

Dark energy, cosmological constant and neutrino mixing

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The today estimated value of dark energy can be achieved by the vacuum condensate induced by neutrino mixing phenomenon. Such a tiny value is recovered for a cut-off of the order of Planck scale and it is linked to the sub-eV neutrino mass scale.

- Neutrino oscillations in Quantum Mechanics;
- Quantum Field Theory of neutrino mixing and oscillations;
- Neutrino mixing and dark energy.

Motivations

- CKM quark mixing, meson mixing (D , B , K , η), massive neutrino mixing (and oscillations) play a crucial role in phenomenology;
- Renewed interest in these topics (evidence of neutrino oscillations, physics of CP violation);
- Theoretical interest: origin of mixing in the Standard Model;
- **Dark energy problem:** Measurements of the cosmic microwave background (CMBR), gravitational lensing, observations of type Ia supernovae suggested that the expansion of the universe is accelerating.

Possible explanation: exists an hypothetical form of energy which permeates all of space and has strong negative pressure: the dark energy.

Proposed forms for dark energy: cosmological constant, quintessence, extended theories of gravity, braneworld, etc...

Fermion mixing in Quantum Mechanics *

- Massive neutrinos.
- Super-weak interaction does not conserve leptonic numbers.
- **Flavor states** $|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\nu_\tau\rangle$ **superpositions of mass states** $|\nu_1\rangle$, $|\nu_2\rangle$, $|\nu_3\rangle$ which propagate with different frequencies due to different masses.
- Neutrinos are electrically neutral \Rightarrow two different types of massive neutrinos may exist: Majorana neutrinos and Dirac neutrinos.

Majorana neutrino: spinor with 2 components, the Hamiltonian does not conserve tau, muon and electron leptonic numbers and $L_e + L_\mu + L_\tau$ it is not conserved.

Dirac neutrino: spinor with 4 components, the Hamiltonian does conserve $L_e + L_\mu + L_\tau$.

*S. M. Bilenky and B. Pontecorvo, *Phys. Rep.* **41** (1978) 225

For two flavor states, we have

$$\begin{aligned} |\nu_e\rangle &= |\nu_1\rangle \cos \theta + |\nu_2\rangle \sin \theta \\ |\nu_\mu\rangle &= -|\nu_1\rangle \sin \theta + |\nu_2\rangle \cos \theta \end{aligned}$$

The Hamiltonian is diagonal in the mass eigenstates, then the time evolution:

$$\begin{aligned} |\nu_e(t)\rangle &= |\nu_1\rangle \cos \theta e^{-i\omega_1 t} + |\nu_2\rangle \sin \theta e^{-i\omega_2 t} \\ |\nu_\mu(t)\rangle &= -|\nu_1\rangle \sin \theta e^{-i\omega_1 t} + |\nu_2\rangle \cos \theta e^{-i\omega_2 t} \end{aligned}$$

• Flavor oscillations:

$$P_{\nu_e \rightarrow \nu_e}(t) = |\langle \nu_e(t) | \nu_e \rangle|^2 = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta\omega}{2} t \right)$$

$$P_{\nu_e \rightarrow \nu_\mu}(t) = |\langle \nu_\mu(t) | \nu_e \rangle|^2 = \sin^2 2\theta \sin^2 \left(\frac{\Delta\omega}{2} t \right).$$

$$\Delta\omega = \omega_1 - \omega_2.$$

Flavor conservation:

$$|\langle \nu_e(t) | \nu_e \rangle|^2 + |\langle \nu_\mu(t) | \nu_e \rangle|^2 = 1$$

Fermion mixing in QFT

The mixing relations for two Dirac fields:

$$\begin{aligned}\nu_e(x) &= \nu_1(x) \cos \theta + \nu_2(x) \sin \theta \\ \nu_\mu(x) &= -\nu_1(x) \sin \theta + \nu_2(x) \cos \theta\end{aligned}$$

ν_i ($i = 1, 2$) are free field operators with definite masses:

$$\nu_i(x) = \sum_{\mathbf{k}, r} \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{\sqrt{V}} \left[u_{\mathbf{k}, i}^r(t) \alpha_{\mathbf{k}, i}^r + v_{-\mathbf{k}, i}^r(t) \beta_{-\mathbf{k}, i}^{r\dagger} \right],$$

with $u_{\mathbf{k}, i}^r(t) = e^{-i\omega_{k, i} t} u_{\mathbf{k}, i}^r$, $v_{\mathbf{k}, i}^r(t) = e^{i\omega_{k, i} t} v_{\mathbf{k}, i}^r$ and $\omega_{k, i} = \sqrt{k^2 + m_i^2}$.

The above mixing transformations connect the two quadratic forms:

$$\mathcal{L} = \bar{\nu}_1 (i \not{\partial} - m_1) \nu_1 + \bar{\nu}_2 (i \not{\partial} - m_2) \nu_2$$

$$\mathcal{L} = \bar{\nu}_e (i \not{\partial} - m_e) \nu_e + \bar{\nu}_\mu (i \not{\partial} - m_\mu) \nu_\mu - m_{e\mu} (\bar{\nu}_e \nu_\mu + \bar{\nu}_\mu \nu_e)$$

with $m_e = m_1 \cos^2 \theta + m_2 \sin^2 \theta$, $m_\mu = m_1 \sin^2 \theta + m_2 \cos^2 \theta$, $m_{e\mu} = (m_2 - m_1) \sin \theta \cos \theta$.

