# Open Effective Field Theories

## Progress and Puzzles

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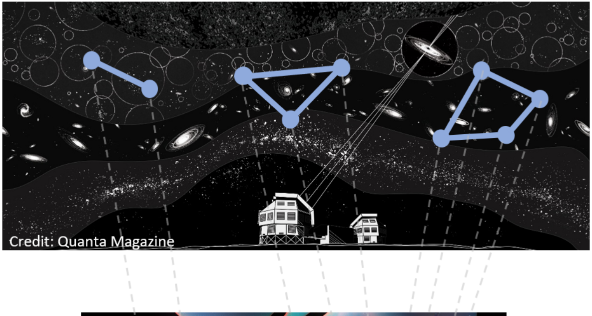
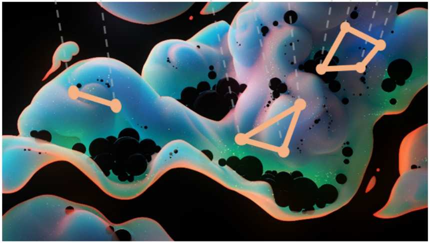
# Outline


1. Motivation
2. Open EFTs
3. Applications

# Cosmological correlators




$$\left\langle \prod_{i=1}^{n} \delta(\boldsymbol{k}_i) \right\rangle$$



$$\left\langle \prod_{i=1}^{n} \hat{\zeta}(\boldsymbol{k}_i, \eta_0) \right\rangle$$

# Schwinger-Keldysh formalism

At first sight, correlators necessitate finite-time QFT

$$\langle\hat{\mathcal{O}}(t)\rangle = \mathrm{Tr}\left[\hat{\rho}(t)\hat{\mathcal{O}}(t)\right] = \int \mathrm{d}\phi \rho_{\phi\phi}(t)\mathcal{O}(t)\,,$$

with

$$\rho_{\phi\phi'}(t) = \int_\Omega^\phi \mathcal{D}\varphi_+ \int_\Omega^{\phi'} \mathcal{D}\varphi_-\ e^{iS_{\mathrm{eff}}[\varphi_+,\varphi_-]}$$

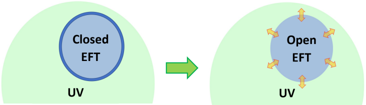
**Physical states** satisfy non-equilibrium constraints [Liu & Glorioso, 2018]

$$\text{i) } \mathrm{Tr}[\hat{\rho}] = 1, \qquad \text{ii) } \hat{\rho}^\dagger = \hat{\rho}, \qquad \text{iii) } \hat{\rho} \geq 0$$

which translate into

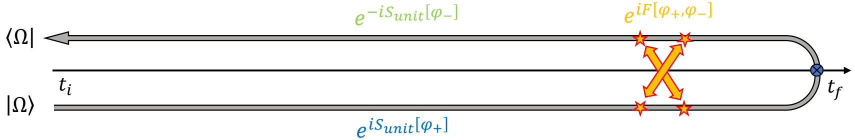
$$\text{i) } S_{\mathrm{eff}}\left[\varphi_+,\varphi_+\right] = 0, \ \ \text{ii) } S_{\mathrm{eff}}\left[\varphi_+,\varphi_-\right] = -S^*_{\mathrm{eff}}\left[\varphi_-,\varphi_+\right], \ \ \text{iii) } \Im\mathrm{m} S_{\mathrm{eff}}\left[\varphi_+,\varphi_-\right] \geq 0$$

*Is* $S_{\mathrm{eff}}[\varphi_+,\varphi_-] = S_{\mathrm{unit}}[\varphi_+] - S_{\mathrm{unit}}[\varphi_-]$ *too restrictive?*



# Non-equilibrium and Open QFT [Kamenev, 2011], [Breuer & Petruccione, 2007]

$$S_{\rm eff}[\varphi_+, \varphi_-] = S_{\rm unit}[\varphi_+] - S_{\rm unit}[\varphi_-] + F[\varphi_+, \varphi_-]\,.$$




Mixing between the branches of the path integral: non-unitary effects:

$\Rightarrow$ Dissipation, decoherence, thermalization, entropy production, $\cdots$

**Top-down**: in cosmology, many situations in which $F[\varphi_+, \varphi_-]$ *does not vanish*.

*Do EFT techniques have something to tell us about these effects?* $\Rightarrow$ *[2404.15416]*

*When do unitary effective evolutions emerge?* $\Rightarrow$ *[2411.09000]*

*How do we do physics with mixed states & information losses?* $\Rightarrow$ [2406.17856]

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# Single scalar

In Keldysh basis: *retarded* $\varphi_r = \frac{1}{2}(\varphi_+ + \varphi_-)$ and *advanced* $\varphi_a = \varphi_+ - \varphi_-$

Unitary part | Dissipation | Noise

$$S_{\rm eff}[\varphi_r, \varphi_a] = \int \mathrm{d}^4 x \left( \dot{\varphi}_r \dot{\varphi}_a - c_s^2 \partial_i \varphi_r \partial^i \varphi_a + \gamma \dot{\varphi}_r \varphi_a + i\beta \varphi_a^2 \right)$$

1. Unitary part: $S_\varphi[\varphi_+] - S_\varphi[\varphi_-]$ with usual kinetic term.
2. Dissipation: $-\frac{\gamma}{2} \int \mathrm{d}^4 x \left( \dot{\varphi}_+ \varphi_- - \varphi_+ \dot{\varphi}_- \right)$: no in-out counterpart.
3. Noise: even in $\varphi_a$, requires *Hubbard-Stratonovich trick*:

$$\exp\left( - \int d^4 x \beta \varphi_a^2 \right) = \mathcal{N}_0 \int \mathcal{D}\xi \; \exp\left[ \int d^4 x \left( -\frac{\xi^2}{4\beta} + i \xi \varphi_a \right) \right].$$

**Langevin equation:** stochastic differential equation

$$\ddot{\varphi}_r + \gamma \dot{\varphi}_r + c_s^2 k^2 \varphi_r = \xi \quad \text{with} \quad \langle \xi^2 \rangle = 2\beta.$$

$$c_s \leq 1, \qquad \gamma \geq 0, \qquad \beta \geq 0$$

# Propagators

*Causality structure*:

$$S_0[\varphi_r, \varphi_a] = -\frac{1}{2}\int \mathrm{d}^4x \int \mathrm{d}^4y \, (\varphi_r(x), \varphi_a(x)) \begin{pmatrix} 0 & \widehat{D}^A \\ \widehat{D}^R & -2i\widehat{D}^K \end{pmatrix} \begin{pmatrix} \varphi_r(y) \\ \varphi_a(y) \end{pmatrix}$$

*Free generating functional*:

$$Z_0 = \exp\left[-\frac{i}{2}\int \mathrm{d}^4x \int \mathrm{d}^4y \, (J_r(x), J_a(x)) \begin{pmatrix} G^K(x,y) & G^R(x,y) \\ G^A(x,y) & 0 \end{pmatrix} \begin{pmatrix} J_r(y) \\ J_a(y) \end{pmatrix}\right]$$

*Retarded/advanced* Green's function: how information **propagates**

$$\widehat{D}^R \circ G^R = \delta^{(4)}(x-y), \quad \widehat{D}^A \circ G^A = \delta^{(4)}(x-y)$$

*Keldysh*-Green's function: how the state is **occupied**

$$G^K = i(G^A \circ \widehat{D}^K \circ G^R + G^R \circ \widehat{D}^K \circ G^A)$$

$\Rightarrow -iG^K(k;t,t)$ *is the* ***power spectrum***.

# Initial conditions [Sieberer *et al.*, 2016]

## $i\epsilon$ prescription

$$\widehat{D}^{R/A} = (\partial_0 \pm \epsilon)^2 + \gamma\partial_0 - c_s^2\partial_i^2, \qquad -2i\widehat{D}^K = if(k)\epsilon + i\beta$$

- When $\gamma = 0$: standard **Feynman prescription**

$$G^{R/A}(k;\omega) = -\frac{1}{(\omega \mp i\epsilon)^2 - c_s^2 k^2} = -\frac{1}{2k}\left[\frac{1}{\omega - (c_s k \pm i\epsilon)} - \frac{1}{\omega - (-c_s k \pm i\epsilon)}\right]$$

with $f(k) = 2n(k) + 1$ and $n(k) = (e^{\frac{c_s k}{T}} - 1)^{-1}$: **initial occupation**.

- When $\gamma \neq 0$: dissipation leads to **initial conditions erasure**

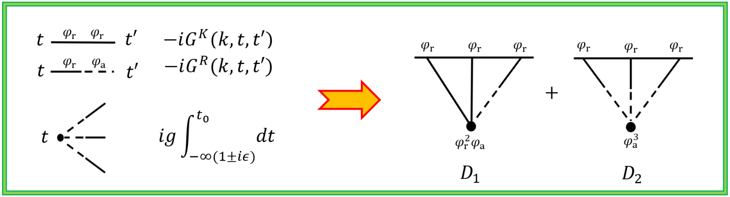
$$G^{R/A}(k;\omega) = -\frac{1}{\omega^2 \pm i\gamma\omega - c_s^2 k^2} = -\frac{1}{(\omega_- - \omega_+)}\left[\frac{1}{\omega - \omega_-} - \frac{1}{\omega - \omega_+}\right]$$

with $\omega_\pm \equiv -i(\gamma/2) \pm \sqrt{c_s^2 k^2 - \gamma^2/4} \Rightarrow$ if $\gamma > 0$, then $\mathrm{Im}\,\omega_\pm < 0$: **stability**.

# Interactions [Ema & Mukaida, 2024]

$$S_{\rm unit}[\varphi] \supset -\int \mathrm{d}^4x \frac{\lambda}{3!}\varphi^3, \quad \Rightarrow \quad S_{\rm eff}[\varphi_r, \varphi_a] \supset \int \mathrm{d}^4x \left(-\frac{\lambda}{2}\varphi_r^2\varphi_a - \frac{\lambda}{24}\varphi_a^3\right)$$

Correlators computed in **perturbation theory** using standard in-in rules:




$$D_1 = -\frac{\lambda}{2}\int_{-\infty(1\pm i\epsilon)}^{t_0} \mathrm{d}t \left[G^K(k_1;t_0,t)G^K(k_2;t_0,t)G^R(k_3;t_0,t)\right] + 5 \text{ perms.}$$

$$D_2 = -\frac{\lambda}{24}\int_{-\infty(1\pm i\epsilon)}^{t_0} \mathrm{d}t \left[G^R(k_1;t_0,t)G^R(k_2;t_0,t)G^R(k_3;t_0,t)\right] + 5 \text{ perms}$$

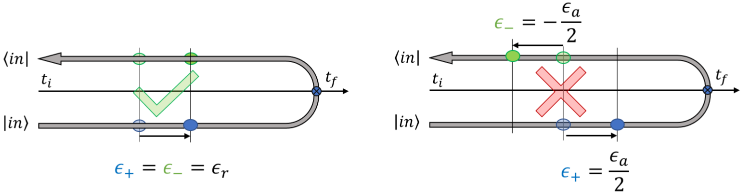
Using *unitary propagators*, recover standard results **upon summing all diagrams**.

# Time translation [Hongo *et al.*, 2018]

$S_{\rm unit}\left[\varphi_\pm\right]$ invariant under

$$\varphi_\pm(t,\boldsymbol{x}) \to \varphi'_\pm(t,\boldsymbol{x}) = \varphi_\pm(t+\epsilon_\pm,\boldsymbol{x}),$$

but $S_{\rm eff}\left[\varphi_+;\varphi_-\right]$ **is not** due to non-unitary effects:




Consider the $\epsilon_a$ broken symmetry:

$$\varphi_r(t,\boldsymbol{x}) \to \varphi'_r(t,\boldsymbol{x}) = \varphi_r(t,\boldsymbol{x}) + \frac{\epsilon_a}{2}\dot\varphi_a(t,\boldsymbol{x}) + \mathcal{O}\left(\epsilon_a^2\right),$$

$$\varphi_a(t,\boldsymbol{x}) \to \varphi'_a(t,\boldsymbol{x}) = \varphi_a(t,\boldsymbol{x}) + \epsilon_a\dot\varphi_r(t,\boldsymbol{x}) + \mathcal{O}\left(\epsilon_a^2\right).$$

- Invariant operators: $\dot\varphi_r\dot\varphi_a, \quad \partial_i\varphi_r\partial^i\varphi_a, \quad \varphi_r^2\varphi_a + \varphi_a^3/12, \quad \cdots$
- Symetry breaking operators: $\dot\varphi_r\varphi_a, \quad \varphi_a^2, \quad \varphi_a^3, \quad \cdots$

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# Open Electromagnetism [Agüí Salcedo, TC & Pajer, to appear]

*Dissipative theory* for a **massless spin** $1$ **photon**: theory of light in a medium.

