Analytic Formulae for Inflationary Correlators with Dynamical Mass

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Cosmological Correlators in Taiwan @ National Taiwan University

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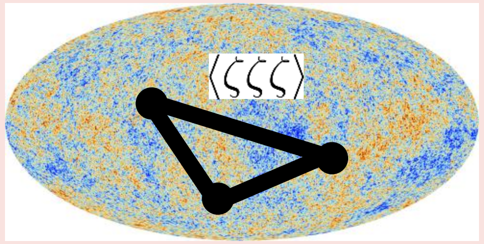
Based on:

2312.09642 with Shuntaro Aoki, Toshifumi Noumi, Masahide Yamaguchi

Cosmological Collider physics

[Chen, Wang '09, Noumi, Yamaguchi, Yokoyama '12, Arkani-Hamed, Maldacena, '15, Lee, Baumann, Pimentel '16 etc.]



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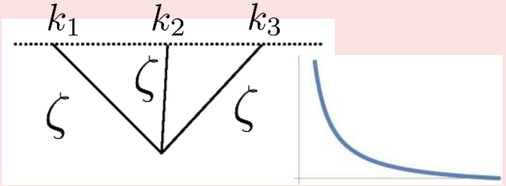

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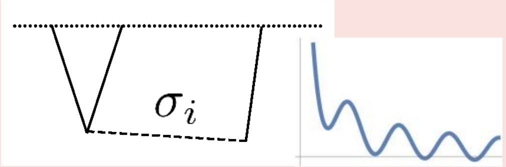

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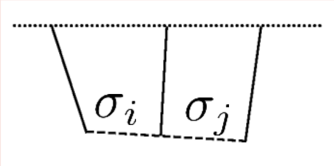

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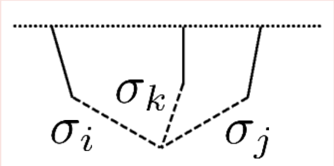


$+\cdots$

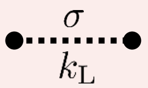
Higher spin, loops, ...

- ❑ **CC Signal**

$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log\frac{k_{\rm L}}{k_{\rm S}} + \delta\right)$$

$$k_{\rm L} \equiv k_3 \ll k_1 \simeq k_2 \equiv k_{\rm S}$$

$$\mu = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$$



$$\Rightarrow e^{-\pi\mu}\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2+i\mu}$$

✓ **Dictionary of particles at the energy scale** $\rho_{\rm inf}^{1/4} \lesssim 10^{15}\ {\rm GeV}$

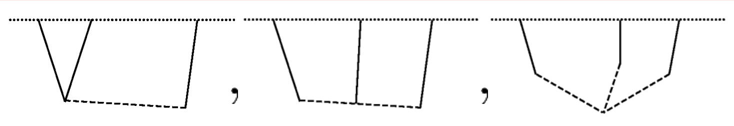
Supersymmetry, [Baumann, Green '12] RH neutrino, [Chen et al. '18] CP violation, [Liu et al. '19] gauge symmetry, [Maru, Okawa '21] swampland, ... [Reece et al. '22]

✓ **Analytical computations**

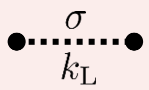
Cosmological bootstrap, [Baumann et al. '18, '19, '20] Mellin-Barnes rep., [Qin and Xianyu '22 etc.] spectral decomp., [Xianyu and Zhang '19] dispersion relation, [Liu et al. '24] polytope, ... [Arkani-Hamed et al. '17]

Interactions in CC signal

❑ Diagrams

➢ 

$$\sim \left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_\mathrm{L}}{k_\mathrm{S}} + \delta\right)$$



$$\Rightarrow e^{-\pi\mu}\left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)^{1/2+i\mu}$$

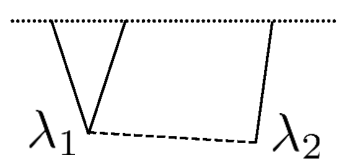
[Chen and Wang ’09, Chen, Wang, Xianyu ’17, etc.]

➢ Phase information δ : $\mathcal{A}(\mu) \times \left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)^{i\mu} = |\mathcal{A}(\mu)| e^{i\mu \ln(k_\mathrm{L}/k_\mathrm{S}) + i\mathrm{Arg}[\mathcal{A}(\mu)]}$

[Qin and Xianyu ’22]

❑ Shift symmetric vs. non-shift symmetric

➢ Signal size



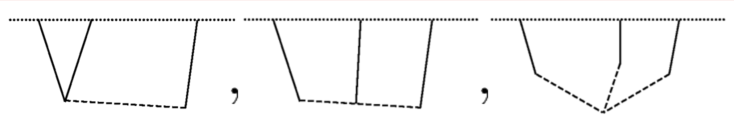
$$S \propto \lambda_1 \lambda_2$$

Exact dS: shift-symmetric in terms of ϕ

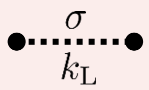
➡ Non-shift sym. ints.: λ_1, λ_2 are bounded by η, ϵ .

Interactions in CC signal

❑ Diagrams

- ➢ 

$$\sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_{\rm L}}{k_{\rm S}} + \delta\right)$$



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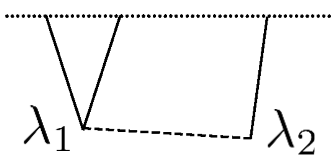


$S \propto \lambda_1 \lambda_2$

Exact dS: shift-symmetric in terms of ϕ

➡ Non-shift sym. ints.: λ_1, λ_2 are bounded by η, ϵ .

