Cosmological Correlators with Double Massive Exchanges

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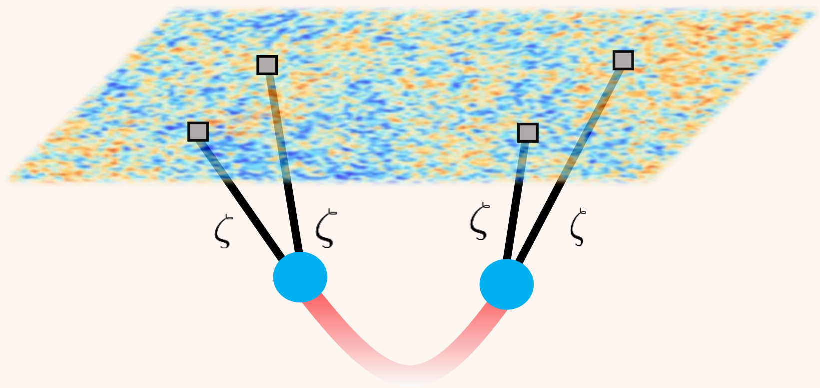
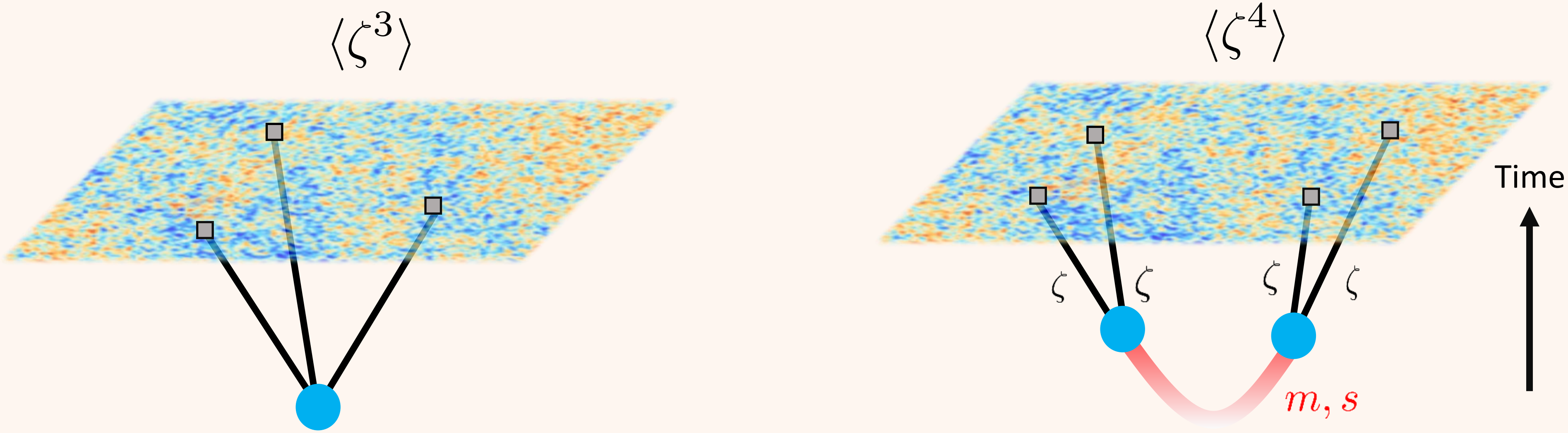
In collaboration with: Shuntaro Aoki, Lucas Pinol,
Fumiya Sano and Masahide Yamaguchi

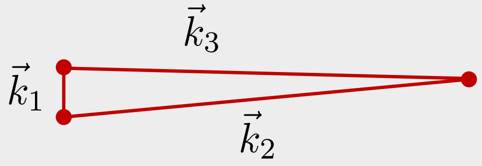
Based on: 2404.09547 [JHEP09(2024)176]



Cosmological Correlator in Taiwan, 02 Dec 2024

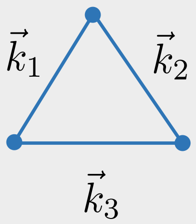
Cosmological Correlators





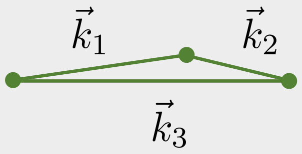
Soft limit

- Inflationary particle spectrum



Equilateral limit

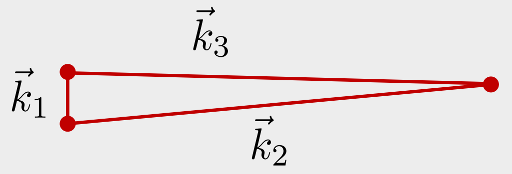
- Higher dimension interactions



Collinear limit

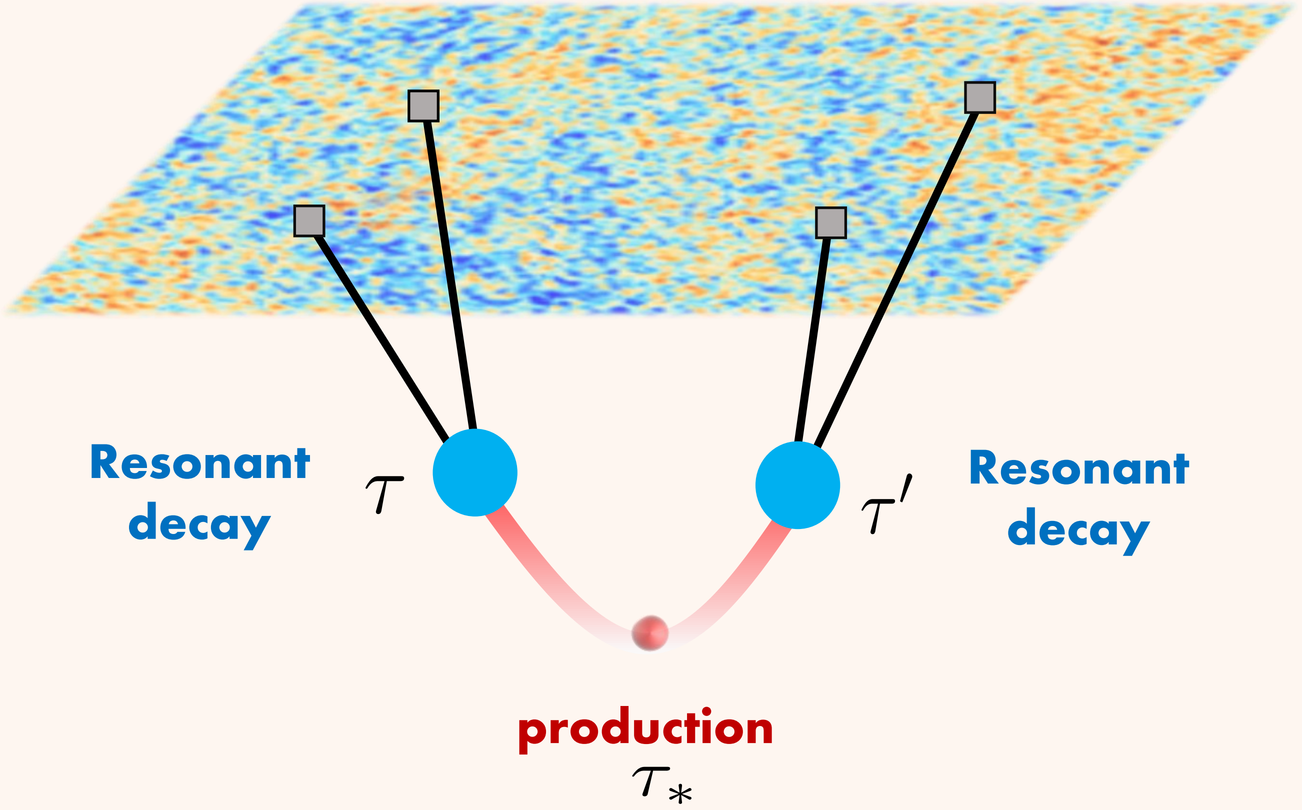
- Probe Initial state

Inflation as the Cosmological Collider



Soft limit

- Inflationary particle spectrum



$$S \sim \mathcal{A}(\lambda, m)\, e^{-\pi\mu} \left(\frac{k_3}{k_1}\right)^{1/2} \sin\left[\mu \log\left(\frac{k_3}{k_1}\right) + \vartheta\right] P_s(\cos\theta)$$

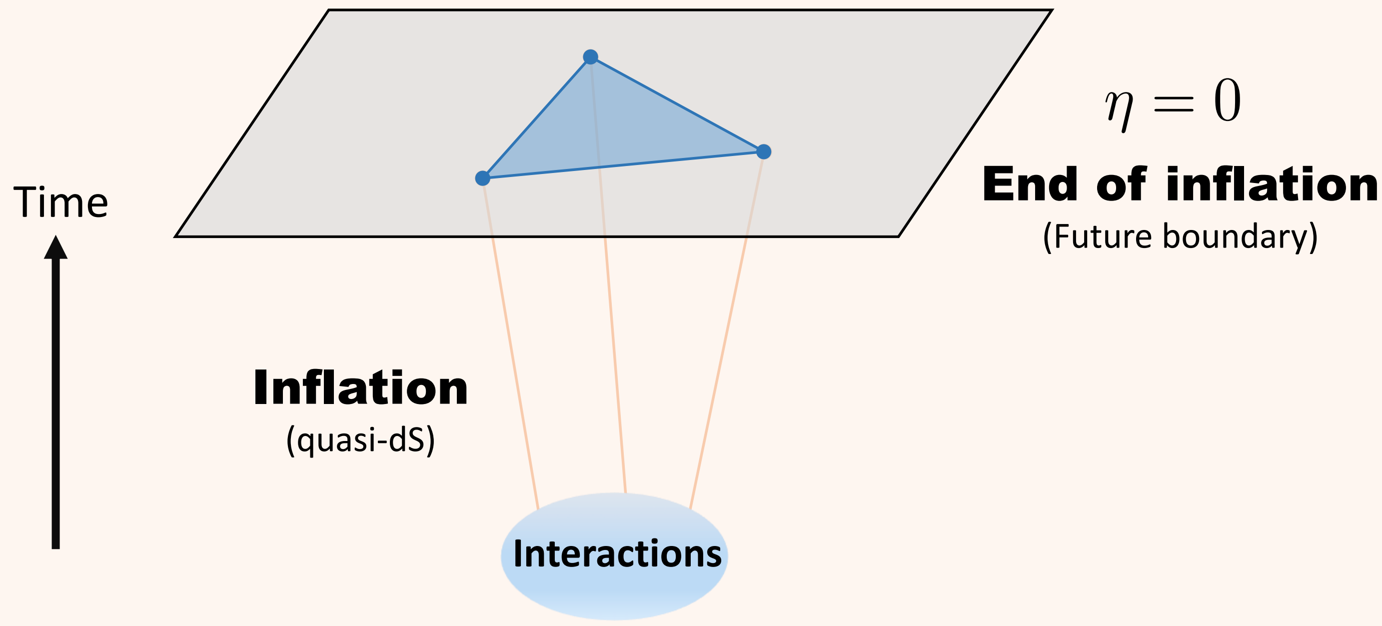
μ → mass

s → spin

$$\mu = \begin{cases} \sqrt{\dfrac{m^2}{H^2} - \dfrac{9}{4}} & s = 0 \\ \sqrt{\dfrac{m^2}{H^2} - \left(s - \dfrac{1}{2}\right)^2} & s \neq 0 \end{cases}$$

dS Bootstrap

Arkani-Hamed, Baumann, Lee, Pimentel 2018
Baumann, Duaso Pueyo, Joyce, Lee, Pimentel 2019 2020
……



