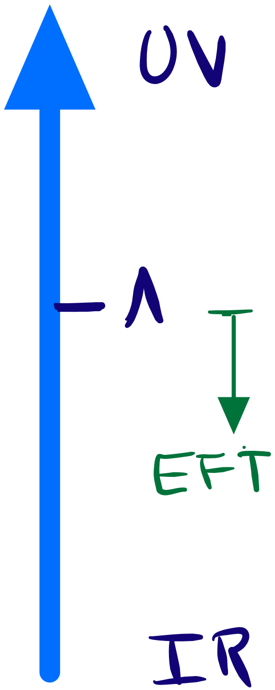
CAUSALITY BOUNDS IN COSMOLOGY

Cosmological Correlators in Taiwan

Mariana Carrillo González

IMPERIAL

CAUSAL EFFECTIVE FIELD THEORIES



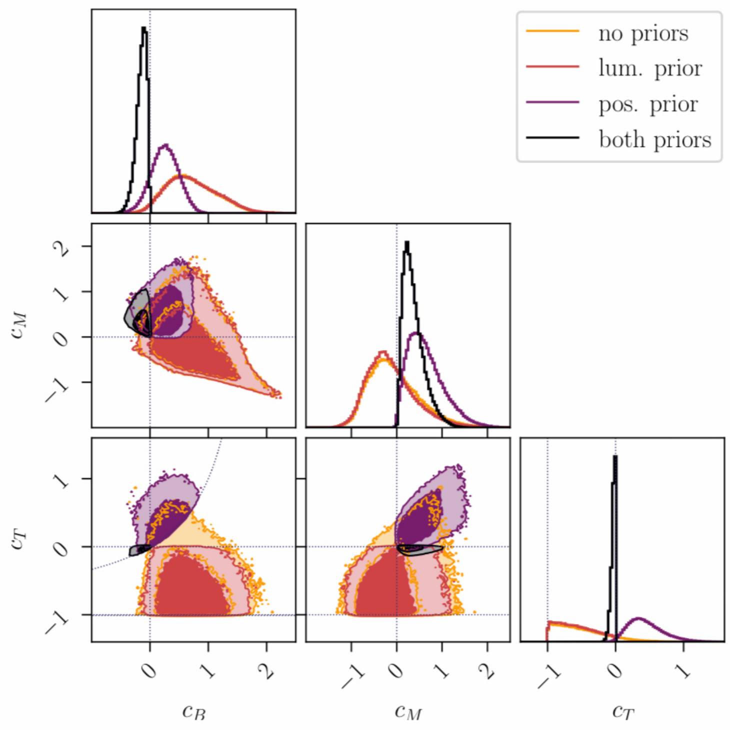
$$\mathcal{L} = \Lambda^4 \sum_n c_n \Lambda^{-n} \mathcal{O}_n$$

e.g. $$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 + \frac{c_8}{\Lambda^4}(\partial\phi)^4 + \dots$$

What are the allowed Wilson coefficients?

CAUSAL EFFECTIVE FIELD THEORIES

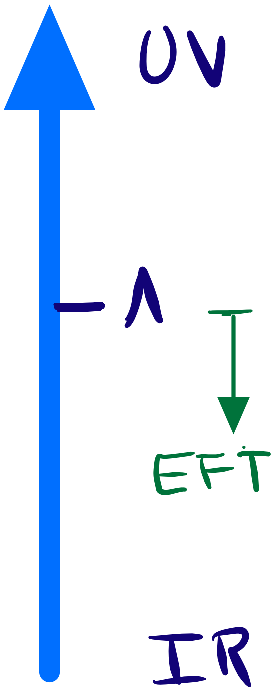
Theoretical priors can drastically change estimations of cosmological parameters



Melville, Noller

Melville, Noller
de Rham, Melville, Noller
Traykova, Bellini, Ferreira, García-García
Noller, Zumalacarregui

CAUSAL EFFECTIVE FIELD THEORIES



$$\mathcal{L} = \Lambda^4 \sum_n c_n \Lambda^{-n} \mathcal{O}_n$$

What are the allowed Cn ?

1) UV = string theory $\rightsquigarrow$ Swampland conjectures

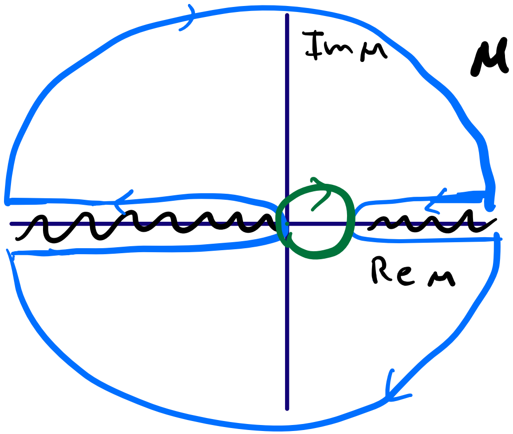
CAUSAL EFFECTIVE FIELD THEORIES

$$\mathcal{L} = \Lambda^4 \sum_n c_n \Lambda^{-n} \mathcal{O}_n$$

What are the allowed Cn ?

2) UV = local, unitary, causal, Lorentz invariant → Positivity bounds

$\qquad\qquad 1 \qquad\quad 2 \qquad\quad 3 \qquad\quad 4$



$$A''(s)\Big|_{t=0} = \oint \frac{d\mu}{2\pi i}\frac{A(\mu)}{(\mu-s)^3} \overset{1,3}{=} \Big(\underbrace{\int_{-\infty}^{0} + \int_{4m^2}^{\infty}}_{\text{related by 4}}\Big) \frac{\mathrm{Im}\,A}{(\mu-s)^3} \overset{2}{>} 0$$

IR

UV

1) UV = string theory

→ Swampland

2) UV = local, unitary, causal, Lorentz invariant

→ Positivity bounds

FLAT SPACE

3) Causal IR propagation

→ CAUSALITY BOUNDS

$\langle \phi \phi \rangle$ in non-trivial backgrounds

Microcausality $[\mathcal{O}(x), \mathcal{O}(y)] = 0$ for $(x-y)^2 > 0$

CAUSALITY

$$\boxed{G_R(x-y) = 0 \text{ for } (x-y)^2 > 0}$$

- Consider local propagation of information around a fixed background -> bound Wilson coefficient.

de Rham, Tolley

- EFT description (low frequency) $\rightarrow$ INFRARED CAUSALITY

Diagnose by looking at time delay

$$\Delta T = -i \langle in | \hat{S}^{\dagger} \frac{\partial}{\partial \omega} \hat{S} | in \rangle$$

CAUSALITY

At L.O. encoded in metric seen by perturbations, φ :

$$\left(g^{eff}(\bar{\phi})\right)^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi + \cdots = 0$$

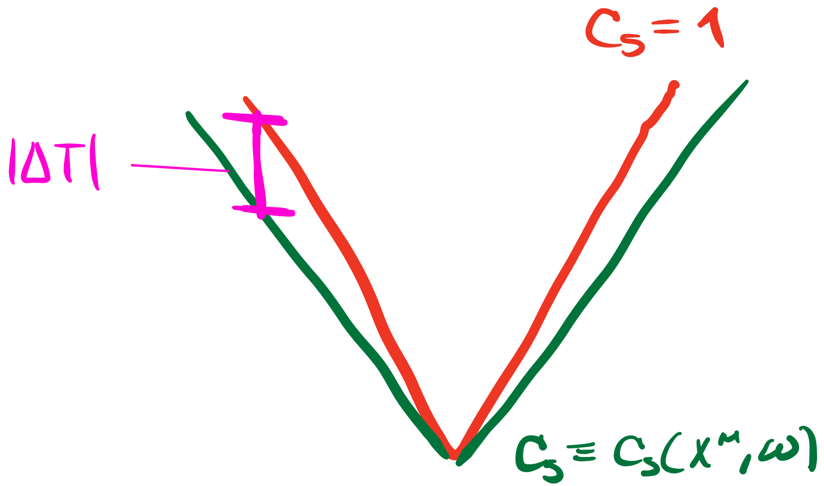
modified lightcone

- Implications on c_s?

$$\cancel{c_s^2(x^\mu,\omega) \leq 1}$$

too restrictive

Acausality needs $c_s>1$ over large regions

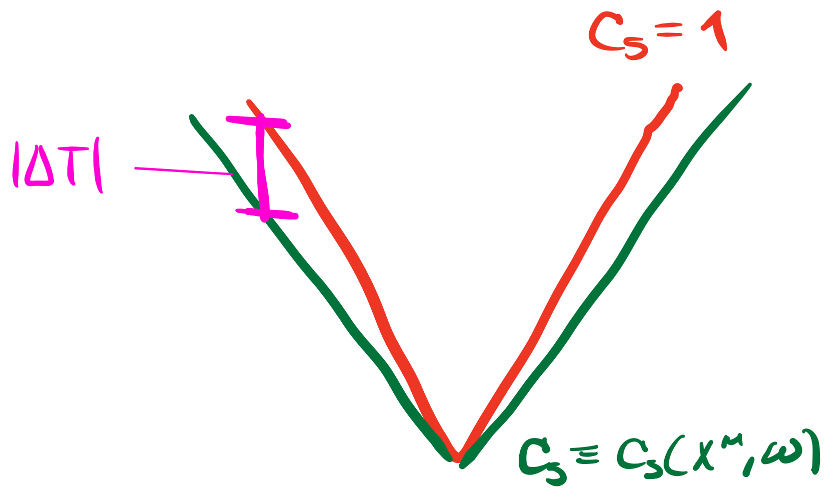


$$|\Delta T| \lesssim \lambda \sim \frac{1}{\omega}$$

Unresolvable

$$\boxed{\Delta T \gtrsim -\frac{1}{\omega}}$$

CAUSALITY
+ GRAVITY



- $\Delta T = \Delta T^{GR} + \Delta T^{EFT} > -1/\omega$ Asymptotic causality
- $\Delta T^{EFT} > -1/\omega$ Infrared causality