✓ *Can we detect/distinguish non-shift symmetric interactions?*

Quick answer: non-shift sym. ➡ scale dependence

Scale dependence of couplings from slow-roll

[Wang '19, Reece, Wang, Xianyu '22]

- **Example: mass of isocurvature modes**

$$\frac{\mathcal{L}_{\rm int}}{\sqrt{-g}} = yH\phi\sigma^2 \quad \Rightarrow \quad m^2_{\sigma,\rm eff} = m^2_{\sigma,0} \underline{\underline{+\, 2yH\phi_0}}$$

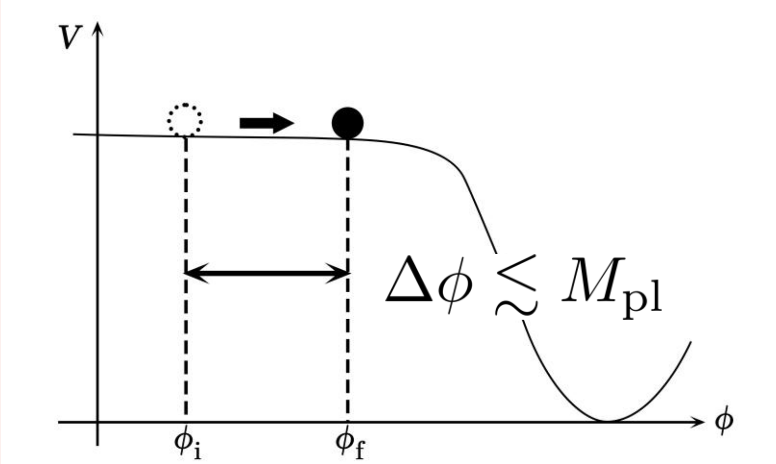
- Slow-roll approximation

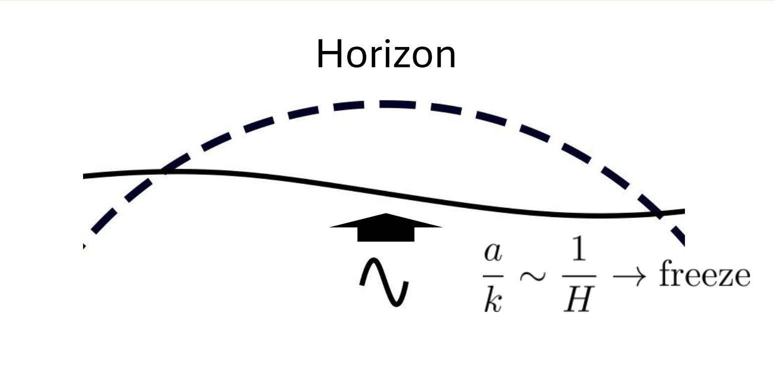
$$|\phi_0| \simeq \sqrt{2\epsilon}M_{\rm pl}H(t-t_{\rm i}) \simeq \sqrt{2\epsilon}M_{\rm pl}\log\left(\frac{\tau_{\rm i}}{\tau}\right)$$

$$\sim \sqrt{2\epsilon}M_{\rm pl}\log\frac{k}{k_{\rm i}}$$

(Horizon crossing $|k\tau| \simeq 1$)

$$\Rightarrow \quad \Delta m^2_\sigma(k) \sim y\sqrt{\epsilon}HM_{\rm pl}\log\frac{k}{k_{\rm i}}$$





* Shift symmetric couplings

$$\frac{\mathcal{L}_{\rm int}}{\sqrt{-g}} = \frac{1}{\Lambda}(\Box\phi)\sigma^2 \quad \Rightarrow \quad \frac{|\partial_t^2\phi_0|}{\Lambda} \sim \epsilon^{3/2}HM_{\rm pl}\frac{H}{\Lambda}\log\frac{k}{k_{\rm i}}$$

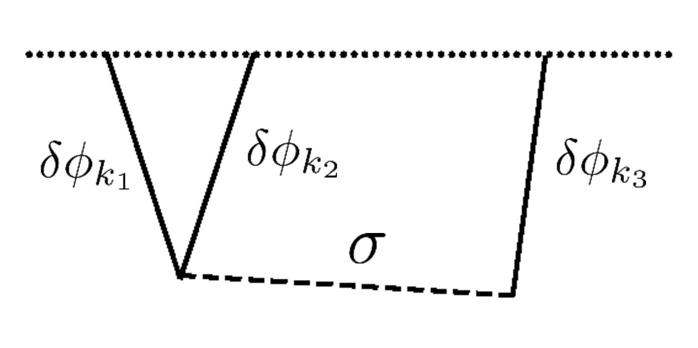
Analytical setup for time-dependent mass

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) - \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}g(\phi)\sigma^2 + \mathcal{L}_{\rm diag}\right]$$

- **Single-exchange with derivative coupling**

$$\mathcal{L}_{\rm diag} \supset \underline{c_2}(-\tau)^{-3}\sigma\delta\phi' + \underline{c_3}(-\tau)^{-2}\sigma(\delta\phi')^2$$

Unsuppressed by slow-roll (shift symmetric)



- **Time-dependent mass**

  - $\sigma_k'' - \frac{2}{\tau}\sigma_k' + \left(k^2 + \frac{m_{\rm eff}^2}{H^2\tau^2}\right)\sigma_k = 0$, $\quad \sigma_k = v_k a_k + v_k^* a_{-k}^\dagger$

Linear approx.

$$v_k = \frac{e^{\pi\gamma/2}}{\sqrt{2k}}(-H\tau)\underline{W_{-i\gamma,i\mu}(2ik\tau)}$$

encodes time dep. of mass

$$\mu^2 = \frac{g_*}{H^2}\left(1 - \frac{\sqrt{2\epsilon}g_{*,\phi}M_{\rm pl}}{g_*}\right) - \frac{9}{4}$$

$$\gamma = -\frac{\sqrt{2\epsilon}g_{*,\phi}M_{\rm pl}}{2H^2}$$

  - Computation: bootstrap eqs. and Mellin integrals [Qin and Xianyu '22, '23]