Anticommutation relations:

$$\{\nu_i^\alpha(x), \nu_j^{\beta\dagger}(y)\}_{t=t'} = \delta^3(\mathbf{x} - \mathbf{y})\delta_{\alpha\beta}\delta_{ij}$$

$$\{\alpha_{\mathbf{k},i}^r, \alpha_{\mathbf{q},j}^{s\dagger}\} = \delta_{\mathbf{kq}}\delta_{rs}\delta_{ij} \quad ; \quad \{\beta_{\mathbf{k},i}^r, \beta_{\mathbf{q},j}^{s\dagger}\} = \delta_{\mathbf{kq}}\delta_{rs}\delta_{ij}$$

Orthonormality and completeness relations:

$$u_{\mathbf{k},i}^{r\dagger}u_{\mathbf{k},i}^s = v_{\mathbf{k},i}^{r\dagger}v_{\mathbf{k},i}^s = \delta_{rs} \quad , \quad u_{\mathbf{k},i}^{r\dagger}v_{-\mathbf{k},i}^s = 0 \quad , \quad \sum_r (u_{\mathbf{k},i}^{r\alpha*}u_{\mathbf{k},i}^{r\beta} + v_{-\mathbf{k},i}^{r\alpha*}v_{-\mathbf{k},i}^{r\beta}) = \delta_{\alpha\beta} .$$

– Perform all computations at finite volume V and only at the end put $V \rightarrow \infty$.

Generator of mixing transformations

Mixing relations can be written as*

$$\nu_e^\alpha(x) = G_\theta^{-1}(t) \nu_1^\alpha(x) G_\theta(t)$$

$$\nu_\mu^\alpha(x) = G_\theta^{-1}(t) \nu_2^\alpha(x) G_\theta(t)$$

with generator given by:

$$G_\theta(t) = \exp[\theta (S_+(t) - S_-(t))]$$

$$S_+(t) \equiv \int d^3\mathbf{x} \nu_1^\dagger(x) \nu_2(x) \quad , \quad S_-(t) \equiv \int d^3\mathbf{x} \nu_2^\dagger(x) \nu_1(x)$$

*M.Blasone and G.Vitiello, Annals Phys. (1995)

Introducing:

$$S_3 \equiv \frac{1}{2} \int d^3\mathbf{x} \left(\nu_1^\dagger(x) \nu_1(x) - \nu_2^\dagger(x) \nu_2(x) \right)$$

$$S_0 \equiv \frac{1}{2} \int d^3\mathbf{x} \left(\nu_1^\dagger(x) \nu_1(x) + \nu_2^\dagger(x) \nu_2(x) \right)$$

the $su(2)$ algebra is closed:

$$[S_+(t), S_-(t)] = 2S_3 \quad , \quad [S_3, S_\pm(t)] = \pm S_\pm(t)$$

Verify above eqs. For ν_e we get

$$\frac{d^2}{d\theta^2} \nu_e^\alpha = -\nu_e^\alpha$$

with the initial conditions

$$\nu_e^\alpha|_{\theta=0} = \nu_1^\alpha \quad , \quad \left. \frac{d}{d\theta} \nu_e^\alpha \right|_{\theta=0} = \nu_2^\alpha$$

- $G_\theta(t)$ is an unitary operator: $G_\theta^{-1}(t) = G_{-\theta}(t) = G_\theta^\dagger(t)$ preserving the canonical anticommutation relations
- $G_\theta^{-1}(t)$ maps $\mathcal{H}_{1,2}$ to $\mathcal{H}_{e,\mu}$: $G_\theta(t) : \mathcal{H}_{1,2} \rightarrow \mathcal{H}_{e,\mu}$.

The vacuum $|0\rangle_{1,2}$ is not invariant under the action of the generator $G_\theta(t)$, at finite volume:

$$|0(t)\rangle_{e,\mu} \equiv G_\theta^{-1}(t) |0\rangle_{1,2} = e^{-\theta(S_+(t)-S_-(t))} |0\rangle_{1,2}$$

The vacuum $|0(t)\rangle_{e,\mu}$ is a $SU(2)$ generalized coherent state.*

*A. Perelomov, *Generalized Coherent States and Their Applications*, (Springer-Verlag, Berlin, 1986)

- Orthogonality between $|0\rangle_{1,2}$ and $|0(t)\rangle_{e,\mu}$ for $V \rightarrow \infty$

$${}_{1,2}\langle 0|0(t)\rangle_{e,\mu} = \prod_{\mathbf{k}} \left(1 - \sin^2 \theta |V_{\mathbf{k}}|^2\right)^2 = \prod_{\mathbf{k}} \Gamma(\mathbf{k}) = e^{\sum_{\mathbf{k}} \ln \Gamma(\mathbf{k})}$$

From the properties of $|V_{\mathbf{k}}|^2$

$$|V_{\mathbf{k}}|^2 \equiv \sum_{r,s} |v_{-\mathbf{k},1}^{r\dagger} u_{\mathbf{k},2}^s|^2, \quad 0 \leq |V_{\mathbf{k}}|^2 \leq \frac{1}{2}$$

we have $\Gamma(\mathbf{k}) < 1, \forall \mathbf{k}$ and $\forall m_1, m_2$, by using $\sum_{\mathbf{k}} \rightarrow \frac{V}{(2\pi)^3} \int d^3\mathbf{k} \Rightarrow$

$$\lim_{V \rightarrow \infty} {}_{1,2}\langle 0|0(t)\rangle_{e,\mu} = \lim_{V \rightarrow \infty} e^{\frac{V}{(2\pi)^3} \int d^3\mathbf{k} \ln \Gamma(\mathbf{k})} = 0$$

- Mass and flavor representations are unitary inequivalent for $V \rightarrow \infty$.

We should use $\mathcal{H}_{e,\mu}$ for deriving oscillation formulas.

- The existence of the two inequivalent vacua for the flavor and the mass eigenstate neutrino fields, respectively, is crucial in order to obtain a non-zero contribution to the dark energy as we show below.

