# Precision Gravity

## Gravitational waves using Feynman diagrams

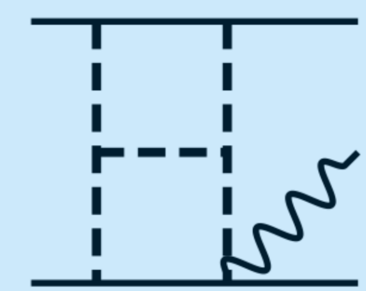

Raj Patil

MPI for Gravitational Physics (AEI) and Humboldt University

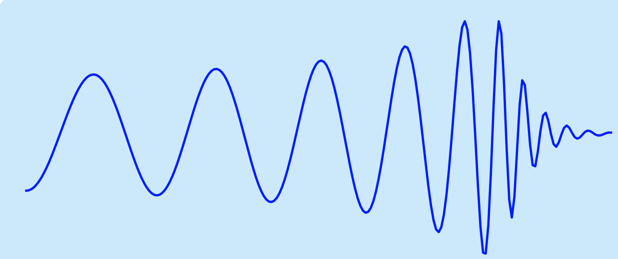

Short talk, Cosmological Correlators in Taiwan
$2^{\text{nd}}$ December 2024

In collaboration with:
Manoj Mandal, Pierpaolo Mastrolia, Hector Silva, and Jan Steinhoff

Talk based on: arXiv 2209.00611, 2210.09176, 2304.02030, 2308.01865, 2412.xxxxx

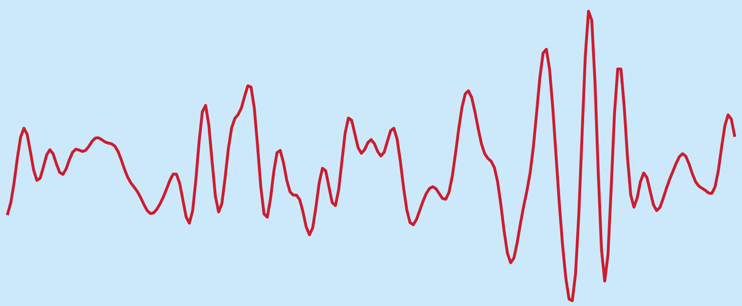


Muti-loop Feynman diagrams



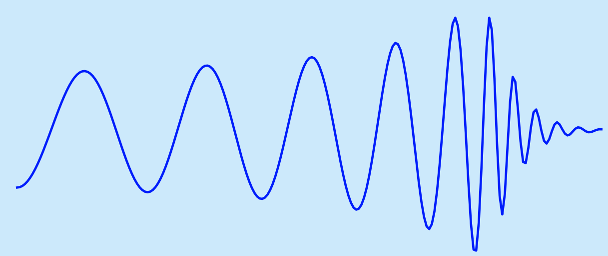
High precision waveform

# Why precision gravity?



Strain in the detectors

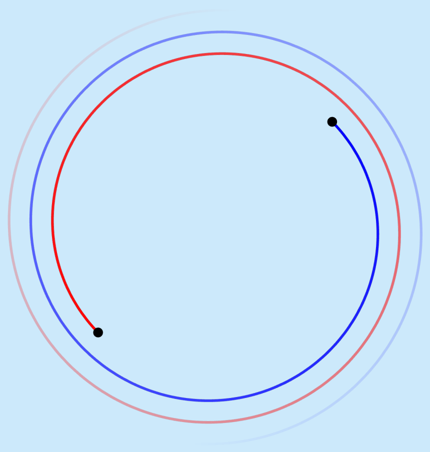
Precise and accurate parameter estimation



Waveform templates

$$\dot{r} = \frac{dH}{dp_r} \qquad \dot{p}_r = -\frac{dH}{dr} + F_r$$

$$\dot{\phi} = \frac{dH}{dp_\phi} \qquad \dot{p}_\phi = -\frac{dH}{d\phi} + F_\phi$$



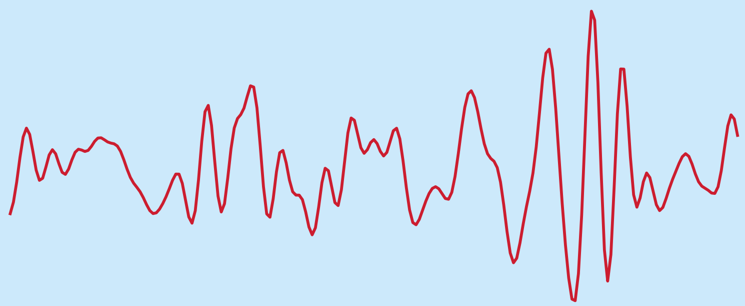
**A typical waveform model needs precise Hamiltonians and Fluxes**

[SEOBNR, TEOBResumS, ...]

| | |
|---|---|
| Primary black hole mass | $36^{+5}_{-4} M_\odot$ |
| Secondary black hole mass | $29^{+4}_{-4} M_\odot$ |
| Final black hole mass | $62^{+4}_{-4} M_\odot$ |
| Final black hole spin | $0.67^{+0.05}_{-0.07}$ |
| Luminosity distance | $410^{+160}_{-180}$ Mpc |
| Source redshift $z$ | $0.09^{+0.03}_{-0.04}$ |

GW150914

# Why precision gravity?



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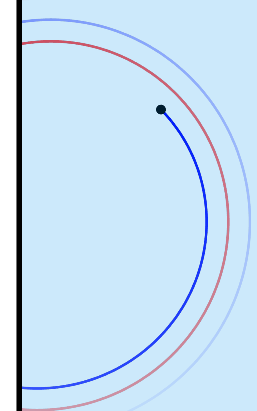
Precise anc
parameter e



| | |
|---|---|
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| Secondary bla | |
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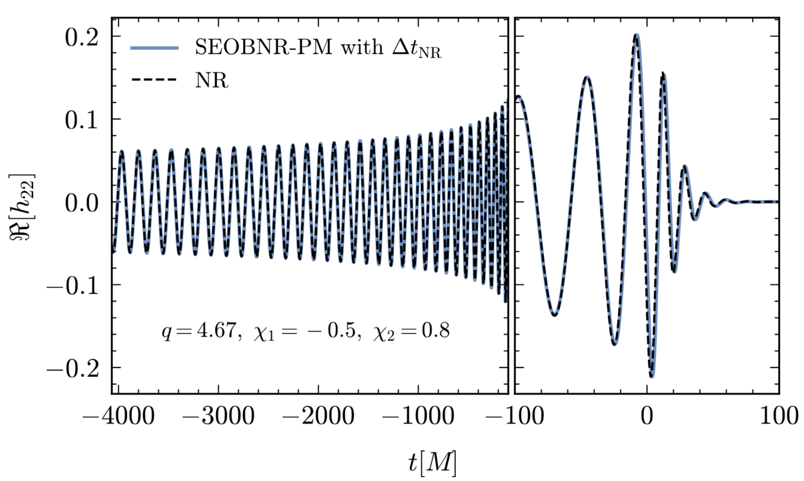
GW150914