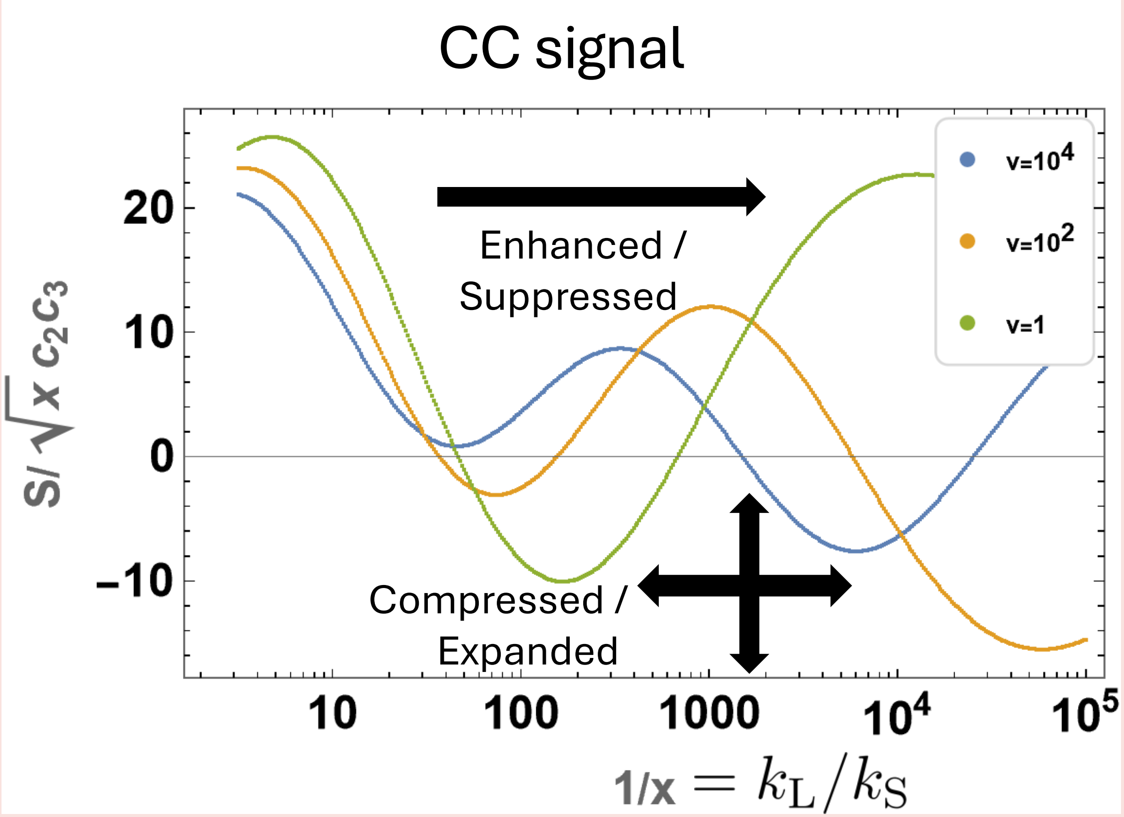
Bispectrum: mass at horizon crossing

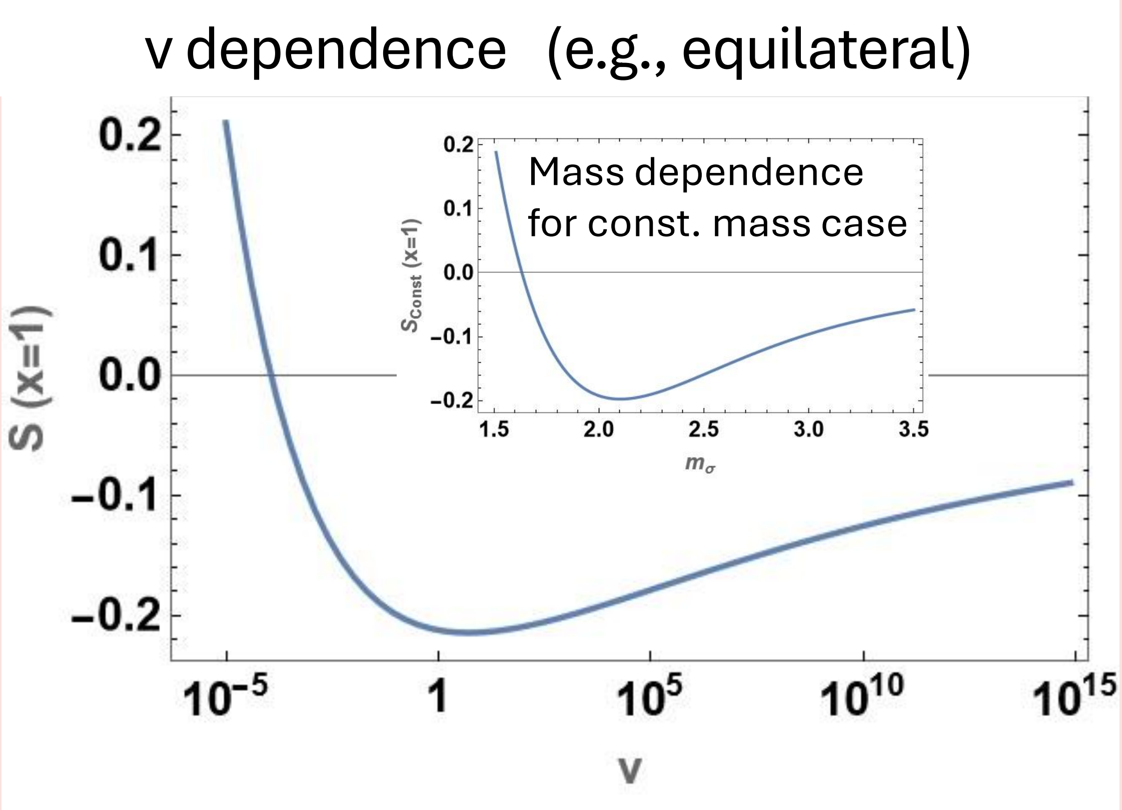
$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_3}{k_1}\right)$$

$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)} \cos\left[\mu\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)\log\frac{k_{\rm L}}{k_{\rm S}} + \delta\left(\mu\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)\right)\right] \qquad \mu^2 = \frac{m_0^2}{H^2}\left(1-\alpha\sqrt{2\epsilon}\left(1+\log\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)\right)-\frac{9}{4}\right)$$

where the interaction is $m_0^2\left(1+\alpha\frac{\phi}{M_{\rm pl}}\right)\sigma^2$.