- Causality bounds on scalar EFTs
  - (2107.11384: MCG, de Rham, Pozsgay, Tolley)

Also for photon EFTs (not this talk)

- Causality in cosmological spacetimes
  - (2312.07651: MCG)
- Causality bounds on the growth of the primordial power spectrum
  - (2412.XXXX: MCG, Céspedes)

CAUSALITY BOUNDS ON SCALAR EFTS

2107.11384:
MCG, de Rham, Pozsgay, Tolley

CAUSALITY INSIGHTS ON SCALAR EFTS

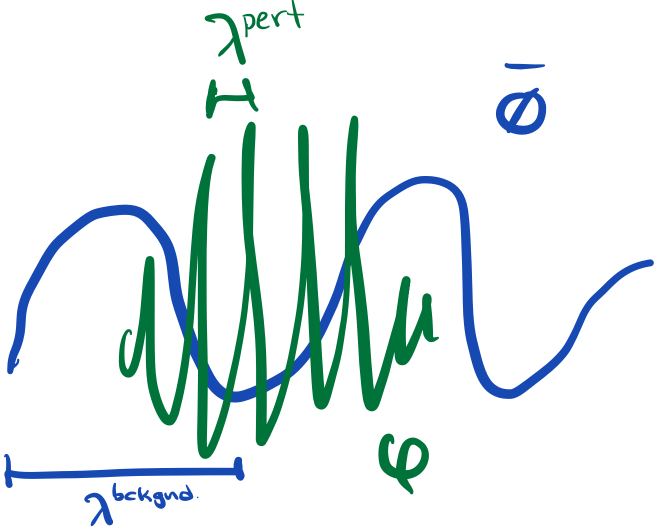
$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{g_8}{\Lambda^4}\left((\partial\phi)^2\right)^2 + \frac{g_{10}}{\Lambda^6}(\partial\phi)^2(\partial\partial\phi)^2 + \frac{g_{12}}{\Lambda^8}\left((\partial\partial\phi)^2\right)^2 - g_{\text{matter}}\,\phi\, J$$

external source

Can probe all local operators of microcausal modes

This can be encoded in different couplings to the source

CAUSALITY INSIGHTS ON SCALAR EFTS



$\phi = \bar{\phi} + \varphi \qquad \partial_\mu \varphi = i k_\mu \varphi$ plane waves

EOM + Disp. rel → c_s

Adams et al

$$c_s^2 \simeq \Big(1 - \# \frac{g_8}{\Lambda^4} \overbrace{\frac{(k\cdot\partial\bar{\phi})^2}{|\bar{k}|^2}}^{>0}$$

$$+ \# \frac{g_{10}}{|k|^2\Lambda^6}\Big((k_\mu \partial^\mu \partial_\rho \bar{\phi})^2 - \Box\phi\,(k_\mu k_\nu \cdot \partial^\mu \partial^\nu \bar{\phi})\Big) - \# \frac{g_{12}}{\Lambda^8} \overbrace{\frac{(kk\cdot\partial\partial\bar{\phi})^2}{|k|^2}}^{>0}\Big)$$

→ Work with

$$g_8 = \begin{cases} 1 \\ 0 \end{cases}$$

CAUSALITY + WKB + EFT

Consider $\phi = \bar{\phi} + \varphi$. Find ΔT experienced by φ.
Solve linearized φ eom using WKB approximation.

$$\boxed{\underbrace{\frac{\lambda^{\text{background}}}{\lambda^{\text{perturbation}}}}_{\gg 1 \ \text{WKB}} \int_{X \subset \mathbb{R}^{1+3}} \underbrace{\left(1 - c_s(\lambda^{\text{pert.}})\right)}_{= -\epsilon \ \text{EFT} \to |\epsilon| \ll 1} \gtrsim -1}$$

$$\boxed{\Delta T \gtrsim -\frac{1}{\omega}}$$

PROPAGATION AROUND SPHERICALLY SYMMETRIC BACKGROUNDS

Spherically-symmetric background

$$\bar{\phi} \equiv \Phi_0 f(R) \,, \quad R = r/r_0$$

Radial perturbations

$$\chi_\ell'' + (\omega r_0)^2 \overbrace{\frac{1}{C_s^2(\omega, r)}\left(1 - \frac{V_\ell^{\text{eff}}(r)}{(\omega r_0)^2}\right)}^{\equiv W_\ell(\omega, r)} \chi_\ell = 0$$

$\rightarrow$ Find $\delta \sim \omega r_0 \int \left(\sqrt{W_\ell} - 1\right) dR \sim \omega r_0 \int (1 - C_s) dR \rightarrow \Delta T = 2\partial_\omega \delta$

WKB: $$\frac{\lambda_{\text{background}}}{\lambda_{\text{perturbation}}} = \omega r_0 \gg 1$$

EFT: $$\frac{\partial\phi}{\Lambda}, \quad \frac{\partial^{p+1}\phi}{\Lambda^{p+2}} \ll 1$$

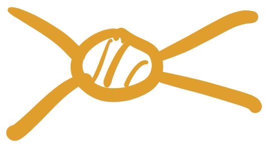
$$\frac{\Phi_0}{r_0\Lambda}, \quad \frac{1}{r_0\Lambda} \sim \mathcal{O}(\epsilon) \qquad \frac{\omega}{r_0\Lambda^2} \sim \mathcal{O}\left(\epsilon\frac{\omega}{\Lambda}\right)$$

Note bound on ω $\longrightarrow$ $$\underbrace{\epsilon < \frac{\omega}{\Lambda}}_{\text{WKB}} \underbrace{< \frac{1}{\epsilon}}_{\text{EFT}}$$

CAUSALITY VS POSITIVITY

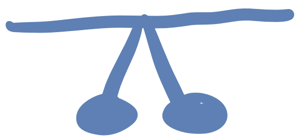
No upper bound on g_{12} from causality due to WKB technical issues.

