Dark energy, cosmological constant and neutrino mixing

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The today estimated value of dark energy can be achieved by the vacuum condensate induced by neutrino mixing phenomenon. Such a tiny value is recovered for a cut-off of the order of Planck scale and it is linked to the sub-eV neutrino mass scale.

- Neutrino oscillations in Quantum Mechanics;
- Quantum Field Theory of neutrino mixing and oscillations;
- Neutrino mixing and dark energy.

Motivations

- CKM quark mixing, meson mixing (D, B, K, η), massive neutrino mixing (and oscillations) play a crucial role in phenomenology;

- Renewed interest in these topics (evidence of neutrino oscillations, physics of CP violation);

- Theoretical interest: origin of mixing in the Standard Model;

- Dark energy problem: Measurements of the cosmic microwave background (CMBR), gravitational lensing, observations of type Ia supernovae suggested that the expansion of the universe is accelerating.

Possible explanation: exists an hypothetical form of energy which permeates all of space and has strong negative pressure: the dark energy.

Proposed forms for dark energy: cosmological constant, quintessence, extended theories of gravity, braneworld, etc...

Fermion mixing in Quantum Mechanics *

- Massive neutrinos.

- Super-weak interaction does not conserve leptonic numbers.

- Flavor states $|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\nu_\tau\rangle$ superpositions of mass states $|\nu_1\rangle$, $|\nu_2\rangle$, $|\nu_3\rangle$ which propagate with different frequencies due to different masses.

- Neutrinos are electrically neutral $\Rightarrow$ two different types of massive neutrinos may exist: Majorana neutrinos and Dirac neutrinos.

Majorana neutrino: spinor with 2 components, the Hamiltonian does not conserve tau, muon and electron leptonic numbers and $L_e+L_\mu+L_\tau$ it is not conserved.

Dirac neutrino: spinor with 4 components, the Hamiltonian does conserve $L_e + L_\mu + L_\tau$.

*S. M. Bilenky and B. Pontecorvo, *Phys. Rep.* **41** (1978) 225

For two flavor states, we have

$$
\begin{aligned}
|\nu_e\rangle &= |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta \\
|\nu_\mu\rangle &= -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta
\end{aligned}
$$

The Hamiltonian is diagonal in the mass eigenstates, then the time evolution:

$$
\begin{aligned}
|\nu_e(t)\rangle &= |\nu_1\rangle \cos\theta e^{-i\omega_1 t} + |\nu_2\rangle \sin\theta e^{-i\omega_2 t} \\
|\nu_\mu(t)\rangle &= -|\nu_1\rangle \sin\theta e^{-i\omega_1 t} + |\nu_2\rangle \cos\theta e^{-i\omega_2 t}
\end{aligned}
$$

- **Flavor oscillations:**

$$
\begin{aligned}
P_{\nu_e\to\nu_e}(t) &= |\langle\nu_e(t)|\nu_e\rangle|^2 = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta\omega}{2}t\right) \\
P_{\nu_e\to\nu_\mu}(t) &= |\langle\nu_\mu(t)|\nu_e\rangle|^2 = \sin^2 2\theta \sin^2\left(\frac{\Delta\omega}{2}t\right).
\end{aligned}
$$

$\Delta\omega = \omega_1 - \omega_2$.

Flavor conservation:

$$
|\langle\nu_e(t)|\nu_e\rangle|^2 + |\langle\nu_\mu(t)|\nu_e\rangle|^2 = 1
$$

Fermion mixing in QFT

The mixing relations for two Dirac fields:

$$\begin{aligned}\nu_e(x) &= \nu_1(x)\,\cos\theta + \nu_2(x)\,\sin\theta \\ \nu_\mu(x) &= -\nu_1(x)\,\sin\theta + \nu_2(x)\,\cos\theta\end{aligned}$$

ν_i $(i=1,2)$ **are free field operators with definite masses:**

$$\nu_i(x) = \sum_{\mathbf{k},r} \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{\sqrt{V}} \left[u^r_{\mathbf{k},i}(t)\,\alpha^r_{\mathbf{k},i} \;+\; v^r_{-\mathbf{k},i}(t)\,\beta^{r\dagger}_{-\mathbf{k},i} \right],$$

with $u^r_{\mathbf{k},i}(t) = e^{-i\omega_{k,i}t}\,u^r_{\mathbf{k},i}$ **,** $v^r_{\mathbf{k},i}(t) = e^{i\omega_{k,i}t}\,v^r_{\mathbf{k},i}$ **and** $\omega_{k,i} = \sqrt{k^2+m_i^2}$**.**

The above mixing transformations connect the two quadratic forms:

$$\mathcal{L} = \bar{\nu}_1\,(i\,\partial\!\!\!/ - m_1)\,\nu_1 + \bar{\nu}_2\,(i\,\partial\!\!\!/ - m_2)\,\nu_2$$

$$\mathcal{L} \;=\; \bar{\nu}_e\,(i\,\partial\!\!\!/ - m_e)\,\nu_e + \bar{\nu}_\mu\,(i\,\partial\!\!\!/ - m_\mu)\,\nu_\mu \;-\; m_{e\mu}\;(\,\bar{\nu}_e\,\nu_\mu + \bar{\nu}_\mu\,\nu_e\,)$$

with $m_e = m_1\cos^2\theta + m_2\sin^2\theta$, $m_\mu = m_1\sin^2\theta + m_2\cos^2\theta$, $m_{e\mu} = (m_2-m_1)\sin\theta\cos\theta$.

Anticommutation relations:

$$\{\nu_i^{\alpha}(x), \nu_j^{\beta\dagger}(y)\}_{t=t'} = \delta^3(\mathbf{x}-\mathbf{y})\delta_{\alpha\beta}\delta_{ij}$$

$$\{\alpha^r_{\mathbf{k},i}, \alpha^{s\dagger}_{\mathbf{q},j}\} = \delta_{\mathbf{kq}}\delta_{rs}\delta_{ij} \quad ; \quad \{\beta^r_{\mathbf{k},i}, \beta^{s\dagger}_{\mathbf{q},j}\} = \delta_{\mathbf{kq}}\delta_{rs}\delta_{ij}$$

Orthonormality and completeness relations:

$$u^{r\dagger}_{\mathbf{k},i}u^s_{\mathbf{k},i} = v^{r\dagger}_{\mathbf{k},i}v^s_{\mathbf{k},i} = \delta_{rs} \quad , \quad u^{r\dagger}_{\mathbf{k},i}v^s_{-\mathbf{k},i} = 0 \quad , \quad \sum_r (u^{r\alpha*}_{\mathbf{k},i}u^{r\beta}_{\mathbf{k},i} + v^{r\alpha*}_{-\mathbf{k},i}v^{r\beta}_{-\mathbf{k},i}) = \delta_{\alpha\beta}\,.$$

– Perform all computations at finite volume V and only at the end put $V \to \infty$.