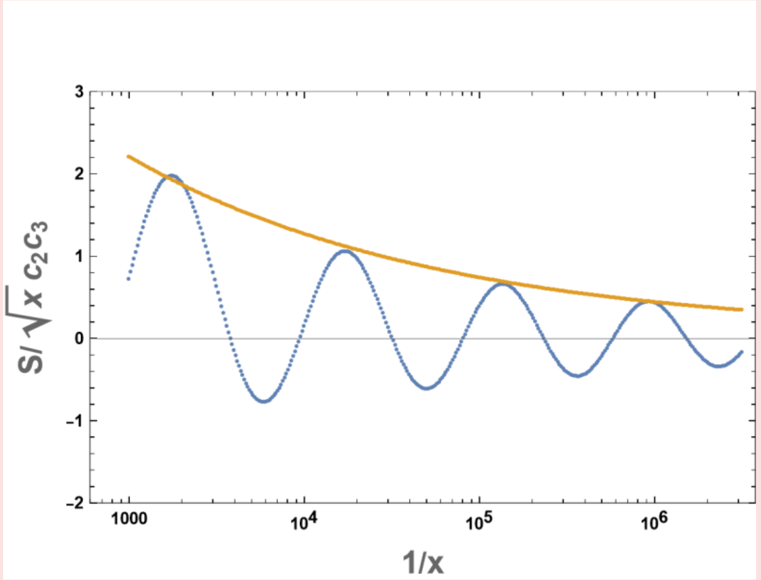
* Scale dependence $v \equiv k_{\rm S}/k_{\rm i}$ determines the value of ϕ_0 at the scale $k_{\rm S}$





Interaction distinction using scale dependence

$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)} \cos\left[\mu\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)\log\frac{k_{\rm L}}{k_{\rm S}} + \delta\left(\mu\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)\right)\right]$$



- ❑ $\overset{\phi_0\sigma^2}{\Delta\mu^2_{\rm NSS}(k)} \lesssim \sqrt{\epsilon}\frac{M_{\rm pl}}{H}$ vs. $\overset{\ddot{\phi}_0\sigma^2}{\Delta\mu^2_{\rm SS}(k)} \lesssim \epsilon^{3/2}\frac{M_{\rm pl}}{\Lambda}$
- ❑ $e^{-\pi\mu} \sim \exp\left[-\frac{\pi}{H}\sqrt{m_0^2 - \frac{9H^2}{4} + g\left(M_{\rm pl}\sqrt{2\epsilon}\log\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)\right)}\right]$ for $\frac{\mathcal{L}_{\rm int}}{\sqrt{-g}} = g(\phi)\sigma^2$

✓ Scale dependence (suppression / enhancement etc.) is characterized by the interaction

Non-shift-sym. ints: detectable through scale-dependence

Summary

❑ Cosmological Collider

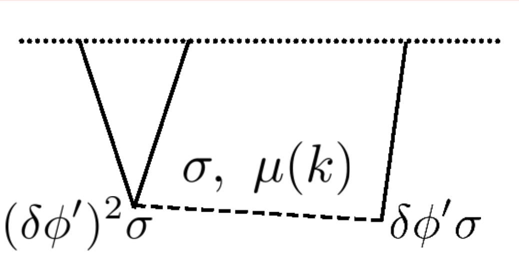
$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log\frac{k_{\rm L}}{k_{\rm S}} + \delta\right)$$

$$k_{\rm L} \equiv k_3 \ll k_1 \simeq k_2 \equiv k_{\rm S}$$

$$\mu = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$$

- Non-shift sym. Interactions: slow-roll suppressed. How to detect? ➡ Scale dependence

❑ Scale dependence from interactions



- Slow-roll of inflaton $\Delta\phi \sim \sqrt{\epsilon} M_{\rm pl} N$, $M_{\rm pl}/H \gtrsim 10^5$

  ➡ Time-dependent mass of σ: $\mathcal{L}_{\rm int} = g(\phi)\sigma^2$

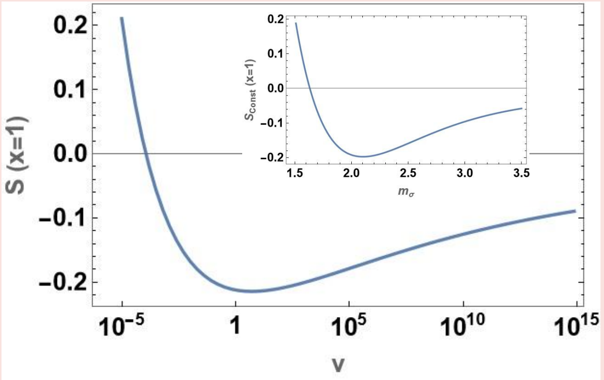
- Bispectrum: evaluating the mass at the horizon crossing scale

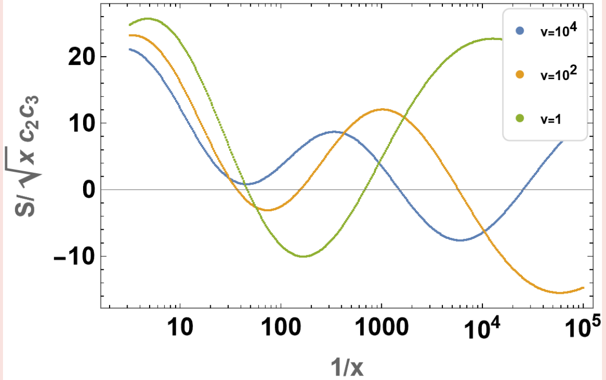
$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)} \cos\left[\mu\left(v\frac{k_{\rm L}}{k_{\rm S}}\right) \log\frac{k_{\rm L}}{k_{\rm S}} + \delta\left(\mu\left(v\frac{k_{\rm L}}{k_{\rm S}}\right)\right)\right]$$

$$\Delta\mu^2 \sim \frac{m_0^2 + g(\phi)}{H^2}$$

$$\log v \equiv \log(k_{\rm S}/k_{\rm i})$$

- Distinguishing interactions: scale dependence is characterized by $g(\phi)$





Back-up

Bispectrum in Single Field Inflation

❑ Maldacena's consistency relation [Maldacena '02]

$$\mathcal{L}_3^\zeta = a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}\partial\zeta\partial\chi + \partial_t\left(-\frac{\epsilon\eta}{2}a^3\zeta^2\dot{\zeta}\right) + \cdots$$