POSITIVITY

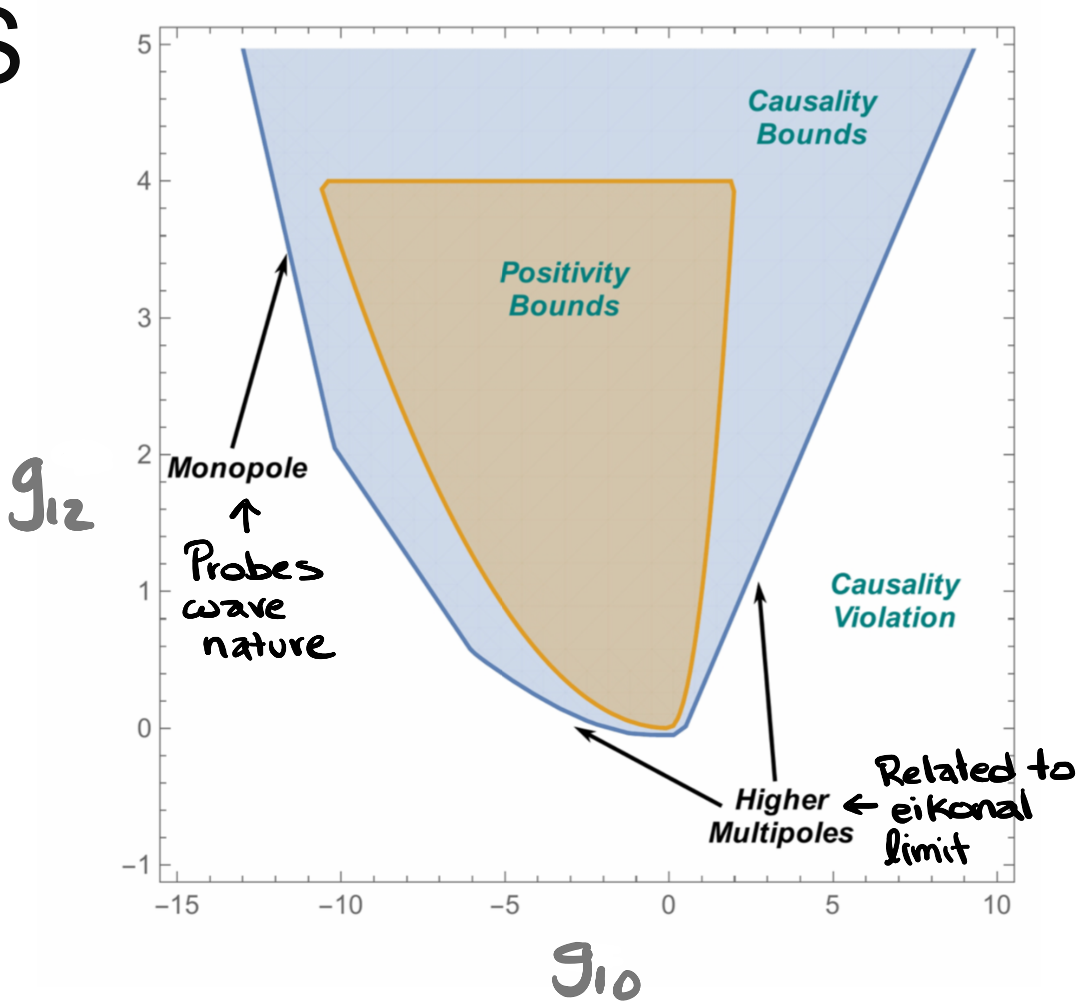


$2 \to 2$

CAUSALITY



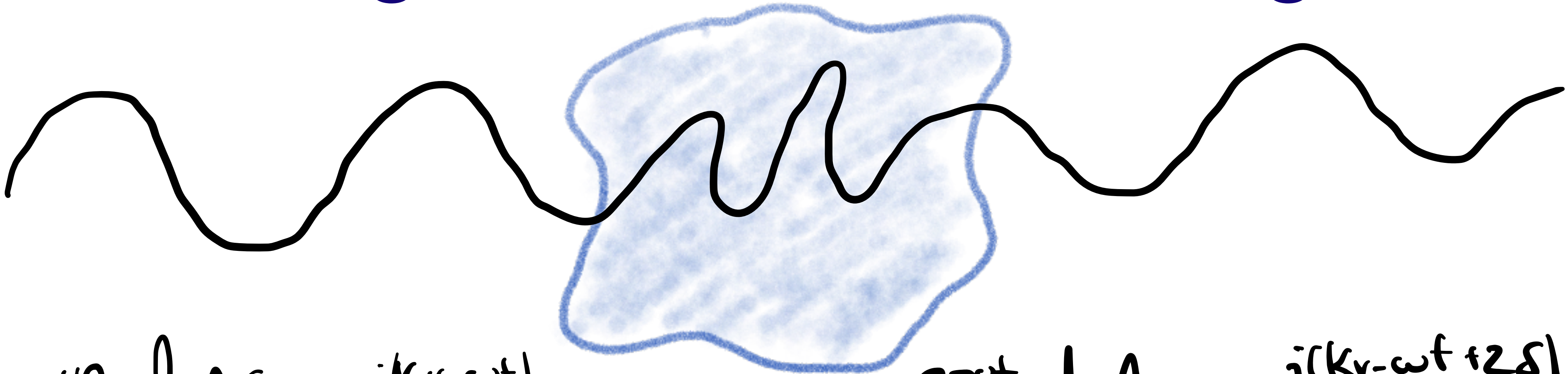
$1 \to 1$



CAUSALITY IN COSMOLOGICAL SPACETIMES

2312.07651: MCG

Scattering off non-trivial background

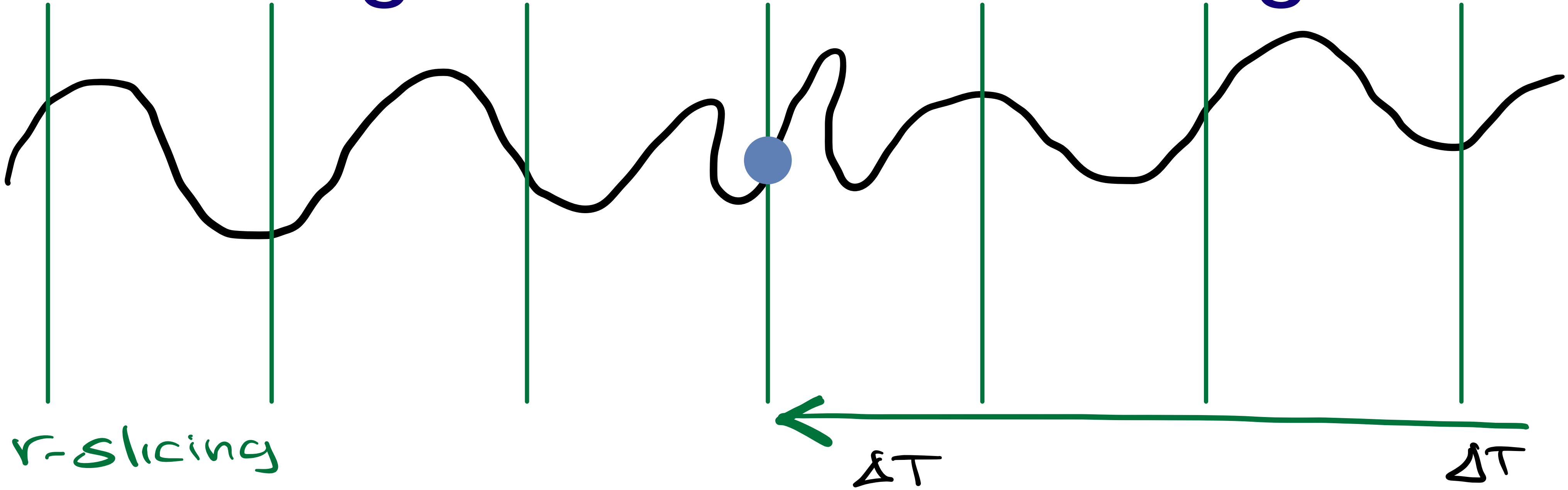


$$\varphi^{in} = \int_k \frac{A(k)}{r} e^{i(k\cdot r - \omega t)} \qquad \varphi^{out} = \int \frac{A(k)}{r} e^{i(kr - \omega t + 2\delta)}$$

At fixed r $\quad \Delta T = 2\,\partial_\omega \delta \quad$ for ω conserved

At fixed t $\quad \Delta r = -2\,\partial_k \delta \quad$ for k conserved

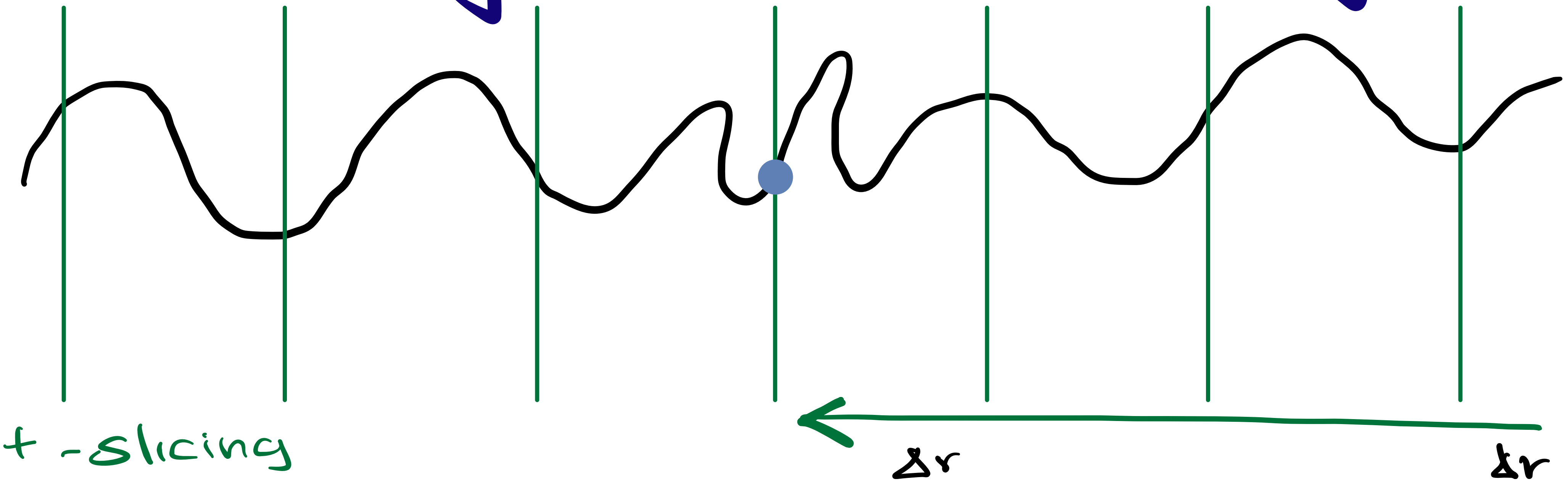
Scattering off non-trivial background



At fixed r $\Delta T = 2\,\partial_\omega \delta$

$\Delta T < 0 \rightarrow$ Wave leaves scatterer before it arrives to it

Scattering off non-trivial background



$$\Delta r = -2\partial_k \delta$$

$\Delta r > 0 \rightarrow$ Wave leaves scatterer before it arrives to it

Applications to Cosmology

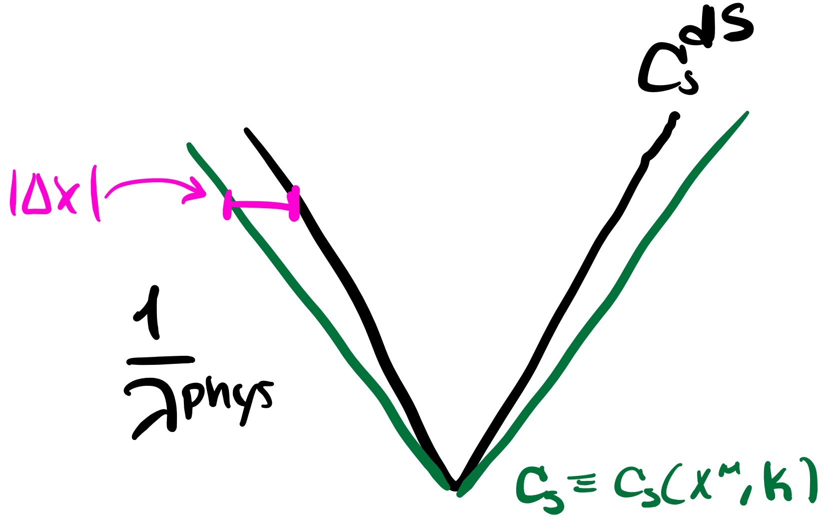
Consider fixed FLRW background

$$ds^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2)$$

Field with background $\bar{\Phi}(t)$

experiences spatial shift

$$\boxed{\frac{k}{a(t_f)}\left(a(t_f)\Delta r\right) \sim k\int_{t_i}^{t_f}\frac{dt}{a(t)}\left(c_s^{EFT}(k,t) - c_s^{FRW}(k,t)\right) < 1}$$



