An on-shell bootstrap method for cosmological correlators

On YM and GR higher points

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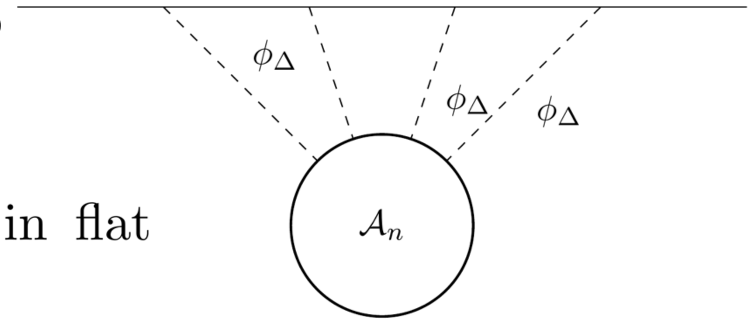
Based on

[Mei, 2305.13894]

[Mei, YM, 2402.09111, 2410.04875]

[Chowdhury, Lipstein, Mei, YM 2407.16052]

Motivation: Features of Correlators



Wave function coefficients/cosmological correlator **VS** amplitudes in flat space

1. **Not** invariant under field redefinition

$$\phi \to \phi + \alpha\phi^3,$$

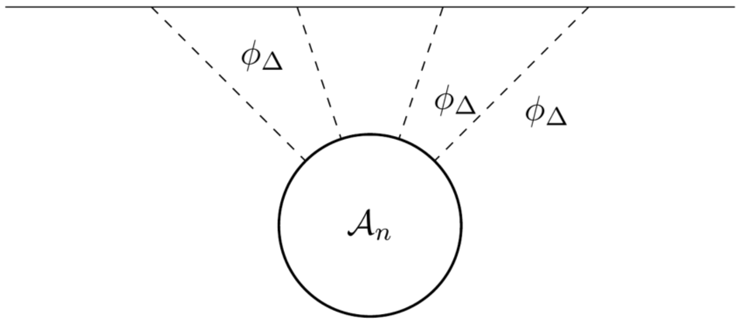
$$\langle \phi(k_1)\phi(k_2)\phi(k_3)\phi(k_4)\rangle \to \langle \phi(k_1)\phi(k_2)\phi(k_3)\phi(k_4)\rangle - \frac{1}{3}\alpha \sum_{i=1}^{4}(k_i^3).$$

2. Ward-Takahashi identity

$$\partial^i \langle J_i(\vec{x}_1) O(\vec{x}_2)\cdots O(\vec{x}_n)\rangle = -\sum_{a=2}^{n} \delta(\vec{x}_1 - \vec{x}_a)\langle O(\vec{x}_2)\cdots \delta O(\vec{x}_a)\cdots O(\vec{x}_n)\rangle$$

boundary local terms

Example: 4-point GR



[Bonifacio, Goodhew, Joyce, Pajer, Stefanyszyn '22]

$$\lim_{\vec{k}_4 \to 0} \langle\langle TTTT \rangle\rangle^{(s)} = -\frac{1}{2}\varepsilon_4^{ij} k_{3i} \partial_{k_{3j}} \langle\langle TTT \rangle\rangle$$

$$-\frac{\varepsilon_4 \cdot k_3 \varepsilon_1 \cdot \varepsilon_2 \varepsilon_3 \cdot \varepsilon_4}{2k_3^2} \left(\varepsilon_1 \cdot \varepsilon_2 \varepsilon_3 \cdot k_1 + \text{cyclic}\right) \left(k_1^2 - k_2^2\right) \left(\frac{1}{z_0} + \frac{k_1 k_2}{k_{12}} - k_1 - k_2\right)$$

boundary local terms

Flat space

Symmetry $\xrightarrow{\text{LSZ}}$ Soft theorem

$$\partial^i \langle J_i(\vec{x}_1) O(\vec{x}_2) \cdots O(\vec{x}_n) \rangle = -\sum_{a=2}^{n} \delta(\vec{x}_1 - \vec{x}_a) \langle O(\vec{x}_2) \cdots \delta O(\vec{x}_a) \cdots O(\vec{x}_n) \rangle$$

$= 0$

Features of on-shell correlators

On-shell part of the wave function coefficients

[Giddings '99] [Melville,Pimentel '23]
[Cheung, Parra-Martinez, Sivaramakrishnan, '22]

$$\Psi_n = \int \frac{dz}{z^{d+1}} \mathcal{A}_n \left(z, \partial_z, \vec{k}_a, \vec{\varepsilon}_a\right) \prod_{a=1}^{n} \phi_\Delta \left(k_a, z\right),$$

