Peculiar Behavior of EFTs in de Sitter (Work in Progress)

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Goal

- We want to see what how positivity of EFT coefficients from integrating out massive particles manifests itself in wavefunction coefficients.

$$\mathcal{L}_{\mathrm{EFT}}^{\mathrm{int}} = \sqrt{-g}\,\phi^2 \left(c_0 + c_1 \frac{\Box}{M^2} + c_2 \frac{\Box^2}{M^4} + \cdots \right) \phi^2$$

Integrating Out Massive Particles

- Start with one light scalar and one heavy scalar coupled together

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}M^2\chi^2 - g\phi^2\chi\right)$$

with $M \gg m$

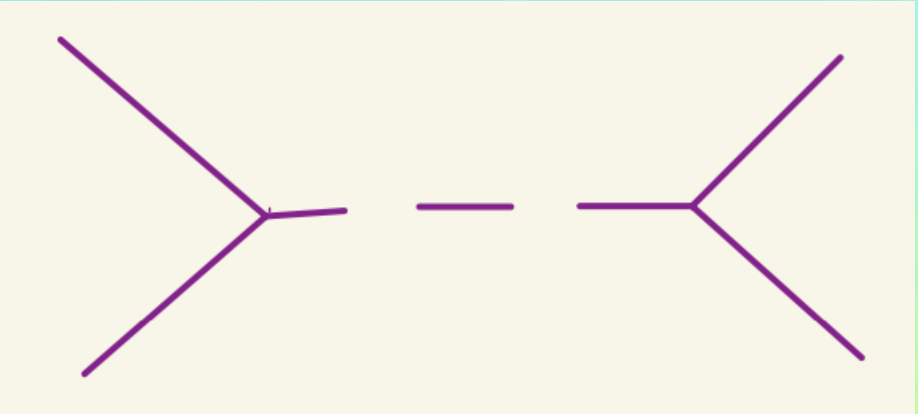
- For energies $E \ll M$, we can integrate out χ to get low energy EFT.

$$\mathcal{L}_{EFT} = \sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{2}\frac{g^2}{M^2}\phi^2\left(1 + \frac{\Box}{M^2} + \frac{\Box^2}{M^4} + \cdots\right)\phi^2\right)$$

Flat Space Amplitudes

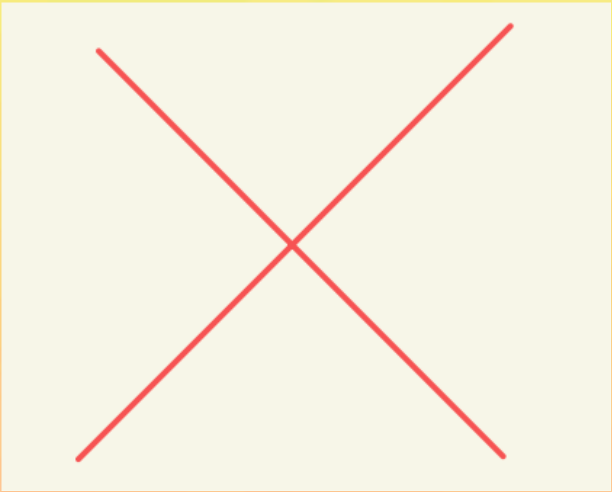
- Amplitude from the "UV completion"

$$\mathcal{A}_4 = -4\frac{g^2}{M^2}\left(\frac{1}{s-M^2}+\frac{1}{t-M^2}+\frac{1}{u-M^2}\right)$$



