A Study of Scalar QED in de Sitter

W.I.P. w/ Victor Gorbenko
Shota Komatsu
Lorenzo di Pietro
Ankur

BROAD GOAL

STUDY GRAVITATIONAL FLUCTUATIONS on a DE SITTER BACKGROUND

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⚠ **HARD !**

IT IS A GAUGE TH.

$\oplus$

GAUGE GROUP $\equiv$ $\mathcal{D}$iffeo

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SIMPLER / TOY MODEL : SCALAR QED

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TECHNIQUE : Rely on AdS technology

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RESULTS

* DEFINITION of GAUGE INVARIANT OBSERVABLES in AdS w/ PHOTON given NEUMANN BDY CONDITIONS ("ALTERNATE QUANTIZATION")

* STUDY of HIGGS MECHANISM in AdS for PHOTON with DIRICHLET BDY CONDITIONS ("CANONICAL QUANTIZATION")

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PREVIOUS WORK : Allen, Baumann, Cacciatori, Jacobson, Moschella, Pimentel, Porrati, Rattazzi, Taronna, Schaub, Sleight, Sun, Zaffaroni

... AND MANY OTHERS

$\boxed{\text{FREE THEORY}}$

$$S_{SQED} = \int_{dS_{d+1}} dx^{d+1} \sqrt{-g} \left\{ -(D^\mu \varphi)(D_\mu \varphi)^* - m^2 \varphi \cdot \varphi^* + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right\}$$

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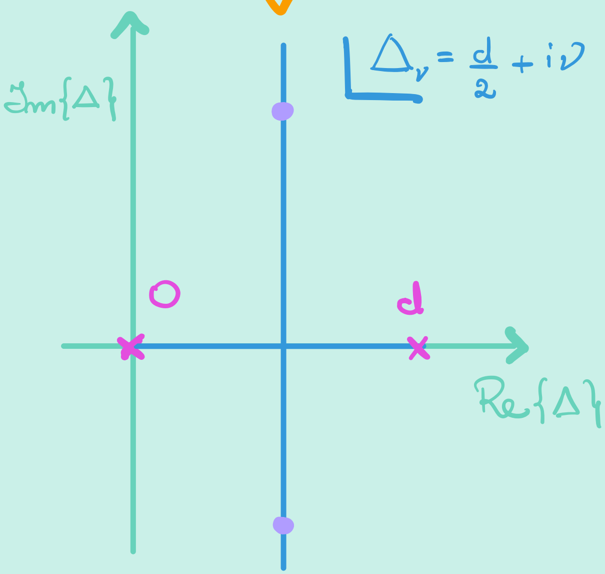

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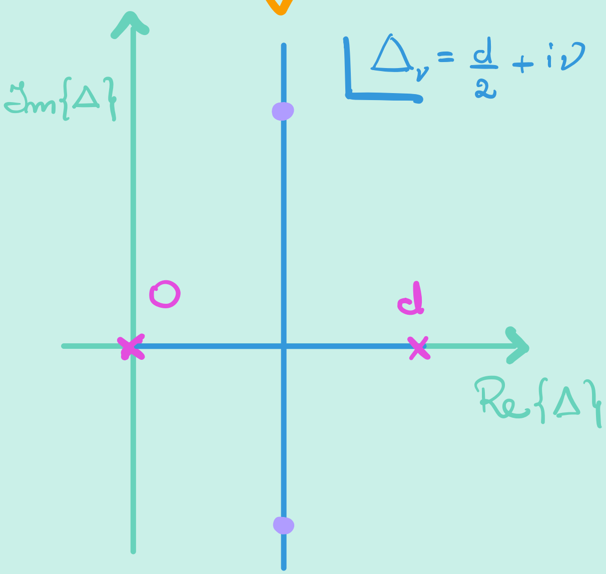

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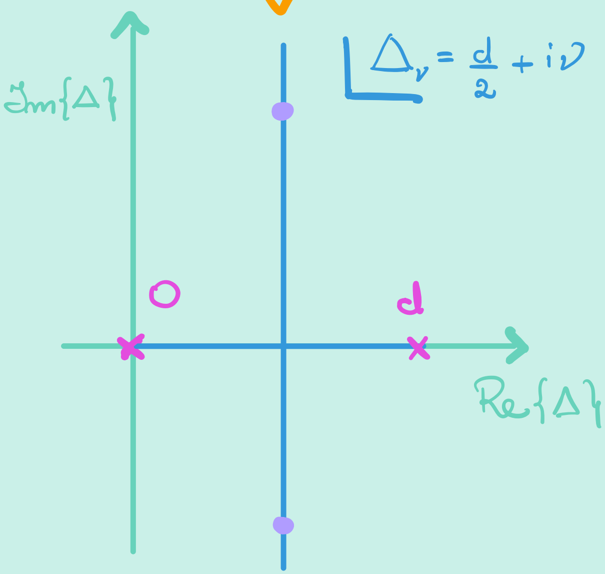

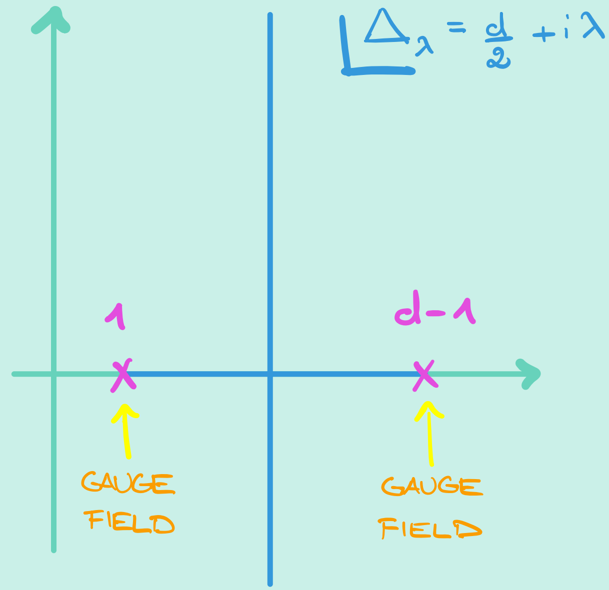

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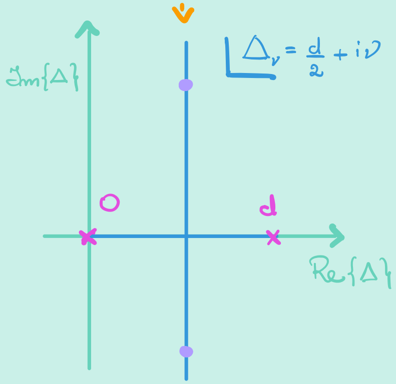
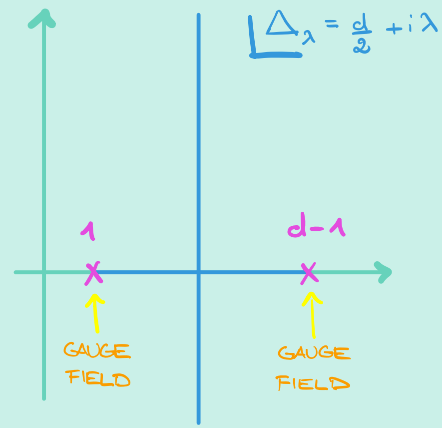


ANALITIC CONTINUATION TO EAdS

[Di Pietro, Gorbenko, Komatsu; '22] for technical details



FREE THEORY

$$S_{SQED} = \int_{dS_{d+1}} d x^{d+1} \sqrt{-g} \left\{ -(D^\mu \varphi)(D_\mu \varphi)^* - m^2 \varphi \cdot \varphi^* + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right\}$$

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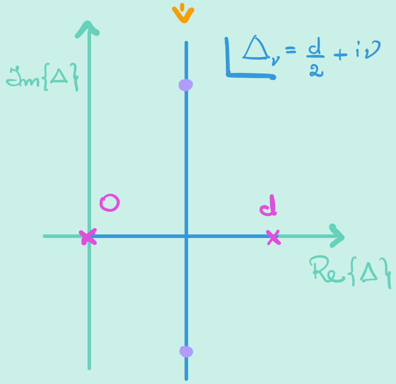

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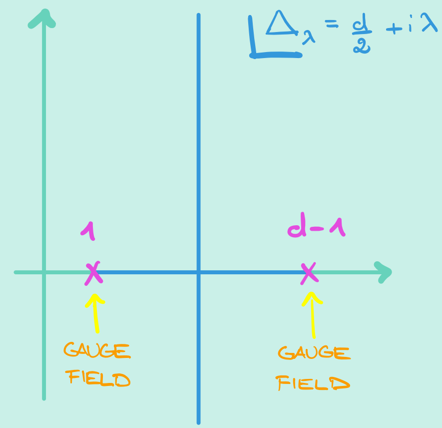

DOUBLING of the FIELDS: TAKE CARE of BOTH LATE TIME dS BEHAVIOUR!

$$S^{[EAdS]} = \text{KINETIC TERMS} + 2ie \left[J^\mu_{++} + J^\mu_{--} \right] \cdot \left(\cosh\pi\nu \cdot A_{\mathcal{N},\mu} + \sinh\pi\nu \cdot A_{\mathcal{D},\mu} \right) - 2ie \, A_{\mathcal{D},\mu} \cdot J^\mu_{+,-}$$