*Keldysh basis*:

$$A^\mu = \frac{1}{2}\left(A^\mu_+ + A^\mu_-\right) , \qquad a^\mu = A^\mu_+ - A^\mu_- .$$

*Retarded gauge transformation* $\epsilon_+ = \epsilon_- = \epsilon$:

$$A^\mu \to A^\mu + \partial^\mu \epsilon , \qquad a^\mu \to a^\mu .$$

*Gauge invariant* functional: constructed out of $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and $a^\mu$.

$$S_1 = \int_{\omega,\boldsymbol{k}} \left[a^0 ik_i F^{0i} + a_i \left(\gamma_2 F^{0i} + \gamma_3 ik_j F^{ij} + \gamma_4 \epsilon^i_{\ jl} F^{jl}\right)\right] \equiv \int_{\omega,\boldsymbol{k}} a^\mu M_{\mu\nu} A^\nu ,$$

The theory is dissipative $\Leftrightarrow$ $M$ is non-Hermitian.

# Retarded and advanced gauges

1. Gauge invariance: $M_{\mu\nu}k^\nu = 0$ where $k^\mu = (\omega, \mathbf{k})$.
2. $\exists$ "right kernel" $\Rightarrow \exists$ "left kernel" such that $v^\mu M_{\mu\nu} = 0$.
3. $M$ is non-Hermitian $\Rightarrow$ different left and right kernels: $v^\mu = (i\gamma_2, \boldsymbol{k})$.

Conclusion: retarded gauge invariance generates advanced gauge invariance.

$S_1$ remains unchanged under

$$A^\mu \to A^\mu + \epsilon_r k^\mu, \qquad a^\mu \to a^\mu + \epsilon_a v^\mu.$$

Advanced gauge redundancy allows us to reduce number of advanced components.

$$\text{Maxwell: } \gamma_2 = -i\omega\,, \qquad \gamma_3 = -c^2 = -1\,, \qquad \gamma_4 = 0\,,$$

$\Rightarrow$ $M$ Hermitian, $v^\mu = k^\mu$: two copies of E&M gauge group [Akyuz, Goon & Penco, 2023].

# Dispersion relations

**Gauge fixing**

- retarded Coulomb gauge: $\partial_i A^i = 0$

$$\exists\ \epsilon_r \quad \text{s.t.} \quad k_i A'^i = 0, \quad \text{where} \quad A'^\mu = A^\mu + \epsilon_r k^\mu .$$

- advanced Coulomb gauge: $\partial_i a^i = 0$

$$\exists\ \epsilon_a \quad \text{s.t.} \quad k_i a'^i = 0, \quad \text{where} \quad a'^\mu = a^\mu + \epsilon_a v^\mu .$$

**Eigenvalues** of the kinetic matrix: constrained dof, 2 propagating dof

$$\left(k^2\,, i\gamma_2\omega + \gamma_3 k^2 + 2\gamma_4 k, i\gamma_2\omega + \gamma_3 k^2 - 2\gamma_4 k\right).$$

Introduce $\gamma_2 = \Gamma - i\omega$, $\gamma_3 = -c_s^2$:

$$\omega^2 + i\Gamma\omega - c_s^2 k^2 \pm 2\gamma_4 k = 0 \quad \Rightarrow \quad \omega = -i\frac{\Gamma}{2} \pm \sqrt{c_s^2 k^2 - (\Gamma/2)^2 \mp 2\gamma_4 k}\,.$$

# Noise constraint

Add some noise: $N_{\mu\nu}$ positive semi-definite

$$S = \int d^4x \, [a^\mu M_{\mu\nu} A^\nu + i a^\mu N_{\mu\nu} a^\nu] \, ,$$

*Hubbard-Stratonovich trick*:

$$\mathcal{Z} = \int [\mathcal{D}A^\mu] \int [\mathcal{D}a^\mu] \int [\mathcal{D}\xi_\mu] \exp \left[ \int d^4x \; i a^\mu \left( M_{\mu\nu} A^\nu - j_\nu - \xi_\nu \right) - \frac{1}{4} \xi_\mu (N^{-1})^{\mu\nu} \xi_\nu \right]$$

Advanced gauge symmetry $a^\mu \to a^\mu + \epsilon_a v^\mu$ induces noise constraint:

$$v^\mu (j_\mu + \xi_\mu) = 0.$$

*Crucial ingredients for understanding stochastic systems*
*in the presence of gauge symmetries.*

$\Rightarrow$ covariant gauges, propagators, interactions, anomaly, $\cdots$

# Recovering electromagnetism in a medium

From $S_{\rm eff}$, obtain modified Gauss and Ampère laws

$$\frac{\delta S_{\rm eff}}{\delta a^0} = 0 \qquad \Rightarrow \qquad \nabla.\mathbf{E} = j_0 + \xi_0,$$

$$\frac{\delta S_{\rm eff}}{\delta a^i} = 0 \qquad \Rightarrow \qquad \gamma_2 \mathbf{E} + \gamma_3 \nabla \times \mathbf{B} - 2\gamma_4 \mathbf{B} = \mathbf{j} + \xi.$$

Properties:

- Dispersive medium: $n \equiv v/c = 1/\sqrt{-\gamma_3}$;
- Dissipative medium: $\gamma_2 = -i\omega + \Gamma$;
- Anisotropic medium: $\gamma_4$;
- Random medium: $\xi_0$ and $\xi$.

*Dark matter & dark energy*: medium through which light/GW propagate:

*From Open E&M to Open GR?*

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# Open Effective Field Theory of Inflation [Agüí Salcedo, TC & Pajer, 2404.15416]

Decoupling limit + **derivative expansion** (up to one $\partial$/field):

*Building blocks:* $t+\pi_r$, $\pi_a$, $\partial_\mu(t+\pi_r)$, $\partial_\mu\pi_a$.

- *Quadratic order:* $1 \to 5$ EFT param (1 tadpole constraint):

$$S_{\rm eff}^{(2)} = \int \mathrm{d}^4x\sqrt{-g}\Big\{\underbrace{\dot\pi_r\dot\pi_a - c_s^2\partial_i\pi_r\partial^i\pi_a}_{\text{Kinetic term}}$$
$$\underbrace{-\gamma\dot\pi_r\pi_a}_{\text{Dissipation}} + i\underbrace{\left[\beta_1\pi_a^2 - (\beta_2-\beta_4)\,\dot\pi_a^2 + \beta_2\,(\partial_i\pi_a)^2\right]}_{\text{Noise}}\Big\}$$

- *Cubic order:* $1 \to 13$ EFT param: EFToI famous for relating operators at different orders because of **non-linearly realised boosts** [López Nacir *et al.*, 2011].

$$\text{EFToI}: \quad \mathcal{L} \supset \left(c_s^2-1\right)\left[-2\dot\pi_r + (\partial_\mu\pi_r)^2\right]\dot\pi_a$$
$$\text{Dissipation}: \quad \mathcal{L} \supset \gamma\left[-2\dot\pi_r + (\partial_\mu\pi_r)^2\right]\pi_a$$
$$\text{Noise}: \quad \mathcal{L} \supset i\beta_4\left(-\dot\pi_a + \partial_\mu\pi_r\partial^\mu\pi_a\right)^2$$

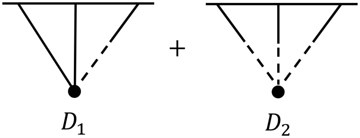
# Standard observables

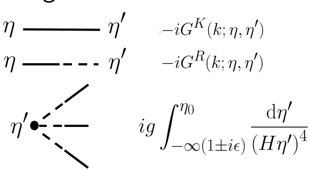
Symmetries ensure existence of nearly scale invariant power spectrum

$$\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle = \frac{H^2}{f_\pi^4} \langle \pi^c_{\boldsymbol{k}} \pi^c_{\boldsymbol{k}'} \rangle \equiv (2\pi)^3 \delta(\boldsymbol{k} + \boldsymbol{k}) \frac{2\pi^2}{k^3} \Delta^2_\zeta(k).$$

$\Rightarrow \Delta^2_\zeta = 10^{-9}$ obtained by imposing hierarchies of scales.

Bispectrum computed in **perturbation theory** using standard in-in rules.

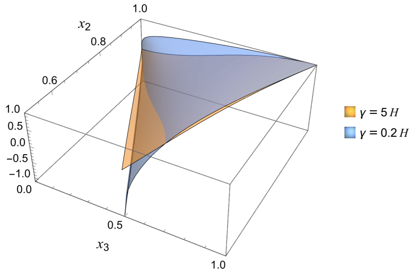



$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle = -\frac{H^3}{f_\pi^6} \langle \pi^c_{\boldsymbol{k}_1} \pi^c_{\boldsymbol{k}_2} \pi^c_{\boldsymbol{k}_3} \rangle \equiv (2\pi)^3 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B(k_1, k_2, k_3).$$

$$S(x_2, x_3) \equiv (x_2 x_3)^2 \frac{B(k_1, x_2 k_1, x_3 k_1)}{B(k_1, k_1, k_1)}, \qquad f_{\mathrm{NL}}(k_1, k_2, k_3) \equiv \frac{5}{6} \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + 2 \text{ perms.}}.$$

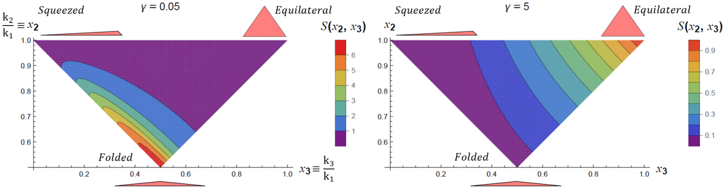
# Bispectrum shapes




**Main features**:

- $\gamma \gg H$: equilateral;
- $\gamma \ll H$: folded;
- Regularized divergence;
- Consistency relations.

Consistent with **flat-space/sub-Hubble** analytic results:

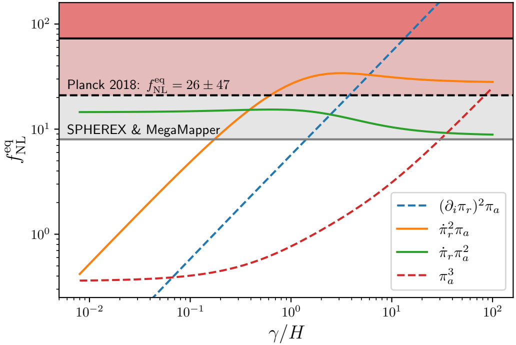


# Matching and $f_{\rm NL}$ with [Creminelli *et al.*, 2305.07695]

*UV completion*: inflaton $\phi$ + massive scalar field $\chi$ with softly-broken $U(1)$:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - |\partial\chi|^2 + M^2|\chi|^2 - \frac{\partial_\mu\phi}{f}\left(\chi\partial^\mu\chi^* - \chi^*\partial^\mu\chi\right) - \frac{1}{2}m^2\left(\chi^2+\chi^{*2}\right)\right].$$

$\Rightarrow$ narrow **instability band** in sub-Hubble regime: *local* particle production.



