Perturbative Unitarity Bounds for Cosmology

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Based on 2410.23709, with

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Cosmological Correlators in Taiwan
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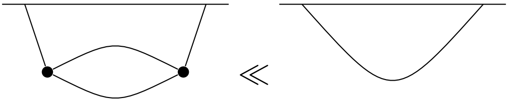
Outline

- Want to know when perturbation theory stops working
- There is a diagonstic in scattering amplitudes from perturbative unitarity.
- We can find a diagnostic for cosmological correlators.
- Some quick applications.

Perturbativity bounds



Estimates are inexact. Exact calculations are hard.

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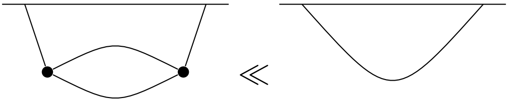

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Scattering amplitudes have partial wave bounds:

coefficients a_l with $|a_l| \leq 1$, $|\operatorname{Re} a_l| \leq 1/2$

Bounded because probabilities add up to 1.

Aplications to cosmology [Baumann & al. '16], [Grall & Melville '20], ...

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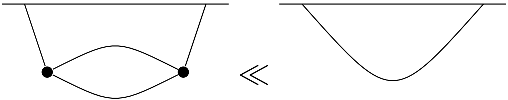

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But we compute correlators at the end of inflation.

Purity in quantum mechanics

System/environment split: $\mathcal{H} = \mathcal{H}_{\mathcal{S}} \otimes \mathcal{H}_{\mathcal{E}}$
Reduced density matrix $\rho_R = \mathrm{Tr}_{\mathcal{E}}\ \rho$.
Define the *purity* $\gamma := \mathrm{Tr}\left(\rho_R^2\right)/(\mathrm{Tr}\,\rho_R)^2$

$$0 \leq \gamma \leq 1$$

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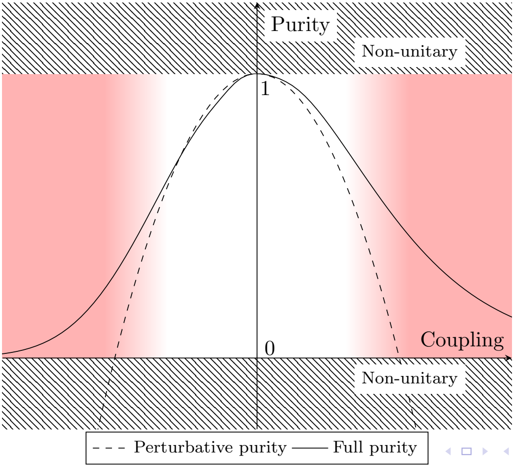
Scalar QFT in infinite volume: $\rho = |\Omega\rangle\,\langle\Omega|$
System: a *single Fourier mode* $\pm\mathbf{p}$,

$$\gamma(\mathbf{p}) = 1 - 2\int_{\mathbf{k}} \frac{\left|\psi_3(\mathbf{p},\mathbf{k},-\mathbf{p}-\mathbf{k})\right|^2}{2\mathrm{Re}\,\psi_2(\mathbf{p})\, 2\mathrm{Re}\,\psi_2(\mathbf{k})\, 2\mathrm{Re}\,\psi_2(\mathbf{p}+\mathbf{k})}\,.$$

Purity bounds

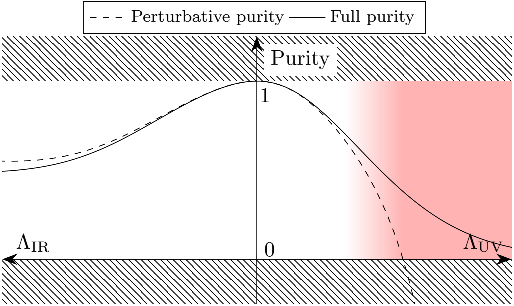
Vary coupling

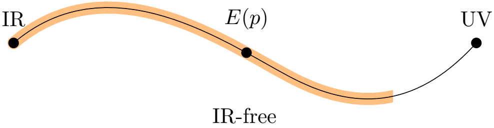
$$\gamma = 1 - 2g^2 I \qquad |g| \leq \frac{1}{\sqrt{2I}}$$



Purity bounds

Vary kinematics





Benchmarking the purity bound

Use the bound on some flat-space theories:

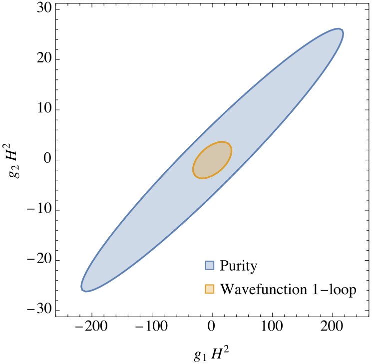
Theory		Partial waves	Purity
$g\phi^3/3!$	Massive	$g^2 < \frac{12\pi}{5} m^2$	$g^2 < 24^2 m^2$
	Massless		$g^2 \lesssim 23^2 \Lambda_{\rm IR}^2$
$g\,\phi\,\partial_\mu\phi\partial^\mu\phi/2$	Massive	$g^2 m^2 < 32\pi/19$	$g^2 m^2 < 64$
	Massless	No bound	$g^2 \lesssim 24\pi^2 \Lambda_{\rm IR}/\Lambda_{\rm UV}^3$
$g\,S\,\partial_\mu\phi\partial^\mu\phi^*/2$	Massive	$g^2 E^2 < 16\pi$	$g^2 \Lambda_{\rm UV}^2 < \frac{768\pi^2}{7}$
	Massless		