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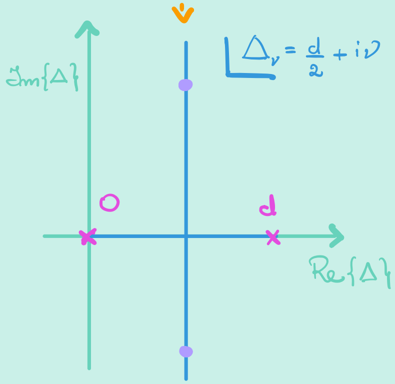

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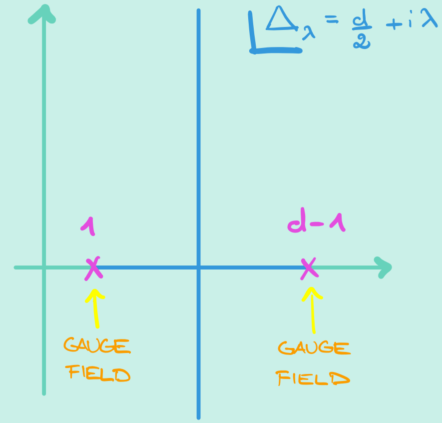
[Di Pietro, Gorbenko, Komatsu; '22]

for TECHNICAL DETAILS



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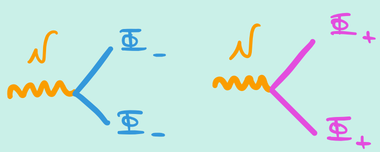

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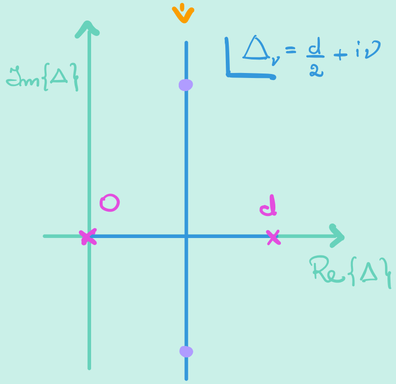

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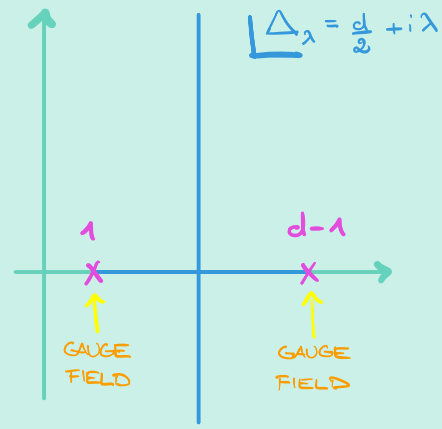

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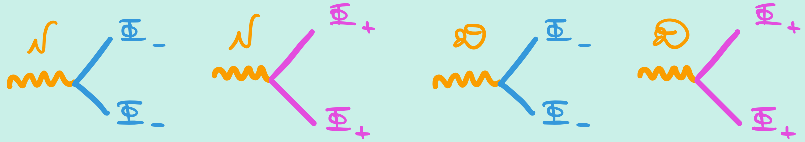

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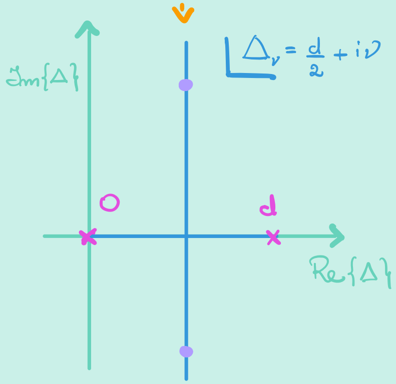

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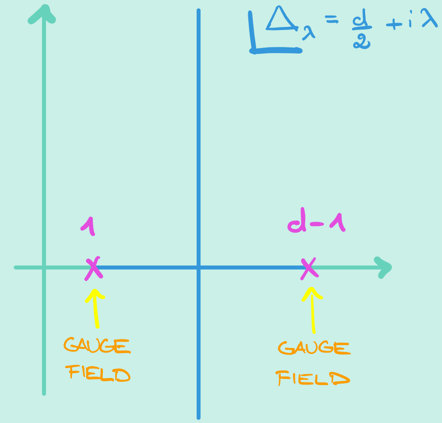

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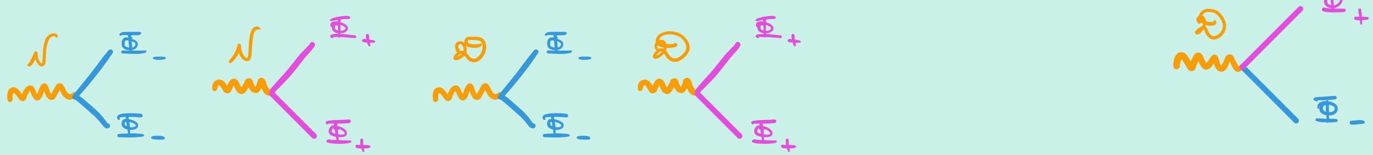

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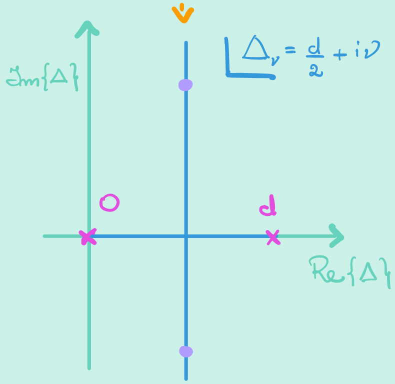

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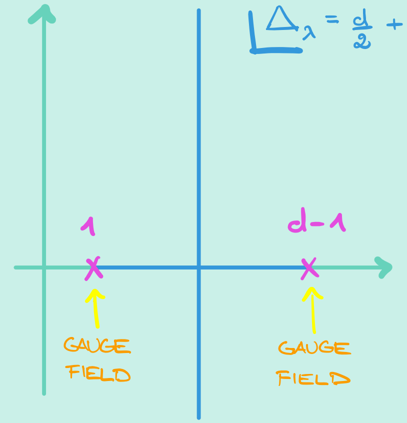

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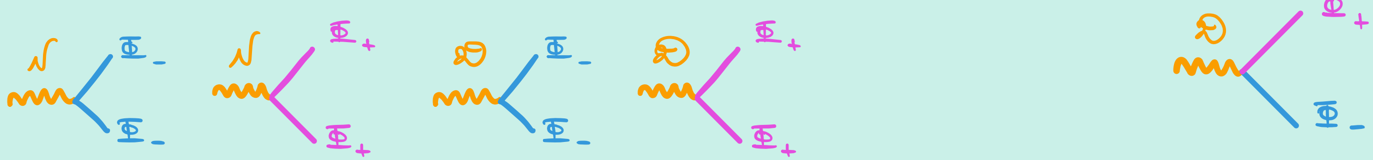

$$+V_{4\text{-points}}: \quad -2e^2 \sinh \pi\nu \left(A_{\mathcal{D}}^2 - A_{\mathcal{N}}^2\right)\left(|\Phi_+|^2 + |\Phi_-|^2\right) - 4e^2 A_{\mathcal{D}} \cdot A_{\mathcal{N}} \left[\cosh \pi\nu \left(|\Phi_+|^2 + |\Phi_-|^2\right) + \Phi_+ \Phi_-^* + \Phi_- \Phi_+^*\right]$$

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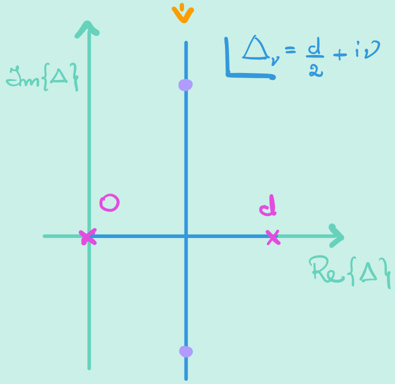

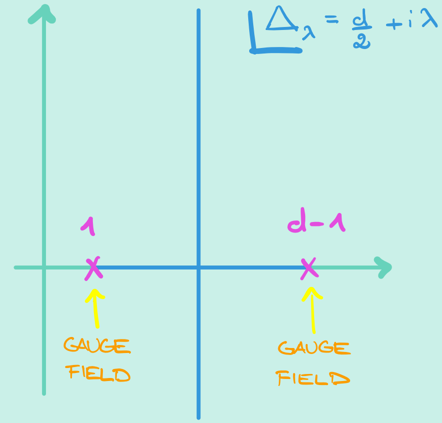

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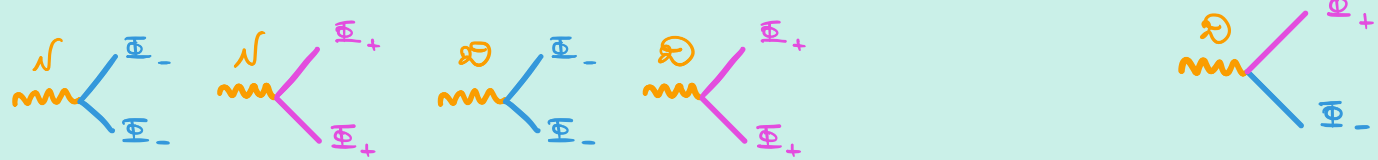

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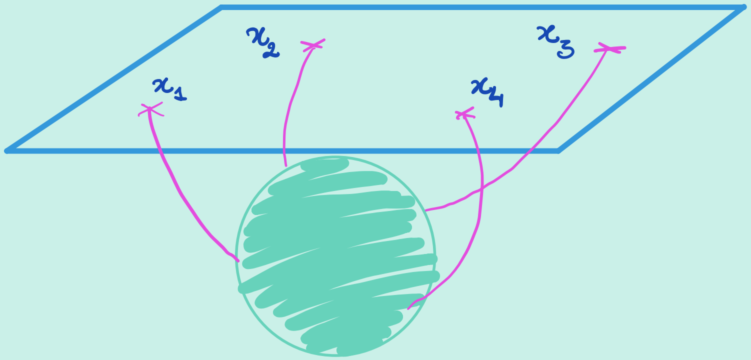
FOLLOWING: ALL COMPUTATIONS ARE IN EAdS!

