The Cosmological CPT Theorem

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Based on 2408.17406 with Ayngaran Thavanesan and Aron Wall

The CPT Theorem

CPT

The CPT Theorem

Charge Conjugation
(Replace Particles with Anti-Particles)

CPT

Time Reversal

The CPT Theorem

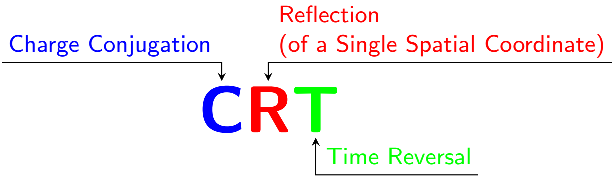
Parity
(Reflection in a Mirror)

Charge Conjugation

CPT

Time Reversal

The CPT Theorem



Reflection
(of a Single Spatial Coordinate)

Charge Conjugation

CRT

Time Reversal

The CPT Theorem

Charge Conjugation Reflection

CRT

Time Reversal

- **CRT** symmetry is guaranteed in any theory that is both Lorentz Invariant and Unitary

A Proof with Converses

Lorentz Invariance

Unitarity

A Proof with Converses

$$\begin{tikzcd}
\text{Lorentz Invariance} \arrow[d, Rightarrow] & \text{Unitarity} \arrow[d, Leftrightarrow] \\
\text{Rotational Invariance} & \text{Reflection Positivity}
\end{tikzcd}$$

A Proof with Converses

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180^\circ \text{ Rotation} & \text{Reflection Reality}
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A Proof with Converses

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\begin{array}{c} 180^\circ \text{ Rotation} \\ \mathcal{O}(\tau, x) = \mathcal{O}(-\tau, -x) \end{array} & \begin{array}{c} \text{Reflection Reality} \\ \mathcal{O}^*(\tau, x) = \mathcal{O}(-\tau, x) \end{array}
\end{tikzcd}$$

Euclidean
Field Operator

A Proof with Converses

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\begin{array}{c}\text{180° Rotation}\\ \mathcal{O}(\tau, x) = \mathcal{O}(-\tau, -x)\end{array} \arrow[dr, Leftrightarrow] & & \begin{array}{c}\text{Reflection Reality}\\ \mathcal{O}^*(\tau, x) = \mathcal{O}(-\tau, x)\end{array} \arrow[dl, Leftrightarrow] \\
& \begin{array}{c}\textbf{CRT}\\ \mathcal{O}^*(\tau, x) = \mathcal{O}(\tau, -x)\end{array} &
\end{tikzcd}$$

Accidental Unitarity

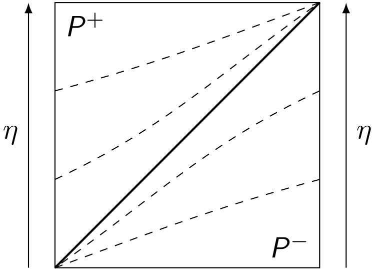
- We can now make converse statements
  - Lorentz Invariance $+$ Unitarity $\rightarrow$ **CRT**
  - **CRT** $+$ Lorentz Invariance $\rightarrow$ Reflection Reality $\neq$ Unitarity
  - **CRT** $+$ Unitarity $\rightarrow$ 180° Rotation $\neq$ Lorentz Invariance

Accidental Unitarity

- We can now make converse statements
    - Lorentz Invariance + Unitarity → **CRT**
    - **CRT** + Lorentz Invariance → Reflection Reality $\neq$ Unitarity
    - **CRT** + Unitarity → 180° Rotation $\neq$ Lorentz Invariance
- However, perturbations around a Unitary theory will remain Unitary provided they are Reflection Real