Generator of mixing transformations

Mixing relations can be written as*

$$\begin{aligned}\nu_e^\alpha(x) &= G_\theta^{-1}(t)\,\nu_1^\alpha(x)\,G_\theta(t)\\ \nu_\mu^\alpha(x) &= G_\theta^{-1}(t)\,\nu_2^\alpha(x)\,G_\theta(t)\end{aligned}$$

with generator given by:

$$G_\theta(t) = \exp[\theta\left(S_+(t) - S_-(t)\right)]$$

$$S_+(t) \equiv \int d^3\mathbf{x}\,\nu_1^\dagger(x)\nu_2(x) \quad , \quad S_-(t) \equiv \int d^3\mathbf{x}\,\nu_2^\dagger(x)\nu_1(x)$$

*M.Blasone and G.Vitiello, Annals Phys. (1995)

Introducing:

$$S_3 \equiv \frac{1}{2}\int d^3\mathbf{x}\,\left(\nu_1^\dagger(x)\nu_1(x) - \nu_2^\dagger(x)\nu_2(x)\right)$$

$$S_0 \equiv \frac{1}{2}\int d^3\mathbf{x}\,\left(\nu_1^\dagger(x)\nu_1(x) + \nu_2^\dagger(x)\nu_2(x)\right)$$

the $su(2)$ algebra is closed:

$$[S_+(t)\,,\,S_-(t)] = 2S_3 \quad , \quad [S_3\,,\,S_\pm(t)] = \pm S_\pm(t)$$

Verify above eqs. For ν_e we get

$$\frac{d^2}{d\theta^2}\,\nu_e^\alpha = -\nu_e^\alpha$$

with the initial conditions

$$\nu_e^\alpha|_{\theta=0} = \nu_{_1}^\alpha \quad , \quad \left.\frac{d}{d\theta}\nu_e^\alpha\right|_{\theta=0} = \nu_{_2}^\alpha$$

- $G_\theta(t)$ **is an unitary operator:** $G_\theta^{-1}(t) = G_{-\theta}(t) = G_\theta^\dagger(t)$ **preserving the canonical anticommutation relations**

- $G_\theta^{-1}(t)$ **maps** $\mathcal{H}_{1,2}$ **to** $\mathcal{H}_{e,\mu}$**:** $\quad G_\theta(t) : \mathcal{H}_{1,2} \to \mathcal{H}_{e,\mu}$.

The vacuum $|0\rangle_{1,2}$ is not invariant under the action of the generator $G_\theta(t)$, at finite volume:

$$|0(t)\rangle_{e,\mu} \equiv G_\theta^{-1}(t)\, |0\rangle_{1,2} = e^{-\theta\left(S_+(t)-S_-(t)\right)}\, |0\rangle_{1,2}$$

The vacuum $|0(t)\rangle_{e,\mu}$ is a $SU(2)$ generalized coherent state.*

*A. Perelomov, *Generalized Coherent States and Their Applications*, (Springer-Verlag, Berlin, 1986)

- Orthogonality between $|0\rangle_{1,2}$ and $|0(t)\rangle_{e,\mu}$ for $V \to \infty$

$$ {}_{1,2}\langle 0|0(t)\rangle_{e,\mu} = \prod_{\mathbf{k}} \left(1 - \sin^2\theta\, |V_{\mathbf{k}}|^2\right)^2 = \prod_{\mathbf{k}} \Gamma(\mathbf{k}) = e^{\sum_{\mathbf{k}} \ln \Gamma(\mathbf{k})} $$

From the properties of $|V_{\mathbf{k}}|^2$

$$ |V_{\mathbf{k}}|^2 \equiv \sum_{r,s} |\, v^{r\dagger}_{-\mathbf{k},1} u^{s}_{\mathbf{k},2}\,|^2 \;, \qquad 0 \leq |V_{\mathbf{k}}|^2 \leq \frac{1}{2} $$

we have $\Gamma(\mathbf{k}) < 1$, $\forall \mathbf{k}$ and $\forall m_1, m_2$, by using $\sum_{\mathbf{k}} \to \frac{V}{(2\pi)^3} \int d^3\mathbf{k} \Rightarrow$

$$ \lim_{V\to\infty} {}_{1,2}\langle 0|0(t)\rangle_{e,\mu} = \lim_{V\to\infty} e^{\frac{V}{(2\pi)^3} \int d^3\mathbf{k}\, \ln \Gamma(\mathbf{k})} = 0 $$

- Mass and flavor representations are unitary inequivalent for $V \to \infty$.

We should use $\mathcal{H}_{e,\mu}$ for deriving oscillation formulas.

- The existence of the two inequivalent vacua for the flavor and the mass eigenstate neutrino fields, respectively, is crucial in order to obtain a non–zero contribution to the dark energy as we show below.

Condensate structure of $|0\rangle_{e,\mu}$ (use $\epsilon^r = (-1)^r$)

$$
\begin{aligned}
|0\rangle_{e,\mu} &= \prod_{\mathbf{k},r} [(1 - \sin^2\theta\, |V_{\mathbf{k}}|^2) - \epsilon^r \sin\theta\, \cos\theta\, |V_{\mathbf{k}}|\, (\alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},2} + \alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},1}) \\
&+ \epsilon^r \sin^2\theta\, |V_{\mathbf{k}}||U_{\mathbf{k}}|\, (\alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},1} - \alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},2}) \\
&+ \sin^2\theta\, |V_{\mathbf{k}}|^2\, \alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},2}\alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},1}]|0\rangle_{1,2}
\end{aligned}
$$

- 4 kinds of particle-antiparticle pairs with zero momentum and spin.