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# Beyond decoupling [Agüí Salcedo, TC & Pajer, to appear]

*Main challenge:* include **metric perturbations**

$$g = \frac{g_+ + g_-}{2} = \bar{g} + \delta g, \qquad \text{and} \qquad a = g_+ - g_- = \delta a$$

## Retarded unitary gauge:

**Retarded gauge fixing removes perturbations in one clock**

Open theory invariant only under diffs that are identical in each branch

$$\pi_r(t, \boldsymbol{x}) \equiv \frac{1}{2}\left[\pi_+(t, \boldsymbol{x}) + \pi_-(t, \boldsymbol{x})\right] = 0$$

$$S_{\text{eff}}[g_{\mu\nu}, a_{\mu\nu}, \pi_a] = \sum_{n=1}^{\infty} S_n \qquad \text{with} \qquad S_n = \mathcal{O}(a^p_{\mu\nu}, \pi^q_a), \quad p + q = n$$

with

$$S_1 = \int \mathrm{d}^4x\sqrt{-g}\Big\{\mathcal{O}\left[\text{diff-inv}\left(g_{\mu\nu}\right) a^{\mu\nu}\right] + \mathcal{O}\left[\text{diff-inv}\left(g_{\mu\nu}\right)\pi_a\right]$$
$$+ \mathcal{O}\left[\text{space-diff-inv}\left(g_{\mu\nu}\right) a^{\mu\nu}\right] + \mathcal{O}\left[\text{space-diff-inv}\left(g_{\mu\nu}\right)\pi_a\right]\Big\}.$$

# A minimal implementation

$$S_{\rm eff} = S_{f(t)} + S_{\Lambda(t)} + S_{c(t)} + S_{\rm SET} + S_{\rm dissip} + S_{\rm noise} + \cdots$$

Universal part:

$$S_{f(t)} \equiv \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_*^2}{2}f(t)G_{\mu\nu}a^{\mu\nu} + \frac{M_*^2}{2}f(t)g^{\mu\nu}\delta_a R_{\mu\nu} + \frac{M_*^2}{2}\dot{f}(t)R\pi_a\right],$$

$$S_{\Lambda(t)} \equiv \int \mathrm{d}^4x\sqrt{-g}\left[\frac{\Lambda(t)}{2}g_{\mu\nu}a^{\mu\nu} - \dot{\Lambda}(t)\pi_a\right],$$

$$S_{c(t)} \equiv \int \mathrm{d}^4x\sqrt{-g}\left[\frac{c(t)}{2}g^{00}g_{\mu\nu}a^{\mu\nu} - c(t)a^{00} - \dot{c}(t)g^{00}\pi_a - 2c(t)g^{0\mu}\partial_\mu\pi_a\right],$$

Minimal coupling:

$$S_{\rm SET} \equiv -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,T_{\mu\nu}a^{\mu\nu},$$

Dissipation and noise in the *scalar sector*:

$$S_{\rm dissip} \equiv -\int \mathrm{d}^4x\sqrt{-g}\,\Gamma(t)g^{00}\pi_a, \qquad S_{\rm noise} \equiv \int \mathrm{d}^4x\sqrt{-g}\,i\beta_\pi\pi_a^2,$$

# A glimpse of what to expect

**Background**: Interacting DE/DM sectors ($\dot{f} = 0$):

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = \Gamma \quad \text{and} \quad \dot{\rho}_m + 3H(\rho_m + p_m) = -\Gamma$$

⇒ an embedding for *Interacting Dark Energy* models (?)

**Perturbations**: modified (dissipative and stochastic) Einstein Equations

- **Clustering** ⇒ redshift space distortion (RSD) and weak lensing (WL)

$$k^2\langle\psi\rangle = -4\pi G\mu(a,k)a^2\rho_m\langle\delta\rangle, \qquad k^2\frac{\langle\psi+\phi\rangle}{2} = -4\pi G\Sigma(a,k)a^2\rho_m\langle\delta\rangle$$

- **Gravitational waves** ⇒ GW production, propagation and dissipation

*Rich phenomenology to explore,*
*eventually **already constrained from data**.*

# Summary

Open EFT are EFTs for finite-time QFT:

- dissipation & noise;
- entropy production & information losses;
- decoherence and (lack of) thermalization.

In this presentation:

1. *Brownian motion*: basic structure, NEQ constraints;
2. *Open E&M*: gauge symetries in stochastic QFT;
3. *Open EFToI*: inflationary phenomenology in the decoupling limit;
4. *Open EFToDE*: away from decoupling: DE and GW phenomenology.

# Outlook

1. Strengthen:
   - Loop corrections, renormalization & power counting;
   - Locality and its eventual breakdown.
2. Explore:
   - Close-from-equilibrium: approach to thermalization;
   - Far-from-equilibrium: non-equilibrium steady states.
3. Extend:
   - Bootstrap non-unitary evolution;
   - Bound EFT coefficients.

*Thank you!*

# Backup

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# Outline

4 Origin

5 Construction

6 Phenomenology

# Outline


4. Origin
5. Construction
6. Phenomenology

# *Ingredient 1:* In-in formalism

*Schrödinger picture*: consider some **observable**

$$\widehat{Q} \equiv \widehat{\zeta}(\boldsymbol{x}_1)\widehat{\zeta}(\boldsymbol{x}_2)\cdots\widehat{\zeta}(\boldsymbol{x}_n)$$

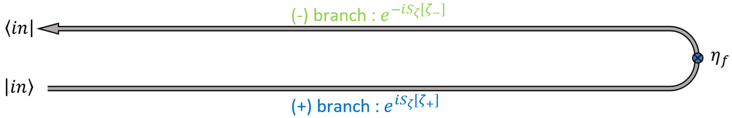
and some unitary **evolution operator** $\widehat{U}(\eta, \eta_0)$ so that

$$|\Psi(\eta)\rangle = \widehat{U}(\eta,\eta_0)\,|\mathrm{BD}\rangle \quad \text{with} \quad \langle\zeta|\,\widehat{U}(\eta,\eta_0)\,|\zeta_1\rangle = \int_{\zeta_1}^{\zeta} \mathcal{D}\left[\Phi\right] e^{iS[\Phi]}.$$

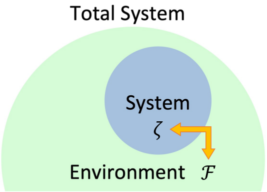
If $S\left[\Phi\right] = S_\zeta\left[\zeta\right]$, see [Donath & Pajer, 2402.05999]:

$$\left\langle \widehat{Q}(\eta) \right\rangle = \int \mathrm{d}\zeta\mathrm{d}\zeta_1\mathrm{d}\zeta_2\left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right]\left[\langle\zeta|\,\widehat{U}(\eta,\eta_0)\,|\zeta_1\rangle\right]\left[\langle\zeta_1|\mathrm{BD}\rangle\,\langle\mathrm{BD}|\zeta_2\rangle\right]\left[\langle\zeta_2|\,\widehat{U}^\dagger(\eta,\eta_0)\,|\zeta\rangle\right]$$

$$= \int \mathrm{d}\zeta\mathrm{d}\zeta_1\mathrm{d}\zeta_2\left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right]\int_{\zeta_1}^{\zeta}\mathcal{D}\left[\zeta_+\right]\int_{\zeta_2}^{\zeta}\mathcal{D}[\zeta_-]e^{iS_\zeta[\zeta_+]-iS_\zeta[\zeta_-]}\,\langle\zeta_1|\mathrm{BD}\rangle\,\langle\mathrm{BD}|\zeta_2\rangle$$



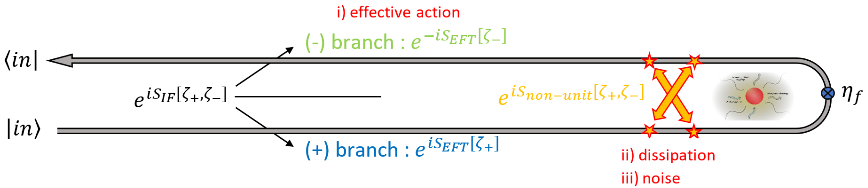
# Integrating out an environment




- $S[\Phi] = S_\zeta[\zeta] + S_\mathcal{F}[\mathcal{F}] + S_{\rm int}[\zeta;\mathcal{F}]$ with $\mathcal{F}$ a **hidden sector**.
- *Goal:* tracing out $\mathcal{F}$, the environment being **unobservable**.

Effects of the environment captured by the **Influence Functional** (IF):

$$\left\langle \widehat{Q}(\eta) \right\rangle = \int \mathrm{d}\zeta \mathrm{d}\zeta_1 \mathrm{d}\zeta_2 \left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right] \int_{\zeta_1}^{\zeta} \mathcal{D}[\zeta_+] \int_{\zeta_2}^{\zeta} \mathcal{D}[\zeta_-] e^{iS_\zeta[\zeta_+] - iS_\zeta[\zeta_-] + i\boldsymbol{S_{\rm IF}[\zeta_+;\zeta_-]}}$$




*What are the rules obeyed by $S_{\rm IF}[\zeta_+;\zeta_-]$?*

# *Ingredient 2*: The EFT of Inflation [Cheung *et al.*, 2008]

1. General perturbed FLRW universe: $\phi(t, \boldsymbol{x}) = \phi_0(t) + \delta\phi(t, \boldsymbol{x})$;
2. *Unitary gauge:* choose slicing $t = t(\phi)$ such that $\delta\phi = 0$;
3. Unit vector perpendicular to slicing: $n_\mu \equiv -\frac{\partial_\mu \phi}{\sqrt{-(\partial\phi)^2}} \to -\frac{\delta^0_\mu}{\sqrt{-g^{00}}}$.

$\phi_0(t) \Rightarrow$ time translation is broken: invariance under $3d$ **spatial diffeo** only!

*Allowed terms*:

- $4d$ covariant terms $(R, \cdots)$;
- time dependent functions $(\Lambda(t), \cdots)$;
- contractions with $n_\mu$ $(g^{00}, R^{00}, \cdots)$;
- extrinsic curvature $K_{\mu\nu} \equiv (g_\mu{}^\sigma + n_\mu n^\sigma)\nabla_\sigma n_\nu$.

$$S = \int \mathrm{d}^4 x \sqrt{-g} F(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t).$$

# Decoupling limit

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big[\frac{1}{2}M_{\mathrm{Pl}}^2 R - \Lambda(t) - c(t)g^{00} + \frac{1}{2}M_2^4(t)(\delta g^{00})^2 + \frac{1}{3!}M_3^4(t)(\delta g^{00})^3$$
$$-\frac{\bar{M}_1^3(t)}{2}(\delta g^{00})\delta K^{\mu}_{\ \mu} - \frac{\bar{M}_2^2(t)}{2}\left(\delta K^{\mu}_{\ \mu}\right)^2 - \frac{\bar{M}_3^2(t)}{2}\delta K^{\mu}_{\ \nu}\delta K^{\nu}_{\ \mu} + \cdots\Big]$$

Reintroduce scalar field by performing a **time-diffeo** $t \to t + \pi(x)$:

- $4d$ covariant terms do not transform under time diffeo;
- $\Lambda(t) \to \Lambda(t+\pi) = \Lambda(t) + \dot{\Lambda}(t)\pi + \frac{1}{2}\ddot{\Lambda}(t)\pi^2 + \cdots;$
- $g^{00} \to g^{00} + 2g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi$ and $\delta K \to \delta K - 3\dot{H}\pi - \partial_\mu\partial^\mu\pi.$

Two **simplifications**:

1. Slow-roll: Mixing $\pi/\delta g$ small as long as $E \sim H \gg E_{\mathrm{mix}} \sim \epsilon^{1/2}H$;
2. Derivative expansion: $\delta K$ tower $\ll \delta g^{00}$ tower.

$\Rightarrow$ **enough** to construct the theory out of $\partial_\mu(t+\pi)$.

*What if $\pi(x)$ also experiences* ***dissipation*** *and* ***noise****?*

# Outline

4 Origin

5 Construction

6 Phenomenology

# Constructing bottom-up Open EFT [Agüí Salcedo, TC & Pajer, 2404.15416]

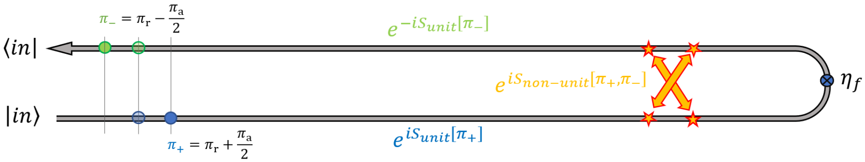
Develop **Open EFT for inflation**, building on [López Nacir, Porto, Senatore & Zaldarriaga, 2011]:

1. *Low-energy* degrees of freedom;
2. Physical principles and symmetries ⇒ *most generic functional*;
3. Radiatively stable power counting scheme ⇒ *finite number* of operators.

*Step 1:* Nambu-Goldstone boson of **spontaneous breaking** of **time-translation symmetry** by inflaton background [Cheung *et al.*, 2007].