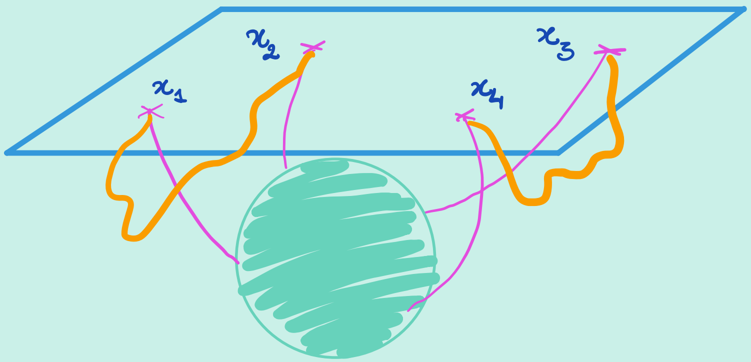
TREE-LEVEL RESULTS

$$\langle \varphi(x_1)\, \varphi(x_2)^\dagger \varphi(x_3)\, \varphi(x_4)^\dagger \rangle$$



TREE-LEVEL RESULTS

$$\left\langle \varphi(x_1)\, e^{ie\int_{x_1}^{x_2} dx^\mu A_\mu}\, \varphi(x_2)^\dagger\, \varphi(x_3)\, e^{ie\int_{x_3}^{x_4} dx^\nu A_\nu}\, \varphi(x_4)^\dagger \right\rangle$$



WILSON LINES

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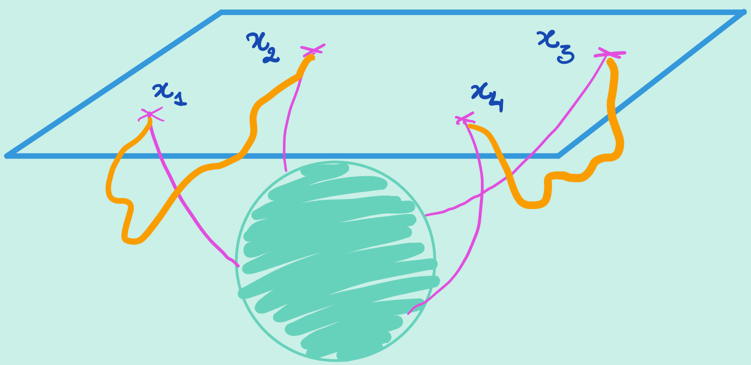
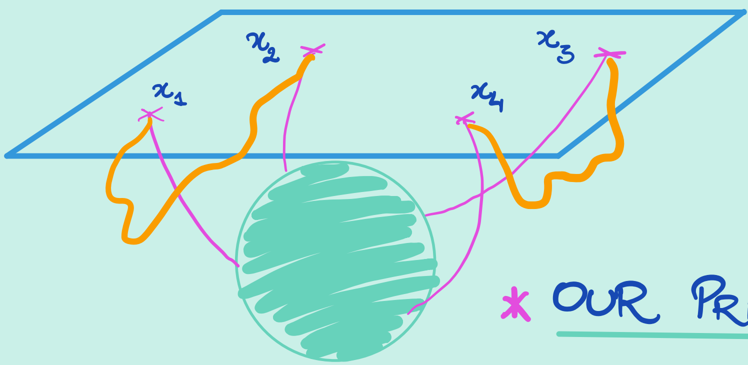


WILSON LINES:

* $\forall$ PATH: 4-POINT FUNCTION BECOMES GAUGE INVARIANT!

TREE - LEVEL RESULTS

$$\langle \varphi(x_1)\, e^{ie\int_{x_1}^{x_2} dX^\mu A_\mu}\, \varphi(x_2)^\dagger \varphi(x_3)\, e^{ie\int_{x_3}^{x_4} dX^\nu A_\nu}\, \varphi(x_4)^\dagger \rangle$$



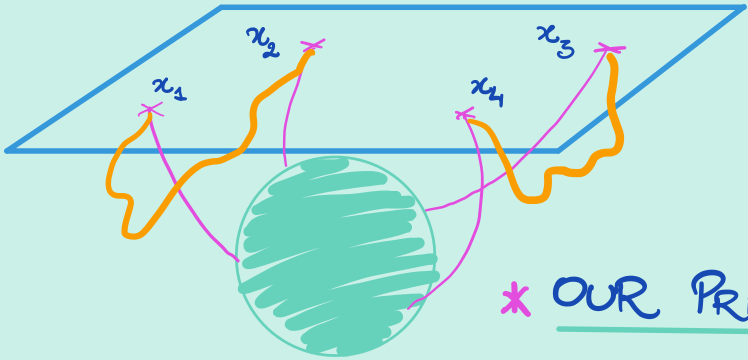
WILSON LINES :

* $\forall$ PATH : 4 - POINT FUNCTION BECOMES GAUGE INVARIANT !

* OUR PRESCRIPTION : SEND W. L. ALONG GEODESICS (in AdS!)

TREE-LEVEL RESULTS

$$\left\langle \varphi(x_1)\, e^{ie\int_{x_1}^{x_2} dx^\mu A_\mu}\, \varphi(x_2)^\dagger\, \varphi(x_3)\, e^{ie\int_{x_3}^{x_4} dx^\nu A_\nu}\, \varphi(x_4)^\dagger \right\rangle$$



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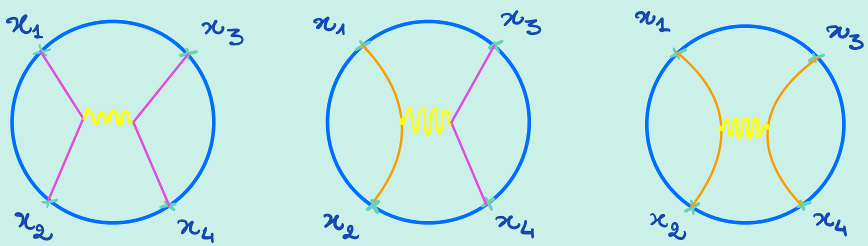
$\Downarrow$

CONFORMAL COVARIANT ANSWER



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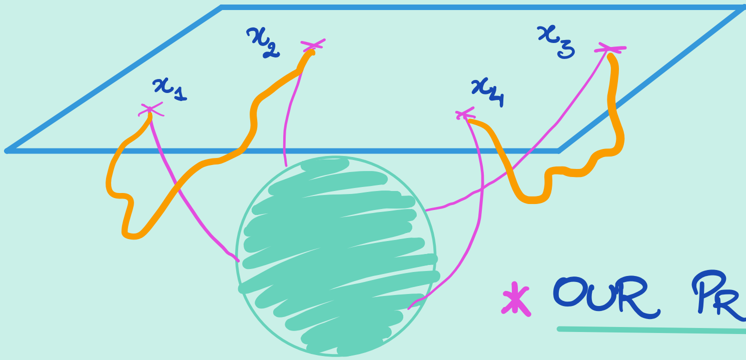

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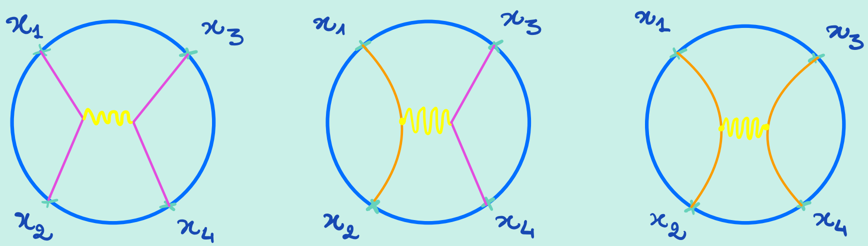
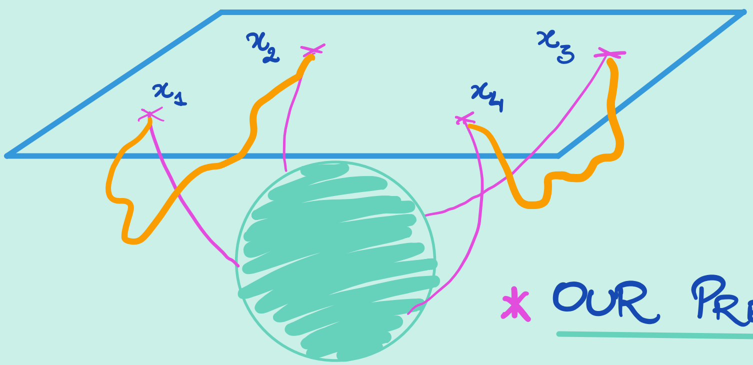


$$\boxed{\text{CONFORMAL COVARIANT ANSWER}}$$

$$e^2\, \mathcal{N}_\nu^{4} \left[\frac{(-\eta_c)^4}{P_{12}^2 \cdot P_{34}^2}\right]^{\frac{d}{2}-i\nu} \cdot \int d\lambda' \, \rho_\nu^{(J=1)}(\lambda') \cdot \mathcal{F}^{(1)}_{\lambda',\nu}(u,v)$$

