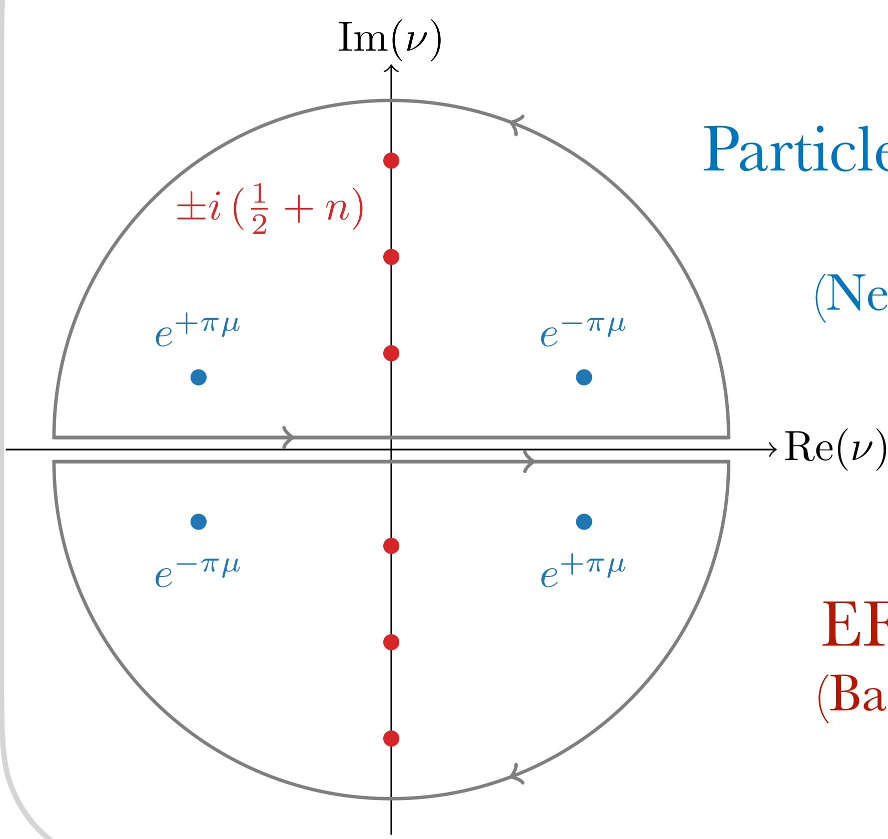
(No nested time integrals) Particle Production

Non-Local EFT



Non-local Vertex

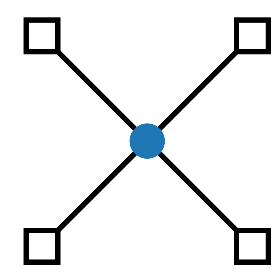
Summing Residues



Particle Production Poles (New Physics)

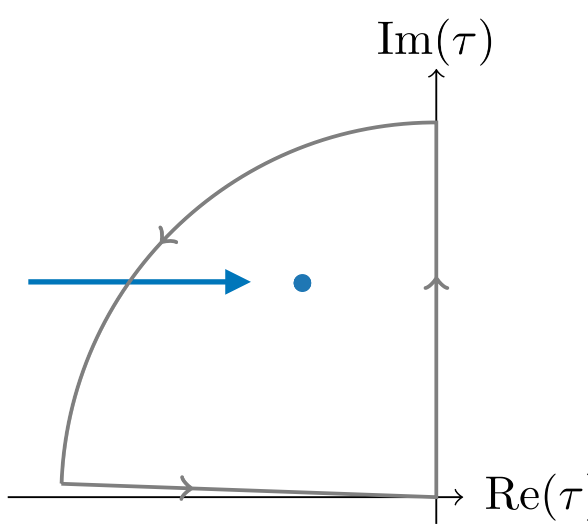
EFT Poles (Background)

Simple & Precise Templates



$$= \int_{-\infty^+}^{0} \mathrm{d}\tau \, \frac{J^2(\tau)}{s^2+m^2}$$

Background Single Pole



Open Problems

(From the theory side)

- Multiple-exchange and loop diagrams ?
- Towards cosmological recursion relations (beyond rational correlators) ?
- (Perturbative) Källén-Lehmann spectral representation and positivity constraints ?
- Numerical Bootstrap to solve crossing equations non-perturbatively ?
- Particle production pole prescription accounting for all in-in branches ?
- Partial-wave expansion, crossing symmetry and Regge limit in de Sitter ?