## SEOBNR-PM




First bound orbit waveform model based on post-Minkowskian data

[Buonanno, Mogull, **RP**, Pompili (2024)]

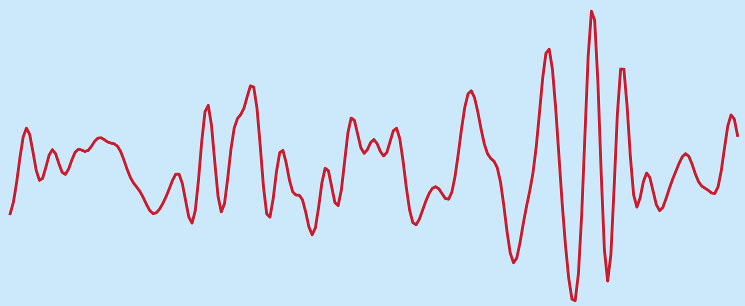
See talk by Alessandra Buonanno

Friday 11.15 am next week!!

A typ al waveform model needs recise Hamiltonians and Fluxes

[SEOBNR, TEOBResumS, ...]

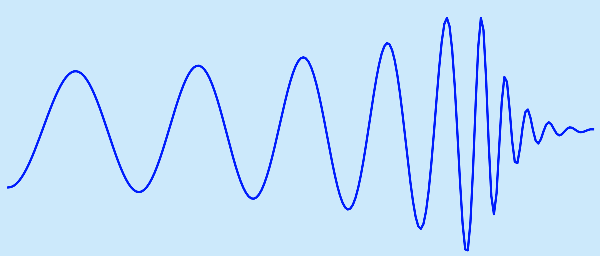
# Why precision gravity?



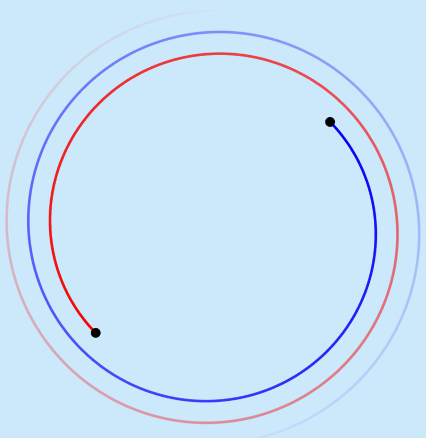
Strain in the detectors

| | |
|---|---|
| **Hamiltonians**<br>conservative dynamics | $\dot{r} = \frac{dH}{dp_r}$ $\quad \dot{p}_r = -\frac{dH}{dr} + F_r$ |
| **Fluxes**<br>dissipative dynamics | $\dot{\phi} = \frac{dH}{dp_\phi}$ $\quad \dot{p}_\phi = -\frac{dH}{d\phi} + F_\phi$ |

Precise and accurate parameter estimation



Waveform templates



**A typical waveform model needs precise Hamiltonians and Fluxes**

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GW150914

# Bound state - Inspiral phase

Inspiral :

the components of the binary are moving at **non-relativistic velocities** and their orbital separation is slowly decaying

Merger :

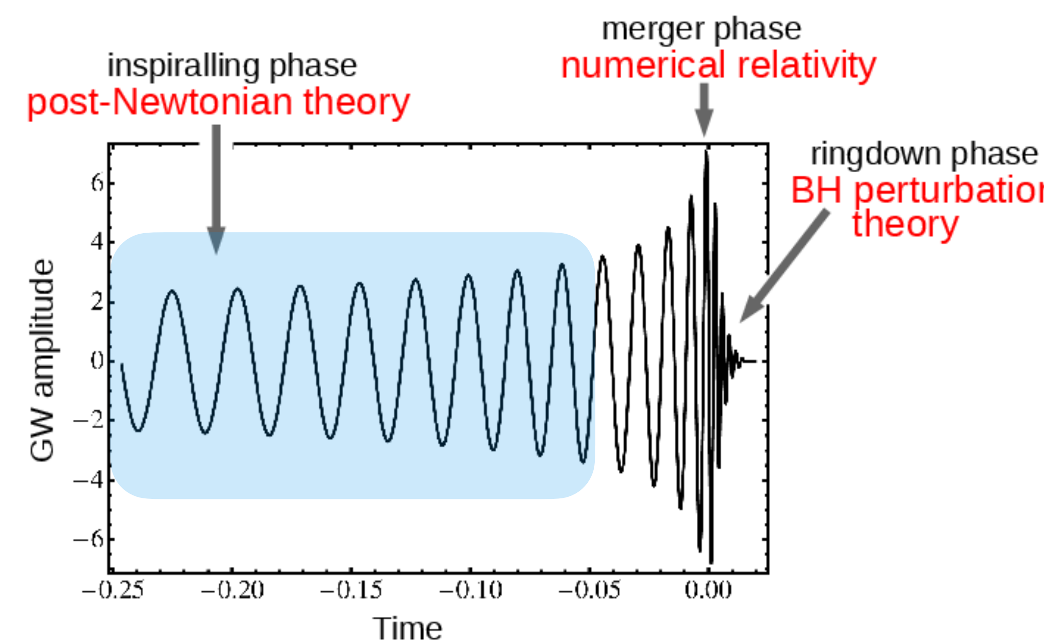
the separation between the components falls roughly below the innermost stable orbit of each other, and the objects reach **relativistic velocities** and **merge into final object**

Ringdown :