- Time dependence:

$$
|0\rangle_{e,\mu} \equiv |0(0)\rangle_{e,\mu} = e^{-iHt}|0(t)\rangle_{e,\mu}
$$

Structure of the annihilation operators for $|0(t)\rangle_{e,\mu}$:

$$\alpha^r_{\mathbf{k},\nu_e}(t) = \cos\theta\,\alpha^r_{\mathbf{k},1} + \sin\theta\left(U^*_{\mathbf{k}}(t)\,\alpha^r_{\mathbf{k},2} + \epsilon^r V_{\mathbf{k}}(t)\,\beta^{r\dagger}_{-\mathbf{k},2}\right)$$

$$\alpha^r_{\mathbf{k},\nu_\mu}(t) = \cos\theta\,\alpha^r_{\mathbf{k},2} - \sin\theta\left(U_{\mathbf{k}}(t)\,\alpha^r_{\mathbf{k},1} - \epsilon^r V_{\mathbf{k}}(t)\,\beta^{r\dagger}_{-\mathbf{k},1}\right)$$

$$\beta^r_{-\mathbf{k},\nu_e}(t) = \cos\theta\,\beta^r_{-\mathbf{k},1} + \sin\theta\left(U^*_{\mathbf{k}}(t)\,\beta^r_{-\mathbf{k},2} - \epsilon^r V_{\mathbf{k}}(t)\,\alpha^{r\dagger}_{\mathbf{k},2}\right)$$

$$\beta^r_{-\mathbf{k},\nu_\mu}(t) = \cos\theta\,\beta^r_{-\mathbf{k},2} - \sin\theta\left(U_{\mathbf{k}}(t)\,\beta^r_{-\mathbf{k},1} + \epsilon^r V_{\mathbf{k}}(t)\,\alpha^{r\dagger}_{\mathbf{k},1}\right)$$

with U_{k}, V_{k} Bogoliubov coefficients:

$$U_{\mathrm{k}}(t) \;=\; u^{r\dagger}_{\mathbf{k},2}u^r_{\mathbf{k},1}\,e^{i(\omega_{k,2}-\omega_{k,1})t}; \qquad V_{\mathrm{k}}(t) = \epsilon^r\,u^{r\dagger}_{\mathbf{k},1}v^r_{-\mathbf{k},2}\,e^{i(\omega_{k,2}+\omega_{k,1})t}, \qquad |U_{\mathrm{k}}|^2 + |V_{\mathrm{k}}|^2 = 1$$

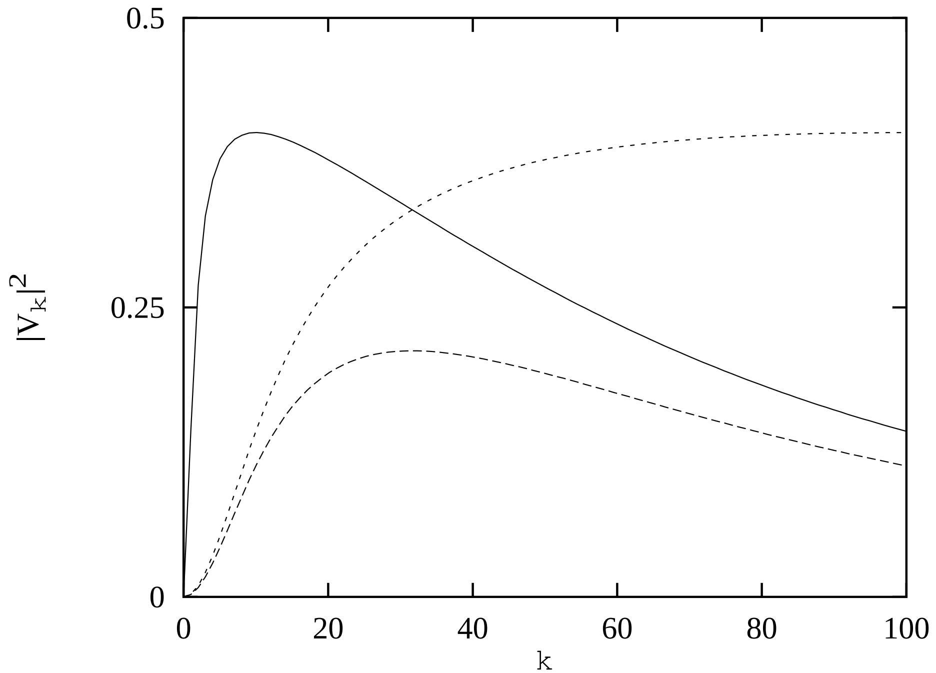
$$|U_{\mathrm{k}}| = \left(\frac{\omega_{k,1}+m_1}{2\omega_{k,1}}\right)^{\frac{1}{2}}\left(\frac{\omega_{k,2}+m_2}{2\omega_{k,2}}\right)^{\frac{1}{2}}\left(1 + \frac{|\mathbf{k}|^2}{(\omega_{k,1}+m_1)(\omega_{k,2}+m_2)}\right) \equiv \cos(\xi^{\mathrm{k}}),$$

$$|V_{\mathrm{k}}| = \left(\frac{\omega_{k,1}+m_1}{2\omega_{k,1}}\right)^{\frac{1}{2}}\left(\frac{\omega_{k,2}+m_2}{2\omega_{k,2}}\right)^{\frac{1}{2}}\left(\frac{|\mathbf{k}|}{(\omega_{k,2}+m_2)} - \frac{|\mathbf{k}|}{(\omega_{k,1}+m_1)}\right) \equiv \sin(\xi^{\mathrm{k}}),$$

Mixing transformation = Rotation $(\cos\theta\,,\,\sin\theta)$ **+ Bogoliubov transformation** $(U_{\mathrm{k}}\,,\,V_{\mathrm{k}})$**.**

$$\alpha_{\nu_e}(t)|0(t)\rangle_{e,\mu} = G^{-1}_\theta(t)\alpha_{{}_1}G_\theta(t)\;G^{-1}_\theta(t)|0\rangle_{1,2} = 0.$$

Condensation density for mixed fermions



Solid line: $m_1 = 1$, $m_2 = 100$; Long dashed line: $m_1 = 10$, $m_2 = 100$; Short dashed line: $m_1 = 10$, $m_2 = 1000$.

$$ {}_{e,\mu}\langle 0(t)|\alpha^{r\dagger}_{\mathbf{k},i}\alpha^{r}_{\mathbf{k},i}|0(t)\rangle_{e,\mu} = {}_{e,\mu}\langle 0(t)|\beta^{r\dagger}_{\mathbf{k},i}\beta^{r}_{\mathbf{k},i}|0(t)\rangle_{e,\mu} = \sin^2\theta\, |V_{\mathbf{k}}|^2, \quad i = 1,2. $$

- $V_{\mathbf{k}} = 0$ when $m_1 = m_2$ and/or $\theta = 0$.
- Max. at $k = \sqrt{m_1 m_2}$ with $V_{max} \to \frac{1}{2}$ for $\frac{(m_2-m_1)^2}{m_1 m_2} \to \infty$.
- $|V_{\mathbf{k}}|^2 \simeq \frac{(m_2-m_1)^2}{4k^2}$ for $k \gg \sqrt{m_1 m_2}$.