TREE-LEVEL RESULTS

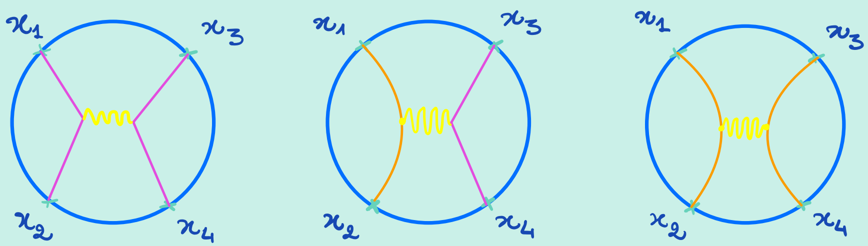
$$\left\langle \varphi(x_1)\, e^{ie\int_{x_1}^{x_2} dx^\mu A_\mu}\, \varphi(x_2)^\dagger\, \varphi(x_3)\, e^{ie\int_{x_3}^{x_4} dx^\nu A_\nu}\, \varphi(x_4)^\dagger \right\rangle$$



WILSON LINES:

* $\forall$ PATH : 4-POINT FUNCTION BECOMES GAUGE INVARIANT !
* OUR PRESCRIPTION : SEND W.L. ALONG GEODESICS (in AdS!)

$\Downarrow$



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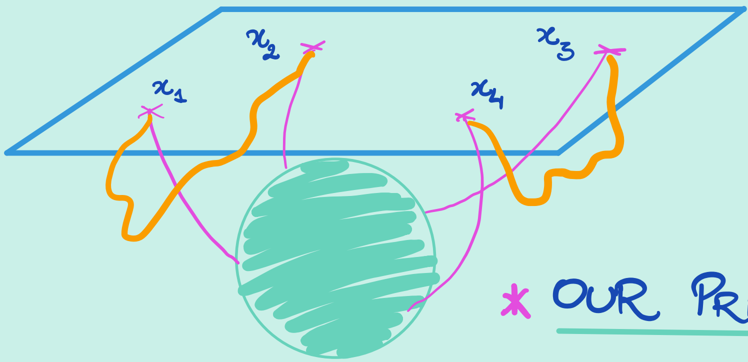
CONFORMAL PARTIAL WAVES

* COMPLETE BASIS
* EIGENFUNCTIONS of CASIMIR
* In terms of CONFORMAL BLOCKS

$$\mathcal{F}^{(1)}_{\lambda',\nu} = K^{(1)}_{d-\Delta_{\lambda'}}\, \hat{\mathcal{G}}^{(1)}_{\Delta_{\lambda'}} + \text{SHADOW}$$

TREE-LEVEL RESULTS

$$\left\langle \varphi(x_1)\, e^{ie\int_{x_1}^{x_2} dx^\mu A_\mu}\, \varphi(x_2)^\dagger\, \varphi(x_3)\, e^{ie\int_{x_3}^{x_4} dx^\nu A_\nu}\, \varphi(x_4)^\dagger \right\rangle$$



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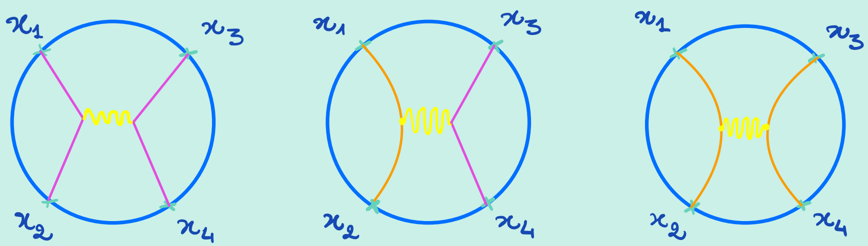
CONFORMAL COVARIANT ANSWER



$$e^2 \mathcal{N}_\nu^4 \left[\frac{(-\eta_c)^4}{P_{12}^2 \cdot P_{34}^2}\right]^{\frac{d}{2}-i\nu} \cdot \int d\lambda' \, \underbrace{\rho_\nu^{(J=1)}(\lambda')} \cdot \underbrace{\mathcal{F}^{(1)}_{\lambda',\nu}(u,v)}$$

TREE-LEVEL DENSITY

* $$\rho(\lambda) \cdot K_{d-\Delta_\lambda} = \underset{\sim\text{NORMALIZATION}}{\#} \cdot \underbrace{\frac{1}{\frac{d}{2}-1-i\lambda}}_{\text{GAUGE FIELD CONTRIBUTION}} \left[\underbrace{\Gamma\left(\# \pm \frac{i\lambda}{2}\right)}_{\text{EXCHANGE of U(1) CURRENT}} - \underbrace{2\Gamma(1+i\nu)\Gamma\left(\frac{d}{2}+i\nu\right)}_{\text{FROM WILSON LINES}}\right]^2$$

CONFORMAL PARTIAL WAVES

* COMPLETE BASIS

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* In terms of CONFORMAL BLOCKS

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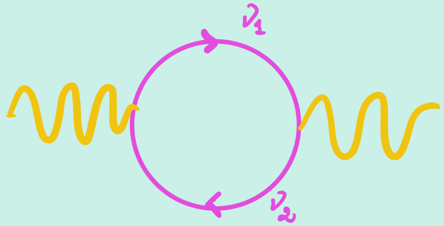
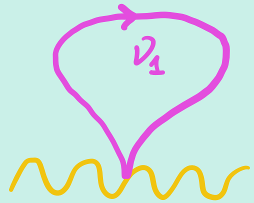
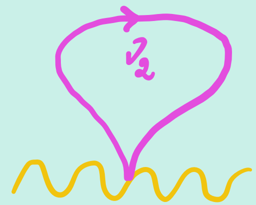
1 - LOOP RESULTS

$B^{(1)}_{\nu_1,\nu_2} =$ 

$+$ 

$+$ 

1-LOOP RESULTS

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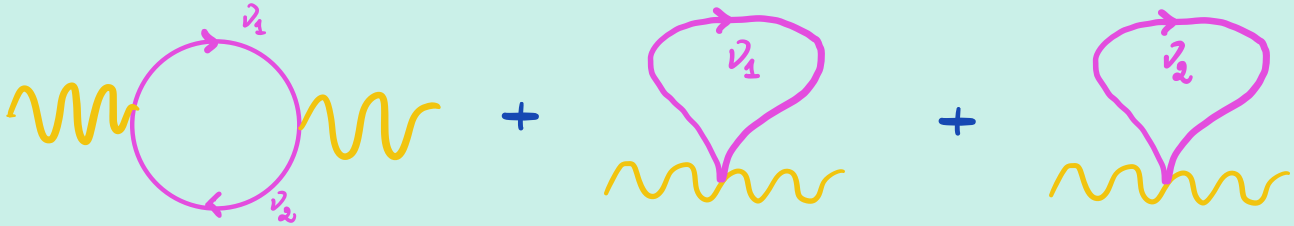
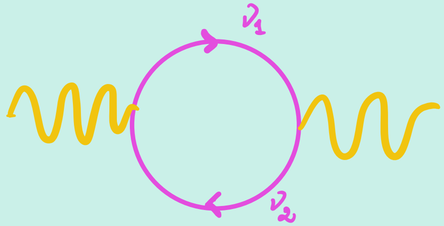


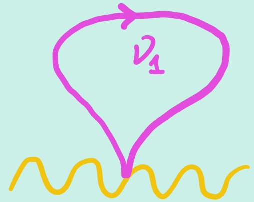
SPECTRAL REPRESENTATION

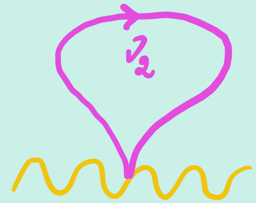
~ "FOURIER TRANSFORM"

$$\langle A_\alpha A_\beta \rangle = \int d\lambda \, \frac{1}{\lambda^2 + \left(\frac{d}{2} - 1\right)^2 - \frac{e^2}{2} B_\nu} \, \Omega^{(1)}_{\lambda;\alpha\beta}$$