Application to de Sitter spacetime

Bounded coupling: EFT of Inflation

$$S = \int \mathrm{d}t\, \mathrm{d}^3x\, a^3 \left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2a^2}(\partial_i \phi)^2 + \frac{g_1}{3!}\dot{\phi}^3 + \frac{g_2}{2a^2}\dot{\phi}(\partial_i\phi)^2 + \dots \right]$$ [Cheung & al. '07]

$$\gamma \geq 0 \qquad \Rightarrow \qquad g_1^2 + 69 g_2^2 - 16 g_1 g_2 \lesssim \frac{3435}{H^4}\,.$$



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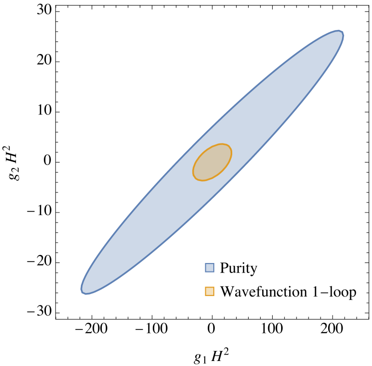

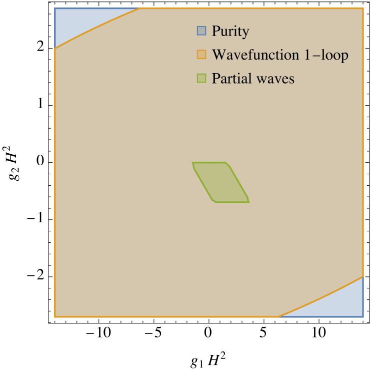

Bounded kinematics: Bogoliubov initial state

Interaction: $g\dot{\phi}^3/3!$

$$\psi_3^{\rm BD}(k_1,k_2,p) = -\frac{2g}{H}\frac{k_1^2k_2^2p^2}{k_T^3}\,,$$

$$\psi_3 = \alpha^3\psi_3^{\rm BD}(k_1,k_2,p) + \alpha^2\beta\left(\psi_3^{\rm BD}(k_1,k_2,-p) + \text{ perms}\right)$$
$$+\,\alpha\beta^2\left(\psi_3^{\rm BD}(k_1,-k_2,-p) + \text{ perms}\right) + \beta^3\psi_3^{\rm BD}(-k_1,-k_2,-p)$$ [Ghosh & al. '24]

Purity has large contributions from folded triangles

folded-singularity regulator: $|k_1+k_2-p| \geq F^{-1}\,\frac{k_T}{3}$ & perms

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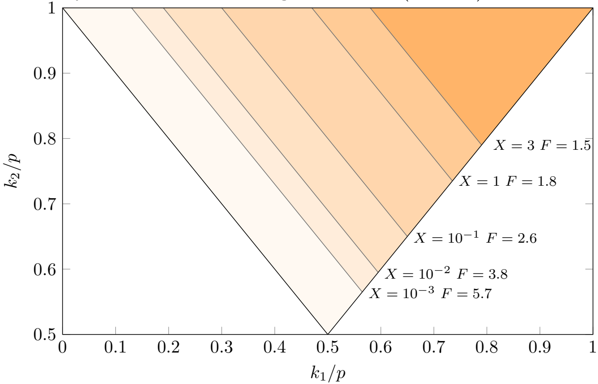
Interaction: $g\dot{\phi}^3/3!$

Folded-singularity regulator: $|k_1 + k_2 - p| \geq F^{-1}\,\frac{k_T}{3}$ & perms

Purity depends only on F and $X := g^2 H^4 \alpha^2 \beta^2 (\alpha - \beta)^2$



Bounded kinematics: $g\phi\dot{\phi}^2/2$

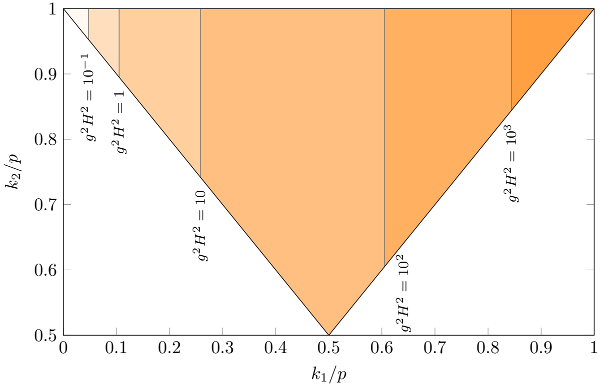
$$\psi_3(k_1, k_2, p) = \frac{g}{H^2}\left(\frac{e_3 e_2}{k_T^2} + \frac{e_2^2}{k_T} - 2e_3\right).$$

Purity has large contributions from squeezed triangles

$$\gamma = 1 - \frac{g^2 H^2}{92\pi^2}\left(\frac{k_s}{k_l}\right)^3 + \mathcal{O}\left(\frac{k_s}{k_l}\right)^2.$$

Bounded kinematics: $g\phi\dot{\phi}^2/2$

$$\gamma \geq 0 \quad \Longrightarrow \quad \left(\frac{k_s}{k_l}\right)^3 \lesssim \left(\frac{30.8}{gH}\right)^2 .$$



Summary

- A new perturbative unitarity bound from momentum-space entanglement
- On flat space, consistent & complementary with partial wave unitarity
- On de Sitter, provides cosmology-native bounds for correlators
- Speculative applications: local-type non-Gaussianity, General Relativity