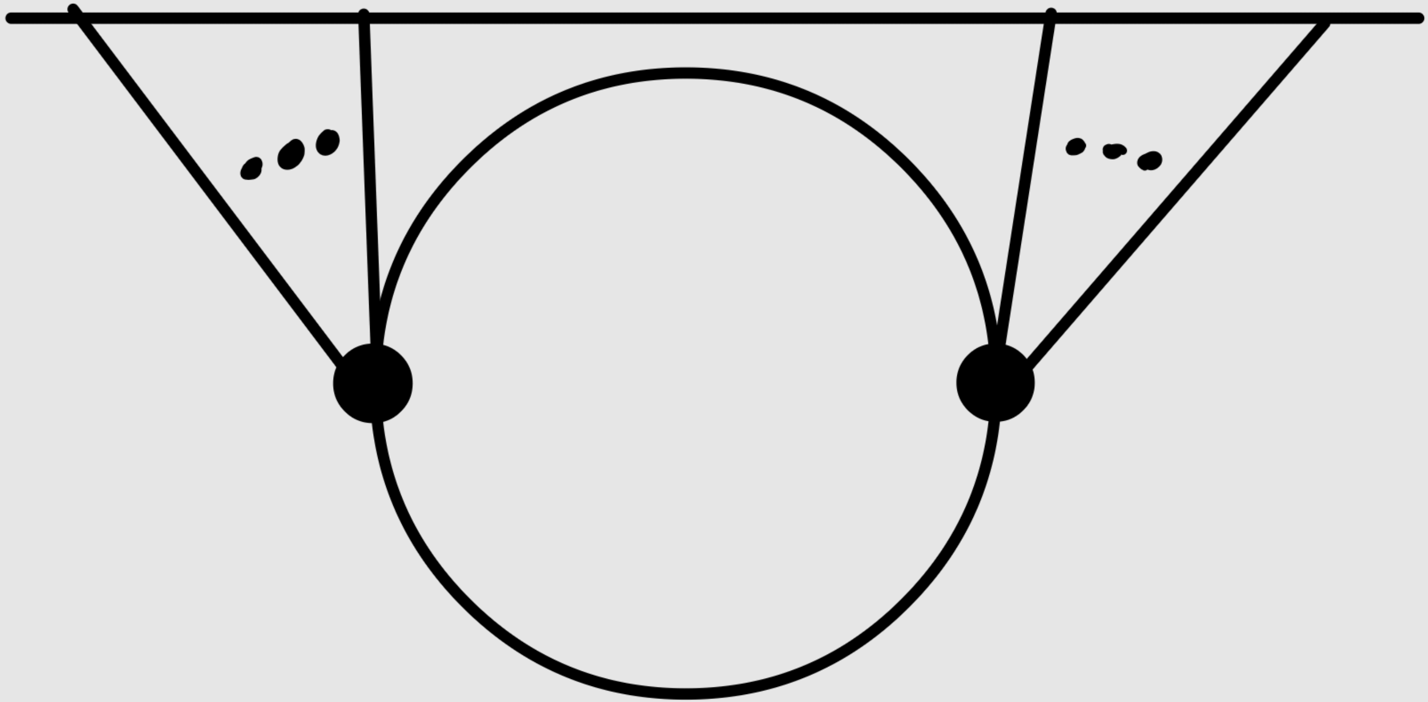
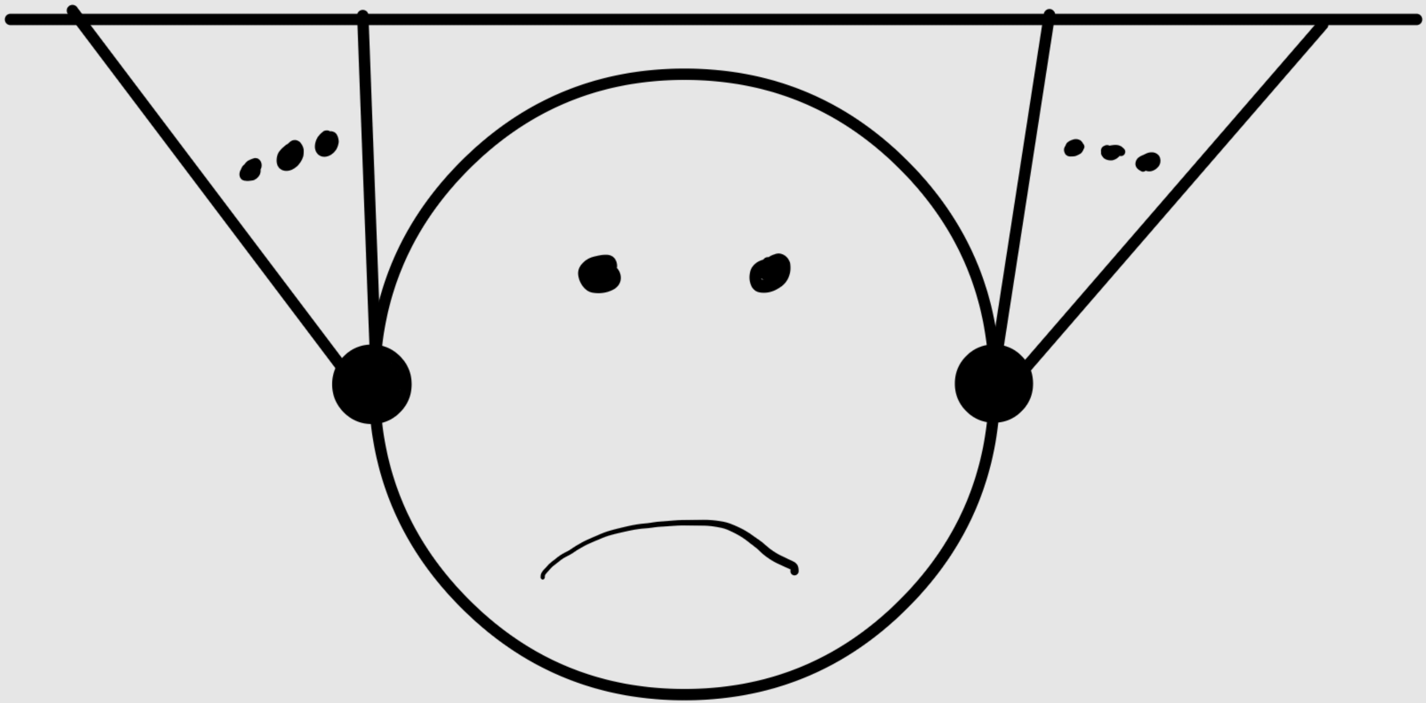