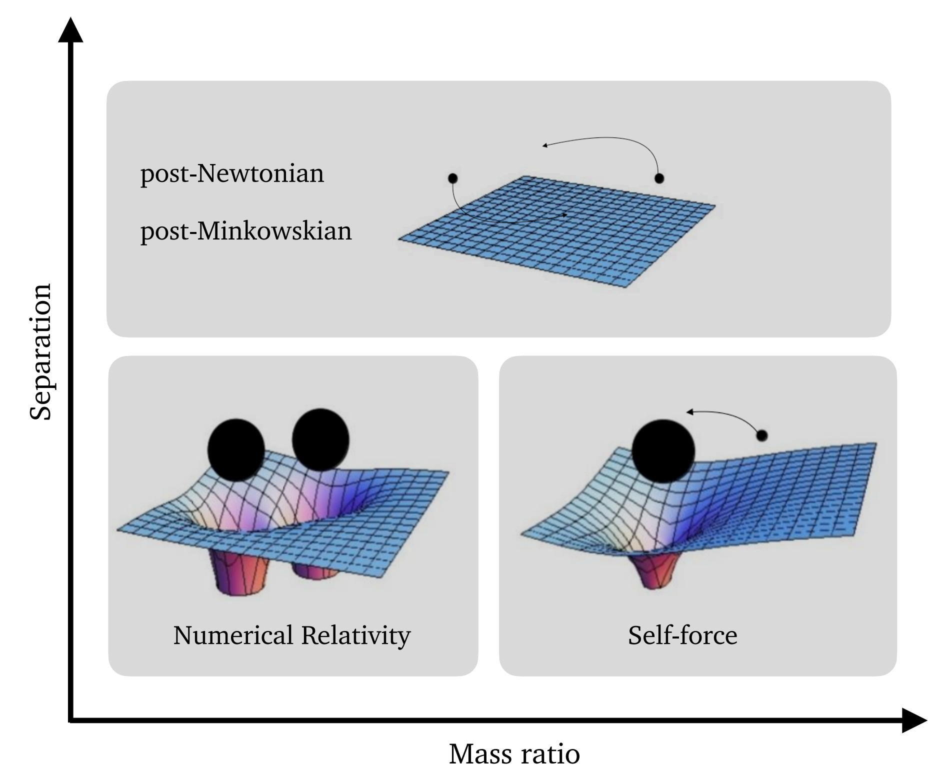
where spacetime **settles down to that of a Kerr** black hole

| | | 0PN (1687) | | 1PN (1938) | | 2PN (1980) | | 3PN (2000) | | 4PN (2014) | | 5PN | | 6PN | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1PM | $G_N$ | 1 | + | $v^2$ | + | $v^4$ | + | $v^6$ | + | $v^8$ | + | $v^{10}$ | + | $v^{12}$ | + | ⋯ |
| 2PM | $G_N^2$ | | | 1 | + | $v^2$ | + | $v^4$ | + | $v^6$ | + | $v^8$ | + | $v^{10}$ | + | ⋯ |
| 3PM | $G_N^3$ | | | | | 1 | + | $v^2$ | + | $v^4$ | + | $v^6$ | + | $v^8$ | + | ⋯ |
| 4PM | $G_N^4$ | | | | | | | 1 | + | $v^2$ | + | $v^4$ | + | $v^6$ | + | ⋯ |
| 5PM | $G_N^5$ | | | | | | | | | 1 | + | $v^2$ | + | $v^4$ | + | ⋯ |

[Newton, Einstein, Infeld, Hoffman, Ohta, Okamura, Kimura, Hiida, Jaranowski, Schäfer, Damour, Buonanno, Blanchet, Faye, Iyer, Will, Wiseman, Poisson, Flanagan, Deruelle, Thorne, Sathyaprakash, Bini, Geralico, . . .]




[Blanchet (2019)]




[adapted from Barack and Pound (2018)]

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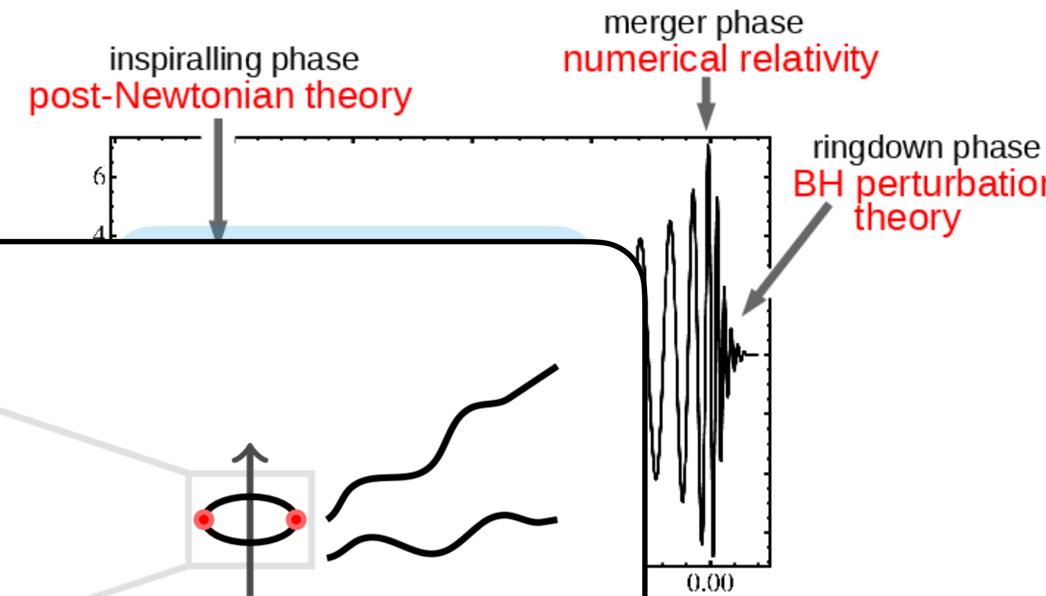
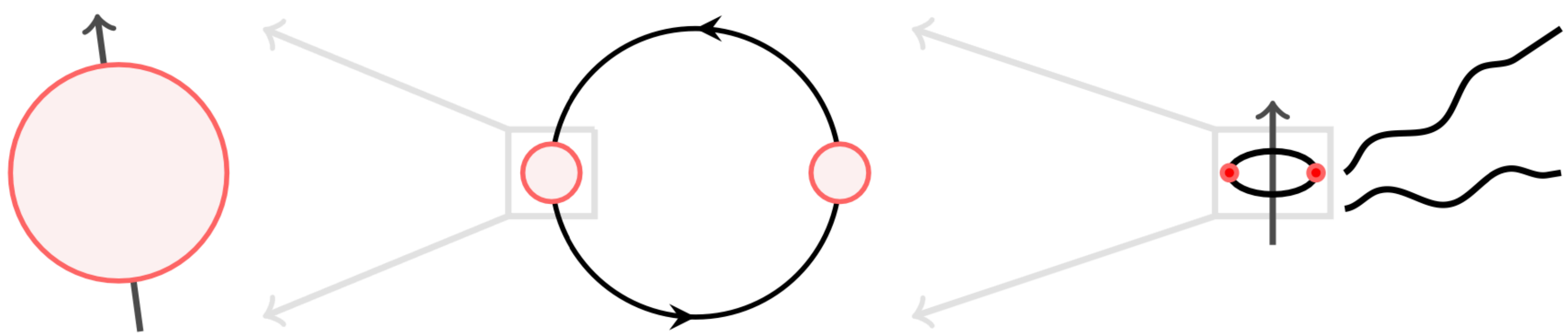