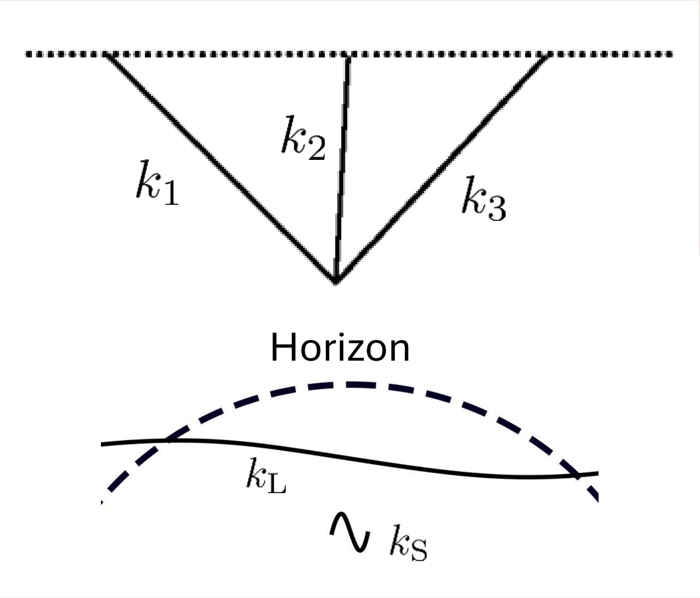
- Squeezed limit $\overset{\equiv k_{\rm L}}{k_3} \ll \overset{\equiv k_{\rm S}}{k_1} \simeq k_2$ with $\delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$

$$S_{\rm F} \longrightarrow \cancel{\frac{k_{\rm S}}{4k_{\rm L}}(1-n_s)} + \mathcal{O}\left(\frac{k_{\rm L}}{k_{\rm S}}\right)$$

Leading

$$\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle_{\rm F} \sim \int \frac{d^3k_1 d^3k_2 d^3k_3}{(2\pi)^9}\frac{S_{\rm F}}{k_1^2k_2^2k_3^2}\delta^3\left(\sum k_i\right) \to \cancel{\int_{k_{\rm L}\ll k_{\rm S}}\frac{dk_{\rm S}dk_{\rm L}}{k_{\rm S}k_{\rm L}}} \to \infty$$

$$\to \int_{k_{\rm L}\ll k_{\rm S}} dk_{\rm S}dk_{\rm L}\frac{k_{\rm L}}{k_{\rm S}^3} \quad : \text{ finite}$$



- Geodesic coordinate (local observer's effect) [Tanaka, Urakawa '11, Pajer et al. '13]

$$ds^2 = -dt^2 + e^{2\zeta}a^2(t)d\boldsymbol{x}^2$$
$$= -dt^2 + a^2(t)d\boldsymbol{x}_{\rm F}^2 + \cdots$$

$$\boldsymbol{x}_{\rm F} \simeq (1+\zeta)\boldsymbol{x},\ \zeta_{\rm F}(\boldsymbol{x}_{\rm F}) = \zeta(\boldsymbol{x}) \simeq \zeta(\boldsymbol{x}_{\rm F}) - \zeta(1 + \boldsymbol{x}\cdot\partial_{\boldsymbol{x}}\zeta)$$

(conformal Fermi normal coordinate)

Quasi-Single Field Inflation

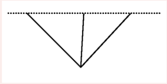
[Chen, Wang '09, Noumi, Yamaguchi, Yokoyama '12 etc.]

❑ Inflaton + *non-inflaton* with *turning trajectory*

$$\mathcal{L} = -\frac{1}{2}(r+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma)$$

$\sigma\dot{\delta\theta}$ Change of slow-roll direction $V'''\sigma^3$ No slow-roll suppression

- UV contribution $\sim$ single field EFT $\left(\frac{1}{\mu^2}\right)^3 \frac{k_{\rm L}}{k_{\rm S}}$



$S \sim \frac{k_{\rm L}}{k_{\rm S}}$

- IR contribution: leading in $k_{\rm L} \ll k_{\rm S}$

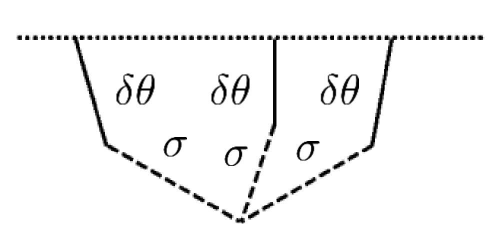
$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log\frac{k_{\rm L}}{k_{\rm S}} + \delta\right) \qquad \mu = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$$

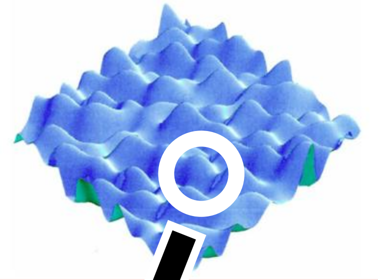
✓ Quantum interference $\left(\frac{k_{\rm S}}{k_{\rm L}}\right)^{i\mu} \sim \left(\frac{\tau_{\rm L}}{\tau_{\rm S}}\right)^{i\mu} \sim e^{im(t_{\rm S}-t_{\rm L})}$