*In-in formalism*: double fields for $+/-$ branches of path integral: $\pi_\pm = \pi_r \pm \frac{\pi_a}{2}$.

$$\mathcal{Z}[J_+, J_-] = \int \mathcal{D}\pi_+ \mathcal{D}\pi_- e^{iS_{\rm unit}[\pi_+] - iS_{\rm unit}[\pi_-] + iS_{\rm non-unit}[\pi_+,\pi_-]} e^{i\int \mathrm{d}^4x\sqrt{-g}J_\pm\pi_\pm}$$



# Non-equilibrium constraints [Liu & Glorioso, 2018]

*Step 2:* Requiring **Open QFT** originates from a unitary “closed” UV theory:

$$\text{i) } \mathrm{Tr}[\widehat{\rho}] = 1, \qquad \text{ii) } \widehat{\rho}^\dagger = \widehat{\rho} \qquad \text{and} \qquad \text{iii) } \widehat{\rho} \geq 0$$

implies constraints on $S_{\rm eff}[\pi_+, \pi_-] \equiv S_{\rm unit}[\pi_+] - S_{\rm unit}[\pi_-] + S_{\rm non-unit}[\pi_+, \pi_-]$:

| | | |
|---|---|---|
| i) | $S_{\rm eff}[\pi_+, \pi_+] = 0,$ | $S_{\rm eff}[\pi_r, \pi_a = 0] = 0;$ |
| ii) | $S_{\rm eff}[\pi_+, \pi_-] = -S^*_{\rm eff}[\pi_-, \pi_+],$ | $S_{\rm eff}[\pi_r, \pi_a] = -S^*_{\rm eff}[\pi_r, -\pi_a];$ |
| iii) | $\mathfrak{Im} S_{\rm eff}[\pi_+, \pi_-] \geq 0,$ | $\mathfrak{Im} S_{\rm eff}[\pi_r, \pi_a] \geq 0.$ |

*Consequences*:

1. $S_{\rm eff}[\pi_r, \pi_a]$ starts **linear** in $\pi_a$;
2. **Odd** powers of $\pi_a$ are purely real; **even** powers of $\pi_a$ purely imaginary;
3. Positivity bounds on the **noise coefficients**.

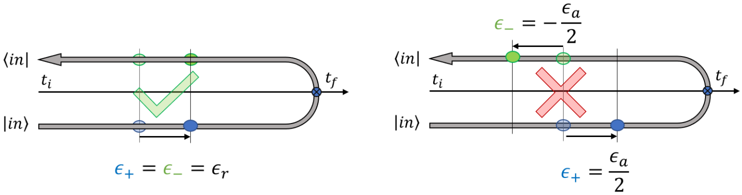
$\Rightarrow$ *Already reduce the scope of available Open EFTs*

# In-in coset construction [Hongo *et al.*, 2018], [Akyuz, Goon & Penco, 2023]

$S_{\rm unit}\,[\pi_\pm]$ invariant under

$$\pi_\pm(t) \to \pi'_\pm(t) = \pi_\pm(t+\epsilon_\pm) + \epsilon_\pm,$$

but $S_{\rm non-unit}\,[\pi_+;\pi_-]$ **is not** due to non-unitary effects $G_+ \times G_- \to G_{\rm diag}$:




$$\pi_r(t) \to \pi'_r(t) = \pi_r(t+\epsilon_r) + \epsilon_r, \qquad \pi_a(t) \to \pi'_a(t) = \pi_a(t+\epsilon_r).$$

*Building blocks:* $\pi_a$, $t+\pi_r$, $\partial_\mu \pi_a$, $\partial_\mu(t+\pi_r)$.

*Step 3:* Locality: strong assumption yet necessary for **truncatable power counting scheme**.

# Open Effective Field Theory of Inflation

Decoupling limit + **derivative expansion** (up to one $\partial$/field):

- *Quadratic order:* 1 $\rightarrow$ 5 EFT param (1 tadpole constraint):

Kinetic term

$$S_{\text{eff}}^{(2)} = \int \mathrm{d}^4x\sqrt{-g}\left\{\dot{\pi}_r\dot{\pi}_a - c_s^2\partial_i\pi_r\partial^i\pi_a \right.$$
$$\left. -\gamma\dot{\pi}_r\pi_a + i\left[\beta_1\pi_a^2 - (\beta_2-\beta_4)\,\dot{\pi}_a^2 + \beta_2\left(\partial_i\pi_a\right)^2\right]\right\}$$

Dissipation — Noise

- *Cubic order:* 1 $\rightarrow$ 13 EFT param: EFToI famous for relating operators at different orders because of **non-linearly realised boosts** [López Nacir *et al.*, 2011].

$$\text{EFToI}: \quad \mathcal{L} \supset \left(c_s^2-1\right)\left[-2\dot{\pi}_r + (\partial_\mu\pi_r)^2\right]\dot{\pi}_a$$
$$\text{Dissipation}: \quad \mathcal{L} \supset \gamma\left[-2\dot{\pi}_r + (\partial_\mu\pi_r)^2\right]\pi_a$$
$$\text{Noise}: \quad \mathcal{L} \supset i\beta_4\left(-\dot{\pi}_a + \partial_\mu\pi_r\partial^\mu\pi_a\right)^2$$

*Recover and extend EFToI construction.*

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# Free theory and propagators

Free theory path integral:

$$\mathcal{Z}[J_r, J_a] = \int_\Omega^\pi \mathcal{D}\pi_r \int_\Omega^0 \mathcal{D}\pi_a e^{i\int d^4x\sqrt{-g}(\pi_r,\pi_a)\begin{pmatrix}0 & \widehat{D}_A \\ \widehat{D}_R & i\widehat{D}_K\end{pmatrix}\begin{pmatrix}\pi_r \\ \pi_a\end{pmatrix} + \int d^4x(J_r\pi_r + J_a\pi_a)}$$

*Propagators*: **retarded/advanced** $G^{R/A}$ and **Keldysh-Green** $G^K$:

1. *Dissipative retarded Green function*:

$$G^R(k;\eta_1,\eta_2) = \frac{\pi}{2}H^2(\eta_1\eta_2)^{\frac{3}{2}}\left(\frac{\eta_1}{\eta_2}\right)^{\frac{\gamma}{2H}}\Im\mathrm{m}\left[H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_1)H^{(2)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_2)\right]\theta(\eta_1-\eta_2)$$

2. *Keldysh-Green function* ($A_\gamma$ and $B_\gamma$ complicated combin of ${}_2F_3$):

$$G^K(k;\eta_1,\eta_2) = i\frac{\pi^2\beta_1^2}{8}(\eta_1\eta_2)^{\frac{3}{2}+\frac{\gamma}{2H}}\Re\mathrm{e}\Big[H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_1)H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_2)A_\gamma(-k\eta_2)$$
$$-H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_1)H^{(2)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_2)B_\gamma(z_2)\Big] + (\eta_1 \leftrightarrow \eta_2)$$

# Dissipative power spectrum

Symmetries ensure existence of nearly scale invariant power spectrum:

$$\Delta_\zeta^2(k) = \frac{1}{c_s^3}\frac{\beta_1}{H^2}\frac{H^4}{f_\pi^4}2^{1+\frac{\gamma}{H}}\frac{\Gamma\left(\frac{1}{2}+\frac{\gamma}{2H}\right)\Gamma\left(\frac{3}{2}+\frac{\gamma}{2H}\right)^2}{\Gamma\left(1+\frac{\gamma}{2H}\right)\Gamma\left(\frac{5}{2}+\frac{\gamma}{H}\right)} \propto \begin{cases} \frac{\beta_1}{H^2}\frac{H^4}{f_\pi^4}\sqrt{\frac{H}{\gamma}}\left[1+\mathcal{O}\left(\frac{H}{\gamma}\right)\right], & \gamma \gg H, \\ \frac{\beta_1}{H^2}\frac{H^4}{f_\pi^4}\left[1+\mathcal{O}\left(\frac{\gamma}{H}\right)\right], & \gamma \ll H. \end{cases}$$

$\Rightarrow \Delta_\zeta^2 = 10^{-9}$ obtained by imposing hierarchies of scales.

- Imposing **thermal equilibrium** at temp. $T$ (*KMS symmetry*: $\beta_1 \propto \gamma T$):

$$\Delta_\zeta^2 \propto \frac{T}{H}\frac{H^4}{f_\pi^4}\sqrt{\frac{\gamma}{H}}$$

  $\Rightarrow$ recover warm inflation predictions [Berera, 1995], [Montefalcone *et al.*, 2023].

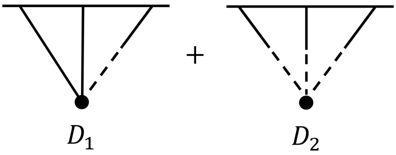
- Extend to **scale-dependent noises** $\beta_2, \beta_4 \neq 0$: also scale invariant

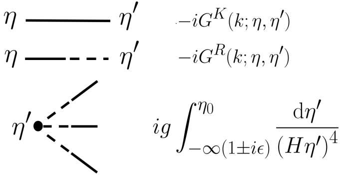
$$\Delta_\zeta^2(k) \supset \begin{cases} \frac{15}{4c_s^3}(\beta_4-\beta_2)\frac{H^4}{f_\pi^4}2^{\frac{\gamma}{H}}\frac{\Gamma\left(-\frac{1}{2}+\frac{\gamma}{2H}\right)\Gamma\left(\frac{3}{2}+\frac{\gamma}{2H}\right)^2}{\Gamma\left(\frac{\gamma}{2H}\right)\Gamma\left(\frac{5}{2}+\frac{\gamma}{H}\right)}, & \text{for} \quad i(\beta_4-\beta_2)\dot{\pi}_a^2, \\ \frac{3}{2c_s^5}\beta_2\frac{H^4}{f_\pi^4}2^{\frac{\gamma}{H}}\frac{\Gamma\left(-\frac{1}{2}+\frac{\gamma}{2H}\right)\Gamma\left(\frac{3}{2}+\frac{\gamma}{2H}\right)^2}{\Gamma\left(\frac{\gamma}{2H}\right)\Gamma\left(\frac{3}{2}+\frac{\gamma}{H}\right)}, & \text{for} \quad i\beta_2(\partial_i\pi_a)^2. \end{cases}$$

# Interactions and non-Gaussianities

$$\langle \zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle = -\frac{H^3}{f_\pi^6}\langle \pi^c_{\boldsymbol{k}_1}\pi^c_{\boldsymbol{k}_2}\pi^c_{\boldsymbol{k}_3}\rangle \equiv (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B(k_1,k_2,k_3).$$

Correlators computed in **perturbation theory** using standard in-in rules.