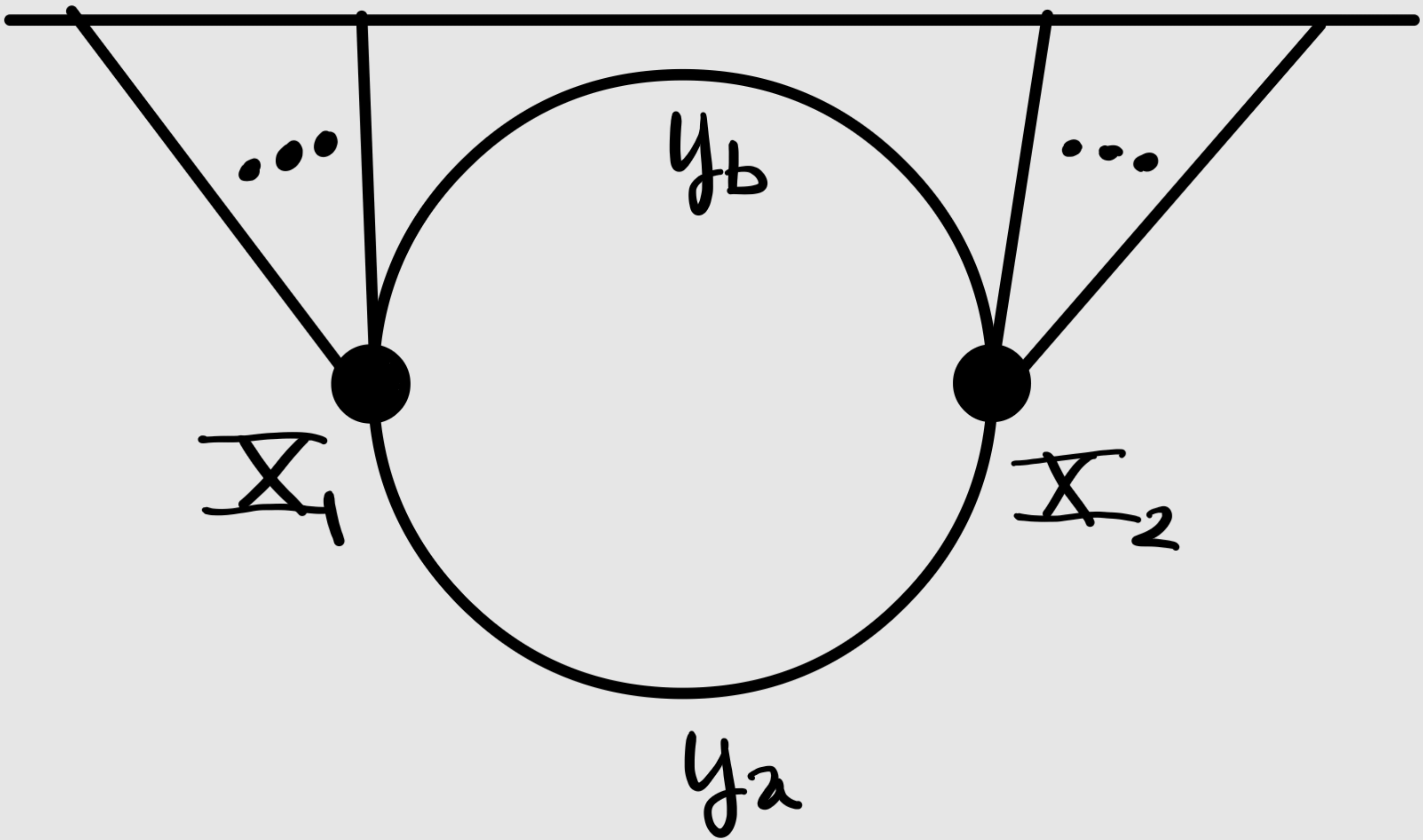
The Bubble