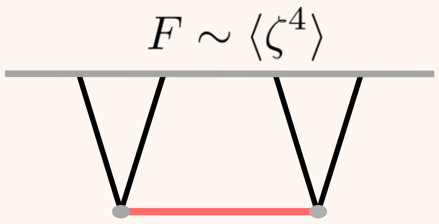
dS Symmetry : $\mathrm{d}s^2 = \dfrac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{H\eta^2}$

Translation: $P_i = \partial_i$

Rotation: $J_{ij} = x_i\partial_j - x_j\partial_i$

Dilation: $D = -\eta\partial_\eta - x_i\partial_i$

dS boosts: $K_i = 2x_i\eta\partial_\eta + \left(2x^j x_i + (\eta^2 - x^2)\delta_i^j\right)\partial_j$

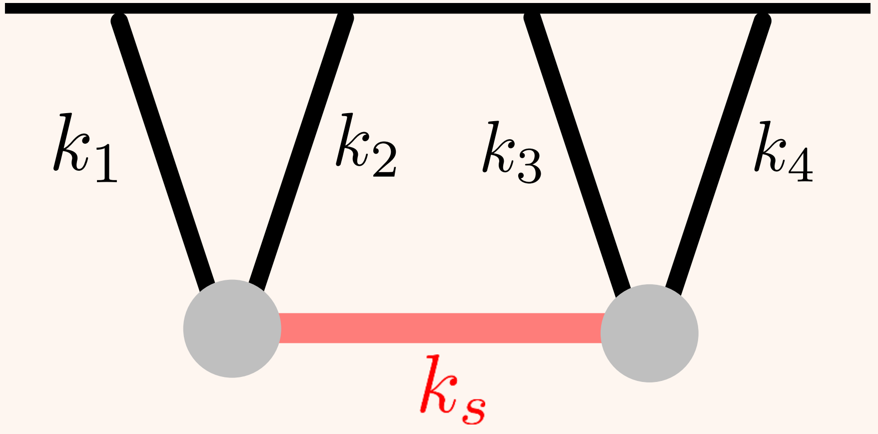


Conformal Ward identities :

$$\left[u^2(1-u^2)\partial_u^2 - 2u^3\partial_u + \mu^2 + \frac{1}{4}\right] F = g^2 \frac{uv}{u+v}$$

\+ **Boundary conditions** = **fix the correlators**

Bootstrap Equation

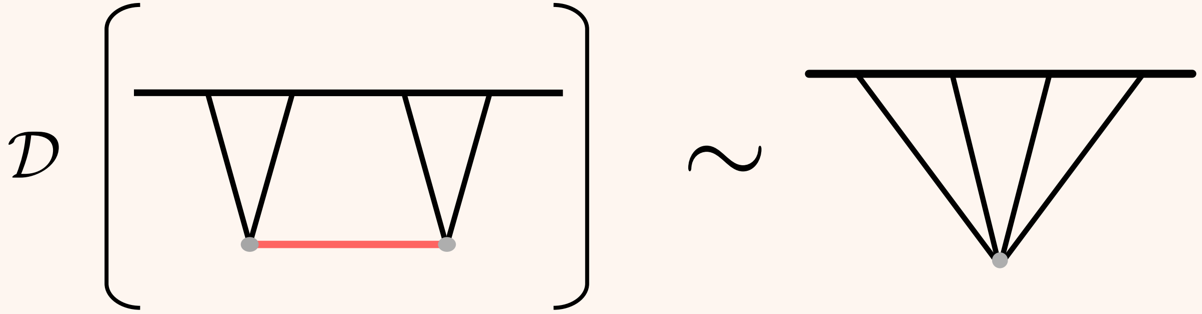


$$v = \frac{k_s}{k_3 + k_4} \qquad u = \frac{k_s}{k_1 + k_2}$$

$$F \sim g^2 \int \frac{\mathrm{d}\eta}{\eta^2}\frac{\mathrm{d}\eta'}{\eta'^2}\, e^{i(k_1+k_2)\eta} e^{i(k_3+k_4)\eta'} G_{++}(k_s, \eta, \eta')$$

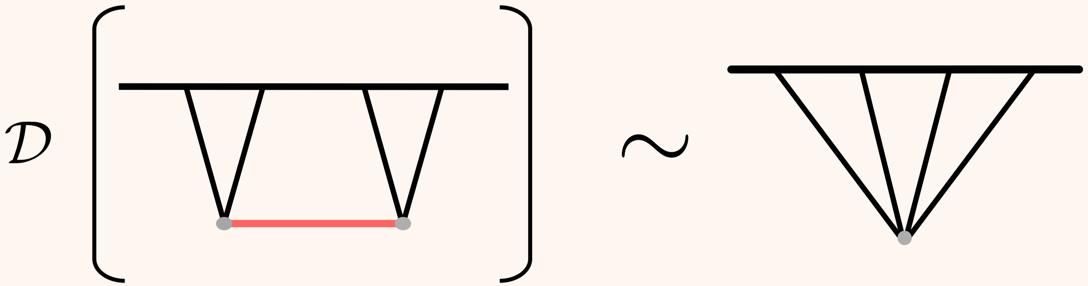
$$(\eta^2\partial_\eta^2 - 2\eta\partial_\eta + k_s^2\eta^2 + m^2)\, G_{++} = -i\eta^2\eta^{\ 2}\delta(\eta - \eta')$$

$$\left[u^2(1-u^2)\partial_u^2 - 2u^3\partial_u + \mu^2 + \frac{1}{4}\right] F = g^2 \frac{uv}{u+v}$$



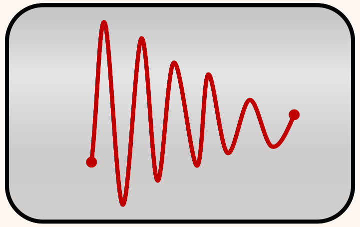
Bootstrap Equation

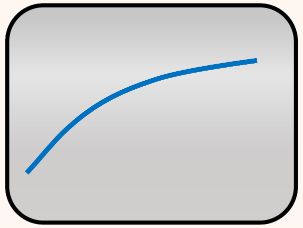
$$\left[u^2(1-u^2)\partial_u^2 - 2u^3\partial_u + \mu^2 + \frac{1}{4}\right] F = g^2 \frac{uv}{u+v}$$



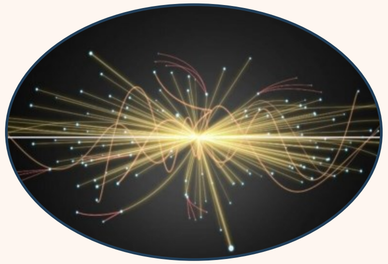
Results:

$$(u\,v)^{i\mu} + (u/v)^{i\mu} + u^m\,v^n$$





CC signals **Background**



Sound speed:

Pimentel, Wang 2022

Jazayeri, Renaux-Petel 2022 2023

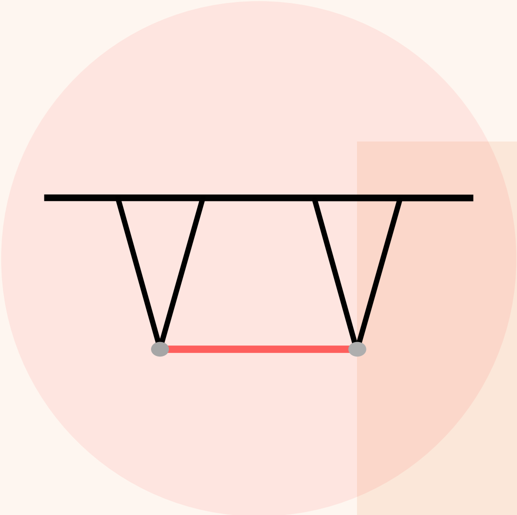
....

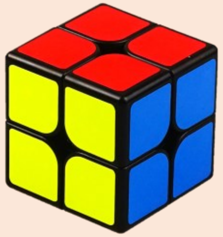
Chemical potential:

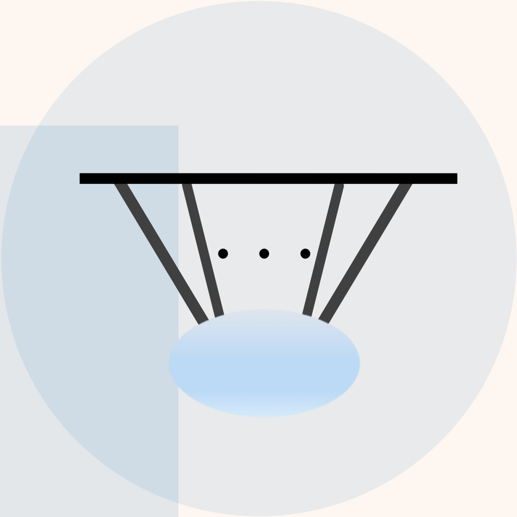
Qin, Xianyu 2022

....