1-LOOP RESULTS

$B^{(1)}_{\nu_1,\nu_2} =$ 

$+$ 

$+$ 

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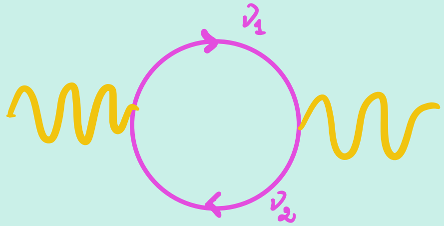
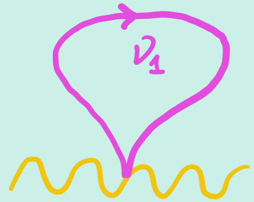
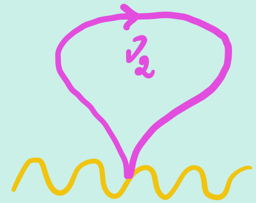
⚠ UV-DIVERGENT:

* DIM-REG ⊕ CHARGE RENORMALIZATION

 $= \frac{1}{4}(Z_3 - 1) F_{\mu\nu} F^{\mu\nu}$

1-LOOP RESULTS

$B^{(1)}_{\nu_1,\nu_2}$ =  +  + 

SPECTRAL REPRESENTATION

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$$\langle A_\alpha A_\beta \rangle = \int d\lambda \, \frac{1}{\lambda^2 + \left(\frac{d}{2}-1\right)^2 - \frac{e^2}{2} B_\nu} \, \Omega^{(1)}_{\lambda;\alpha\beta}$$

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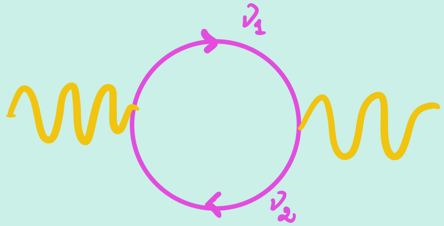
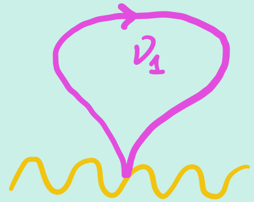
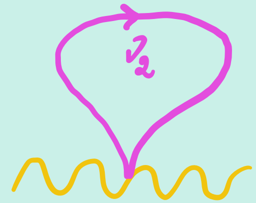
 $= \frac{1}{4}(Z_3 - 1) F_{\mu\nu} F^{\mu\nu}$

$$\boxed{(Z_3 - 1)_\infty = \frac{e^2}{48\pi^2} \cdot \frac{1}{d-3}}$$

1 - LOOP RESULTS

$B^{(1)}_{\nu_1,\nu_2} =$  $+$  $+$ 

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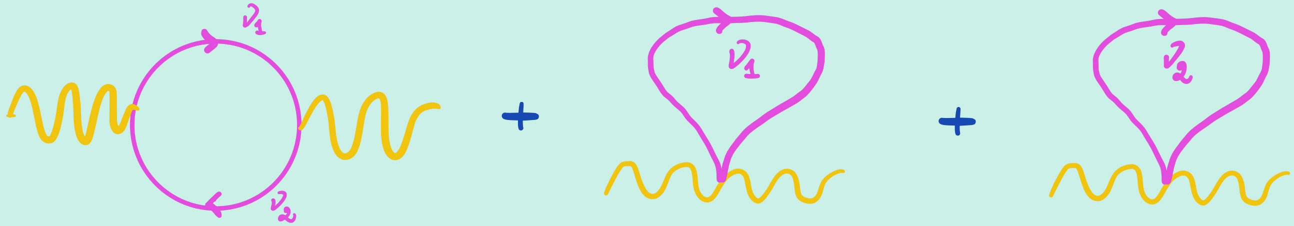
✓ SAME β FUNCTION AS FLAT SPACE : UV PHYSICS DOESN'T SEE CURVATURE

1 - LOOP RESULTS

$B^{(1)}_{\nu_1,\nu_2} =$



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FINITE PART

* NEUMANN PHOTON : $-\frac{e^2}{2} B^{REN}_{\nu,\nu}(\lambda) \Big|_{\lambda = \pm i\left(\frac{d}{2}-1\right)} = 0$

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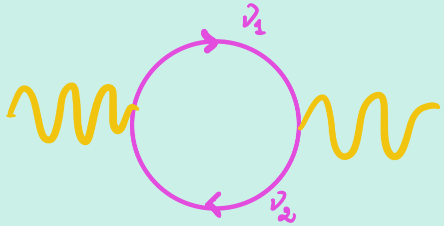
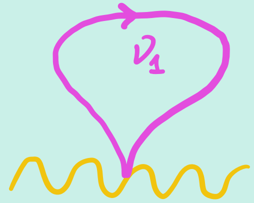
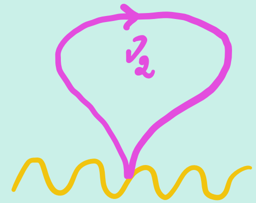
$B^{(1)}_{\nu_1,\nu_2} =$ 

$+$ 

$+$ 

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$\Downarrow$

NO POLE SHIFT $\Longleftrightarrow$ GAUGE INVARIANCE

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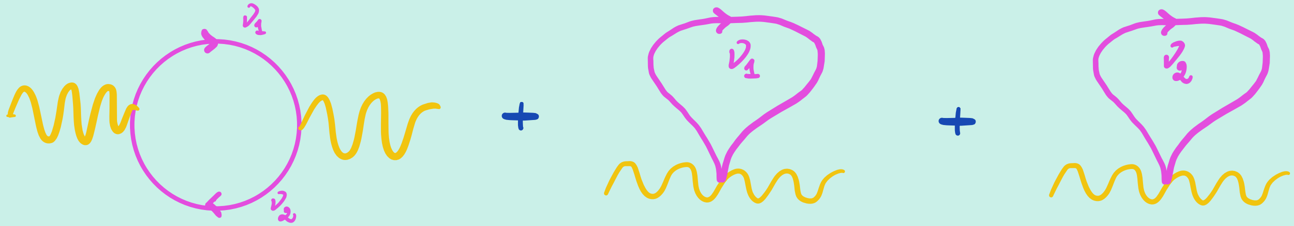

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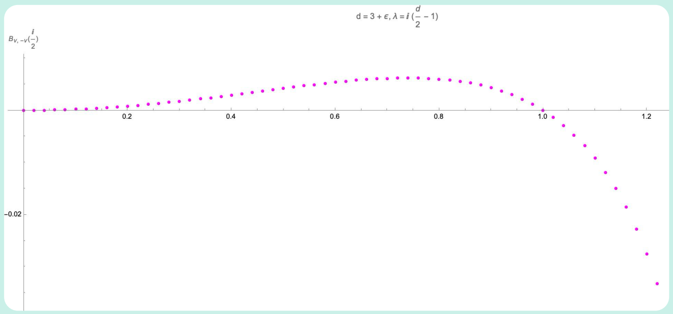

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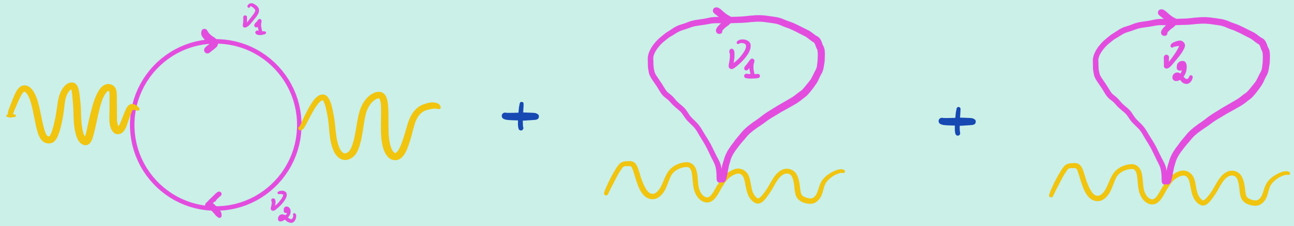

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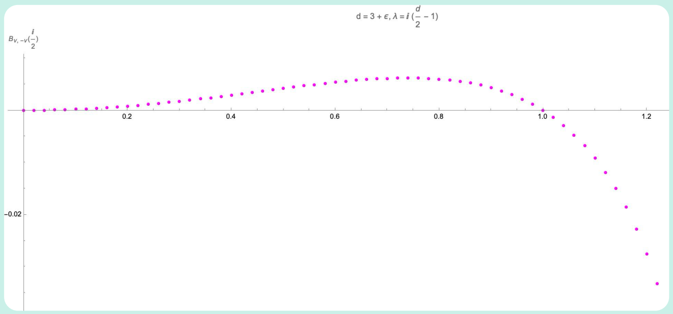

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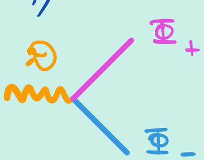
$\mathcal{L}_{INT} \supseteq -2ie \, A_{\mathcal{D},\mu} \cdot J^\mu_{+,-}$