- Organising principle to construct non-local EFTs ?
- Bounds on non-local EFTs ?
- Integrating out beyond tree level ?
- How to resum asymptotic time-derivative expansions ?

(A lot to be done also from the phenomenological and observational sides)

Thank you

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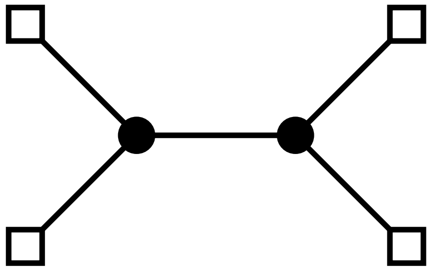
State-of-the-Art Perturbative Calculations

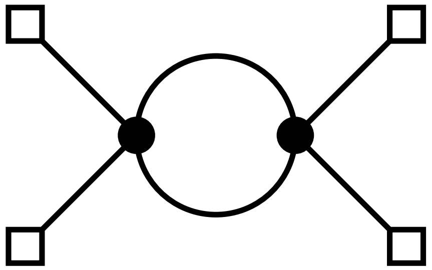
Cosmological Correlators (full results)



Tree-level single massive exchange diagram

"Generalised hypergeometric" (Kampé de Fériet)

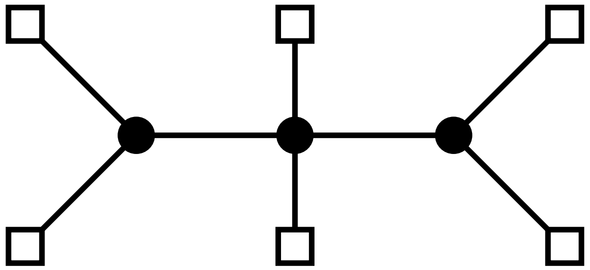
Arkani-Hamed, Baumann, Lee, Pimentel [2018]



One-loop massive diagram

Not-yet-named nor studied function

Xianyu, Zhang [2022]
Qin [2024]

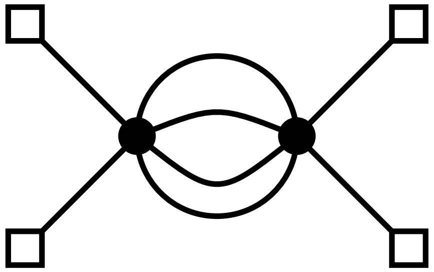


Tree-level double massive exchange diagram

Not-yet-named nor studied function

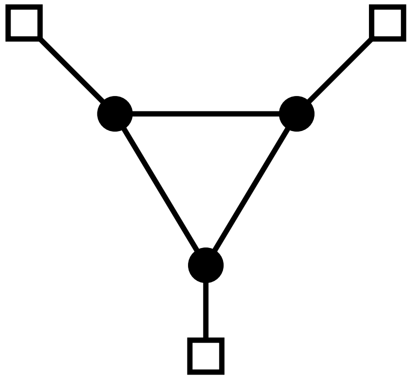
Aoki, Pinol, Sano, Yamaguchi, Zhu [2024]

Cosmological Correlators (signal only)



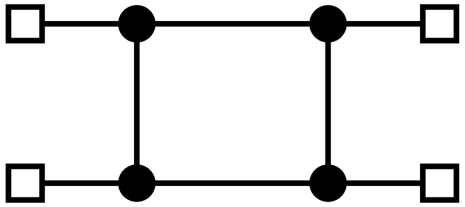
Melon diagrams

Qin, Xianyu [2023]