More is Difficult

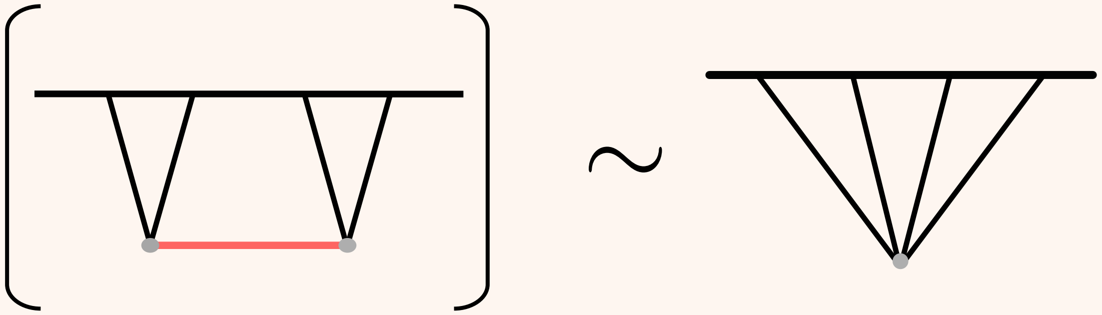






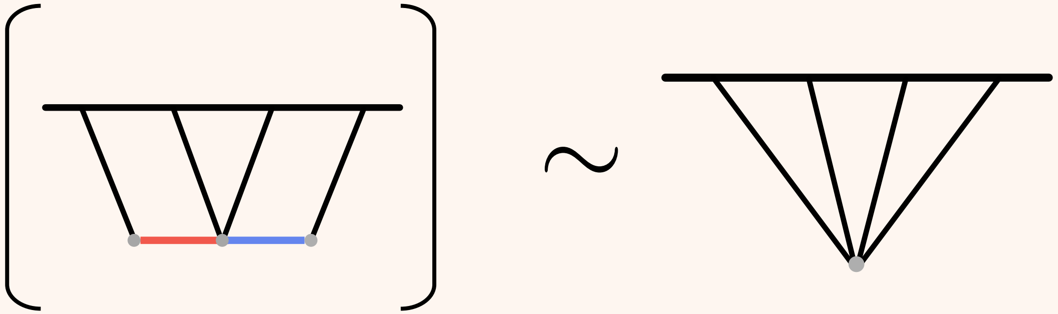
Single Massive Exchange Diagram

$\mathcal{D}$



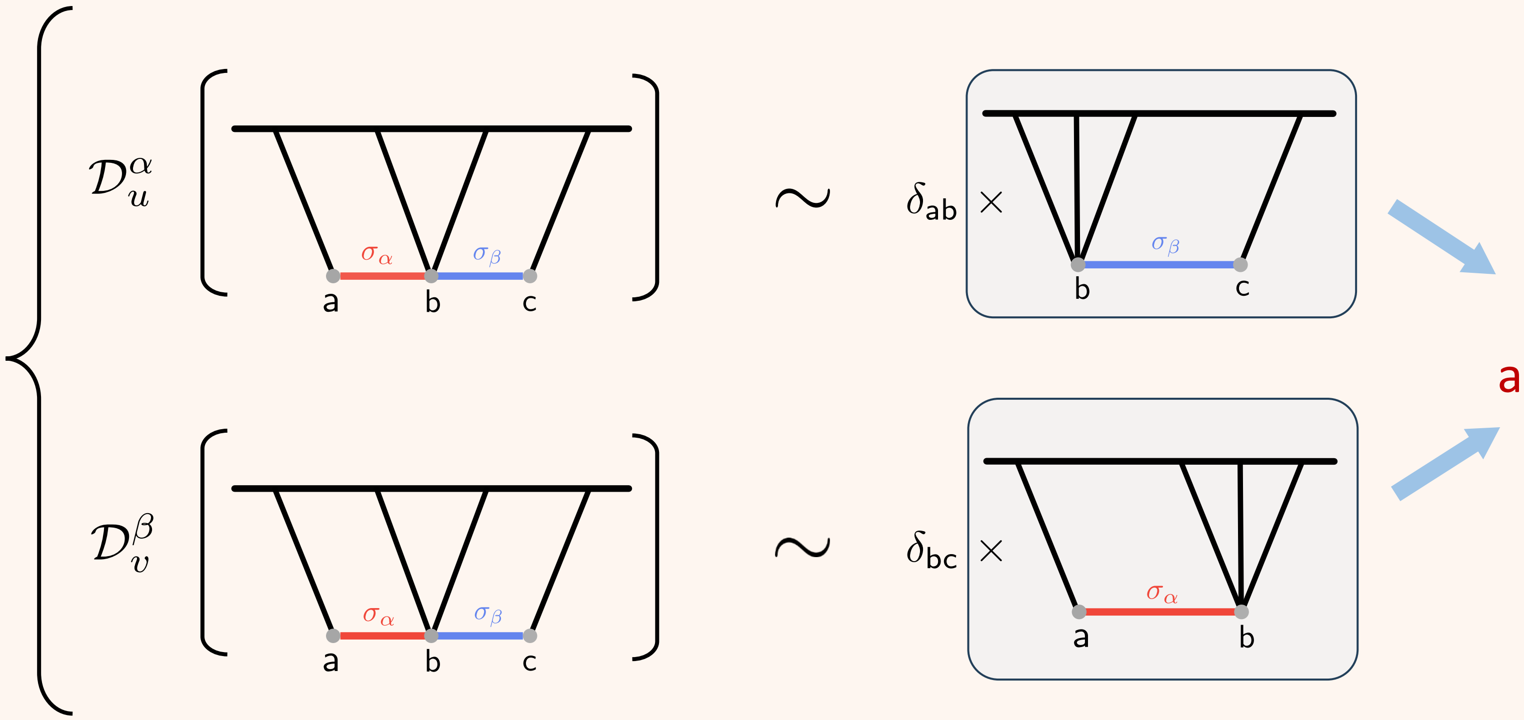
Double Massive Exchange Diagram

$\mathcal{D} \cdot \mathcal{D} \cdot$



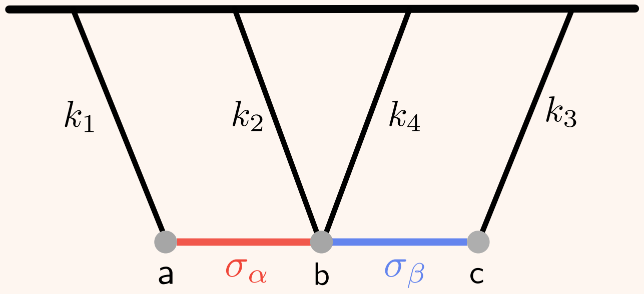
→ Fourth order differential equations ☹

Double Massive Exchange Diagram



Exact solution has already been found !

Double Massive Exchange: Equations



Seed integral:

$$\mathcal{I}^{p_1p_2p_3}_{\mathsf{abc},\alpha\beta} = H^{-4}k_{24}^{9+p_{123}}(-\mathrm{iabc})\int_{-\infty}^{0}\mathrm{d}\tau_1\mathrm{d}\tau_2\mathrm{d}\tau_3(-\tau_1)^{p_1}(-\tau_2)^{p_2}(-\tau_3)^{p_3}$$
$$\times\, e^{\mathrm{i}\mathsf{a}k_1\tau_1+\mathrm{i}\mathsf{b}k_{24}\tau_2+\mathrm{i}\mathsf{c}k_3\tau_3}D^{\alpha}_{\mathsf{ab}}(k_1;\tau_1,\tau_2)D^{\beta}_{\mathsf{bc}}(k_3;\tau_2,\tau_3)$$

Differential Equations:

$$\mathcal{D}^{\alpha}_{u}\widehat{\mathcal{I}}^{p_1p_2p_3}_{\pm\pm\pm,\alpha\beta}(u,v) = \frac{u^{p_1+4}}{v^{p_{12}+5}}\mathcal{I}^{p_{12}+4,p_3}_{\pm\pm,\beta}\ ,$$

$$\mathcal{D}^{\beta}_{v}\widehat{\mathcal{I}}^{p_1p_2p_3}_{\pm\pm\pm,\alpha\beta}(u,v) = \frac{v^{p_3+4}}{u^{p_{23}+5}}\mathcal{I}^{p_{23}+4,p_1}_{\pm\pm,\alpha}\ .$$

Differential Operators:

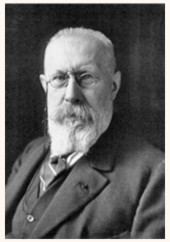
$$\mathcal{D}^{\alpha}_{u} = u^2\left(1-u^2\right)\partial_u^2 - u^2v^2\partial_v^2 - 2u\Big[1+u^2\left(p_2+2\right)\Big]\partial_u - 2u^2v\left(p_2+2\right)\partial_v$$
$$-\,2u^3v\partial_u\partial_v + \left(\mu_\alpha^2+\frac{9}{4}\right) - u^2(p_2+2)(p_2+1)\ ,$$

Double Massive Exchange: Solutions

$$\mathcal{D}_u^{\alpha}\, \widehat{\mathcal{I}}^{p_1p_2p_3}_{\pm\mp\pm,\alpha\beta}(u,v)=0\,,\qquad \mathcal{D}_v^{\beta}\, \widehat{\mathcal{I}}^{p_1p_2p_3}_{\pm\mp\pm,\alpha\beta}(u,v)=0$$