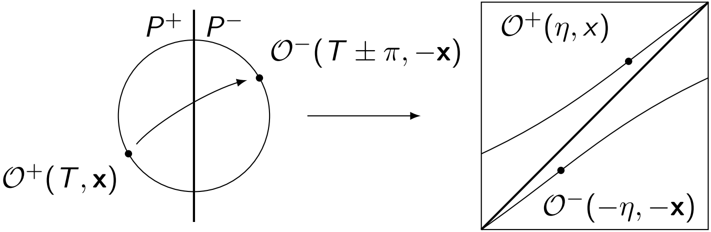
Cosmological CPT

- Expansion breaks time reversal in a single Poincaré patch
- Global dS inherits the embedding space symmetries which map between 2 patches
- Spacetime symmetries convert this into a local CRT symmetry



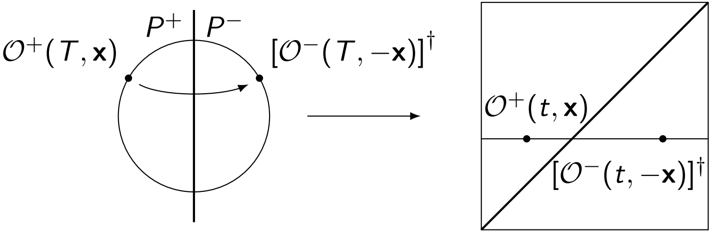
Symmetries

- Consider the 180° Euclidean rotation we used previously



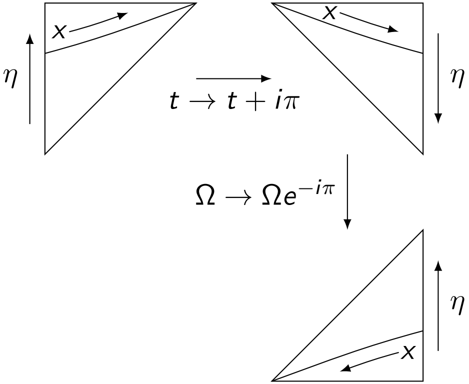
Bunch-Davies condition: $\mathcal{O}^-(-\eta, -\mathbf{x}) = \mathcal{O}^+(-\eta, -\mathbf{x})$

- Similarly, consider the **CRT** transformation



Relating the Patches

- Analytically continuing t in P^+ does not give P^-
- An extra complex Weyl transformation can relate P^+ and P^-



The Cosmological CPT Theorem

- To fully exploit these results we need an observable
- Here we deal with the wavefunction of the universe coefficients, evaluated on the future boundary

$$\Psi[\phi;\eta=0]=\exp\left[-\sum_{n=2}^{\infty}\int\prod_{a=1}^{n}d^d k_a\phi_{k_a}\,\psi_n(\mathbf{k})\,\hat{\delta}^d\left(\sum_a k_a\right)\right]$$

- These obey relationships which we label according to the symmetries:
  - **CRT**: $\psi_n^*(\mathbf{k})=e^{i\pi[(d+1)(L-1)-\sum_\alpha(\Delta_\alpha-d)]}\,\psi_n(\mathbf{k})$
  - **D**$^{\pm}$: $\psi_n(\mathbf{k})=e^{\mp i\pi[d+\sum_\alpha(\Delta_\alpha-d)]}\,\psi_n(\mathbf{k})$
- We can combine these together:
  - **D**$^-$**·CPT**: $\psi_n^*(\mathbf{k})=e^{i\pi((d+1)L-1)}\psi_n(e^{-i\pi}\mathbf{k})$

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  - **RR**: $\psi_n^*(\mathbf{k})=e^{i\pi((d+1)L-1)}\psi_n(e^{-i\pi}\mathbf{k})$

Conclusions

- There exists a group structure describing the relationship between **CPT**, Lorentz Invariance and Unitarity in both flat space and de Sitter
- At the boundary of inflation **CRT** symmetry fixes the phase of the wavefunction coefficients
- This is particularly interesting in the search for parity violation which depends exclusively on the imaginary part of the wavefunction coefficient
- Thank you for listening