Condensate structure of $|0\rangle_{e,\mu}$ (use $\epsilon^r = (-1)^r$)

$$\begin{aligned}
 |0\rangle_{e,\mu} &= \prod_{\mathbf{k},r} [(1 - \sin^2 \theta |V_{\mathbf{k}}|^2) - \epsilon^r \sin \theta \cos \theta |V_{\mathbf{k}}| (\alpha_{\mathbf{k},1}^{r\dagger} \beta_{-\mathbf{k},2}^{r\dagger} + \alpha_{\mathbf{k},2}^{r\dagger} \beta_{-\mathbf{k},1}^{r\dagger}) \\
 &+ \epsilon^r \sin^2 \theta |V_{\mathbf{k}}| |U_{\mathbf{k}}| (\alpha_{\mathbf{k},1}^{r\dagger} \beta_{-\mathbf{k},1}^{r\dagger} - \alpha_{\mathbf{k},2}^{r\dagger} \beta_{-\mathbf{k},2}^{r\dagger}) \\
 &+ \sin^2 \theta |V_{\mathbf{k}}|^2 \alpha_{\mathbf{k},1}^{r\dagger} \beta_{-\mathbf{k},2}^{r\dagger} \alpha_{\mathbf{k},2}^{r\dagger} \beta_{-\mathbf{k},1}^{r\dagger}] |0\rangle_{1,2}
 \end{aligned}$$

- 4 kinds of particle-antiparticle pairs with zero momentum and spin.

- Time dependence:

$$|0\rangle_{e,\mu} \equiv |0(0)\rangle_{e,\mu} = e^{-iHt} |0(t)\rangle_{e,\mu}$$

Structure of the annihilation operators for $|0(t)\rangle_{e,\mu}$:

$$\alpha_{\mathbf{k},\nu_e}^r(t) = \cos\theta \alpha_{\mathbf{k},1}^r + \sin\theta \left(U_{\mathbf{k}}^*(t) \alpha_{\mathbf{k},2}^r + \epsilon^r V_{\mathbf{k}}(t) \beta_{-\mathbf{k},2}^{r\dagger} \right)$$

$$\alpha_{\mathbf{k},\nu_\mu}^r(t) = \cos\theta \alpha_{\mathbf{k},2}^r - \sin\theta \left(U_{\mathbf{k}}(t) \alpha_{\mathbf{k},1}^r - \epsilon^r V_{\mathbf{k}}(t) \beta_{-\mathbf{k},1}^{r\dagger} \right)$$

$$\beta_{-\mathbf{k},\nu_e}^r(t) = \cos\theta \beta_{-\mathbf{k},1}^r + \sin\theta \left(U_{\mathbf{k}}^*(t) \beta_{-\mathbf{k},2}^r - \epsilon^r V_{\mathbf{k}}(t) \alpha_{\mathbf{k},2}^{r\dagger} \right)$$

$$\beta_{-\mathbf{k},\nu_\mu}^r(t) = \cos\theta \beta_{-\mathbf{k},2}^r - \sin\theta \left(U_{\mathbf{k}}(t) \beta_{-\mathbf{k},1}^r + \epsilon^r V_{\mathbf{k}}(t) \alpha_{\mathbf{k},1}^{r\dagger} \right)$$

with $U_{\mathbf{k}}, V_{\mathbf{k}}$ Bogoliubov coefficients:

$$U_{\mathbf{k}}(t) = u_{\mathbf{k},2}^{r\dagger} u_{\mathbf{k},1}^r e^{i(\omega_{\mathbf{k},2} - \omega_{\mathbf{k},1})t}; \quad V_{\mathbf{k}}(t) = \epsilon^r u_{\mathbf{k},1}^{r\dagger} v_{-\mathbf{k},2}^r e^{i(\omega_{\mathbf{k},2} + \omega_{\mathbf{k},1})t}, \quad |U_{\mathbf{k}}|^2 + |V_{\mathbf{k}}|^2 = 1$$

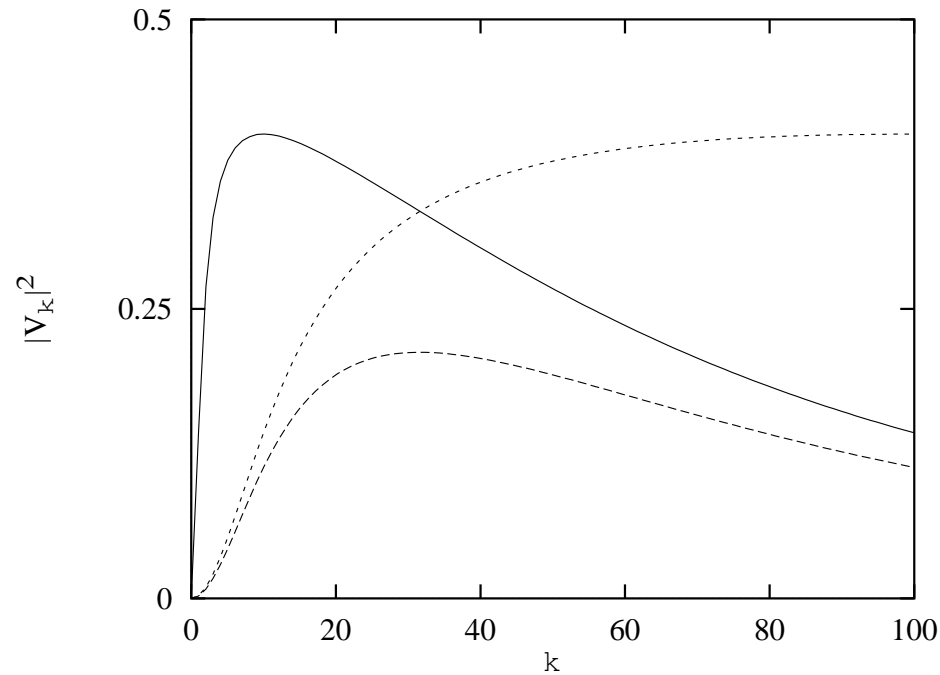
$$|U_{\mathbf{k}}| = \left(\frac{\omega_{\mathbf{k},1} + m_1}{2\omega_{\mathbf{k},1}} \right)^{\frac{1}{2}} \left(\frac{\omega_{\mathbf{k},2} + m_2}{2\omega_{\mathbf{k},2}} \right)^{\frac{1}{2}} \left(1 + \frac{|\mathbf{k}|^2}{(\omega_{\mathbf{k},1} + m_1)(\omega_{\mathbf{k},2} + m_2)} \right) \equiv \cos(\xi^{\mathbf{k}}),$$