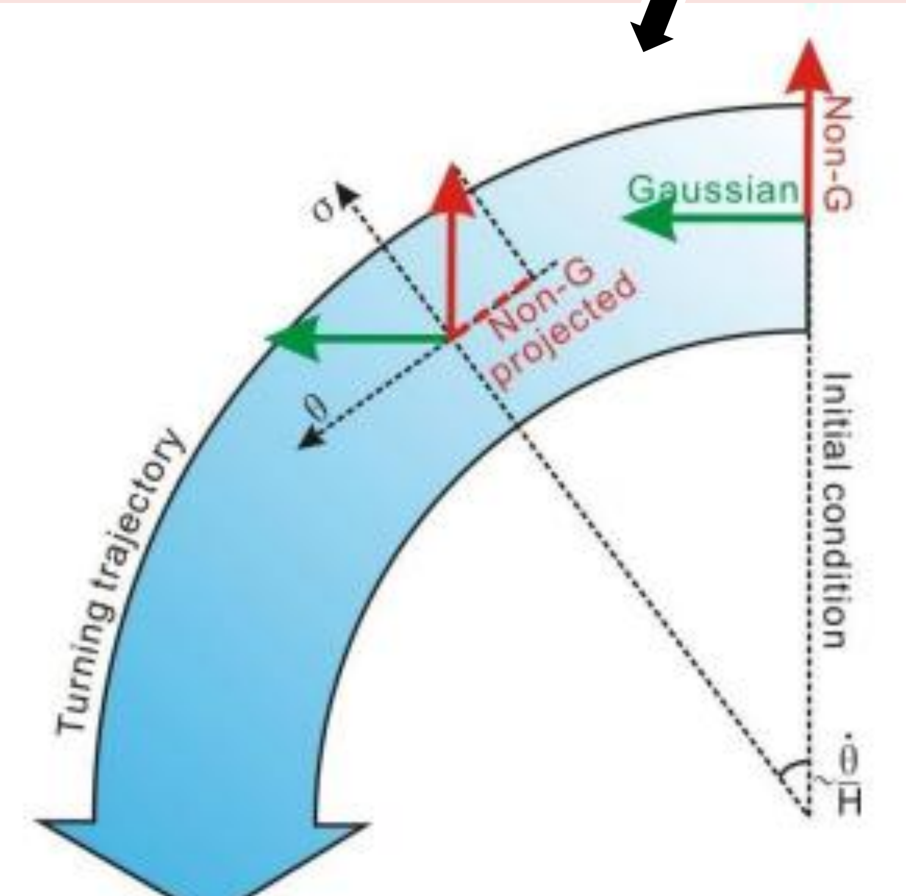
✓ Gravitational Boltzmann suppression

$$e^{-E/T} \sim e^{-m/T_{\rm H}} \sim e^{-2\pi\mu} \qquad T_{\rm H} = \frac{H}{2\pi}$$

➡ Dominant: interference term $e^{-\pi\mu}$







[Chen, Wang '09]

Quick Review of Important Aspects

❑ Mass spectra [Copeland et al. '94, Chen, Wang, Xianyu '16 etc.]

➢ Hubble scale mass

✓ "Thermal" correction $T_{\rm H} = H/2\pi \longrightarrow \Delta m^2 \propto T_{\rm H}^2$

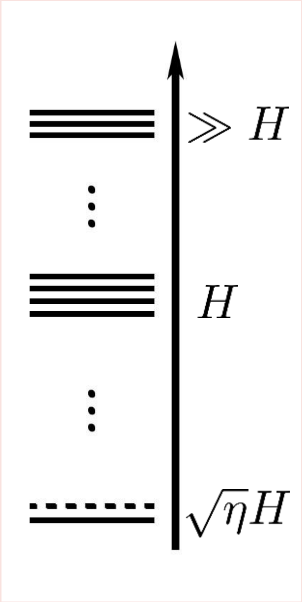
✓ SUGRA $\mathcal{L} \supset e^K V(\phi) \simeq V + \frac{c\sigma^2}{M_{\rm pl}^2} V \simeq V + 3cH^2\sigma^2$

✓ Non-minimal coupling $\mathcal{L} \supset \xi\sigma^2 R \simeq 12\xi H^2 \sigma^2$

Fields in UV theory (decouple)

Hubble induced (quasi-single)

Hubble induced? (inflaton(s))



❑ Observable range of the amplitude [Wang, Xianyu '19, Snowmass '21, Yin '23 etc.]

➢ CMB: $f_{\rm NL}^{\rm sq} \sim \mathcal{O}(1)$, galaxy survey: $f_{\rm NL}^{\rm sq} \sim \mathcal{O}(0.1)$, 21cm line from dark age: $f_{\rm NL}^{\rm sq} \sim \mathcal{O}(0.01)$?

➢ Theoretical predictions: $f_{\rm NL}^{\rm CC} \sim (\text{coupling consts.}) \times e^{-\pi\mu} \times (k_{\rm L}/k_{\rm S})^{3/2} \times \mathcal{O}(1)$ $\quad (f_{\rm NL} \sim (k_{\rm S}/k_{\rm L})S)$

Large amplitude: particle production

$\partial_\mu \phi \mathcal{J}^\mu \to \dot{\phi}\mathcal{Q}$

✓ Chemical potential $\mathcal{H} \longrightarrow \mathcal{H} - \alpha\mathcal{Q} \implies e^{-\pi\mu} \longrightarrow e^{-\pi(\mu\pm\alpha)}$

✓ Thermal state $\langle 0|\sigma\sigma|0\rangle \sim e^{-\pi\mu} \longrightarrow \langle\alpha_\sigma|\sigma\sigma|\alpha_\sigma\rangle \sim e^0$

Less Boltzmann suppression

Mellin-Barnes Representation for Boundary Conditions

[Qin, Xianyu '22 and '23]

- **Direct integration using MB rep.**

$$W_{\kappa,i\mu}(z) = e^{z/2}\int_{-i\infty}^{i\infty}\frac{ds}{2\pi i}\frac{\Gamma(s-i\mu)\Gamma(s+i\mu)}{\Gamma(s-\kappa+1/2)}z^{-s+1/2}$$

➡ Trivialize time integral

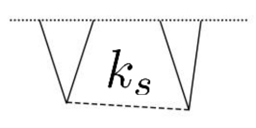
$$\mathcal{I} \sim \int d\tau_1 d\tau_2 e^{ik\tau_1+ik'\tau_2}(-\tau_1)^{p_1}(-\tau_2)^{p_2}W_{i\gamma,i\mu}(2ik\tau_1)W^*_{i\gamma,i\mu}(2ik'\tau_2)\theta(\tau_1-\tau_2)$$

$$\sim \sum_{n_1,n_2}\mathcal{A}_{n_1,n_2}(k,k')\mathrm{Res}[\Gamma(s_1\pm i\mu)]\mathrm{Res}[\Gamma(s_2\pm i\mu)]$$