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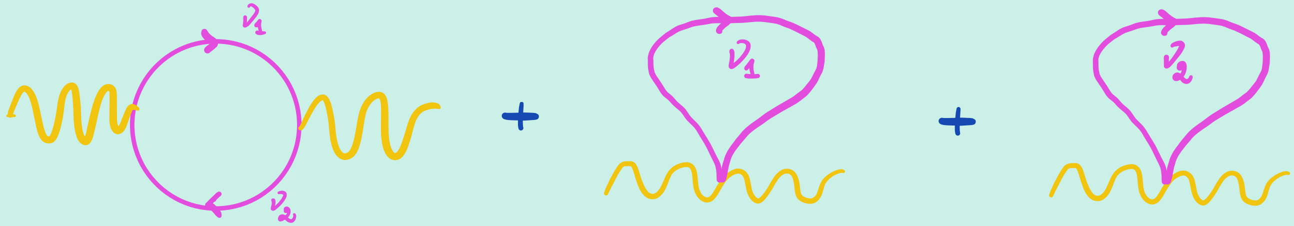
$$B^{(1)}_{\nu_1,\nu_2} = $$ 

SPECTRAL REPRESENTATION

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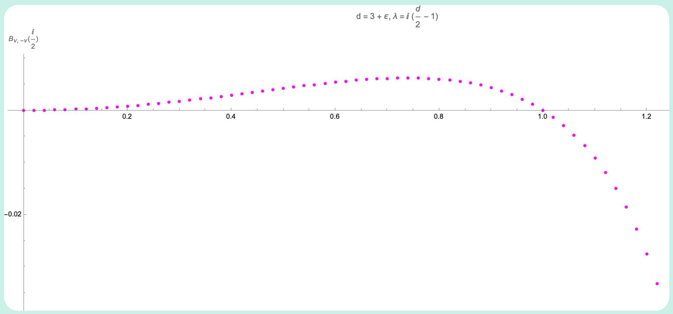

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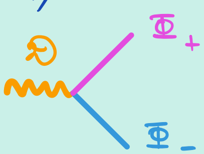
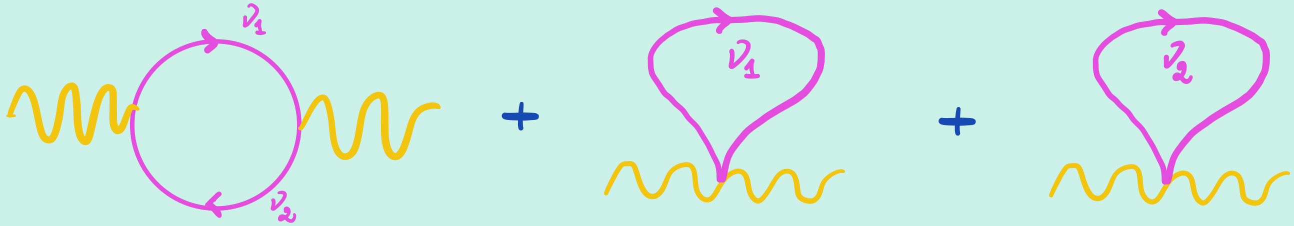


$\Rightarrow$

SPONTANEOUS BREAKING of BULK GAUGE SYMM.

[EXPLICIT BREAKING of BOUNDARY THEORY]

1 – LOOP RESULTS

$B^{(1)}_{\nu_1,\nu_2} =$ 

SPECTRAL REPRESENTATION

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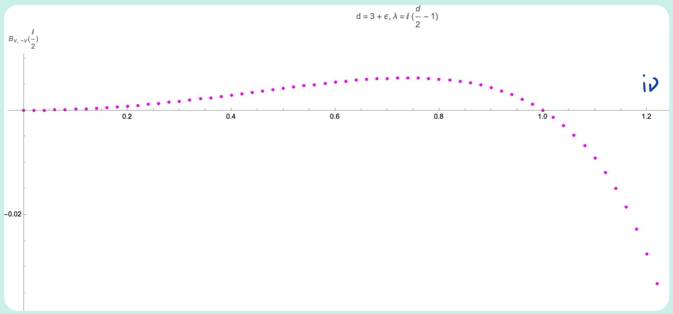

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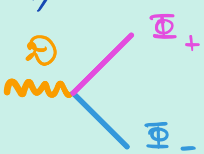


$\Rightarrow$

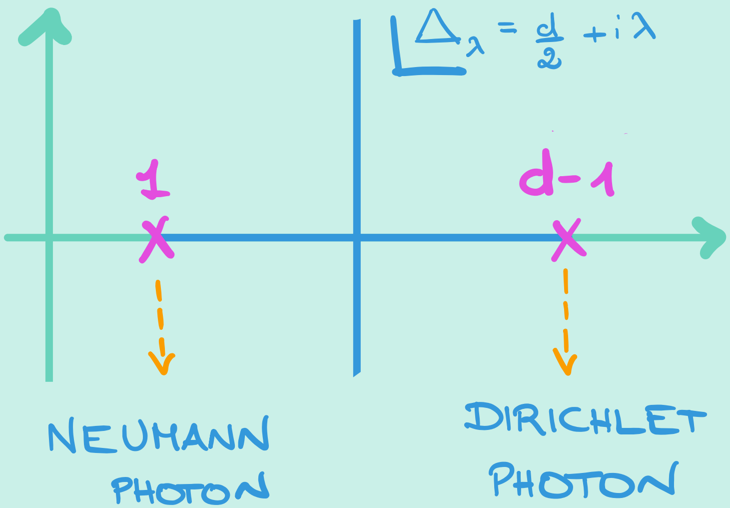
SPONTANEOUS BREAKING of BULK GAUGE SYMM.

[EXPLICIT BREAKING of BOUNDARY THEORY]

DIRICHLET PHOTON ACQUIRES A MASS VIA A HIGGS MECHANISM

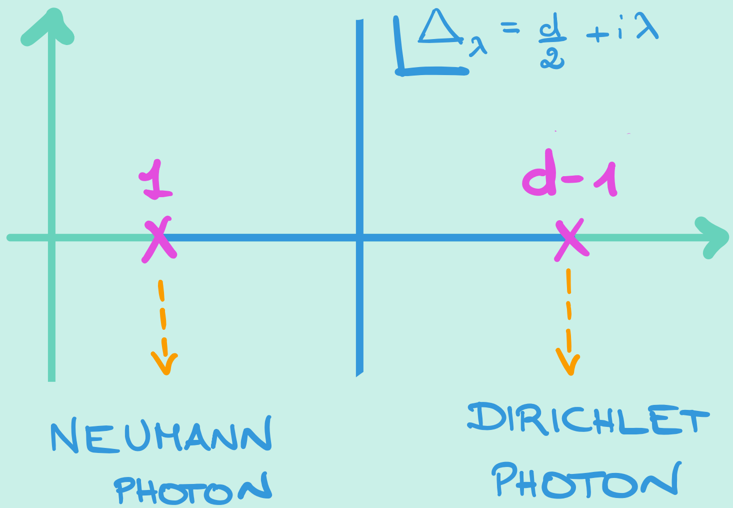
OVERVIEW & SUMMARY

dS: $A_\mu(\eta,\vec{x}) \sim \eta\, a_{N,\mu}(\vec{x}) + \eta^{d-1+e^2\Delta_A}\, a_{D,\mu}(\vec{x})$



OVERVIEW & SUMMARY

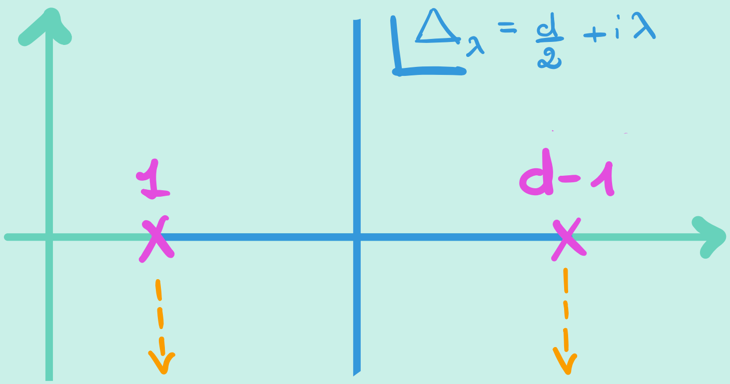
dS: $A_\mu(\eta, \vec{x}) \sim \eta \cdot a_{N,\mu}(\vec{x}) + \eta^{d-1+e^2\Delta_A} a_{D,\mu}(\vec{x})$



- We can build gauge invariant observables with Wilson lines

OVERVIEW & SUMMARY

dS: $A_\mu(\eta,\vec{x}) \sim \eta\, a_{N,\mu}(\vec{x}) + \eta^{d-1+e^2\Delta_A}\, a_{D,\mu}(\vec{x})$



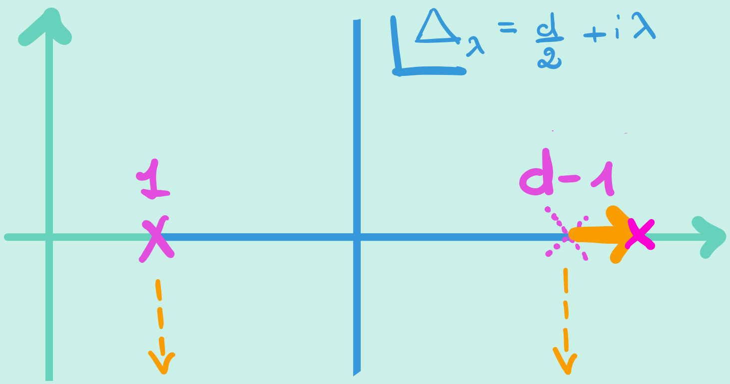
NEUMANN PHOTON → GAUGE INVARIANCE ⇓ NO 1-LOOP CORRECTIONS

DIRICHLET PHOTON

- WE CAN BUILD GAUGE INVARIANT OBSERVABLES WITH WILSON LINES
- GAUGE INVARIANCE PROTECTS THE LEADING MODE @ LATE TIME

OVERVIEW & SUMMARY

dS: $A_\mu(\eta,\vec{x}) \sim \eta\, a_{\mathcal{N},\mu}(\vec{x}) + \eta^{d-1+e^2\Delta_A}\, a_{\mathcal{D},\mu}(\vec{x})$