$$|V_{\mathbf{k}}| = \left(\frac{\omega_{\mathbf{k},1} + m_1}{2\omega_{\mathbf{k},1}} \right)^{\frac{1}{2}} \left(\frac{\omega_{\mathbf{k},2} + m_2}{2\omega_{\mathbf{k},2}} \right)^{\frac{1}{2}} \left(\frac{|\mathbf{k}|}{(\omega_{\mathbf{k},2} + m_2)} - \frac{|\mathbf{k}|}{(\omega_{\mathbf{k},1} + m_1)} \right) \equiv \sin(\xi^{\mathbf{k}}),$$

Mixing transformation = Rotation $(\cos\theta, \sin\theta)$ + Bogoliubov transformation $(U_{\mathbf{k}}, V_{\mathbf{k}})$.

$$\alpha_{\nu_e}(t)|0(t)\rangle_{e,\mu} = G_\theta^{-1}(t)\alpha_1 G_\theta(t) G_\theta^{-1}(t)|0\rangle_{1,2} = 0.$$

Condensation density for mixed fermions



Solid line: $m_1 = 1, m_2 = 100$; Long dashed line: $m_1 = 10, m_2 = 100$; Short dashed line: $m_1 = 10, m_2 = 1000$.

$$e, \mu \langle 0(t) | \alpha_{\mathbf{k}, i}^{r\dagger} \alpha_{\mathbf{k}, i}^r | 0(t) \rangle_{e, \mu} = e, \mu \langle 0(t) | \beta_{\mathbf{k}, i}^{r\dagger} \beta_{\mathbf{k}, i}^r | 0(t) \rangle_{e, \mu} = \sin^2 \theta |V_{\mathbf{k}}|^2, \quad i = 1, 2.$$

- $V_{\mathbf{k}} = 0$ when $m_1 = m_2$ and/or $\theta = 0$.
- Max. at $k = \sqrt{m_1 m_2}$ with $V_{max} \rightarrow \frac{1}{2}$ for $\frac{(m_2 - m_1)^2}{m_1 m_2} \rightarrow \infty$.
- $|V_{\mathbf{k}}|^2 \simeq \frac{(m_2 - m_1)^2}{4k^2}$ for $k \gg \sqrt{m_1 m_2}$.

The current structure for field mixing

Mass basis: Let us consider the Lagrangian describing two free Dirac fields with masses m_1, m_2 :

$$\mathcal{L}(x) = \bar{\nu}_m(x) (i \not{\partial} - \mathbf{M}_d) \nu_m(x),$$

with $\nu_m^T = (\nu_1, \nu_2)$ and $\mathbf{M}_d = \text{diag}(m_1, m_2)$

• $\mathcal{L}(x)$ is invariant under global $U(1)$ phase transformations of the type

$$\nu'_m(x) = e^{i\alpha} \nu_m(x),$$

then, we have the conservation of the Noether charge

$$Q = \int I^0(x) d^3\mathbf{x}$$

(with $I^\mu(x) = \bar{\nu}_m(x) \gamma^\mu \nu_m(x)$) which is indeed the total charge of the system, i.e. the total lepton number.

Consider now the global $SU(2)$ transformation:

$$\nu'_m(x) = e^{i\alpha_j \tau_j} \nu_m(x), \quad j = 1, 2, 3.$$

with α_j real constants, $\tau_j = \sigma_j/2$ and σ_j being the Pauli matrices.

- Since m_1 and m_2 are different, $\mathcal{L}(x)$ is not invariant under $SU(2)$

The variation of $\mathcal{L}(x)$ is:

$$\delta\mathcal{L} = i\alpha_j \bar{\nu}_m(x) [\tau_j, M_d] \nu_m(x) = -\alpha_j \partial_\mu J_{m,j}^\mu(x),$$

where the currents are:

$$J_{m,j}^\mu(x) = \bar{\nu}_m(x) \gamma^\mu \tau_j \nu_m(x), \quad j = 1, 2, 3.$$

The related charges,

$$Q_{m,j}(t) = \int J_{m,j}^0(x) d^3\mathbf{x} = \frac{1}{2} \int d^3\mathbf{x} \nu_m^\dagger(x) \tau_j \nu_m(x), \quad j = 1, 2, 3,$$

satisfy the $su(2)$ algebra:

$$[Q_{m,i}(t), Q_{m,j}(t)] = i\varepsilon_{ijk} Q_{m,k}(t).$$

- $Q_{m,3}$ is conserved \Rightarrow charge conserved separately for ν_1 and ν_2 .

Then the Noether charges associated with ν_1 and ν_2 can be expressed as

$$Q_1 \equiv \frac{1}{2}Q + Q_{m,3} \quad ; \quad Q_2 \equiv \frac{1}{2}Q - Q_{m,3}.$$

with Q total (conserved) charge.

- The normal ordered charge operators are:

$$: Q_i : \equiv \int d^3\mathbf{x} : \nu_i^\dagger(x) \nu_i(x) := \sum_r \int d^3\mathbf{k} \left(\alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r - \beta_{-\mathbf{k},i}^{r\dagger} \beta_{-\mathbf{k},i}^r \right), \quad i = 1, 2.$$

where the $: \dots :$ denotes normal ordering with respect to the vacuum $|0\rangle_{1,2}$.