The current structure for field mixing

Mass basis: Let us consider the Lagrangian describing two free Dirac fields with masses m_1, m_2:

$$\mathcal{L}(x) = \bar{\nu}_m(x) \left(i \not\partial - \mathbf{M}_d\right) \nu_m(x) \, ,$$

with $\nu_m^T = (\nu_1, \nu_2)$ and $\mathbf{M}_d = diag(m_1, m_2)$

- $\mathcal{L}(x)$ is invariant under global $U(1)$ phase transformations of the type

$$\nu_m^{'}(x) = e^{i\alpha} \nu_m(x),$$

then, we have the conservation of the Noether charge

$$Q = \int I^0(x) d^3\mathbf{x}$$

(with $I^\mu(x) = \bar{\nu}_m(x) \gamma^\mu \nu_m(x)$) which is indeed the total charge of the system, i.e. the total lepton number.

Consider now the global $SU(2)$ transformation:

$$\nu_m^{'}(x) = e^{i\alpha_j \cdot \tau_j} \nu_m(x), \qquad\qquad j = 1,2,3.$$

with α_j real constants, $\tau_j = \sigma_j/2$ and σ_j being the Pauli matrices.

- Since m_1 and m_2 are different, $\mathcal{L}(x)$ is not invariant under $SU(2)$

The variation of $\mathcal{L}(x)$ is:

$$\delta\mathcal{L} = i\alpha_j \bar{\nu}_m(x) \left[\tau_j,\ M_d\right] \nu_m(x) = -\alpha_j \partial_\mu J^\mu_{m,j}(x),$$

where the currents are:

$$J^\mu_{m,j}(x) = \bar{\nu}_m(x)\, \gamma^\mu\, \tau_j\, \nu_m(x), \qquad\qquad j = 1,2,3.$$

The related charges,

$$Q_{m,j}(t) = \int J^0_{m,j}(x) d^3\mathbf{x} = \frac{1}{2}\int d^3\mathbf{x}\, \nu_m^\dagger(x)\, \tau_j\, \nu_m(x), \qquad\qquad j = 1,2,3,$$

satisfy the $su(2)$ algebra:

$$\left[Q_{m,i}(t), Q_{m,j}(t)\right] = i\varepsilon_{ijk} Q_{m,k}(t).$$

- $Q_{m,3}$ is conserved $\Rightarrow$ charge conserved separately for ν_1 and ν_2.

Then the Noether charges associated with ν_1 and ν_2 can be expressed as

$$Q_1 \equiv \frac{1}{2}Q + Q_{m,3} \qquad ; \qquad Q_2 \equiv \frac{1}{2}Q - Q_{m,3}.$$

with Q total (conserved) charge.

- The normal ordered charge operators are:

$$: Q_i :\equiv \int d^3\mathbf{x} \; : \nu_i^\dagger(x)\, \nu_i(x) := \sum_r \int d^3\mathbf{k} \left(\alpha^{r\dagger}_{\mathbf{k},i} \alpha^r_{\mathbf{k},i} - \beta^{r\dagger}_{-\mathbf{k},i} \beta^r_{-\mathbf{k},i} \right) \;, \qquad i = 1, 2.$$

where the $: .. :$ denotes normal ordering with respect to the vacuum $|0\rangle_{1,2}$.

- Then the neutrino states with definite masses defined as

$$|\nu^r_{\mathbf{k},i}\rangle = \alpha^{r\dagger}_{\mathbf{k},i}|0\rangle_{1,2}, \qquad i = 1, 2,$$

are eigenstates of the above conserved charges.

Flavor basis: Let us consider the Lagrangian written in the flavor basis

$$\mathcal{L}(x) \,=\, \bar{\nu}_f(x)\,(i\,\partial\!\!\!/ - \mathbf{M})\,\nu_f(x)\,,$$

where $\nu_f^T = (\nu_e,\ \nu_\mu)$, $\mathbf{M} = \begin{pmatrix} m_e & m_{e\mu} \\ m_{e\mu} & m_\mu \end{pmatrix}$ and the flavor fields ν_σ are

$$\nu_\sigma(x) \;=\; \sum_{\mathbf{k},r} \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{\sqrt{V}} \left[u^r_{\mathbf{k},i}(t)\,\alpha^r_{\mathbf{k},\nu_\sigma}(t) + v^r_{-\mathbf{k},i}(t)\,\beta^{r\dagger}_{-\mathbf{k},\nu_\sigma}(t)\right], \qquad (\sigma,i) = (e,1),(\mu,2).$$

Consider the $SU(2)$ transformation: $\nu'_f(x) = e^{i\alpha_j\cdot\tau_j}\nu_f(x)$, with $j = 1,2,3.$

$$\delta\mathcal{L}(x) = i\alpha_j\bar{\nu}_f(x)\left[\tau_j, \mathbf{M}\right]\nu_f(x) = -\alpha_j\partial_\mu J^\mu_{f,j}(x),$$

$$J^\mu_{f,j}(x) = \bar{\nu}_f(x)\,\gamma^\mu\,\tau_j\,\nu_f(x)$$

Again, the charges

$$Q_{f,j}(t) = \int J^0_{f,j}(x) d^3\mathbf{x} = \frac{1}{2}\int d^3\mathbf{x}\,\nu_f^\dagger(x)\,\tau_j\,\nu_f(x), \qquad f \,=\, e,\mu, \qquad j = 1,2,3,$$

satisfy the $su(2)$ algebra

- $Q_{f,3}$ is not conserved $\Rightarrow$ exchange of charge between ν_e and ν_μ. Thus the flavor charges defined as

$$Q_{\nu_e}(t) = \frac{1}{2}Q + Q_{f,3}(t) \qquad ; \qquad Q_{\nu_\mu}(t) = \frac{1}{2}Q - Q_{f,3}(t),$$

are the time-dependent lepton charges in presence of mixing.

- The normal ordered charges $:: Q_{\nu_\sigma}(t) ::$ with respect to $|0\rangle_{e,\mu}$ are

$$\begin{aligned} :: Q_{\nu_\sigma}(t) :: \;&\equiv\; \int d^3\mathbf{x} \; :: \nu_\sigma^\dagger(x)\, \nu_\sigma(x) :: \\ &= \; \sum_r \int d^3\mathbf{k} \left(\alpha^{r\dagger}_{\mathbf{k},\nu_\sigma}(t)\alpha^{r}_{\mathbf{k},\nu_\sigma}(t) \;-\; \beta^{r\dagger}_{-\mathbf{k},\nu_\sigma}(t)\beta^{r}_{-\mathbf{k},\nu_\sigma}(t) \right), \end{aligned}$$

where $\sigma = e, \mu$, and $:: \ldots ::$ is the normal ordering with respect to $|0\rangle_{e,\mu}$. We have $:: Q_{\nu_e}(t) :: + :: Q_{\nu_\mu}(t) ::=: Q_1 : + : Q_2 :=: Q :$

- The flavor charges are related to the Noether charges Q_i by:

$$:: Q_{\nu_\sigma}(t) ::= G_\theta^{-1}(t) \; : Q_j : \; G_\theta(t) \,, \qquad \textit{with} \qquad (\sigma, j) = (e,1), (\mu,2),$$

