Effective field theory in de Sitter space and the Method of Regions

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with Martin Beneke, Patrick Hager
based on arXiv:2312.06766 and work in progress

Cosmological Correlators in Taiwan

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Observables are in-in correlation functions at equal and late times:

$$\lim_{k_i/(a(t)H) \to 0} \langle \phi(t, \boldsymbol{k}_1) ... \phi(t, \boldsymbol{k}_n) \rangle \,.$$

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$\Rightarrow$ Standard perturbation theory is ill-defined, it fails due to superhorizon field modes with

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$$\phi(t,\boldsymbol{k}) = \underbrace{H\Big[\varphi_+(t,\boldsymbol{k}) + [a(t)H]^{-3}\varphi_-(t,\boldsymbol{k})\Big]}_{\text{EFT fields, } k/\Lambda(t)<1} + \underset{\substack{\uparrow \\ k/\Lambda(t)>1}}{\phi_{\mathsf{UV}}}(t,\boldsymbol{k}) \,,$$

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ϕ_{UV} is integrated out, its effects are captured by Wilson coefficients and non-Gaussian initial conditions (IC's).

Matching SdSET to the full theory

To determine the IC's and Wilson coefficients need to carry out matching computations, schematically:

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Caveat: For the method to work, need to use an analytic or dimensional regulator.

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In each region can expand the integrand in the quantities which are small, sum of all regions reproduces expansion of the full result [Beneke, Hager, AFS 2023].

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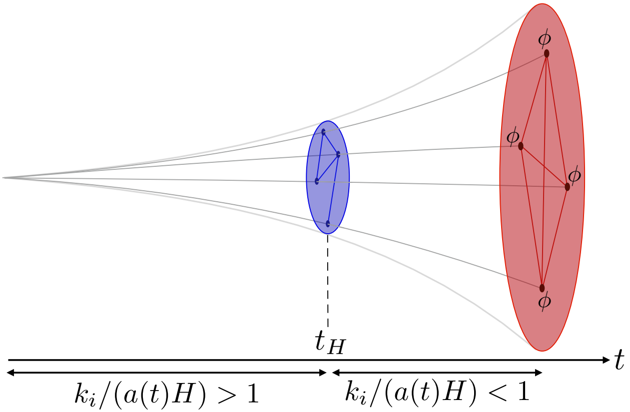
Thank you for your attention!

Backup slides

Physical picture for late-time correlators

$$\lim_{k_i/(a(t)H)\to 0} \langle \phi(t,\boldsymbol{k}_1)\ldots\phi(t,\boldsymbol{k}_n)\rangle \,.$$



- Start at $t = -\infty$, subhorizon evolution.
- Horizon crossing at t_H, where

$$\frac{k_i}{a(t_H)H} \sim 1 \,.$$

- Superhorizon evolution, correlator measured at fixed time t.