Merger :
the separa
the innerr
reach **rela**

Ringdown :
where spa



EFTs provide a simple framework
to capture the essential physics at different scales

[Goldberger and Rothstein (2006)]

1687

0PN

1PM $G_N$ 1

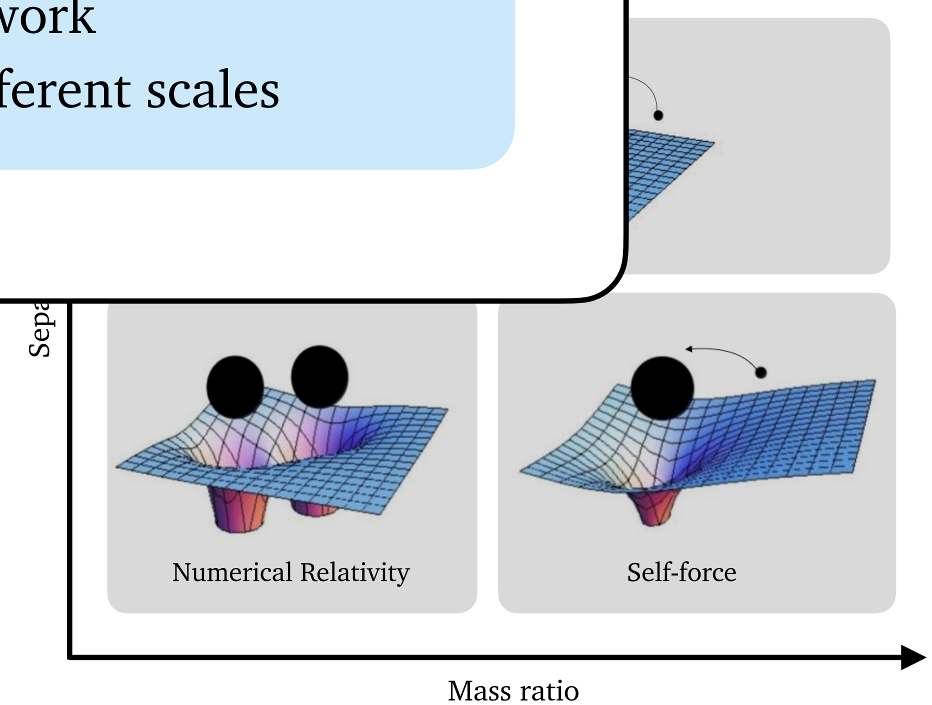
2PM $G_N^2$

3PM $G_N^3$ 1 + $v^2$ + $v^4$ + $v^6$ + $v^8$ + ⋯

4PM $G_N^4$ 1 + $v^2$ + $v^4$ + $v^6$ + ⋯

5PM $G_N^5$ 1 + $v^2$ + $v^4$ + ⋯

[Newton, Einstein, Infeld, Hoffman, Ohta, Okamura, Kimura, Hiida, Jaranowski, Schäfer, Damour, Buonanno, Blanchet, Faye, Iyer, Will, Wiseman, Poisson, Flanagan, Deruelle, Thorne, Sathyaprakash, Bini, Geralico, . . .]




[adapted from Barack and Pound (2018)]

# post-Newtonian pipeline

**EFT around a point particle** for compact objects with internal structure

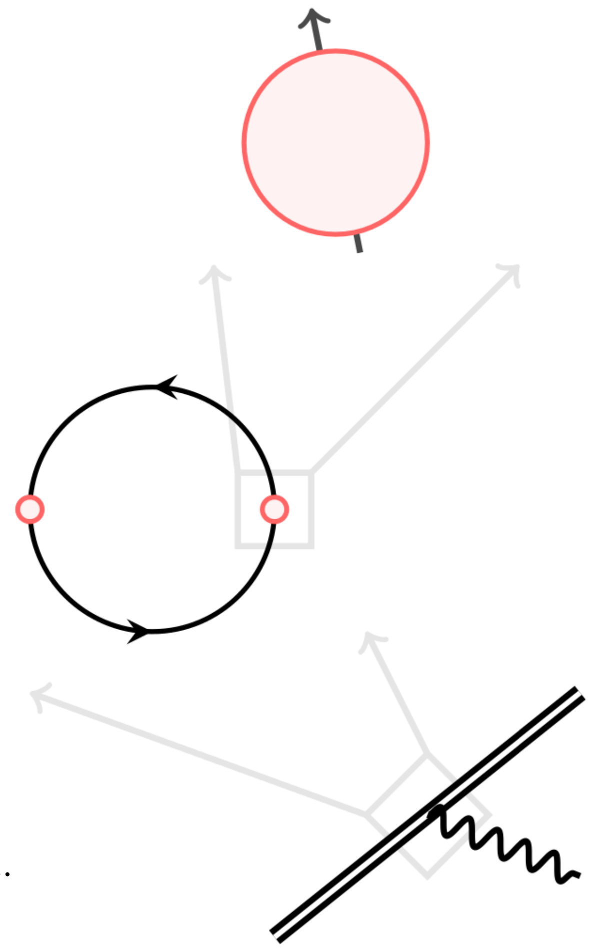
$$\underbrace{m\sqrt{u^2}}_{\text{pp}} + \underbrace{\frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu}}_{\text{spin}} + \underbrace{c_{E^2}\, E_{\mu\nu}E^{\mu\nu}}_{\text{tides}} + \cdots$$

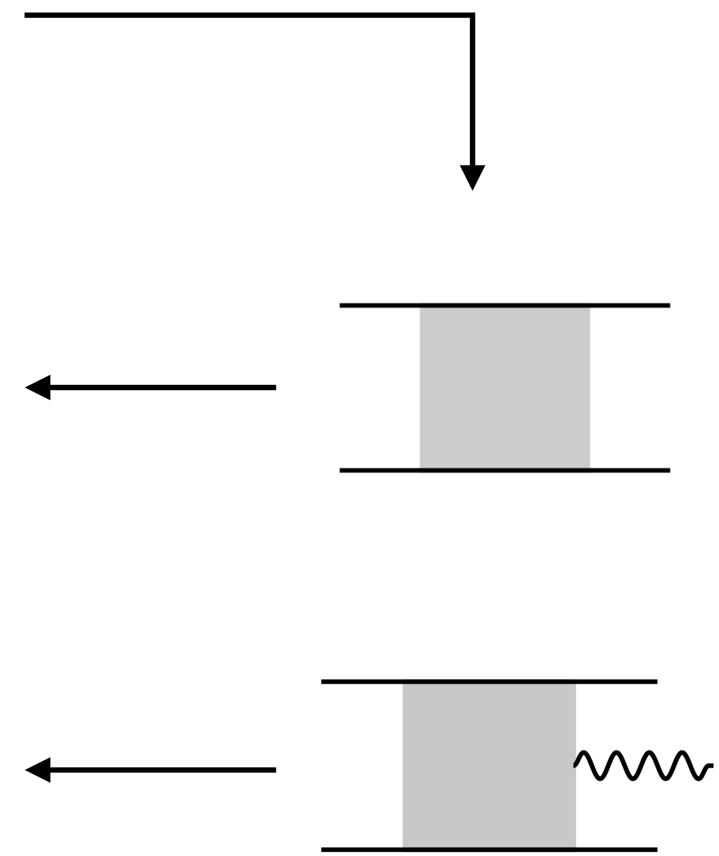
**Two point particle EFT** to describe a bound state like binary at scales r