- Then the neutrino states with definite masses defined as

$$|\nu_{\mathbf{k},i}^r\rangle = \alpha_{\mathbf{k},i}^{r\dagger} |0\rangle_{1,2}, \quad i = 1, 2,$$

are eigenstates of the above conserved charges.

Flavor basis: Let us consider the Lagrangian written in the flavor basis

$$\mathcal{L}(x) = \bar{\nu}_f(x) (i \not{\partial} - \mathbf{M}) \nu_f(x),$$

where $\nu_f^T = (\nu_e, \nu_\mu)$, $\mathbf{M} = \begin{pmatrix} m_e & m_{e\mu} \\ m_{e\mu} & m_\mu \end{pmatrix}$ and the flavor fields ν_σ are

$$\nu_\sigma(x) = \sum_{\mathbf{k}, r} \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{\sqrt{V}} \left[u_{\mathbf{k}, i}^r(t) \alpha_{\mathbf{k}, \nu_\sigma}^r(t) + v_{-\mathbf{k}, i}^r(t) \beta_{-\mathbf{k}, \nu_\sigma}^{r\dagger}(t) \right], \quad (\sigma, i) = (e, 1), (\mu, 2).$$

Consider the $SU(2)$ transformation: $\nu'_f(x) = e^{i\alpha_j \cdot \tau_j} \nu_f(x)$, with $j = 1, 2, 3$.

$$\delta\mathcal{L}(x) = i\alpha_j \bar{\nu}_f(x) [\tau_j, \mathbf{M}] \nu_f(x) = -\alpha_j \partial_\mu J_{f,j}^\mu(x),$$

$$J_{f,j}^\mu(x) = \bar{\nu}_f(x) \gamma^\mu \tau_j \nu_f(x)$$

Again, the charges

$$Q_{f,j}(t) = \int J_{f,j}^0(x) d^3\mathbf{x} = \frac{1}{2} \int d^3\mathbf{x} \nu_f^\dagger(x) \tau_j \nu_f(x), \quad f = e, \mu, \quad j = 1, 2, 3,$$

satisfy the $su(2)$ algebra

- $Q_{f,3}$ is not conserved \Rightarrow exchange of charge between ν_e and ν_μ .
Thus the **flavor charges** defined as

$$Q_{\nu_e}(t) = \frac{1}{2}Q + Q_{f,3}(t) \quad ; \quad Q_{\nu_\mu}(t) = \frac{1}{2}Q - Q_{f,3}(t),$$

are the time-dependent lepton charges in presence of mixing.

- **The normal ordered charges** $:: Q_{\nu_\sigma}(t) ::$ with respect to $|0\rangle_{e,\mu}$ are

$$\begin{aligned} :: Q_{\nu_\sigma}(t) :: &\equiv \int d^3\mathbf{x} :: \nu_\sigma^\dagger(x) \nu_\sigma(x) :: \\ &= \sum_r \int d^3\mathbf{k} \left(\alpha_{\mathbf{k},\nu_\sigma}^{r\dagger}(t) \alpha_{\mathbf{k},\nu_\sigma}^r(t) - \beta_{-\mathbf{k},\nu_\sigma}^{r\dagger}(t) \beta_{-\mathbf{k},\nu_\sigma}^r(t) \right), \end{aligned}$$

where $\sigma = e, \mu$, and $:: \dots ::$ is the normal ordering with respect to $|0\rangle_{e,\mu}$.

We have $:: Q_{\nu_e}(t) :: + :: Q_{\nu_\mu}(t) :: =: Q_1 : + : Q_2 :=: Q :$ • **The flavor charges are related to the Noether charges Q_i by:**

$$:: Q_{\nu_\sigma}(t) :: = G_\theta^{-1}(t) : Q_j : G_\theta(t), \quad \text{with} \quad (\sigma, j) = (e, 1), (\mu, 2),$$

- The flavor neutrino states defined as:

$$|\nu_{\mathbf{k},\sigma}^r\rangle \equiv \alpha_{\mathbf{k},\nu_\sigma}^{r\dagger}(0)|0(0)\rangle_{e,\mu}, \quad \sigma = e, \mu$$

are eigenstates of the flavor charges Q_{ν_σ} at a reference time $t = 0$:

$$\begin{aligned} \therefore Q_{\nu_e}(0) \therefore |\nu_{\mathbf{k},e}^r\rangle &= |\nu_{\mathbf{k},e}^r\rangle \\ \therefore Q_{\nu_\mu}(0) \therefore |\nu_{\mathbf{k},\mu}^r\rangle &= |\nu_{\mathbf{k},\mu}^r\rangle \\ \therefore Q_{\nu_e}(0) \therefore |\nu_{\mathbf{k},\mu}^r\rangle &= \therefore Q_{\nu_\mu}(0) \therefore |\nu_{\mathbf{k},e}^r\rangle = 0. \\ \therefore Q_{\nu_\sigma}(0) \therefore |0\rangle_{e,\mu} &= 0. \end{aligned}$$

- These results are not trivial since the usual Pontecorvo states:

$$\begin{aligned} |\nu_{\mathbf{k},e}^r\rangle_P &= \cos\theta |\nu_{\mathbf{k},1}^r\rangle + \sin\theta |\nu_{\mathbf{k},2}^r\rangle \\ |\nu_{\mathbf{k},\mu}^r\rangle_P &= -\sin\theta |\nu_{\mathbf{k},1}^r\rangle + \cos\theta |\nu_{\mathbf{k},2}^r\rangle, \end{aligned}$$

are not eigenstates of the flavor charges, as can be easily checked.

- Thus the correct flavor state and normal ordered operators are*:

$$|\nu_{\mathbf{k},e}^r\rangle \equiv \alpha_{\mathbf{k},\nu_\sigma}^{r\dagger}(0)|0(0)\rangle_{e,\mu} \quad \therefore A \therefore \equiv A - e,\mu\langle 0|A|0\rangle_{e,\mu}$$

*M.Blasone, A.Capolupo, F.Terranova and G.Vitiello, Phys. Rev. **D** (2005).
M.Blasone, A.Capolupo, C.R.Ji and G.Vitiello, submitted to J.Phys. **G** (2007).