Similar ideas (same at LO) Bittermann, Mc. Loughlin, Rosen

Applications to Cosmology

$$\boxed{\frac{k}{a(t_f)}\left(a(t_f)\Delta r\right) \sim k \int_{t_i}^{t_f} \frac{dt}{a(t)} \left(c_s^{EFT}(k,t) - c_s^{FRW}(k,t) \right) < 1}$$

Similar ideas (same at LO) Bittermann, Mc. Loughlin, Rosen

Causality determined by fastest null geodesic

$(\Delta r)^{LO} \sim$ geodesic distance in $g^{eff}(\bar{\phi}(t))$

Applications to Cosmology

Consider fixed FLRW background

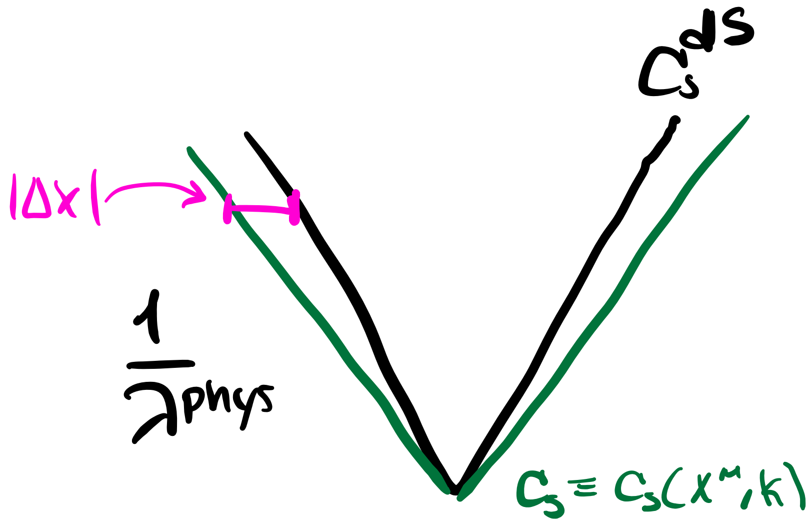
$$ds^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2)$$

Field with background $\bar{\phi}(t)$

experiences spatial shift

$$\boxed{k\int_{\tau_i}^{\tau_f} \partial_q \omega(q(\tau),\tau)\, d\tau \lesssim k(\tau_f - \tau_i)}$$

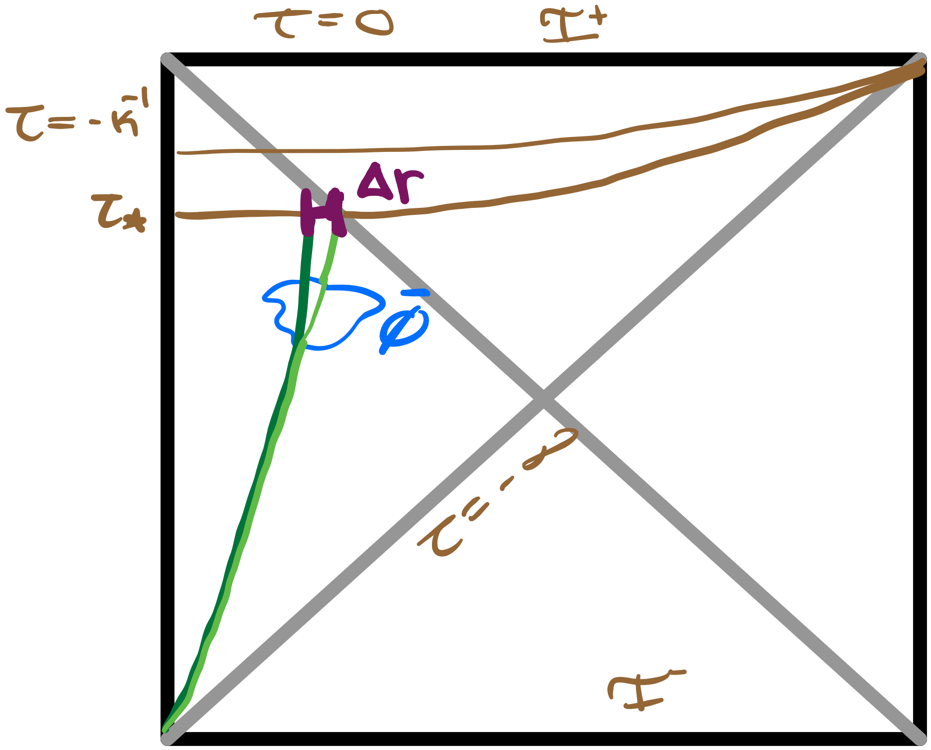
$q \equiv k/(aH)$



Dispersion relation

$$\omega^2 = a_2(\bar{\phi})\left(\frac{k}{aH}\right)^2 + a_4(\bar{\phi})\left(\frac{k}{aH}\right)^4 + \cdots$$

Propagation in de sitter



Propagation around

$\bar{\phi}(t)$ from

$\tau=-\infty$ to $\tau_* < -1/k$

$$\varphi = e^{i\bar{k}\cdot\bar{x}} f(t)$$

Compute Δr with WKB approximation.

Work in Poincare patch, i. e. , conformally flat cord.

Shift-symmetric scalar

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{g_8}{\Lambda^4}\left((\partial\phi)^2\right)^2$$

$$+ \frac{g_{10}}{\Lambda^6}(\partial\phi)^2(\partial\partial\phi)^2 + \frac{g_{12}}{\Lambda^8}\left((\partial\partial\phi)^2\right)^2 - g_{\text{matter}}\,\phi\, J(t)$$