- **The flavor neutrino states defined as:**

$$|\nu^r_{\mathbf{k},\sigma}\rangle \equiv \alpha^{r\dagger}_{\mathbf{k},\nu_\sigma}(0)|0(0)\rangle_{e,\mu}, \qquad \sigma = e, \mu$$

are eigenstates of the flavor charges Q_{ν_σ} at a reference time $t = 0$:

$$\begin{aligned} :: Q_{\nu_e}(0) :: |\nu^r_{\mathbf{k},e}\rangle &= |\nu^r_{\mathbf{k},e}\rangle \\ :: Q_{\nu_\mu}(0) :: |\nu^r_{\mathbf{k},\mu}\rangle &= |\nu^r_{\mathbf{k},\mu}\rangle \\ :: Q_{\nu_e}(0) :: |\nu^r_{\mathbf{k},\mu}\rangle &= :: Q_{\nu_\mu}(0) :: |\nu^r_{\mathbf{k},e}\rangle = 0. \\ :: Q_{\nu_\sigma}(0) :: |0\rangle_{e,\mu} &= 0. \end{aligned}$$

- **These results are not trivial since the usual Pontecorvo states:**

$$\begin{aligned} |\nu^r_{\mathbf{k},e}\rangle_P &= \cos\theta\,|\nu^r_{\mathbf{k},1}\rangle + \sin\theta\,|\nu^r_{\mathbf{k},2}\rangle \\ |\nu^r_{\mathbf{k},\mu}\rangle_P &= -\sin\theta\,|\nu^r_{\mathbf{k},1}\rangle + \cos\theta\,|\nu^r_{\mathbf{k},2}\rangle\,, \end{aligned}$$

are not eigenstates of the flavor charges, as can be easily checked.

- **Thus the correct flavor state and normal ordered operators are*:**

$$|\nu^r_{\mathbf{k},e}\rangle \equiv \alpha^{r\dagger}_{\mathbf{k},\nu_\sigma}(0)\,|0(0)\rangle_{e,\mu} \qquad\qquad :: A :: \equiv A - {}_{e,\mu}\langle 0|A|0\rangle_{e,\mu}$$

*M.Blasone, A.Capolupo, F.Terranova and G.Vitiello, Phys. Rev. **D** (2005).
M.Blasone, A.Capolupo, C.R.Ji and G.Vitiello, submitted to J.Phys. **G** (2007).

Oscillation formulae

In the Heisenberg picture: we have

$$ {}_{e,\mu}\langle 0| :: Q_{\nu_\sigma}(t) :: |\, 0\rangle_{e,\mu} = 0 $$

$$ \begin{aligned} \mathcal{Q}^{\mathbf{k}}_{\nu_e\rightarrow\nu_\sigma}(t) &\equiv \langle \nu^r_{\mathbf{k},e}| :: Q_{\nu_\sigma}(t) :: |\nu^r_{\mathbf{k},e}\rangle \\ &= \left|\left\{\alpha^r_{\mathbf{k},\nu_\sigma}(t), \alpha^{r\dagger}_{\mathbf{k},\nu_e}(0)\right\}\right|^2 + \left|\left\{\beta^{r\dagger}_{-\mathbf{k},\nu_\sigma}(t), \alpha^{r\dagger}_{\mathbf{k},\nu_e}(0)\right\}\right|^2 \end{aligned} $$

Charge conservation is ensured at any time:

$$ \langle \nu^r_{\mathbf{k},e}| \left[:: Q_{\nu_e}(t) :: \; + \; :: Q_{\nu_\mu}(t) ::\right] |\nu^r_{\mathbf{k},e}\rangle \; = \; 1 $$

Neutrino oscillation formulae (exact result)*:

$$\mathcal{Q}^{\mathbf{k}}_{\nu_e\to\nu_e}(t) \;=\; 1 - |U_{\mathbf{k}}|^2 \sin^2(2\theta) \sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}\,t\right) - |V_{\mathbf{k}}|^2 \sin^2(2\theta) \sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}\,t\right)$$

$$\mathcal{Q}^{\mathbf{k}}_{\nu_e\to\nu_\mu}(t) \;=\; |U_{\mathbf{k}}|^2 \sin^2(2\theta) \sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}\,t\right) + |V_{\mathbf{k}}|^2 \sin^2(2\theta) \sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}\,t\right)$$

- **Correction to amplitudes + new oscillating term !**
- **"Statistical" interpretation of flavor oscillations: only average values make sense.**
- **For $k \gg \sqrt{m_1 m_2}$ we have: $|V_{\mathbf{k}}|^2 \to 0$ and $|U_{\mathbf{k}}|^2 \to 1$ the Pontecorvo formulae are reobtained in the relativistic limit.**

Similar results are obtained for three flavor neutrino fields and for boson fields. For the $\eta - \eta'$ system, the correction may be as large as 20%.†

*M.Blasone, P.Henning and G.Vitiello, Phys. Lett. **B** (1999).

†M.Blasone, A.Capolupo, O.Romei and G.Vitiello, Phys. Rev. **D** (2001);
M.Blasone, A.Capolupo and G.Vitiello, Phys. Rev. **D** (2002)
A.Capolupo, C.R.Ji, Y.Mischenko and G.Vitiello, Phys. Lett.**B** (2004)

Neutrino mixing contribution to the dark energy*

Experimental data support the picture that some form of *dark energy*, evolving from early epochs, induces the today observed acceleration of the universe.

There are many proposals to achieve cosmological models justifying such a dark component.

Our result is: The non–perturbative vacuum structure associated with neutrino mixing leads to a non–zero contribution to the dark energy.

*A.Capolupo, S.Capozziello, G.Vitiello, PLA (2007)
A.Capolupo, S.Capozziello, G.Vitiello, submitted to PLB (2007).