Oscillation formulae

In the Heisenberg picture: we have

$${}_{e,\mu}\langle 0 | \because Q_{\nu_\sigma}(t) \because | 0 \rangle_{e,\mu} = 0$$

$$\begin{aligned} Q_{\nu_e \rightarrow \nu_\sigma}^{\mathbf{k}}(t) &\equiv \langle \nu_{\mathbf{k},e}^r | \because Q_{\nu_\sigma}(t) \because | \nu_{\mathbf{k},e}^r \rangle \\ &= \left| \{ \alpha_{\mathbf{k},\nu_\sigma}^r(t), \alpha_{\mathbf{k},\nu_e}^{r\dagger}(0) \} \right|^2 + \left| \{ \beta_{-\mathbf{k},\nu_\sigma}^{r\dagger}(t), \alpha_{\mathbf{k},\nu_e}^{r\dagger}(0) \} \right|^2 \end{aligned}$$

Charge conservation is ensured at any time:

$$\langle \nu_{\mathbf{k},e}^r | \left[\because Q_{\nu_e}(t) \because + \because Q_{\nu_\mu}(t) \because \right] | \nu_{\mathbf{k},e}^r \rangle = 1$$

Neutrino oscillation formulae (exact result)*:

$$Q_{\nu_e \rightarrow \nu_e}^k(t) = 1 - |U_k|^2 \sin^2(2\theta) \sin^2\left(\frac{\omega_{k,2} - \omega_{k,1}}{2} t\right) - |V_k|^2 \sin^2(2\theta) \sin^2\left(\frac{\omega_{k,2} + \omega_{k,1}}{2} t\right)$$

$$Q_{\nu_e \rightarrow \nu_\mu}^k(t) = |U_k|^2 \sin^2(2\theta) \sin^2\left(\frac{\omega_{k,2} - \omega_{k,1}}{2} t\right) + |V_k|^2 \sin^2(2\theta) \sin^2\left(\frac{\omega_{k,2} + \omega_{k,1}}{2} t\right)$$

- **Correction to amplitudes + new oscillating term !**
- **“Statistical” interpretation of flavor oscillations: only average values make sense.**
- **For $k \gg \sqrt{m_1 m_2}$ we have: $|V_k|^2 \rightarrow 0$ and $|U_k|^2 \rightarrow 1$ the Pontecorvo formulae are reobtained in the relativistic limit.**

Similar results are obtained for three flavor neutrino fields and for boson fields. For the $\eta - \eta'$ system, the correction may be as large as 20%.[†]

*M.Blasone, P.Henning and G.Vitiello, Phys. Lett. **B** (1999).

†M.Blasone, A.Capolupo, O.Romei and G.Vitiello, Phys. Rev. **D** (2001);
M.Blasone, A.Capolupo and G.Vitiello, Phys. Rev. **D** (2002)
A.Capolupo, C.R.Ji, Y.Mischenko and G.Vitiello, Phys. Lett.**B** (2004)

Neutrino mixing contribution to the dark energy*

Experimental data support the picture that some form of *dark energy*, evolving from early epochs, induces the today observed acceleration of the universe.

There are many proposals to achieve cosmological models justifying such a dark component.

Our result is: The non-perturbative vacuum structure associated with neutrino mixing leads to a non-zero contribution to the dark energy.

*A.Capolupo, S.Capozziello, G.Vitiello, PLA (2007)
A.Capolupo, S.Capozziello, G.Vitiello, submitted to PLB (2007).

We consider the Minkowski metric

Lorentz invariance of $|0\rangle$ implies that $|0\rangle$ is the zero eigenvalue eigenstate of the normal ordered energy, momentum and angular momentum operators. Therefore $\mathcal{T}_{\mu\nu}^{vac} = \langle 0 | : \mathcal{T}_{\mu\nu} : | 0 \rangle = 0$.

the $(0,0)$ component of $\mathcal{T}_{\mu\nu}(x)$ is

$$: \mathcal{T}_{00}(x) := \frac{i}{2} : \left(\bar{\Psi}_m(x) \gamma_0 \overleftrightarrow{\partial}_0 \Psi_m(x) \right) :$$

In terms of the annihilation and creation operators of fields ν_1 and ν_2 , the energy momentum tensor $: T_{00} := \int d^3x : \mathcal{T}_{00}(x) :$ is given by

$$: T_{00}^{(i)} := \sum_r \int d^3\mathbf{k} \omega_{k,i} \left(\alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r + \beta_{-\mathbf{k},i}^{r\dagger} \beta_{-\mathbf{k},i}^r \right),$$

we note that $T_{00}^{(i)}$ is time independent, moreover

$${}_{e,\mu} \langle 0 | : T_{00}^{(i)} : | 0 \rangle_{e,\mu} = {}_{e,\mu} \langle 0(t) | : T_{00}^{(i)} : | 0(t) \rangle_{e,\mu}, \quad \forall t.$$

- **Early universe epochs:** Lorentz invariance of the vacuum condensate is broken, ρ_{vac}^{mix} presents also space-time dependent condensate contributions. The contribution of the neutrino mixing to the vacuum energy density ρ_{vac}^{mix} is given by the expectation value of $:T_{00}^{(i)}:$ in the flavor vacuum $|0\rangle_{e,\mu}^*$:

$$\rho_{vac}^{mix} = \frac{1}{V} \eta_{00} \langle 0 | \sum_i :T_{00}^{(i)}: | 0 \rangle_{e,\mu} \Rightarrow$$

$$\Rightarrow \rho_{vac}^{mix} = \sum_{i,r} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \omega_{k,i} \left({}_{e,\mu} \langle 0 | \alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r | 0 \rangle_{e,\mu} + {}_{e,\mu} \langle 0 | \beta_{\mathbf{k},i}^{r\dagger} \beta_{\mathbf{k},i}^r | 0 \rangle_{e,\mu} \right).$$