NEUMANN PHOTON

$\Downarrow$

GAUGE INVARIANCE

$\Downarrow$

NO 1-LOOP CORRECTIONS

DIRICHLET PHOTON

$\Downarrow$

SPONTANEOUS BREAKING of GAUGE U(1)

$\Downarrow$

HIGGS-LIKE MECHANISM IN AdS

- We can build gauge invariant observables with Wilson lines
- Gauge invariance protects the leading mode @ late time
- Next-to-leading mode at late time acquires an anomalous dimension

OVERVIEW & SUMMARY

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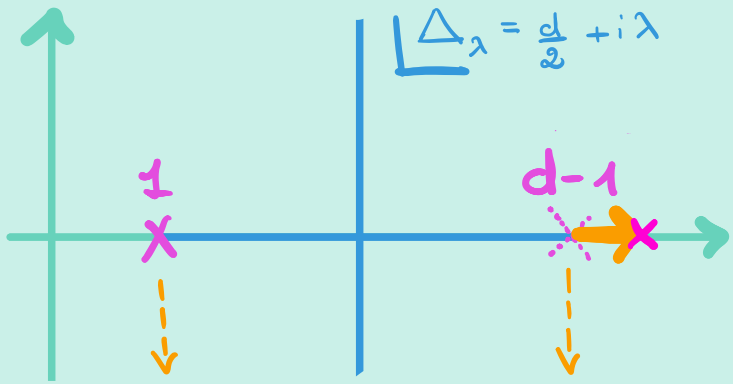

NEUMANN PHOTON

$\downarrow$

GAUGE INVARIANCE

$\Downarrow$

NO 1-LOOP CORRECTIONS

$\downarrow$

MAGNETIC FIELD $\vec{B}$ @ LATE TIME

$\downarrow$

UNSCREENED IN PLASMA

DIRICHLET PHOTON

$\downarrow$

SPONTANEOUS BREAKING of GAUGE U(1)

$\Downarrow$

HIGGS-LIKE MECHANISM IN AdS

- WE CAN BUILD GAUGE INVARIANT OBSERVABLES WITH WILSON LINES
- GAUGE INVARIANCE PROTECTS THE LEADING MODE @ LATE TIME
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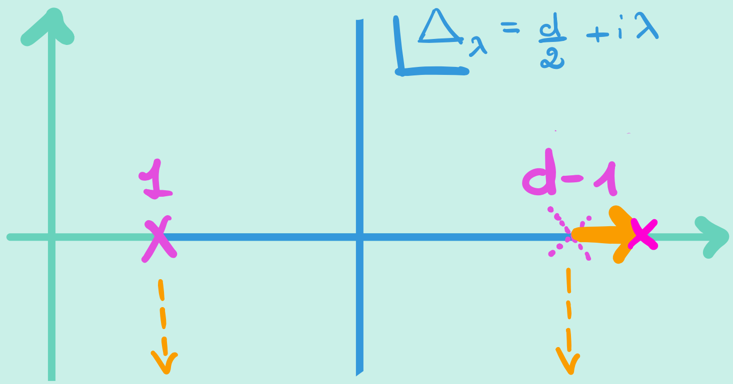
OVERVIEW & SUMMARY

dS: $A_\mu(\eta,\vec{x}) \sim \eta\, a_{\mathcal{N},\mu}(\vec{x}) + \eta^{d-1+e^2\Delta_A}\, a_{\mathcal{D},\mu}(\vec{x})$



NEUMANN PHOTON

GAUGE INVARIANCE

$\Downarrow$

NO 1-LOOP CORRECTIONS

MAGNETIC FIELD $\vec{B}$ @ LATE TIME

$\downarrow$

UNSCREENED IN PLASMA

DIRICHLET PHOTON

SPONTANEOUS BREAKING of GAUGE U(1)

$\Downarrow$

HIGGS-LIKE MECHANISM IN AdS

ELECTRIC FIELD $\vec{E}$ @ LATE TIME

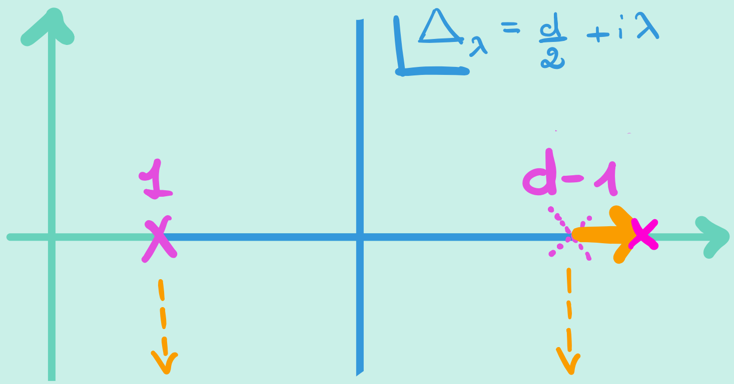
$\downarrow$

UNDERGOES DEBYE SCREENING IN PLASMA of CHARGED PARTICLES

- WE CAN BUILD GAUGE INVARIANT OBSERVABLES WITH WILSON LINES
- GAUGE INVARIANCE PROTECTS THE LEADING MODE @ LATE TIME
- NEXT-TO-LEADING MODE AT LATE TIME ACQUIRES AN ANOMALOUS DIMENSION
- IT CAN BE INTERPRETED AS THE DEBYE SCREENING GENERATED BY THE PLASMA OF CHARGED φ PARTICLES

OVERVIEW & SUMMARY

$$dS: \quad A_\mu(\eta,\vec{x}) \sim \eta\, a_{\mathcal{N},\mu}(\vec{x}) + \eta^{d-1+e^2\Delta_A}\, a_{\mathcal{D},\mu}(\vec{x})$$



NEUMANN PHOTON

GAUGE INVARIANCE

⇓

NO 1-LOOP CORRECTIONS

MAGNETIC FIELD $\vec{B}$ @ LATE TIME

↓

UNSCREENED IN PLASMA

DIRICHLET PHOTON

SPONTANEOUS BREAKING of GAUGE U(1)

⇓

HIGGS-LIKE MECHANISM IN AdS

ELECTRIC FIELD $\vec{E}$ @ LATE TIME

↓

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Thank you very much!

PROPAGATORS

$$\Pi^{(1)}_{dS}(y_1,y_2;\,W_1,W_2) = \oint_{\lambda=\pm i\left(\frac{d}{2}-1\right)} d\lambda\, \frac{1}{\lambda^2+\left(\frac{d}{2}-1\right)^2}\,\Omega^{(1)}_{\lambda}(y_1\cdot y_2;\,W_1,W_2) + \xi \oint_{\lambda=\pm i\frac{d}{2}} d\lambda\,(W_1\cdot\nabla_1)(W_2\cdot\nabla_2)\,\Omega^{(0)}_{\lambda}(y_1\cdot y_2)$$

$$\Pi^{(1);\,AdS}_{\mathcal{D}}(x_1,x_2;\,W_1,W_2) = \int_{-\infty}^{+\infty} d\lambda\, \frac{1}{\lambda^2+\left(\frac{d}{2}-1\right)^2}\,\Omega^{(1)}_{\lambda}(x_1\cdot x_2;\,W_1,W_2) + \xi \int_{-\infty}^{+\infty} d\lambda\,(W_1\cdot\nabla_1)(W_2\cdot\nabla_2)\,\Omega^{(0)}_{\lambda}(x_1\cdot x_2)$$

$$\Pi^{(1);\,AdS}_{\mathcal{N}}(x_1,x_2;\,W_1,W_2) = \int_{\mathbb{R}+\circlearrowleft} d\lambda\, \frac{1}{\lambda^2+\left(\frac{d}{2}-1\right)^2}\,\Omega^{(1)}_{\lambda}(x_1\cdot x_2;\,W_1,W_2) + \xi \int_{\mathbb{R}+\circlearrowleft} d\lambda\,(W_1\cdot\nabla_1)(W_2\cdot\nabla_2)\,\Omega^{(0)}_{\lambda}(x_1\cdot x_2)$$

$$\lim_{u\to+\infty} \Pi^{(1);\,AdS}_{\mathcal{D}} = -\frac{\Gamma\left(\frac{d+1}{2}\right)}{2\pi^{\frac{d+1}{2}}}\cdot\frac{1}{d-2}\cdot\frac{1}{u^d}\,\frac{\partial^2 u}{\partial x_1^\mu\,\partial x_2^\nu} + \frac{\Gamma\left(\frac{d+1}{2}\right)}{2\pi^{\frac{d+1}{2}}}\cdot\frac{1}{d-2}\cdot\frac{1}{u^{d+1}}\,\frac{\partial u}{\partial x_1^\mu}\,\frac{\partial u}{\partial x_2^\nu}$$

$$\lim_{u\to+\infty} \Pi^{(1);\,AdS}_{\mathcal{N}} = \frac{\Gamma\left(\frac{d-2}{2}\right)}{4\pi^{\frac{d+2}{2}}}\sin\left(\frac{\pi d}{2}\right)\cdot\frac{1}{u}\left[\log\frac{u}{2}+\#\right]\frac{\partial^2 u}{\partial x_1^\mu\,\partial x_2^\nu} - \frac{\Gamma\left(\frac{d-2}{2}\right)}{4\pi^{\frac{d+2}{2}}}\sin\left(\frac{\pi d}{2}\right)\cdot\frac{1}{u}\left[\log\frac{u}{2}+\#\right]\frac{\partial u}{\partial x_1^\mu}\,\frac{\partial u}{\partial x_2^\nu}$$