$$\sum_{a=1,2}\frac{1}{2}m_a\mathbf{v}_a^2 + \frac{G_N m_1 m_2}{|\mathbf{r}|} + 1\text{PN} + 2\text{PN} + \cdots$$

EFT of **multipole moments** to describe the binary at scale $\lambda$

$$\underbrace{M}_{\substack{\text{total}\\ \text{mass}}}\ \bar{h}_{00} + \underbrace{\mathbf{L}^i}_{\substack{\text{angular}\\ \text{mom.}}}\ \epsilon_{ijk}\partial_j\bar{h}_{0k} + \underbrace{I^{ij}}_{\substack{\text{mass}\\ \text{quad.}}}\ E_{ij} + \underbrace{J^{ij}}_{\substack{\text{current}\\ \text{quad.}}}\ B_{ij} + \cdots$$





# post-Newtonian pipeline

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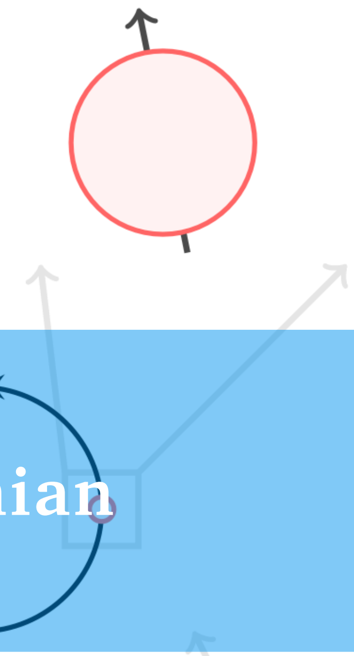

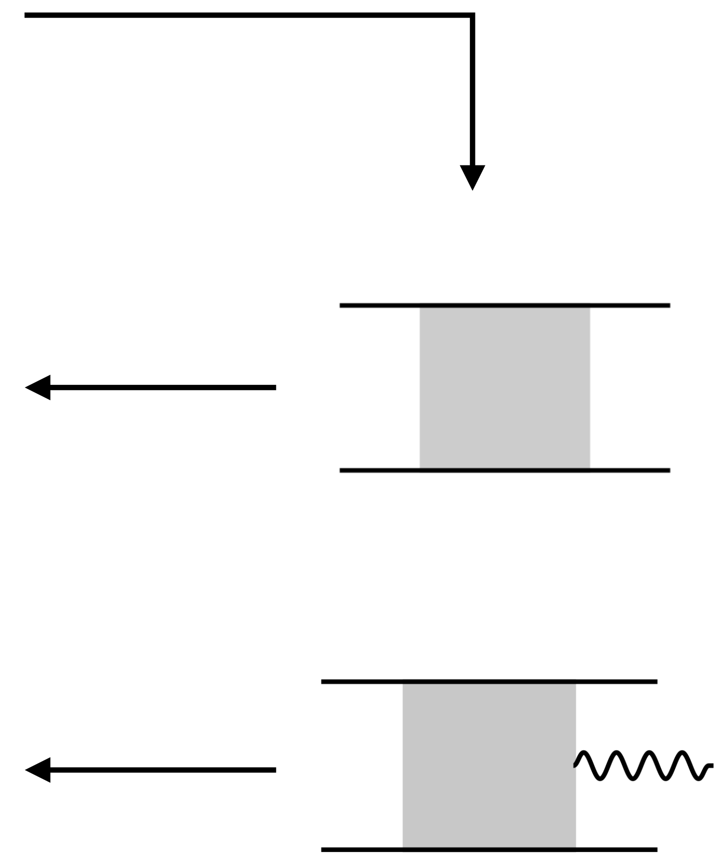

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$$\sum_{a=1,2}\frac{1}{2}m_a\mathbf{v}_a^2 + \frac{G_N m_1 m_2}{|\mathbf{r}|} + 1\text{PN} + 2\text{PN} + \cdots$$

Computing Hamiltonian

EFT of **multipole moments** to describe the binary at scale $\lambda$

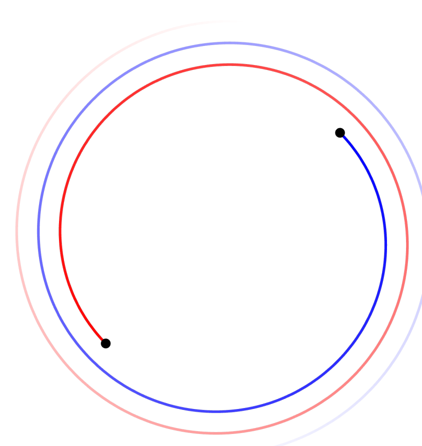
$$\underbrace{M}_{\text{total mass}}\bar{h}_{00} + \underbrace{\mathbf{L}^i}_{\text{angular mom.}}\epsilon_{ijk}\partial_j h_{0k} + \underbrace{I^{ij}}_{\text{mass quad.}}E_{ij} + \underbrace{J^{ij}}_{\text{current quad.}}B_{ij} + \cdots$$