$$s=-n\pm i\mu$$

- ➢ Double integration ➡ double summation ...... still complicated

- **Boundary conditions**

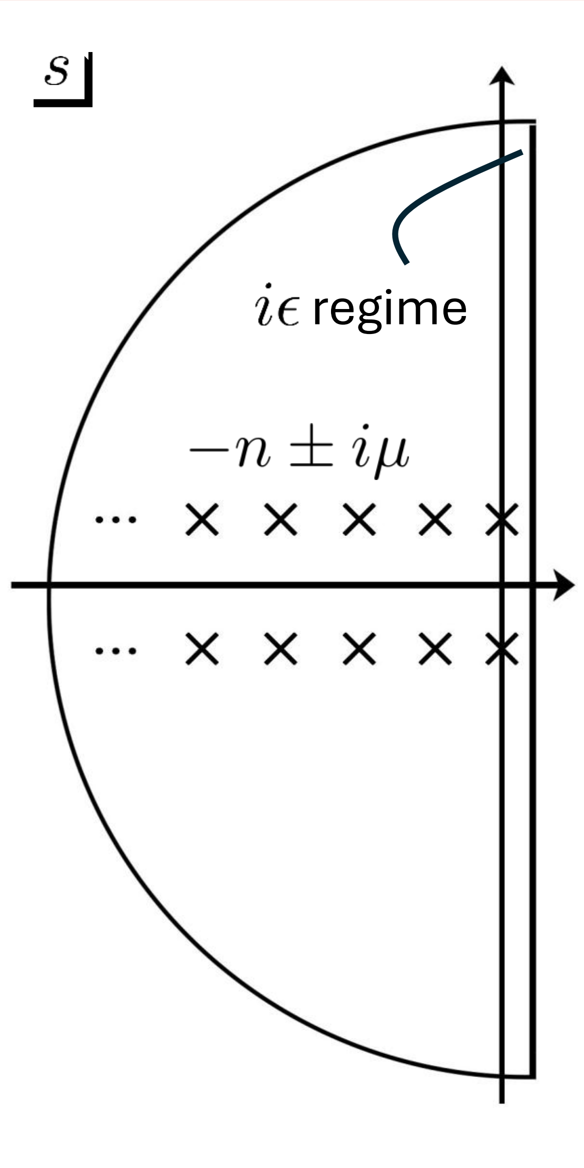


$k_s \to 0$ ➡ Bootstrapped $\mathcal{I}$, MB-integrated $\mathcal{I}$ } Resummed to closed form

➡ Boundary conditions of bootstrap eq.: fixed by MB-integrated one

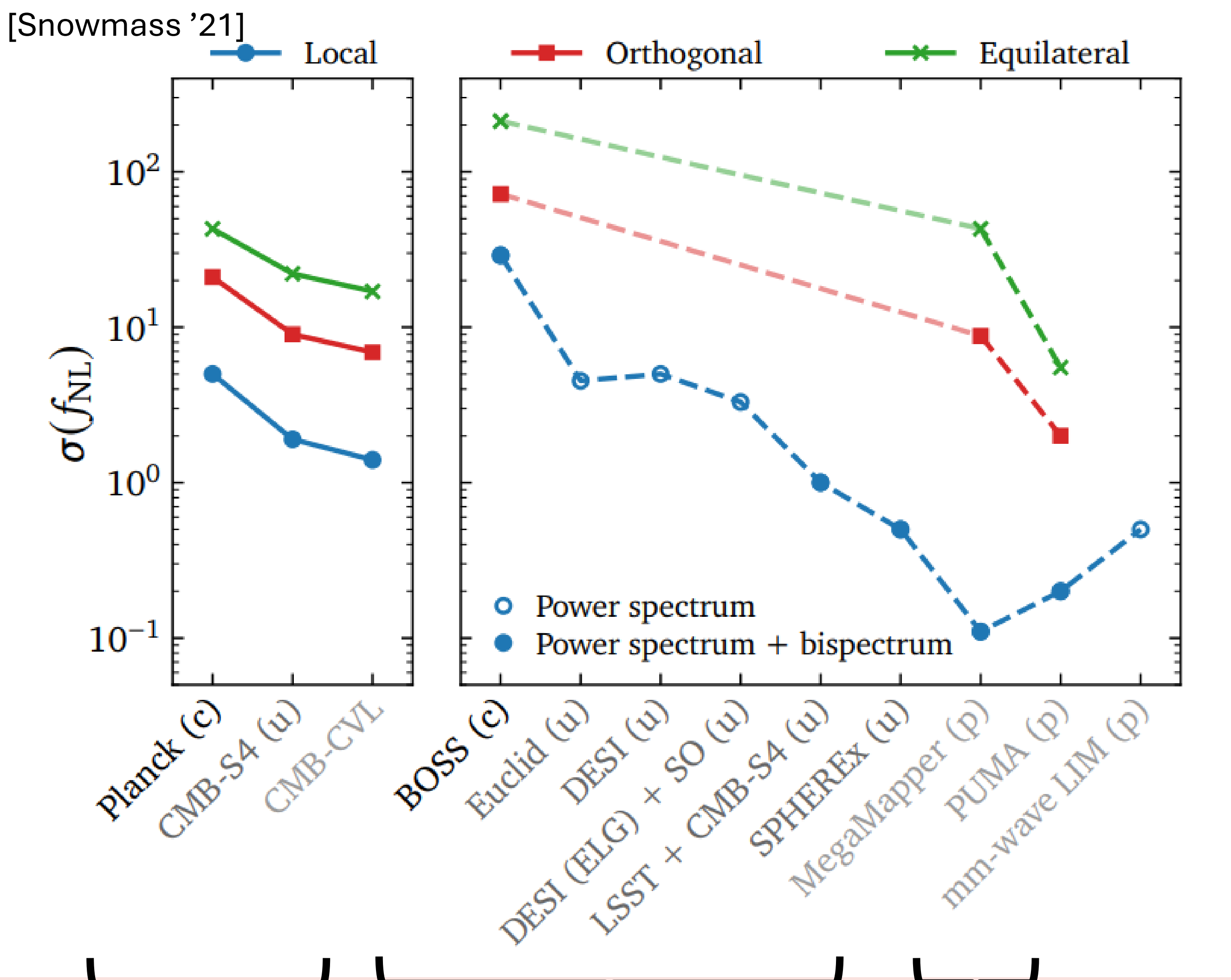
➡ Bispectrum limit: $\lim_{k_4\to 0}\mathcal{I} \sim {}_2F_1,\ {}_3F_2$ closed form !!