Onshell $\mathcal{D}_k^\Delta \phi_\Delta(k, z) = 0$

$\mathcal{A}_n$

Exactly invariant
field redefinition
symmetry transformation

Known [Mei, YM '24]

$\mathcal{A}_n$ → cosmological correlator

YM GR Tree-level N points

Bootstrap

$$\mathcal{M}_4 = \frac{a^{\mathrm{GR}}(1,2,3,4)}{\mathcal{D}^d_{k_s}} + \frac{b^{(2,\mathrm{GR})}(1,2,3,4)}{k_s^2} + \frac{b^{(4,\mathrm{GR})}(1,2,3,4)}{k_s^4} + c^{\mathrm{GR}}(1,2,3,4) + \mathcal{P}(2,3,4),$$

- By **factorization**

$$a^{\mathrm{GR}}(1,2,3,4) = \sum_{h=\pm} \mathcal{M}_3\left(1,2,-k_s^h\right) \cdot \mathcal{M}_3\left(k_s^{-h},3,4\right)$$

- Simply taking the **residue** of the **OPE pole**, we obtain:

$$b^{\mathrm{GR}}(1,2,3,4) = -\operatorname*{Res}_{k_s^2\to 0} \frac{a^{\mathrm{GR}}(1,2,3,4)}{\mathcal{D}^d_{k_s}}$$

- Finally, the contact terms $c^{\mathrm{GR}}(1,2,3,4)$ are determined by **flat space limits** and the **Adler zero**.

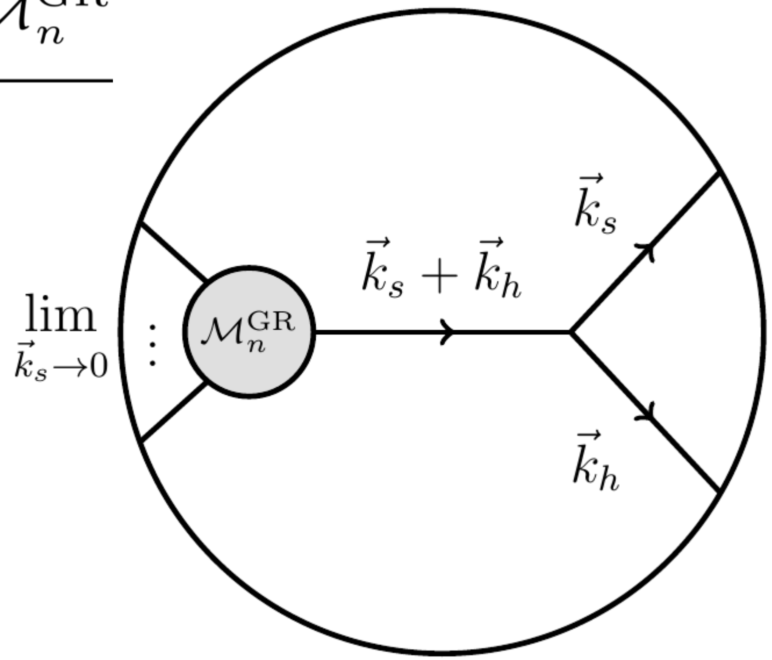
Match with [Bonifacio, Goodhew, Joyce, Pajer, Stefanyszyn '22]

First 5-graviton correlator!

Soft limit

[Chowdhury, Lipstein, Mei, YM, 24]

$$\lim_{\vec{k}_s \to 0} \mathcal{M}^{\mathrm{GR1}}_{n+1} = \frac{z^2(\varepsilon_s \cdot k_h)^2 \mathcal{M}^{\mathrm{GR}}_n}{\mathcal{D}^d_{k_h}} + \frac{\varepsilon_s \cdot \varepsilon_h \varepsilon_s \cdot k_h \varepsilon^i_h k^j_h \partial_{\varepsilon^{ij}_h} \mathcal{M}^{\mathrm{GR}}_n}{k^2_h}$$



Easily map to wave function coefficients

$$\lim_{\vec{k}_{n+1} \to 0} \langle\langle T \dots T \rangle\rangle_{n+1} = -\frac{\mathcal{N}_d}{2} \sum_{a=1}^{n} \varepsilon^{ij}_{n+1} k_{ai} \partial_{k_{aj}} \langle\langle T \dots T \rangle\rangle_n$$

So we construct a very powerful on-shell formalism for higher points spinning cosmological correlators.

Thank you