- Amplitude from the low energy EFT, valid for $s < M$

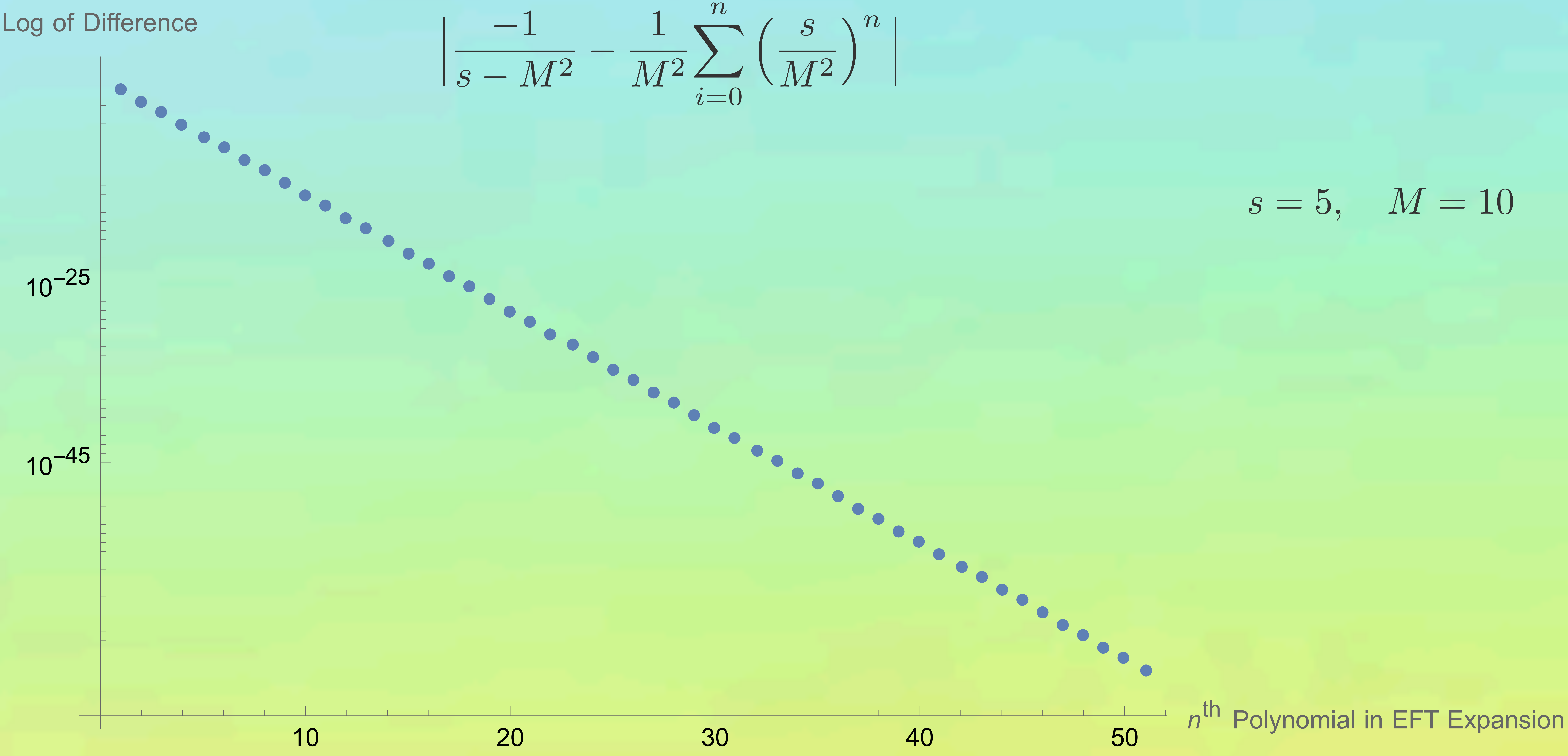
$$\mathcal{A}_4^{\mathrm{EFT}} = 4\frac{g^2}{M^2}\left(3+4\frac{m^2}{M^2}+\frac{s^2+t^2+u^2}{M^4}+\cdots\right)$$



Difference Between UV Completion & n$^{\text{th}}$ Polynomial in EFT Expansion

$$\left|\frac{-1}{s-M^2} - \frac{1}{M^2}\sum_{i=0}^{n}\left(\frac{s}{M^2}\right)^n\right|$$

$$s = 5, \quad M = 10$$



Wavefunction Coefficients in dS

- In dS we choose to focus on wavefunction coefficients ψ, instead of scattering amplitudes, with the external legs having momenta k_1, k_2, k_3, k_4 being conformally coupled $m = 2H^2$.