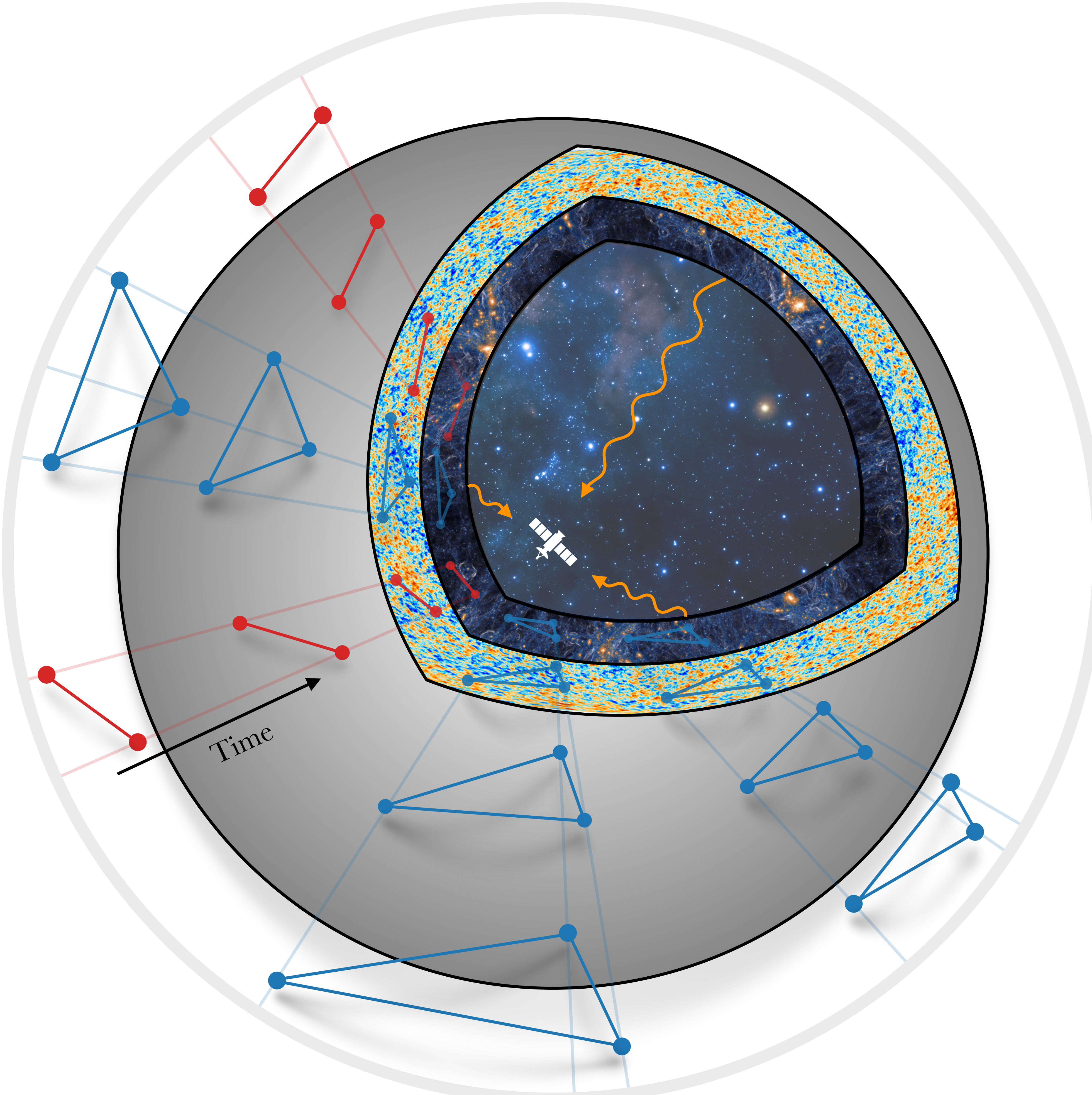
Triangle diagram

Qin, Xianyu [2023]



Box diagram

Qin, Xianyu [2023]


Time