We consider the Minkowski metric

Lorentz invariance of $|0\rangle$ implies that $|0\rangle$ is the zero eigenvalue eigenstate of the normal ordered energy, momentum and angular momentum operators. Therefore $\mathcal{T}^{vac}_{\mu\nu} = \langle 0| : \mathcal{T}_{\mu\nu} : |0\rangle = 0$.

the **(0,0)** component of $\mathcal{T}_{\mu\nu}(x)$ is

$$: \mathcal{T}_{00}(x) := \frac{i}{2} : \left(\bar{\Psi}_m(x) \gamma_0 \overleftrightarrow{\partial}_0 \Psi_m(x) \right) :$$

In terms of the annihilation and creation operators of fields ν_1 and ν_2, the energy momentum tensor $: T_{00} := \int d^3x : \mathcal{T}_{00}(x) :$ is given by

$$: T^{(i)}_{00} := \sum_r \int d^3\mathbf{k}\, \omega_{k,i} \left(\alpha^{r\dagger}_{\mathbf{k},i} \alpha^r_{\mathbf{k},i} + \beta^{r\dagger}_{-\mathbf{k},i} \beta^r_{-\mathbf{k},i} \right),$$

we note that $T^{(i)}_{00}$ is time independent, moreover

$${}_{e,\mu}\langle 0| : T^{(i)}_{00} : |0\rangle_{e,\mu} = {}_{e,\mu}\langle 0(t)| : T^{(i)}_{00} : |0(t)\rangle_{e,\mu}, \qquad \forall t.$$

• **Early universe epochs: Lorentz invariance of the vacuum condensate is broken, ρ_{vac}^{mix} presents also space-time dependent condensate contributions. The contribution of the neutrino mixing to the vacuum energy density ρ_{vac}^{mix} is given by the expectation value of $:T_{00}^{(i)}:$ in the flavor vacuum $|0\rangle_{e,\mu}$*:**

$$\rho_{vac}^{mix} = \frac{1}{V}\eta_{00\; e,\mu}\langle 0|\sum_i :T_{00}^{(i)}: |0\rangle_{e,\mu} \Rightarrow$$

$$\Rightarrow \rho_{vac}^{mix} = \sum_{i,r}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\omega_{k,i}\left({}_{e,\mu}\langle 0|\alpha_{\mathbf{k},i}^{r\dagger}\alpha_{\mathbf{k},i}^{r}|0\rangle_{e,\mu} + {}_{e,\mu}\langle 0|\beta_{\mathbf{k},i}^{r\dagger}\beta_{\mathbf{k},i}^{r}|0\rangle_{e,\mu}\right).$$

Since

$${}_{e,\mu}\langle 0|\alpha_{\mathbf{k},i}^{r\dagger}\alpha_{\mathbf{k},i}^{r}|0\rangle_{e,\mu} = {}_{e,\mu}\langle 0|\beta_{\mathbf{k},i}^{r\dagger}\beta_{\mathbf{k},i}^{r}|0\rangle_{e,\mu} = \sin^2\theta |V_{\mathbf{k}}|^2 \Rightarrow$$

$$\Rightarrow \rho_{vac}^{mix} = 4\sin^2\theta \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left(\omega_{k,1}+\omega_{k,2}\right)|V_{\mathbf{k}}|^2,$$

$$\rho_{vac}^{mix} = \frac{2}{\pi}\sin^2\theta \int_0^K dk\, k^2(\omega_{k,1}+\omega_{k,2})|V_{\mathbf{k}}|^2,$$

where K is the cut-off.

*M.Blasone, A.Capolupo, S.Capozziello, S.Carloni G.Vitiello, PLA (2004).

• In a similar way, the contribution p^{mix}_{vac} of the neutrino mixing to the vacuum pressure is:

$$p^{mix}_{vac} \quad = \frac{1}{V}\, \eta_{jj}\ {}_{e,\mu}\langle 0| \sum_i : T^{(i)}_{jj}(0) : |0\rangle_{e,\mu}$$

(no summation on the index j is intended). Being

$$: T^{jj}_{(i)} := \sum_r \int d^3\mathbf{k}\, \frac{k^j k^j}{\omega_{k,i}} \left(\alpha^{r\dagger}_{\mathbf{k},i}\alpha^{r}_{\mathbf{k},i} + \beta^{r\dagger}_{-\mathbf{k},i}\beta^{r}_{-\mathbf{k},i}\right),$$

in the case of the isotropy of the momenta: $T^{11} = T^{22} = T^{33}$

$$p^{mix}_{vac} = \frac{2}{3\,\pi} \sin^2\theta \int_0^K dk\, k^4 \left[\frac{1}{\omega_{k,1}} + \frac{1}{\omega_{k,2}}\right] |V_{\mathbf{k}}|^2$$

• $\rho^{mix}_{vac} \neq -p^{mix}_{vac} \Rightarrow$ violation of the Lorentz invariance originates from the neutrino-antineutrino condensate structure of the vacuum ($\rho^{mix}_{vac} = p^{mix}_{vac} = 0$ when $|V_{\mathbf{k}}|^2 = 0$) for any m_1, m_2 and K.

• $w = p^{mix}_{vac}/\rho^{mix}_{vac} \simeq 1/3$ when the cut-off is chosen to be $K \gg m_1, m_2$.

ρ_{vac}^{mix} is time-independent since, for simplicity, we are considering Minkowski metric. When the curved background metric is considered, ρ_{vac}^{mix} is time-dependent, but the essence of the result is the same.*

At the present epoch, the breaking of the Lorentz invariance is negligible $\Rightarrow \rho_{vac}^{mix}$ comes from space-time independent condensate contributions (i.e. the contributions carrying a non-vanishing $\partial_\mu \sim k_\mu = (\omega_k, k_j)$ are missing). This means that the stress energy tensor of the vacuum condensate is

$$T_{\mu\nu}^{cond} = V^{cond}\,\eta_{\mu\nu}$$

that is

$$_{e,\mu}\langle 0| : T_{\mu\nu} : |0\rangle_{e,\mu} = \eta_{\mu\nu}\,\sum_i m_i \int \frac{d^3x}{(2\pi)^3}\,_{e,\mu}\langle 0| : \bar{\nu}_i(x)\nu_i(x) : |0\rangle_{e,\mu} = \eta_{\mu\nu}\,\rho_\Lambda^{mix}.$$

*A.Capolupo, S.Capozziello, G.Vitiello, work in progress.

Since

$$\eta_{\mu\nu} = diag(1, -1, -1, -1)$$

and, in a homogeneous and isotropic universe, $T_{\mu\nu}$ is

$$T_{\mu\nu} = diag(\rho\,, p\,, p\,, p\,),$$

$\Rightarrow$ The state equation is now $\rho_\Lambda^{mix} \sim -p_\Lambda^{mix}$, where*

$$\rho_\Lambda^{mix} = \frac{2}{\pi} \sin^2\theta \int_0^K dk\, k^2 \left[\frac{m_1^2}{\omega_{k,1}} + \frac{m_2^2}{\omega_{k,2}}\right] |V_{\mathbf{k}}|^2.$$

<u>Present epoch:</u> ρ_Λ^{mix} has a behavior similar to that of Λ.

- neutrino oscillation length $\ll$ radius of curvature of the universe $\Rightarrow$ mixing treatment in the flat space-time is a good approximation of that in FRW space-time.

*A.Capolupo, S.Capozziello, G.Vitiello, PLA(2007).