Since

$${}_{e,\mu} \langle 0 | \alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r | 0 \rangle_{e,\mu} = {}_{e,\mu} \langle 0 | \beta_{\mathbf{k},i}^{r\dagger} \beta_{\mathbf{k},i}^r | 0 \rangle_{e,\mu} = \sin^2 \theta |V_{\mathbf{k}}|^2 \Rightarrow$$

$$\Rightarrow \rho_{vac}^{mix} = 4 \sin^2 \theta \int \frac{d^3\mathbf{k}}{(2\pi)^3} (\omega_{k,1} + \omega_{k,2}) |V_{\mathbf{k}}|^2,$$

$$\rho_{vac}^{mix} = \frac{2}{\pi} \sin^2 \theta \int_0^K dk k^2 (\omega_{k,1} + \omega_{k,2}) |V_{\mathbf{k}}|^2,$$

where K is the cut-off.

*M.Blasone, A.Capolupo, S.Capozziello, S.Carloni G.Vitiello, PLA (2004).

- In a similar way, the contribution p_{vac}^{mix} of the neutrino mixing to the vacuum pressure is:

$$p_{vac}^{mix} = \frac{1}{V} \eta_{jj} e_{,\mu} \langle 0 | \sum_i : T_{jj}^{(i)}(0) : | 0 \rangle_{e,\mu}$$

(no summation on the index j is intended). Being

$$: T_{(i)}^{jj} := \sum_r \int d^3\mathbf{k} \frac{k^j k^j}{\omega_{k,i}} \left(\alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r + \beta_{-\mathbf{k},i}^{r\dagger} \beta_{-\mathbf{k},i}^r \right),$$

in the case of the isotropy of the momenta: $T^{11} = T^{22} = T^{33}$

$$p_{vac}^{mix} = \frac{2}{3\pi} \sin^2 \theta \int_0^K dk k^4 \left[\frac{1}{\omega_{k,1}} + \frac{1}{\omega_{k,2}} \right] |V_{\mathbf{k}}|^2$$

- $\rho_{vac}^{mix} \neq -p_{vac}^{mix} \Rightarrow$ violation of the Lorentz invariance originates from the neutrino-antineutrino condensate structure of the vacuum ($\rho_{vac}^{mix} = p_{vac}^{mix} = 0$ when $|V_{\mathbf{k}}|^2 = 0$) for any m_1, m_2 and K .

- $w = p_{vac}^{mix} / \rho_{vac}^{mix} \simeq 1/3$ when the cut-off is chosen to be $K \gg m_1, m_2$.

ρ_{vac}^{mix} is time-independent since, for simplicity, we are considering Minkowski metric. When the curved background metric is considered, ρ_{vac}^{mix} is time-dependent, but the essence of the result is the same.*

At the present epoch, the breaking of the Lorentz invariance is negligible $\Rightarrow \rho_{vac}^{mix}$ comes from space-time independent condensate contributions (i.e. the contributions carrying a non-vanishing $\partial_\mu \sim k_\mu = (\omega_k, k_j)$ are missing). This means that the stress energy tensor of the vacuum condensate is

$$T_{\mu\nu}^{cond} = V^{cond} \eta_{\mu\nu}$$

that is

$$e_{,\mu} \langle 0 | : T_{\mu\nu} : | 0 \rangle_{e,\mu} = \eta_{\mu\nu} \sum_i m_i \int \frac{d^3x}{(2\pi)^3} e_{,\mu} \langle 0 | : \bar{\nu}_i(x) \nu_i(x) : | 0 \rangle_{e,\mu} = \eta_{\mu\nu} \rho_\Lambda^{mix}.$$

*A.Capolupo, S.Capozziello, G.Vitiello, work in progress.

Since

$$\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

and, in a homogeneous and isotropic universe, $T_{\mu\nu}$ is

$$T_{\mu\nu} = \text{diag}(\rho, p, p, p),$$

⇒ **The state equation is now** $\rho_{\Lambda}^{mix} \sim -p_{\Lambda}^{mix}$, **where***

$$\rho_{\Lambda}^{mix} = \frac{2}{\pi} \sin^2 \theta \int_0^K dk k^2 \left[\frac{m_1^2}{\omega_{k,1}} + \frac{m_2^2}{\omega_{k,2}} \right] |V_{\mathbf{k}}|^2.$$

Present epoch: ρ_{Λ}^{mix} has a behavior similar to that of Λ .

- **neutrino oscillation length** \ll **radius of curvature of the universe** \Rightarrow **mixing treatment in the flat space-time is a good approximation of that in FRW space-time.**

*A.Capolupo, S.Capozziello, G.Vitiello, PLA(2007).

Solving the integral ^{*}, we obtain

$$\begin{aligned}
\rho_{\Lambda}^{mix} &= \frac{2}{\pi} \sin^2 \theta \{ (m_2^2 - m_1^2) K (\sqrt{K^2 + m_2^2} - \sqrt{K^2 + m_1^2}) \\
&+ \frac{2(m_2 - m_1)}{\sqrt{m_2^2 - m_1^2}} [m_1^4 \arctan \left(\frac{\sqrt{m_2^2 - m_1^2}}{m_1 \sqrt{K^2 + m_2^2}} K \right) - m_2^4 \arctan \left(\frac{\sqrt{m_2^2 - m_1^2}}{m_2 \sqrt{K^2 + m_1^2}} K \right)] \\
&+ (2m_1^4 - 2m_1^3 m_2 + m_1^2 m_2^2 - m_2^4) \log \left(K + \sqrt{K^2 + m_2^2} \right) \\
&+ (2m_2^4 - 2m_2^3 m_1 + m_1^2 m_2^2 - m_1^4) \log \left(K + \sqrt{K^2 + m_1^2} \right) \\
&- (2m_1^4 - 2m_1^3 m_2 + m_1^2 m_2^2 - m_2^4) \log (m_2) \\
&- (2m_2^4 - 2m_2^3 m_1 + m_1^2 m_2^2 - m_1^4) \log (m_1) \}.
\end{aligned}$$