$$+ \frac{h_{12}}{\Lambda^8}\left((\partial\phi)^2\right)^3 + \cdots$$

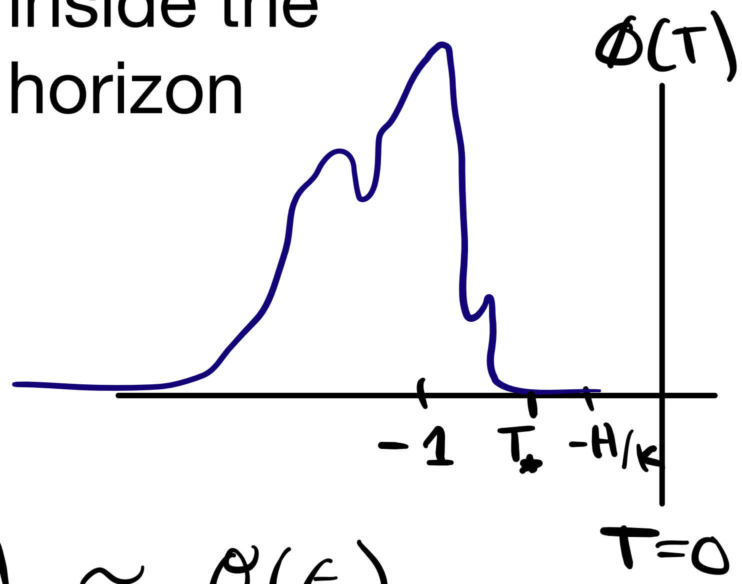
external source

Consider background localized near $T \equiv H\tau \sim -1$

WKB : $\dfrac{\lambda_{bckgnd}}{\lambda_{pert}} = \dfrac{k|T|}{H} \gg 1$

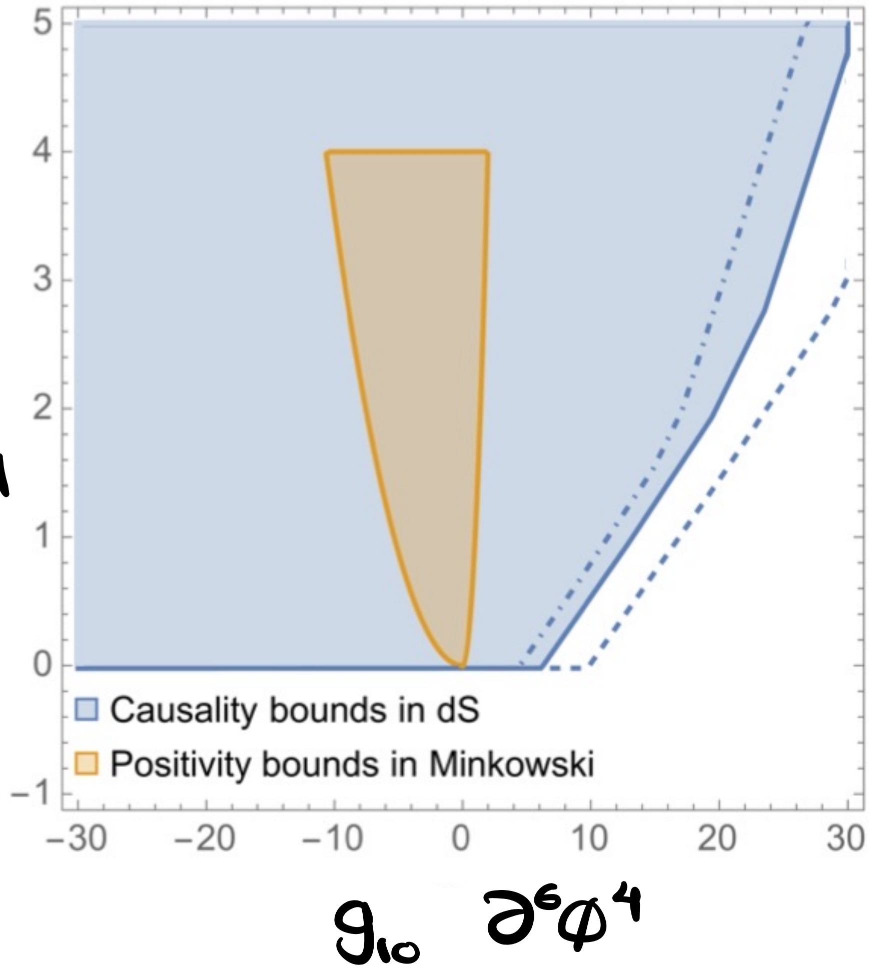
Modes inside the horizon

EFT : $\dfrac{\nabla\phi}{M}, \; \dfrac{\nabla^{p+1}\phi}{M^{p+2}} \ll 1$



$\Rightarrow \; \tau_\star \ll -\dfrac{1}{k}, \quad \dfrac{\Phi_0}{r_0 M}|T\phi'|, \; \dfrac{H}{M} \sim \mathcal{O}(\epsilon)$

Shift symmetric scalar



Bound on higher-order operator

$\partial^6\phi^6$: $h_{12} < 1.64$

v.s. flat space

$h_{12} < 0$

Chandrasekaran, Remmen, Shahbazi-Moghaddam

assuming $\mathcal{L} = P(X = (\partial\phi)^2)$

and only 1 term at a time

CAUSALITY BOUNDS ON THE GROWTH OF THE PRIMORDIAL POWER SPECTRUM

2412.XXXX:

MCG, Céspedes

EFT of inflation + decoupling limit

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - M_{\rm Pl}^2(3H^2+\dot{H}) + M_{\rm Pl}^2\dot{H}g^{00} + \underbrace{\frac{1}{2}M_2^4(\delta g^{00})^2 - \frac{1}{2}\hat{M}_1^3\delta g^{00}\delta K - \frac{1}{2}\bar{M}_2^2(\delta K)^2}_{} + \dots\right]$$

Extrinsic curvature

In zeta gauge, $\delta g_{ij} = a^2 e^{2\zeta}\delta_{ij}$

Real Wilson coeff. $\Rightarrow \alpha > 0$

$$S_\zeta^{(2)} = \int \mathrm{d}^4x\, A\, a^3\left[\dot{\zeta}^2 - c_s^2\frac{(\nabla\zeta)^2}{a^2} - \alpha\frac{(\nabla^2\zeta)^2}{H^2a^4}\right] \rightarrow \boxed{\omega^2 = c_s^2k^2 + \alpha\frac{k^4}{a^2H^2},}$$

In zeta gauge, $\delta g_{ij} = a^2 e^{2\zeta} \delta_{ij}$

$$S_\zeta^{(2)} = \int \mathrm{d}^4x\, A\, a^3 \left[\dot{\zeta}^2 - c_s^2 \frac{(\nabla\zeta)^2}{a^2} - \alpha \frac{(\nabla^2\zeta)^2}{H^2 a^4}\right] \rightarrow \boxed{\omega^2 = c_s^2 k^2 + \alpha \frac{k^4}{a^2 H^2}\,,}$$

Add weakly broken Galileon symmetry

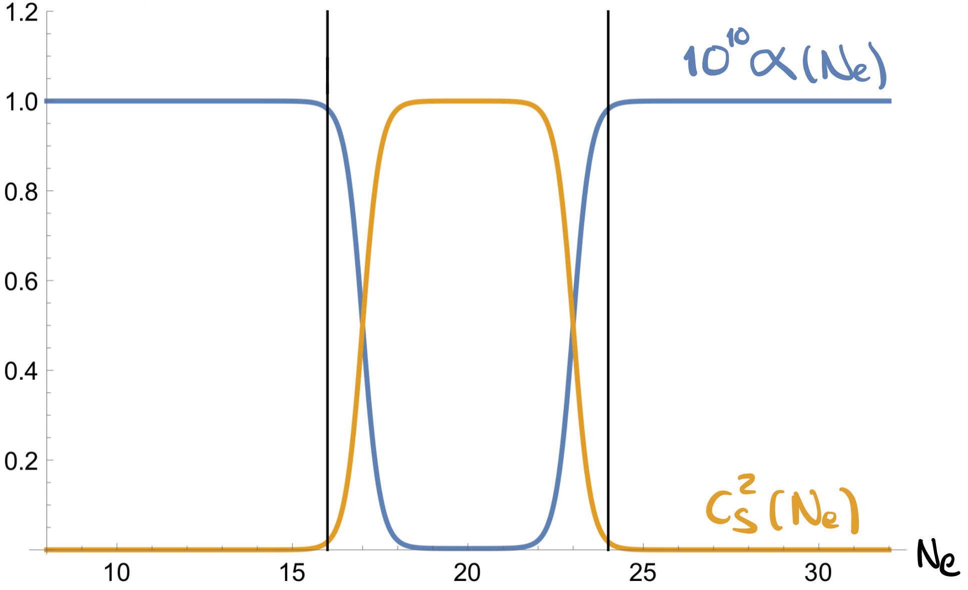
$$M_2^4 \sim \hat{M}_1^3 H \gg \bar{M}_2^2 H^2\,, \rightarrow \boxed{c_s^2 = \frac{-2M_{\rm Pl}^2 \dot{H} + \hat{M}_1^3 H + \partial_t(\hat{M}_1^3)}{2(2M_2^4 - M_{\rm Pl}^2 \dot{H})} \gg \alpha = \frac{\bar{M}_2^2 H^2}{2(2M_2^4 - M_{\rm Pl}^2 \dot{H})}.}$$

Growth of primordial power spectrum

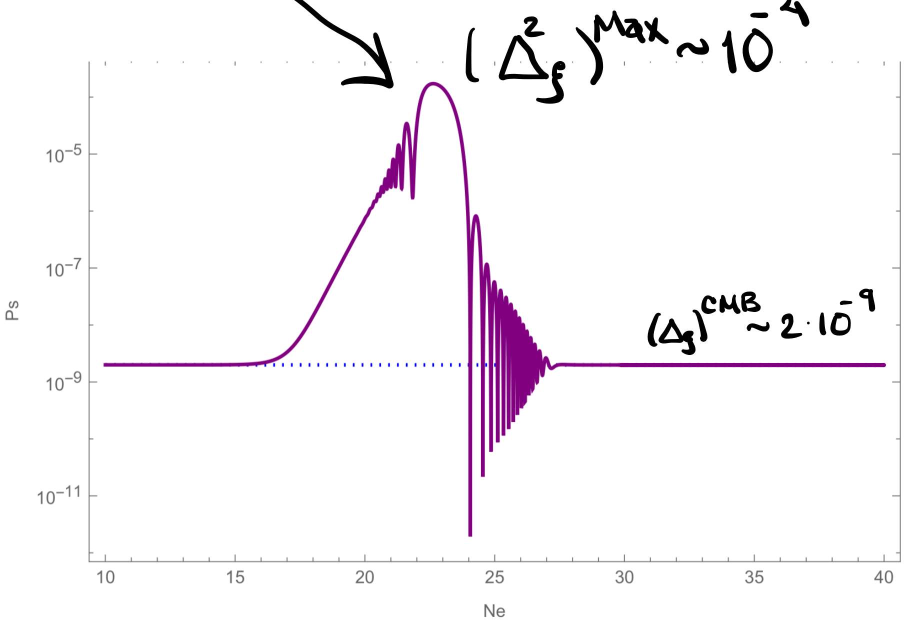
Ballesteros, Céspedes, Santoni

$$\omega^2 = c_s^2 k^2 + \alpha \frac{k^4}{a^2 H^2},$$

When αk^4 dominates



width = 6 e-folds



$\rightarrow$ Primordial black holes

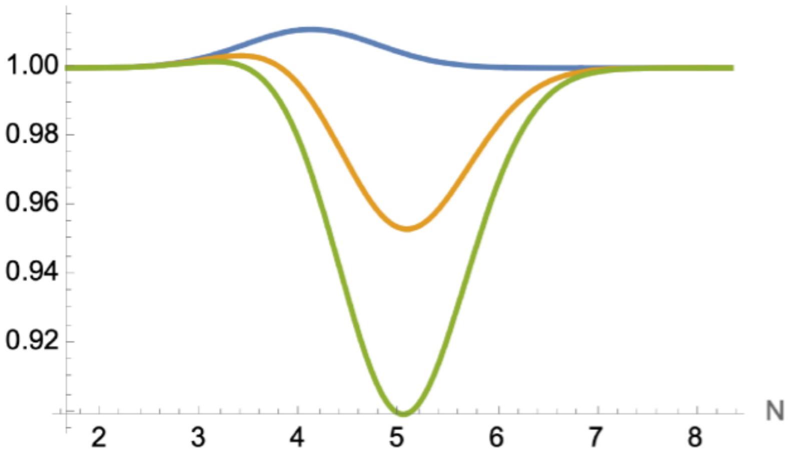
Bounds on primordial power spectrum

$$\omega^2 = c_s^2 \left(\frac{k}{aH}\right)^2 + \alpha \left(\frac{k}{aH}\right)^4 \equiv |c_s^{eff}|^2 \left(\frac{k}{aH}\right)^2$$