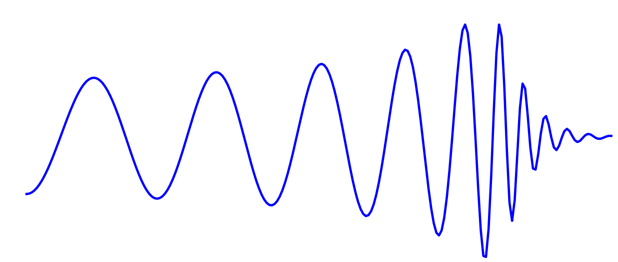
Radiation reaction force

Equation of motion

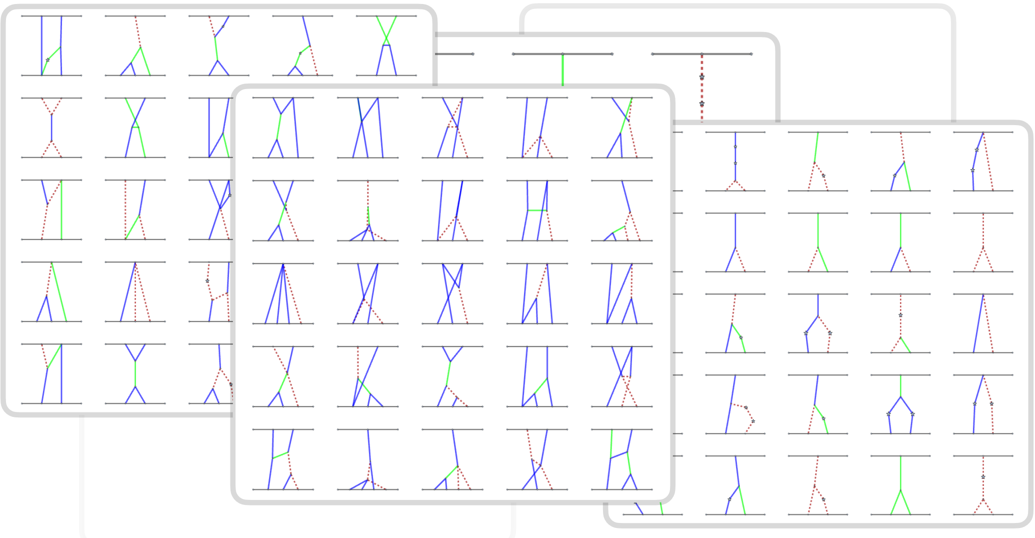
$$\dot{r} = \frac{dH}{dp_r} \qquad \dot{p}_r = -\frac{dH}{dr} + F_r$$

$$\dot{\phi} = \frac{dH}{dp_\phi} \qquad \dot{p}_\phi = -\frac{dH}{d\phi} + F_\phi$$





# Fast and Flexible code



$$\mathcal{FD}^{(l)}_{\{g\}} = \underbrace{N_{\mathrm{C}}^{\mu_1,\mu_2,\cdots}\left(x_{(a)}, S_{(a)}\right)}_{\text{Coefficient that depends on orbital variables}} \underbrace{\int_p e^{ip_\mu x^\mu_{(12)}} N_{\mathrm{F}}^{\alpha_1,\alpha_2,\cdots}(p)}_{\text{Fourier integral}} \underbrace{\prod_{i=1}^{l} \int_{k_i} \frac{N_{\mathrm{M}}^{\nu_1,\nu_2,\cdots}(k_i)}{\prod_{\sigma\in g} D_\sigma(p,k_i)}}_{\text{Multi-loop integral}}$$

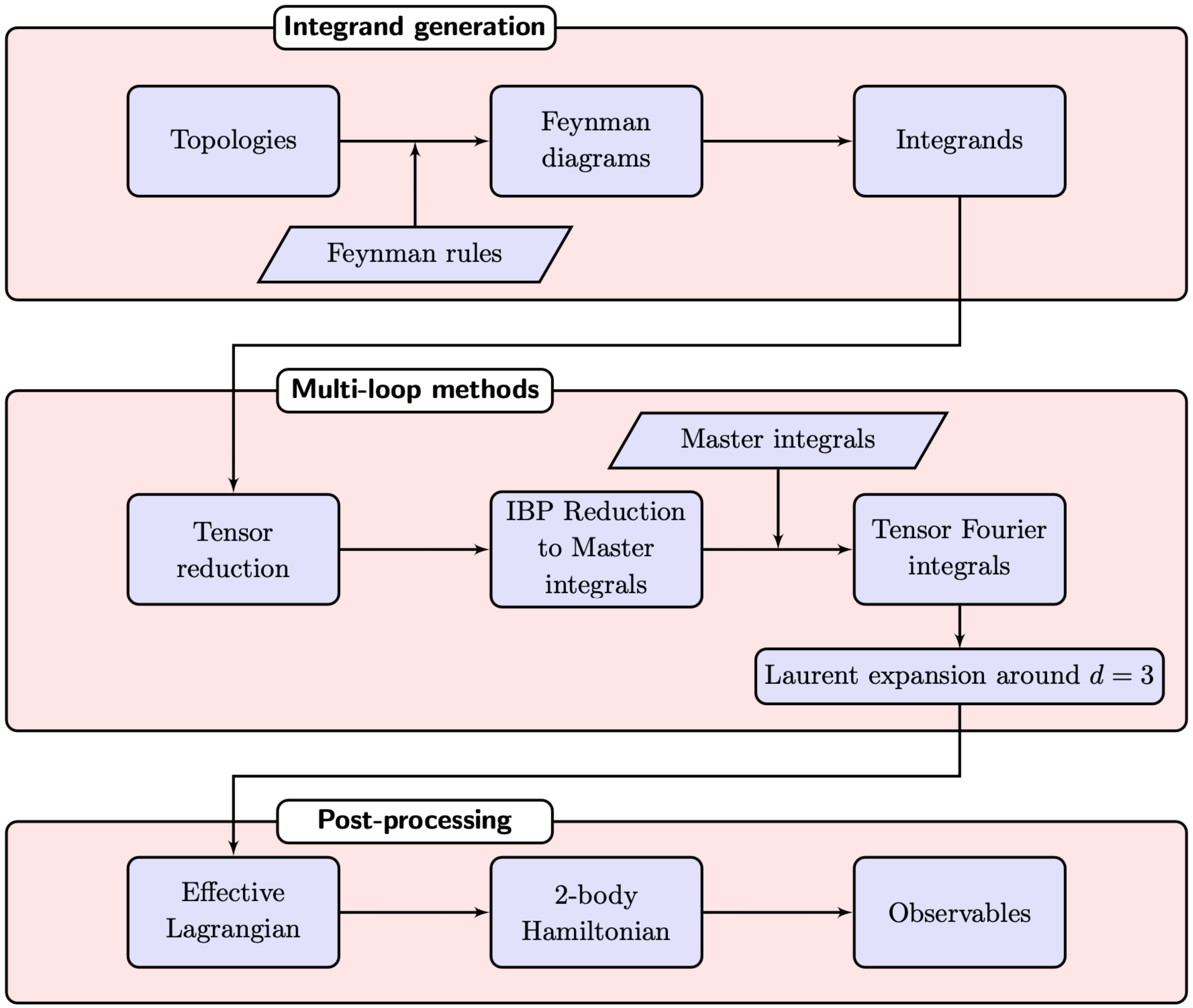
**Computing effective Lagrangian and Stress-energy**