The Bubble



The Bubble



Bubble

- FRW power-law : ε_C

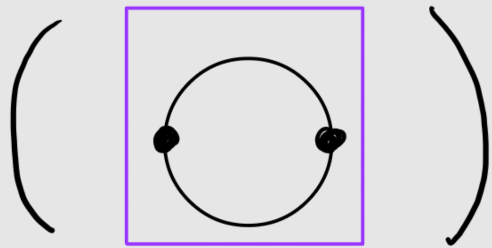
- Dim-reg : ε_D

$$\psi_{\varepsilon_C,\varepsilon_D} = \int dx_1 dx_2 (x_1 x_2)^{\varepsilon_C} \int dy_a dy_b \left(f(y_a, y_b)\right)^{\varepsilon_D} y_a y_b \times$$

$$\boxed{\frac{1}{X_1+x_1+X_2+x_2}\,\frac{1}{X_1+x_1+y_a+y_b}\,\frac{1}{X_2+x_2+y_a+y_b}\left(\frac{1}{X_1+x_1+X_2+x_2+2y_a}+\frac{1}{X_1+x_1+X_2+x_2+2y_b}\right)}$$

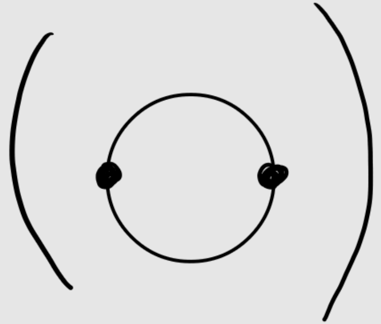
Bubble

$$\psi_{\varepsilon_C,\varepsilon_D} = \int dx_1 dx_2 \left(x_1 x_2\right)^{\varepsilon_C} \int dy_a dy_b \left(f(y_a, y_b)\right)^{\varepsilon_D} y_a y_b \;\times$$