- We write them as functions of u and v, defined as

$$u = \frac{k_I}{k_1 + k_2}, \qquad u = \frac{k_I}{k_3 + k_4}$$

with $\quad k_I = |\mathbf{k}_1 + \mathbf{k}_2|$

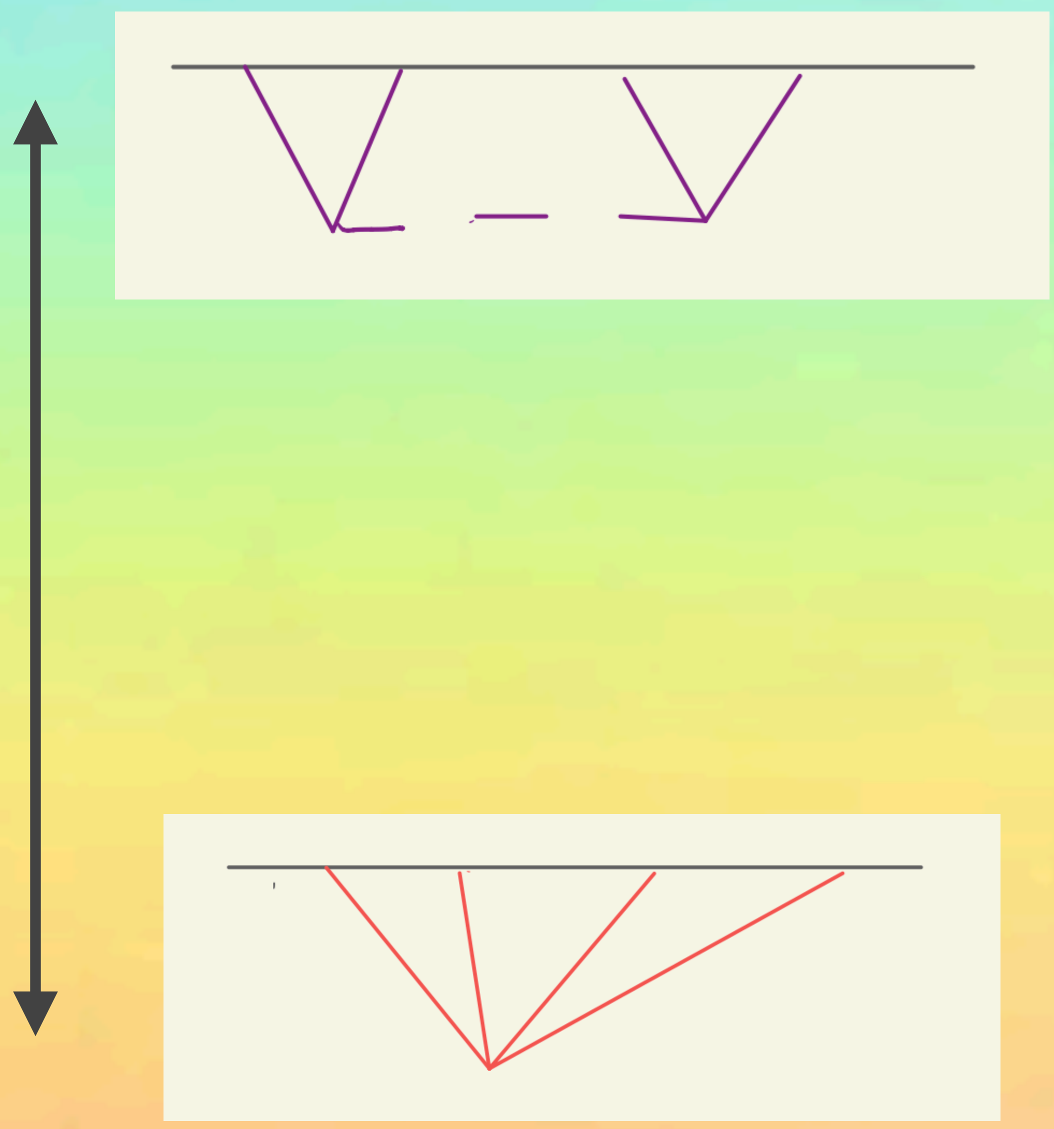
- In the wavefunction coefficients, the mass shows up more naturally in the combinations defined by

$$\tilde{M}^2 = \mu^2 + \frac{1}{4} = \frac{M^2}{H^2} - 2$$

dS Wavefunction Coefficients

- s-channel wavefunction coefficient from the "UV completion"

$$\psi_4 = k_I^{-1} \times \begin{cases} \sum_{m,n=0}^{\infty} c_{m,n} u^{2m+1} (u/v)^n + \frac{\pi}{2\cosh(\pi\mu)} \hat{g}(u,v) & u \le v \\ \sum_{m,n=0}^{\infty} c_{m,n} v^{2m+1} (v/u)^n + \frac{\pi}{2\cosh(\pi\mu)} \hat{g}(v,u) & u \ge v \end{cases},$$