**Contact bispectrum:**

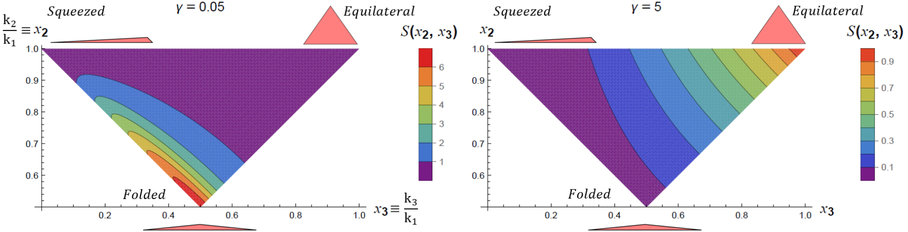
$$B(k_1,k_2,k_3) = (-i)^{n_K+n_R+1}\frac{H^3}{f_\pi^6}\frac{g}{H^{4-n_d}}\int_{-\infty(1\pm i\epsilon)}^{0^-}\frac{\mathrm{d}\eta}{\eta^{4-n_d}}$$
$$\widehat{\mathcal{D}}(\{\boldsymbol{k}_i\},\partial_\eta)\left[G^{K/R}(k_1,0,\eta)G^{K/R}(k_2,0,\eta)G^R(k_3,0,\eta)+5\text{ perms.}\right]$$

$$S(x_2,x_3)\equiv(x_2x_3)^2\frac{B(k_1,x_2k_1,x_3k_1)}{B(k_1,k_1,k_1)},\qquad f_{\mathrm{NL}}(k_1,k_2,k_3)\equiv\frac{5}{6}\frac{B(k_1,k_2,k_3)}{P(k_1)P(k_2)+2\text{ perms.}}.$$

# Flat space intuition

**Contact bispectrum** $B(k_1, k_2, k_3) \sim \mathrm{Poly}_\gamma / \mathrm{Sing}_\gamma$ with $E_k^\gamma \equiv \sqrt{c_s^2 k^2 - \gamma^2/4}$

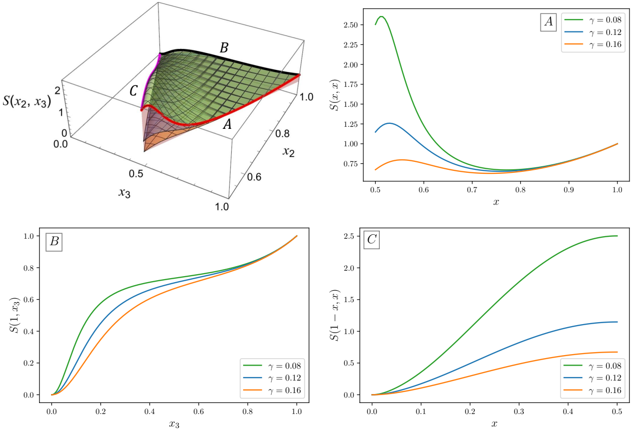
$$\mathrm{Sing}_\gamma = |E_1^\gamma + E_2^\gamma + E_3^\gamma + \frac{3}{2} i\gamma|^2 \times |-E_1^\gamma + E_2^\gamma + E_3^\gamma + \frac{3}{2} i\gamma|^2 \times |E_1^\gamma - E_2^\gamma + E_3^\gamma + \frac{3}{2} i\gamma|^2 \times |E_1^\gamma + E_2^\gamma - E_3^\gamma + \frac{3}{2} i\gamma|^2,$$




$\Rightarrow$ at **small dissipation**, peaks when $k_1 \pm k_2 \pm k_3 = 0$, *i.e.* folded triangles.

# Fingerprints

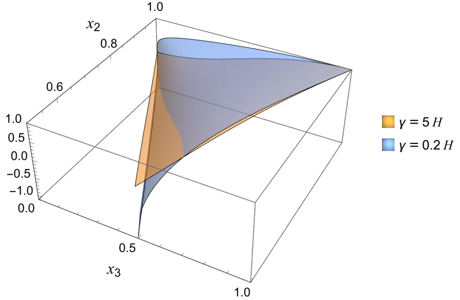
**Smoking gun**: peaks near folded triangles when $\gamma \ll H$




1. May $\exists$ intermediate peak in the **small dissipation regime**;
2. Sing$_\gamma$ regulated: different from **non-BD ICs** [Chen *et al.*, 2007], [Holman & Tolley, 2008];
3. **Squeezed limit** goes to zero because of symmetries (consistency relations).

# de Sitter bispectrum: shape function

Analytical results **hard to reach** $\Rightarrow$ mostly rely on numerics.




**Main features unchanged**:

- $\gamma \gg H$: equilateral;
- $\gamma \ll H$: folded;
- Consistency relations;
- Regularized divergence.

Convergence in the sub-Hubble regime:

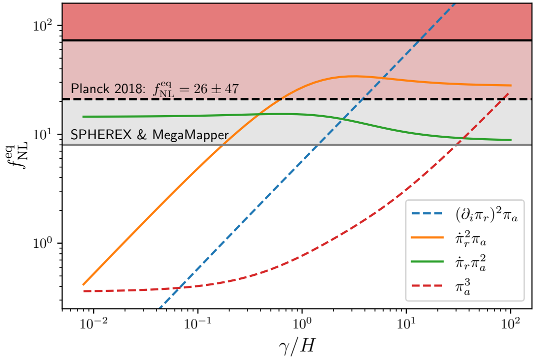
$$\left| \int \mathrm{d}\eta \frac{e^{-i(\pm k_1 \pm k_2 \pm k_3)\eta}}{\eta^{1+\frac{3}{2}\frac{\gamma}{H}}} \right| < \infty \qquad \text{when} \qquad \frac{\gamma}{H} > 0.$$

# de Sitter bispectrum: $f_{\rm NL}$ heuristic estimate

1. Adiabatic perturbations $\pi_r \sim \frac{f_\pi^2 \Delta_\zeta}{H}$ **freeze** at $\frac{c_s k}{a_* H} \sim \sqrt{\frac{H+\gamma}{H}}$;
2. Noise-sourced dynamics $\pi_r \sim \frac{\beta_1}{H(H+\gamma)}\pi_a \Rightarrow$ **dominant** quadratic term: $a^4\beta_1\pi_a^2$.

Estimate non-Gaussianities from $f_{\rm NL}\Delta_\zeta \sim \mathcal{L}_3/(a^4\beta_1\pi_a^2)$:

$$\frac{\gamma}{f_\pi^2}(\partial_i\pi_r)^2\pi_a \to f_{\rm NL} \sim \frac{1}{c_s^2}\frac{\gamma}{H}, \qquad \frac{\gamma}{f_\pi^2}\pi_r'^2\pi_a \to f_{\rm NL} \sim \frac{\gamma}{H+\gamma}, \qquad \frac{i\beta_5}{f_\pi^2}\pi_r'\pi_a^2 \to f_{\rm NL} \sim \frac{\beta_5}{\beta_1}.$$



# Matching with [Creminelli *et al.*, 2305.07695]

*UV completion*: inflaton $\phi$ + massive scalar field $\chi$ with softly-broken $U(1)$:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\mathrm{Pl}}^2 R - \frac{1}{2}\left(\partial\phi\right)^2 - V(\phi) - \left|\partial\chi\right|^2 + M^2\left|\chi\right|^2 \right.$$
$$\left. -\frac{\partial_\mu\phi}{f}\left(\chi\partial^\mu\chi^* - \chi^*\partial^\mu\chi\right) - \frac{1}{2}m^2\left(\chi^2 + \chi^{*2}\right)\right].$$

$\Rightarrow$ narrow **instability band** in sub-Hubble regime: *local* particle production.

Dynamics described in terms of a **non-linear Langevin equation**

$$\pi'' + (2H+\gamma)\,a\pi' - \partial_i^2\pi \simeq \frac{\gamma}{2\rho f}\left[\left(\partial_i\pi\right)^2 - 2\pi\xi\pi'^2\right] - \frac{a^2m^2}{f}\left(1 + 2\pi\xi\frac{\pi'}{a\rho f}\right)\delta\mathcal{O}_S\,,$$

with non-Gaussian noise $\delta\mathcal{O}_S$ $\Rightarrow$ completely equivalent to

$$S_{\mathrm{eff}} = \int \mathrm{d}^4x\left[a^2\pi_r'\pi_a' - c_s^2a^2\partial_i\pi_r\partial^i\pi_a - a^3\gamma\pi_r'\pi_a + i\beta_1a^4\pi_a^2\right.$$
$$\left. + \frac{(8\gamma_2-\gamma)}{2f_\pi^2}a^2\pi_r'^2\pi_a + \frac{\gamma}{2f_\pi^2}a^2\left(\partial_i\pi_r\right)^2\pi_a - 2i\frac{\beta_5}{f_\pi^2}a^3\pi_r'\pi_a^2 + \frac{\delta_1}{f_\pi^2}a^4\pi_a^3\right].$$

# Summary and prospects

- We developed a systematic Open EFT for inflation building on **previous work** by [López Nacir, Porto, Senatore & Zaldarriaga, 2011].
- Assuming locality, the resulting EFT is easy to write and can be studied in **perturbation theory**.
- **Smoking gun** signal are peaks in the bispectrum near folded triangles in the small dissipation regime.
- Our formalism is a **starting point** to study dissipative and diffusive effects in primordial cosmology and beyond.

Future directions:

- *Theory*: beyond decoupling, tensor modes, **dark energy** $\cdots$
- *Observations*: **constraints** on EFT parameters (*e.g. folded bispectrum*).

# Appendices

Thomas Colas

# Outline


7. More motivations
8. More construction
9. More phenomenology

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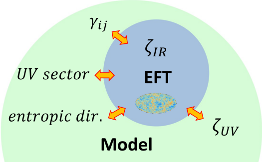
7 More motivations

8 More construction

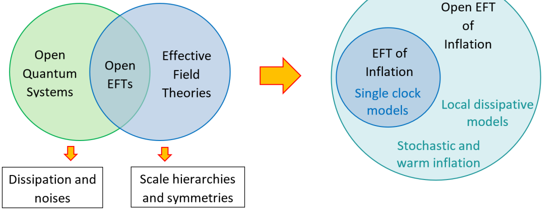
9 More phenomenology

# Why Open EFTs?

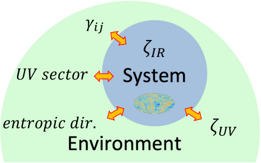



**EFTs** focus on observable dof while encoding unknown physics in free coefficients.

1. EFTs are useful middle grounds between observations and models;
2. EFT of Inflation [Cheung *et al.*, 2007] embed single-clock models. **Extensions?**
3. Relaxing unitarity open doors to **dissipation and noises**.



# Why Open EFTs?




**EFTs** focus on observable dof while encoding unknown physics in free coefficients.

1. EFTs are useful middle ground between observations and models:
   - A convenient parametrization for **degenerate models**;
   - **Test physical principles** rather than microphysics: bootstrap.
2. The EFT of Inflation [Cheung *et al.*, 2007]:
   - Provides an **embedding** for many single-clock inflationary models;
   - ∃ **well-motivated classes** of models which do not fit this description.
3. To embed them in an EFT, we need to relax assumptions:
   - **Main inputs**: symmetries, locality, unitarity;
   - Relaxing unitary open the door to **dissipation and noises**.

# *Ingredient 1:* In-in formalism

*Schrödinger picture*: consider some **observable**

$$\widehat{Q} \equiv \widehat{\zeta}(\boldsymbol{x}_1)\widehat{\zeta}(\boldsymbol{x}_2)\cdots\widehat{\zeta}(\boldsymbol{x}_n)$$

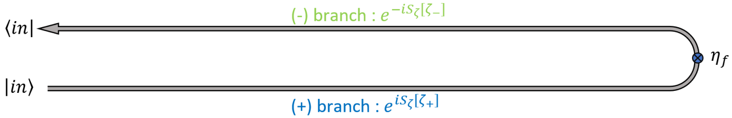
and some unitary **evolution operator** $\widehat{U}(\eta,\eta_0)$ so that

$$|\Psi(\eta)\rangle = \widehat{U}(\eta,\eta_0)\,|\mathrm{BD}\rangle \quad \text{with} \quad \langle\zeta|\,\widehat{U}(\eta,\eta_0)\,|\zeta_1\rangle = \int_{\zeta_1}^{\zeta}\mathcal{D}\left[\Phi\right]e^{iS[\Phi]}.$$

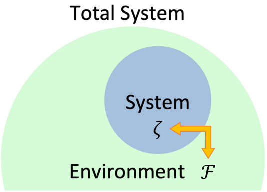
If $S\left[\Phi\right] = S_\zeta\left[\zeta\right]$, see [Donath & Pajer, 2402.05999]:

$$\left\langle\widehat{Q}(\eta)\right\rangle = \int \mathrm{d}\zeta\mathrm{d}\zeta_1\mathrm{d}\zeta_2\left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right]\left[\langle\zeta|\,\widehat{U}(\eta,\eta_0)\,|\zeta_1\rangle\right]\left[\langle\zeta_1|\mathrm{BD}\rangle\,\langle\mathrm{BD}|\zeta_2\rangle\right]\left[\langle\zeta_2|\,\widehat{U}^\dagger(\eta,\eta_0)\,|\zeta\rangle\right]$$

$$= \int \mathrm{d}\zeta\mathrm{d}\zeta_1\mathrm{d}\zeta_2\left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right]\int_{\zeta_1}^{\zeta}\mathcal{D}\left[\zeta_+\right]\int_{\zeta_2}^{\zeta}\mathcal{D}[\zeta_-]e^{iS_\zeta[\zeta_+]-iS_\zeta[\zeta_-]}\,\langle\zeta_1|\mathrm{BD}\rangle\,\langle\mathrm{BD}|\zeta_2\rangle$$



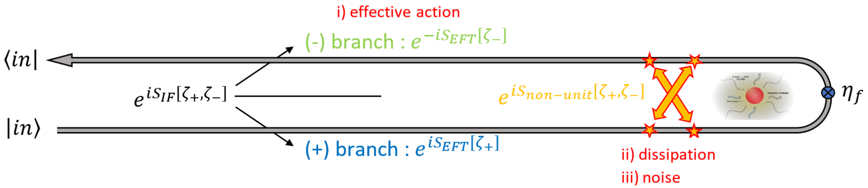
# Integrating out an environment




- $S[\Phi] = S_\zeta[\zeta] + S_{\mathcal{F}}[\mathcal{F}] + S_{\rm int}[\zeta;\mathcal{F}]$ with $\mathcal{F}$ a **hidden sector**.
- *Goal:* tracing out $\mathcal{F}$, the environment being **unobservable**.

