De Sitter correlators in the stochastic approach

(work in progress)

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Cosmological Correlators in Taiwan, December 2024

Scalar fields in de Sitter (dS)

Consider: Light ($m \ll \sqrt{\lambda} H$) scalar field ϕ with $V(\phi) \sim \lambda \phi^4$ on dS

→ perturbatively computed correlation functions IR-divergent

Intuition: Scalar EOM in Fourier space

$$\cancel{\ddot{\phi}_k} + 3H\dot{\phi}_k + \left(\cancel{\frac{k^2}{a^2}} - \cancel{m^2} \right) \phi_k = \frac{\lambda}{3!}[\phi^3]_k \tag{1}$$

→ long superhorizon modes ($k < aH$): evolution *ultra-local*

$$\langle \phi_k(t)\phi_{k'}(t)\rangle' \sim \lambda \frac{H}{k^3} \underbrace{\log\left(\frac{k}{aH}\right)}_{\text{secular divergence}} \quad \underbrace{\log(kaHL^2)}_{\text{IR div. with cutoff } L}$$

⇒ PT breaks down, **non-perturbative treatment** needed

The stochastic approach

[V. Gorbenko and L. Senatore, "$\lambda\phi^4$ in dS", arXiv:1911.00022]

1. Mode splitting: Split scalar field using a comoving momentum cutoff

$$\Lambda(t) \equiv \varepsilon a(t) H \tag{2}$$

(with ε an artificial parameter) into long- and short-wavelength contributions

$$\phi(x) := \phi_\ell(\vec{x}, t) + \phi_s(\vec{x}, t) \tag{3}$$

$$= \int_0^{\Lambda(t)} \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \phi_k + \int_{\Lambda(t)}^{\infty} \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \phi_k \tag{4}$$

$\rightarrow$ **treat long field ϕ_ℓ non-perturbatively**

2. Correlators from probability distribution: $P[\phi] \equiv \Psi^*_{BD}[\phi]\Psi_{BD}[\phi]$

$$\langle \phi(x_1)...\phi(x_n)\rangle = \int \mathcal{D}\phi \, P[\phi]\phi(x_1)...\phi(x_n) \qquad \text{hard!} \tag{5}$$

with $P[\phi]$ determined from the functional continuity equation

- focus on **long modes**:

$$P_\ell[\phi_\ell] \equiv \int \mathcal{D}\phi \, \delta\left[\phi_\ell(x) - \int_0^{\Lambda(t)} \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}}\phi_k\right] P[\phi] \tag{6}$$

- use **ultra-locality of long modes**: different long field points $\phi_\ell(x_i)$ do not interact and behave as independent DOF ϕ_i

$$\rightarrow \langle \phi_\ell(x_1)...\phi_\ell(x_n)\rangle = \int d\phi_1...d\phi_n \, P_n(\phi_1,..,\phi_n;t)\phi_1...\phi_n \tag{7}$$

with n-point distribution P_n determined from local Fokker-Planck PDE

The stochastic approach

[V. Gorbenko and L. Senatore, "$\lambda\phi^4$ in dS", arXiv:1911.00022]

3. Evolution equation for long mode distribution P_n:

$$\frac{\partial}{\partial t} P_n(\phi_1, ..., \phi_n; t) = -\left(H_0^{(n)} + \frac{1}{2} \sum_{i \neq j}^{n} j_0(\epsilon a H x_{ij}) H'_{ij} \right) P_n \quad (8)$$

with a **"late-time free Hamiltonian"**

$$H_0^{(n)} \equiv -\sum_{i=1}^{n} \Gamma_{\phi_i}, \qquad \Gamma_{\phi_i} \equiv \frac{\partial}{\partial \phi_i} \frac{V'(\phi_i)}{3H} + \frac{H^3}{8\pi^2} \frac{\partial^2}{\partial \phi_i^2} \quad (9)$$

and **"interaction Hamiltonians"**

$$H'_{ij} \equiv -\frac{H^3}{4\pi^2} \frac{\partial^2}{\partial \phi_i \partial \phi_j} \quad (10)$$