- Diagrams with QGRAF, Tensor Algebra with xTensor, IBPs with LiteRed
- Can compute up to 3 loops or up to $(G_N)^4$

**Post processing**

- Removing higher order time derivatives
- Removing spurious divergences
- Computing observables
  - Binding energy on circular orbits
  - Scattering angle
  - Mode amplitude and phase

**Very easy to modify for extra degrees of freedom (spin, tides, beyond GR, etc)**

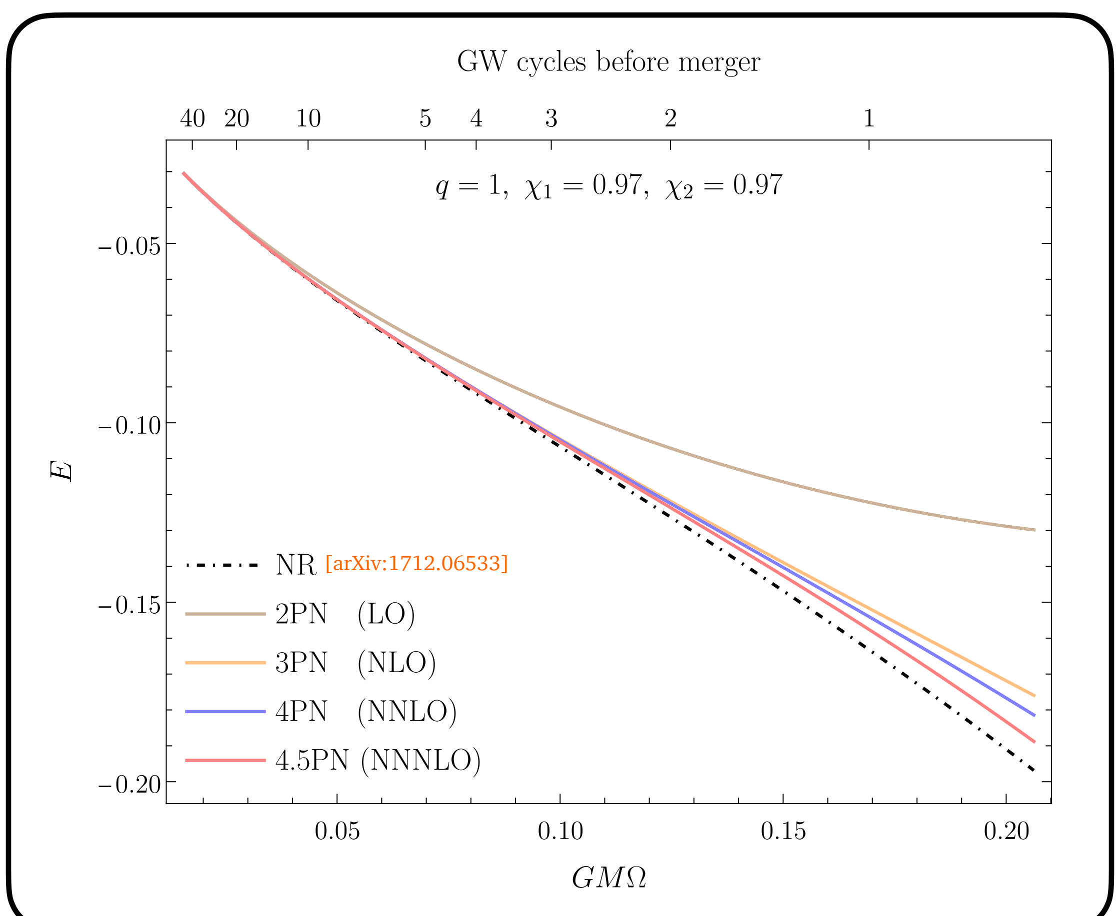



[Mandal, Mastrolia, **RP**, Steinhoff]

# Conservative results - NNNLO spin

$$\int dt \left\{ m\sqrt{g^L_{\mu\nu}u^\mu u^\nu} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + \cdots \right\}$$

| PN order | 0 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 | 6.5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| no spin | N | 1PN | | 2PN | | 3PN | | 4PN | | 5PN | | 6PN | |
| spin-orbit | | | LO SO | | NLO SO | | N2LO SO | | N3LO SO | | N4O SO | | |
| spin^2 | | | | LO S2 | | NLO S2 | | N2LO S2 | | N3LO S2 | | | |
| spin^3 | | | | | | | LO S3 | | NLO S3 | | NNLO S3 | | |
| spin^4 | | | | | | | | LO S4 | | NLO S4 | | NNLO S4 | |
| spin^5 | | | | | | | | | | | LO S5 | | NLO S5 |
| spin^6 | | | | | | | | | | | | LO S6 | |




## Spin-orbit coupling at 4.5PN: $\left(S_{(a)} \cdot L\right)$

arXiv:2209.00611 [Mandal, Mastrolia, **RP**, Steinhoff (2022)]

- ➡ Analogous to fine structure correction to Hydrogen atom
- ➡ 894 Feynman diagrams up to 3 loops

## Quadratic-in-spin coupling at 5PN: $\left(S^2_{(a)}\right), \left(S_{(1)} \cdot S_{(2)}\right)$

arXiv:2210.09176 [Mandal, Mastrolia, **RP**, Steinhoff (2022)]

- ➡ Analogous to hyperfine structure correction to Hydrogen atom
- ➡ 723 Feynman diagrams up to 3 loops

Wilson coefficients

$$L^{(R,S^2)} = -\frac{1}{2mc}\left(C^{(0)}_{\mathrm{ES}^2}\right)\frac{E_{\mu\nu}}{u}\left[S^\mu S^\nu\right]_{\mathrm{STF}} + \cdots$$

$$L^{(R^2,S^0)} = \frac{1}{2}\left(C^{(2)}_{\mathrm{E}^2}\right)\frac{G_N^2 m}{c^5}\frac{E_{\mu\nu}E^{\mu\nu}}{u^3}S^2 + \cdots$$

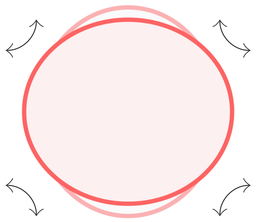
$$L^{(R^2,S^2)} = \frac{1}{2}\left(C^{(0)}_{\mathrm{E}^2\mathrm{S}^2}\right)\frac{G_N^2 m}{c^5}\frac{E_{\mu\alpha}E^{\alpha}_{\nu}}{u^3}\left[S^\mu S^\nu\right]_{\mathrm{STF}} + \cdots$$

- ➡ $C^{(0)}_{\mathrm{ES}^2} = 1$ for Kerr BHs.