Effects of the environment captured by the **Influence Functional** (IF):

$$\left\langle \widehat{Q}(\eta)\right\rangle = \int \mathrm{d}\zeta \mathrm{d}\zeta_1 \mathrm{d}\zeta_2 \left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right] \int_{\zeta_1}^{\zeta} \mathcal{D}[\zeta_+] \int_{\zeta_2}^{\zeta} \mathcal{D}[\zeta_-] e^{iS_\zeta[\zeta_+] - iS_\zeta[\zeta_-] + i\boldsymbol{S}_{\rm IF}[\boldsymbol{\zeta}_+;\boldsymbol{\zeta}_-]}$$




*What are the rules obeyed by* $S_{\rm IF}[\zeta_+;\zeta_-]$*?*

# *Ingredient 2*: The EFT of Inflation [Cheung *et al.*, 2008]

1. General perturbed FLRW universe: $\phi(t, \boldsymbol{x}) = \phi_0(t) + \delta\phi(t, \boldsymbol{x})$;
2. *Unitary gauge:* choose slicing $t = t(\phi)$ such that $\delta\phi = 0$;
3. Unit vector perpendicular to slicing: $n_\mu \equiv -\frac{\partial_\mu \phi}{\sqrt{-(\partial\phi)^2}} \to -\frac{\delta^0_\mu}{\sqrt{-g^{00}}}$.

$\phi_0(t) \Rightarrow$ time translation is broken: invariance under $3d$ **spatial diffeo** only!

*Allowed terms*:

- $4d$ covariant terms $(R, \cdots)$;
- time dependent functions $(\Lambda(t), \cdots)$;
- contractions with $n_\mu$ $(g^{00}, R^{00}, \cdots)$;
- extrinsic curvature $K_{\mu\nu} \equiv ({g_\mu}^\sigma + n_\mu n^\sigma)\nabla_\sigma n_\nu$.

$$S = \int \mathrm{d}^4 x \sqrt{-g} F(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t).$$

# Decoupling limit

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big[\frac{1}{2}M_{\rm Pl}^2 R - \Lambda(t) - c(t)g^{00} + \frac{1}{2}M_2^4(t)(\delta g^{00})^2 + \frac{1}{3!}M_3^4(t)(\delta g^{00})^3$$
$$-\frac{\bar{M}_1^3(t)}{2}(\delta g^{00})\delta K^\mu_{\ \mu} - \frac{\bar{M}_2^2(t)}{2}\left(\delta K^\mu_{\ \mu}\right)^2 - \frac{\bar{M}_3^2(t)}{2}\delta K^\mu_{\ \nu}\delta K^\nu_{\ \mu} + \cdots\Big]$$

Reintroduce scalar field by performing a **time-diffeo** $t \to t + \pi(x)$:

- $4d$ covariant terms do not transform under time diffeo;
- $\Lambda(t) \to \Lambda(t+\pi) = \Lambda(t) + \dot{\Lambda}(t)\pi + \frac{1}{2}\ddot{\Lambda}(t)\pi^2 + \cdots$;
- $g^{00} \to g^{00} + 2g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi$ and $\delta K \to \delta K - 3\dot{H}\pi - \partial_\mu\partial^\mu\pi$.

Two **simplifications**:

1. Slow-roll: Mixing $\pi/\delta g$ small as long as $E \sim H \gg E_{\rm mix} \sim \epsilon^{1/2}H$;
2. Derivative expansion: $\delta K$ tower $\ll \delta g^{00}$ tower.

$\Rightarrow$ **enough** to construct the theory out of $\partial_\mu(t+\pi)$.

*What if $\pi(x)$ also experiences **dissipation** and **noise**?*

# The problem

- Start with unitary evolution:

$$\rho_{\pi\sigma,\pi'\sigma'}(\eta) \equiv \langle\pi'| \otimes \langle\psi'| \,\widehat{\rho}(\eta)\, |\pi\rangle \otimes |\psi\rangle = \Psi\left[\pi,\sigma\right] \Psi^*\left[\pi',\sigma'\right].$$

- Integrate out $\sigma$: cannot write the state of $\pi$ as a wavefunction

$$\rho_{\pi\pi'}(\eta) \neq \Psi_{\mathrm{red}}\left[\pi\right] \Psi^*_{\mathrm{red}}\left[\pi'\right].$$

- There is an extra piece which does not obey the rules of unitary EFTs.

*How can we understand it?*

# Top-down approach

$$e^{iS_{\rm IF}[\zeta_+;\zeta_-]} = \sum_{\mathcal{F},\mathcal{F}_1,\mathcal{F}_2} \int_{\mathcal{F}_1}^{\mathcal{F}} \mathcal{D}[\mathcal{F}_+] \int_{\mathcal{F}_2}^{\mathcal{F}} \mathcal{D}[\mathcal{F}_-]$$

$$e^{iS_{\mathcal{F}}[\mathcal{F}_+]+iS_{\rm int}[\zeta_+;\mathcal{F}_+]-iS_{\mathcal{F}}[\mathcal{F}_-]-iS_{\rm int}[\zeta_-;\mathcal{F}_-]} \left\langle \mathcal{F}_1 \right| \widehat{\rho}_{\mathcal{F},0} \left| \mathcal{F}_2 \right\rangle$$

Suitable for **perturbative expansion**, e.g. $S_{\rm int}[\zeta;\mathcal{F}] = g\int \mathrm{d}^4 x J_{\mathcal{S}}[\zeta] J_{\mathcal{E}}[\mathcal{F}]$

$$iS_{\rm IF}[\zeta_+;\zeta_-] = -\frac{g^2}{2}\int \mathrm{d}^4x \int \mathrm{d}^4y \Big[ J_{\mathcal{S}}^+(x) G_{++}(x,y) J_{\mathcal{S}}^+(y) + J_{\mathcal{S}}^-(x) G_{--}(x,y) J_{\mathcal{S}}^-(y)$$

$$- J_{\mathcal{S}}^+(x) G_{+-}(x,y) J_{\mathcal{S}}^-(y) - J_{\mathcal{S}}^-(x) G_{-+}(x,y) J_{\mathcal{S}}^+(y) \Big]$$

with

$$G_{+-}(x,y) \equiv \langle \widehat{J}_{\mathcal{E}}(x) \widehat{J}_{\mathcal{E}}(y) \rangle_0 = G_{-+}^*(x,y)$$

$$G_{++}(x,y) \equiv \langle \mathcal{T}[\widehat{J}_{\mathcal{E}}(x) \widehat{J}_{\mathcal{E}}(y)] \rangle_0 = G_{--}^*(x,y)$$

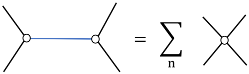
Environment encoded in **unequal-time correlators**: $\left\langle \widehat{v}^{\mathcal{E}}_{-\boldsymbol{k}}(\eta) \widehat{v}^{\mathcal{E}}_{-\boldsymbol{k}_1}(\eta_1) \widehat{v}^{\mathcal{E}}_{-\boldsymbol{k}_2}(\eta_2) \right\rangle' , \cdots$

# An example from [Agüí Salcedo, Gordon Lee, Melville & Pajer, 2022]

$$\mathcal{L}_{\rm UV}[\phi,\chi] = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}M^2\chi^2 - gM\phi^2\chi$$

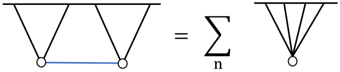
*Amplitudes*

$$A_4 \sim \frac{g^2}{s+M^2} \sim \frac{g^2}{M^2}\sum_n \left(-\frac{s}{M^2}\right)^n$$




*Correlators*

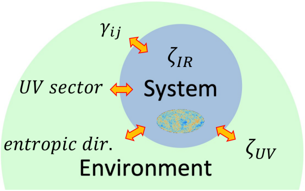
$$B_4 \sim \sum_n \frac{B_{4,n}^{\rm even}}{M^{2n}} + \frac{B_{4,n}^{\rm odd}}{M^{2n+1}}$$




+ ???

*No unitary local EFT can reproduce the low energy expansion for $B_4$.*

# Relevance for cosmology




Integrate out:

- high-energy extensions;
- multifield hidden sectors;
- short and soft modes;

and write an **EFT** for adiabatic dof $\zeta_{\rm IR}$.

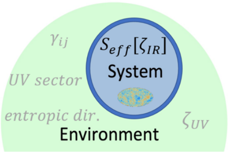
*Which properties depend on the microphysical details?*

*Physical principles* strongly constrain the system dynamics [Baumann *et al*, 2022]:

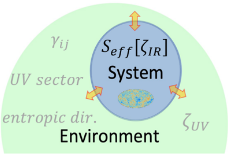
- Symmetries: near scale invariance, soft theorems [Hui *et al*, 2022];
- Locality: Manifestly Local Test [Jazayeri, Pajer & Stefanyszyn, 2021];
- Unitarity: Cosmological Optical Theorem [Goodhew, Jazayeri & Pajer, 2021].

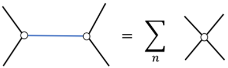
# Unitary *vs.* non-unitary dynamics

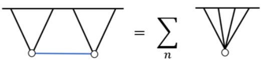
**Closed/Unitary**




**Open/Non-unitary**

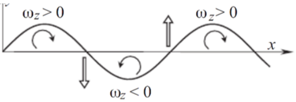


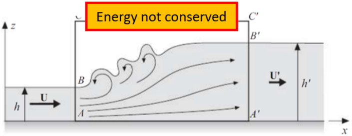

+ energy and information losses and gains

*Perfect fluid:*




$$S_{\text{fluid}} = \int d^4x F(\det B) + \text{higher } \partial\text{'s}$$

*Imperfect fluid:*



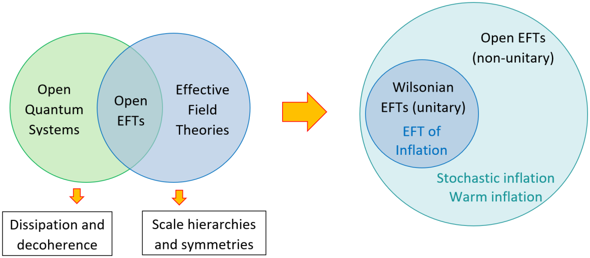

Credit: Guyon *et al*, 2012

$S_{\text{eff}}[\zeta_{\text{IR}}]$ local and unitary might not be enough: *Reheating, BBN, EFTofLSS*, $\cdots$

# Open Effective Field Theories

- When integrate out hidden sector, $S_{\rm eff}[\zeta_{\rm IR}]$ local and unitary might not be enough to describe visible evolution.
- **Non-unitary effects** (dissipation & decoherence) capture energy and information loss (or gain);
- Many **generic models**, once coarse-grained, exhibit **non-unitarity evolutions** (e.g. decoherence).