$\rightarrow$ solve analogously to Schrödinger equation

4. Solve evolution equation for long mode distribution P_n:

$$\frac{\partial}{\partial t} P_n(\phi_1, \dots, \phi_n; t) = -\left(H_0^{(n)} + \frac{1}{2} \sum_{i \neq j}^{n} \cancel{j_0(\epsilon a H x_{ij})} H'_{ij} \right) P_n \qquad (11)$$

Focus on late-time free evolution $H_0^{(n)} = -\sum_{i=1}^{n} \Gamma_{\phi_i}$ (as $j_0(t \to \infty) \sim 0$)

$$\to P_n(\phi_1, \dots, \phi_n; t) = \sum_{p_1, \dots, p_n = 0}^{\infty} c_{p_1, \dots, p_n}(t) \Phi_{p_1}(\phi_1) \dots \Phi_{p_n}(\phi_n) \qquad (12)$$

with $\{\Phi_p\}_{p=0}^{\infty}$ a complete ONB solving the EV problem

$$\Gamma_{\phi_i} \Phi_p(\phi_i) = -\lambda_p \Phi_p(\phi_i) \qquad \to \quad \lambda_p \sim p^{\alpha} \sqrt{\lambda} H \qquad (13)$$

4. Solve evolution equation for long mode distribution P_n:

$$\frac{\partial}{\partial t} P_n(\phi_1, ..., \phi_n; t) = -\left(H_0^{(n)} + \frac{1}{2} \sum_{i \neq j}^{n} j_0(\epsilon a H x_{ij}) H'_{ij} \right) P_n \qquad (14)$$

Treat interaction $\sim H'_{ij}$ with **sudden perturbation theory** (PT)

ansatz: $P_n(\phi_1, ..., \phi_n; t) = \sum_{p_1,...,p_n=0}^{\infty} c_{p_1,...,p_n}(t) \Phi_{p_1}(\phi_1) ... \Phi_{p_n}(\phi_n)$

At late times, we find **conformal correlators** (up to subleading order in sudden PT):

$\rightarrow n = 2$: $c_{p_1 p_2}(t) = \langle \Phi_{p_1}(\phi_1) \Phi_{p_2}(\phi_2) \rangle = \frac{\delta_{p_1 p_2}}{(aHx_{12})^{2\lambda_{p_1}/H}}$

$\rightarrow n = 3$: $\langle \Phi_{p_1}(\phi_1) \Phi_{p_2}(\phi_2) \Phi_{p_3}(\phi_3) \rangle = \prod_{i<j\neq k}^{3} \frac{I_{p_1 p_2 p_3}}{(aHx_{ij})^{(\lambda_{p_i} + \lambda_{p_j} - \lambda_{p_k})/H}}$

Correlators on superhorizon scales

Result

The eigenfunctions Φ_p of the "late-time free Hamiltonian" Γ_ϕ behave as late-time conformal primaries with conformal weights given by the eigenvalues $\lambda_p/H \rightarrow$ dS boundary symmetries ✓

Another consistency check: 4-point function in the squeezed limit decomposes into product of two squeezed 3-point functions in Fourier space

$$\langle \Phi_{p_1}^{\vec{k}_1} \Phi_{p_2}^{\vec{k}_2} \Phi_{p_3}^{\vec{k}_3} \Phi_{p_4}^{\vec{k}_4} \rangle_{\rm sq.} = (2\pi)^4 \delta\left(\sum_{i=1}^{4} \vec{k}_i\right) \sum_{p=0}^{\infty} \frac{\langle \Phi_{p_1}^{\vec{k}_1} \Phi_{p_2}^{\vec{k}_2} \Phi_{p}^{\vec{k}} \rangle'_{\rm sq.} \langle \Phi_{p}^{\vec{k}} \Phi_{p_2}^{\vec{k}_3} \Phi_{p}^{\vec{k}_4} \rangle'_{\rm sq.}}{\langle \Phi_p^{\vec{k}} \Phi_p^{-\vec{k}} \rangle'} \tag{15}$$