$$1 \ll \frac{k}{aH} \ll \alpha^{-1/2}$$

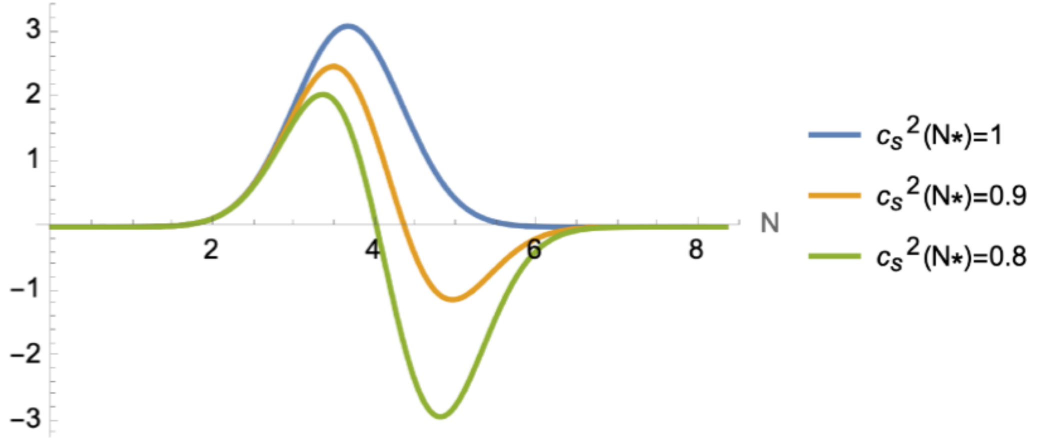
WKB EFT

c_s^{eff}

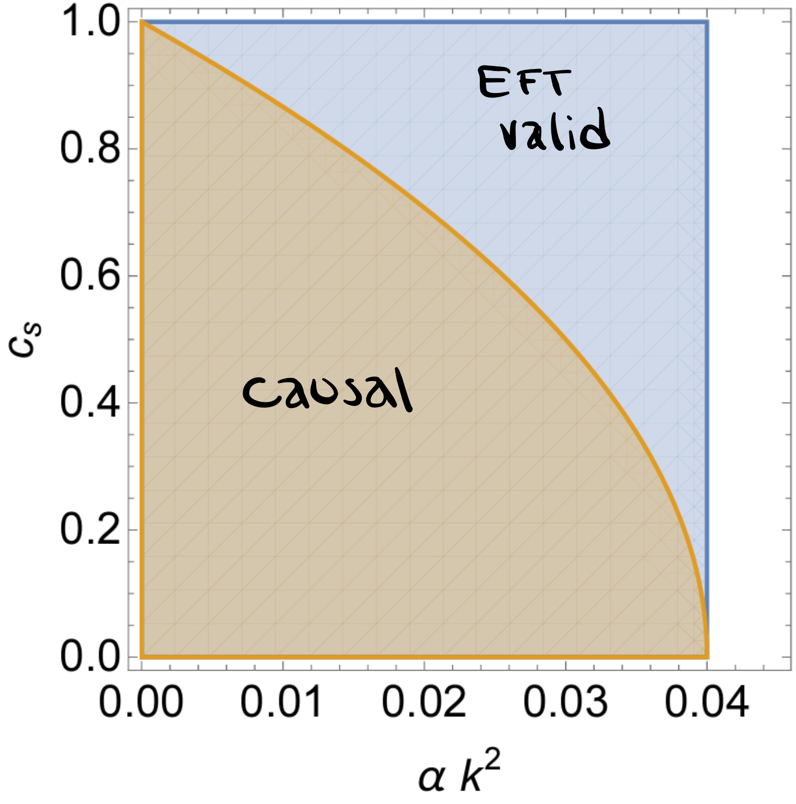


$k(\partial_k \omega - 1/aH)$

$a = e^N$



Bounds on constant c_s & α



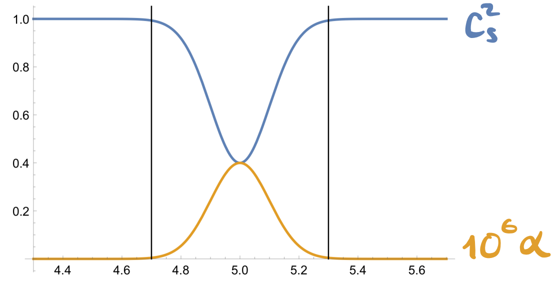
$c_s = 1$

$\rightarrow \alpha = 0$

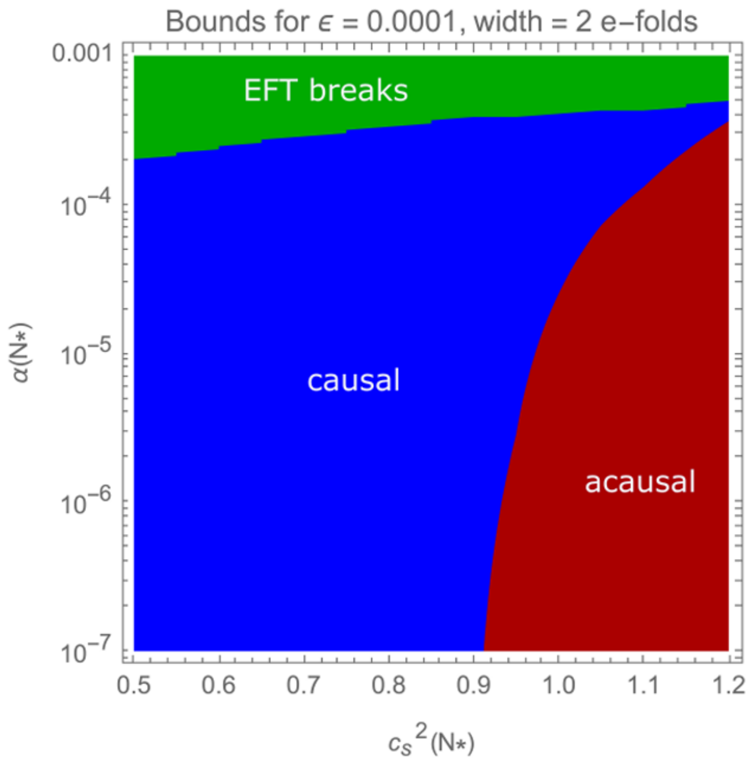
No higher-derivative corrections

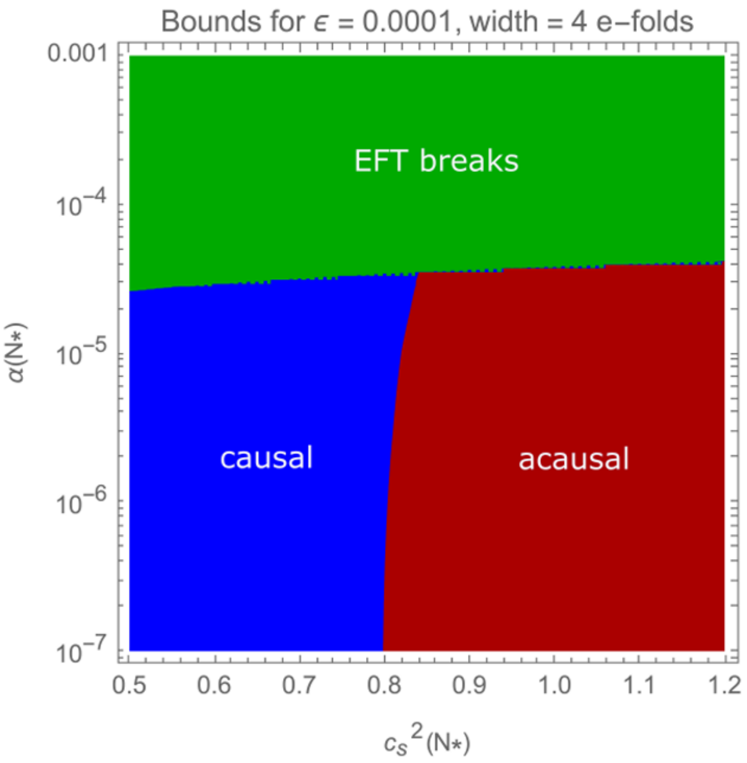
Bounds on c_s & α

w.i.p. w/ S. Céspedes

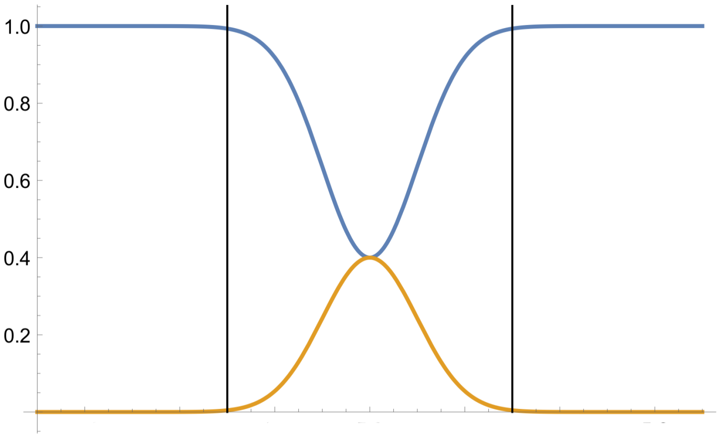


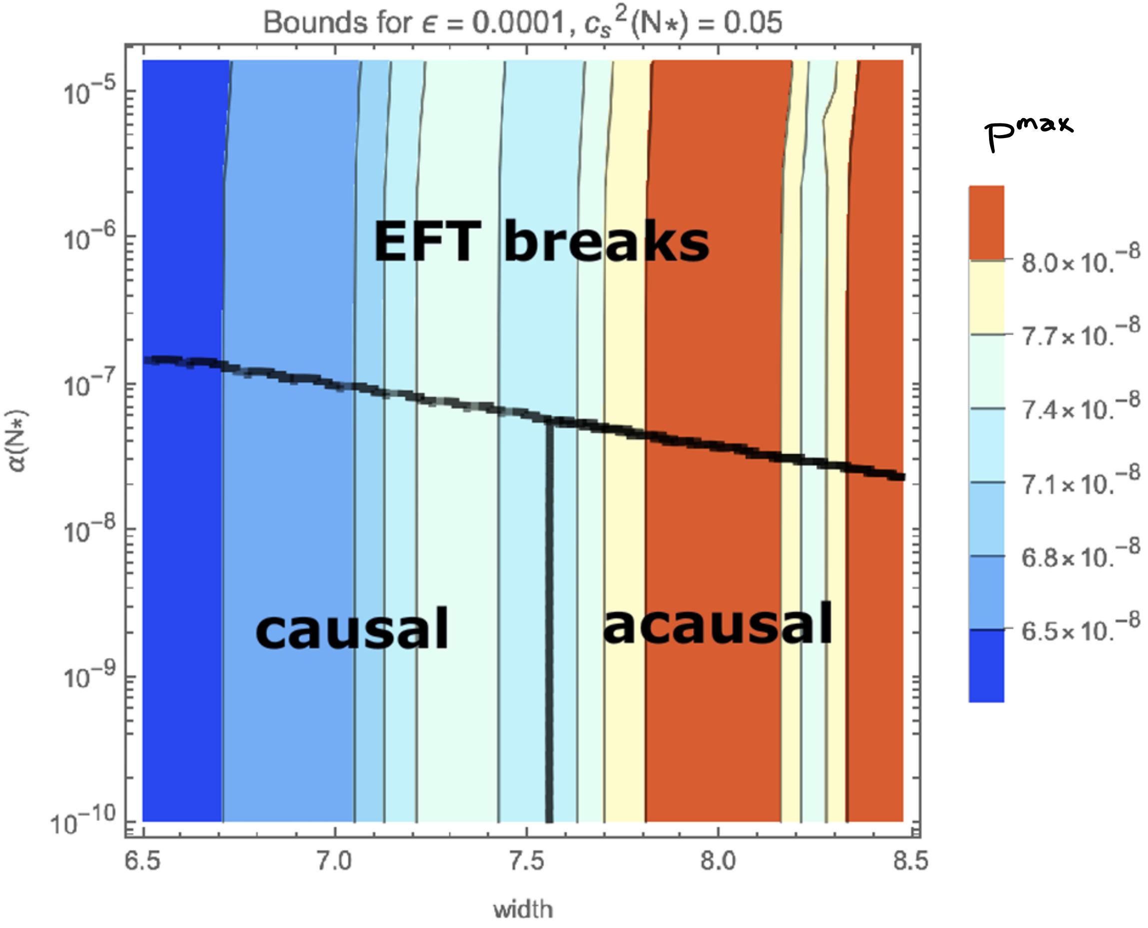