*How do we incorporate these effects in EFT dictionnary?*



# Outline


7 More motivations

8 More construction

9 More phenomenology

# Non-equilibrium constraints [Liu & Glorioso, 2018]

*Step 2:* Requiring **Open QFT** originates from a unitary "closed" UV theory:

$$\text{i) } \mathrm{Tr}[\widehat{\rho}] = 1, \qquad \text{ii) } \widehat{\rho}^\dagger = \widehat{\rho} \qquad \text{and} \qquad \text{iii) } \widehat{\rho} \geq 0$$

implies constraints on $S_{\rm eff}[\pi_+, \pi_-] \equiv S_{\rm unit}[\pi_+] - S_{\rm unit}[\pi_-] + S_{\rm non-unit}[\pi_+, \pi_-]$:

| | | |
|---|---|---|
| i) | $S_{\rm eff}[\pi_+, \pi_+] = 0,$ | $S_{\rm eff}[\pi_r, \pi_a = 0] = 0;$ |
| ii) | $S_{\rm eff}[\pi_+, \pi_-] = -S^*_{\rm eff}[\pi_-, \pi_+],$ | $S_{\rm eff}[\pi_r, \pi_a] = -S^*_{\rm eff}[\pi_r, -\pi_a];$ |
| iii) | $\mathfrak{Im} S_{\rm eff}[\pi_+, \pi_-] \geq 0,$ | $\mathfrak{Im} S_{\rm eff}[\pi_r, \pi_a] \geq 0.$ |

**Influence functional as a transition probability** [Glorioso & Liu, 1612.07705]

$$e^{iS_{\rm eff}[\pi_+, \pi_-]} = \langle \Omega_\sigma^{\{\pi_-\}}(t) | \Omega_\sigma^{\{\pi_+\}}(t) \rangle$$

where $|\Omega_\sigma^{\{\pi_+\}}(t)\rangle = \widehat{\mathcal{U}}(t, t_0; \{\pi_+\}) |\Omega_\sigma\rangle, \quad \langle \Omega_\sigma^{\{\pi_-\}}(t)| = \langle \Omega_\sigma | \widehat{\mathcal{U}}^\dagger(t, t_0; \{\pi_-\}).$

$$\text{i) } \langle \Omega_\sigma^{\{\pi_+\}}(t) | \Omega_\sigma^{\{\pi_+\}}(t) \rangle = 1, \qquad \text{ii) } [\langle \Omega_\sigma^{\{\pi_+\}}(t) | \Omega_\sigma^{\{\pi_-\}}(t) \rangle]^\dagger = \langle \Omega_\sigma^{\{\pi_-\}}(t) | \Omega_\sigma^{\{\pi_+\}}(t) \rangle,$$

$$\text{and} \qquad \text{iii) } || \langle \Omega_\sigma^{\{\pi_-\}}(t) | \Omega_\sigma^{\{\pi_+\}}(t) \rangle ||^2 \leq 1.$$

# Broken time-translation: a bottom-up Wilsonian EFT

A scalar field breaking time-translation symmetry

$$\langle\phi(t,\boldsymbol{x})\rangle = \bar{\phi}(t) \quad \text{with} \quad \dot{\bar{\phi}} \neq 0.$$

**Nambu-Goldstone mode** [Cheung *et al.*, 2007]

$$\phi(t,\boldsymbol{x}) = \bar{\phi}[t+\pi(t,\boldsymbol{x})]$$

transforms under time-translations as

$$\pi(t,\boldsymbol{x}) \to \pi'(t,\boldsymbol{x}) = \pi(t+\epsilon^0,\boldsymbol{x}) + \epsilon^0.$$

**Effective action** is constructed from $t+\pi(x)$ and its derivatives:

$$S_{\mathrm{eff}} = -\frac{1}{2}\int \mathrm{d}^4x \left\{\alpha_1(\partial_\mu\pi)^2 + \sum_{n\geq 2}\alpha_n\left[-2\dot{\pi}+(\partial_\mu\pi)^2\right]^n\right\}$$

# Time-symmetries and open systems [Hongo *et al.*, 2018]

Influence functional:

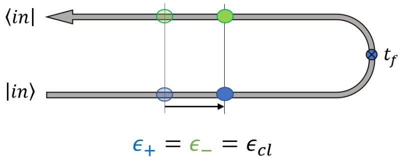
$$\rho_{\pi\pi'} = \int \mathrm{d}\pi_1 \mathrm{d}\pi_2 \int_{\pi_1}^{\pi} \mathcal{D}[\pi_+] \int_{\pi_2}^{\pi'} \mathcal{D}[\pi_-]\, e^{iS_\pi[\pi_+] - iS_\pi[\pi_-] + iS_{\mathrm{IF}}[\pi_+;\pi_-]} \langle \pi_1 | \, \widehat{\rho}_{\mathcal{S}}^{(0)} \, | \pi_2 \rangle$$

**Microscopic action** $S_\pi[\pi_\pm]$ invariant under

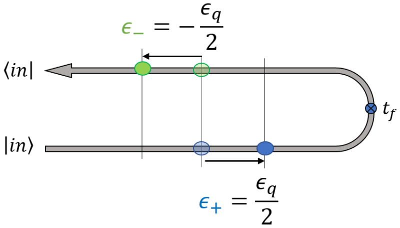
$$\pi_\pm(t) \to \pi'_\pm(t) = \pi_\pm(t+\epsilon_\pm),$$

but $S_{\mathrm{IF}}[\pi_+;\pi_-]$ **is not** due to non-unitary effects:

Detailed balance: conserved




Time reversal: broken



# Constraining the open dynamics

**cl-q basis** transform as [Hongo *et al.*,2018]

$$\pi_r(t) \equiv [\pi_+(t) + \pi_-(t)]/2 \rightarrow \pi_r'(t) = \pi_r(t+\epsilon_r^0) + \epsilon_r^0$$
$$\pi_a(t) \equiv \pi_+(t) - \pi_-(t) \rightarrow \pi_a'(t) = \pi_a(t+\epsilon_r^0),$$

1. Non-unitary Lagrangian constructed out of

   $\pi_a$, $t + \pi_r$, and their derivatives

2. Semiclassical expansion (MSR formalism): $\pi_r = \mathcal{O}(\hbar^0)$ and $\pi_a = \mathcal{O}(\hbar)$

$$\mathcal{L}_{\mathrm{eff}} = \sum_{n=1}^{\infty} \mathcal{L}_n \quad \text{s.t.} \quad \mathcal{L}_n = \mathcal{O}(\pi_a^n) = \mathcal{O}(\hbar^n)$$

3. Physical constraints: [Glorioso & Liu, 2018]

| | |
|---|---|
| $S_{\mathrm{eff}}\left[\pi_r; \pi_a = 0\right] = 0$ | Normalization |
| $S_{\mathrm{eff}}\left[\pi_r; \pi_a\right] = -S_{\mathrm{eff}}\left[\pi_r; -\pi_a\right]^*$ | Self-adjointness |
| $\mathrm{Im}\left\{S_{\mathrm{eff}}\left[\pi_r; \pi_a\right]\right\} > 0$ | Positivity |

# Effective open dynamics

1. Dissipation $\mathcal{O}(\hbar)$: $f_{\rm NL}^{\rm eq} \sim \gamma_1/H$ in [López Nacir *et al.*, 2011]

Kinetic term / Dissipation / NL ext.

$$\mathcal{L}_1^{\rm LO} = -\alpha_1 \partial^\mu \pi_{\rm cl} \partial_\mu \pi_{\rm q} + \gamma_1 \left[ -2\dot{\pi}_{\rm cl} + \left(\partial_\mu \pi_{\rm cl}\right)^2 \right] \pi_{\rm q}.$$

2. Diffusion $\mathcal{O}(\hbar^2)$: $f_{\rm NL}^{\rm eq} \gtrsim \mathcal{O}(10)$ in [Creminelli *et al.*, 2023]

Diffusion / Non-standard noises / NL ext.

$$\mathcal{L}_2^{\rm LO} = i \left[ \beta_1 \pi_{\rm q}^2 + \beta_2 \left(\partial_\mu \pi_{\rm q}\right)^2 + \beta_3 \left( -\dot{\pi}_{\rm q} + \partial^\mu \pi_{\rm cl} \partial_\mu \pi_{\rm q} \right) \pi_{\rm q} + \cdots \right]$$

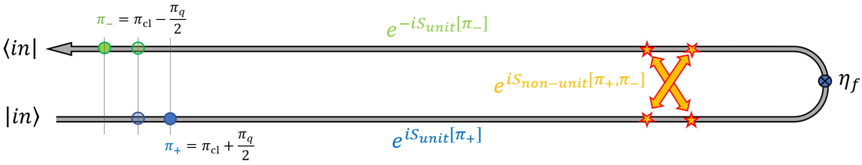
**Higher order**: multiply operators by powers of $-2\dot{\pi}_r + \left(\partial_\mu \pi_r\right)^2$, e.g.

$$\mathcal{L}_1 \supset \mathcal{L}_1^{\rm LO} - \sum_{n=2}^{\infty} \alpha_n \left( -\dot{\pi}_a + \partial^\mu \pi_r \partial_\mu \pi_a \right) \left[ -2\dot{\pi}_r + \left(\partial_\mu \pi_r\right)^2 \right]^{n-1}$$

$\alpha_n$'s dynamics invariant under both $\epsilon_r$ and $\epsilon_a$: unitary evolution.

# Constructing bottom-up Open EFT [Agüí Salcedo, TC & Pajer, in prep.]

Develop **Open EFT for inflation**, building on [López Nacir, Porto, Senatore & Zaldarriaga, 2011].




*In-in formalism*: double fields for $+/-$ branches of path integral: $\pi_\pm = \pi_r \pm \frac{\pi_a}{2}$.

$$\mathcal{Z}[J_+, J_-] = \int \mathcal{D}\pi_+ \mathcal{D}\pi_- e^{iS_{\mathrm{unit}}[\pi_+] - iS_{\mathrm{unit}}[\pi_-] + iS_{\mathrm{non-unit}}[\pi_+,\pi_-]} e^{i\int \mathrm{d}^4x\sqrt{-g}J_\pm \pi_\pm}$$

Consistency and physical principles:

- unitary “closed” UV theory [Liu & Glorioso, 2018]: i) $\mathrm{Tr}[\widehat{\rho}] = 1$, ii) $\widehat{\rho}^\dagger = \widehat{\rho}$ and iii) $\widehat{\rho} \geq 0$;
- coset construction for in-in [Hongo *et al.*, 2018], [Akyuz, Goon & Penco, 2023]: $G_+ \times G_- \to G_{\mathrm{diag}}$;
- locality: truncatable power counting scheme.

# Outline

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# Heuristic estimate

$$S_{\text{eff}}^{(2)} = \int \mathrm{d}^4x \left( a^2\pi_r'\pi_a' - c_s^2 a^2 \partial_i\pi_r\partial^i\pi_a - a^3\gamma\pi_r'\pi_a + i\beta_1 a^4\pi_a^2 \right)$$

1. Adiabatic perturbations $\pi_r \sim \frac{f_\pi^2\Delta_\zeta}{H}$ **freeze** at $\frac{c_s k}{a_* H} \sim \sqrt{\frac{H+\gamma}{H}}$;
2. Noise-sourced dynamics $\pi_r \sim \frac{\beta_1}{H(H+\gamma)}\pi_a$

$\Rightarrow$ *Driven-dissipative* harmonic oscillator:

$$\frac{a^2\pi_r'\pi_a'}{a^4\beta_1\pi_a^2} \sim \frac{H}{H+\gamma}, \qquad \frac{c_s^2 a^2\partial_i\pi_r\partial^i\pi_a}{a^4\beta_1\pi_a^2} \sim 1, \qquad \frac{a^3\gamma\pi_r'\pi_a}{a^4\beta_1\pi_a^2} \sim \frac{\gamma}{H+\gamma}.$$

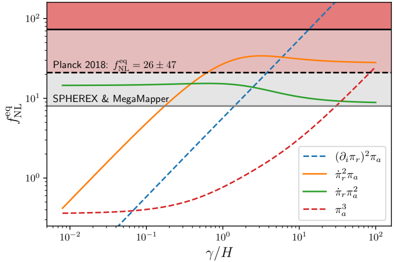
Estimate non-Gaussianities from $f_{\text{NL}}\Delta_\zeta \sim \mathcal{L}_3/(a^4\beta_1\pi_a^2)$:

$$\frac{\gamma}{f_\pi^2} a^2(\partial_i\pi_r)^2\pi_a \quad \rightarrow \quad f_{\text{NL}} \sim \frac{1}{c_s^2}\frac{\gamma}{H},$$

$$\frac{\gamma}{f_\pi^2} a^2\pi_r'^2\pi_a \quad \rightarrow \quad f_{\text{NL}} \sim \frac{\gamma}{H+\gamma},$$

$$\frac{i\beta_5}{f_\pi^2} a^3\pi_r'\pi_a^2 \quad \rightarrow \quad f_{\text{NL}} \sim \frac{\beta_5}{\beta_1},$$

$$\frac{\delta_1}{f_\pi^2} a^4\pi_a^3 \quad \rightarrow \quad f_{\text{NL}} \sim \frac{\delta_1}{\beta_1^2}(H+\gamma).$$