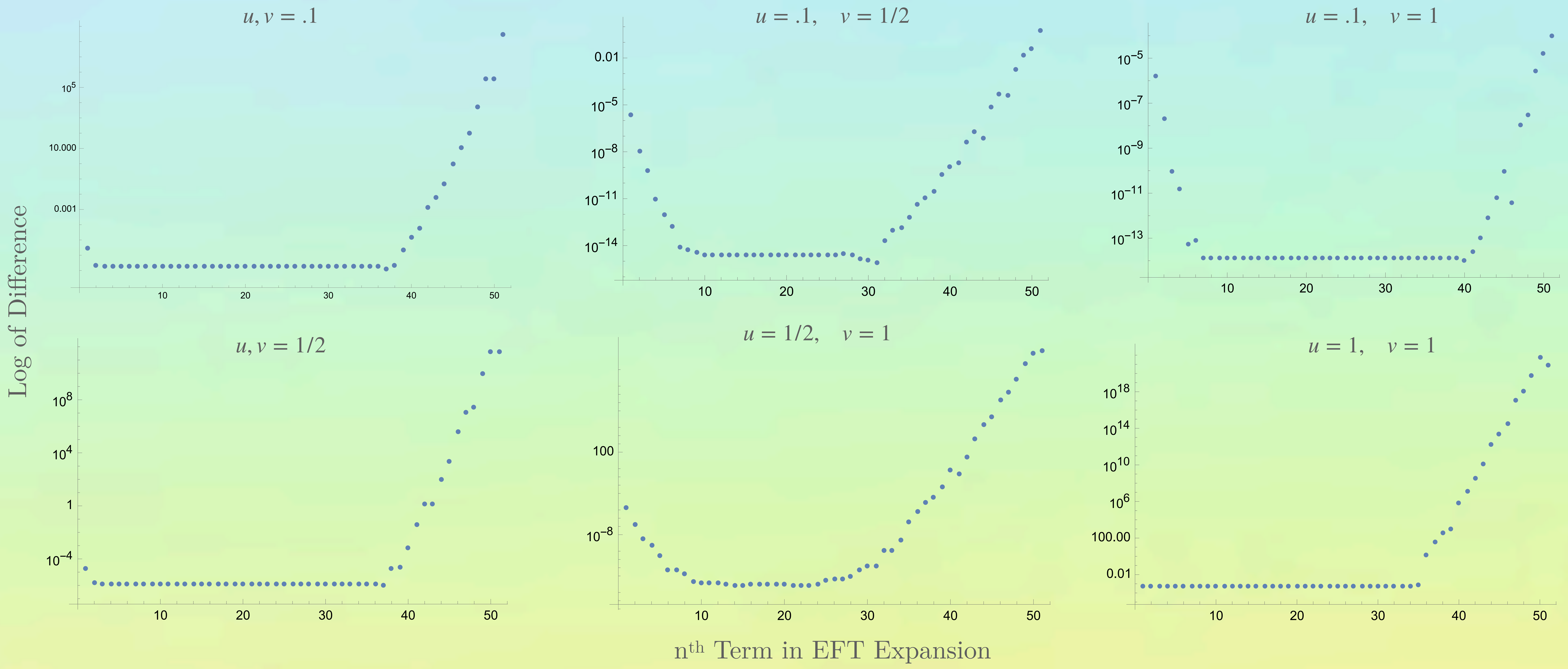
- s-channel wavefunction coefficient from the low energy EFT

$$\psi_4^{\mathrm{EFT}} = k_I^{-1} \sum_{n=0}^{\infty} \frac{(-1)^n}{\tilde{M}^{2n+2}} \Delta_u^n \left(\frac{uv}{u+v} \right)$$

with $\Delta_u = u^2(1-u^2)\partial_u^2 - 2u^3\partial_u$

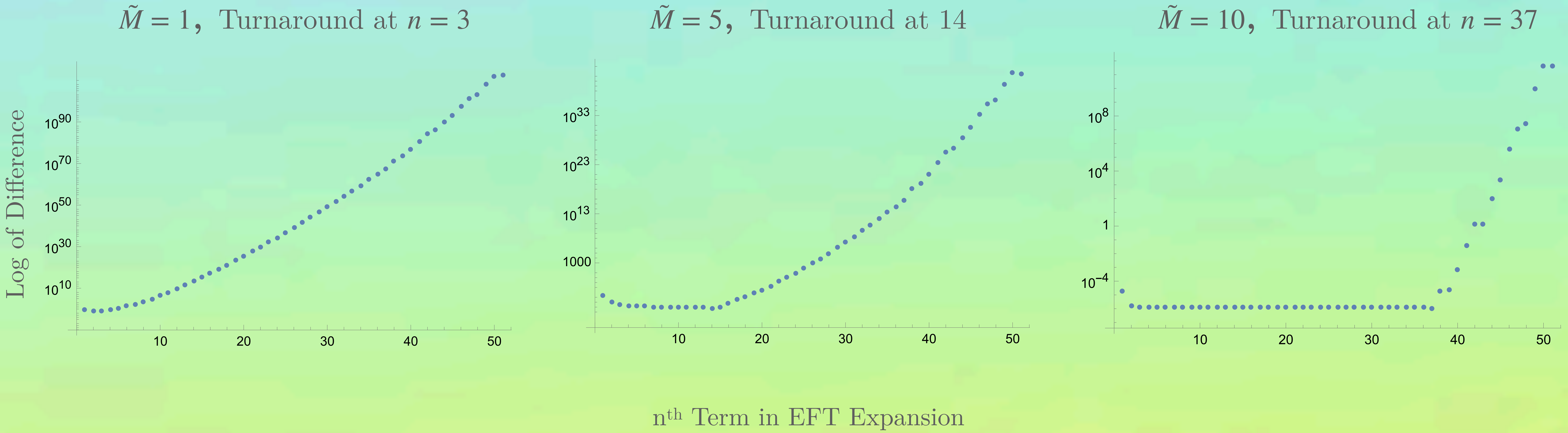
Difference Between UV Completion & n^{th} Term in EFT Expansion in dS