Gaussian transition



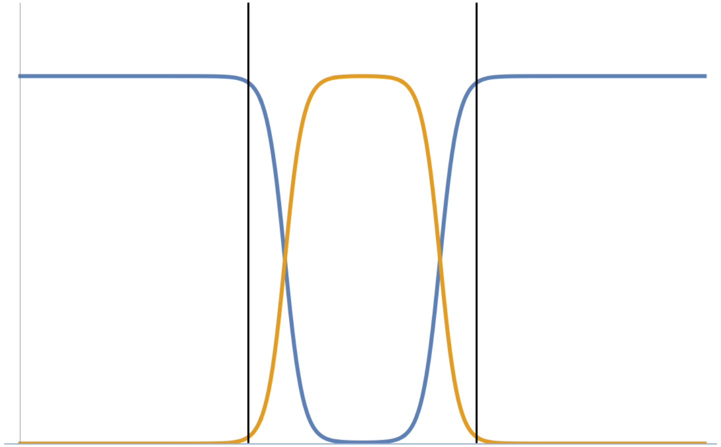


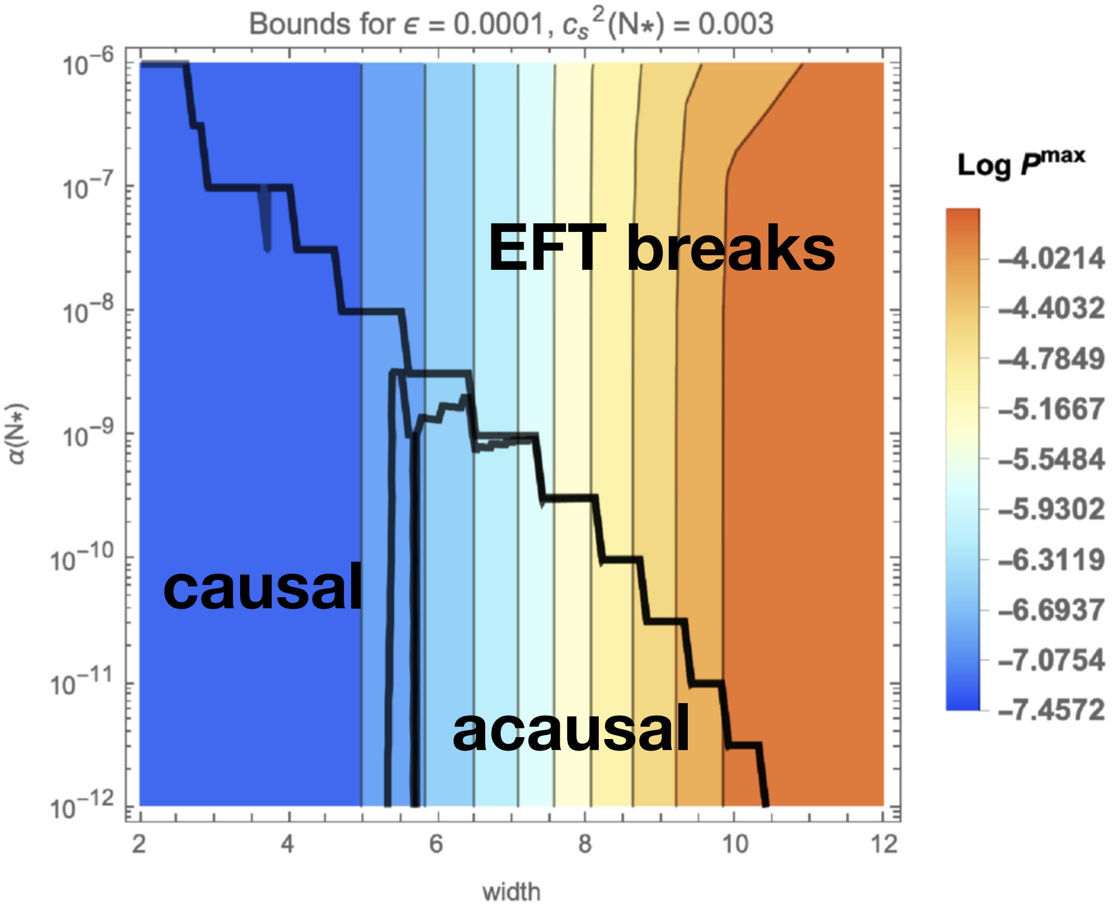
Bounds on primordial power spectrum





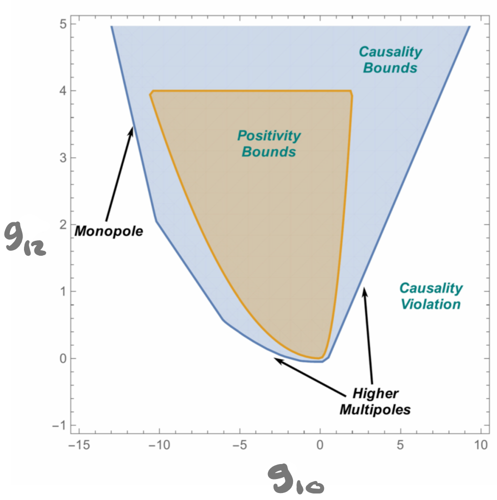
Bounds on primordial power spectrum

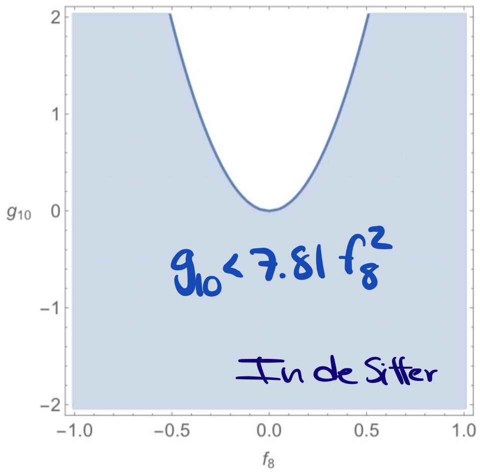


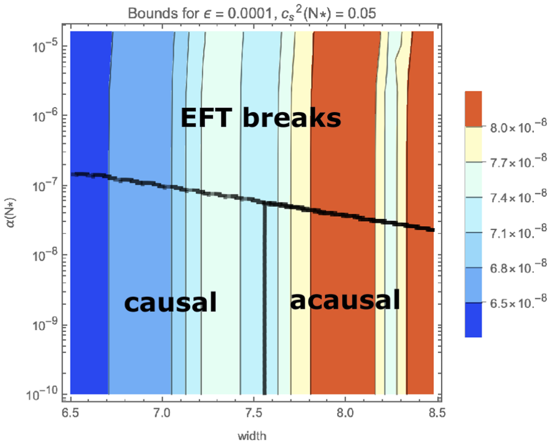


Infrared Causality

- Use low energy information
- Can test beyond 4-field operators (Lorentz invariant)
- Can be applied to Cosmological backgrounds
- Bounds on the growth of primordial power spectrum