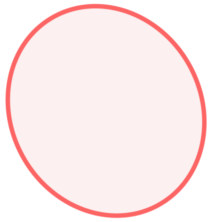
  $C^{(2)}_{\mathrm{E}^2}$ and $C^{(0)}_{\mathrm{E}^2\mathrm{S}^2}$ are yet unknown for Kerr BHs

Results also computed by [Kim, Levi, Yin (2022)] using EFTs, and [Antonelli, Kavanagh, Khalil, Steinhoff, Vines (2020)] using self-force

# Conservative results - 3PN tides



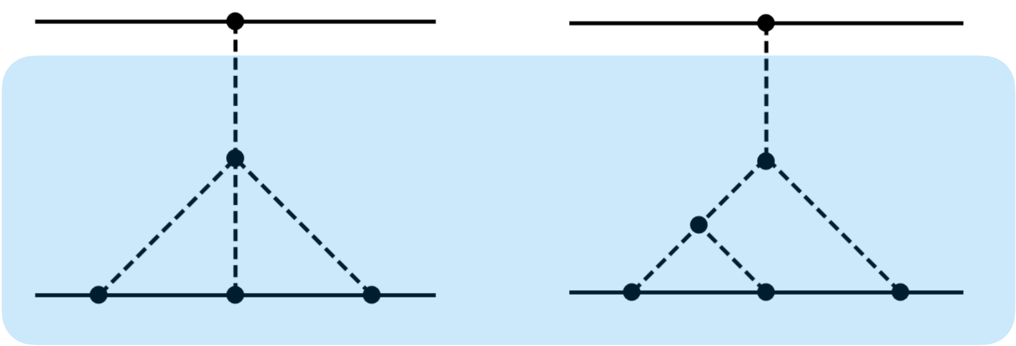
## Dynamic tides



## Adiabatic tides

$$\mathscr{L}_{\rm dy} = \frac{z}{4\lambda\omega_f^2}\left[\frac{c^2}{z^2}\dot{Q}_{\mu\nu}\dot{Q}^{\mu\nu} - \omega_f^2 Q_{\mu\nu}Q^{\mu\nu}\right] - \frac{z}{2}Q^{\mu\nu}E_{\mu\nu} - \kappa\frac{G_N^2 m^2}{c^6}\frac{z}{2}Q_{\mu\nu}\ddot{E}^{\mu\nu}$$

$$\mathscr{L}_{\rm ad} = \lambda\,\frac{z}{4}E_{\mu\nu}E^{\mu\nu} - \lambda\kappa\frac{G_N^2 m^2}{c^6}\frac{z}{2}\dot{E}_{\mu\nu}\dot{E}^{\mu\nu}$$



### Adiabatic and Dynamic tides up to 3PN

arXiv:2304.02030 [Mandal, Mastrolia, Silva, **RP**, Steinhoff (2023)]
arXiv:2308.01865 [Mandal, Mastrolia, Silva, **RP**, Steinhoff (2023)]

- ➡ Modelling of mode oscillation of NS
- ➡ 290 Feynman diagrams up to 3 loops

- ➡ Radiation by a single NS produces a divergent metric [Blanchet (1998), Goldberger, Ross (2010)]
- ➡ This divergent metric shows up in the two-body potential

$$\left(L_{\rm eff}\right)_{1/\epsilon} = \left(\frac{107}{105}m_{(1)}^2 G_N^2 \frac{1}{c^6}\frac{1}{\epsilon}\right)\left(\frac{3}{2}\frac{G_N m_{(2)}}{r^3}\left(\ddot{Q}_{(1)}^{ij}n^i n^j\right)\right) + (1 \leftrightarrow 2)$$

### Hence requires renormalisation!!

$$\beta(\kappa) \equiv R\,\frac{d\kappa}{dR} = -\frac{214}{105}$$

$$\kappa(R) = \kappa(R_0) - \frac{214}{105}\log\left(\frac{R}{R_0}\right)$$

# Summary - PN pipeline

EFT around a point particle for compact objects

## State-of-the-art results!

**Spin-orbit Hamiltonian at 4.5PN (NNNLO):** $\left(S_{(a)} \cdot L\right)$

arXiv:2209.00611 [Mandal, Mastrolia, **RP**, Steinhoff (2022)]

**Quadratic-in-spin Hamiltonian at 5PN (NNNLO):** $\left(S_{(a)}^2\right), \left(S_{(1)} \cdot S_{(2)}\right)$

arXiv:2210.09176 [Mandal, Mastrolia, **RP**, Steinhoff (2022)]

**Adiabatic and Dynamic tides Hamiltonian up to 3PN:**

arXiv:2304.02030 [Mandal, Mastrolia, Silva, **RP**, Steinhoff (2023)]
arXiv:2308.01865 [Mandal, Mastrolia, Silva, **RP**, Steinhoff (2023)]

**Adiabatic tides Fluxes and modes up to 2PN:** Coming out tomorrow! [Mandal, Mastrolia, **RP**, Steinhoff (2024)]

**Automatic computational framework of QFT/EFT prove very effective in going to higher order corrections!!**

| PN order | 0 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 | 6.5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| no spin | N | 1PN | | 2PN | | 3PN | | 4PN | | 5PN | | 6PN | |
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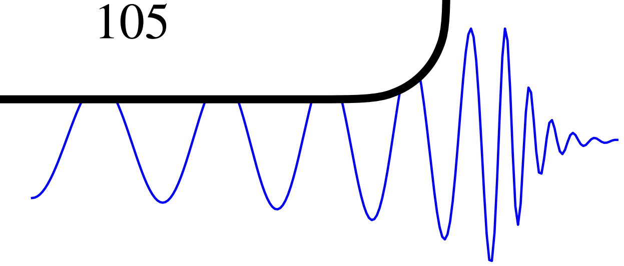
| spin^3 | | | | | | | LO S3 | | NLO S3 | | NNLO S3 | | |
| spin^4 | | | | | | | | LO S4 | | NLO S4 | | NNLO S4 | |
| spin^5 | | | | | | | | | | | LO S5 | | NLO S5 |
| spin^6 | | | | | | | | | | | | LO S6 | |

**Renormalisation of post-adiabatic Love number!!**

$$\beta(\kappa) \equiv R\,\frac{d\kappa}{dR} = -\frac{214}{105}$$

Equa…
of motion

$$\dot{\phi} = \frac{dH}{dp_\phi} \qquad \dot{p}_\phi = -\frac{dH}{d\phi} + F_\phi$$



# Thank you