Solving the integral *, **we obtain**

$$\begin{aligned}
\rho_\Lambda^{mix} &= \frac{2}{\pi}\sin^2\theta\{(m_2^2-m_1^2)K(\sqrt{K^2+m_2^2}-\sqrt{K^2+m_1^2})\\
&+ \frac{2(m_2-m_1)}{\sqrt{m_2^2-m_1^2}}[m_1^4\arctan(\frac{\sqrt{m_2^2-m_1^2}}{m_1\sqrt{K^2+m_2^2}}K)-m_2^4\arctan(\frac{\sqrt{m_2^2-m_1^2}}{m_2\sqrt{K^2+m_1^2}}K)]\\
&+ (2m_1^4-2m_1^3m_2+m_1^2m_2^2-m_2^4)\log\left(K+\sqrt{K^2+m_2^2}\right)\\
&+ (2m_2^4-2m_2^3m_1+m_1^2m_2^2-m_1^4)\log\left(K+\sqrt{K^2+m_1^2}\right)\\
&- (2m_1^4-2m_1^3m_2+m_1^2m_2^2-m_2^4)\log(m_2)\\
&- (2m_2^4-2m_2^3m_1+m_1^2m_2^2-m_1^4)\log(m_1)\}.
\end{aligned}$$

The behavior of ρ_Λ^{mix} **for** $K \gg m_1, m_2$:

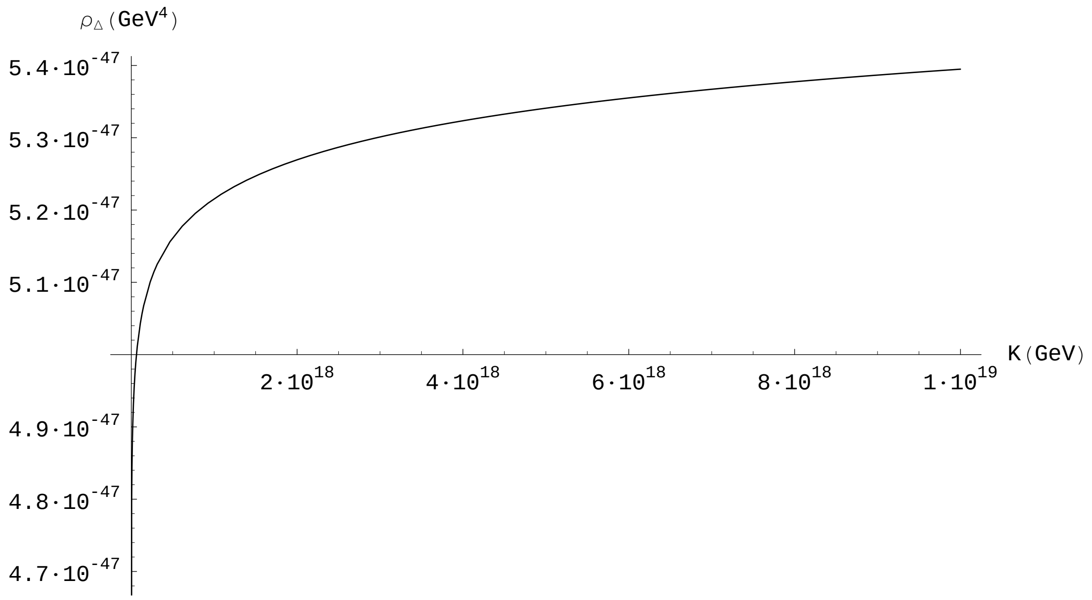
$$\begin{aligned}
\rho_\Lambda^{mix} &\approx \frac{2}{\pi}\sin^2\theta\{\frac{2(m_2-m_1)}{\sqrt{m_2^2-m_1^2}}[m_1^4\arctan(\frac{\sqrt{m_2^2-m_1^2}}{m_1})-m_2^4\arctan(\frac{\sqrt{m_2^2-m_1^2}}{m_2})]\\
&- (2m_1^4-2m_1^3m_2+m_1^2m_2^2-m_2^4)\log(m_2)-(2m_2^4-2m_2^3m_1+m_1^2m_2^2-m_1^4)\log(m_1)\\
&+ (m_1^4+m_2^4+2m_1^2m_2^2-2m_1^3m_2-2m_2^3m_1)\log(2K)\}.
\end{aligned}$$

The integral diverges in K **as** $m_i^4\log(K)$.

*A.Capolupo, S.Capozziello, G.Vitiello, submitted to PLB(2007).

For m_i of order of $10^{-3} eV$ we have $\rho_\Lambda^{mix} = 5.4 \times 10^{-47} GeV^4$ for a value of the cut-off of order of the Planck scale $K = 10^{19} GeV$.: agreement with the observed value of cosmological constant.

Moreover $\frac{d\rho_\Lambda^{mix}(K)}{dK} \propto \frac{1}{K} \to 0$ for large K.



The neutrino mixing dark energy as a function of cut-off K.

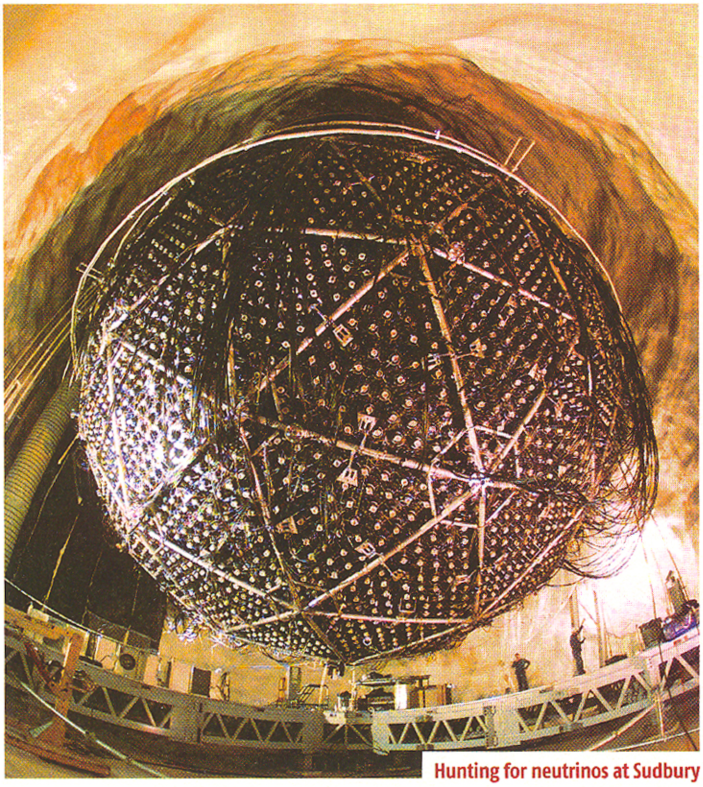
At present epoch, the characteristic oscillation length of the neutrino is much smaller than the universe curvature radius, the mixing treatment in the flat space-time, in such an epoch, is a good approximation of that in FRW space-time.