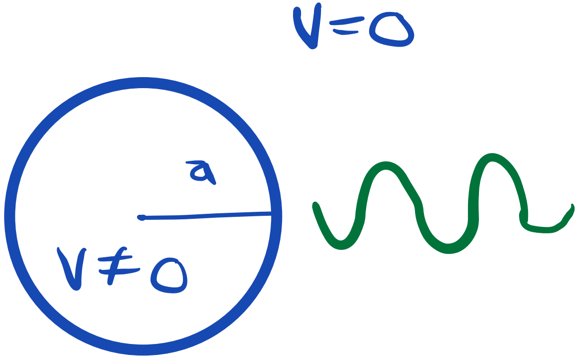


Back-up slides

Assumptions

Property	Causality Bounds	Positivity Bounds
Lorentz invariance	• Lorentz invariant EFT	• Invariant EFT and UV completion • Crossing symmetry
Unitarity	• Hermitian Hamiltonian: real Wilson coefficients	• Positive discontinuity of the EFT and UV amplitude
Causality	• No resolvable time advance	• Analyticity of amplitude in the complex s plane for fixed t
Locality	• IR theory is local	• IR and UV theories are local • Froissart-like bound in the UV
Other assumptions	• EFT and WKB expansions under control • Background generated by localized external source	• IR EFT is under perturbative control

EXAMPLES



QM: s-wave scattering in $V(r)$

(Wigner Eisenbud 50's)

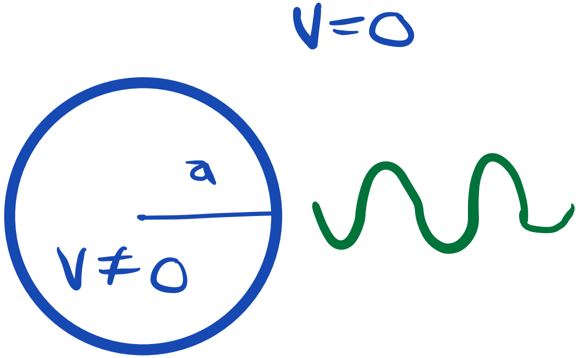
$$\Delta T_{l=0} \geq -\frac{2a}{v} + \frac{1}{kv}\sin(2ka + \delta_0) \gtrsim -\frac{2a}{v} - \frac{1}{kv}$$

hard-shell scattering

Wave nature contribution: Uncertainty Principle

$\sim -\frac{1}{\omega}$

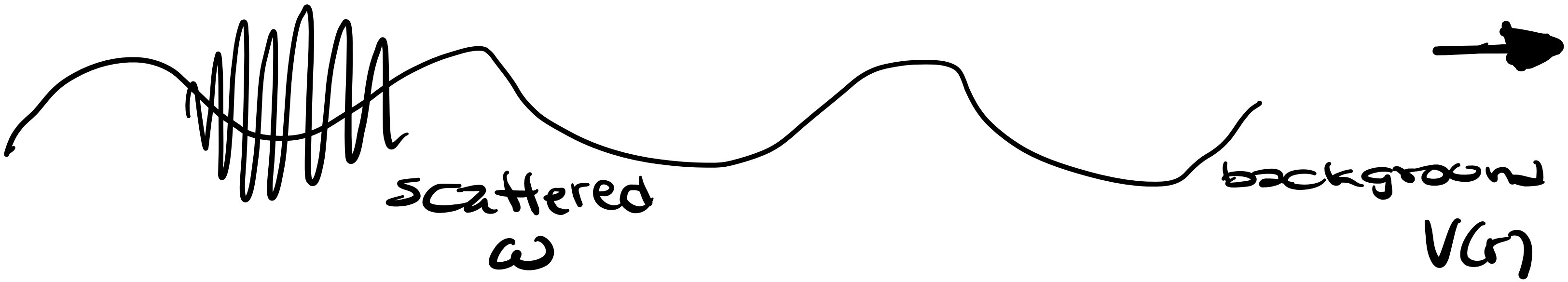
EXAMPLES



QFT: s-wave scattering of complex Φ

Same as above unless: $\omega \gg \lambda_b^{-1}, m < |\omega - V|, \omega$

$\longrightarrow$ WKB



then scattered wave doesn't see the boundary $\longrightarrow$ $\boxed{\Delta\tau \geq -\frac{1}{\omega}}$

EXAMPLES

Integrate out electron

QED + GRAVITY $\rightarrow \dfrac{\alpha}{m_e^2} R_{\mu\nu\rho\lambda} F^{\mu\nu} F^{\rho\lambda}$

Drummond Hathrell 80's

$\underline{c_s > 1}$

Around Schwarzschild

$$c_s^{\phi} = 1 + \frac{\beta_p}{m_e^2}\frac{MG}{r^3}, \quad \beta_p > 0$$

$$\boxed{-\frac{1}{\omega} < \Delta T^{EFT} \simeq -\frac{4GM}{b^2 m_e^2}\beta_p < 0}$$

Measures support of G_R w.r.t. curved background

For Shockwaves: Hollowood, Shore 2015

de Rham Tolley 2020

Galileon Generalizations

Covariant Galileon (Defayet, Esposito-Farase, Vikman)

- Keep 2nd order eom
- Weakly broken galileon symmetry $\phi \to \phi + c + b_\mu x^\mu$
- Any curved background

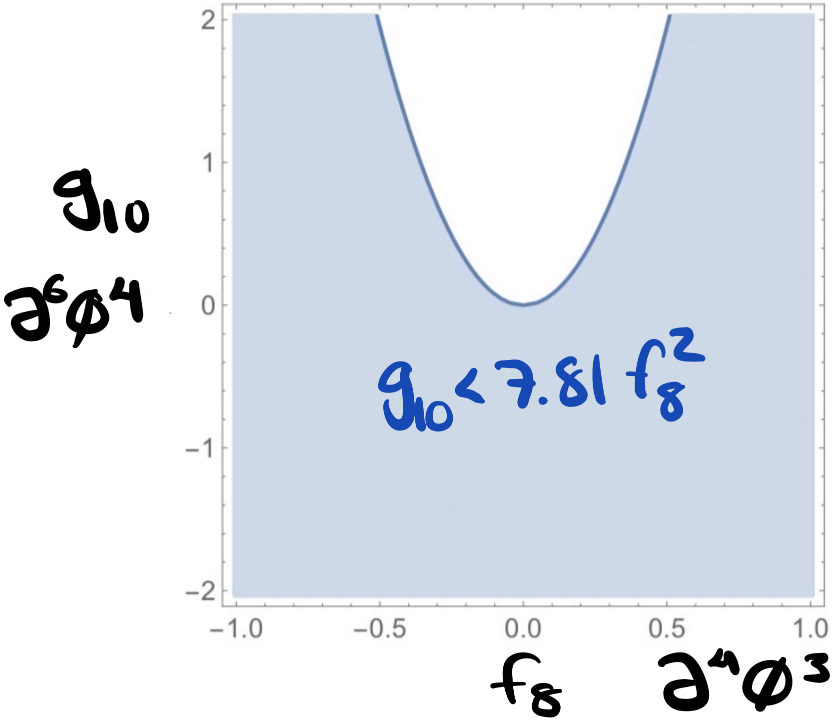
de Sitter Galileon (Goon, Hinterbichler, Trodden; Burrage, de Rham, Heisenberg)

- Invariant under

$$\phi \to \phi - \frac{1}{H\tau}\left(c + v_i x^i + v_0 H (x_i x^i - \tau^2)\right)$$

- Has symmetry breaking potential

Bounding the de Sitter Galileon



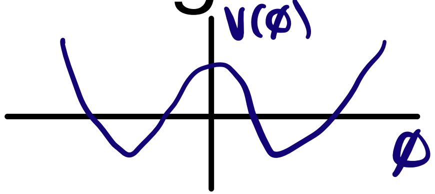
☆ Distinguish between

Galileon generalizations *

- covariant galileon $g_{10} > 0$

☆ Potential terms don't

change causal structure



$$K|\Delta r| \sim \frac{m_{eff}^2}{KH} \ll 1$$

Unresolvable