First approach

$$\psi^{flat}_{\varepsilon_D} = \int dy_a \, dy_b \left(f(y_a, y_b) \right)^{\varepsilon_D} y_a \, y_b \times$$



Differential equations $\psi^{flat}_{\varepsilon_D}$

$$d\vec{I}_{\varepsilon_D} = \varepsilon_D A \cdot \vec{I}_{\varepsilon_D}$$

5 Master integrals

Solution

$$\psi^{flat}_{\epsilon_D \to 0} = \frac{1/\epsilon_D + \log\text{'s}}{X_1 + X_2} + Li_2 + \cdots$$
$$+ \mathcal{O}(\epsilon_D)$$

Cosmo Bubble

$$\psi_{\varepsilon_C,\varepsilon_D \to \phi} = \int dx_1 dx_2 (x_1 x_2)^{\varepsilon_C} \left[\psi^{flat}_{\varepsilon_D \to \phi} \right]$$

$$= \text{Mathematica} \left[\psi^{flat}_{\varepsilon_D \to \phi} \right]$$

Not enough

$$\psi_{\varepsilon_C \to 0,\, \varepsilon_D \to 0} = \#\left(\frac{1}{\varepsilon_C^{\,2}} - \frac{1}{\varepsilon_C\,\varepsilon_D}\right) + \mathcal{O}\left(\frac{1}{\varepsilon_C}\right)$$

$$\boxed{\text{What if} \quad \varepsilon_C \sim \varepsilon_D \quad ?}$$

$$\int dx_1 dx_2 \left(x_1 x_2\right)^{\varepsilon_C} \mathcal{O}\left(\varepsilon_D\right) \sim \frac{\mathcal{O}(\varepsilon_D)}{\varepsilon_C^{\,2}} \sim \frac{1}{\varepsilon_C}$$

$$\boxed{\text{What if } \quad \varepsilon_c \sim \varepsilon_D \quad ?}$$

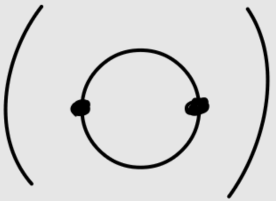
$$\psi^{flat}_{\varepsilon_D \to 0} = \frac{1/\varepsilon_D}{X_1 + X_2} + \mathcal{O}(1) + \mathcal{O}(\varepsilon_D)$$

$$\int dx_1 dx_2 (x_1 x_2)^{\varepsilon_c}\, \mathcal{O}(\varepsilon_D) \sim \frac{\mathcal{O}(\varepsilon_D)}{\varepsilon_c^2} \sim \frac{1}{\varepsilon_c}$$

Preliminary Results

Differential equations for

$$\psi_{\varepsilon_{C_1}\varepsilon_D} = \int dx_1 dx_2 \left(x_1 x_2\right)^{\varepsilon_C} \int dy_a dy_b \left(f(y_a, y_b)\right)^{\varepsilon_D} y_a y_b$$



Preliminary Results

Equivalences for $\psi = \psi^{(1)} + \psi^{(2)}$:

$$\psi^{(1)}_{\varepsilon_{C_1}, \varepsilon_D \to 0} \xleftrightarrow{\text{DE}} \psi^{\varepsilon+1}_{2\text{-site}}$$

$$\psi^{(2)}_{\varepsilon_{C_1}, \varepsilon_D} \longleftrightarrow \psi^{\text{Strongly Coupled}}_{\varepsilon_1,\, \varepsilon_2}$$

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Conclusion

$$d\,\mathcal{I}_{\varepsilon_C,\varepsilon_D} = A_{\varepsilon_C,\varepsilon_D} \cdot \mathcal{I}_{\varepsilon_C,\varepsilon_D}$$

<u>not</u>

$$\psi_{\varepsilon_C,\varepsilon_D \to \phi} = \int dx_1 dx_2 (x_1 x_2)^{\varepsilon_C} \left[\psi^{flat}_{\varepsilon_D \to \phi} \right]$$