LIMIT $x_1 \to \partial AdS\,/\,\partial dS$

$z_1 \longrightarrow 0$

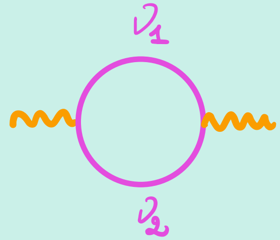
	"Dirichlet"	"Neumann"
$\langle E_j(x_1)\,E_k(x_2)\rangle$	$\mathcal{O}(z_1^{d-3})$	$\mathcal{O}(z_1)$
$\langle B_j(x_1)\,B_k(x_2)\rangle$	$\mathcal{O}(z_1^{d-2})$	$\mathcal{O}(z_1^0)$

$\Longrightarrow$

$\langle EE\rangle \gg \langle BB\rangle$ for DIRICHLET

$\langle BB\rangle \gg \langle EE\rangle$ for Neumann

THE U(1) CURRENT 2-POINT FUNCTION



$$J^{\mu}_{\nu_1,\nu_2} = \Phi_{\nu_1} \nabla^{\mu} \Phi_{\nu_2} - \Phi_{\nu_2} \nabla^{\mu} \Phi_{\nu_1}$$

$$\langle JJ\rangle_{\nu_1,\nu_2}(\lambda) = \sum_{n=0}^{+\infty} \frac{8\pi a^{d+2}_{\nu_1,\nu_2}(n)}{2i\nu_n^-} \left[\frac{1}{i(\lambda-\nu_n^-)} + \frac{1}{i(-\lambda-\nu_n^-)}\right] \tag{5.3}$$

$$\nu_n^{\pm} = \pm i \left(\frac{d}{2} + i\nu_1 + i\nu_2 + 2n + 1\right) \tag{5.4}$$

$$a^{d+2}_{\nu_1,\nu_2}(n) = \frac{\left(\frac{d+2}{2}\right)_n (1+i\nu_1+i\nu_2+n)_n \,(2n+2+d+i\nu_1+i\nu_2)_{-\frac{d}{2}}}{2\pi^{\frac{d+2}{2}} n! \left(\frac{d}{2}+i\nu_1+n+1\right)_{-\frac{d}{2}} \left(\frac{d}{2}+i\nu_2+n+1\right)_{-\frac{d}{2}} \left(\frac{d}{2}+i\nu_1+i\nu_2+n+1\right)_n} \tag{5.5}$$

$$\begin{aligned} \langle JJ\rangle^{(1),\perp}_{\nu,\nu}(\lambda) = {} & \frac{4}{i\pi^{\frac{d}{2}}\lambda} \cdot \frac{1}{\lambda^2 + \left(\frac{d}{2}+2i\nu+1\right)^2} \cdot \frac{(d+2i\nu+2)_{-\frac{d}{2}}}{\left(\frac{d}{2}+i\nu+1\right)^2_{-\frac{d}{2}}} \cdot \\ & \left[\left(\frac{d}{2} - i\lambda + 2i\nu + 1\right) {}_6F_5\left(\begin{matrix} \frac{d}{2}+1, i\nu+\frac{1}{2}, \frac{d}{4}+i\nu+\frac{3}{2}, \frac{d}{2}+i\nu+1, \frac{d}{4}+\frac{i\lambda}{2}+i\nu+\frac{1}{2}, \frac{d}{2}+2i\nu+1 \\ i\nu+1, \frac{d}{4}+i\nu+\frac{1}{2}, \frac{d}{2}+i\nu+\frac{3}{2}, \frac{d}{4}+\frac{i\lambda}{2}+i\nu+\frac{3}{2}, 2i\nu+1 \end{matrix} ;1\right)\right. \\ & \left. - \left(\frac{d}{2} + i\lambda + 2i\nu + 1\right) {}_6F_5\left(\begin{matrix} \frac{d}{2}+1, i\nu+\frac{1}{2}, \frac{d}{4}+i\nu+\frac{3}{2}, \frac{d}{2}+i\nu+1, \frac{d}{4}+i\nu-\frac{i\lambda}{2}+\frac{1}{2}, \frac{d}{2}+2i\nu+1 \\ i\nu+1, \frac{d}{4}+i\nu+\frac{1}{2}, \frac{d}{2}+i\nu+\frac{3}{2}, \frac{d}{4}+i\nu-\frac{i\lambda}{2}+\frac{3}{2}, 2i\nu+1 \end{matrix} ;1\right)\right] \end{aligned} \tag{E.4}$$

$$\begin{aligned} \langle JJ\rangle_{\nu,-\nu}(\lambda) = {} & \frac{1}{(4\pi)^{\frac{d+1}{2}}} \Gamma\left(\frac{d}{2} - i\nu + 1\right) \Gamma\left(\frac{d}{2} + i\nu + 1\right) \cdot \\ & \left[\Gamma\left(\frac{1}{4}(d+2i\lambda+2)\right) {}_5\tilde{F}_4\left(\begin{matrix} \frac{1}{2}, \frac{d+2}{2}, \frac{1}{2}(d+2i\nu+2), \frac{1}{2}(d-2i\nu+2), \frac{1}{4}(d+2i\lambda+2) \\ \frac{d+3}{2}, i\nu+1, 1-i\nu, \frac{1}{4}(d+2i\lambda+6) \end{matrix} ;1\right)\right. \\ & \left. + \Gamma\left(\frac{1}{4}(d-2i\lambda+2)\right) {}_5\tilde{F}_4\left(\begin{matrix} \frac{1}{2}, \frac{d+2}{2}, \frac{1}{2}(d+2i\nu+2), \frac{1}{2}(d-2i\nu+2), \frac{1}{4}(d-2i\lambda+2) \\ \frac{d+3}{2}, i\nu+1, 1-i\nu, \frac{1}{4}(d-2i\lambda+6) \end{matrix} ;1\right)\right] \end{aligned} \tag{E.5}$$

Dimensional Regularization of $B_{\nu_1,\nu_2}(\lambda)$

$$ {}_{q+1}F_q\left(\begin{matrix} a_1,\dots,a_{q+1} \\ b_1,\dots,b_q \end{matrix};1\right) = -\frac{1}{Z}\sum_{j=1}^{q}\frac{\Pi_{k=1}^{q+1}(b_j-a_k)}{b_j\Pi_{k=1,k\neq j}^{q}(b_j-b_k)}\,{}_{q+1}F_q\left(\begin{matrix} a_1,\dots,a_{q+1} \\ b_1,\dots,b_j+1,\dots,b_q \end{matrix};1\right), $$

[CACCIATORI, EPSTEIN, MOSCHELLA 2024]

CONVERGENT FOR

$\mathrm{Re}\{Z\} = \mathrm{Re}\{\sum a_i - \sum b_i\} < 0$

CONVERGENT FOR

$\mathrm{Re}\{Z'\} < 0 \iff \mathrm{Re}\{Z\} < 1$

$\Rightarrow$ $\boxed{B_{\nu_1,\nu_2}}$ CONVERGENT UP TO $d<0$

APPLY RECURSION RELATION 4 TIMES

$$ \langle JJ\rangle^{(1),\perp}_{\nu,\nu}(\lambda) = \frac{1}{d(d-1)(d-2)(d-3)}\cdot\langle\widetilde{JJ}\rangle^{(1),\perp}_{\nu,\nu}(\lambda), $$
$$ \langle JJ\rangle^{(1),\perp}_{\nu,-\nu}(\lambda) = \frac{1}{(d-1)(d-2)(d-3)}\cdot\langle\widetilde{JJ}\rangle^{(1),\perp}_{\nu,-\nu}(\lambda), $$

$\Downarrow$

DIVERGENT PART CAN BE EXTRACTED ANALYTICALLY AND IS "SIMPLE"

$$ \langle JJ\rangle^{(1),\perp}_{\nu_1,\nu_2}(\lambda)\Big|_{d=3+\epsilon} = \frac{a\lambda^2 + c(\nu_1,\nu_2)}{d-3} \qquad a = -\frac{1}{24\pi^2}. $$

Finite part analytically is a combination of 1350 HYPERGEOMETRIC FUNCTIONS

$\Downarrow$

WE ONLY EVALUATE IT NUMERICALLY