Full expression: see our paper



Future Observations

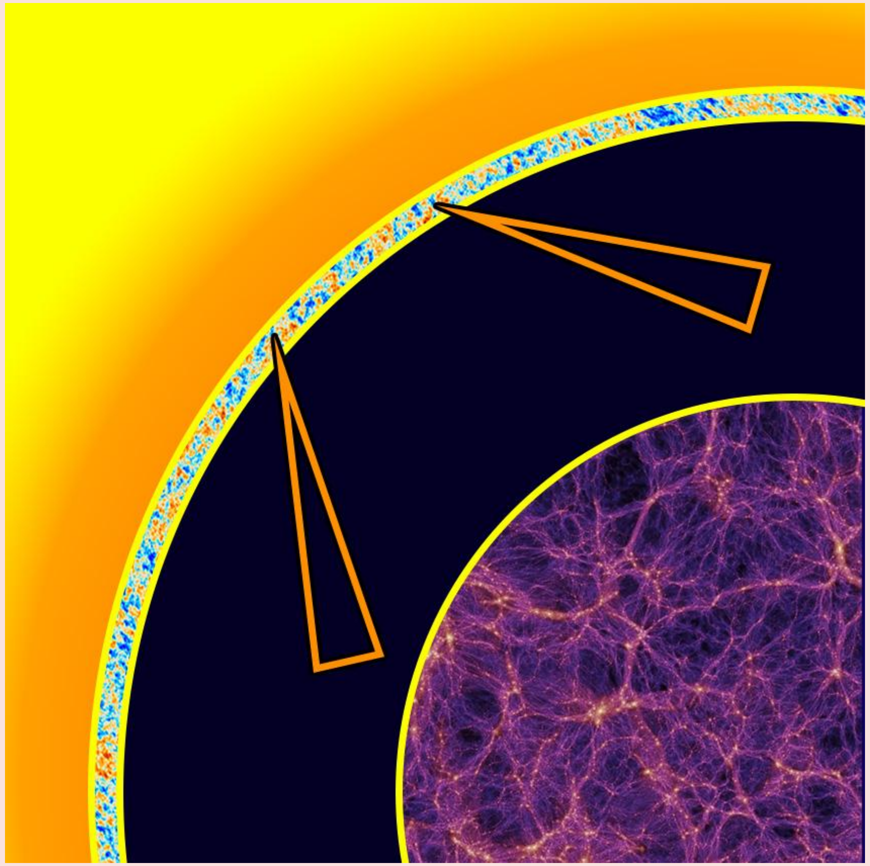
(c): completed
(u): upcoming
(p): projected



CMB

Galaxy survey

21 cm



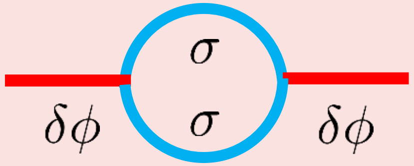
CMB Dark age Galaxies

21cm-21cm-CMB cross-correlation

$f_{\mathrm{NL}}^{\mathrm{local}} \sim 6 \times 10^{-3}$ [Orlando et al. '23]

Naturalness Conditions for Non-shift-symmetric Couplings

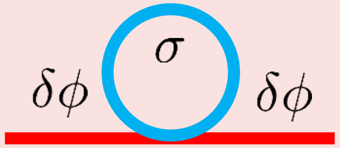
- ❑ $yH\phi\sigma^2$



$$\sim y^2 H^2 \lesssim \mathcal{O}(\eta,\epsilon) H^2 \longrightarrow y \lesssim \sqrt{\epsilon} \longrightarrow \Delta m_\sigma^2(k) \lesssim \epsilon H M_{\rm pl} \log\frac{k}{k_{\rm i}}$$

- ❑ $\lambda\phi^2\sigma^2$

$$\Delta m_{\phi_0}^2 \sim \lambda \langle\sigma^2\rangle \sim \lambda H^2 \lesssim \mathcal{O}(\eta,\epsilon) H^2 \longrightarrow \lambda \lesssim \epsilon$$



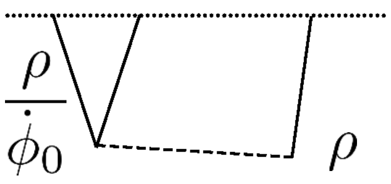
$$\sim \lambda\Lambda^2 \lesssim \mathcal{O}(\eta,\epsilon) H^2 \longrightarrow \lambda \lesssim \epsilon\frac{H^2}{\Lambda^2}$$

$$\Rightarrow \Delta m_\sigma^2(k) \lesssim \epsilon^2 H M_{\rm pl}\frac{H M_{\rm pl}}{\Lambda^2}\log^2\frac{k}{k_{\rm i}}$$

* Shift sym. Couplings: $\frac{|\partial_t^2\phi|}{\Lambda} \sim \epsilon^{3/2} H M_{\rm pl}\frac{H}{\Lambda}\log\frac{k}{k_{\rm i}}, \quad \frac{|\partial_t\phi_0|^2}{\Lambda^2} \sim \epsilon H M_{\rm pl}\frac{H M_{\rm pl}}{\Lambda^2}\log^2\frac{k}{k_{\rm i}}$

Size Estimation of Single-exchange Diagrams

$$\frac{\rho}{\dot\phi_0}(\partial_\mu\phi)^2\sigma \longrightarrow \rho\ \delta\phi'\sigma + \frac{\rho}{\dot\phi_0}\ (\delta\phi')^2\sigma$$
$$= c_2\delta\phi'\sigma + c_3(\delta\phi')^2\sigma$$

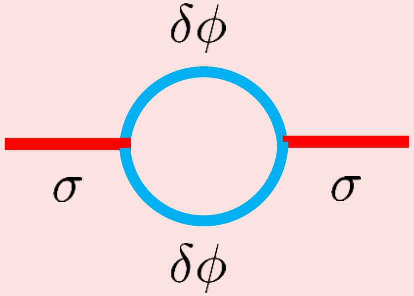


$$S_{\rm SE} \sim \frac{\rho^2}{\dot\phi_0} P_\zeta^{-1/2} e^{-\pi\mu}\mathcal{O}(1)$$

$$\left.\begin{aligned}\dot\phi_0 &\sim H^2 P_\zeta^{-1/2}\\ \rho &\equiv \alpha H\end{aligned}\right\}\ \frac{\rho^2}{\dot\phi_0} \sim \alpha^2 P_\zeta^{1/2}$$

Naturalness $\alpha \lesssim 1$

[Pinol, Renaux-Petel, Werth ’23]



$$\Longrightarrow S_{\rm SE} \lesssim e^{-\pi\mu} \times \mathcal{O}(1)$$

$$m_\sigma = 2.5H \to e^{-\pi\mu} \sim 10^{-3}$$