# Free theory and propagators

Free theory path integral:

$$\mathcal{Z}[J_r, J_a] = \int_\Omega^\pi \mathcal{D}\pi_r \int_\Omega^0 \mathcal{D}\pi_a e^{i\int d^4x\sqrt{-g}(\pi_r,\pi_a)\begin{pmatrix} 0 & \widehat{D}_A \\ \widehat{D}_R & i\widehat{D}_K \end{pmatrix}\begin{pmatrix}\pi_r \\ \pi_a\end{pmatrix} + \int d^4x(J_r\pi_r + J_a\pi_a)}$$

*Propagators*: **retarded/advanced** $G^{R/A}$ and **Keldysh-Green** $G^K$:

1. *Dissipative retarded Green function*:

$$G^R(k;\eta_1,\eta_2) = \frac{\pi}{2}H^2(\eta_1\eta_2)^{\frac{3}{2}}\left(\frac{\eta_1}{\eta_2}\right)^{\frac{\gamma}{2H}}\Im\mathrm{m}\left[H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_1)H^{(2)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_2)\right]\theta(\eta_1-\eta_2)$$

2. *Keldysh-Green function* ($A_\gamma$ and $B_\gamma$ complicated combin of ${}_2F_3$):

$$G^K(k;\eta_1,\eta_2) = i\frac{\pi^2\beta_1^2}{8}(\eta_1\eta_2)^{\frac{3}{2}+\frac{\gamma}{2H}}\Re\mathrm{e}\Big[H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_1)H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_2)A_\gamma(-k\eta_2)$$
$$-H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_1)H^{(2)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_2)B_\gamma(z_2)\Big] + (\eta_1\leftrightarrow\eta_2)$$

# Dissipative power spectrum

Symmetries ensure existence of nearly scale invariant power spectrum:

$$\Delta_\zeta^2(k) = \frac{1}{c_s^3}\frac{\beta_1}{H^2}\frac{H^4}{f_\pi^4}2^{1+\frac{\gamma}{H}}\frac{\Gamma\left(\frac{1}{2}+\frac{\gamma}{2H}\right)\Gamma\left(\frac{3}{2}+\frac{\gamma}{2H}\right)^2}{\Gamma\left(1+\frac{\gamma}{2H}\right)\Gamma\left(\frac{5}{2}+\frac{\gamma}{H}\right)} \propto \begin{cases} \frac{\beta_1}{H^2}\frac{H^4}{f_\pi^4}\sqrt{\frac{H}{\gamma}}\left[1+\mathcal{O}\left(\frac{H}{\gamma}\right)\right], & \gamma \gg H, \\ \frac{\beta_1}{H^2}\frac{H^4}{f_\pi^4}\left[1+\mathcal{O}\left(\frac{\gamma}{H}\right)\right], & \gamma \ll H. \end{cases}$$

$\Rightarrow \Delta_\zeta^2 = 10^{-9}$ obtained by imposing hierarchies of scales.

- Imposing **thermal equilibrium** at temp. $T$ (*KMS symmetry*: $\beta_1 \propto \gamma T$):

$$\Delta_\zeta^2 \propto \frac{T}{H}\frac{H^4}{f_\pi^4}\sqrt{\frac{\gamma}{H}}$$

  $\Rightarrow$ recover warm inflation predictions [Berera, 1995], [Montefalcone *et al.*, 2023].

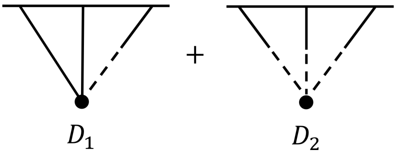
- Extend to **scale-dependent noises** $\beta_2, \beta_4 \neq 0$: also scale invariant

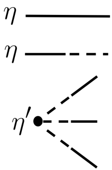
$$\Delta_\zeta^2(k) \supset \begin{cases} \frac{15}{4c_s^3}(\beta_4-\beta_2)\frac{H^4}{f_\pi^4}2^{\frac{\gamma}{H}}\frac{\Gamma\left(-\frac{1}{2}+\frac{\gamma}{2H}\right)\Gamma\left(\frac{3}{2}+\frac{\gamma}{2H}\right)^2}{\Gamma\left(\frac{\gamma}{2H}\right)\Gamma\left(\frac{5}{2}+\frac{\gamma}{H}\right)}, & \text{for} \quad i(\beta_4-\beta_2)\dot{\pi}_a^2, \\ \frac{3}{2c_s^5}\beta_2\frac{H^4}{f_\pi^4}2^{\frac{\gamma}{H}}\frac{\Gamma\left(-\frac{1}{2}+\frac{\gamma}{2H}\right)\Gamma\left(\frac{3}{2}+\frac{\gamma}{2H}\right)^2}{\Gamma\left(\frac{\gamma}{2H}\right)\Gamma\left(\frac{3}{2}+\frac{\gamma}{H}\right)}, & \text{for} \quad i\beta_2(\partial_i\pi_a)^2. \end{cases}$$

# Interactions and non-Gaussianities

$$\langle \zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle = -\frac{H^3}{f_\pi^6}\langle \pi^c_{\boldsymbol{k}_1}\pi^c_{\boldsymbol{k}_2}\pi^c_{\boldsymbol{k}_3}\rangle \equiv (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B(k_1,k_2,k_3).$$

Correlators computed in **perturbation theory** using standard in-in rules.




$-iG^K(k;\eta,\eta')$

$-iG^R(k;\eta,\eta')$

$$ig\int_{-\infty(1\pm i\epsilon)}^{\eta_0}\frac{\mathrm{d}\eta'}{(H\eta')^4}$$

**Contact bispectrum:**

$$B(k_1,k_2,k_3) = (-i)^{n_K+n_R+1}\frac{H^3}{f_\pi^6}\frac{g}{H^{4-n_d}}\int_{-\infty(1\pm i\epsilon)}^{0^-}\frac{\mathrm{d}\eta}{\eta^{4-n_d}}$$

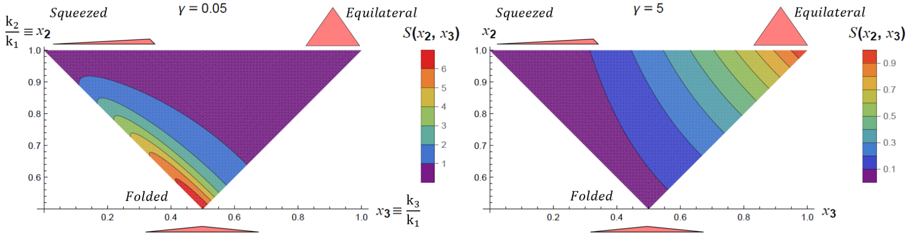
$$\widehat{\mathcal{D}}(\{\boldsymbol{k}_i\},\partial_\eta)\left[G^{K/R}(k_1,0,\eta)G^{K/R}(k_2,0,\eta)G^R(k_3,0,\eta)+5\ \mathrm{perms.}\right]$$

$$S(x_2,x_3)\equiv(x_2x_3)^2\frac{B(k_1,x_2k_1,x_3k_1)}{B(k_1,k_1,k_1)},\qquad f_{\mathrm{NL}}(k_1,k_2,k_3)\equiv\frac{5}{6}\frac{B(k_1,k_2,k_3)}{P(k_1)P(k_2)+2\ \mathrm{perms.}}.$$

# Flat space intuition

**Contact bispectrum** $B(k_1, k_2, k_3) \sim \mathrm{Poly}_\gamma / \mathrm{Sing}_\gamma$ with $E_k^\gamma \equiv \sqrt{c_s^2 k^2 - \gamma^2/4}$

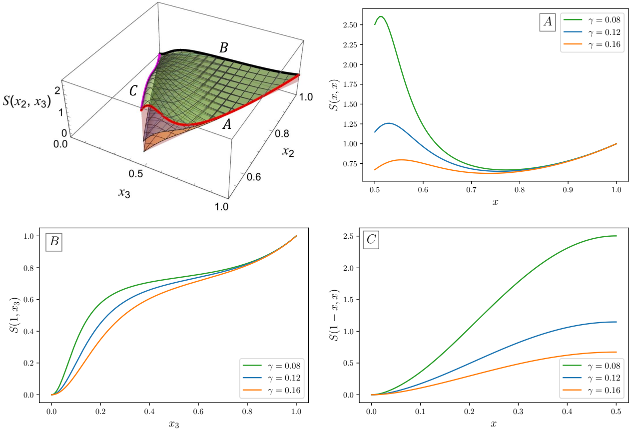
$$\mathrm{Sing}_\gamma = |E_1^\gamma + E_2^\gamma + E_3^\gamma + \frac{3}{2} i\gamma|^2 \times |-E_1^\gamma + E_2^\gamma + E_3^\gamma + \frac{3}{2} i\gamma|^2 \times |E_1^\gamma - E_2^\gamma + E_3^\gamma + \frac{3}{2} i\gamma|^2 \times |E_1^\gamma + E_2^\gamma - E_3^\gamma + \frac{3}{2} i\gamma|^2,$$




⇒ at **small dissipation**, peaks when $k_1 \pm k_2 \pm k_3 = 0$, *i.e.* folded triangles.

# Fingerprints

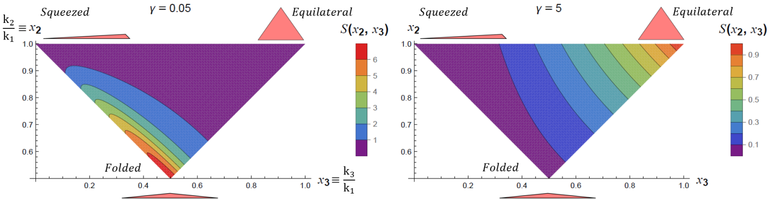
**Smoking gun**: peaks near folded triangles when $\gamma \ll H$




1. May $\exists$ intermediate peak in the **small dissipation regime**;
2. $\text{Sing}_\gamma$ regulated: different from **non-BD ICs** [Chen *et al.*, 2007], [Holman & Tolley, 2008];
3. **Squeezed limit** goes to zero because of symmetries (consistency relations).

# Summary on interactions and non-Gaussianities

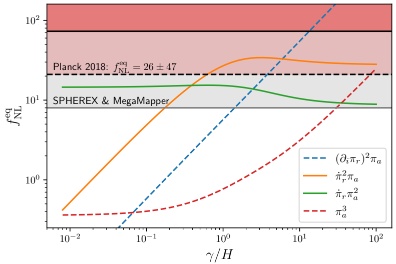
$$\langle \zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle = -\frac{H^3}{f_\pi^6}\langle \pi_{\boldsymbol{k}_1}\pi_{\boldsymbol{k}_2}\pi_{\boldsymbol{k}_3}\rangle \equiv (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B(k_1,k_2,k_3)$$




**Constrained** class of models:

- in $\gamma \gg H$: $f_{NL}^{\rm eq} = -26 \pm 47$
- in $\gamma \ll H$: $f_{NL}^{\rm folded} = ?$

Matching with [Creminelli *et al.*, 2305.07695]



# Details of matching with [Creminelli *et al.*, 2305.07695]

| | *Parameters:* | | | | | | |
|---|---|---|---|---|---|---|---|
| UV completion | $M$ | $m$ | $f$ | $\rho$ | $\xi$ | $\gamma$ | $\nu_{\mathcal{O}}/\nu_{\mathcal{O}^3}$ |
| Open EFT | $f_\pi^2$ | $c_s$ | $\gamma$ | $\gamma_2$ | $\beta_1$ | $\beta_5$ | $\delta_1$ |

| | |
|---|---|
| *Matching:* | $f_\pi^2 = \rho f$ |
| $c_s = 1$ | $\gamma = \frac{\xi m^4}{\pi M f^2} e^{2\pi\xi}$ |
| $\Gamma = \pi\xi\gamma$ | $\beta_1 = \frac{\nu_{\mathcal{O}}}{2\rho f} \frac{m^4}{f^2}$ |
| $\beta_5 = 2\pi\xi\beta_1$ | $\delta_1 = \frac{\nu_{\mathcal{O}^3}}{6\sqrt{\rho f}} \frac{m^6}{f^3}$ |