with $k \equiv |\vec{k}_1 + \vec{k}_2|$ the exchanged momentum

Correlators on superhorizon scales

Conjecture

$\lambda\phi^4$-theory in dS "hadronises" on superhorizon scales into another weakly coupled EFT of scalar hadrons $\Psi_{p_i}(t,\phi_i)$

$$\mathcal{L}(\Psi_p) \sim -\frac{1}{2}\sum_{p=0}^{\infty}[(\partial\Psi_p)^2 + M_p^2\Psi_p^2] - \sum_{p_1,p_2,p_3}\lambda_{p_1p_2p_3}\Psi_{p_1}\Psi_{p_2}\Psi_{p_3}$$

$$- \sum_{p_1,p_2,p_3,p_4}\lambda_{p_1p_2p_3p_4}\Psi_{p_1}\Psi_{p_2}\Psi_{p_3}\Psi_{p_4} + \ldots \tag{16}$$

such that late-time correlators $\langle\Psi_{p_1}\Psi_{p_2}\ldots\rangle(t\to\infty) = \langle\Phi_{p_1}\Phi_{p_2}\ldots\rangle$ yield the conformal primaries.

From dS mass formula: $\frac{\lambda_p}{H}\left(\frac{\lambda_p}{H}-3\right) = -\frac{M_p^2}{H^2} \to M_p^2 \sim \mathcal{O}(\sqrt{\lambda})$ **light!**

Correlators on superhorizon scales

Cubic coupling $\lambda_{p_1 p_2 p_3}$: Matching procedure with massive 3-point scalar contact diagram at late times (e.g. using in-in-formalism)

(a) (b)

$$\to \lambda_{p_1 p_2 p_3} = I_{p_1 p_2 p_3} \frac{64\pi^5}{H^2} \frac{\Gamma\left(\frac{1}{2} - \frac{\sum_j^3 \lambda_{p_j}}{2H}\right)}{\prod_j \Gamma(\frac{3}{2} - \frac{\lambda_{p_j}}{H})} \frac{\left(\frac{\sum_j^3 \lambda_{p_j}}{2H} - \frac{1}{2}\right)\left(\frac{\sum_j^3 \lambda_{p_j}}{2H} - \frac{3}{2}\right)}{\prod_{i<j\neq k}^3 \Gamma\left(\frac{\lambda_{p_i} + \lambda_{p_j} - \lambda_{p_k}}{2H}\right)}$$
$$\sim \mathcal{O}(\lambda^{3/2}) \qquad (17)$$

$\Rightarrow \Psi_p$'s **more weakly coupled** than original $\lambda\phi^4$-theory!

Correlators on superhorizon scales

Quartic coupling $\lambda_{p_1 p_2 p_3 p_4}$: Interesting testing ground for the conjecture, since correlators at 3-point level pinned down by dS symmetry

From stochastic approach: At subleading order in sudden PT

$$\langle \Phi_{p_1}^{(\phi_1)} \Phi_{p_2}^{(\phi_2)} \Phi_{p_3}^{(\phi_3)} \Phi_{p_4}^{(\phi_4)} \rangle \sim I_{p_1 p_2 p_3 p_4} u^\alpha v^\beta \prod_{i<j}^{4} (aHx_{ij})^{\left(\frac{\sum_k \lambda_{p_k}}{3} - \lambda_{p_i} - \lambda_{p_j}\right)/H} \quad (18)$$

with invariant cross ratios $u \equiv \frac{x_{12}x_{34}}{x_{13}x_{24}}$, $v \equiv \frac{x_{12}x_{34}}{x_{14}x_{23}}$ and const. α, $\beta \sim \mathcal{O}(\sqrt{\lambda})$

→ problem: 4-point function from stochastic approach **too simple**!

Owing to finite precision $\sim \sqrt{\lambda}$*? Only valid in certain kinematical regime? Does it satisfy the differential equation for contact interactions?*

→ There's more to discover, stay tuned!