Homogenous Solutions: ***Appell series***

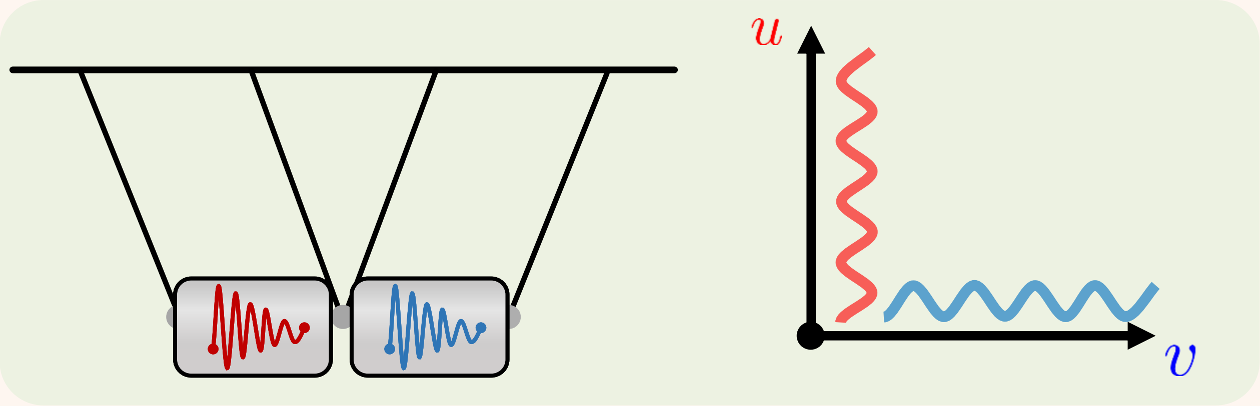
$$\widehat{\mathcal{I}}^{p_1p_2p_3}_{\pm\mp\pm,\alpha\beta}=\sum_{\mathrm{a},\mathrm{b}=\pm} c_{\pm\mp\pm,\mathrm{ab}}\, u^{\frac{3}{2}-\mathrm{i}\mathrm{a}\mu_\alpha}\, v^{\frac{3}{2}-\mathrm{i}\mathrm{b}\mu_\beta}\; \mathcal{F}_4\left[\begin{matrix}\frac{4+p_2-\mathrm{i}(\mathrm{a}\mu_\alpha+\mathrm{b}\mu_\beta)}{2},\ \frac{5+p_2-\mathrm{i}(\mathrm{a}\mu_\alpha+\mathrm{b}\mu_\beta)}{2}\\ 1-\mathrm{i}\mathrm{a}\mu_\alpha,\ 1-\mathrm{i}\mathrm{b}\mu_\beta\end{matrix}\,\middle|\, u^2,v^2\right],$$



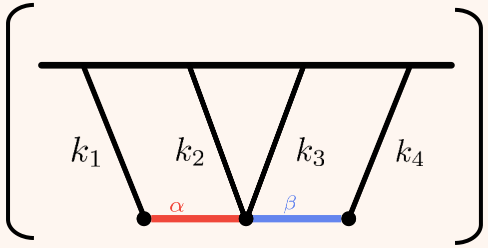
Undetermined coefficients

+

Boundary conditions



Double Massive Exchange: Solutions

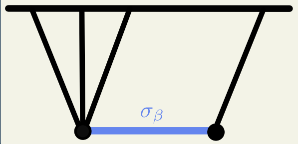


$$\frac{2k_1}{k_T} \equiv r_1, \qquad \frac{2k_3}{k_T} \equiv r_3,$$

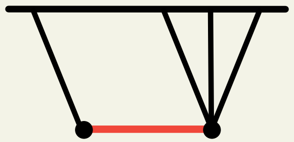
Bootstrap Equations:

$$\mathcal{D}_u^{\alpha}\widehat{\mathcal{P}}^{p_1p_2p_3}_{\pm\pm\pm,\alpha\beta}(u,v) = \frac{u^{p_1+4}}{v^{p_{12}+5}}\mathcal{I}^{p_{12}+4,p_3}_{\pm\pm,\beta}\ ,$$

$$\mathcal{D}_v^{\beta}\widehat{\mathcal{P}}^{p_1p_2p_3}_{\pm\pm\pm,\alpha\beta}(u,v) = \frac{v^{p_3+4}}{u^{p_{23}+5}}\mathcal{I}^{p_{23}+4,p_1}_{\pm\pm,\alpha}\ .$$



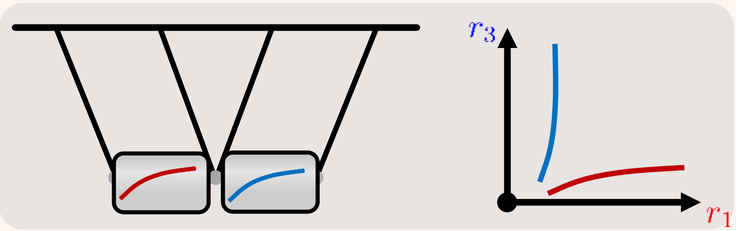
$$\sim \quad r_3^{\mathrm{i}\mu_\beta} + r_3^{n}$$

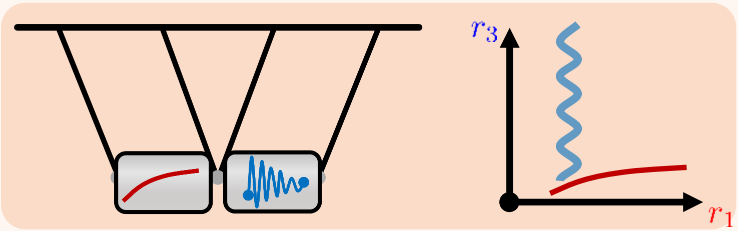


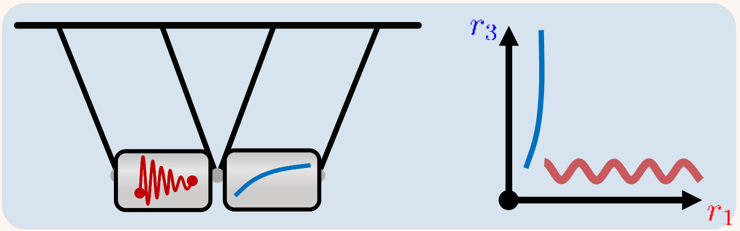
$$\sim \quad r_1^{\mathrm{i}\mu_\beta} + r_1^{m}$$

Ansatz of particular solution:

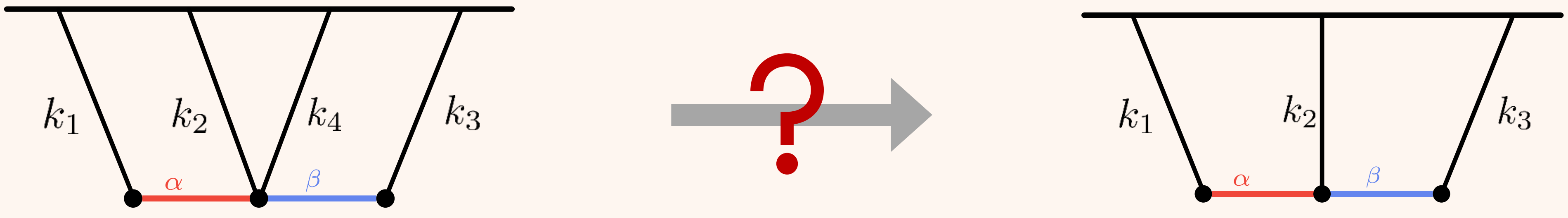
$$\mathcal{P}^{p_1p_2p_3}_{\pm\pm\pm,\alpha\beta}(r_1,r_3) \sim \mathcal{C}^{\pm\pm\pm}_{m,n} r_1^m r_3^{n+9} + \mathcal{A}^{\pm\pm\pm(\mathrm{a})}_{m,n} r_1^m r_3^{n-p_3+\frac{13}{2}+\mathrm{i}a\mu_\beta} + \mathcal{B}^{\pm\pm\pm(\mathrm{a})}_{m,n} r_1^{n-p_1-\frac{5}{2}+\mathrm{i}a\mu_\alpha} r_3^{m+9}$$

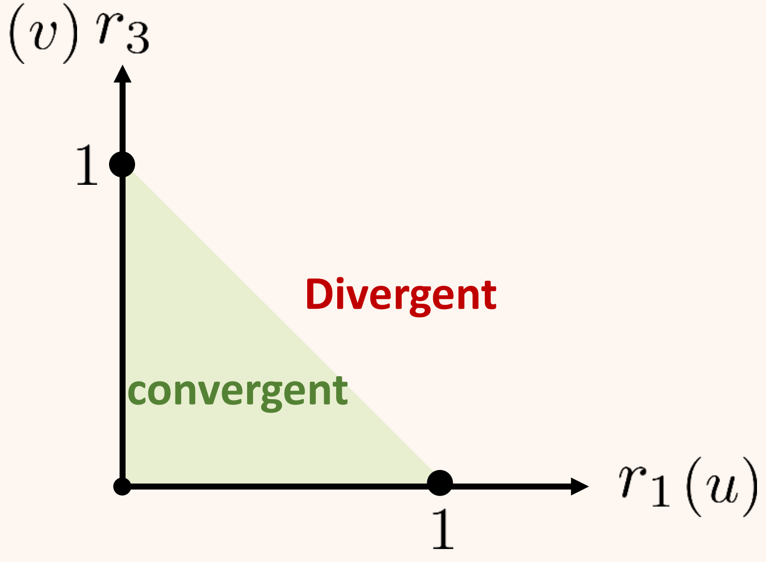




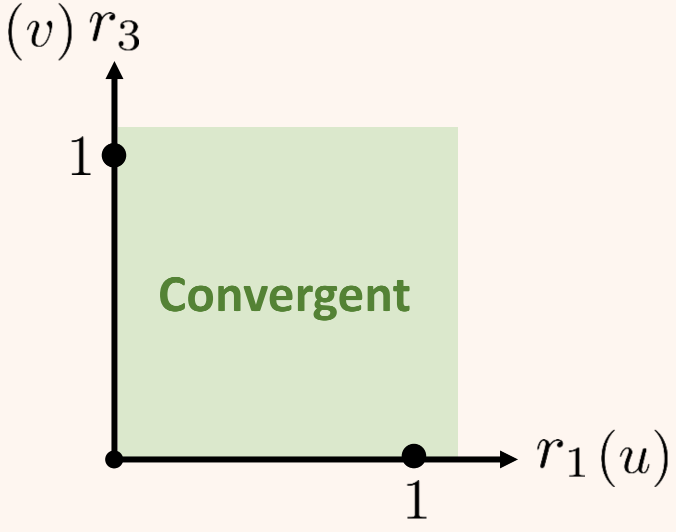


Double Massive Exchange： Towards the bispectrum





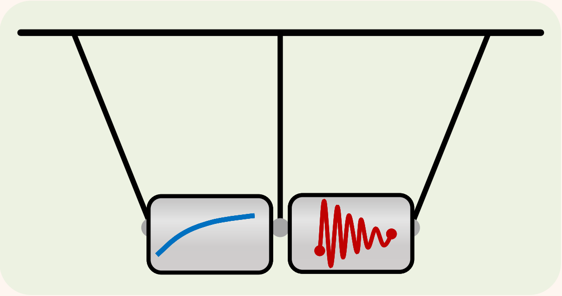
- Transformation
- Resummation



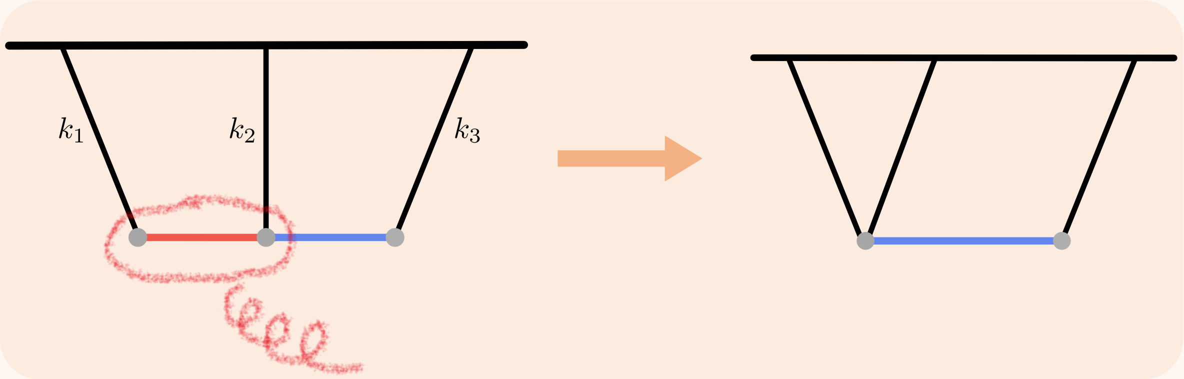
- Convergent only in part of kinematic regions