$\tilde{M} = 10$



Turnaround Point Increases with $\tilde{M}$

$u, v = 1/2$



Failure of Convergence

- To achieve convergence, we would want $\left|\frac{\Delta_u}{\tilde{M}^2}\right| \ll 1$, but this is not guaranteed since Δ_u is an operator.

- We find there is always an n_* such that

$$\left(\frac{\Delta_u}{\tilde{M}^2}\right)^{n_*}\left(\frac{uv}{u+v}\right) > \left(\frac{\Delta_u}{\tilde{M}^2}\right)^{n_*-1}\left(\frac{uv}{u+v}\right)$$

Why is this happening?

- The variables u and v are not eigenstates of Δ_u, unlike in Minkowski space where $\Box \to p^2$, and we can directly take the limit $p^2 \ll M^2$.
- We want $\Box \ll M^2$, but in dS, turning this into a range of u and v for which the EFT is valid is not straightforward.
- Fixed u and v correspond to a superposition of all possibly values of Δ_u, probing beyond the EFT's cutoff.
- Future work will focus on how to proceed.

Wavefunction Coefficients in dS

- The s-channel four-point wave function coefficient can be written as a series solution

$$\psi_4 = k_I^{-1} \times \begin{cases} \sum_{m,n=0}^{\infty} c_{m,n} u^{2m+1} (u/v)^n + \frac{\pi}{2\cosh(\pi\mu)} \hat{g}(u,v) & u \leq v \\ \sum_{m,n=0}^{\infty} c_{m,n} v^{2m+1} (v/u)^n + \frac{\pi}{2\cosh(\pi\mu)} \hat{g}(v,u) & u \geq v \end{cases},$$

where $$c_{m,n} = \frac{(-1)^n (n+1)(n+2)\cdots(n+2m)}{[(n+\frac{1}{2})^2+\mu^2][(n+\frac{5}{2})^2+\mu^2]\cdots[(n+\frac{1}{2}+2m)^2+\mu^2]}$$

$$\begin{aligned} \hat{g}(u,v) =& \hat{F}_+(u)\hat{F}_-(v) - \hat{F}_-(u)\hat{F}_+(v) - \frac{\alpha_-}{\alpha_+}(\beta_0+1)\hat{F}_+(u)\hat{F}_+(v) - \frac{\alpha_+}{\alpha_-}(\beta_0-1)\hat{F}_-(u)\hat{F}_-(v) \\ &+ \beta_0[\hat{F}_-(u)\hat{F}_+(v) + \hat{F}_-(u)\hat{F}_+(v)] \end{aligned}$$

$$\alpha_\pm = -\left(\frac{i}{2\mu}\right)^{\frac{1}{2}\pm i\mu} \frac{\Gamma(1\pm i\mu)}{\Gamma(\frac{1}{4}\pm\frac{i\mu}{2})\Gamma(\frac{3}{4}\pm\frac{i\mu}{2})}, \qquad \beta_0 \equiv \frac{1}{i\sinh(\pi\mu)}$$

Wavefunction Coefficients in dS

- The s-channel wave function coefficient of the low energy EFT is given by

$$\psi_4^{EFT} = k_I^{-1} \sum_{n=0}^{\infty} \frac{(-1)^n}{M^{2n+2}} \Delta_u^n \left(\frac{uv}{u+v} \right)$$

where

$$\Delta_u = u^2(1-u^2)\partial_u^2 - 2u^3\partial_u$$

- which captures the solution for large M^2, but fails to capture non-perturbative pieces describing particle production.