The vacuum condensate from neutrino mixing can give rise to the *observed* value of the cosmological constant. Exotic components to dark energy are not necessary in this approach.

- **Dark energy gets non-zero contribution induced from the neutrino mixing.**

- **Such a contribution is zero in the no-mixing limit: $\theta = 0$ and/or $m_1 = m_2$.**

- **The contribution is absent in the QM mixing treatment.**


LAWRENCE BERKELEY NATIONAL LAB/SPL

Hunting for neutrinos at Sudbury

Humble origins of force that rules the universe

DARK energy, the mysterious stuff invoked to explain why the expansion of the universe is accelerating, could have a simple explanation. The energy may be coming from neutrinos that were created in copious quantities just after the big bang.

The leading candidate for dark energy is Einstein's "cosmological constant", which proposes that the vacuum of space has an inherent energy that counters gravity. But if you calculate the density of this energy using quantum theory, it works out at nearly 120 orders of magnitude greater than what would fit with cosmological observations. So physicists have proposed ever more exotic explanations for dark energy that require, for instance, the existence of extra dimensions.

Now a team of Italian physicists says the answer has been under our noses all along. Antonio Capolupo and Giuseppe Vitiello of the University of Salerno and Salvatore Capozziello at the University of Naples claim that dark energy can be explained by neutrinos, particles that have no charge and little or no mass. Vast numbers were created just after the big bang, and many remain today because they barely interact with matter.

Neutrinos come in three "flavours", and recent experiments have confirmed they can switch flavours. According to the researchers, this neutrino "mixing" contributes just the right amount of energy to the vacuum of space.

"The explanation for dark energy might have been under our noses, in a particle created in vast numbers by the big bang"

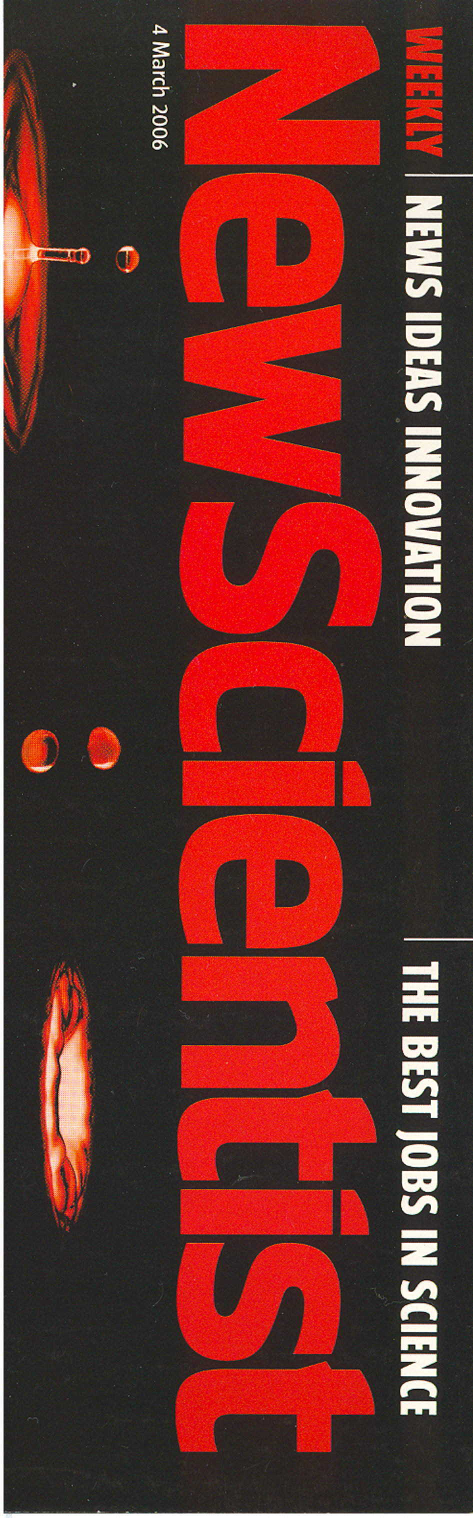
"Neutrino mixing may solve the problem of dark energy," says Capolupo.

Several years ago, the Italian trio, along with Massimo Blasone of the University of Salerno, developed a model to better explain neutrino mixing. Recently, the trio used the model to calculate how much energy the neutrinos contribute to the vacuum of space. The result fitted very well with the observational values for dark energy – assuming that they are the only significant contributors to the energy of vacuum (www.arxiv.org/astro-ph/0602467).

Intriguingly, there have been indications that every cubic centimetre of space contains about 300 neutrinos, and that the energy density of these particles is roughly equal to the energy density required to drive the acceleration caused by dark energy. However, no one could explain how the energy of neutrinos translates into the vacuum energy of space. The Italians' model of neutrino mixing now provides a mechanism. The vacuum is thought to be a cauldron of particles that pop in and out of existence and the neutrinos are transmuted into energy by their interactions with the vacuum. "It reveals an elegant, deep connection between particle physics and cosmology," says Capolupo.

Other cosmologists are cautious. "We are far from a complete understanding of both dark energy and the problem of [neutrino] mixing," says Nikolaos Mavromatos at King's College London. Scott Dodelson at particle research lab Fermilab in Chicago agrees. "If their explanation works theoretically, it is extremely interesting," he says. "Their model in principle is testable by looking at how dark energy evolves over time."

Once the team works out exactly how dark energy changes with time, projects such as the Supernova Cosmology Project can test the predictions. The model also predicts how neutrinos change flavours, which could be tested by experiments such as at the Sudbury Neutrino Observatory in Ontario, Canada. Amarendra Swarup



Conclusions

• Mixing transformations are not trivial in Q.F.T. $\Leftrightarrow$ they are associated to inequivalent representations (orthogonal Hilbert spaces).

• QFT oscillation formulas for both fermions and bosons exhibit non–perturbative corrections with respect to the usual QM ones, due to the condensate structure of the flavor vacuum.

• The effects are more relevant in the non-relativistic region, i.e. $k \simeq \sqrt{m_1 m_2}$ for fermions and $k \simeq 0$ for bosons. For the $\eta - \eta'$ system, the non-perturbative correction may be as large as 20%.

• The vacuum condensate generated by neutrino mixing can be interpreted as an evolving dark energy that, at present epoch, behaves as the cosmological constant, giving rise to its observed value.

• Its origin is completely different from the one of the ordinary contribution of a massive spinor field. It cames from the property of QFT: infinitely many representations of the canonical (anti-)commutation relations in the infinite volume limit.

• The mixing phenomenon provides the vacuum energy contribution, since the field mixing involves unitary inequivalent representations.