Double Massive Exchange： Phenomenology

The leading order contributions



$\sim \mathcal{O}(e^{-\pi\mu})$



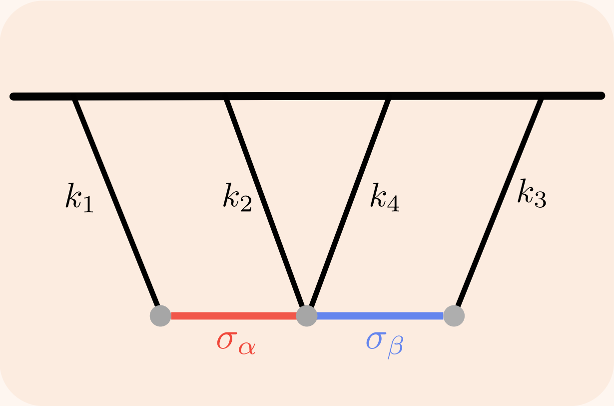
Effectively, like a single exchange?

Tong, Wang, **YH**
2112.03448

How to distinguish between *double*-exchange and *single*-exchange channels

Double Massive Exchange: double VS single

Towards the primordial trispectrum

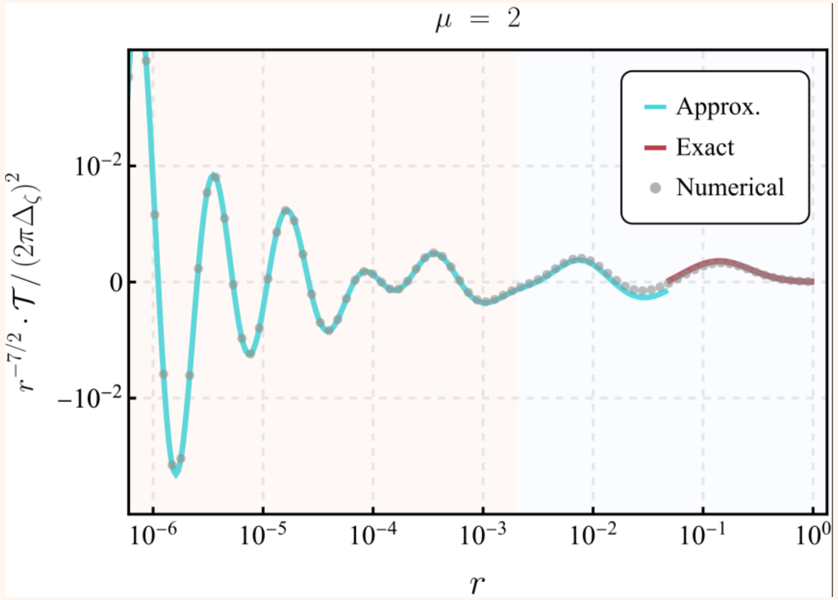


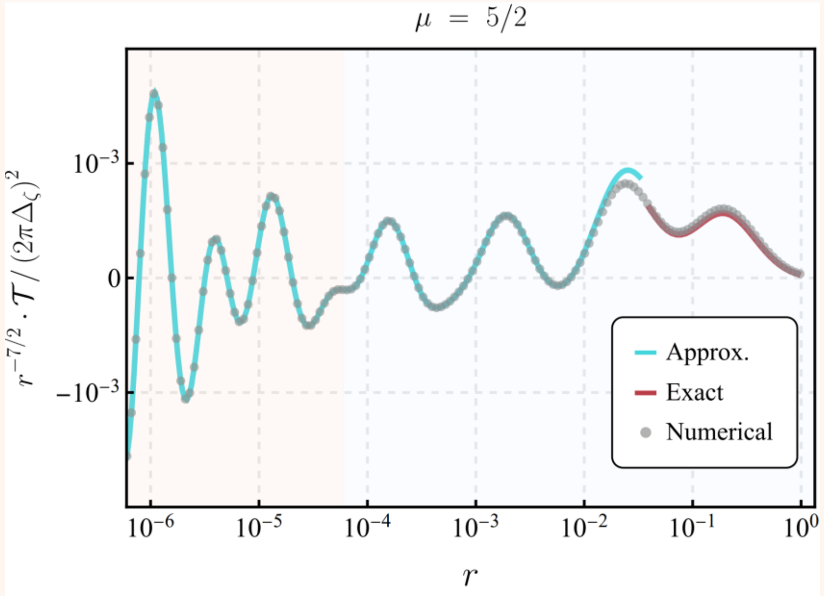
$$\langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\varphi_{\mathbf{k}_4}\rangle' = \frac{H^8}{f_\pi^4}\frac{(k_{1234}/4)^3}{(k_1k_2k_3k_4)^3}\mathcal{T}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)$$

Consider channel: $\mathcal{T}(rk,k,rk,k)$

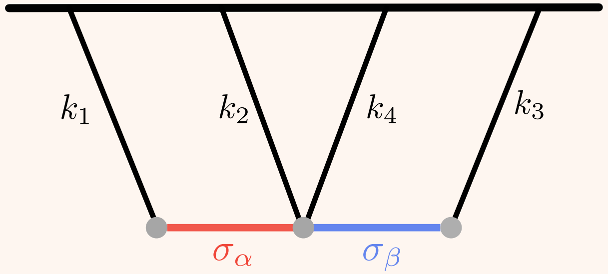
$$\mathcal{T} = \mathcal{T}_{\rm ss} + \mathcal{T}_{\rm sb} + \mathcal{T}_{\rm bb}$$

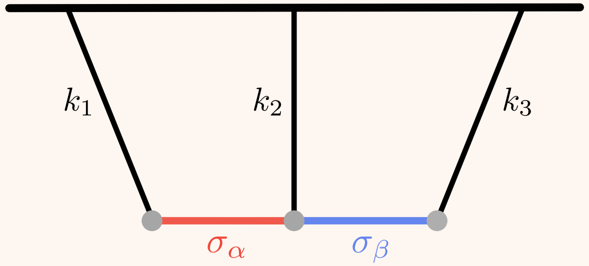
different terms	mass dependence	r dependence
$\mathcal{T}_{\rm ss}$	$\mu^{3/2}e^{-2\pi\mu}$	$r^{3+2i\mu}$
$\mathcal{T}_{\rm sb}$	$\mu^{3/2}e^{-\pi\mu}$	$r^{7/2+i\mu}$
$\mathcal{T}_{\rm bb}$	μ^{-4}	r^4





Double Massive Exchange： Summary



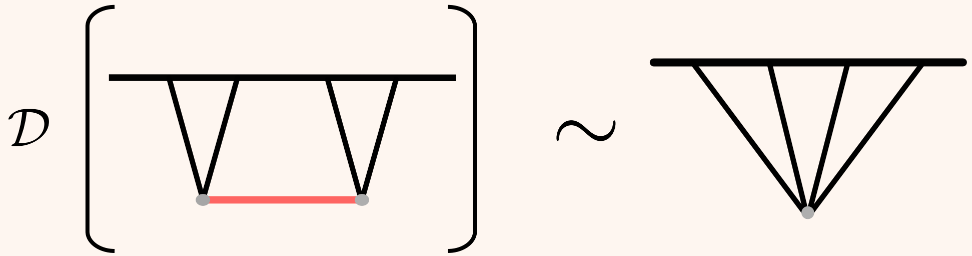


- **Bootstrap equations and analytical solutions of double-exchange**
- **Extend the results to three-point correlators**
- **The Phenomenology of double-exchange diagrams**
  - Size, shape, CC signals
  - Double VS Single (phase, multiple species, trispectrum …)

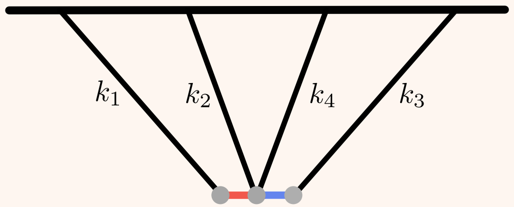
Back up

Double Massive Exchange: Boundary conditions

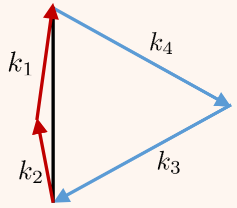
Single Massive Exchange:



Double Massive Exchange:



Boundary conditions:



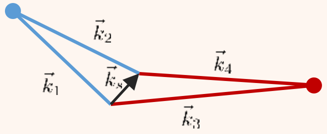
- This singularity should be absent under folded limit in the standard vacuum
- + Unitarity (COT) …

$$\lim_{u,v\ll 1} \mathcal{I}^{p_1p_2p_3}_{\mathsf{abc},\alpha\beta} = \sum_{\mathsf{d,e}=\pm} c_{\mathsf{abc,de}} u^{-\frac{5}{2}-p_1-\mathrm{i}\mathsf{d}\mu_\alpha} v^{-\frac{5}{2}-p_3-\mathrm{i}\mathsf{e}\mu_\beta}\, \Gamma\left[\begin{matrix} \frac{4+p_2-\mathrm{i}(\mathsf{d}\mu_\alpha+\mathsf{e}\mu_\beta)}{2}, \frac{5+p_2-\mathrm{i}(\mathsf{d}\mu_\alpha+\mathsf{e}\mu_\beta)}{2} \\ 1-\mathrm{i}\mathsf{d}\mu_\alpha, 1-\mathrm{i}\mathsf{e}\mu_\beta \end{matrix}\right]$$

$$\begin{aligned} c_{\pm\mp\pm,\mathsf{de}} &= -e^{\mp\mathrm{i}\frac{\pi}{2}(p_{13}-p_2)}\,\mathrm{csch}\,(\pi\mathsf{d}\mu_\alpha)\,\mathrm{csch}\,(\pi\mathsf{e}\mu_\beta) \times \widetilde{\Gamma}(p_1,p_2,p_3,\mu_\alpha,\mu_\beta)\ , \\ c_{\pm\pm\mp,\mathsf{de}} &= \mp\mathrm{i}\, e^{\mp\mathrm{i}\frac{\pi}{2}(p_{12}-p_3)}\,\mathrm{csch}\,(\pi\mathsf{d}\mu_\alpha)\,\mathrm{csch}\,(\pi\mathsf{e}\mu_\beta)\, e^{\mp\pi\mathsf{d}\mu_\alpha} \times \widetilde{\Gamma}(p_1,p_2,p_3,\mu_\alpha,\mu_\beta)\ , \\ c_{\pm\mp\mp,\mathsf{de}} &= \ c_{\mp\mp\pm,\mathsf{ed}} \quad \text{with} \quad (p_1\leftrightarrow p_3) \quad \text{and} \quad (\alpha\leftrightarrow\beta)\ , \\ c_{\pm\pm\pm,\mathsf{de}} &= e^{\mp\mathrm{i}\frac{\pi}{2}p_{123}}\,\mathrm{csch}\,(\pi\mathsf{d}\mu_\alpha)\,\mathrm{csch}\,(\pi\mathsf{e}\mu_\beta)\, e^{\mp\pi(\mathsf{d}\mu_\alpha+\mathsf{e}\mu_\beta)} \times \widetilde{\Gamma}(p_1,p_2,p_3,\mu_\alpha,\mu_\beta)\ , \end{aligned}$$

with the new defined function,

$$\widetilde{\Gamma}(p_1,p_2,p_3,\mu_\alpha,\mu_\beta) \equiv \frac{\pi^{\frac{1}{2}}}{2^{4+p_{13}-p_2}}\Gamma\left[\begin{matrix} \frac{5}{2}+p_1-\mathrm{i}\mu_\alpha, \frac{5}{2}+p_1+\mathrm{i}\mu_\alpha, \frac{5}{2}+p_3-\mathrm{i}\mu_\beta, \frac{5}{2}+p_3+\mathrm{i}\mu_\beta \\ 3+p_1, 3+p_3 \end{matrix}\right]$$



Soft limit, which can be easily found

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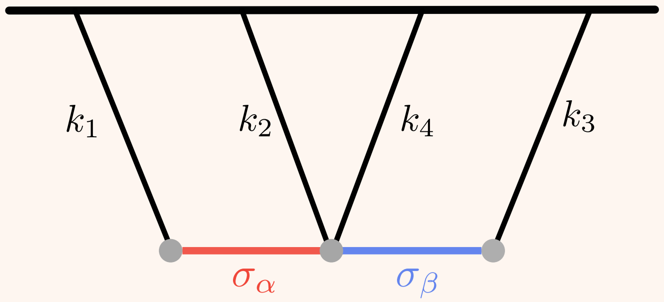
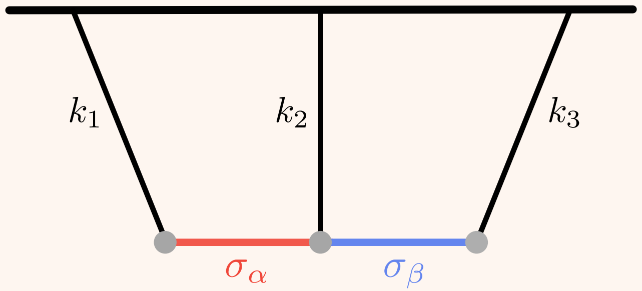
Double Massive Exchange： Consistency checks





- **Folded limit** Spurious divergence is canceled out

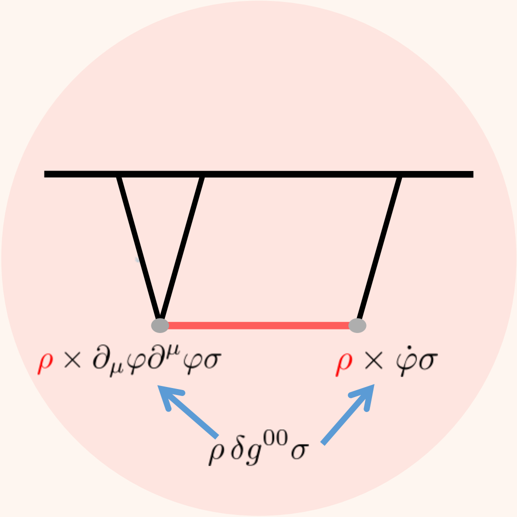
- **Total energy limit**

$$\lim_{k_T\to 0}\mathcal{I} = \frac{r_{13}^3}{8\,r_1\,r_3}\log(r_1)\,\log(r_3) \sim \frac{k_{13}^3}{4k_1k_3k_T}\log^2(k_T) \qquad \frac{2k_1}{k_T}\equiv r_1, \quad \frac{2k_3}{k_T}\equiv r_3,$$

- **Squeezed limit**

$$\lim_{k_1\to 0}\langle\varphi_{\mathbf{k_1}}\varphi_{\mathbf{k_2}}\varphi_{\mathbf{k_3}}\rangle'$$
$$= -\frac{\rho^2\lambda H}{(k_1k_2k_3)^2}\cdot \mathrm{Re}\left\{\left[\frac{\pi^{1/2}}{2^{4+2\mathrm{i}\mu}}\frac{2\mathrm{i}\mu+5}{2\mu-3\mathrm{i}}\Gamma\left[\frac{1}{2}+\mathrm{i}\mu,-\mathrm{i}\mu\right]\left(1+\tanh(\pi\mu)\right)+\mathcal{O}\left(e^{-2\pi\mu}\right)\right]\left(\frac{k_1}{k_3}\right)^{\frac{1}{2}+\mathrm{i}\mu}+\mathcal{O}\left(\frac{k_1}{k_3}\right)\right\},$$

Why we want more ?



too small?

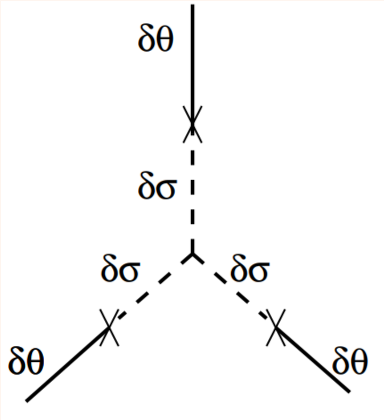
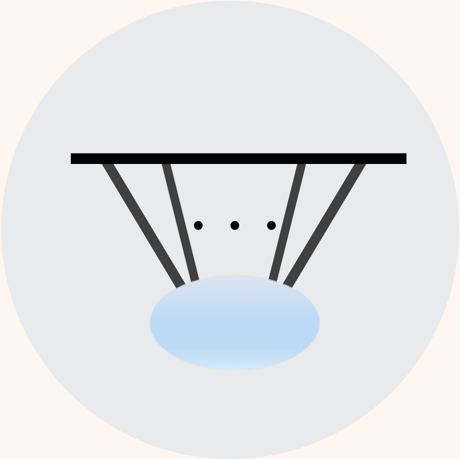


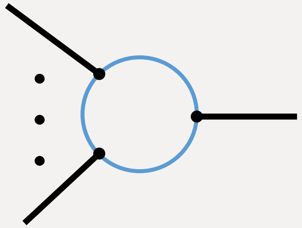
Diagram in QSFI

We NEED more !

More is Difficult

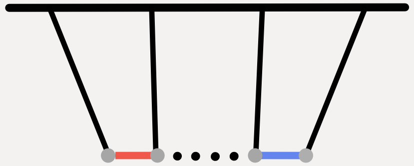


Loop diagrams



Xianyu, Zhang 2022
Qin, Xianyu 2023
.....

Tree diagrams with more vertices



Double Massive Exchange：Phenomenology （Size）

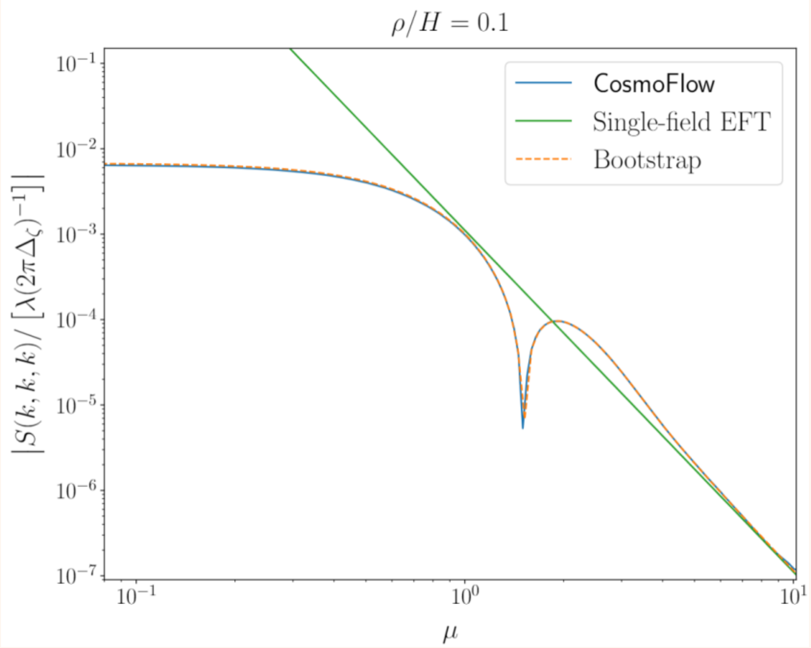
$$\langle \varphi_{\mathbf{k_1}} \varphi_{\mathbf{k_2}} \varphi_{\mathbf{k_3}} \rangle' = -\frac{H^5}{f_\pi^2 (k_1 k_2 k_3)^2} S(k_1, k_2, k_3)$$