The behavior of ρ_{Λ}^{mix} for $K \gg m_1, m_2$:

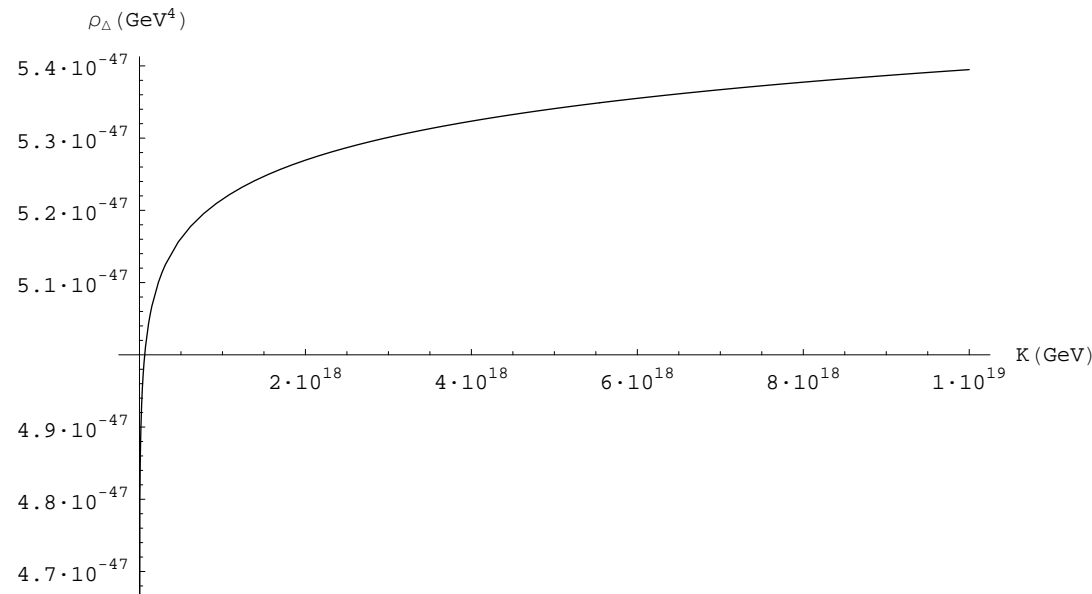
$$\begin{aligned}
\rho_{\Lambda}^{mix} &\approx \frac{2}{\pi} \sin^2 \theta \left\{ \frac{2(m_2 - m_1)}{\sqrt{m_2^2 - m_1^2}} \left[m_1^4 \arctan \left(\frac{\sqrt{m_2^2 - m_1^2}}{m_1} \right) - m_2^4 \arctan \left(\frac{\sqrt{m_2^2 - m_1^2}}{m_2} \right) \right] \right. \\
&- (2m_1^4 - 2m_1^3 m_2 + m_1^2 m_2^2 - m_2^4) \log (m_2) - (2m_2^4 - 2m_2^3 m_1 + m_1^2 m_2^2 - m_1^4) \log (m_1) \\
&\left. + (m_1^4 + m_2^4 + 2m_1^2 m_2^2 - 2m_1^3 m_2 - 2m_2^3 m_1) \log (2K) \right\}.
\end{aligned}$$

The integral diverges in K as $m_i^4 \log (K)$.

^{*}A.Capolupo, S.Capozziello, G.Vitiello, submitted to PLB(2007).

For m_i of order of $10^{-3}eV$ we have $\rho_{\Lambda}^{mix} = 5.4 \times 10^{-47} GeV^4$ for a value of the cut-off of order of the Planck scale $K = 10^{19} GeV$.: agreement with the observed value of cosmological constant.

Moreover $\frac{d\rho_{\Lambda}^{mix}(K)}{dK} \propto \frac{1}{K} \rightarrow 0$ for large K .



The neutrino mixing dark energy as a function of cut-off K .

At present epoch, the characteristic oscillation length of the neutrino is much smaller than the universe curvature radius, the mixing treatment in the flat space-time, in such an epoch, is a good approximation of that in FRW space-time.

The vacuum condensate from neutrino mixing can give rise to the *observed* value of the cosmological constant. Exotic components to dark energy are not necessary in this approach.

- Dark energy gets non-zero contribution induced from the neutrino mixing.
- Such a contribution is zero in the no-mixing limit: $\theta = 0$ and/or $m_1 = m_2$.
- The contribution is absent in the QM mixing treatment.

Conclusions

- Mixing transformations are not trivial in Q.F.T. \Leftrightarrow they are associated to inequivalent representations (orthogonal Hilbert spaces).
- QFT oscillation formulas for both fermions and bosons exhibit non-perturbative corrections with respect to the usual QM ones, due to the condensate structure of the flavor vacuum.
- The effects are more relevant in the non-relativistic region, i.e. $k \simeq \sqrt{m_1 m_2}$ for fermions and $k \simeq 0$ for bosons. For the $\eta-\eta'$ system, the non-perturbative correction may be as large as 20%.
- The vacuum condensate generated by neutrino mixing can be interpreted as an evolving dark energy that, at present epoch, behaves as the cosmological constant, giving rise to its observed value.
- Its origin is completely different from the one of the ordinary contribution of a massive spinor field. It comes from the property of QFT: infinitely many representations of the canonical (anti-)commutation relations in the infinite volume limit.
- The mixing phenomenon provides the vacuum energy contribution, since the field mixing involves unitary inequivalent representations.