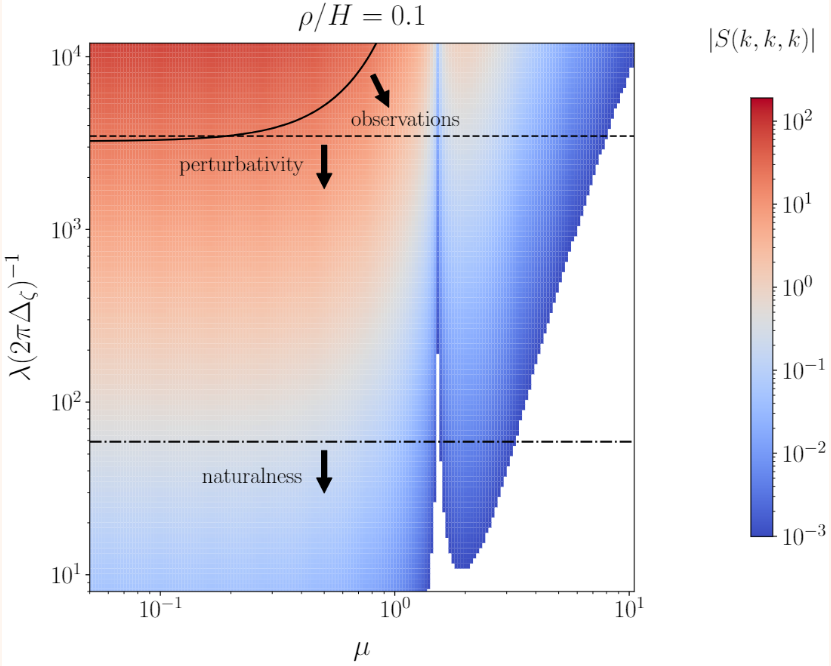
$$S \sim f_{\rm NL} \sim \underbrace{(\Delta_\zeta)^{-1}}_{10^4} \times (\rho/H)^2 \times \lambda$$

$$S_{\rm bootstrap}(k,k,k) \underset{\mu \gg 1}{\simeq} -0.11 \times \left(\frac{\rho}{H}\right)^2 \times \frac{\lambda f_\pi^2}{H^2} \times \frac{1}{\mu^4}$$

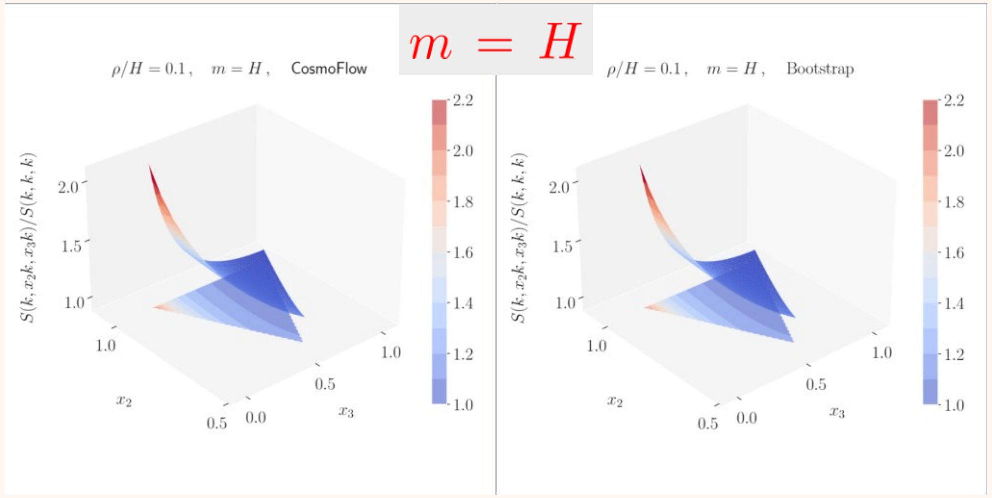
Single-field effective theory：

$$f_{\rm NL}^{\rm eq} = -\frac{1}{9} \left(\frac{\rho}{H}\right)^2 \frac{1}{1+\rho^2/m^2} \times \frac{\lambda f_\pi^2}{H^2} \times \frac{H^4}{m^4}$$

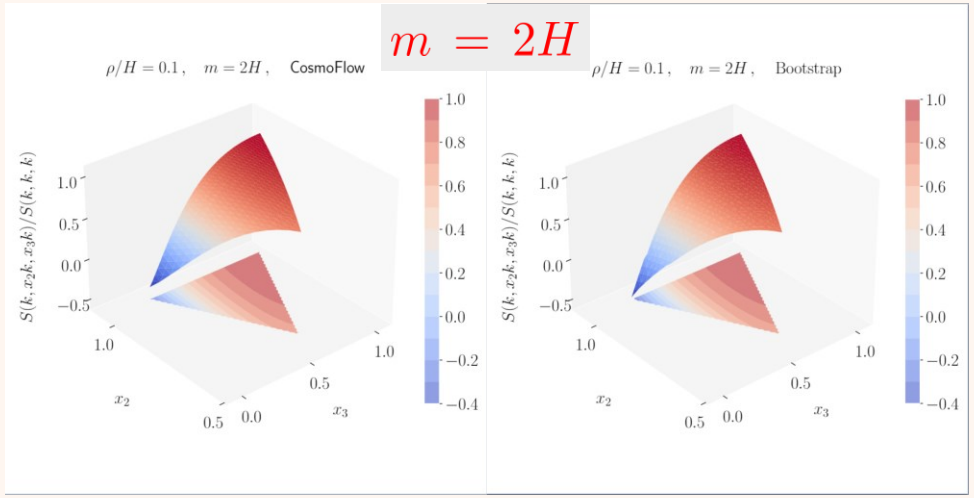




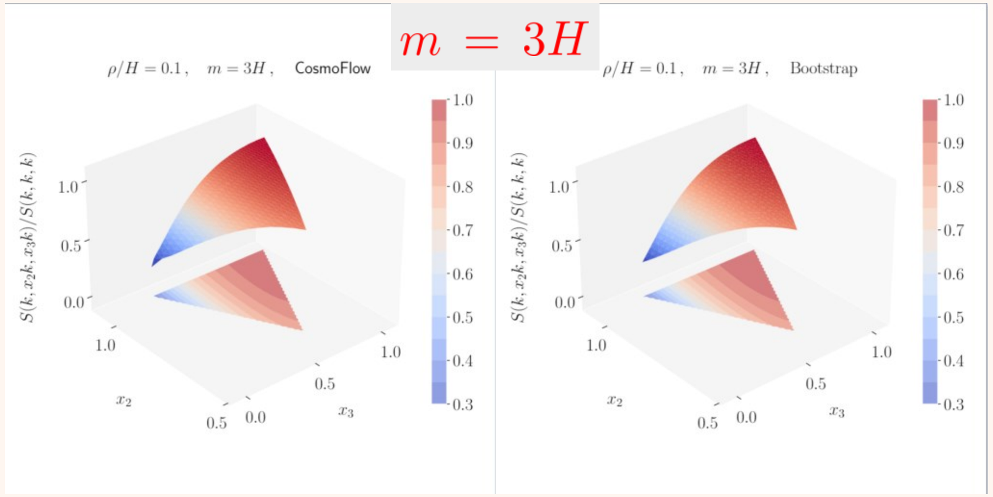
Double Massive Exchange： Phenomenology （Shape）



∼ **Local**



∼ **equilateral**



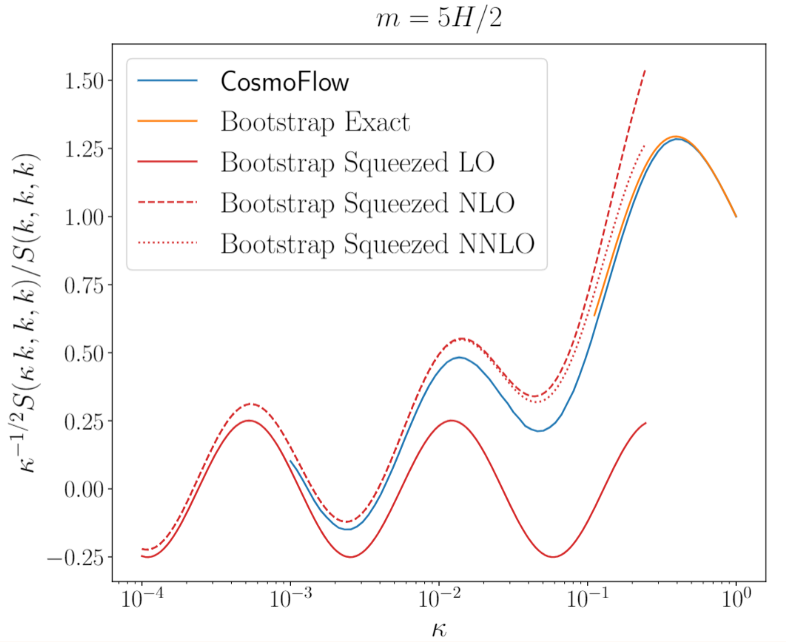
∼ **equilateral**

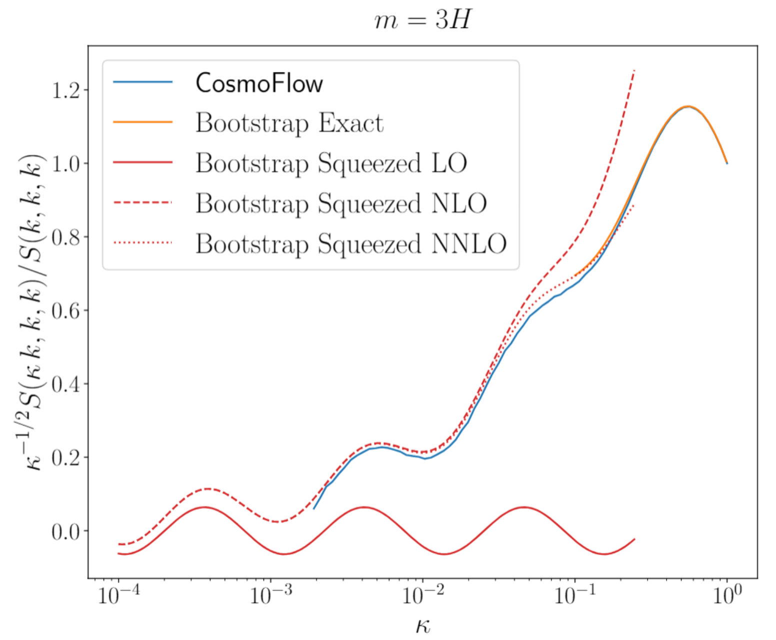
Double Massive Exchange: Phenomenology （CC signals）

Squeezed limit approximation:

$\sim \mathcal{O}(e^{-\pi\mu})$

$$\lim_{k_1\to 0}\langle\varphi_{\mathbf{k_1}}\varphi_{\mathbf{k_2}}\varphi_{\mathbf{k_3}}\rangle'$$
$$= -\frac{\rho^2\lambda H}{(k_1k_2k_3)^2}\cdot \mathrm{Re}\left\{\left[\frac{\pi^{1/2}}{2^{4+2\mathrm{i}\mu}}\frac{2\mathrm{i}\mu+5}{2\mu-3\mathrm{i}}\Gamma\left[\frac{1}{2}+\mathrm{i}\mu,-\mathrm{i}\mu\right](1+\tanh(\pi\mu))+\mathcal{O}\left(e^{-2\pi\mu}\right)\right]\left(\frac{k_1}{k_3}\right)^{\frac{1}{2}+\mathrm{i}\mu}+\mathcal{O}\left(\frac{k_1}{k_3}\right)\right\},$$





Double Massive Exchange： double VS single

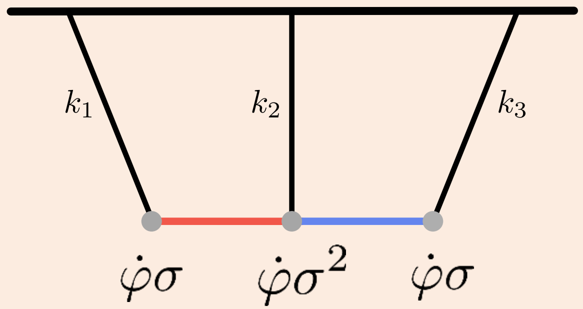
(1) Phase information in CC signals

$$S \sim \mathcal{A}(\lambda, m)\, e^{-\pi\mu} \left(\frac{k_3}{k_1}\right)^{1/2} \sin\left[\mu \log\left(\frac{k_3}{k_1}\right) + \vartheta\right]$$

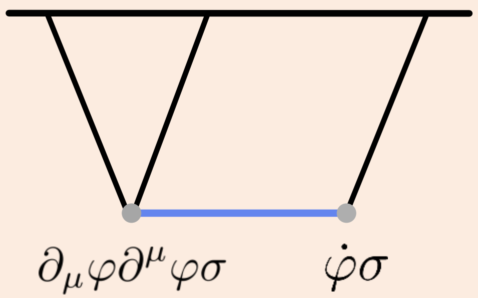
Qin, Xianyu 2205.01692

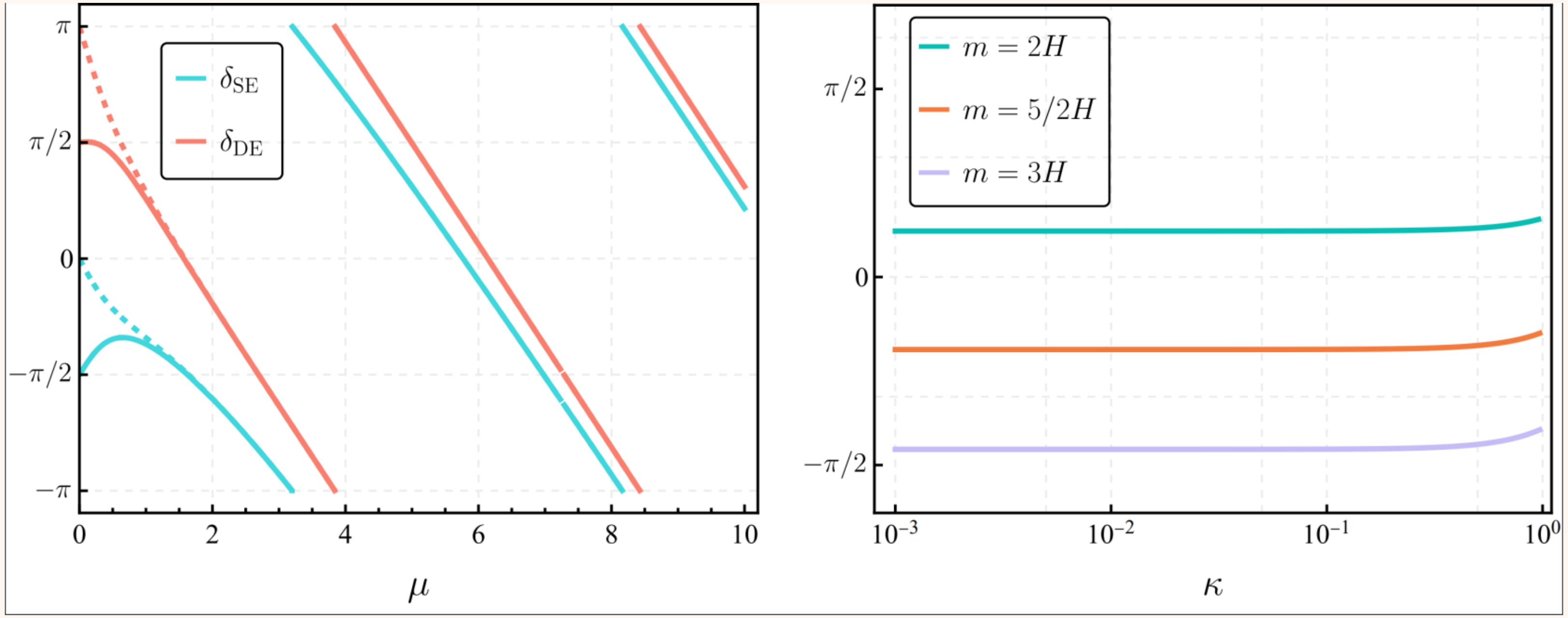
Phase

Double:



Single:

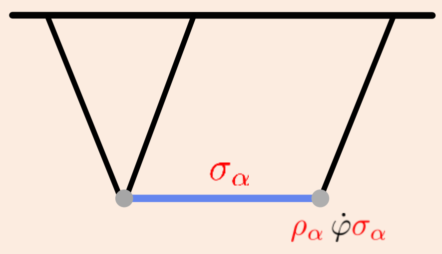




Double Massive Exchange： double VS single

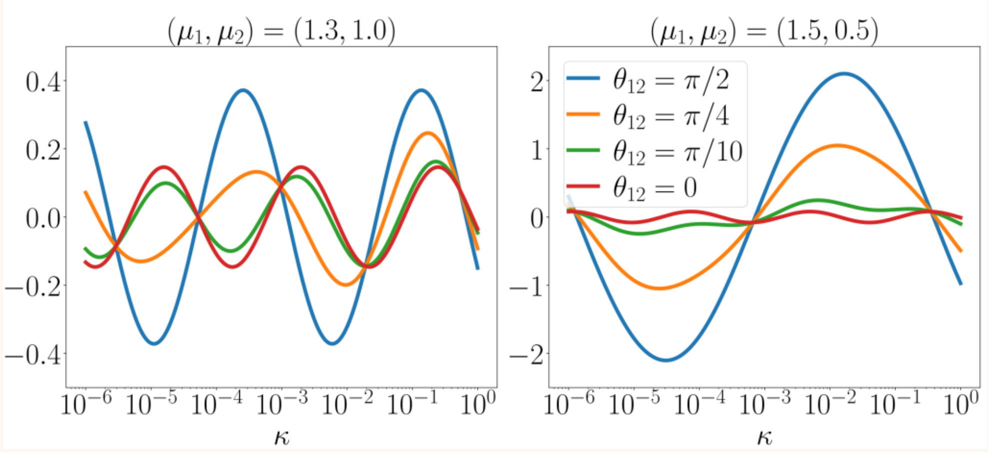
(2) Multiple isocurvature species

Single:



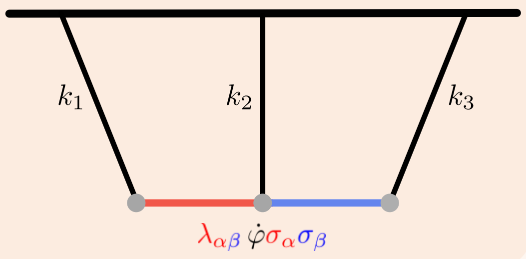
$$S^{\rm SE,multi}_{\rm CC,LO}=\sum_\alpha\left(\frac{\rho_\alpha}{H}\right)^2{\rm Re}\left[\kappa^{1/2+{\rm i}\mu_\alpha}\mathcal{A}_{\rm SE}(\mu_\alpha)e^{{\rm i}\delta_{\rm SE}(\mu_\alpha)}\right]$$

$$(\rho_1,\rho_2)=(\rho\cos(\theta_{12})\,,\rho\sin(\theta_{12}))$$



Pinol, Aoki, Renaux-Petel, Yamaguchi 2112.05710

Double:



$$S^{\rm DE,multi}_{\rm CC,LO}=\sum_{\alpha,\beta}\frac{\rho_\alpha\rho_\beta}{H^2}\times\lambda_{\alpha\beta}(2\pi\Delta_\zeta)^{-1}\times{\rm Re}\left[\kappa^{1/2+{\rm i}\mu_\alpha}\mathcal{A}^{\rm multi}_{\rm DE}(\mu_\alpha,\mu_\beta)e^{{\rm i}\delta^{\rm multi}_{\rm DE}(\mu_\alpha,\mu_\beta)}\right]$$

$$\lambda_{\alpha\beta}\equiv\lambda\times e_{\alpha\beta}$$

Case 1: $e_{11}=1$

Case 2: $e_{11}=e_{22}=1$

Case 3: $e_{11}=e_{12}=e_{21}=1$

Case 4: $e_{11}=e_{22}=e_{12}=e_{21}=1$